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Published in:
Journal of Numerical Cognition

DOI:
10.5964/jnc.v4i2.115

Journal of Numerical Cognition
jnc.psychopen.eu | 2363-8761

PsychOpen GOLD

Special Thematic Section on "Tracking the Continuous Dynamics of Numerical Processing"

Strategy Use on Bounded and Unbounded Number Lines in Typically Developing Adults and Adults With Dyscalculia: An Eye-Tracking Study

Fae Aimée van der Weijden[a], Erica Kamphorst[a], Robin Hella Willemsen[a], Evelyn H. Kroesbergen[ab], Anne H. van Hoogmoed*[ac]

[a] Department of Special Education, Utrecht University, Utrecht, the Netherlands. **[b]** Behavioural Science Institute, Radboud University, Nijmegen, the Netherlands. **[c]** Department of Special Education and Youth Care, University of Groningen, Groningen, the Netherlands.

Abstract

Recent research suggests that bounded number line tasks, often used to measure number sense, measure proportion estimation instead of pure number estimation. The latter is thought to be measured in recently developed unbounded number line tasks. Children with dyscalculia use less mature strategies on unbounded number lines than typically developing children. In this qualitative study, we explored strategy use in bounded and unbounded number lines in adults with (N = 8) and without dyscalculia (N = 8). Our aim was to gain more detailed insights into strategy use. Differences in accuracy and strategy use between individuals with and without dyscalculia on both number lines may enhance our understanding of the underlying deficits in individuals with dyscalculia. We combined eye-tracking and Cued Retrospective Reporting (CRR) to identify strategies on a detailed level. Strategy use and performance were highly similar in adults with and without dyscalculia on both number lines, which implies that adults with dyscalculia may have partly overcome their deficits in number sense. New strategies and additional steps and tools used to solve number lines were identified, such as the use of the previous target number. We provide gaze patterns and descriptions of strategies that give important first insights into new strategies. These newly defined strategies give a more in-depth view on how individuals approach a number lines task, and these should be taken into account when studying number estimations, especially when using the unbounded number line.

Keywords: dyscalculia, number line estimation, unbounded number line, strategy use, eye-tracking, CRR

Journal of Numerical Cognition, 2018, Vol. 4(2), 337–359, doi:10.5964/jnc.v4i2.115

Received: 2017-03-29. Accepted: 2018-03-12. Published (VoR): 2018-09-07.

Handling Editors: Matthias Witte, University of Graz, Graz, Austria; Matthias Hartmann, University of Bern, Bern, Switzerland; Swiss Distance Learning University, Brig, Switzerland; Thomas J. Faulkenberry, Tarleton State University, Stephenville, TX, USA

*Corresponding author at: University of Groningen, dept. Special Needs Education and Youth Care, Grote Rozenstraat 38, 9712 TJ Groningen, the Netherlands. E-mail: a.h.van.hoogmoed@rug.nl

Dyscalculia is a specific learning disorder with impairments in mathematics, characterized by difficulties in number sense, memorization of arithmetic facts, accurate or fluent calculation and/or accurate math reasoning (American Psychiatric Association, 2013; Butterworth & Varma, 2013; Kucian & Von Aster, 2015). The prevalence of dyscalculia is between 3 and 6% (Kucian & Von Aster, 2015). Although most dyscalculia research focusses on childhood, studies have revealed that problems with mathematics persist into adulthood (Mejias, Grégoire, & Noël, 2012; Rubinsten & Henik, 2005; Wilson et al., 2015). Therefore, it is important to acquire more insights into the expression of dyscalculia and the underlying deficits in (early) adulthood.

Over the last years, researchers have found that children with dyscalculia have difficulties understanding and manipulating numerical quantities, which led them to propose that dyscalculia is associated with a deficit in numerical processing or number sense (Chu, van Marle, & Geary, 2013; Mazzocco, Feigenson, & Halberda, 2011; Moeller, Fischer, Cress, & Nuerk, 2012; Mussolin, Mejias, & Noël, 2010; Piazza et al., 2010). Number sense is the ability to quickly understand and manipulate numerical quantities, which can be processed in a non-symbolic format (analogue representation of quantities) or a symbolic format (either an auditory verbal number word or visual Arabic number; Dehaene, 1992). Due to the assumed deficit in number sense in individuals with dyscalculia, inaccurate mapping between different formats can occur (Kucian & Von Aster, 2015). These inaccurate mappings are a result of an impairment in the access to, or the development of, these mapping skills (Kucian et al., 2011; Kucian & Von Aster, 2015). These inaccurate mappings result in decreased performance on many number sense tasks including number line estimation tasks (e.g. van Viersen, Slot, Kroesbergen, van 't Noordende, & Leseman, 2013).

Number line (estimation) tasks (NLTs) are tasks in which participants need to position a target number on a visually presented number line (Siegler & Opfer, 2003). These number lines are typically presented with two labeled endpoints and as such have been named bounded number lines (Cohen & Blanc-Goldhammer, 2011; Cohen & Sarnecka, 2014), see Figure 1. Recently, it has been argued that these number lines measure proportion-judgement and strategies associated with proportion-judgement, rather than number representations (Barth & Paladino, 2011; Slusser, Santiago, & Barth, 2013). Therefore, an alternative number line, the unbounded number line has been introduced (Cohen & Blanc-Goldhammer, 2011). Instead of a begin-point and end-point (e.g. 1-100), in the unbounded number line task only the begin-point and scaling unit (e.g. 0-1) are given, see Figure 2. This unbounded number line has been shown to be a more pure measure of number representation (Cohen & Blanc-Goldhammer, 2011).

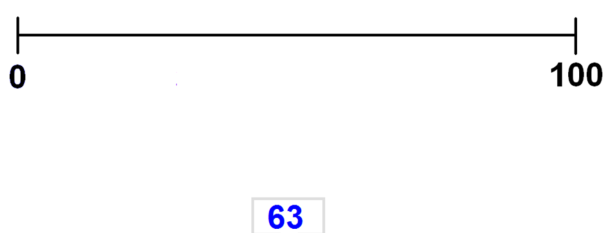

Figure 1. Illustration of the bounded number line estimation task.

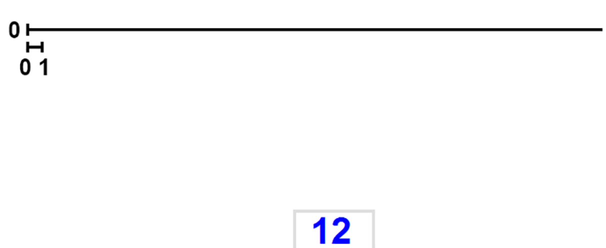

Figure 2. Illustration of the unbounded number line estimation task.

Over the last decades, strategy use in bounded NLTs has received much attention. In early research children were asked to report their estimation strategy (Newman & Berger, 1984; Petitto, 1990). Three strategies were identified: the 'counting up strategy' (i.e., counting forward from the begin-point to the target), 'counting down strategy' (i.e., counting backward from the end-point to the target), and the 'mid-point strategy' (i.e., counting either forward or backward from the mid-point to the target). More recently, in order to better explore strategy use in individuals with and without dyscalculia, eye-tracking has been employed as a validated instrument to measure strategy use during number line estimation tasks (Heine et al., 2010; Schneider et al., 2008; Sullivan, Juhasz, Slattery, & Barth, 2011). By using eye-tracking individuals were found to use also combinations of reference points (i.e., begin-mid, begin-end, mid-end, or all). The use of reference point combinations is suggested to reflect proportion based judgement (Barth & Paladino, 2011). For example, when employing the begin- and mid-point to estimate target number 25, an individual needs to know that 25 is half of 50 to divide a line into equal proportions.

Eye-tracking studies have revealed differences in strategy use on the bounded NLT between children with and without dyscalculia. For example, children with dyscalculia make less use of reference points, such as the begin-point or the end-point, as compared to typically developing children (van 't Noordende, van Hoogmoed, Schot, & Kroesbergen, 2016; van Viersen et al., 2013). More specifically, the majority of typically developing children used strategies including reference points, whereas children with dyscalculia (9- to 12- years of age) showed many 'undefined' or 'other' strategies that did not include reference points (e.g. van 't Noordende et al., 2016). Additionally, when using proportion based strategies, children with dyscalculia used less functional reference point combinations (van 't Noordende et al., 2016; van Viersen et al., 2013). The use of a reference point is considered functional (or 'adaptive', as it has been referred to in previous studies, e.g. van 't Noordende et al., 2016) when it includes the reference point that is most proximal to the target number. This means that it is functional to use the begin-point (and mid-point) for estimating a small number, whereas it is functional to use the end-point (and mid-point) for estimating a large number. Children with dyscalculia more often used reference points that were less functional; i.e., others than the one closest to the target. It is assumed that they use less proportion based strategies and less functional reference point combinations because of their deficits in number sense, especially in the development of mapping skills (cf. Kucian & Von Aster, 2015). Since dyscalculia has been shown to be persistent (Mejias et al., 2012; Rubinsten & Henik, 2005; Wilson et al., 2015), one may also expect inaccurate mapping skills and thus less functional strategy use in adults with dyscalculia. The larger number of undefined strategies in children with dyscalculia is also expected in adults with dyscalculia. In previous research these undefined strategies were based on the absence of the use of reference points (van 't Noordende et al., 2016). However, the question is whether other strategies are used instead, or whether the undefined strategies reflect guessing.

In contrast to bounded NLTs, strategy use in unbounded NLTs has received little attention. Some first studies have shown that strategy use on unbounded NLTs differs from strategy use on bounded NLTs in typically developing children and adults (Link, Huber, Nuerk, & Moeller, 2014; Reinert, Huber, Nuerk, & Moeller, 2015a). Whereas reference points were typically used in the bounded NLTs, strategies used in the unbounded NLT consisted of direct estimation and dead-reckoning. Direct estimation involves a direct move to the target location based on the given unit, whereas dead-reckoning involves the estimation of a working window of a specific size [e.g. 5 units] based on the given unit and then use multiples of this working window to estimate the position of larger numbers (Link et al., 2014; Reinert et al., 2015a; Reinert, Huber, Nuerk, & Moeller, 2015b). As an example of the latter, consider the situation where the target number 200 should be estimated with 10 as the given unit. Dead-reckoning would involve estimating a working window of for example 50 and then take subsequent steps of 50 on the number line (starting at the begin-point) until 200 is reached.

The strategies used for the unbounded number line reflect low level mensuration skills, such as (repeated) addition and counting, while strategies used for the bounded number line reflect complex mensuration skills, such as subtraction and division (Cohen & Sarnecka, 2014). Therefore it has been suggested that the unbounded NLT is a purer measure of number estimation than the bounded NLT (Cohen & Blanc-Goldhammer, 2011). This idea is supported by differences in developmental trajectories between number estimation skills at both lines (Cohen & Sarnecka, 2014). Typically, a logarithmic-to-linear shift occurs in the development of children's number line estimation on bounded NLTs. That is, whereas estimations on the bounded NLT are logarithmic early in development, they become more precise and more linear with age. The logarithmic-to-linear shift in performance on bounded NLTs is suggested to represent an advancement in the mastering of more complex mensuration skills, such as subtraction and division (Cohen & Sarnecka, 2014). Compared to the bounded NLTs, unbounded NLTs require rather low level mensuration skills such as (repeated) addition and counting, which are already present in young children. Children with severe mathematical learning difficulties have been shown to be less precise in bounded number line estimations, and their precision has been shown to increase at a lower rate (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Even adults with dyscalculia show slightly less linear number line representations than typically developing adults (Wilson et al., 2015). Remarkably, no research as to yet has explored differences in accuracy and/or strategy use between individuals with and without dyscalculia on unbounded NLTs. Assuming that dyscalculia is due to difficulties in number sense (Butterworth & Varma, 2013), one would expect adults with dyscalculia to perform worse on the unbounded number line than typically developing (TD) adults. However, if adults with dyscalculia mainly have difficulties in memorization of arithmetic facts, calculation, or accurate math reasoning, also included in the definition in the DSM-V, one would not necessarily expect worse performance of the dyscalculia group as compared to the TD group.

In the current study, we explored strategy use and performance of adults with dyscalculia and typically developing adults on bounded and unbounded NLTs. First, we focused on the predefined proportion based strategies used by adults with dyscalculia in contrast to those used by TD adults. In the bounded NLT, the proportion based strategies were divided in different combinations of reference points based on prior eye-tracking studies (i.e., ‘begin’, ‘begin & mid’, ‘mid’, ‘begin & end’, ‘mid & end’, ‘end’, or ‘all’). Based on previous research in children (van ’t Noordende et al., 2016; van Viersen et al., 2013) in combination with impaired performance in adulthood (Wilson et al., 2015), we expected that adults with dyscalculia would show less functional reference point combinations than TD adults, as well as less use of reference points in general (previously classified as undefined strategies). For the unbounded NLT we expected to find the dead-reckoning strategy as well as direct estimation, since they have already been identified among TD individuals (Link et al., 2014). However, we expected that the adults with dyscalculia would use more direct estimation and less dead-reckoning, since dead-reckoning relies on arithmetic skills and places a larger burden on the working memory, which both have been shown to be impaired in individuals with dyscalculia (Mammarella, Hill, Devine, Caviola, & Szűcs, 2015; Peng & Fuchs, 2016; Rotzer et al., 2009; Schuchardt, Maehler, & Hasselhorn, 2008).

Second, the substantive amount of ‘undefined’ or ‘other’ strategies found in previous studies, incited us to take on a more exploratory approach. Our aims and expectations in this regard were twofold. On the one hand, we expected to find additional new strategies alongside the already defined strategies for the bounded and unbounded NLT. On the other hand, a more detailed description of the already defined strategies (e.g., beyond merely ‘looking at a certain reference point’) could further our understanding of strategy use during number estimation. To accomplish this, instead of classifying strategies based solely on gaze data, we used a qualitative method called cued retrospective reporting (CRR), in which we asked participants to describe their strategies while replaying their eye movements. Because this method requires insight in and verbalization of the strategies one uses, we chose to conduct this study with adults, since children may have difficulties describing the strategies they used. We expected that adults with dyscalculia may use other strategies than reference points, which were previously undiscovered because only strategies based on reference points were included in eye-tracking research.

In addition to strategy use, we qualitatively explored differences in accuracy between TD adults and adults with dyscalculia on the bounded and unbounded NLTs. We hypothesized that adults with dyscalculia would perform less accurately than TD adults on the bounded NLT (Geary et al., 2008; van 't Noordende et al., 2016; Wilson et al., 2015). Since bounded NLTs require both low level and complex mensuration skills, worse performance could reflect a deficit in complex mensuration skills, a deficit in number sense, or both. Either one of these deficits would result in less functional strategy use and worse performance on the bounded NLT. Poor performance on the unbounded NLT on the other hand, would specifically imply a deficit in low level mensuration skills, more closely related to poor number representation. Therefore, investigating both strategy use and performance of adults with dyscalculia is highly interesting. In this respect, the unbounded NLT is of particular interest, given the paucity in research on the unbounded NLT in individuals with dyscalculia.

Method

Participants

The initial sample consisted of eight participants (four adults with dyscalculia and four TD adults). This small group enabled us to thoroughly explore strategy use in a qualitative manner. To validate our findings, we tested an additional group of eight participants. Since we found highly similar results between these two groups, in the remainder of this paper we will treat the samples as one sample. The final sample consisted of eight adults with dyscalculia (dyscalculia group) and eight TD adults (TD group). The dyscalculia group included participants with educational levels ranging from vocational education (two participants) to (pre-)university (six participants). The group consisted mainly of females (seven, one male), and the average age was 22.85 years (*SD* = 3.85 years). All were diagnosed with dyscalculia according to the Dutch guidelines (van Luit, Bloemert, Ganzinga, & Mönch, 2012; see Appendix). Several participants were diagnosed with comorbid neurodevelopmental disorders; that is, one with dyslexia, one with Attention Deficit Disorder (ADD), one with both dyslexia and dysorthography, and finally one participant with both dyslexia and ADD. Given that 8 to 65% of individuals with dyscalculia also has dyslexia and 26% has symptoms of ADHD (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Dirks, Spyer, van Lieshout, & de Sonneville, 2008; Gross-Tsur, Manor, & Shalev, 1996), we included participants with those comorbid conditions in order to keep the sample as representative as possible. As we aimed for an individual matching design, the final TD group was highly similar to the dyscalculia group with respect to gender (two male, six female), age (M_{age} = 22.05, *SD* = 2.66), and educational level (seven from university level, one from vocational education). Participants were all native speakers of Dutch. Participants all had normal or corrected-to-normal vision.

Procedure

All participants gave informed consent before participating in the study. Participants were tested individually in a quiet room. This room was depleted of distracting stimuli. Light conditions were held stable and direct light on the tracker or in the eyes was prevented. The experiment consisted of two tasks and the Cued Retrospective Reporting (CRR). After calibration, an unbounded NLT was administered, followed by a bounded NLT. Both NLTs (i.e., unbounded and bounded) started with a practice trial, which was excluded from the analyses. After both tasks were carried out, the CRR was administered. In total, the experiment took about 50 minutes per participant. During the tasks one of the researchers was present to answer the participants' questions and to administer the CCR.

Tasks

Unbounded NLT

The unbounded NLT consisted of two subtasks, consisting of 15 trials each. Trials were randomized within each subtask and presented to the participants in a fixed order. A trial consisted of a presentation of a number line and a number, depicted below the line, that had to be estimated. All number lines were 757 pixels (20 cm) long and the numerical length ranged from 0 to 40 in the first subtask (Figure 3a) and from 0 to 400 in the second subtask (Figure 3b), but only the begin-point was labeled. The unit indicating the distance between 0 and 1 in the 0-40 line and the distance between 0 and 10 in the 0-400 line was depicted below the begin-point and included 19 pixels. Target numbers were selected by picking a random number from each segment, after dividing the number line into 15 segments (corresponding with the number of stimuli; cf. van 't Noordende et al., 2016). The selected numbers were 2, 3, 7, 10, 12, 14, 17, 19, 23, 26, 28, 30, 34, 35, and 39 for the 0-40 line and 15, 38, 80, 96, 113, 154, 177, 201, 232, 269, 273, 315, 346, 358, and 392 for the 0-400 line. Participants were instructed to place the cursor at the location corresponding the target number and click the mouse-button.

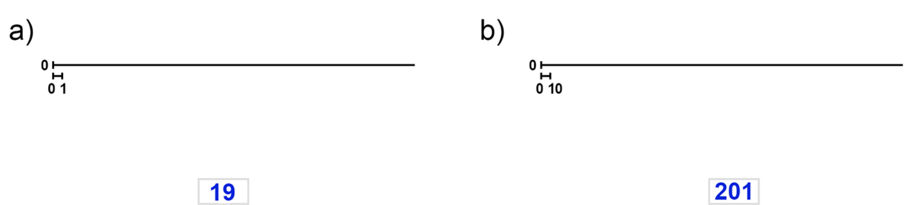

Figure 3. Examples of the 0-40 unbounded NLT (a) and the 0-400 unbounded NLT (b).

Bounded NLT

The bounded NLT was similar to the unbounded NLT, with the exception that the number lines were 816 pixels long and both begin-point and end-point were labeled. The first subtask ranged from 0-100 and the second from 0-1000. The target numbers were selected in a similar way as the unbounded number line. The selected numbers were 3, 12, 17, 25, 29, 37, 44, 51, 58, 63, 69, 76, 84, 88, and 99 for the 0-100 line and 36, 104, 153, 277, 308, 385, 422, 542, 594, 684, 723, 804, 880, 919, and 996 for the 0-1000 line.

CRR

After finishing the NLTs, eye movements and mouse clicks of some trials were replayed to the participants. After replaying the trial, the participants were asked to explain what strategy they used to position the target number ("can you explain what you did during this trial?"). Follow-up open-ended questions were asked when participant responses were unclear (e.g. "what do you mean by this?") and/or to evoke more comprehensive reactions (e.g. "is there anything you would like to add?"). To be sure participants knew how to interpret the replays, participants were first shown an introductory replay and were explained how eye movements and mouse clicks were displayed. Subsequently, ten trials from both subtasks of the bounded NLT were replayed, and five trials from both subtasks of the unbounded NLT, since less variation in strategy use was expected in the unbounded NLT. For one participant with dyscalculia, no eye movements were recorded for the bounded number line tasks, due to equipment failure. As such, CRR data were available for eight participants with dyscalculia in the unbounded NLT and for seven participants in the bounded NLT. The CRR was audio-taped and transcribed afterwards.

Eye-Tracker

During the NLTs, eye movement data were collected with a Tobii T60 Eye Tracker (60 Hz, manufactured: Dec 2008), containing a 17 inch screen (resolution: 1280 x 1024). A nine-point calibration was performed, and accuracy of calibration was assessed based upon visual inspection of length of error vectors and possible missing calibration points. Eye movements were analyzed using Tobii Studio (Version 3.1.2). Default fixation filter settings (Tobii I_VT fixation filter) were used (Olsen, 2012).

Data Coding and Analysis

Strategy Use

To examine strategy use during both the bounded and the unbounded NLTs in adults with dyscalculia versus TD adults, we first analyzed the predefined strategies on the bounded NLTs based on the strategies defined by van ’t Noordende et al. (2016) and after that explored the use of other strategies using the transcripts from the CRR on both the bounded and unbounded NLTs.

Predefined strategies — First, we defined areas of interest (AOIs), corresponding with predefined reference point combinations within the proportion based strategy: begin-point, mid-point, and end-point. The size of the AOIs was defined with a margin of 5% of the length of each number line on each side of the particular point. We retrieved the number of fixations within each AOI for each trial. To evaluate which reference point combinations were used in each trial (i.e., (1) begin, (2) begin-mid, (3) mid, (4) mid-end, (5) end, (6) begin-end, or (7) all, based on van ‘t Noordende et al., 2016), we determined in which AOIs fixations had occurred. Subsequently, we coded each trial based on the reference point combinations that were used (code 1-7, see above). For example, when a participant’s eye fixations were located in the AOIs around both the begin-point and the midpoint (i.e. they ‘looked’ at both the begin- and midpoint of the NLT), their use of reference points would be classified as ‘begin-mid’. When no fixations were located in either one of the AOIs, the strategy was coded as ‘undefined’. Per participant and group we computed the mean percentage of trials in which each of the strategies were used.

Next to general differences in the use of proportion based strategies, we determined the functionality of reference point combinations for both groups. We distinguished target number intervals: (1) 0-25 and 0-250, (2) 26-50 and 251-500, (3) 51-75 and 501-750, and (4) 76-100 and 751-1000. We computed the mean percentage of trials in which each reference point (combination) was used, separately for each interval, participant, and group. We defined functionality for each interval the following way: begin and begin-mid as functional for the first interval, begin-mid and mid for the second interval, mid and mid-end for the third interval, mid-end and end for the fourth interval, and other combinations as dysfunctional.

Due to equipment failure, for one participant in the dyscalculia group eye-tracking data were unavailable. As a result, this participant’s use of predefined strategies and functionality could not be calculated. When computing group percentages of the use of predefined strategies and reference point combinations and their functionality, missing values were excluded.

In-depth analysis of strategy use — For the in-depth analysis of strategies we qualitatively explored strategy use in both the bounded and the unbounded NLTs. Utilizing an inductive thematic analysis (Braun & Clarke, 2006), we analyzed the verbatim text of the CRR from both NLTs for each participant using Nvivo 10 (QSR International Pty Ltd.). During the thematic analysis we used the step-by-step approach as described by Braun and Clarke (2006): (1) familiarization with data, (2) generating initial codes, (3) searching for themes, (4) reviewing themes, and (5) defining and naming themes. Additionally, to identify themes we used four scrutiny techniques: repetitions, indigenous typologies (i.e., unfamiliar terms), transitions, and similarities and differences (Ryan & Bernard, 2003). During step two, some initial codes for the unbounded NLT were used: the 'dead-reckoning' and 'direct estimation' strategies (Link et al., 2014). The first three steps of the thematic analyses were conducted on all CRR data of the first sub-sample by three coders independently. Subsequently, themes were reviewed by discussing the results of step three between coders until agreement was reached and themes were named and defined. All CRR data (including the second sub-sample) were then divided between coders and reanalyzed, based on the defined themes. When an individual coder was uncertain about a specific classification this was discussed between all coders until consensus was attained.

Accuracy

To explore differences in accuracy of number estimation between bounded and unbounded NLTs and between TD adults and adults with dyscalculia, we defined accuracy as 'the percentage of absolute error (PAE)'. To obtain the PAE for each trial we carried out several steps. First, we retrieved the horizontal pixel of the mouse click for each stimulus. Second, we computed absolute deviation scores by subtracting the horizontal coordinate of the mouse click from the horizontal coordinate of the correct location of the target number. In order to correct for different number ranges of the number lines in the tasks, we computed the deviations in pixels, rather than in numbers. Lastly, PAE was computed by means of next formula: (absolute deviation/length of line [in pixels]) * 100. These separate PAE scores were used to compute mean PAE for each participant, group, and subtask separately.

Results

Strategy Use

Proportion Based Strategies in the Bounded NLT

The use of reference points looked similar between the dyscalculia and TD group (see Figure 4).

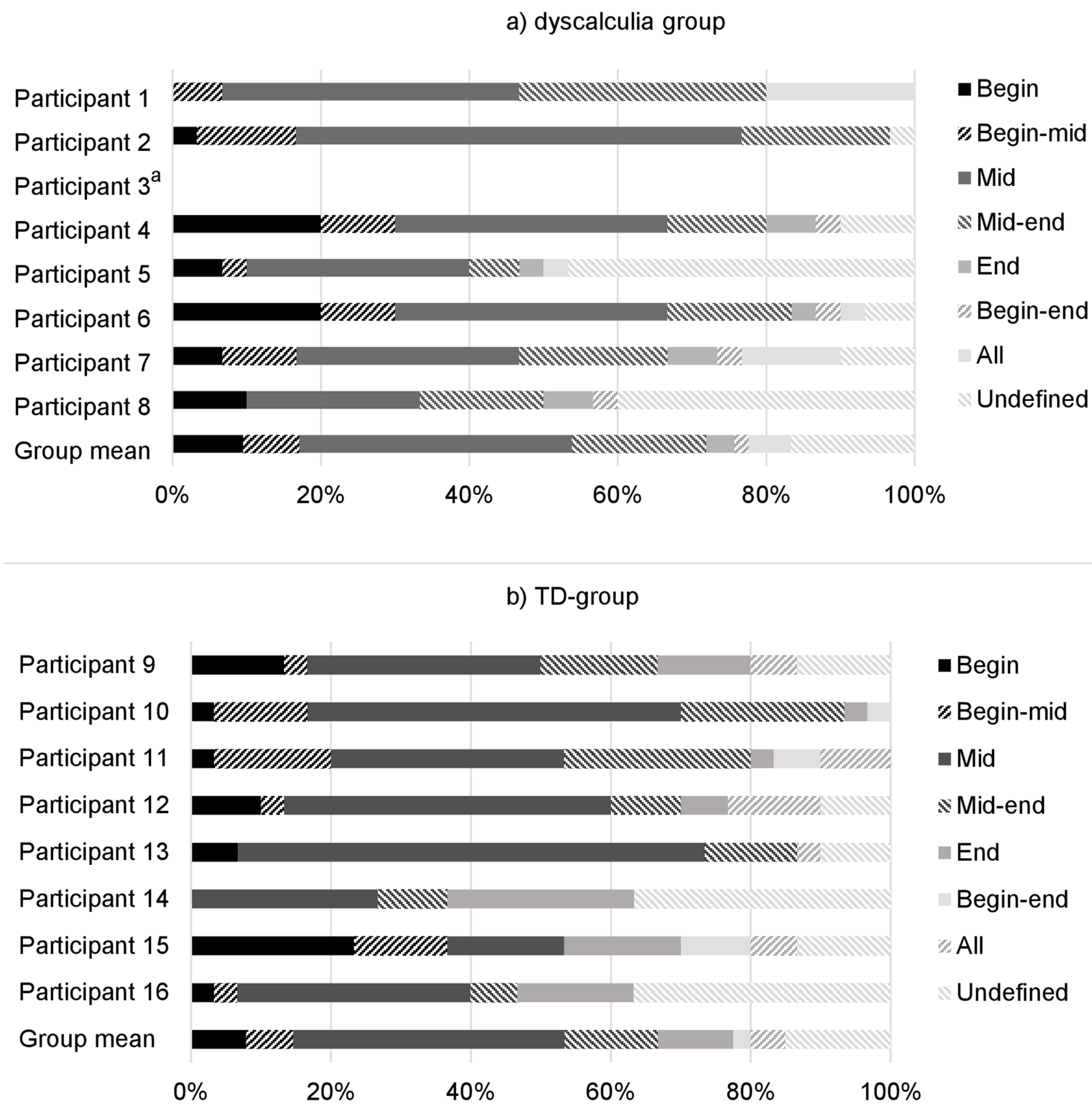

Figure 4. Percentage of reference point (Combinations) used on the bounded NLT per group.

[a]Data on strategy use by Participant 3 from the dyscalculia group were missing, due to equipment failure.

Moreover, the percentage of trials that could not be specified as one of the reference point combinations was similar for both groups, and concerned around one-sixth (15%) of all trials. Results regarding the functionality of the use of reference points in both groups are presented in Figure 5. In general, adults with dyscalculia showed similar functionality as compared to TD adults, mainly using begin or begin-mid for the lowest interval, using begin-mid or mid for the second interval, using mid or mid-end for the third interval and using mid-end or end for the last interval. Although not defined as functional within the lowest and highest interval, the mid-strategy was employed in approximately one-fifth of trials within these intervals by both groups. The TD group seemed to use more functional reference point combinations for numbers close to the mid-point (i.e., interval 26-50 and 251-500, and interval 51-75 and 501-750) than for numbers close to the beginning or end of the number line. In addition, the groups seemed to differ in the percentage of the use of mid-point only (dyscalculia: 26%, TD: 52%) as a reference point in the 26-50 and 251-500 interval and end-point only (dyscalculia: 13%, TD: 21.9%) as a reference point in the interval 76-100 and 751-1000. A more general finding for the last three intervals was that the dyscalculia group seemed to rely mainly on a combination of reference points including a reference point below the target number, whereas the TD group more often only used a reference point above the target number.

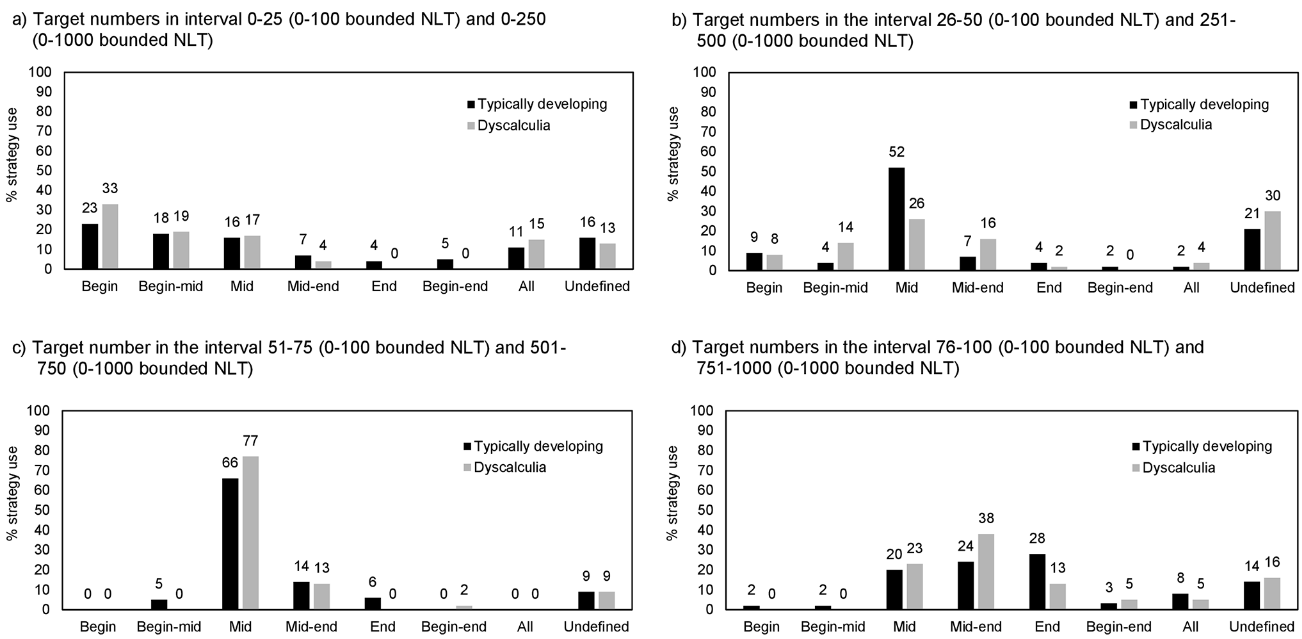

Figure 5. Percentages of trials in which each proportion based strategy was used, seperated per target number interval.

In-Depth Analysis of Strategy Use in the Bounded NLT

The thematic analysis of the CRR on the bounded NLT resulted in several themes and subthemes. We found two substantive main themes: strategies, including the already established use of reference points as well as new strategies, and tools or additional steps to help with determining the target position. In Table 1, examples of responses within each subtheme are displayed.

Figure 6 represents examples of gaze patterns of the strategies. Four subthemes were interpreted within the main theme strategies, which were sometimes combined to estimate the target number: the first strategy consisted of the direct estimation of the number, without gazes towards other parts of the NLT (i.e., direct estimation), the second strategy consisted of use of the reference points to make a proportion based judgement (i.e., proportion), the third strategy was the explicit use of steps to estimate the place of the target number (i.e., taking steps), and the fourth strategy refers to establishing the target number through the use of a previously estimated number (i.e. use of previous number). The strategy of taking steps was interpreted in three different ways: taking steps from the begin-, mid-, or end-point; taking steps from the previous number; and taking steps from a number that was presented even before the previous one.

Table 1

Examples of the Subthemes Derived From the Qualitative Analysis of the Strategy Use in the Bounded NLT for the Both Groups

		Number of participants who used the strategy		Percentage of trials in which the strategy was used	
Themes	**Examples**	**TD Group**	**Dyscalculia Group**	**TD Group**	**Dyscalculia Group**
Newly defined strategies					
Direct estimation	Just on intuition, that should be here.	8	5	15.72	14.38
Proportion	First I looked at the half, but on the other side: so 25, divided in two. Then I thought: it has to be a little bit back.	8	8	58.49	56.25
Taking steps	I think I started at 100 (…). I think: [I used] steps of 100 till 400.	6	6	30.82	29.38
Use of previous number	This is a little bit further than the previous number.	4	3	4.40	8.75
Additional tools and steps					
Orientation on the line	I first looked where the line ended, [and I saw it was] at 100. Then I looked at the [target] number.	7	7	10.69	9.38
Check	I often check whether I have the right number [in mind].	4	5	10.06	5.63
Estimation of small units[a]	I looked at 50, and then [moved] a little bit back.	8	8	55.35	41.88
Rounding off	The 88, rounded off to 90	7	6	16.35	16.25
Use of cursor	I think I (...) tried to move the cursor to 900. (...) Actually, I used it to create a new [vertical] bar on the line.	3	2	2.52	3.13
Decision based on multiple strategies	I think I started at 100, where I ended the previous one. I think [I used] steps of 100 till 400. Then I checked whether this [position] matched with [the place] where I think the 500 should be. And then 400 and a little bit further.	3	2	3.77	2.50
Remaining theme					
No strategy defined	I don't know either. [No further comments]	1	4	0.63	2.50

Note. All examples are translated from Dutch to comprehensible English.

[a]Units refers to small estimations in general (e.g. 15 in 115), not only to estimating the last unit in the decimal system (e.g. 5 in 115).

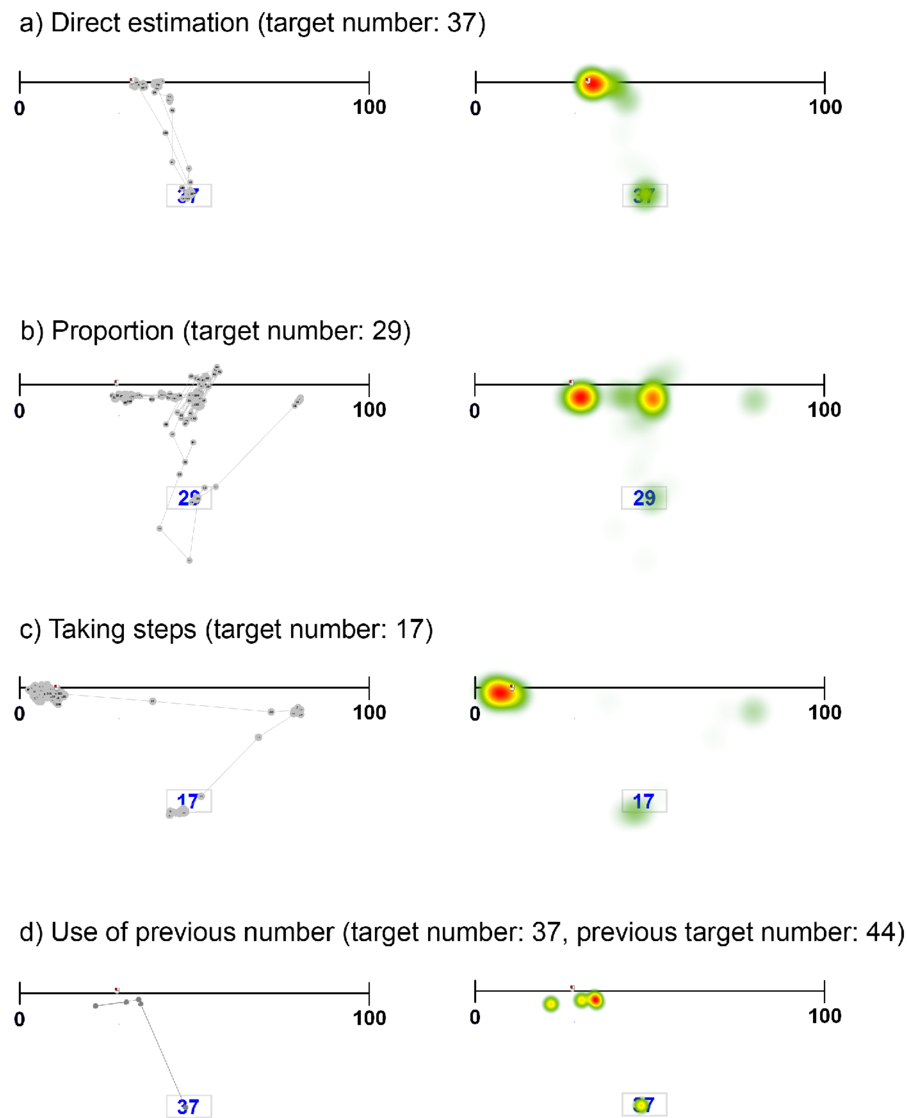

Figure 6. Representative examples of strategies used during the bounded NLT. On the left, gaze patterns are displayed, in which dots represent eye fixations and lines represent saccades (eye movements). On the right, heat maps are displayed, in which the color indicates the duration of fixations at a specific location. Green indicates short fixations, yellow indicates fixations of medium duration, and red indicates long fixations.

Next to strategies, six subthemes were interpreted as additional tools and steps. *Orientation on the line* refers to quickly reviewing the NLT before estimating the target number. *Check* refers to checking of the target number after establishing where it should be situated, or re-conducting the used strategy to verify that the number is correctly estimated. *Estimation of small units* refers to the situation in which participants divided the target number in two pieces, of which the major part is estimated quite precisely and a rest-part (i.e., small unit) is estimated loosely. *Rounding off* refers to rounding the target number off to a number that is more easily estimated. *Use of cursor* refers to the use of the cursor as an extra visual aid, e.g. as a reference point. Lastly, *decision based on multiple strategies* refers to estimating the target number in multiple ways to determine the location based on a consideration of multiple aforementioned strategies. These themes were similar for the TD and the dyscalculia group. In addition to these substantive themes, trials for which participants were unable to describe a strategy were coded as 'no strategy defined'.

Subsequently, we explored differences between the dyscalculia and TD group in the use of these newly defined strategies, and additional steps and tools. For this purpose, the percentage of trials in which the strategies and additional tools or steps were used was calculated. The results (see Table 1) show that the dyscalculia group tended to prefer the use of *proportion* (56.25%) over *taking steps* (29.38%), over *direct estimation* of the number (14.38%), and over *use of previous number* (8.75%). In addition, adults in the dyscalculia group made use of several additional tools and steps. They often estimated small units (e.g. 'and then a little to the right') after they used the strategies of taking steps and proportion (41.88%). Furthermore, sometimes they *rounded off* the target numbers upwards or downwards (16.25%) and used an *orientation on the line* (i.e., determining the length of the line) (9.38%). *Checking* whether the target was positioned correctly, *using the cursor* as visual aid, and using *multiple strategies* were scarce in this group (resp. 5.63%, 3.13%, and 1.25%).

A similar pattern of strategies was found in the TD group in which *proportion* (58.49%) was used most often, followed by *taking steps* (30.82%), *direct estimation* (15.72%), and *use of previous number* (4.40%). Regarding the additional tools and steps, Table 1 shows that in comparison to the dyscalculia group, the TD group more often *checked* whether they correctly remembered the target number or whether the target number was correctly positioned (10.06%) and more often *estimated* the *small units* (55.35%). The frequency of all other tools was similar to the dyscalculia group. Furthermore, individual differences in the use of strategies and additional tools are shown in Table 1, which reveals that not all strategies and tools are employed by every participant.

In-Depth Analysis of Strategy Use in the Unbounded NLT

For the unbounded NLT we found main themes similar to those found in the bounded line for the dyscalculia and TD group (i.e., independent strategies, additional steps and tools, and being not able to define the strategy). In addition to the already known *dead-reckoning* and *direct estimation* strategy, we found two more independent strategies: *explicit use of unit* and *use of previous number*. The first additional strategy, *explicit use of unit*, refers to explicitly and continuously using the 0-1 or 0-10 unit until the target number is established. The second additional strategy, *use of previous number*, refers to establishing the target number through the use of a previously estimated number. Examples of these defined strategies are displayed in Table 2. Corresponding gaze pattern examples are represented in Figure 7. Also, we found that several additional tools and steps were used: *rounding off*, *using the cursor as reference point*, *orientation on line*, *estimation of small units*, *decision based on multiple strategies*, *looking back at target number or unit*, and *proportion* (see Table 2 for examples). Some of the themes did not occur among the TD group (i.e., *rounding off* was not used in this group).

The use of each strategy was calculated, as shown in Table 2. In the dyscalculia group, *explicit use of unit* was used most often (39.76%), followed by *use of previous number* (31.33%). *Direct estimation* was used in less trials (24.10%) and *dead-reckoning* was used even less (16.87%). Similarly, in the TD group *explicit use of unit* was used most often (39.51%), followed by *use of previous number* (25.93%). However, *dead-reckoning* was used slightly more often in the TD group (20.99%) and the use of *direct estimation* was scarce (1.23%) The results indicate that the dyscalculia group uses *dead-reckoning* less often, and *direct estimation* more often than the TD group.

Table 2

Examples of the Subthemes Derived From the Qualitative Analysis of the Strategy Use in the Unbounded NLT for Both Groups

Themes	Examples	Number of participants who used the strategy		Percentage of trials in which the strategy was used	
		TD Group	Dyscalculia Group	TD Group	Dyscalculia Group
Previously established strategies					
Direct estimation	I first looked at the unit, and then I determined where it approximately has to be (...).	1	4	1.23	24.10
Dead-reckoning	I first took a step of five, and then I added this step of five again, and once again, and after that [I moved] a little bit back	4	5	20.99	16.87
Newly defined strategies					
Explicit use of unit	I used this unit to draw it repeatedly in my head, until I arrived at 14.	7	8	39.51	39.76
Use of previous number	Yes. I still remembered that the previous number was 26, so I thought: this number has to be right after it. I tried to determine this, how many parts of one fitted behind this.	7	8	25.93	31.33
Additional tools and steps					
Orientation on the line	At the start I constantly looked at the begin-point, and then to the end, and back again, because I constantly tried to estimate what the length of the line is.	3	5	11.11	9.64
Cursor as reference point	I repeatedly moved the cursor a bit further. I used the cursor as a measuring tape.	1	4	2.47	8.43
Estimation of small units	… I took three steps of 10 and then again ten and after that a little bit back [target number 38].	5	7	14.81	19.28
Rounding off	I rounded off 38 to 40.	0	2	0.00	3.61
Decision based onmultiple strategies	I thought I have to place it a little bit before the previous one. (…) then I thought I have to place it at one-third (…), because I thought it would end at 300.	2	4	4.94	6.02
Looking back at targetnumber or unit	I often look back to the number, otherwise I forget which number it is	3	3	4.94	4.82
Proportion[a]	So I thought one-third is approximately 100. 154… If the [end of the] line is a little more than 300, than 154 is a little bit before half [of the line].	1	2	9.88	4.82
Remaining theme					
No strategy defined	10. [no further comments]	0	3	0.00	3.61

Note. All examples are translated from Dutch to comprehensible English, and are drawn both from the dyscalculia and the TD group.
[a]Before using 'proportion' the participant determined the length of the line with the use of dead-reckoning, which helped the participant to form reference points.

Regarding the tools, participants in the TD group seemed to be less likely to use *rounding off* and *the cursor as a reference point,* and more likely to use *proportion* as compared to participants in the dyscalculia group. Additionally, as becomes clear from the number of participants who use specific s*trategies* and *steps and tools* shown in Table 2, there is much variation within groups in the use of additional tools and steps, and -to a lesser degree- in the use of strategies.

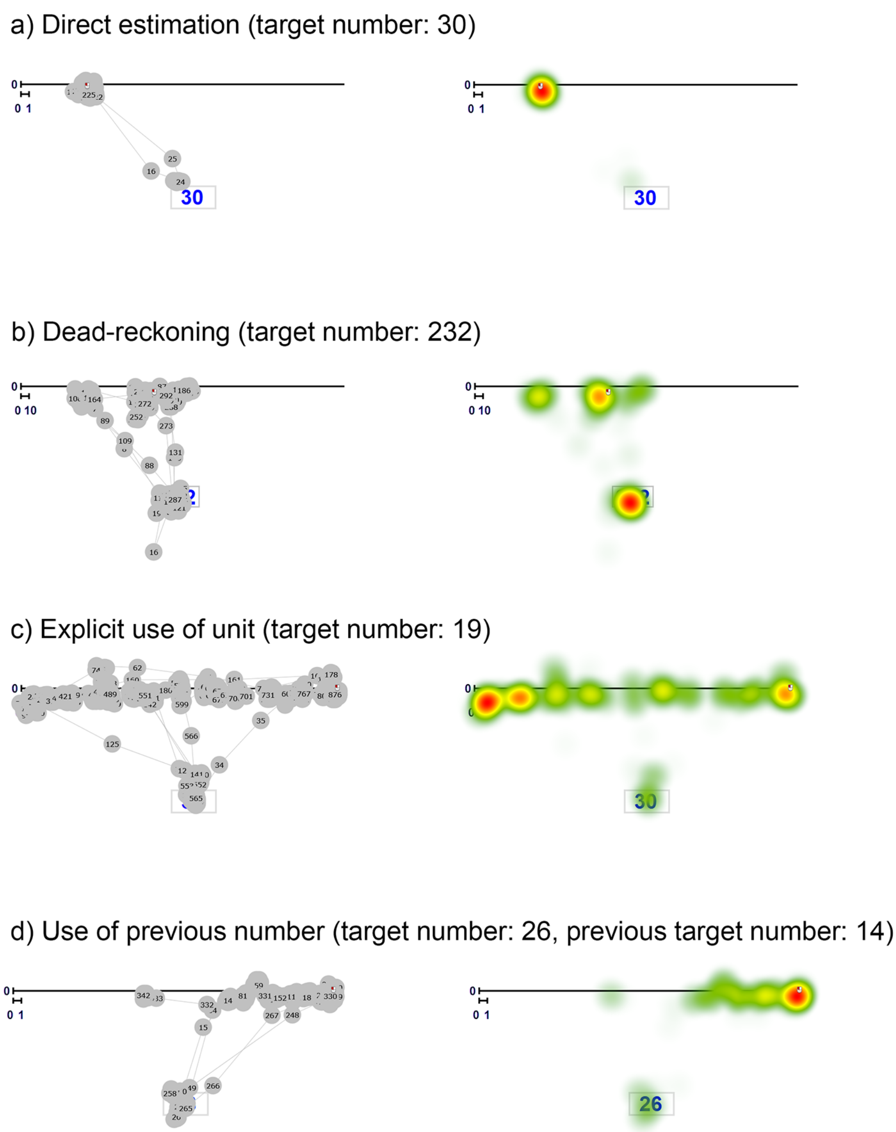

Figure 7. Gaze patterns examples of strategies used during the unbounded NLT. On the left, gaze patterns are displayed, in which dots represent eye fixations and lines represent utterances (eye movements). On the right, heat maps are displayed, in which the color indicates the duration of fixations at a specific location. Green indicates short fixations, yellow indicates fixations of medium duration, and red indicates long fixations

Accuracy

The accuracy on both the unbounded and bounded NLTs is displayed in Figure 8. In contrast to our expectations, the results indicate no major differences between the dyscalculia group and the TD group in positioning the target number in both the unbounded and bounded NLTs. Regarding the bounded NLT, results showed that in 11 out of 16 participants, performance was better on the 0-1000 task than the 0-100 task. Whereas the group means were slightly lower for the TD group, indicating slightly better performance on the NLTs, individual variation within both groups was large. That is, when ranking participants from most to least accurate, participants with dyscalculia and TD participants were mixed. With regard to differences between the bounded and unbounded NLTs, overall PAE was higher for the unbounded NLT ($M_{unbounded}$ = 12.72, *SD* = 13.50) as compared to the bounded NLT ($M_{bounded}$ = 5.59, *SD* = 7.46), indicating lower accuracy on the unbounded NLT than on the bounded NLT.

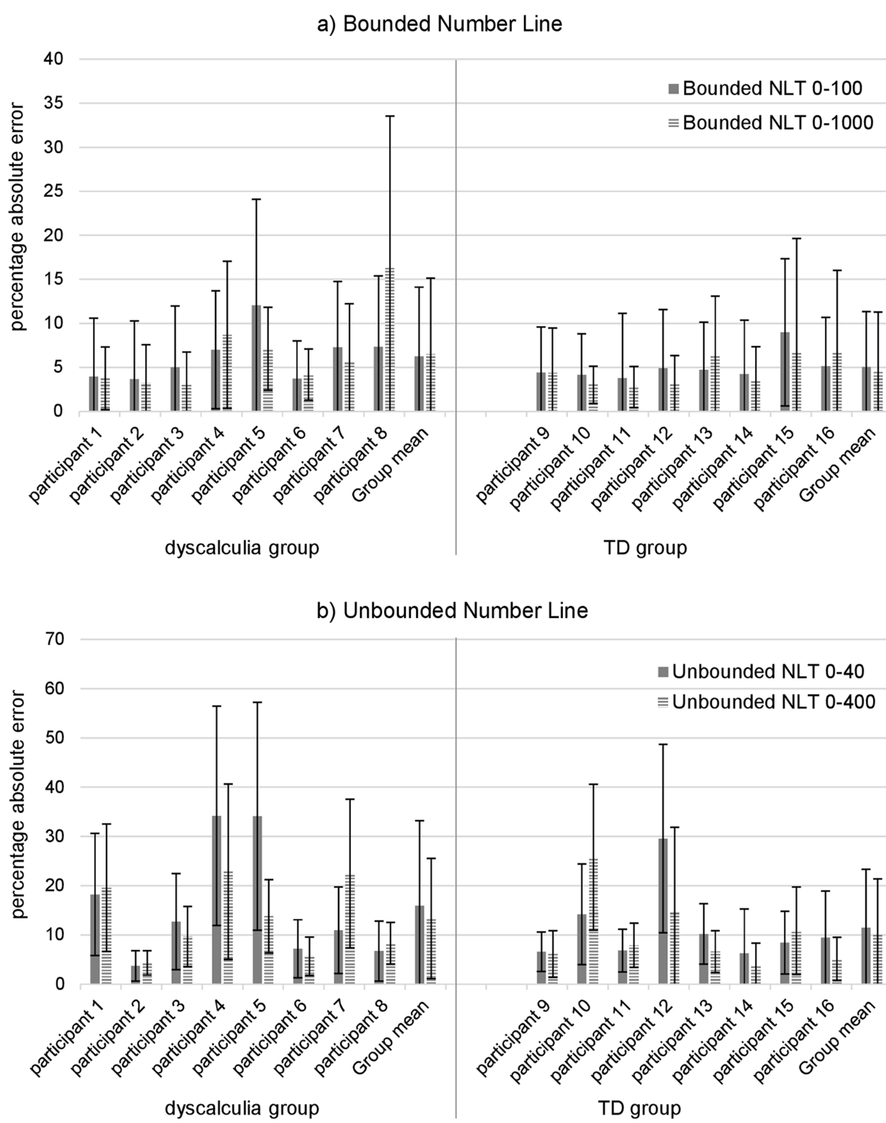

Figure 8. Percentage absolute error per group and per NLT. Error bars display standard deviations in positive and negative direction.

Discussion

The aim of our study was to explore strategy use and accuracy in both bounded and unbounded NLTs in adults with dyscalculia and typically developing adults. A combination of eye-tracking recordings and CRR enabled us to explore strategies more in-depth and as such establish additional strategies that were not captured in previous research. In addition, our study is the first to provide insight into the unbounded NLT in individuals with dyscalculia. We will first discuss the results with regards to strategy use, followed by the results on accuracy in both tasks.

Predefined strategy use on the bounded NLT was examined first based on the use of reference point combinations, i.e. proportion based strategies, as defined in previous research (van ‘t Noordende et al., 2016). Reference points were used to a similar extent by individuals with and without dyscalculia. Contrary to findings in children (van ‘t Noordende et al., 2016), the use of undefined strategies did not differ either, which may imply that children with dyscalculia use more immature strategies in number line estimation, but adults with dyscalculia have learned to use strategies similar to those used by typically developing adults. Our in-depth exploration of strategies did reveal the use of the additional strategies *direct estimation, proportion based judgement, taking steps*, and *use of previous number*, which were used by adults with and without dyscalculia. Note that *proportion based judgement* includes the previously defined use of reference points. Moreover, additional steps and tools, such as *checking* and *rounding off*, were used in combination with these strategies (see Table 2). The results suggest that research solely focusing on the use of reference points may not capture the complete range of processes taking place during estimations on the bounded NLT.

With regards to the functionality of the use of reference points, results showed mainly similarities between the individuals with and without dyscalculia. One difference between the groups on the bounded NLT was the tendency of the dyscalculia group to include a reference point below (i.e., smaller than) the target number in their estimations, whereas the TD group more often relied on a reference point larger than the target number only. An explanation for this difference could be that counting down requires more working memory capacity than counting up (Baroody, 1984). Since working memory is impaired in individuals with dyscalculia (Mammarella et al., 2015; Peng & Fuchs, 2016; Rotzer et al., 2009; Schuchardt et al., 2008), they may have chosen to use counting up more often, since this is less demanding for working memory. Taken together, our results suggest that adults with dyscalculia mainly use the same (combinations of) reference points as adults without dyscalculia on bounded number lines.

In the unbounded NLT, the predefined strategies *dead-reckoning* and *direct estimation* were examined first. Individuals with dyscalculia differed from individuals without dyscalculia in the use of these strategies on the unbounded NLT. In line with our expectations, individuals with dyscalculia used the predefined strategy *dead-reckoning* less often and *direct estimation* more often than typically developing individuals. Accuracy results show little differences between groups, suggesting that *direct estimation* is not necessarily a poor strategy that is only used when not knowing how to estimate a number.

In-depth exploration of the strategies on the unbounded NLT revealed two additional strategies next to *dead reckoning* and *direct estimation*. One is the *explicit use of unit* in which the given 0-1 or 0-10 step is used to estimate the target number. This was used by 15 out of 16 participants, indicating that this is a strategy that is commonly used. The other one is the *use of previous number* in which the previously estimated number is taken as an anchor point to estimate the current number. This strategy was also frequently used by both groups. Several additional steps and tools were reported for the unbounded NLT. These include for example *orientation on the line* (quickly reviewing the NLT before estimating the target number) and *looking back to the target number or unit*. Thus, four different strategies are commonly used to estimate numbers on the unbounded number line, two of which were not described in previous research (Link et al., 2014; Reinert et al., 2015a). Future research should use these four strategies to classify strategy use based on eye movements in larger samples. Using that approach, differences in strategy use between individuals with and without dyscalculia can be quantified, and strategy use and performance can be related to each other to investigate which strategies are more or less successful. An advantage of predefining these strategies in future research is that participants do not have to report on their strategies. Therefore, it is easily applicable in larger groups of participants as well as easily implemented in research with children.

With regards to accuracy, based on previous research, we expected that adults with dyscalculia would perform less well than adults without dyscalculia, at least on the bounded NLT (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary et al., 2008; van ‘t Noordende et al., 2016; van Viersen et al., 2013). However, our descriptive results indicate quite similar performance in adults with and without dyscalculia on either type of number line. Whereas the mean PAE was slightly larger in the dyscalculia group, individual differences were much larger than differences between groups. This finding, in combination with the similarities in strategy use, may suggest that problems in both bounded and unbounded number line estimation in individuals with dyscalculia can possibly be largely overcome in adulthood. This idea is supported by the similar performance in the dyscalculia group compared to the TD on the number lines with small numbers (bounded 0-100, unbounded unit 0-1) and the lines with larger numbers (bounded 0-1,000, unbounded unit 0-10). Booth and Siegler (2006) suggest that estimations become more linear and more precise with increasing familiarity with the numbers. Similar performance on both lines suggests that these adults are familiar with the numbers of 0 to 1,000.

Taken together, our results suggest that successful estimations on number lines up to 1,000 are reached in adults with dyscalculia with the use of similar strategies as typically developing adults. Previous research has shown that mathematical difficulties in individuals with dyscalculia do persist into adulthood (Mejias et al., 2012; Wilson et al., 2015). In these studies, timed responses were often required (Ashkenazi & Henik, 2010; Rubinsten & Henik, 2005; Mejias et al., 2012). In the current study, as in other studies on number lines, only accuracy was taken into account. Therefore, it may be that adults with dyscalculia overcome their deficits in number sense, if measured with tasks that do not require speeded processing. Problems in memorization of arithmetic facts, fluent calculation and math reasoning may still persist in adults with dyscalculia. These may not (solely) be due to deficits in number sense itself (Ashkenazi & Henik, 2010). They could however be due to deficits in fast access to numerical information. Similar findings were already shown in studies on dyslexia. Whereas young children with dyslexia have problems in accurately decoding words, older children are often able to accurately decode words (in regular languages), but lag behind in decoding speed (e.g. de Jong & van der Leij, 2003). Whether the same holds for individuals with dyscalculia in tasks with numbers requires further research.

Limitations

Concerning possible limitations, one could argue that posing guiding questions during CRR evokes certain answers, by directing the attention of the participant, which could have influenced the in-depth analysis of strategy use (but not strategy use and performance itself since it was administered after the task). However, it has recently been suggested that combining specific questions with CRR and eye-tracking does have added value in comparison to retrospective think aloud methods without eye-tracking, because the questions are based on the eyemovement data observed during replay (Elling, Lents, & De Jong, 2011; van Gog, Paas, van Merriënboer, & Witte, 2005).

A second remark that could be made is the small sample used in this study. However, our main aim was to explore additional strategies used in bounded and unbounded number lines and see whether these strategies were used by both individuals with and without dyscalculia. As such, very detailed examination of the eye movements had to take place. Since identification of strategies that were not defined beforehand based on the eye movements was the main aim, we specifically choose an exploratory instead of experimental approach. The additional strategies that were discovered using this method serve as a basis for future quantitative research.

Conclusion

To conclude, our findings give some first insights into strategy use and performance of adults with dyscalculia on unbounded number lines. The results suggest highly similar strategy use and performance in adults with and without dyscalculia. Herewith, neither our hypothesis that adults with dyscalculia have difficulties with more complex mensuration skills (i.e., proportion based judgement in bounded NLTs), nor our hypothesis that adults with dyscalculia have difficulties with low level mensuration skills was confirmed. We suggest that adults with dyscalculia may be mainly impaired in fast access to numerical knowledge instead of lacking number knowledge per se. However, this suggestion requires further research including a broader range of tasks and a larger sample.

Most importantly, this study was the first to combine the use of eye-tracking with CRR to capture strategy use in number line estimation. We revealed multiple additional strategies, steps and tools as compared to earlier research on both the bounded and unbounded NLT. Combining results from quantitative and qualitative analyses also led to more specific insights in what should be considered functional strategy use. Alternative strategies in the bounded NLT were revealed, showing that the lack of use of the begin-, mid-, and end-points could still indicate the use of functional reference points when, for example, *use of previous number* is used to estimate the target number. Therefore, functionality should not be determined based on the use of known reference points only. We suggest that the additional strategies revealed in this study, should be taken into account when studying number line estimations quantitatively in larger samples. Such research may shed a light on differences between individuals with and without dyscalculia on a larger scale.

Funding

The authors have no funding to report.

Competing Interests

The authors have declared that no competing interests exist.

Acknowledgments

We thank Margot van Wermeskerken, Willemijn Schot, and Tamara van Gog for their helpful comments on an earlier version of this manuscript, and Jos Jaspers for his help with the eye-tracking apparatus. We also want to show our gratitude towards Diedre Batjes who greatly assisted us in data collection.

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Appendix: Criteria for Diagnosing Dyscalculia

Dutch guidelines for diagnosing dyscalculia are based on criteria of van Luit, Bloemert, Ganzinga, and Mönch (2012):

- Criterion of severity.
  The level of mathematics is substantially below that of peers, and interferes with every day life.
- Criterion of discrepancy.
  The proficiency in mathematical skills is substantially below the level expected based on the cognitive development of the individual.
- Criterion of didactic resistance.
  Improvement in math skills based on individualized instruction is limited.

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