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Tracking Control of Marine Craft in the port-Hamiltonian Framework: A Virtual Differential Passivity Approach

Rodolfo Reyes-Báez, Alejandro Donaire, Arjan van der Schaft, Bayu Jayawardhana, Tristán Pérez

Abstract—In this work we propose a *virtual contraction-based control* (v-CBC) design approach to the tracking problem in marine craft modeled as port-Hamiltonian (pH) systems. The design method consists of three main steps: *i*) construct a virtual control system which has all the original marine craft pH model's solutions embedded; *ii*) design a control law that makes the virtual control system contracting with a desired steady-state trajectory; *iii*) close the loop of the original marine craft pH model with above controllers.

Due to the *rigid body* nature of marine craft, two v-CBC schemes are proposed; one in a body frame and another in an inertial frame. We show how the *intrinsic structure* of pH models and their *workless* forces can be exploited to construct virtual control systems for marine craft in both frames. The closed-loop system's performance is evaluated on simulations.

I. INTRODUCTION

The recent advances of incremental methods in systems and control have enabled the development of a new control paradigm [13], [20], [16], [12], [6]. The extension of the differential Lyapunov framework [6] to open systems for contraction analysis, the so-called *differential passivity* [22], resembles the relation between the standard passivity property and Lyapunov stability. In other words, the fact that differential passivity implies contraction (for a suitable input), is analogous to the fact that passivity implies Lyapunov stability (with zero input). This gives us the possibility of exploiting systems's interconnection properties in a differential passivity preserving manner [22]. Contraction analysis (respectively, differential passivity) has been generalized by considering contracting (respectively, differentially passive) *virtual (control) systems* [24], [12], [6] whose solutions include all the original system's state trajectories. Roughly speaking, when both states (the virtual and original) are initialized at the same point (and input) then both systems produce identical state trajectories.

The latter approach has motivated us to propose a v-CBC technique called *virtual differential passivity-based control* (v-dPBC) where we combine the concept of virtual systems and of differential passivity for designing stabilizing/tracking controllers [18]. The v-dPBC method consists basically of three main steps. In the first step, we need to define an admissible virtual control system (the precise definition of

this will come later). Subsequently, in the second step, we design a controller such that the closed-loop virtual system is differentially passive and has a desired steady-state behaviour or trajectory. Finally, in the third step, we close the loop of the original system using the control law from the previous step where the virtual state is replaced by the original state; this last step solves the tracking problem. The method has been applied before to classes of mechanical systems [17], [18], [19]. Here, we extend the method to marine craft which are mechanical systems on *moving frames*.

The dynamic models of marine craft and hydrodynamic forces possess intrinsic passivity properties inherited from its physical nature [8], [9]. These properties have been widely used for motion control design of ships and underwater vehicles, see for example [10], [21], [26]. In the same spirit, port-Hamiltonian models for marine craft have been proposed and used for passivity-control design in a structure preserving manner; for further details see [4], [5] and references therein. Specifically, in [5], two marine craft pH models are presented, one in a body-fixed frame and another in an inertial one¹. The body-frame pH model is later used for designing a passivity based tracking control scheme. However, the trajectory tracking control problem for the inertial-frame pH model remains *open*.

In this work we solve the tracking problem of marine craft *via the v-dPBC method* [19]. For this, we first recall the existing pH models for marine craft [5] in order to construct associated virtual control systems, which will be used later in the v-CBC procedure. The paper is organized as follows: in Section II the preliminaries of the v-dPBC method are introduced; the notation and nomenclature for marine systems are presented in Section III together with a *remark* on the *inertial dynamic model* reported in the literature [8]; in Section IV we state our main results where two v-dPBC schemes for marine craft pH models are designed, one on the body-frame and the other on the inertial one. The performance of the closed-loop system is briefly indicated in Section V due to space limitations. Finally, in Section VI, the conclusions and future research directions are presented.

II. PRELIMINARIES

A. Contraction, differential passivity and virtual systems

1) *Differential analysis: contraction and passivity*: Let Σ_u be a nonlinear control system with N -dimensional state space

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¹From a practical point of view, the relation between these two models is very useful since the attitude and velocities are usually measured by IMU (Inertial Measurement Unit) and GPS (Global Position System) sensors.

\mathcal{X} , affine in the input u ,

$$\Sigma_u : \begin{cases} \dot{x} = f(x,t) + \sum_{i=1}^n g_i(x,t)u_i, \\ y_i = h(x,t), \end{cases} \quad (1)$$

where $x \in \mathcal{X}$, $u \in \mathcal{U} \subset \mathbb{R}^n$ and $y \in \mathcal{Y}$. The vector fields $f, g_i : \mathcal{X} \times \mathbb{R}_{\geq 0} \rightarrow T\mathcal{X}$ and $h : \mathcal{X} \times \mathbb{R}_{\geq 0} \rightarrow \mathcal{Y}$ are assumed to be smooth. The input space \mathcal{U} and output space \mathcal{Y} are assumed to be open subsets of \mathbb{R}^n . System (1) in closed-loop with the feedback control law $u = \gamma(x,t)$ will be denoted by

$$\Sigma : \begin{cases} \dot{x} = F(x,t), \\ y = h(x,t), \end{cases} \quad (2)$$

The *variational system* along the trajectory $(u, x, y)(t)$ is the time-varying system given by

$$\begin{cases} \delta\dot{x} = \frac{\partial f}{\partial x}(x,t)\delta x + \sum_{i=1}^n u_i \frac{\partial g_i}{\partial x}(x,t)\delta x + \sum_{i=1}^n g_i \delta u_i, \\ \delta y = \frac{\partial h}{\partial x}(x,t)\delta x. \end{cases} \quad (3)$$

Definition 1 ([3]): The *prolonged system* Σ_u^δ corresponding with Σ_u in (1) is the system described by

$$\begin{aligned} \dot{x} &= f(x,t) + \sum_{i=1}^n g_i(x,t)u_i, \\ y &= h(x,t), \\ \delta\dot{x} &= \frac{\partial f}{\partial x}(x,t)\delta x + \sum_{i=1}^n u_i \frac{\partial g_i}{\partial x}(x,t)\delta x + \sum_{i=1}^n g_i(x,t)\delta u_i, \\ \delta y &= \frac{\partial h}{\partial x}(x,t)\delta x. \end{aligned} \quad (4)$$

The prolonged system of (2) is

$$\begin{cases} \dot{x} = F(x,t), \\ y = h(x,t), \\ \delta\dot{x} = \frac{\partial F}{\partial x}(x,t)\delta x, \\ \delta y = \frac{\partial h}{\partial x}(x,t)\delta x. \end{cases} \quad (5)$$

Definition 2 ([6]): A function $V : T\mathcal{X} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ is a candidate *differential/Finsler Lyapunov function* if it satisfies

$$c_1 \mathcal{F}(x, \delta x, t)^p \leq V(x, \delta x, t) \leq c_2 \mathcal{F}(x, \delta x, t)^p, \quad (6)$$

where $c_1, c_2 \in \mathbb{R}_{>0}$, p is a positive integer and $\mathcal{F}(x, \delta x, t)$ is a Finsler structure, uniformly in t .

Definition 3: Consider a candidate differential Lyapunov function on \mathcal{X} and the associated Finsler structure \mathcal{F} . For any subset $\mathcal{C} \subseteq \mathcal{X}$ and any $x_1, x_2 \in \mathcal{C}$, let $\Gamma(x_1, x_2)$ be the collection of piecewise C^1 curves $\gamma : I \rightarrow \mathcal{X}$ connecting $\gamma(0) = x_1$ and $\gamma(1) = x_2$. The *Finsler distance* $d : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}_{\geq 0}$ induced by \mathcal{F} is defined by

$$d(x_1, x_2) := \inf_{\Gamma(x_1, x_2)} \int_{\gamma} \mathcal{F} \left(\gamma(s), \frac{\partial \gamma}{\partial s}(s), t \right) ds. \quad (7)$$

The following result gives a sufficient condition for contraction in terms of differential Lyapunov functions

Theorem 1: Consider system (5), a connected and forward invariant set $\mathcal{C} \subseteq \mathcal{X}$, and a function $\alpha : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$. Let V be a candidate differential Lyapunov function satisfying

$$\dot{V}(x, \delta x, t) \leq -\alpha(V(x, \delta x, t)) \quad (8)$$

for all $(x, \delta x) \in T\mathcal{X}$ and all $t > t_0$. Then, system (2) is

- incrementally stable (IS) if $\alpha(s) = 0$ for each $s \geq 0$;
- asymptotically IS if α is of class \mathcal{K} ;
- exponentially IS with rate β if $\alpha(s) = \beta s, \forall s \geq 0$.

Definition 4 (Contracting system): We say that Σ contracts V in \mathcal{C} if (8) is satisfied for α of class \mathcal{K} . Function V is the *contraction measure*, and \mathcal{C} is the *contraction region*.

In analogy to the standard notion of dissipativity [25], [23], The differential Lyapunov framework for contraction analysis was extended to open systems [22].

Definition 5 ([22]): Consider a control system Σ_u in (1) together with its prolonged system Σ_u^δ in (4). Then, Σ_u is called *differentially passive* if there exist a *differential storage function* $W : T\mathcal{X} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ satisfying

$$\frac{dW}{dt}(x, \delta x, t) \leq \delta y^\top \delta u, \quad (9)$$

for all $x, \delta x, u, \delta u$ and uniformly in t . Furthermore, system (1) is called *differentially lossless* if (9) holds with equality. If additionally, the differential storage function is required to be a differential Lyapunov function, then differential passivity implies contraction when the input is $\delta u = 0$.

Lemma 1: Consider system Σ_u in (1). Suppose there exists a differential transformation $\delta\tilde{x} = \Theta(x,t)\delta x$ such that the variational dynamics $\delta\Sigma_u$ in (3) takes the form

$$\begin{cases} \delta\dot{\tilde{x}} = [\Xi(\tilde{x}, t) - \Upsilon(\tilde{x}, t)]\Pi(\tilde{x}, t)\delta\tilde{x} + \Psi(\tilde{x}, t)\delta u, \\ \delta\tilde{y} = \Psi(\tilde{x}, t)^\top \Pi(\tilde{x}, t)\delta\tilde{x}, \end{cases} \quad (10)$$

where $\Pi(\tilde{x}, t)$ defines a Riemannian metric, $\Xi(\tilde{x}, t) = -\Xi^\top(\tilde{x}, t)$, $\Upsilon(\tilde{x}, t) = \Upsilon^\top(\tilde{x}, t)$ and $\delta\tilde{y}$ the variational output. If the following inequality holds

$$\delta\tilde{x}^\top \left[\dot{\Pi}(\tilde{x}, t) - \Pi(\tilde{x}, t)(\Upsilon + \Upsilon^\top)\Pi(\tilde{x}, t) \right] \delta\tilde{x} \leq -\alpha(W(\tilde{x}, \delta\tilde{x}, t)), \quad (11)$$

with α of class \mathcal{K} . Then, Σ_u is differentially passive from δu to $\delta\tilde{y}$ with respect to the differential storage function

$$W(\tilde{x}, \delta\tilde{x}, t) = \frac{1}{2} \delta\tilde{x}^\top \Pi(\tilde{x}, t) \delta\tilde{x}. \quad (12)$$

2) *Contracting virtual systems:* A generalization of contraction was introduced in [24] to study the convergence between solutions of two or more systems.

Definition 6 (Virtual system): Consider systems Σ_u and Σ , given by (1) and (2), respectively. Suppose that $\mathcal{C}_v \subseteq \mathcal{X}$ and $\mathcal{C}_x \subseteq \mathcal{X}$ are connected and forward invariant. A *virtual control system* of Σ_u is defined as the time-varying system

$$\Sigma_v : \begin{cases} \dot{x}_v = \Gamma_v(x_v, x, u, t), \\ y_v = h_v(x_v, x, t), \quad \forall t \geq t_0, \end{cases} \quad (13)$$

with state $x_v \in \mathcal{X}$ and parametrized by $x \in \mathcal{X}$, where $h_v : \mathcal{C}_v \times \mathcal{C}_x \times \mathbb{R}_{\geq 0} \rightarrow \mathcal{Y}$ and $\Gamma_v : \mathcal{C}_v \times \mathcal{C}_x \times \mathcal{U} \times \mathbb{R}_{\geq 0} \rightarrow T\mathcal{X}$ are such that

$$\Gamma(x, x, u, t) = f(x, t) + \sum_{i=1}^n g_i(x, t)u_i, \quad (14)$$

$$h_v(x, x, t) = h(x, t), \quad \forall u, \forall t \geq t_0.$$

Similarly, a *virtual system* associated to Σ is defined as

$$\Sigma^v : \begin{cases} \dot{x}_v = \Phi_v(x_v, x, t), \\ y_v = h_v(x_v, x, t), \end{cases} \quad (15)$$

with state $x_v \in \mathcal{C}_v$ and parametrized by $x \in \mathcal{C}_x$, where $\Phi_v : \mathcal{C}_v \times \mathcal{C}_x \times \mathbb{R}_{\geq 0} \rightarrow T\mathcal{X}$ and $h_v : \mathcal{C}_v \times \mathcal{C}_x \times \mathbb{R}_{\geq 0} \rightarrow \mathcal{Y}$ satisfying uniformly the condition

$$\Phi_v(x, x, t) = F(x, t) \quad \text{and} \quad h_v(x, x, t) = h(x, t). \quad (16)$$

Theorem 2 (Virtual contraction [24], [7]): Consider Σ and Σ_v in (2) and (15), respectively. Let $\mathcal{C}_v \subseteq \mathcal{X}$ and $\mathcal{C}_x \subseteq \mathcal{X}$ be two connected and forward invariant sets. Suppose that Σ_v is uniformly contracting with respect to x_v . Then, for any $x_0 \in \mathcal{C}_x$ and $x_{v,0} \in \mathcal{C}_v$, each solution to Σ_v asymptotically converges to the solution of Σ .

If Theorem 2 holds, then the original system Σ is said to be *virtually contracting*². On the other hand, if the virtual control system Σ_u^v is differentially passive, then the original control system Σ_u is said to be *virtually differentially passive*.

3) *Virtual differential passivity based control (v-dPBC):* The design method³ is divided in three main steps:

- Design the virtual system control (13) for system (1).
- Design the feedback $u_v = \zeta(x_v, x, t) + \omega$ for (13) such that the closed-loop virtual system is differentially passive for the input-output pair $(\delta y_v, \delta \omega)$ and has a desired trajectory $x_d(t)$ as steady-state solution for $\delta \omega$.
- Define the controller for system (1) as $u = \zeta(x, x, t)$.

If we are able to design a controller following the above steps, then all closed-loop actual system trajectories will converge asymptotically to $x_d(t)$.

B. Mechanical port-Hamiltonian and virtual systems

Previous ideas will be applied to mechanical pH systems.

Definition 7 ([23]): A port-Hamiltonian system with N -dimensional state space \mathcal{X} , input and output spaces $\mathcal{U} = \mathcal{Y} \subset \mathbb{R}^m$ and Hamiltonian function $H: \mathcal{X} \rightarrow \mathbb{R}$, is given by

$$\begin{aligned} \dot{x} &= [J_H(x) - R(x)] \frac{\partial H}{\partial x}(x) + G(x)u \\ y &= G^\top(x) \frac{\partial H}{\partial x}(x), \end{aligned} \quad (17)$$

where $G(x)$ is a $N \times m$ input matrix, and $J_H(x)$, $R(x)$ are the interconnection and dissipation $N \times N$ matrices which satisfy $J_H(x) = -J_H^\top(x)$ and $R(x) = R^\top(x) \geq 0$, respectively.

In the specific case of a standard mechanical system with generalized coordinates q on the n -dimensional configuration space \mathcal{Q} and velocity $\dot{q} \in T_q\mathcal{Q}$, the Hamiltonian function is given by the total energy

$$H(q, p) = \frac{1}{2} p^\top M^{-1}(q) p + P(q), \quad (18)$$

where $x = (q, p) \in T^*\mathcal{Q} := \mathcal{X}$ is the state, $P(q)$ is the potential energy, $p := M(q)\dot{q}$ is the momentum and the inertia matrix $M(q)$ is symmetric, positive definite and bounded. Then, the pH system (17) takes the form

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -I & -D(q) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q}(q, p) \\ \frac{\partial H}{\partial p}(q, p) \end{bmatrix} + \begin{bmatrix} 0 \\ B(q) \end{bmatrix} u, \\ y &= B^\top(q) \frac{\partial H}{\partial p}(q, p), \end{aligned} \quad (19)$$

with matrices

$$J_H(x) = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}; R(x) = \begin{bmatrix} 0 & 0 \\ 0 & D(q) \end{bmatrix}; G(x) = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad (20)$$

²The concept was originally called *partial contraction* in [24]. However, to avoid confusion with future research we use the adjective *virtual*.

³Virtual systems for control design were first considered in [12], [14].

where $D(q) = D^\top(q) \geq 0$ is the damping matrix and I and 0 are the $n \times n$ identity, respectively, zero matrices. The input force matrix $B(q)$ has rank $m \leq n$.

III. MARINE CRAFT'S PORT-HAMILTONIAN MODELING

The goal of this section is to express the marine craft pH models developed by [9] in terms of its workless forces explicitly; this form will be instrumental for the construction of associated virtual systems in body and inertial frames.

To this end, we adopt the notation of *SNAME (1950)* for marine vessels. From a guidance, navigation and control point of view, in the modeling of marine craft, four different coordinate frames are considered: the Earth Centered inertial (ECI) frame $\{i\}$, whose origin O_i is located at the center of mass of the Earth; the Earth-centered-Earth-fixed (ECEF) frame $\{e\}$ that rotates with the Earth; the North-East-Down (NED) frame $\{n\}$ with origin O_n defined relative to the Earth's reference ellipsoid (WGS84); and the body frame $\{b\}$ which is a moving coordinate frame that is fixed to the craft⁴. In this work we take the following modeling assumptions:

Assumption 1 (Flat navigation): The operating radius of a marine craft is limited. We assume $\{n\}$ to be inertial.

Assumption 2 (Maneuvering theory): The hydrodynamic coefficients are frequency independent (no wave excitation).

Under the above assumptions, by the Newton-Euler (N-E) approach, the equation of motion in the body-frame are [9]:

$$\begin{aligned} \dot{\eta} &= J(\eta)v, \\ M\dot{v} + C(v)v + D(v)v + g(\eta) &= \tau \end{aligned} \quad (21)$$

where $\eta = [x, y, z, \phi, \theta, \psi]^\top \in \mathcal{Q} := \mathbb{R}^3 \times S^3$ describes the marine craft's position and orientation; $v = [v_1^\top, v_2^\top]^\top$ is the (quasi-)velocity in $\{b\}$, with $v_1 = [u, v, w]^\top$ (surge, sway, heave) and $v_2 = [p, q, r]^\top$ (roll, pitch, yaw); and τ are the force and torque inputs. The matrix function $J(\eta)$ is a well defined transformation if $\theta \neq \pm\pi/2$, due to the Euler angles representation [9]. The inertia matrix $M = M^\top > 0$ is

$$M := \begin{bmatrix} mI & -m\mathcal{S}_\times(r_g^b) \\ m\mathcal{S}_\times(r_g^b) & I_b \end{bmatrix} \quad (22)$$

where m is the total mass due to the craft's mass and fluid added mass, I_b the moment of inertia in $\{b\}$, r_g^b the constant vector between O_b and the center of gravity (CG) in $\{b\}$ -coordinates, and the matrix $\mathcal{S}_\times(\cdot)$ is defined as

$$\mathcal{S}_\times(a) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \quad (23)$$

According to Kirchhoff's equations of motion, the Coriolis-centripetal matrix $C(v)$ is any matrix satisfying

$$C(v)v = \begin{bmatrix} \mathcal{S}_\times(v_2) \frac{\partial T^b}{\partial v_1}(v) \\ \mathcal{S}_\times(v_2) \frac{\partial T^b}{\partial v_2}(v) + \mathcal{S}_\times(v_1) \frac{\partial T^b}{\partial v_1}(v) \end{bmatrix}, \quad (24)$$

⁴For a marine craft, the origin O_b is usually chosen to coincide with a point midship CO in the water line; while the body-frame axes are chosen to coincide with the principal axes of inertia [9].

where $T^b(\mathbf{v}) = \frac{1}{2}\mathbf{v}^\top M\mathbf{v}$ is the kinetic (co-)energy; $D(\mathbf{v}) = D^\top(\mathbf{v}) > 0$ is the total hydrodynamic damping matrix; and $g(\boldsymbol{\eta})$ is the vector of hydrostatic forces and torques due to gravity and buoyancy. Clearly the force $F_{gr}^b(\mathbf{v}) := C(\mathbf{v})\mathbf{v}$ in (24) is *workless*; that is $\mathbf{v}^\top F_{gr}^b(\mathbf{v}) = 0$ for every \mathbf{v} .

Following [2], any workless force $F(\mathbf{v})$ can be expressed as $F(\mathbf{v}) = S_L(\mathbf{v})\mathbf{v}$ for some skew-symmetric matrix $S_L(\mathbf{v})$. Indeed, $F_{gr}^b(\mathbf{v}) = S_L^b(\mathbf{v})\mathbf{v}$ with $S_L^b(\mathbf{v}) = C(\mathbf{v})$ skew-symmetric. This was first shown in [9] for system (21).

A. A remark on marine craft's dynamics in the inertial frame

The body-frame equations of motion (21) can be also expressed in the inertial frame $\{n\}$ by performing the kinematic transformation $\dot{\boldsymbol{\eta}} = J(\boldsymbol{\eta})\mathbf{v}$ as follows [8, p.48]:

$$M_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} + D_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} + g_\eta(\boldsymbol{\eta}) = \boldsymbol{\tau}_\eta \quad (25)$$

where

$$\begin{aligned} M_\eta(\boldsymbol{\eta}) &= J^{-\top}(\boldsymbol{\eta})MJ^{-1}(\boldsymbol{\eta}), \\ C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) &= J^{-\top} [C(J^{-1}\dot{\boldsymbol{\eta}}) - MJ^{-1}(\boldsymbol{\eta})J(\boldsymbol{\eta})] J^{-1}, \\ D_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) &= J^{-\top}(\boldsymbol{\eta})D(J^{-1}\dot{\boldsymbol{\eta}})J^{-1}(\boldsymbol{\eta}), \\ g_\eta(\boldsymbol{\eta}) &= J^{-\top}(\boldsymbol{\eta})g(\boldsymbol{\eta}), \\ \boldsymbol{\tau}_\eta &= J^{-\top}(\boldsymbol{\eta})\boldsymbol{\tau}. \end{aligned} \quad (26)$$

Alternatively, the Lagrange equations can be used to derive (25) since in $\{n\}$ the vector $\boldsymbol{\eta}$ qualifies as generalized coordinate. Then, the kinetic (co-)energy in $\{n\}$ is given by

$$T_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \frac{1}{2}\dot{\boldsymbol{\eta}}^\top M_\eta(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}, \quad (27)$$

and the potential energy $P(\boldsymbol{\eta})$ is defined as the solution to

$$J^\top(\boldsymbol{\eta})\frac{\partial P}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}) = g(\boldsymbol{\eta}). \quad (28)$$

Hence, the vehicle-ambient Lagrangian function is [8]:

$$L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = T_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) - P(\boldsymbol{\eta}). \quad (29)$$

For the dissipative hydrodynamic forces a Rayleigh function $F_R(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \frac{1}{2}\dot{\boldsymbol{\eta}}^\top D_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}}$ is considered.

With the Lagrangian approach, the Coriolis-centripetal matrix is defined as any matrix $C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ satisfying [15]:

$$C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} = \dot{M}_\eta(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} - \frac{\partial T}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}). \quad (30)$$

Notice that the following work relation holds [1]

$$\dot{\boldsymbol{\eta}}^\top \underbrace{\left[\frac{1}{2}\dot{M}_\eta(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} - \frac{\partial T}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) \right]}_{F_{gr}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})} = 0. \quad (31)$$

Substitution of (31) in (30) implies the well-know identity⁵

$$\dot{\boldsymbol{\eta}}^\top [\dot{M}_\eta(\boldsymbol{\eta}) - 2C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})] \dot{\boldsymbol{\eta}} = 0.$$

Thus, $F_{gr}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ is workless and there exists a skew-symmetric matrix such that $F_{gr}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = S_\eta^L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}}$ [24]. This

⁵Furthermore, the equality $r^\top [\dot{M}_\eta(\boldsymbol{\eta}) - 2C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})]r = 0$ holds for any $r \in T_\eta\mathcal{Q}$ due to the Levi-Civita connection is torsion free and compatible with the Riemannian metric associated to $M_\eta(\boldsymbol{\eta})$; see [23] for further details.

matrix can be expressed in terms of the Christoffel symbols corresponding to the Levi-Civita connection of $M_\eta(\boldsymbol{\eta})$ as

$$S_\eta^L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = \frac{1}{2} \sum_{i=1}^n \left\{ \frac{\partial M_{\eta ki}}{\partial \eta_j}(\boldsymbol{\eta}) - \frac{\partial M_{\eta ij}}{\partial \eta_k}(\boldsymbol{\eta}) \right\} \dot{\eta}_i. \quad (32)$$

or equivalently in explicit terms of $J(\boldsymbol{\eta})$ as

$$S_\eta^L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) = J^{-\top}CJ^{-1} + \frac{1}{2} \left[J^{-\top}MJ^{-1} - (J^{-\top}MJ^{-1})^\top \right]. \quad (33)$$

Hence, expression (30) takes the form

$$C_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}} = \left[\frac{1}{2}\dot{M}_\eta(\boldsymbol{\eta}) + S_\eta^L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) \right] \dot{\boldsymbol{\eta}}, \quad (34)$$

Remark 1: We point out that with $S_\eta^L(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})$ as in (33), the correspondence between the Coriolis matrices in the Newton-Euler formalism in (26) and in the Lagrange framework in (30) is *clear* by relation (34). *This is not the case* in reference [8, p. 54] for the Lagrangian model.

B. Craft's pH model in body-frame and workless forces

The next assumption is made on (21) [4], [5]:

Assumption 3: There exists $P: \mathcal{Q} \rightarrow \mathbb{R}$ satisfying (28).

Under Assumption 3, the marine craft dynamics (21) can be written in port-Hamiltonian form as follows [5]:

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\mathbf{p}}^b \end{bmatrix} = \begin{bmatrix} \mathbf{0} & J(\boldsymbol{\eta}) \\ -J^\top(\boldsymbol{\eta}) & -F_2(\mathbf{p}^b) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}, \mathbf{p}^b) \\ \frac{\partial H}{\partial \mathbf{p}^b}(\boldsymbol{\eta}, \mathbf{p}^b) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ I_n \end{bmatrix} \boldsymbol{\tau}, \quad (35)$$

where the Hamiltonian function is

$$H(\boldsymbol{\eta}, \mathbf{p}^b) = \frac{1}{2}\mathbf{p}^{b\top}M^{-1}\mathbf{p}^b + P(\boldsymbol{\eta}), \quad (36)$$

the *quasi momentum*⁶ is defined as

$$\mathbf{p}^b = M\mathbf{v}, \quad (37)$$

and $F_2(\mathbf{p}^b) = C(M^{-1}\mathbf{p}^b) + D(M^{-1}\mathbf{p}^b)$. System (35) is passive with (36) as storage function. We define $E^b(\boldsymbol{\eta}, \mathbf{p}^b) := C(M^{-1}\mathbf{p}^b)$ as the workless forces matrix for future purposes.

C. Craft's pH model in inertial-frame and workless forces

A pH model in inertial coordinates was developed in [5] as well under Assumption 3. To this end, let \mathbf{p}^b be transformed into the *true-momentum* of $\boldsymbol{\eta}$ in the frame $\{n\}$ as

$$\mathbf{p} = J^{-\top}(\boldsymbol{\eta})\mathbf{p}^b \iff \mathbf{p} = M_\eta(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\dot{\boldsymbol{\eta}}. \quad (38)$$

Then, the dynamics (37) in coordinates $(\boldsymbol{\eta}, \mathbf{p})$ is given by

$$\begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I \\ -I & -L(\boldsymbol{\eta}, \mathbf{p}) \end{bmatrix} \begin{bmatrix} \frac{\partial H_\eta}{\partial \boldsymbol{\eta}}(\boldsymbol{\eta}, \mathbf{p}) \\ \frac{\partial H_\eta}{\partial \mathbf{p}}(\boldsymbol{\eta}, \mathbf{p}) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ I \end{bmatrix} \boldsymbol{\tau}_\eta, \quad (39)$$

where the Hamiltonian function is

$$H_\eta(\boldsymbol{\eta}, \mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M_\eta^{-1}(\boldsymbol{\eta})\mathbf{p} + P(\boldsymbol{\eta}), \quad (40)$$

⁶Since \mathbf{v} are velocities measured in $\{b\}$ (*quasi-velocities*), \mathbf{p}^b is not the true momentum of $\boldsymbol{\eta}$. For details see [11, p. 193] and [9].

and the matrix

$$L(\eta, p) = \left[\sum_{i=1}^n \frac{\partial J^{-\top}}{\partial \eta_i} J^\top p e_i^\top \right]^\top - \left[\sum_{i=1}^n \frac{\partial J^{-\top}}{\partial \eta_i} J^\top p e_i^\top \right] + J^{-\top} F_2(M^{-1} J^\top p) J^{-1}, \quad (41)$$

with e_i is the i -th element of the Euclidean canonical base.

In the following proposition we present an alternative form of the inertial model (39) in which *the workless forces are decoupled* from the gradient of the Hamiltonian function.

Proposition 1: The pH system (39) takes the form

$$\begin{bmatrix} \dot{\eta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & -(E_\eta(\eta, p) + D_\eta^H(\eta, p)) \end{bmatrix} \begin{bmatrix} \frac{\partial P}{\partial \eta}(\eta) \\ M_\eta^{-1} p \end{bmatrix} + \begin{bmatrix} 0 \\ J^{-\top} \end{bmatrix} \tau, \quad (42)$$

where $D_\eta^H = D_\eta(\eta, M_\eta^{-1}(\eta)p)$ and $E_\eta = S_\eta^H(\eta, p) - \frac{1}{2} \dot{M}_\eta(\eta)$ with $S_\eta^H(\eta, p) := S_\eta^L(\eta, M_\eta^{-1}(\eta)p)$ from (33).

IV. TRAJECTORY TRACKING CONTROL DESIGN

A. Control design in the body-fixed frame

1) *Virtual control system design:* The structure of (35) motivates the definition of a virtual system for it, in the state (η_v, p_v^b) and parametrized by the trajectory (η, p^b) , as

$$\begin{bmatrix} \dot{\eta}_v \\ \dot{p}_v^b \end{bmatrix} = \begin{bmatrix} 0 & J(\eta) \\ -J^\top(\eta) & -F_2(p^b) \end{bmatrix} \begin{bmatrix} \frac{\partial H_v}{\partial \eta_v}(\eta_v, p_v^b) \\ \frac{\partial H_v}{\partial p_v^b}(\eta_v, p_v^b) \end{bmatrix} + \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tau_v, \quad (43)$$

with

$$H_v(\eta_v, p_v^b) = \frac{1}{2} p_v^{b\top} M^{-1} p_v^b + P_v(\eta_v) \quad (44)$$

where $P_v(\eta_v)$ also fulfills Assumption 3 and $P_v(\eta) = P(\eta)$. Remarkably, (43) is also a pH system and passive with (44) as storage function and supply rate $y_{b\eta}^\top \tau_\eta$, uniformly in $(\eta(t), p^b(t))$, for all $t > t_0$; where $y_\eta^b = M^{-1} p_v^b$.

2) *Differential passivity based control design:* In this step of the method v-dPBC, we will design $\tau_v = \zeta^b(x_v^b, x^b, t) + \omega^b$ such that (43) is differentially passive in the closed-loop.

Proposition 2: Consider system (43) and a smooth trajectory $x_d^b = (\eta_d, p_d^b)$ in $\{b\}$ with $p_d^b = M^{-1} \dot{\eta}_d$. Let us introduce the following error coordinates

$$\tilde{x}_v^b := \begin{bmatrix} \tilde{\eta}_v^b \\ \tilde{\sigma}_v^b \end{bmatrix} = \begin{bmatrix} \eta_v - \eta_d^b \\ p_v^b - p_r^b \end{bmatrix}, \quad (45)$$

where the auxiliary momentum reference p_r^b is given by

$$p_r^b(\tilde{\eta}_v^b, t) := M(\dot{\eta}_d^b - \phi^b(\tilde{\eta}_v^b, t)), \quad (46)$$

$\phi^b : \mathcal{Q} \rightarrow T\mathcal{Q}$ is such that $\phi^b(0) = 0$; and $\Pi_{\tilde{\eta}_v^b}^b : \mathcal{Q} \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^{n \times n}$ is a positive definite Riemannian metric tensor satisfying the inequality

$$\begin{aligned} & \Pi_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t) - \Pi_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t) \frac{\partial \phi^b}{\partial \tilde{\eta}_v^b}(\tilde{\eta}_v^b) \\ & - \frac{\partial \phi^{b\top}}{\partial \tilde{\eta}_v^b}(\tilde{\eta}_v^b) \Pi_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t) \leq -2\beta_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t) \Pi_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t), \end{aligned} \quad (47)$$

with $\beta_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t) > 0$, uniformly. Assume also that the i -th row of $\Pi_{\tilde{\eta}_v^b}^b(\tilde{\eta}_v^b, t)$ is a conservative vector field⁷. Consider also the composite control law given by

$$\tau_v(x_v^b, x^b, t) := \tau_{ff}(x_v^b, x^b, t) + \tau_{fb}(x_v^b, x^b, t) + \omega^b, \quad (48)$$

where $x^b = (\eta, p^b)$, $x_v^b = (\eta_v, p_v^b)$ and

$$\begin{aligned} \tau_{ffv} &= \dot{p}_r^b + \frac{\partial P_v}{\partial \eta_v} + [E^b(\eta, p^b) + D(M^{-1} p^b)] M^{-1} p_r^b, \\ \tau_{fbv} &= - \int_0^{\tilde{\eta}_v^b} \Pi_{\tilde{\eta}_v^b}^b(\xi, t) d\xi - K_d M^{-1} \sigma_v^b, \end{aligned} \quad (49)$$

with $K_d > 0$ and ω^b an external input. Then, system (43) in closed-loop with (48) is differentially passive from $\delta \omega^b$ to $\delta y_{\sigma_v}^b = M^{-1} \delta \sigma_v^b$ with respect to the storage function

$$W_{x^b}(\tilde{x}_v^b, \delta \tilde{x}_v^b, t) = \frac{1}{2} \delta \tilde{x}_v^{b\top} \begin{bmatrix} \Pi_{\tilde{\eta}_v^b}^b & 0 \\ 0 & M^{-1} \end{bmatrix} \delta \tilde{x}_v^b. \quad (50)$$

3) *Actual system's controller:* Here we show that the controller which is designed in the previous item can be used as a trajectory tracking controller for the actual Hamiltonian system (35). This is stated in the following corollary.

Corollary 1: Consider the controller (48). Then, all solutions of system (35) in closed-loop with the control law

$$\tau(x^b, x^b, t) = \tau_{ffv}(x^b, x^b, t) + \tau_{fbv}(x^b, x^b, t)$$

converge exponentially to the trajectory $x_d^b(t)$ with rate

$$\beta^b = \min\{\beta_{\tilde{\eta}_v^b}^b, \lambda_{\min}\{D + K_d\} \lambda_{\min}\{M^{-1}\}\}, \quad (51)$$

where $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of its argument.

B. Control design in the inertial frame

The control design procedure in the inertial frame is almost identical to that in the body-fixed frame. Thus, due to space limitation we only present the virtual system dynamics.

1) *Virtual control system design:* With the alternative form (42) in Proposition 1, we define a virtual system associated with the pH system (39), in the state (η_v, p_v) and parametrized by (η, p) , as the time-varying system

$$\begin{bmatrix} \dot{\eta}_v \\ \dot{p}_v \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & -(E_\eta + D_\eta^H) \end{bmatrix} \begin{bmatrix} \frac{\partial P_v}{\partial \eta_v}(\eta_v) \\ M_\eta^{-1} p_v \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \tau_{\eta_v}, \quad (52)$$

This system also inherits the passivity property of the actual one in (42), with storage function

$$H_{\eta_v}(\eta_v, p_v, t) = \frac{1}{2} p_v^\top M_\eta^{-1}(\eta) p_v + P_v(\eta_v), \quad (53)$$

for any trajectory $(\eta(t), p(t))$ and $t > t_0$ and supply rate $y_\eta^\top \tau_\eta$, where the output is given by $y_\eta = M_\eta^{-1}(\eta) p_v$.

⁷This ensures that the integral in (49) is well defined and independent of the path connecting 0 and \tilde{q}_v .

V. EXAMPLE: OPEN-FRAME UUV

We consider the example in [5] The vehicle has four thrusters in an x-type configuration such that the system is fully-actuated in the degrees of freedom of interest, i.e., surge, sway and yaw. The corresponding inertia, Coriolis and damping matrices in the body frame, respectively, are

$$M = \begin{bmatrix} 290 & 0 & 0 \\ 0 & 404 & 50 \\ 0 & 50 & 132 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & -404v - 50r \\ 0 & 0 & 290u \\ 404v + 50r & -290u & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 95 + 268|v| & 0 & 0 \\ 0 & 613 + 164|u| & 0 \\ 0 & 0 & 105 \end{bmatrix}.$$

Due to space limitations, we only present the performance of controller (48) in body frame, with $\phi^b(\tilde{\eta}_v^b) = \Lambda \tilde{\eta}_v^b$ where $\Lambda = \text{diag}\{0.6, 0.8, 0.2\}$, $\Pi_{\eta_v}^b = \Lambda$ and $K_d = \text{diag}\{300, 100, 200\}$. The time performance of system configuration η and the desired reference η_d is shown in Figure 1. After a short transient, system's position tracks asymptotically to η_d .

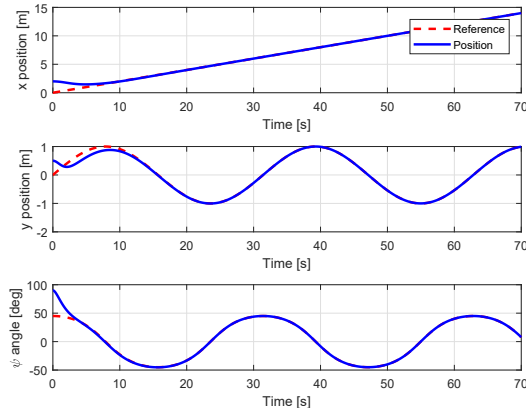


Fig. 1. Vehicle's position vector η against η_d .

VI. CONCLUSIONS AND FUTURE RESEARCH

In this work we have applied the v-dPBC method to solve the trajectory tracking problem in marine craft. The pH structure and its workless forces have been exploited to propose the associated virtual systems. The exponential convergence to a unique predefined steady-state trajectory is guaranteed by the differential passivity property the virtual system. We have developed two families of control schemes based on body-fixed attitude and velocity measurements, one for the pH model in $\{b\}$ and another for the pH model $\{n\}$. Simulations confirm the expected performance of the closed-loop system.

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