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22. February 2014

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ABSTRACT

This paper introduces a new model called the buffered autoregressive model with generalized autoregressive conditional heteroskedasticity (BAR-GARCH). The proposed model, as an extension of the BAR model in Li et al. (2013), can capture the buffering phenomenon of time series in both conditional mean and conditional variance. Thus, it provides us a new way to study the nonlinearity of a time series. Compared with the existing AR-GARCH and threshold AR-GARCH models, an application to several exchange rates highlights an interesting interpretation of the buffer zone determined by the fitted BAR-GARCH models.

Some key words: Buffered AR model; Buffered AR-GARCH model; Exchange rate; GARCH model; Nonlinear time series; Threshold AR model.

1. INTRODUCTION

After the seminal work of Tong (1978), the class of threshold autoregressive (TAR) models has achieved a great success in practice; see, e.g., Tong (1990) for earlier works and Tong (2011) and the references therein for more recent ones. The TAR model with single threshold r basically says that the structure of an AR model shifts from one regime to another, if the status of the threshold variable z_t crosses above or below the threshold r. Due to this piecewise linear character, the TAR model is able to mimic many nonlinear features in time series, such as resonance, limit cycles, time-irreversibility, and many others; see, e.g., Li and Lam (1995), Pesaran and Potter (1997), and Hansen (2011) for an overview. However, in the two-regime situation, there are some cases that the two-way switching of regimes may not happen at the same threshold. A typical example is the hoisting of a typhoon warning signal mentioned in Li et al. (2013). It is observed that a typhoon warning signal is hoisted when the average wind speed up-crosses a certain threshold r_U, but it may only be canceled when the average wind speed down-crosses another threshold r_L which is smaller than r_U. In view of this, it is reasonable to treat the zone $(r_L, r_U]$ as a buffer zone, in which the weather station keeps the warning signal unchanged when the average wind speed (a threshold variable in this case) lies in the buffer zone.

In order to describe such buffering phenomenon in the real world, Li et al. (2013) first investigated a buffered AR (BAR) model. Unlike the TAR model, this BAR model says that the structure of an AR model remains unchanged when the value of z_t lies inside the buffer zone $(r_L, r_U]$. Thus, the BAR model provides us a new way to study nonlinearity in time series. As expected, the buffering phenomenon not only exists in meteorology but also in finance and economics. A nice illustrative example is the study of the quarterly U.S. GNP data set in Li et al. (2013) and Zhu et al. (2013). Their empirical studies showed that the BAR model provides a better fit than TAR model for the GNP series. Based on the fitted BAR model, they discovered an interval of GNP which does not shift in terms of the probabilistic structure unless we have experienced a big 'contraction' or 'expansion' two years before. This finding is clearly of practical interest and can not be detected by the TAR model.

The above-mentioned BAR model focuses on the buffering phenomenon that is present in the conditional mean. A natural but important extension is to consider the buffering phenomenon in both the conditional mean and variance or in the conditional variance only. It is well known that many financial and economic time series exhibit conditional heteroscedasticity, and they can generally be fitted by the generalized autoregressive conditional heteroskedasticity (GARCH) models proposed by Engle (1982) and Bollerslev (1986). Moreover, many variants of the GARCH model have since been developed to capture the asymmetry and leverage effects in the market volatility. Among them, the threshold GARCH (TGARCH) model proposed by Rabemanjara and Zakoïan (1993) and Zakoïan (1994) emphasizes that the structures of volatilities are different for positive and negative return series. In their settings, the threshold r is fixed at zero. Generally, we are interested in cases where the threshold is unknown. Some pioneer works in this context can be found in Li and Li (1996) and Liu et al. (1997), in which the double threshold AR-ARCH and AR-GARCH models were proposed respectively to capture the piecewise linear conditional mean and variance. Motivated by the BAR model, this paper introduces a new model called the buffered AR-GARCH (BAR-GARCH) model. The new model includes the AR-GARCH, double threshold AR-ARCH, threshold AR-GARCH, or TGARCH model as special cases, and can capture the buffering phenomenon of time series in both conditional mean and variance. To highlight the importance of the BAR-GARCH model, we apply it to several exchange rate series. In most of these cases, the BAR-GARCH model gives a better fit than the AR-GARCH or TAR-GARCH model. Therefore, the participants in these exchange rate markets should not ignore this buffering phenomenon in the conditional mean/variance or both of them.

This paper is organized as follows. Sections 2 and 3 review the TAR and BAR models, respectively. Section 4 proposes our new BAR-GARCH model, with a specified estimation and model-selection procedure. The application of the BAR-GARCH model to several exchange rate series is presented in Section 5. Section 6 concludes with some suggested future work.

2. THE CLASSIC THRESHOLD AUTOREGRESSIVE MODEL

Let $\{y_t\}$ be a stationary time series. A p-th order threshold autoregressive [TAR(p)] model for $\{y_t\}$ can be defined as

$$y_t = \begin{cases} \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t, & \text{if } R_t = 1, \\ \psi_0 + \sum_{i=1}^{p} \psi_i y_{t-i} + \varepsilon_t, & \text{if } R_t = 0, \end{cases} \tag{1}$$

where $R_t = I(y_{t-d} \leq r)$ is the regime indicator of y_t, r is the threshold parameter, $d(\geq 1)$ is the delay parameter, $I(\cdot)$ is the indicator function, and ε_t is an uncorrelated error term with zero mean and variance $\sigma^2(> 0)$. Under such a model, there are two separate regimes for y_t, and the regime of y_t will shift when the value of y_{t-d} crosses above or below the threshold r. This model allows for a non-linear behavior of y_t, since the generating process of y_t is different in each regime. For example, if y_t is the daily return on a certain financial asset, model (1) with $d = 1$ and $r = 0$ describes that the structure of y_t today depends on whether the price rose or fell on the previous day. This kind of asymmetric phenomenon has been well documented in Li and Lam (1995); see also Tsay (1989, 1998) and Tong (2011) for other empirical studies based on TAR models. Moreover, the TAR model can be treated as a special case of the threshold ARMA (TARMA) model, which also nests TMA model as another special case. A sophisticated study of TMA or TARMA model can be found in Ling and Tong (2005), Li and Li (2008), Li et al. (2011) and Li et al. (2013).

Since the non-linearity of y_t happens only when there exists a threshold structure, it is important for us to detect the existence of a threshold structure. The related works are the Likelihood ratio (LR) test in Chan (1990, 1991), Li and Li (2011) and Zhu and Ling (2012), and the Wald test and Lagrange multiplier (LM) test in Hansen (1996). Once the null hypothesis of no threshold structure is rejected, the least squares estimation procedure in Li and Ling (2012) can be used to estimate model (1).

3. THE BUFFERED AUTOREGRESSIVE MODEL

In view of model (1), the regime of y_t shifts immediately no matter which direction y_{t-d} crosses (above or below) the threshold r . However, this may not always be the case. Li et al. (2013) gave several empirical examples to show that the switch in regime may be delayed when y_{t-d} lies in a buffer zone. To capture this new non-linear feature of time series, they proposed a p-th order buffered autoregressive [BAR(p)] model with R_t in (1) satisfying

$$R_t = \begin{cases} 1 & \text{if } y_{t-d} \leq r_L \\ 0 & \text{if } y_{t-d} > r_U \\ R_{t-1} & \text{otherwise} \end{cases}, \tag{2}$$

where r_L and r_U are two threshold parameters such that $r_L \leq r_U$. This BAR(p) model stipulates that the regime of y_t is unchanged when y_{t-d} falls into the buffer zone $(r_L, r_U]$. When $r_L = r_U$, the BAR(p) model reduces to the TAR(p) model. As shown in Li et al. (2013), a sufficient condition for the geometrical ergodicity of the BAR(p) model is

$$\sum_{i=1}^{p} |\phi_i| < 1 \quad \text{and} \quad \sum_{i=1}^{p} |\psi_i| < 1,$$

provided that ε_t has a density function that is positive everywhere on $\mathbb{R}$ and $E|\varepsilon_t| < \infty$.

Although both TAR(p) and BAR(p) models have two regimes, the regime indicator in BAR(p) model depends on past observations infinitely far away. This can be illustrated by the fact that

$$R_t(\gamma) = I(y_{t-d} \le r_L) + \sum_{j=1}^{\infty} I(y_{t-j-d} \le r_L) \prod_{i=1}^{j} I(r_L < y_{t-i+1-d} \le r_U) \text{ a.s.}$$

in BAR(p) models, where $\gamma = (r_L, r_U)'$. Thus, unlike TAR(p) model, the states of all past observations $\{y_j; j \le t-d\}$ have impact on determining the regime of the current observation y_t in BAR(p) models. However, given a finite realizations of y_t, the regimes of the first few observations may not be well identified, since no observations exist for $t \le 0$ in practice. To circumvent this problem, it is natural to assume that $[r_L, r_U]$ is a subset of $[a, b]$, where a, b are set to some empirical quantiles of the data sample $\{y_t\}_{t=1}^n$ as in Chan (1991) and Andrews (1993). By doing so, we can always find a smallest integer $n_0 (\ge p)$ such that y_{n_0-d} stays outside the region $[a, b]$. Hence, the regime indicator $R_{n_0}(\gamma)$ is well identified, and the regime indicators for observations $\{y_t\}_{t=n_0+1}^n$ can be iteratively calculated by

$$R_t(\gamma) = I(y_{t-d} \le r_L) + R_{t-1}(\gamma) I(r_L < y_{t-d} \le r_U).$$

For the remaining observations $\{y_t\}_{t=1}^{n_0-1}$ whose regimes are not well identified, we then set their regime indicators to be zeros. Thus, we should use $\tilde{R}_t(\gamma)$ rather than $R_t(\gamma)$ in practice, where

$$\tilde{R}_t(\gamma) = \begin{cases} 0 & \text{for } t = 1, \cdots, n_0 - 1, \\ R_t(\gamma) & \text{for } t = n_0, \cdots, N. \end{cases} \tag{3}$$

Although the first $(n_0 - 1)$ regime indicators are artificially chosen in (3), efficiency loss is negligible because Zhu et al. (2013) showed that n_0, an integer depending on $\{y_t\}_{t=1}^n$, is bounded in probability.

Even though the BAR(p) model has two thresholds, it has only two regimes. If we treat the buffer zone as the “middle regime”, the BAR(p) model is similar to the classical three-regime TAR(p) model, but the parameters in its “middle regime” inherit their values from the two outer regimes. Hence, the buffering phenomenon captured by the BAR(p) model can not be handled by the three-regime TAR(p) model. Finally, it is worth noting that the non-linearity of y_t in BAR(p) model exists only when the threshold variables r_L and r_U are present. Like the TAR(p) model, it is of interest to detect the existence of these two thresholds. Under certain conditions, the LR test in Zhu et al. (2013) is applicable. Once the BAR(p) model is preferable, the least squares estimation procedure in Li et al. (2013) can be used to estimate this model.

4. THE BUFFERED AR-GARCH MODEL

Conditional heteroscedasticity is a key character in most of economic and financial real data. Basically, it says that the conditional variance of the data is changing over time. So far, this phenomenon has been well modelled by the ARCH model in Engle (1982) and its huge variants; see, e.g., Bollerslev et al. (1992) and Francq and Zakoïan (2010). In many applications, a conditional mean model along with an ARCH-type conditional variance model is necessary to fit the real data (see, e.g., Tsay (2005)). Bollerslev et al. (1992) showed that ignoring the ARCH effect would lead to inefficient estimates and suboptimal statistical inference in the conditional mean model. In view of this, it is of interest to consider the following buffered AR(p)-GARCH(m, s) (BAR-GARCH) model:

$$y_t = \begin{cases} \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t, & \text{if } R_t = 1, \\ \psi_0 + \sum_{i=1}^{p} \psi_i y_{t-i} + \varepsilon_t, & \text{if } R_t = 0, \end{cases} \tag{4}$$

where R_t is defined as in (2), and

$$\varepsilon_t = \sqrt{h_t}\eta_t \quad \text{with} \quad h_t = \begin{cases} \alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{s} \beta_i h_{t-i}, & \text{if } R_t = 1, \\ \pi_0 + \sum_{i=1}^{m} \pi_i \varepsilon_{t-i}^2 + \sum_{i=1}^{s} \delta_i h_{t-i}, & \text{if } R_t = 0, \end{cases} \tag{5}$$

and η_t is a sequence of iid random variables with mean zero and variance one. Here, we assume that $\alpha_0, \pi_0 > 0$ and other $\alpha_i, \beta_i, \pi_i, \delta_i \geq 0$ for the positivity of h_t. Particularly, when $r_L = r_U$, we call models (4)-(5) the threshold AR-GARCH (TAR-GARCH) model which includes the double threshold AR-ARCH model in Li and Li (1996) and Wong and Li (1997, 2000) as a special case. When both r_L and r_U are absent, model (4)-(5) becomes the classical AR-GARCH model. Also, when $p = 0$, we have a buffered GARCH model which can be viewed as a natural extension of the threshold GARCH models in Liu, et al. (1997) and Brooks (2001). Thus, when two threshold variables r_L and r_U in R_t are present and different, the BAR-GARCH model captures the buffering phenomenon of y_t in terms of both conditional mean and variance. Note that our BAR-GARCH model assumes that both conditional mean and variance models switch regime at the same time. This assumption seems to be the most likely situation in practice, although allowing for different regime-switching rules in the conditional mean and variance models could easily be done.

Next, we consider the estimation for the BAR-GARCH model. Let $\theta = (\theta_1', \theta_2')'$ with $\theta_1 = (\phi', \alpha', \beta')'$, $\theta_2 = (\psi', \pi', \delta')'$, $\phi = (\phi_0, \cdots, \phi_p)'$, $\psi = (\psi_0, \cdots, \psi_p)'$, $\alpha = (\alpha_0, \cdots, \alpha_m)'$, $\beta = (\beta_1, \cdots, \beta_s)'$, $\pi = (\pi_0, \cdots, \pi_m)'$ and $\delta = (\delta_1, \cdots, \delta_s)'$. Given n observations of y_t, -2 times quasi-log-likelihood of models (4)-(5) (ignoring some constants) can be written as

$$L_n(\theta) = \sum_{t=1}^{n} \left[\log h_t(\theta, \gamma) + \frac{\varepsilon_t^2(\theta, \gamma)}{h_t(\theta, \gamma)} \right], \tag{6}$$

where $\varepsilon_t(\theta, \gamma)$ and $h_t(\theta, \gamma)$ are iteratively calculated as

$$\begin{aligned}
\varepsilon_t(\theta, \gamma) &= \left[y_t - \phi_0 - \sum_{i=1}^{p} \phi_i y_{t-i} \right] \tilde{R}_t(\gamma) + \left[y_t - \psi_0 - \sum_{i=1}^{p} \psi_i y_{t-i} \right] (1 - \tilde{R}_t(\gamma)), \\
h_t(\theta, \gamma) &= \left[\alpha_0 + \sum_{i=1}^{m} \alpha_i \varepsilon_{t-i}^2(\theta, \gamma) + \sum_{i=1}^{s} \beta_i h_{t-i}(\theta, \gamma) \right] \tilde{R}_t(\gamma) \\
&\quad + \left[\pi_0 + \sum_{i=1}^{m} \pi_i \varepsilon_{t-i}^2(\theta, \gamma) + \sum_{i=1}^{s} \delta_i h_{t-i}(\theta, \gamma) \right] (1 - \tilde{R}_t(\gamma))
\end{aligned}$$

with $\tilde{R}_t(\gamma)$ being defined as in (3) and the initial values $Y_0 \equiv \{y_i; i \leq 0\}$ being zeros.

Given (p, m, s, d, γ), the quasi-maximum-likelihood estimator $\hat{\theta}$ of θ is obtained by minimizing $L_n(\theta)$ with the Newton-Raphson method. Then, the estimation of d and γ is employed by considering

$$\min_{(d,\gamma) \in A_d \times A_\gamma} L_n(\hat{\theta}),$$

where $A_d = \{1, \cdots, D\}$ and $A_\gamma = \{(r_L, r_U); a \leq r_L \leq r_U \leq b\}$ for a user-chosen integer D and two user-chosen real numbers a and b. The space $A_d \times A_\gamma$ contains all potential candidates for (d, γ). Furthermore, the estimation of p, m and s is performed by considering

$$\min_{(p,m,s) \in A_p \times A_m \times A_s} AIC(p, m, s),$$

where $AIC(p, m, s) =: L_n(\hat{\theta}) + 2(p + m + s + 1)$ is the Akaike information criterion (AIC), and $A_p \times A_m \times A_s =: \{0, \cdots, P\} \times \{0, \cdots, M\} \times \{0, \cdots, S\}$ for some user-chosen integers P, M and S, contains all potential candidates for (p, m, s).

Although the aforementioned estimation procedure of θ and (p, m, s, d, γ) is efficient, it will be very time-consuming. A simple but less efficient way is to first select an efficient TAR-GARCH model (i.e., $r_L = r_U$), and then use these estimators for (p, d, m, s) in the TAR-GARCH model to estimate the parameter θ in the BAR-GARCH model. The detailed steps are as follows:

1. Apply the foregoing estimation procedure in model (4)-(5) with $r_L = r_U = r_0$ to get the estimators $(\hat{p}, \hat{m}, \hat{s}, \hat{d}, \hat{r}_0, \hat{r}_0)$ for (p, m, s, d, r_L, r_U).

2. Based on $(\hat{p}, \hat{m}, \hat{s}, \hat{d}, \hat{r}_0)$ from step 1 and each $(r_L, r_U) \in [a, \hat{r}_0] \times [\hat{r}_0, b]$, calculate the estimator $\hat{\theta}_{(r_L.r_U)}$ for θ by minimizing $L_n(\theta)$ in (6) with the Newton-Raphson method.

3. Obtain the estimators $(\hat{r}_L, \hat{r}_U)$ for (r_L, r_U) and estimator $\hat{\theta}_{(\hat{r}_L, \hat{r}_U)}$ for θ via

$$(\hat{r}_L, \hat{r}_U) := \min_{(r_L, r_U) \in [a, \hat{r}_0] \times [\hat{r}_0, b]} L_n(\hat{\theta}_{(r_L, r_U)}).$$

Note that for the AR-GARCH model, the aforementioned procedure is not needed. It is because we do not need to estimate the threshold and delay parameters in this model. Thus, the AR-GARCH model can be estimated efficiently. Also, it is worth noting that although the estimation procedure in steps 1-3 is sub-efficient for the BAR-GARCH model, the TAR-GARCH model estimated from step 1 is nested within the fitted BAR-GARCH model obtained from steps 1-3. Hence, statistical inference about comparing TAR and BAR models in terms of their log-likelihood values becomes standard.

5. APPLICATION TO EXCHANGE RATES

In this section, we apply the BAR-GARCH model to fit several exchange rate series. Our main objective is to check whether the BAR-GARCH gives a better fit to the data sets than the AR-GARCH or TAR-GARCH model. If this is the case, practitioners in the exchange rate markets can get more insight from the BAR-GARCH model, which demonstrates that the asymmetric property, caused by a novel buffered mechanism, should not be ignored.

The exchange rate series we studied are the six daily currencies against the U.S. dollar, the Argentine Peso(USD/ARS), Bulgarian Lev(USD/BGN), Philippine Peso(USD/PHP), Polish Zloty(USD/PLN), Russian Ruble(USD/RUB) and Taiwan Dollar(USD/TWD), over the period from January 1, 2006 to December 31, 2011. They are mostly currencies from developing countries, one from Latin America, two from Asia and three from Eastern Europe. Each series has a total of 2191 observations. The log-return ($\times$100) y_t of each series is plotted in Figure 1. A simple visual inspection of the sample autocorrelation plots of y_t and y_t^2 (not reported here) implies that all return series are highly correlated with possible ARCH effect. Thus, it is natural to consider fitting AR-GARCH, TAR-GARCH and BAR-GARCH models for these exchange rates. We employ the estimation procedure stated in Section 4. The maximum orders (P, M, S, D) for these models are set to be $(8, 1, 1, 8)$, which is generally enough for selecting the best model in most financial applications. The value of a (or b) is set to be the 10th (or 90th) percentile of the data.

Table 1 reports the estimates for (p, m, s, d, γ) of the best fitted AR-GARCH, TAR-GARCH and BAR-GARCH models for all six exchange rate return series, together with the corresponding values of -2 times the maximized log-likelihood function (LLF), AIC, BIC, and AICc, where LLF is the $L_n(\hat{\theta})$ defined in (6), and

$$\begin{aligned} AIC &= L_n(\hat{\theta}) + 2k, \\ BIC &= L_n(\hat{\theta}) + k\log(n), \\ AICc &= L_n(\hat{\theta}) + 2k(k+1)/(n-k-1). \end{aligned}$$

Here, k is the number of estimated parameters and n is the sample size. Meanwhile, it is worth mentioning that all best fitted models are adequate by looking at the the ACF and PACF plots (not depicted here) of the residuals and squared residuals. From Table 1, we find that the TAR-GARCH model nests the AR-GARCH model in all cases. Thus, it is meaningful to use the LR test statistic

$$\text{LR}_{1n} := \text{LLF}_{\text{AR-GARCH}} - \text{LLF}_{\text{TAR-GARCH}}$$

to test for no threshold structure (i.e., the null is H_{10}: the AR-GARCH model and the alternative is H_{11}: the TAR-GARCH model). Conventionally, the p-value of LR_{1n} is $p_1 = 1 - F_{k_1}(\text{LR}_{1n})$ with $k_1 = p + m + s + 4$, where $F_k(\cdot)$ is the cdf of a chi-square distribution with degrees of freedom k. Similarly, we can use the following LR test statistic

$$\text{LR}_{2n} := \text{LLF}_{\text{TAR-GARCH}} - \text{LLF}_{\text{BAR-GARCH}}$$

to test for no buffer zone (i.e., the null is H_{20}: the TAR-GARCH model and the alternative is H_{21}: the BAR-GARCH model), and its p-value is $p_2 = 1 - F_1(\text{LR}_{2n})$. The values of p_1 and p_2 for all return series are also given in Table 1, from which we can see that at the 5% level of significance, the TAR-GARCH model is always superior to the AR-GARCH in all cases while the BAR-GARCH model is superior to the TAR-GARCH model except the case of USD/PHP.

To look for further evidence, Table 1 also reports the values of n_L, n_B and n_U, where n_B, n_L and n_U are the number of observations in the buffer zone (i.e., $r_L < y_{t-d} \leq r_U$), the lower regime outside the buffer zone (called lower outer regime) (i.e., $y_{t-d} \leq r_L$), and the upper regime outside the buffer zone (called upper outer regime) (i.e., $y_{t-d} > r_U$), respectively. Clearly, n_B is zero for all TAR-GARCH models, and n_B for a fitted BAR-GARCH model can be decomposed into n_{BL} and n_{BU}, which are the number of observations in the buffer zone belonging to the lower and upper regimes, respectively. For the TAR-GARCH models, neither n_L nor n_U is small, indicating that the threshold effect tends to exist, as also evidenced by significant testing results based on LR_{1n}. For the BAR-GARCH model, most exchange rates except USD/PHP generally have a large buffer zone, particularly for USD/PLN, USD/RUB and USD/TWD. This implies that the buffer zone exists in most exchange rates except USD/PHP, and this result is consistent with the one based on the likelihood ratio test statistic LR_{2n}. Suppose for instance, the USD/RUB series is now in the lower regime. According to its fitted BAR-GARCH model, it will move to the upper regime whenever its previous day's log-return exceeds 0.4202% and then return back to the lower regime only when its previous day's log-return drops below -0.3381%. However, the TAR-GARCH model will wrongly assign it to the lower regime once its previous day's log-return drops below 0.4202%.

Figures 2-7 plot the time points of each return series belonging to lower outer regime ($i.e., y_{t-d} \leq r_L$), lower buffer regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 1$), upper buffer regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 0$), and upper outer regime ($i.e., y_{t-d} > r_U$) based on the best fitted BAR-GARCH model. It can be seen that the USD/PLN, USD/RUB and USD/TWD return series stayed in the buffer zone (lower or upper buffer regimes) most of the time before the financial crisis in 2008 but they switched regimes and moved outside the buffer zone more frequently during and after the crisis. This finding reveals that the buffer zone reflects a normal range of exchange rate variation under normal market condition, and the market intervention by the central bank will be invoked only when the exchange rate move outside the buffer zone.

As the hypothesis of no buffer zone is not rejected for USD/PHP, it is natural to see that USD/PHP had very few observations located inside the buffer zone throughout the whole time period. Noted from Figure 3 that USD/BGN stayed in the lower outer regime (return less than 0.2249%) most of the time, particular in 2006-2007. Once it reached the upper outer regime (return greater than 0.5048%), it would often bounce back to the lower outer regime. This indicates that relative to the Bulgarian Lev, the U.S. dollar was weakening in 2006-2007.

Table 1. *Model selection results for all six exchange rate return series.*

	Log-return Series ($\times 100$)					
	USD/ARS			USD/BGN		
	AR-GARCH	TAR-GARCH	BAR-GARCH	AR-GARCH	TAR-GARCH	BAR-GARCH
Orders	$p=2$	$p=2$ $d=5$	$p=2$ $d=5$	$p=4$	$p=4$ $d=7$	$p=4$ $d=7$
	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$
Thresholds		$r=-0.0761$	$r_L=-0.0761$		$r=0.5048$	$r_L=0.2249$
			$r_U=-0.0025$			$r_U=0.5048$
LLF†	-4828.0	-4914.3	-4919.2	-1476.3	-1512.0	-1519.0
AIC	-4816.0	-4886.3	-4889.2	-1460.3	-1476.0	-1481.0
BIC	-4781.9	-4806.6	-4803.8	-1414.8	-1373.6	-1372.9
AICc	-4828.0	-4914.1	-4919.0	-1476.3	-1511.7	-1518.7
LR tests‡	$p_1=0.0000$	$p_2=0.0269$		$p_1=0.0001$	$p_2=0.0082$	
#observations		$n_L=523$	$n_L=523$		$n_L=1949$	$n_L=1665$
		$n_U=1667$	$n_{BL}=114$		$n_U=241$	$n_{BL}=229$
			$n_{BU}=325$			$n_{BU}=55$
			$n_U=1228$			$n_U=241$
	USD/PHP			USD/PLN		
	AR-GARCH	TAR-GARCH	BAR-GARCH	AR-GARCH	TAR-GARCH	BAR-GARCH
Orders	$p=3$	$p=3$ $d=5$	$p=3$ $d=5$	$p=4$	$p=4$ $d=1$	$p=4$ $d=1$
	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$
Thresholds		$r=0.2129$	$r_L=0.1700$		$r=0.4305$	$r_L=-0.5396$
			$r_U=0.2129$			$r_U=0.7347$
LLF	-2648.4	-2683.2	-2683.6	390.9	359.7	351.3
AIC	-2634.4	-2651.2	-2649.6	406.9	395.7	389.3
BIC	-2594.5	-2560.1	-2552.9	452.4	498.1	497.5
AICc	-2648.3	-2682.9	-2683.3	391.0	360.0	351.7
LR tests	$p_1=0.0001$	$p_2=0.5271$		$p_1=0.0005$	$p_2=0.0038$	
#observations		$n_L=1753$	$n_L=1665$		$n_L=1751$	$n_L=394$
		$n_U=437$	$n_{BL}=74$		$n_U=439$	$n_{BL}=988$
			$n_{BU}=14$			$n_{BU}=544$
			$n_U=437$			$n_U=264$
	USD/RUB			USD/TWD		
	AR-GARCH	TAR-GARCH	BAR-GARCH	AR-GARCH	TAR-GARCH	BAR-GARCH
Orders	$p=1$	$p=1$ $d=1$	$p=1$ $d=1$	$p=5$	$p=5$ $d=7$	$p=5$ $d=7$
	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$	$m=1$ $s=1$
Thresholds		$r=0.4202$	$r_L=-0.3381$		$r=0.1804$	$r_L=-0.2542$
			$r_U=0.4202$			$r_U=0.2679$
LLF	-2834.1	-2887.8	-2901.7	-3771.6	-3841.6	-3874.6
AIC	-2824.1	-2863.8	-2875.7	-3753.6	-3801.6	-3832.6
BIC	-2795.6	-2795.5	-2801.7	-3702.3	-3687.8	-3713.0
AICc	-2834.1	-2887.6	-2901.5	-3771.5	-3841.2	-3874.1
LR tests	$p_1=0.0000$	$p_2=0.0002$		$p_1=0.0000$	$p_2=0.0000$	
#observations		$n_L=1970$	$n_L=285$		$n_L=1861$	$n_L=247$
		$n_U=220$	$n_{BL}=1250$		$n_U=329$	$n_{BL}=986$
			$n_{BU}=435$			$n_{BU}=737$
			$n_U=220$			$n_U=220$

† The smaller value of LLF (AIC, BIC, or AICc), the better fitted model.

‡ p_1 and p_2 are the p-values of the test statistics LR_{1n} and LR_{2n} respectively.

Table 2. *Estimated results for the best fitted model of each return series.*

y_t	Fitted model	Regime	
USD/ARS	AR-GARCH	———	$\phi_0 = 0.0088\,(0.0025)$ $\phi_1 = -0.0838\,(0.0193)$ $\phi_2 = -0.1835\,(0.0198)$ $\alpha_0 = 0.0044\,(0.0004)$ $\alpha_1 = 0.2857\,(0.0192)$ $\beta_1 = 0.6754\,(0.0161)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = 0.0241\,(0.0067)$ $\phi_1 = -0.0476\,(0.0398)$ $\phi_2 = -0.2510\,(0.0398)$ $\alpha_0 = 0.0203\,(0.0017)$ $\alpha_1 = 0.3018\,(0.0392)$ $\beta_1 = 0.5656\,(0.0344)$
		$R_t = 0$	$\psi_0 = 0.0041\,(0.0026)$ $\psi_1 = -0.0639\,(0.0201)$ $\psi_2 = -0.1714\,(0.0176)$ $\pi_0 = 0.0012\,(0.0003)$ $\pi_1 = 0.2698\,(0.0209)$ $\delta_1 = 0.6924\,(0.0208)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = 0.0202\,(0.0058)$ $\phi_1 = -0.0016\,(0.0404)$ $\phi_2 = -0.2265\,(0.0346)$ $\alpha_0 = 0.0154\,(0.0014)$ $\alpha_1 = 0.3911\,(0.0418)$ $\beta_1 = 0.5734\,(0.0314)$
		$R_t = 0$	$\psi_0 = 0.0036\,(0.0026)$ $\psi_1 = -0.0821\,(0.0198)$ $\psi_2 = -0.1759\,(0.0180)$ $\pi_0 = 0.0012\,(0.0003)$ $\pi_1 = 0.2539\,(0.0209)$ $\delta_1 = 0.7005\,(0.0213)$
USD/BGN	AR-GARCH	———	$\phi_0 = -0.0154\,(0.0059)$ $\phi_1 = 0.2187\,(0.0156)$ $\phi_2 = -0.0325\,(0.0145)$ $\phi_3 = 0.0029\,(0.0132)$ $\phi_4 = -0.0593\,(0.0149)$ $\alpha_0 = 0.0016\,(0.0003)$ $\alpha_1 = 0.0379\,(0.0040)$ $\beta_1 = 0.9554\,(0.0043)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = -0.0161\,(0.0060)$ $\phi_1 = 0.2195\,(0.0185)$ $\phi_2 = -0.0204\,(0.0169)$ $\phi_3 = 0.0086\,(0.0159)$ $\phi_4 = -0.0511\,(0.0157)$ $\alpha_0 = 0.0000\,(0.0002)$ $\alpha_1 = 0.0234(0.0037)$ $\beta_1 = 0.9758\,(0.0037)$
		$R_t = 0$	$\psi_0 = 0.0334\,(0.0208)$ $\psi_1 = 0.2492\,(0.0459)$ $\psi_2 = -0.1679\,(0.0462)$ $\psi_3 = -0.1246\,(0.0486)$ $\psi_4 = -0.0419\,(0.0455)$ $\pi_0 = 0.0189\,(0.0031)$ $\pi_1 = 0.0487\,(0.0104)$ $\delta_1 = 0.8893\,(0.0204)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = -0.0151\,(0.0062)$ $\phi_1 = 0.2118\,(0.0168)$ $\phi_2 = -0.0121\,(0.0168)$ $\phi_3 = 0.0129\,(0.0165)$ $\phi_4 = -0.0517\,(0.0163)$ $\alpha_0 = 0.0001\,(0.0002)$ $\alpha_1 = 0.0206(0.0044)$ $\beta_1 = 0.9764\,(0.0044)$
		$R_t = 0$	$\psi_0 = 0.0212\,(0.0171)$ $\psi_1 = 0.2766\,(0.0384)$ $\psi_2 = -0.1993\,(0.0431)$ $\psi_3 = -0.1152\,(0.0416)$ $\psi_4 = -0.0350\,(0.0363)$ $\pi_0 = 0.0176\,(0.0035)$ $\pi_1 = 0.0460\,(0.0105)$ $\delta_1 = 0.9030\,(0.0205)$
USD/PHP	AR-GARCH	———	$\phi_0 = -0.0125\,(0.0049)$ $\phi_1 = 0.0631\,(0.0171)$ $\phi_2 = -0.0812\,(0.0163)$ $\phi_3 = 0.0512\,(0.0163)$ $\alpha_0 = 0.0020\,(0.0005)$ $\alpha_1 = 0.0689(0.0081)$ $\beta_1 = 0.9177\,(0.0097)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = -0.0127\,(0.0062)$ $\phi_1 = 0.0791\,(0.0262)$ $\phi_2 = -0.0782\,(0.0187)$ $\phi_3 = 0.0423\,(0.0130)$ $\alpha_0 = 0.0009\,(0.0005)$ $\alpha_1 = 0.0698(0.0100)$ $\beta_1 = 0.9201\,(0.0104)$
		$R_t = 0$	$\psi_0 = -0.0042\,(0.0130)$ $\psi_1 = 0.0035\,(0.0373)$ $\psi_2 = -0.0926\,(0.0398)$ $\psi_3 = 0.0731\,(0.0384)$ $\pi_0 = 0.0082\,(0.0026)$ $\pi_1 = 0.0000\,(0.0267)$ $\delta_1 = 0.9559\,(0.0460)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = -0.0126\,(0.0052)$ $\phi_1 = 0.0791\,(0.0188)$ $\phi_2 = -0.0777\,(0.0189)$ $\phi_3 = 0.0446\,(0.0180)$ $\alpha_0 = 0.0007\,(0.0005)$ $\alpha_1 = 0.0729(0.0085)$ $\beta_1 = 0.9222\,(0.0104)$
		$R_t = 0$	$\psi_0 = -0.0058\,(0.0116)$ $\psi_1 = 0.0064\,(0.0317)$ $\psi_2 = -0.0817\,(0.0340)$ $\psi_3 = 0.0676\,(0.0330)$ $\pi_0 = 0.0081\,(0.0024)$ $\pi_1 = 0.0008\,(0.0078)$ $\delta_1 = 0.9453\,(0.0251)$

y_t	Fitted model	Regime	
USD/PLN	AR-GARCH	———	$\phi_0 = -0.0280\,(0.0091)$ $\phi_1 = 0.2368\,(0.0168)$ $\phi_2 = -0.0287\,(0.0176)$ $\phi_3 = -0.0115\,(0.0167)$ $\phi_4 = -0.0145\,(0.0178)$ $\alpha_0 = 0.0050\,(0.0009)$ $\alpha_1 = 0.0620\,(0.0061)$ $\beta_1 = 0.9306\,(0.0064)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = -0.0264\,(0.0104)$ $\phi_1 = 0.2109\,(0.0223)$ $\phi_2 = -0.0055\,(0.0178)$ $\phi_3 = -0.0101\,(0.0161)$ $\phi_4 = 0.0022\,(0.0164)$ $\alpha_0 = 0.0047\,(0.0007)$ $\alpha_1 = 0.0339\,(0.0076)$ $\beta_1 = 0.9289\,(0.0071)$
		$R_t = 0$	$\psi_0 = 0.0437\,(0.0460)$ $\psi_1 = 0.2198\,(0.0461)$ $\psi_2 = -0.1060\,(0.0342)$ $\psi_3 = -0.0057\,(0.0420)$ $\psi_4 = -0.0840\,(0.0367)$ $\pi_0 = 0.0378\,(0.0092)$ $\pi_1 = 0.0621\,(0.0098)$ $\delta_1 = 0.9379\,(0.0223)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = -0.0325\,(0.0912)$ $\phi_1 = 0.1949\,(0.1087)$ $\phi_2 = 0.0043\,(0.1860)$ $\phi_3 = -0.0560\,(0.2370)$ $\phi_4 = -0.0063\,(0.5004)$ $\alpha_0 = 0.0065\,(0.0891)$ $\alpha_1 = 0.0195\,(0.1203)$ $\beta_1 = 0.9579\,(0.4704)$
		$R_t = 0$	$\psi_0 = -0.0410\,(0.1197)$ $\psi_1 = 0.2701\,(0.5639)$ $\psi_2 = -0.0718\,(0.0278)$ $\psi_3 = 0.0600\,(0.2954)$ $\psi_4 = -0.0420\,(0.3987)$ $\pi_0 = 0.0000\,(0.0989)$ $\pi_1 = 0.0768\,(0.0118)$ $\delta_1 = 0.9232\,(0.9632)$
USD/RUB	AR-GARCH	———	$\phi_0 = -0.0130\,(0.0035)$ $\phi_1 = 0.2807\,(0.0162)$ $\alpha_0 = 0.0005\,(0.0001)$ $\alpha_1 = 0.0687\,(0.0063)$ $\beta_1 = 0.9313\,(0.0061)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = -0.0111\,(0.0036)$ $\phi_1 = 0.2702\,(0.0178)$ $\alpha_0 = 0.0003\,(0.0001)$ $\alpha_1 = 0.0336(0.0055)$ $\beta_1 = 0.9575\,(0.0048)$
		$R_t = 0$	$\psi_0 = -0.0722\,(0.0536)$ $\psi_1 = 0.3497\,(0.0711)$ $\pi_0 = 0.0040\,(0.0060)$ $\pi_1 = 0.1443\,(0.0195)$ $\delta_1 = 0.7254\,(0.0361)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = -0.0088\,(0.0038)$ $\phi_1 = 0.2766\,(0.0202)$ $\alpha_0 = 0.0002\,(0.0001)$ $\alpha_1 = 0.0265(0.0055)$ $\beta_1 = 0.9626\,(0.0048)$
		$R_t = 0$	$\psi_0 = -0.0123\,(0.0114)$ $\psi_1 = 0.2768\,(0.0291)$ $\pi_0 = 0.0053\,(0.0009)$ $\pi_1 = 0.1088\,(0.0118)$ $\delta_1 = 0.8791\,(0.0130)$
USD/TWD	AR-GARCH	———	$\phi_0 = 0.0001\,(0.0031)$ $\phi_1 = -0.0018\,(0.0165)$ $\phi_2 = -0.0801\,(0.0174)$ $\phi_3 = -0.0409\,(0.0171)$ $\phi_4 = 0.0450\,(0.0155)$ $\phi_5 = 0.0214\,(0.0165)$ $\alpha_0 = 0.0007\,(0.0001)$ $\alpha_1 = 0.0879(0.0070)$ $\beta_1 = 0.9121\,(0.0062)$
	TAR-GARCH	$R_t = 1$	$\phi_0 = 0.0013\,(0.0038)$ $\phi_1 = 0.0105\,(0.0253)$ $\phi_2 = -0.0661\,(0.0208)$ $\phi_3 = -0.0239\,(0.0186)$ $\phi_4 = 0.0227\,(0.0167)$ $\phi_5 = 0.0468\,(0.0157)$ $\alpha_0 = 0.0003\,(0.0001)$ $\alpha_1 = 0.0712(0.0083)$ $\beta_1 = 0.9243\,(0.0089)$
		$R_t = 0$	$\psi_0 = 0.0152\,(0.0110)$ $\psi_1 = -0.0695\,(0.0461)$ $\psi_2 = -0.1064\,(0.0452)$ $\psi_3 = -0.0184\,(0.0482)$ $\psi_4 = 0.0797\,(0.0365)$ $\psi_3 = -0.1455\,(0.0459)$ $\pi_0 = 0.0110\,(0.0017)$ $\pi_1 = 0.1902\,(0.0311)$ $\delta_1 = 0.7587\,(0.0407)$
	BAR-GARCH	$R_t = 1$	$\phi_0 = -0.0002\,(0.0046)$ $\phi_1 = 0.0132\,(0.0265)$ $\phi_2 = -0.0245\,(0.0360)$ $\phi_3 = -0.0328\,(0.0243)$ $\phi_4 = 0.0300\,(0.0235)$ $\phi_5 = 0.0669\,(0.0218)$ $\alpha_0 = 0.0008\,(0.0005)$ $\alpha_1 = 0.1348(0.0168)$ $\beta_1 = 0.8652\,(0.0406)$
		$R_t = 0$	$\psi_0 = 0.0091\,(0.0063)$ $\psi_1 = -0.0439\,(0.0370)$ $\psi_2 = -0.1015\,(0.0282)$ $\psi_3 = -0.0192\,(0.0243)$ $\psi_4 = 0.0218\,(0.0211)$ $\psi_5 = -0.0284\,(0.0320)$ $\pi_0 = 0.0199\,(0.0058)$ $\pi_1 = 0.2496\,(0.0462)$ $\delta_1 = 0.5678\,(0.0934)$

† The standard deviations in parentheses are calculated as in Bollerslev and Wooldridge (1992).

Now we consider the parameter estimates of the best fitted models as shown in Table 2. For ease of comparison, we also compute the volatility persistence in each regime derived from these models (see Table 3).

Table 3. *Persistence measure in each regime derived from the best fitted models.*

		USD/ARS	USD/BGN	USD/PHP	USD/PLN	USD/RUB	USD/TWD
AR-GARCH		0.9611	0.9933	0.9866	0.9926	1.0000	1.0000
TAR-GARCH	$R_t = 1$	0.8674	0.9992	0.9899	0.9628	0.9911	0.9955
	$R_t = 0$	0.9622	0.9380	0.9559	1.0000	0.8697	0.9489
BAR-GARCH	$R_t = 1$	0.9645	0.9970	0.9951	0.9774	0.9891	1.0000
	$R_t = 0$	0.9544	0.9490	0.9461	1.0000	0.9879	0.8174

It can be seen from Table 3 that there was a strong persistence in volatility in all these models, with a slightly higher persistence in the lower regime in most exchange rates except the case of USD/PLN where it had an explosive volatility in the upper regime. This might be because Poland is the only European country to have avoided the recession in the late 2000s (Buckley, 2012).

One may argue that as the BAR-GARCH model is nested within the three-regime TAR-GARCH (denoted by 3R-TAR-GARCH) model, it is therefore interesting to see whether the 3R-TAR-GARCH model can give a better fit to our data sets than the BAR-GARCH model. Table 4 shows the results of the best fitted 3R-TAR-GARCH model, which is obtained by using the same estimation procedure for the BAR-GARCH model. We also calculate the LR test statistic

$$\text{LR}_{3n} := \text{LLF}_{\text{BAR-GARCH}} - \text{LLF}_{\text{3R-TAR-GARCH}}$$

to test for the null hypothesis H_{30}: the BAR-GARCH model against the alternative H_{31}: the 3R-TAR-GARCH, and its p-value $p_3 = 1 - F_{k_2}(\text{LR}_{3n})$ with $k_2 = p + m + s + 2$.

We can see from Table 4 that for the USD/BGN, USD/PLN and USD/TWD return series, their best fitted 3R-TAR-GARCH models are simply their corresponding two-regime TAR-GARCH models. This indicates that a two-regime model is enough for these three series. For the other three return series, the results of the likelihood ratio test reveal that, at the 5% significance level, the BAR-GARCH model is superior to the 3R-TAR-GARCH model except the USD/ARS return series. However, there are only $n_M = 21$ observations in the middle regime of 3R-TAR-GARCH model for the USD/ARS return series. In view of this, it is believed that the BAR-GARCH model is a plausible alternative to TAR-GARCH model.

Table 4. *Model selection results for 3-R-TAR-GARCH model in all six log-return series.*

	Log-return Series ($\times 100$)					
	USD/ARS	USD/BGN	USD/PHP	USD/PLN	USD/RUB	USD/TWD
Orders	$p = 2$ $d = 5$	$p = 4$ $d = 7$	$p = 3$ $d = 5$	$p = 4$ $d = 1$	$p = 1$ $d = 1$	$p = 5$ $d = 7$
	$m = 1$ $s = 1$	$m = 1$ $s = 1$	$m = 1$ $s = 1$	$m = 1$ $s = 1$	$m = 1$ $s = 1$	$m = 1$ $s = 1$
Thresholds	$r_L = -0.0838$	$r_L = 0.5048$	$r_L = 0.0886$	$r_L = 0.4305$	$r_L = 0.0038$	$r_L = 0.1804$
	$r_R = -0.0761$	$r_R = 0.5048$	$r_R = 0.2129$	$r_R = 0.4305$	$r_R = 0.4202$	$r_R = 0.1804$
LLF†	-4958.8	-1512.0	-2696.6	359.7	-2905.0	-3841.6
AIC	-4916.8	-1476.0	-2648.6	395.7	-2869.0	-3801.6
BIC	-4797.2	-1373.6	-2512.0	498.1	-2766.5	-3687.8
AICc	-4958.3	-1511.7	-2696.1	360.0	-2904.7	-3841.2
LR test‡	0.0000	—	0.0721	—	0.6538	—
#observations	$n_L = 502$	$n_L = 1949$	$n_L = 1491$	$n_L = 1751$	$n_L = 1270$	$n_L = 1861$
	$n_M = 21$	$n_M = 0$	$n_M = 262$	$n_M = 0$	$n_M = 700$	$n_M = 0$
	$n_U = 1667$	$n_U = 241$	$n_U = 437$	$n_U = 439$	$n_U = 220$	$n_U = 329$

† The smaller value of LLF (AIC, BIC, or AICc), the better fitted model.

‡ The p-values the test statistic LR_{3n}.

6. Concluding remarks

This paper proposes a new BAR-GARCH model, which captures the buffering phenomenon of time series in both conditional mean and variance. The empirical study on the six exchange rates series shows that the BAR-GARCH model could provide a better fit than AR-GARCH and TAR-GARCH models in most of cases. It is interesting to find that the buffer zone obtained in these series can be interpreted as a normal region of the exchange rates so that possibly there is no market intervention. All of our findings imply that the property of asymmetry, caused by a novel buffered mechanism from BAR-GARCH model, should not be ignored. Due to the empirical importance of the BAR-GARCH model, a theoretical exploration on its stationarity, estimation, statistical inference and model-diagnostic, should be considered, and we leave it for a future study.

Acknowledgement

This work is supported by Research Grants Council of the Hong Kong SAR Government, GRF grant HKU703711P, and National Natural Science Foundation of China (No.11201459). The authors also thank Mr. Lam Wai William for providing the data sets.

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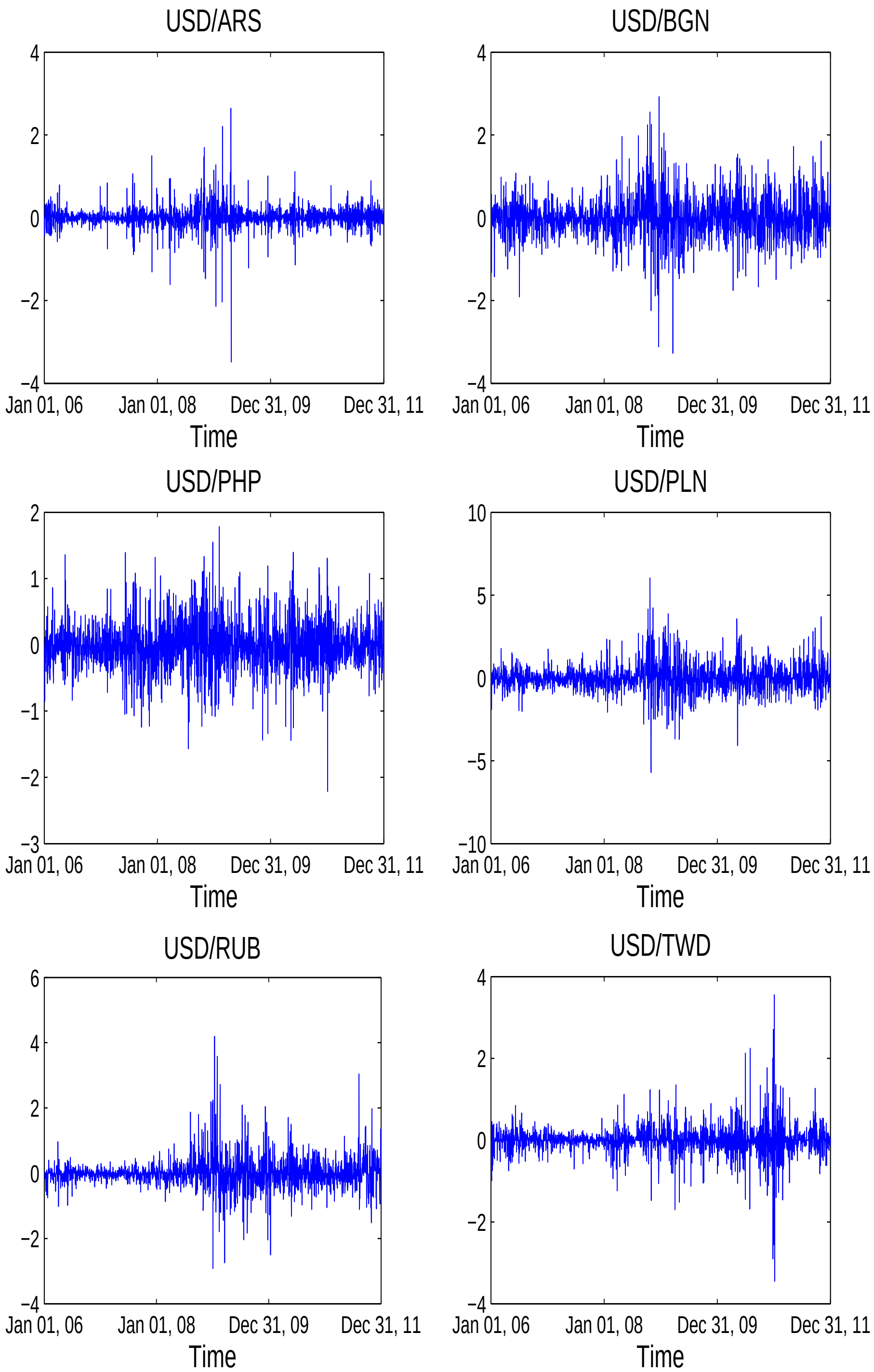

Fig. 1. The log-return ($\times 100$) of all six exchange rates.

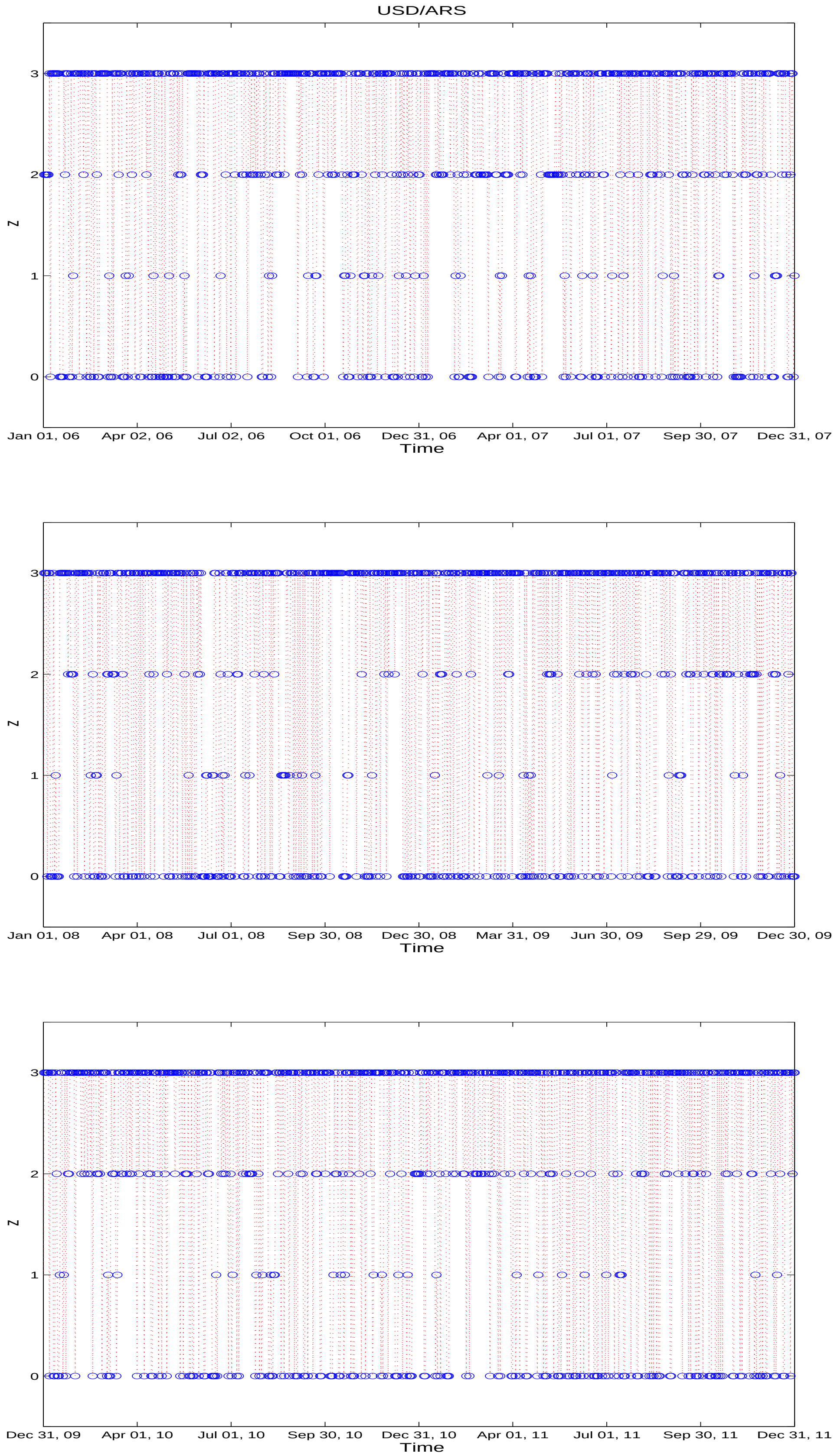

Fig. 2. The time points (represented by circles) in the lower outer regime ($i.e., y_{t-d} \leq r_L$), the lower buffering regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 1$), the upper buffering regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 0$), and the upper outer regime ($i.e., y_{t-d} > r_U$) of the BAR-GARCH model for USD/ARS return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.

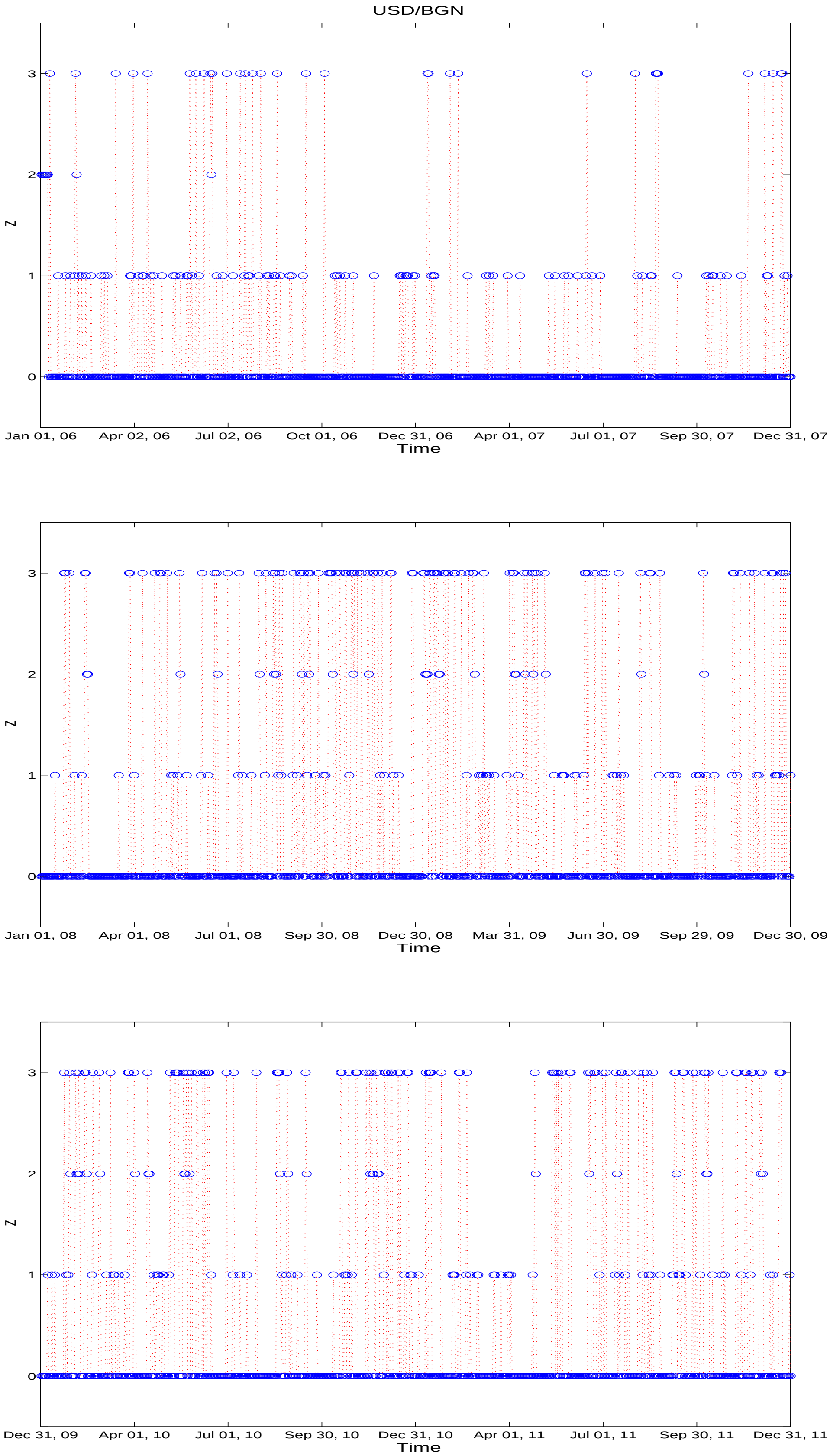

Fig. 3. The time points (represented by circles) in the lower outer regime ($i.e., y_{t-d} \le r_L$), the lower buffering regime ($i.e., r_L < y_{t-d} \le r_U, R_t = 1$), the upper buffering regime ($i.e., r_L < y_{t-d} \le r_U, R_t = 0$), and the upper outer regime ($i.e., y_{t-d} > r_U$) of the BAR-GARCH model for USD/BGN return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.

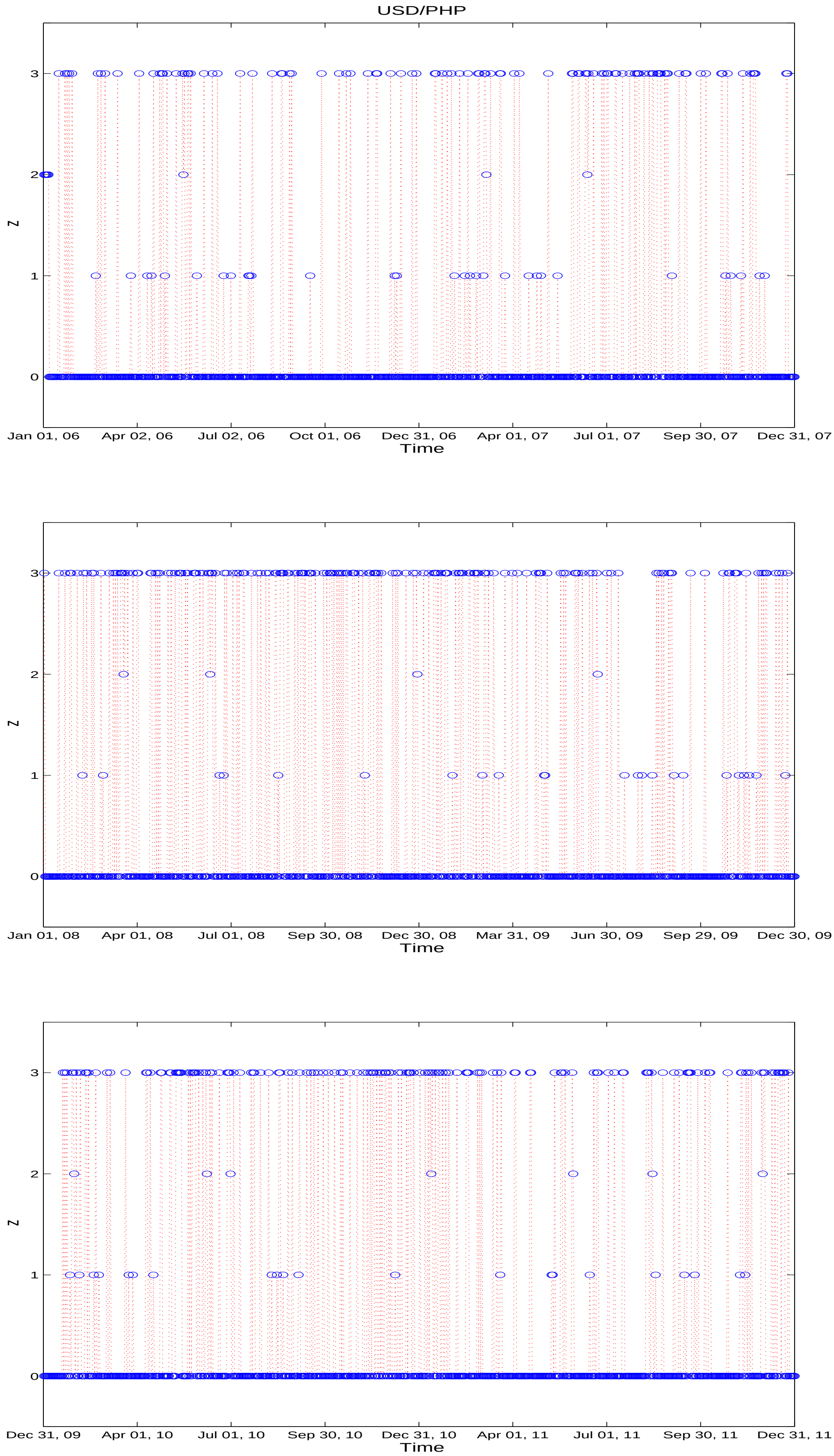

Fig. 4. The time points (represented by circles) in the lower outer regime (*i.e.*, $y_{t-d} \leq r_L$), the lower buffering regime (*i.e.*, $r_L < y_{t-d} \leq r_U, R_t = 1$), the upper buffering regime (*i.e.*, $r_L < y_{t-d} \leq r_U, R_t = 0$), and the upper outer regime (*i.e.*, $y_{t-d} > r_U$) of the BAR-GARCH model for USD/PHP return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.

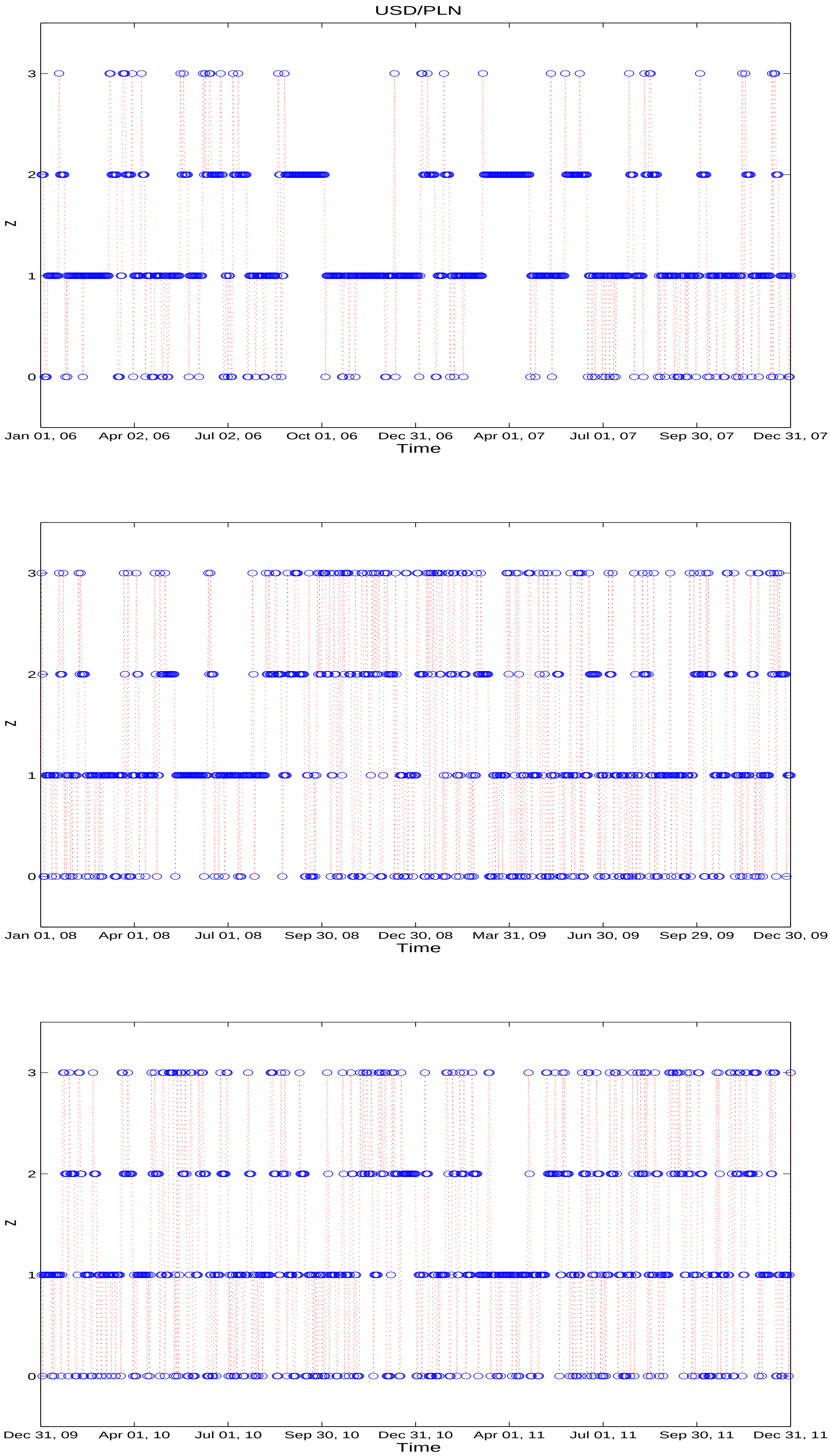

Fig. 5. The time points (represented by circles) in the lower outer regime ($i.e., y_{t-d} \le r_L$), the lower buffering regime ($i.e., r_L < y_{t-d} \le r_U, R_t = 1$), the upper buffering regime ($i.e., r_L < y_{t-d} \le r_U, R_t = 0$), and the upper outer regime ($i.e., y_{t-d} > r_U$) of the BAR-GARCH model for USD/PLN return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.

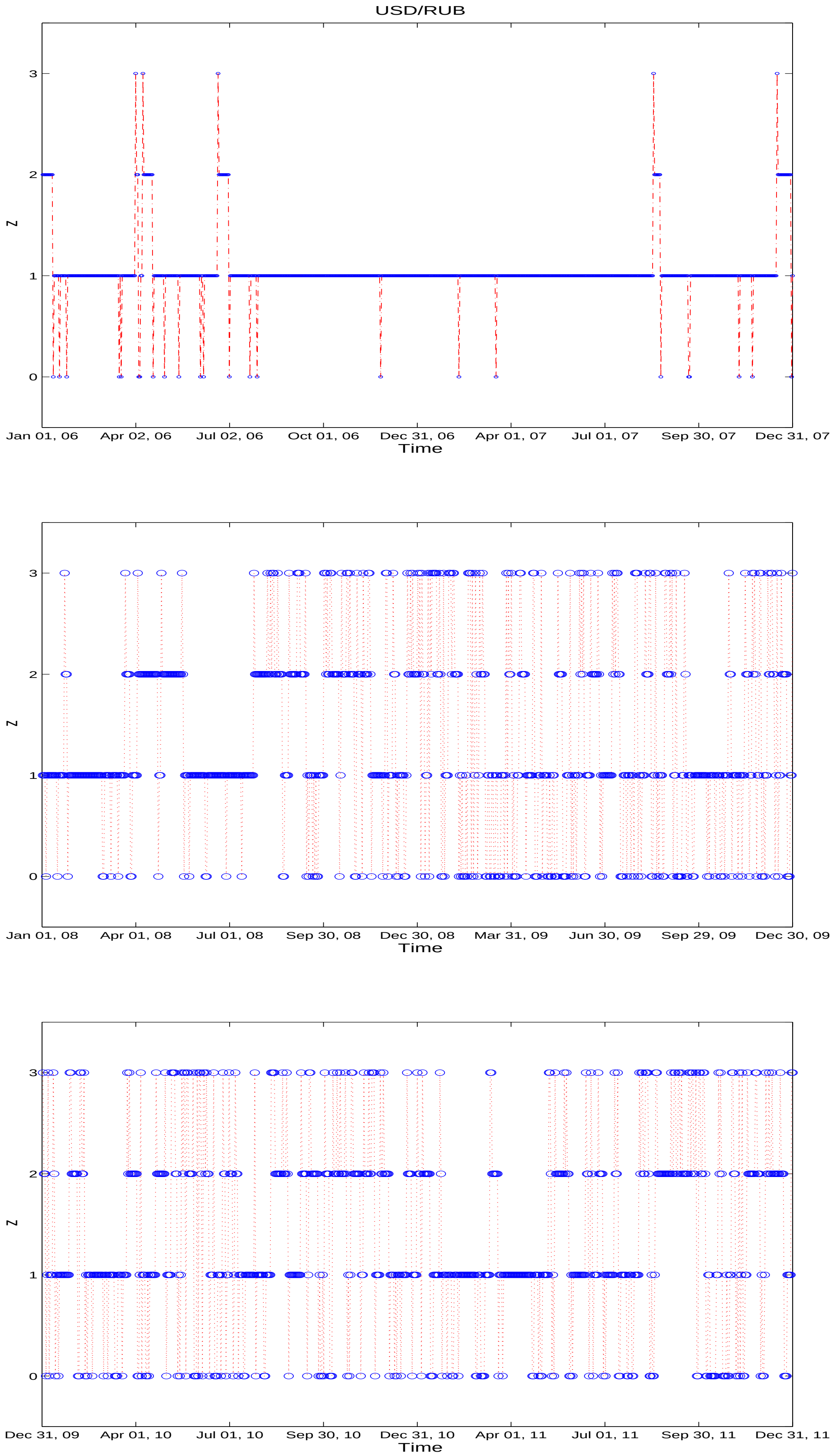

Fig. 6. The time points (represented by circles) in the lower outer regime (*i.e.*, $y_{t-d} \leq r_L$), the lower buffering regime (*i.e.*, $r_L < y_{t-d} \leq r_U, R_t = 1$), the upper buffering regime (*i.e.*, $r_L < y_{t-d} \leq r_U, R_t = 0$), and the upper outer regime (*i.e.*, $y_{t-d} > r_U$) of the BAR-GARCH model for USD/RUB return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.

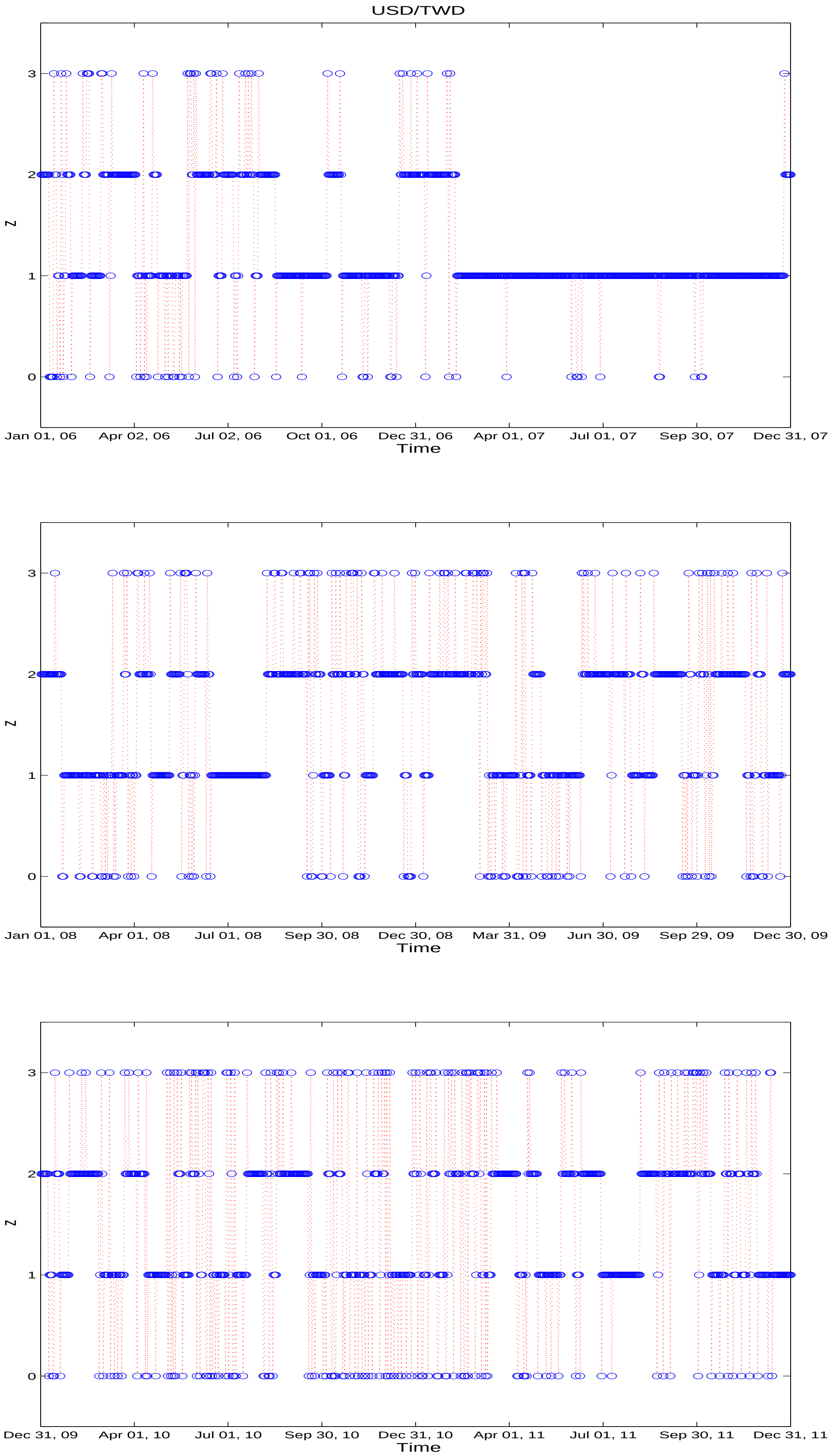

Fig. 7. The time points (represented by circles) in the lower outer regime ($i.e., y_{t-d} \leq r_L$), the lower buffering regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 1$), the upper buffering regime ($i.e., r_L < y_{t-d} \leq r_U, R_t = 0$), and the upper outer regime ($i.e., y_{t-d} > r_U$) of the BAR-GARCH model for USD/TWD return series are located at $z = 0$, $z = 1$, $z = 2$, and $z = 3$, respectively. Here, every two consecutive time points are linked by a dotted line.