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Distributed Averaging Control for Voltage Regulation and Current Sharing in DC Microgrids

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Abstract—In this letter we propose a new distributed control scheme, achieving current sharing and average voltage regulation in Direct Current (DC) microgrids. The considered DC microgrid is composed of several Distributed Generation Units (DGUs) interconnected through resistive-inductive power lines. Each DGU includes a generic energy source that supplies a local current load through a DC-DC buck converter. The proposed distributed control scheme achieves current sharing and average voltage regulation, independently of the initial condition of the controlled microgrid. Moreover, the proposed solution requires only measurements of the generated currents, and is independent of the microgrid parameters and the topology of the used communication network, facilitating Plug-and-Play capabilities. Global convergence to a desired steady state is proven and simulations indicate a good performance.

Index Terms—Distributed control, control of networks, control over communications, power systems.

I. INTRODUCTION

MICROGRIDS are relatively small electricity networks, wherein loads, sources and storage systems require careful coordination [1]. Generally, microgrids are either Alternating Current (AC) or Direct Current (DC) networks, where each type requires dedicated control strategies. The vast experience with high voltage networks, that are almost exclusively AC, has caused an initial focus on adapting control paradigms to AC microgrids [2]–[5]. It is however realized that many sources and loads (e.g. photovoltaic panels, batteries, electronic appliances) can be directly connected to DC microgrids by using DC-DC converters, making DC microgrids in some situations more efficient than AC microgrids [6]. Due to their simplicity with respect to their AC counterparts, DC microgrids are often deployed where reliability is essential, such as aircrafts and trains.

A. Literature review

The aforementioned careful coordination is foremost needed to ensure that voltages at the loads are around desired values and to ensure that providing power to the network is shared fairly among the various sources [7]–[11]. A popular approach

to obtain this fair sharing, is to design controllers that aim for (proportional) current sharing, wherein the total current that has to be generated is (proportionally) allocated to the various sources. Conventionally, hierarchical control schemes are proposed to achieve both objectives [12], where e.g. voltage setpoints at the converters are determined to achieve appropriate current sharing. Due to required scalability and Plug-and-Play capabilities of possible control schemes, while providing a fast response to changing loads, there has been a growing interest in the development of distributed controllers, particularly aiming at current (power) sharing [13]–[21]. Achieving simultaneously a form of voltage regulation appears to be more challenging and provided solutions often rely on simplifying assumptions. Since the currents in the network are tightly related to the voltages, it is not possible to freely adjust the voltages while still expecting a proper allocation of the generated currents. To alleviate this concern the voltage requirements are generally relaxed and for example only the average voltage across the whole microgrid is regulate towards a global voltage set point. We will follow the same approach, which is often referred to as 'average voltage regulation', 'global voltage regulation' or 'voltage balancing' [14], [17].

B. Main contributions

Although the design and analysis of distributed control schemes for DC microgrids have received a significant amount of attention, we notice that results on *provably* achieving *simultaneously* voltage regulation and current (power) sharing are still lacking. Indeed, results with theoretical stability assurance often focus on only one aspect [16], [20]. In case the DC microgrid comprises buck-converters and current loads, we provide novel insights. In particular, we show that it is possible to achieve a form of voltage regulation *without* the need of voltage measurements. We elaborate on some contributions below, where we also provide a brief comparison with some existing theoretical results considering both a form of voltage regulation and current (power) sharing.

1) Although the considered microgrid model is fairly standard, the presented results take particularly into account a possible meshed microgrid topology, incorporating dynamic resistive-inductive lines, which are neglected in e.g. [17], [18], where purely resistive lines are considered.

2) The proposed control scheme is distributed and only local measurements of the generated currents are needed, as well as current measurements of connected DGUs exchanged over a communication network. Notably, the proposed solution
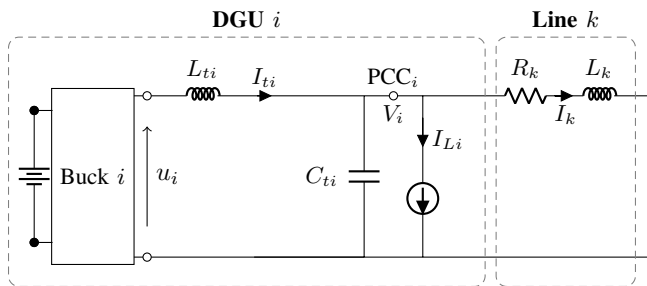
Fig. 1. Electrical scheme of DGU i and line k.

TABLE I
DESCRIPTION OF THE USED SYMBOLS

	State variables
I_{ti}	Generated current
V_i	Load voltage
I_k	Line current
	Parameters
L_{ti}	Filter inductance
C_{ti}	Shunt capacitor
R_k	Line resistance
L_k	Line inductance
	Inputs
u_i	Control input
I_{Li}	Unknown current demand

achieves average voltage regulation without voltage measurements, which are needed in e.g. [17], [21]. The topology of the communication can be designed without requiring any particular knowledge on the topology of the microgrid. This is in contrast to [17], where an additional assumption is introduced on the product between the Laplacian matrices associated to the microgrid and communication networks [17, Assumption 4]. Finally, the design of the distributed controller is independent of the parameters of the microgrid and is consequently more flexible than the controller proposed in [21], where bounds on the parameters are needed.

3) Existence of a desired steady state that corresponds to current sharing and average voltage regulation is proven. Furthermore, global convergence to a desired steady state is established, independently from the initial condition of the physical system and the controller state, facilitating Plug-and-Play capabilities. This is in contrast to e.g. [18], where a suitable initialization of the voltages are assumed, or [17], [21] where a suitable initialization of the controller state is required. The obtained theoretical results in this work rely partly on the assumption of constant *current* loads that can represent in some circumstances, in particular when voltages are substantially constant, e.g. LED-lighting, battery chargers or linearized nonlinear loads [11]. Extending the theoretical results towards incorporating the more common constant power loads remains an important challenge. However, the rationale of the proposed solution is independent of the exact load model and we believe that the presented exposition is an important step towards incorporating more general load models, considered e.g. in [7], [20] and [18].

C. Outline

The remainder of this letter is organized as follows: In Section II, the microgrid model is presented, while in Section III the control problem is formulated. In Section IV, the distributed control scheme is introduced, whereafter the stability of the controlled microgrid is studied in Section V. In Section VI, the simulation results are illustrated and discussed, and finally, conclusions are gathered in Section VII.

II. DC MICROGRID MODEL

In this letter we study a typical DC microgrid composed of n Distributed Generation Units (DGUs) connected to each other through m resistive-inductive (RL) power lines (see

Fig. 1). The energy source of each DGU is represented by a DC voltage source that supplies a local load through a DC-DC buck converter. The local DC load is connected to the so-called Point of Common Coupling (PCC) and we assume that current* demand I_{Li} is not measurable. The equations describing the dynamic behaviour of the DGU i are given by

$$\begin{aligned} L_{ti}\dot{I}_{ti} &= -V_i + u_i \\ C_{ti}\dot{V}_i &= I_{ti} - I_{Li} - \sum_{k \in \mathcal{E}_i} I_k, \end{aligned} \quad (1)$$

where $\mathcal{E}_i$ is the set of power lines incident to the DGU i, while the control input u_i represents the buck converter output voltage†. The current from DGU i to DGU j is denoted by I_k, and its dynamic is given by

$$L_k \dot{I}_k = (V_i - V_j) - R_k I_k. \quad (2)$$

The symbols used in (1) and (2) are described in Table I. The overall DC power network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes, $\mathcal{V} = \{1, ..., n\}$, represent the DGUs and the edges, $\mathcal{E} = \{1, ..., m\}$, represent the power lines interconnecting the DGUs. The network topology is described by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n \times m}$. The ends of edge k are arbitrarily labeled with a $+$ and a $-$, and the entries of $\mathcal{B}$ are given by

$$\mathcal{B}_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise.} \end{cases}$$

Consequently, the overall microgrid system can be written compactly for all DGUs $i \in \mathcal{V}$ as

$$\begin{aligned} L_t \dot{I}_t &= -V + u \\ C_t \dot{V} &= I_t + \mathcal{B}I - I_L \\ L\dot{I} &= -\mathcal{B}^T V - RI, \end{aligned} \quad (3)$$

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*More general load models include the so-called ZIP loads, comprising constant impedance, constant current and constant power loads. These can be incorporated by replacing I_{Li} with $Z_{Li}V_i + I_{Li} + \frac{P_{Li}}{V_i}$. However, since constant power loads would render the microgrid model nonlinear, extending the results of this work towards these loads is left as an interesting future endeavor.

†Note that u_i in (1) can be expressed as $\delta_i V_{DC_i}$, where δ_i is the duty cycle of the buck i and V_{DC_i} is the DC voltage source provided by a generic energy source at node i.

where $I_t, V, I_L, u \in \mathbb{R}^n$, and $I \in \mathbb{R}^m$. Moreover, $C_t, L_t \in \mathbb{R}^{n \times n}$ and $R, L \in \mathbb{R}^{m \times m}$ are positive definite diagonal matrices, e.g., $C_t = \text{diag}(C_{t1}, \ldots, C_{tn})$.

Remark 1: (**Kron reduction**) Note that in (1), the load currents are located at the PCC of each DGU (see also Figure 1). This situation is generally obtained by a Kron reduction of the original network, yielding an equivalent representation of the network [16].

III. PROBLEM FORMULATION

In this section we formulate two common control objectives in DC microgrids. First, we notice that for given demand I_L and constant inputs $\overline{u}$, a steady state solution $(\overline{I}_t, \overline{V}, \overline{I})$ to system (3) satisfies

$$\overline{V} = \overline{u} \tag{4a}$$
$$I_L - \overline{I}_t = \mathcal{B}\overline{I} \tag{4b}$$
$$\overline{I} = -R^{-1}\mathcal{B}^T\overline{V}. \tag{4c}$$

Equation (4b) implies[‡] that at the steady state the total generated current $\mathbb{1}^T\overline{I}_t$ is equal to the total current demand $\mathbb{1}^T I_L$. To avoid the overstressing of a source and to improve the generation efficiency, it is generally desired that the total demand of the network is shared among all the various DGUs proportionally to the generation capacity of their corresponding energy sources (proportional current sharing). This desire can be formulated as $w_i\overline{I}_{ti} = w_j\overline{I}_{tj}$ for all $i, j \in \mathcal{V}$, where a relatively large value of w_i corresponds to a relatively small generation capacity of DGU i. This leads to the first control objective concerned with the steady state value of the generated currents $\overline{I}_t$.

Objective 1: (**Current sharing**)

$$\lim_{t \to \infty} I_t(t) = \overline{I}_t = W^{-1}\mathbb{1}i_t^*, \tag{5}$$

with $W = \text{diag}(w_1, \ldots, w_n)$, $w_i > 0$, for all $i \in \mathcal{V}$ and i_t^* any scalar.

Note that the steady state requirement $\mathbb{1}^T\overline{I}_t = \mathbb{1}^T I_L$ necessarily prescribes that $i_t^* = \frac{\mathbb{1}^T I_L}{\mathbb{1}^T W^{-1}\mathbb{1}}$. Before introducing the second control objective considered in this work, we assume that for every DGU i, there exists a desired reference voltage $V_i^\star$.

Assumption 1: (**Desired voltages**) There exists a reference voltage[§] $V_i^\star$ at the PCC, for all $i \in \mathcal{V}$.

Generally, achieving Objective 1 does not permit a steady state voltage $\overline{V} = V^*$. Particularly, equations (4b) and (4c) require that the steady state voltage $\overline{V}$ satisfies $\mathcal{B}R^{-1}\mathcal{B}^T\overline{V} = W^{-1}\mathbb{1}i_t^* - I_L$. However, this admits the freedom to shift all steady state voltages with the same constant value, since $\mathcal{B}^T\overline{V} = \mathcal{B}^T(\overline{V} + a\mathbb{1})$, with $a \in \mathbb{R}$ any scalar. We therefore aim at an *average voltage regulation*, where the weighted average value of $\overline{V}$ is identical to the weighted average value

[‡]The incidence matrix $\mathcal{B}$, satisfies $\mathbb{1}^T\mathcal{B} = \mathbf{0}$, where $\mathbb{1} \in \mathbb{R}^n$ is the vector consisting of all ones.

[§]Often the values for $V_i^\star$ are chosen identical for all $i \in \mathcal{V}$. However, the control strategy proposed later in this work permits to select also non-identical values for $V_i^\star$.

of the desired reference voltages $V^\star$. Following the standard practise where the sources with the largest generation capacity determine the grid voltage, we select a weight of $\frac{1}{w_i}$ for all $i \in \mathcal{V}$, leading to the second objective.

Objective 2: (**Average voltage regulation**)

$$\lim_{t \to \infty} \mathbb{1}^T W^{-1}V(t) = \mathbb{1}^T W^{-1}\overline{V} = \mathbb{1}^T W^{-1}V^\star. \tag{6}$$

IV. DISTRIBUTED AVERAGING CONTROL WITH DAMPING

Before proposing a distributed controller achieving the objectives discussed in the previous section, we make the following assumption on the available measurements:

Assumption 2: (**Available measurements**) The generated current I_{ti} is measurable at converter $i \in \mathcal{V}$.

Note that we neither require that the voltage V is measurable, nor information on the system parameters. We now suggest a possible control scheme in an ad-hoc manner, and provide the rationale of it later in this section. To this end, consider a distributed controller at node $i \in \mathcal{V}$ of the form

$$T_{\theta i}\dot{\theta}_i = - \sum_{j \in \mathcal{N}_i^{com}} \gamma_{ij}(w_i I_{ti} - w_j I_{tj})$$
$$T_{\phi i}\dot{\phi}_i = -\phi_i + I_{ti} \tag{7}$$
$$u_i = -K_i(I_{ti} - \phi_i) + w_i \sum_{j \in \mathcal{N}_i^{com}} \gamma_{ij}(\theta_i - \theta_j) + V_i^\star.$$

The parameters $T_{\theta i}, T_{\phi i}, K_i \in \mathbb{R}_{>0}$ permit appropriate tuning of the transient response. The set $\mathcal{N}_i^{com}$ is the set of nodes connected to node i via a communication network, with edge weights $\gamma_{ij} = \gamma_{ji} \in \mathbb{R}_{>0}$. Consequently, the controller is distributed as it prescribes the exchange of information on I_t and θ among neighbouring nodes. Similar to the topology of the microgrid, the overall communication network is represented by a connected and undirected graph $\mathcal{G}^{com} = (\mathcal{V}^{com}, \mathcal{E}^{com})$, where $\mathcal{V}^{com} = \mathcal{V}$ and the edges, $\mathcal{E}^{com} = \{1, ..., m_c\}$, represent the communication links between the DGUs. The communication network topology is described by its corresponding incidence matrix $\mathcal{B}^{com} \in \mathbb{R}^{n \times m_c}$, which is defined similarly as $\mathcal{B}$. Then, the overall control scheme can be compactly written for all $i \in \mathcal{V}$ as

$$T_\theta\dot{\theta} = -\mathcal{L}^{com}W I_t \tag{8a}$$
$$T_\phi\dot{\phi} = -\phi + I_t \tag{8b}$$
$$u = -K(I_t - \phi) + W\mathcal{L}^{com}\theta + V^\star, \tag{8c}$$

where $T_\theta, T_\phi, K \in \mathbb{R}^{n \times n}$ are positive definite diagonal matrices. Furthermore, $\mathcal{L}^{com} = \mathcal{B}^{com}\Gamma(\mathcal{B}^{com})^T$ is the (weighted) Laplacian matrix associated to the communication network and $\Gamma \in \mathbb{R}^{m_c \times m_c}$ is a positive definite diagonal matrix describing the weights on the edges. Interconnecting the microgrid (3) with the distributed controller (8) then yields the overall closed-loop system

$$L_t\dot{I}_t = -V - K(I_t - \phi) + W\mathcal{L}^{com}\theta + V^\star$$
$$C_t\dot{V} = I_t + \mathcal{B}I - I_L$$
$$L\dot{I} = -\mathcal{B}^T V - RI \tag{9}$$
$$T_\theta\dot{\theta} = -\mathcal{L}^{com}W I_t$$
$$T_\phi\dot{\phi} = -\phi + I_t.$$

First, we show that a steady state solution to system (9) always exists.

Lemma 1: (**Existence of a steady state solution**) Let Assumptions 1 and 2 hold. There exists a steady state solution $(\overline{I}_t, \overline{V}, \overline{I}, \overline{\theta}, \overline{\phi})$ to system (9), satisfying

$$0 = -\overline{V} - K(\overline{I}_t - \overline{\phi}) + W\mathcal{L}^{com}\overline{\theta} + V^\star \tag{10a}$$

$$0 = \overline{I}_t + \mathcal{B}\overline{I} - I_L \tag{10b}$$

$$0 = -\mathcal{B}^T\overline{V} - R\overline{I} \tag{10c}$$

$$0 = -\mathcal{L}^{com}W\overline{I}_t \tag{10d}$$

$$0 = -\overline{\phi} + \overline{I}_t. \tag{10e}$$

Proof: We have from (10b) that $\mathbb{1}_n^T I_t = \mathbb{1}_n^T I_L$ and from (10d) that $\overline{I}_t = \alpha W^{-1}\mathbb{1}$, with α any scalar. It follows that necessarily

$$\overline{I}_t = \sigma_W^{-1} W^{-1}\mathbb{1}\mathbb{1}^T I_L, \tag{11}$$

with $\sigma_W := \mathbb{1}^T W^{-1}\mathbb{1} \in \mathbb{R}$. Then, (10e) leads to

$$\overline{\phi} = \overline{I}_t. \tag{12}$$

Substituting (11) in (10b) yields

$$\mathcal{B}\overline{I} = \mathcal{L}_W W I_L, \tag{13}$$

where $\mathcal{L}_W := W^{-1} - \sigma_W^{-1} W^{-1}\mathbb{1}\mathbb{1}^T W^{-1}$. If the graph of the microgrid network $\mathcal{G}$ is a tree graph, i.e., $m = n - 1$, then the inverse of the edge–Laplacian matrix $\mathcal{B}^T\mathcal{B}$ exists and the solution for $\overline{I}$ is given by

$$\overline{I} = (\mathcal{B}^T\mathcal{B})^{-1}\mathcal{B}^T\mathcal{L}_W W I_L. \tag{14}$$

In case the network $\mathcal{G}$ is not a tree graph, $m \geq n$, and we rewrite, without loss of generality, $\mathcal{B} := [\mathcal{B}_t, \mathcal{B}_r]$, where $\mathcal{B}_t \in \mathbb{R}^{n \times (n-1)}$ is the incidence matrix of a spanning tree of $\mathcal{G}$, such that a solution of (13) is constructed as

$$\overline{I} = \begin{bmatrix} \overline{I}_T \\ \mathbf{0} \end{bmatrix}, \text{ with } \overline{I}_T = (\mathcal{B}_t^T\mathcal{B}_t)^{-1}\mathcal{B}_t^T\mathcal{L}_W W I_L. \tag{15}$$

Therefore, for any connected graph $\mathcal{G}$, there exists a solution for $\overline{I}$. Pre-multiplying both sides of (10a) by $\mathbb{1}^T W^{-1}$, yields $\mathbb{1}^T W^{-1}V = \mathbb{1}^T W^{-1}V^\star$. Together with (10c), we have consequently that

$$\underbrace{\begin{bmatrix} \mathcal{B}^T \\ \mathbb{1}^T W^{-1} \end{bmatrix}}_{Z}\overline{V} = \begin{bmatrix} -R\overline{I} \\ \mathbb{1}^T W^{-1}V^\star \end{bmatrix}. \tag{16}$$

Since Z is full column rank, the pseudo-inverse $Z^\dagger$ constitutes a left inverse, such that

$$\overline{V} = Z^\dagger \begin{bmatrix} -R\overline{I} \\ \mathbb{1}^T W^{-1}V^\star \end{bmatrix}. \tag{17}$$

By (12), the equation (10a) becomes

$$\mathcal{L}^{com}\overline{\theta} = W^{-1}(\overline{V} - V^*). \tag{18}$$

Consider a nonsingular matrix $\mathcal{U} := \left[1/\sqrt{n}, \tilde{\mathcal{U}} \right]$, where $\tilde{\mathcal{U}} \in \mathbb{R}^{n \times (n-1)}$ is a matrix such that $\mathcal{U}$ is unitary (i.e., $\mathcal{U}^{-1} = \mathcal{U}^T$). We rewrite (18) as

$$\mathcal{U}^T\mathcal{L}^{com}\mathcal{U}\mathcal{U}^T\overline{\theta} = \mathcal{U}^T W^{-1}(\overline{V} - V^\star)$$

$$\Leftrightarrow \begin{bmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \tilde{\mathcal{U}}^T\mathcal{L}^{com}\tilde{\mathcal{U}} \end{bmatrix}\mathcal{U}^T\overline{\theta} = \begin{bmatrix} \frac{1}{\sqrt{n}}\mathbb{1}^T W^{-1}(\overline{V} - V^\star) \\ \tilde{\mathcal{U}}^T W^{-1}(\overline{V} - V^\star) \end{bmatrix}. \tag{19}$$

Bearing in mind that $\mathbb{1}^T W^{-1}(\overline{V} - V^\star) = 0$, this results in

$$\overline{\theta} = \mathcal{U}\begin{bmatrix} \beta \\ (\tilde{\mathcal{U}}^T\mathcal{L}^{com}\tilde{\mathcal{U}})^{-1}\tilde{\mathcal{U}}^T W^{-1}(\overline{V} - V^\star) \end{bmatrix}$$

$$= \beta/\sqrt{n}\mathbb{1} + \tilde{\mathcal{U}}(\tilde{\mathcal{U}}^T\mathcal{L}^{com}\tilde{\mathcal{U}})^{-1}\tilde{\mathcal{U}}^T W^{-1}(\overline{V} - V^\star), \tag{20}$$

where $\beta \in \mathbb{R}$ is arbitrary. In conclusion, given the value of I_L, there exist solutions for the variables $\overline{I}_t$, $\overline{\phi}$, $\overline{I}$, $\overline{V}$ and $\overline{\theta}$ due to the equations (11), (12), (14) or (15), (17) and (20). ∎

In the coming section we show the exponential stability of the microgrid controlled by proposed control scheme. We conclude this section by remarking on the rationale of the proposed control scheme.

Remark 2: (**Rationale behind the controller design**) The consensus dynamics (8a) are frequently used in distributed controller designs aiming at current or power sharing in power networks. Indeed, from the steady state equation (10d), it follows immediately that at steady state (proportional) current sharing is achieved since $W\overline{I}_t \in \mathcal{R}(\mathbb{1})$, with $\mathcal{R}(\mathbb{1})$ denoting the range of $\mathbb{1}$, i.e. all elements of $W\overline{I}_t$ are identical (Objective 1). The dynamics of ϕ given by (8b) in combination with the additional term $-K(I_t - \phi)$ in the controller output (8c) do not alter the steady state of the system, as at steady state $\overline{\phi} = \overline{I}_t$ holds. However, these terms are useful to prevent the occurrence of oscillations. Indeed, in the proof of Thereom 1 in the next section, these terms appear essential to infer that the solutions to the controlled microgrid (9) converge to a constant steady state. Finally, the term $W\mathcal{L}^{com}\theta + V^\star$ in (8c) is added to the controller output (8c) to have a suitable (passive) interconnection with controller state θ, guaranteeing average voltage regulation at steady state. Indeed, notice that after pre-multiplying both sides of (10a) with $\mathbb{1}_n^T W^{-1}$, realizing that $\overline{\phi} = \overline{I}_t$, yields $\mathbb{1}_n^T W^{-1}\overline{V} = \mathbb{1}^T W^{-1}V^\star$ (Objective 2).

V. STABILITY ANALYSIS

In this section we show that all solutions to (9) converge to a steady state, achieving current sharing (Objective 1) and average voltage regulation (Objective 2).

Theorem 1: (**Main result**) Let Assumptions 1 and 2 hold. Consider system (9). The solutions to (9) converge exponentially to a steady steady[¶] $(\overline{I}_t', \overline{V}', \overline{I}', \overline{\theta}', \overline{\phi}')$, achieving current sharing (Objective 1) and voltage balancing (Objective 2).

Proof: Consider the incremental storage function

$$\mathcal{S} = \frac{1}{2}(I_t - \overline{I}_t)^T L_t(I_t - \overline{I}_t) + \frac{1}{2}(V - \overline{V})^T C_t(V - \overline{V})$$

$$+ \frac{1}{2}(I - \overline{I})^T L(I - \overline{I}) + \frac{1}{2}(\theta - \overline{\theta})^T T_\theta(\theta - \overline{\theta})$$

$$+ \frac{1}{2}(\phi - \overline{\phi})^T T_\phi K(\phi - \overline{\phi}), \tag{21}$$

where $(\overline{I}_t, \overline{V}, \overline{I}, \overline{\theta}, \overline{\phi})$ is a steady state solution of (9), satisfying (10)[‖]. It is immediate to see that $\mathcal{S}$ is radially unbounded and that $\mathcal{S}$ attains a minimum at $(\overline{I}_t, \overline{V}, \overline{I}, \overline{\theta}, \overline{\phi})$. Furthermore, a straightforward calculation[**] shows that $\mathcal{S}$

[¶]Following [22], this is equivalent to the system (9) being semi-stable. Consequently, the system is also Lyapunov stable [22, Proposition 1].

[‖]Note that $(\overline{I}_t, \overline{V}, \overline{I}, \overline{\theta}, \overline{\phi})$ exists according to Lemma 1 and that possibly $(\overline{I}_t, \overline{V}, \overline{I}, \overline{\theta}, \overline{\phi}) \neq (\overline{I}_t', \overline{V}', \overline{I}', \overline{\theta}', \overline{\phi}')$.

[**]We used that $-(I_t - \overline{I}_t)^T K(I_t - \overline{I}_t) + 2(I_t - \overline{I}_t)^T K(\phi - \overline{\phi}) - (\phi - \overline{\phi})^T K(\phi - \overline{\phi}) = -(I_t - \phi)^T K(I_t - \phi)$.

satisfies $\dot{\mathcal{S}} = -(I - \overline{I})^T R(I - \overline{I}) - (I_t - \phi)^T K(I_t - \phi) \leq 0$, along the solutions to (9). As an intermediate result, we can conclude that all solutions to system (9) are bounded. According to LaSalle's invariance principle, the solutions to (9) approach the largest invariant set contained entirely in the set $\Upsilon = \{I_t, V, I, \theta, \phi : I = \overline{I}, I_t = \phi\}$, such that on this set I is a constant, i.e. $I = \overline{I} := \overline{I}'$. Furthermore, on this set Υ, system (9) satisfies

$$L_t \dot{I}_t = -V + W\mathcal{L}^{com}\theta + V^\star \tag{22a}$$

$$C_t \dot{V} = I_t + \mathcal{B}\overline{I} - I_L \tag{22b}$$

$$\mathbf{0} = -\mathcal{B}^T V - R\overline{I} \tag{22c}$$

$$T_\theta \dot{\theta} = -\mathcal{L}^{com} W I_t \tag{22d}$$

$$T_\phi \dot{\phi} = \mathbf{0}. \tag{22e}$$

From (22e) it follows that on the largest invariant set ϕ is a constant, i.e. $\phi := \overline{\phi}'$. Since in the set Υ it holds that $I_t = \phi$, we have that on the largest invariant set also I_t is constant, i.e. $I_t = \phi = \overline{\phi}' := \overline{I}'_t$. Therefore, the right hand side of (22d) is a constant vector, such that θ grows unbounded if $\dot{\theta} \neq \mathbf{0}$, contradicting the previously established boundedness of the solutions to system (9). Therefore, on the largest invariant set θ is constant, i.e. $\theta := \overline{\theta}'$. From $\mathbf{0} = -\mathcal{L}^{com} W\overline{I}'_t$, it then follows that current sharing is achieved (Objective 1). Pre-multiplying both sides of (22a) by $\mathbb{1}^T W^{-1}$, and realizing that on the largest invariant set $\dot{I}_t = \mathbf{0}$, yields $\mathbb{1}^T W^{-1} V = \mathbb{1}^T W^{-1} V^\star$. Together with (22c), we have consequently that on the invariant set

$$\underbrace{\begin{bmatrix} \mathcal{B}^T \\ \mathbb{1}^T W \end{bmatrix}}_{Z} V = \begin{bmatrix} -R\overline{I} \\ \mathbb{1}^T W^{-1} V^\star \end{bmatrix}. \tag{23}$$

Since Z is full column rank, the pseudo-inverse $Z^\dagger$ constitutes a left inverse, such that on the largest invariant set

$$V = Z^\dagger \begin{bmatrix} -R\overline{I} \\ \mathbb{1}^T W^{-1} V^\star \end{bmatrix} := \overline{V}'. \tag{24}$$

Since on the largest invariant set $V = \overline{V}'$ is a constant, satisfying $\mathbb{1}^T W^{-1} \overline{V}' = \mathbb{1}^T W^{-1} V^\star$, also voltage balancing (Objective 2) is achieved. We can conclude that the solutions to (9) converge to a steady state $(\overline{I}'_t, \overline{V}', \overline{I}', \overline{\theta}', \overline{\phi}')$, achieving current sharing (Objective 1) and voltage balancing (Objective 2). Furthermore, since the system is linear the convergence is exponential. ■

Remark 3: (**Plug-and-Play**) The main results in this work assume a constant network topology. Nevertheless, an interesting extension is to consider the plugging in or out of various converters. The analysis of the corresponding switched/hybrid system is outside the scope of this work. However, since the convergence result of Theorem 1 holds globally, independent of the system parameters, and since the controller at the i-th node requires only measurements of the generated current I_{ti} and information from nodes connected via a communication network, the proposed solution is expected to scale well and suitable for Plug-and-Play operation.
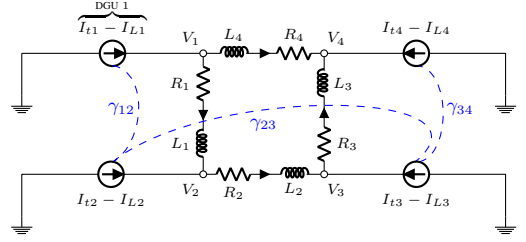

Fig. 2. Scheme of the considered (Kron reduced) microgrid with 4 power converters. The dashed lines represent the communication network.

VI. SIMULATION RESULTS

In this section, the proposed distributed consensus algorithm is assessed in simulation. We consider a microgrid composed of 4 DGUs interconnected as shown in Figure 2, where also the communication network is depicted. For the parameters of the DGUs and the lines we refer to [21, Tables II, III]. The weights associated with the edges of the communication graph are $\gamma_{12} = \gamma_{23} = \gamma_{34} = 1 \times 10^2$. In the controller (8), we have selected $T_\theta = \mathbb{I}_4$, $T_\phi = 1 \times 10^{-2} \mathbb{I}_4$ and $K = 0.5\mathbb{I}_4$, $\mathbb{I}_4 \in \mathbb{R}^{4\times4}$ being the identity matrix. The system is initially at a steady state with current demand $I_L(0) = [30, 15, 30, 26]^T$ A. Then, consider a current demand variation $\Delta I_L = [10, 7, -10, 5]^T$ A at the time instant $t = 1$ s. The PCC voltages and the generated currents are illustrated in Figure 3. One can appreciate that the steady state weighted average of the PCC voltages (denoted by V_{av}) is equal to the weighted average of the corresponding references (see Objective 2), and that the current generated by each DGU converges to the desired value, achieving proportional current sharing (see Objective 1). Moreover, we report in Figure 3 the time evolution of the voltage at the PCC of DGU 3 (similar behaviours are present at all other DGUs) when the control parameter K is set to zero. As discussed in Remark 2, in this case the voltage response is characterized by the presence of high frequency oscillations during the transient. For the sake of comparison, we also report in Figure 4 the results obtained by implementing the sliding mode (SM) controller proposed in [21]. One can observe that the behaviour of the generated currents is similar, while the voltage transient is improved when the SM controller is used. This is reasonable considering that the proposed controller in this work does not require voltage measurements, while the SM controller uses not only the voltage, but also the estimates of its first and second time derivative, resulting in possible robustness issues in the presence of measurement noise.

VII. CONCLUSIONS AND FUTURE RESEARCH

In this letter, a distributed control scheme is proposed for proportional current sharing and average voltage regulation in DC microgrids with unknown current loads. The suggested controller only requires measurements of generated currents, without knowledge on the microgrid parameters. The controlled microgrid is proven to converge globally to a desired steady state, independently of the initial conditions of the physical system and the controller state, facilitating Plug-and-
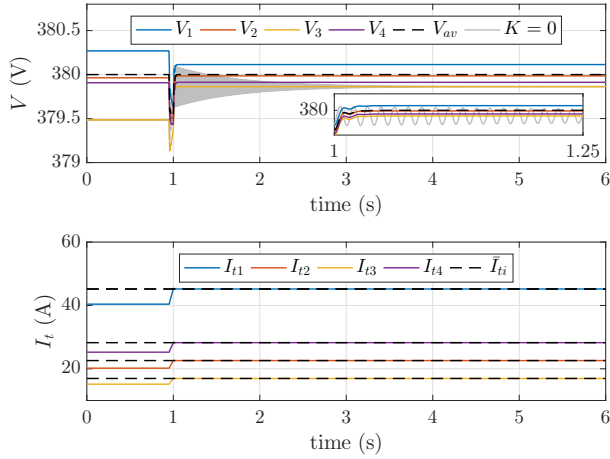
Fig. 3. (Proposed controller) From the top: voltage at the PCC of each DGU together with the weighted average value (dashed line) and the voltage at the PCC of DGU 3 when $K = 0$ (grey line); generated currents together with the corresponding values (dashed lines) that correspond to (proportional) current sharing for $t > 1$.
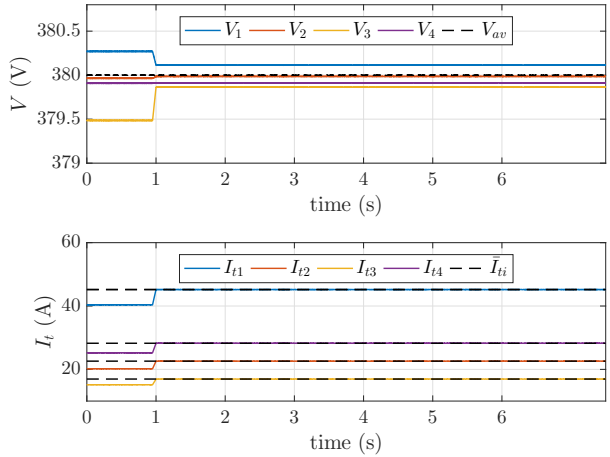

Fig. 4. (Controller proposed in [21]) From the top: voltage at the PCC of each DGU together with the weighted average value (dashed line); generated currents together with the corresponding values (dashed lines) that correspond to (proportional) current sharing for $t > 1$.

Play capabilities. Interesting future research includes extensions towards incorporating power sharing and nonlinear (ZIP) loads. Furthermore, allowing for different converter types, such as boost converters [8], [9], is a meaningful future endeavor.

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