# Switching from Patents to an Intertemporal Bounty in a Non-Scale Growth Model

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**Abstract**

The prize system for innovation has been criticized as impractical due to the lack of any workable formula or algorithm to determine the size of prizes. In this paper, a decentralized market mechanism via the intertemporal bounty (IB) system can function to duplicate Pareto optimality. Under this system, any bountiable innovation is placed in the public domain, and the prize of innovation is dynamically amortized in an infinite time domain as periodic bounties paid to holders of bounty claims. Periodic bounties are calculated using a government-determined bounty rate times observed market sales. Two formulas are derived to calculate *"long-run Pareto optimal bounty rate"* and *"long-run suboptimal bounty rate."* The former can correct monopoly distortions and externalities, while the latter can only address monopoly distortions. They are empirically computable and can serve as an upper bound and the lower bound of the bounty rate. This paper provides a dynamic general-equilibrium analysis of replacing finitely-lived patents with the IB system using either of these two bounty rates. Based on a non-scale growth model calibrated to the US economy, transition paths are worked out to compute welfare gains. [JEL Classification: C63, O31, O34]

Key Words: Patent, Prize, Bounty Claim, Innovation, Growth, Transitional Dynamics



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# 1 Introduction

There are alternative reward systems including patent and prize designed to promote technological innovation.[1] Patent is a commercial reward system for the government to confer upon the innovator short-lived intellectual property rights. This allows the innovator to turn the shadow price of a patent-protected innovation into a reward in the form of monopoly profits accruing during the granted patent life. Prize is a tax-financed public reward system for the government to award a prize for the innovator to place an innovation in the public domain. The prize is awarded, once and for all, to reflect its social value. Hence, a prize system can spur innovation without creating monopoly distortions.[2] The patent system, however, must always seek to balance short-run monopoly distortion against long-run dynamic innovation [Nordhaus (1969)], and tends to provide inadequate incentives for innovation in that the shadow price of a patent-protected innovation is usually below the social value.[3] If the government knows how to set prizes to the social value, a prize system can be Pareto optimal and outshine the patent system as suboptimal.[4]

Polanvyi (1944) trumpeted the prize system informally. Four decades later, Wright (1983) developed a formal model to compare patents and prizes. This seminal paper emphasizes asymmetric information between governments and innovators, showing that prizes can be superior to patents if the government is appropriately informed about an innovation's social surplus and costs. Shavell and Van Ypersele (2001) restricts asymmetric information to market demand, presuming that the innovator knows the exact market demand curve and that the government is only informed of its probability distribution. They demonstrate that an optional patent-or-prize system can dominate the patent system. Under this optional system, the prize is determined at the smallest possible social surplus, and the innovator is free to choose either a patent or a prize.[5]

In principle, prize systems — either optional or mandatory — are subject to the government's informational readiness to determine the social value of innovations. This concern has been raised in Wright (1983), Scotchmer (1999), Shavell and Van Ypersele (2001) and Hopenhayn et al. (2006). To resolve this problem, Kremer (1998) suggests a patent buyouts mechanism to determine the private value or shadow price of

[1]This paper focuses on patent and prize as two competing reward systems. To avoid terminological confusions, we note that some economists such as Shavell and Van Ypersele (2001) distinguish intellectual property rights (patents, copyrights, etc.) from all reward systems.

[2]Some tax-induced distortions may take place if lump-sum taxes are unavailable. However, technical innovations have public-good features. Prizes for innovation can be financed in a manner that results in no additional distortion; see Kaplow (1996) and Footnote 6 of Kremer (1998).

[3]Whether patents had contributed to innovation and productivity has recently been questioned in Boldrin and Levine (2013) and Williams (2013). In theory, patents may lead to over-investment in research and development, if externatlities such as research congestion or patent races are significant enough to diminish an innovation's social value. This possibility appears in the present paper , Jones and Williams (2000) and others. Empirically, under-investment in R&D is however so obvious: the social rate return to R&D is at least twice the private rate of return according to Mansfield et al. (1977) and Mansfield et al. (1981) and Jones and Williams (2000).

[4]Many studies have sought to improve efficiency by fine-tuning a patent's dimensions such as *length*, or *scope*, or both; see, for instance, Judd (1985), Gilbert and Shapiro (1990), Klemperer (1990), Matutes et al. (1996), and O'Donoghue and Zweimuller (2004). But patents, designed to function by introducing monopolies, are doomed to be suboptimal, no matter how one fine-tunes their dimensions. Besides, it is difficult to harmonize patents and other form of intellectual property rights between developed and developing economies; see Helpman (1993) and Grinols and Lin (2006). Romer (1990) shows that subsidizing R&D input can achieve the Pareto optimum in his seminal growth model where all firms are noncompetitive. But this subsidization policy is at most second-best in a more general context where competitive and noncompetitive firms co-exit, as in the present paper.

[5]See Proposition 5 in Shavell and Van Ypersele (2001).

patents through an auction process.[6] The government then offers to purchase patents at this private value times some constant markup to match up to the social value, and these purchased patents would be mostly placed in the public domain.[7] Kremer's idea is clever because auctioning patents allows the government to harness private information.

True, the above-cited studies have provided valuable insights into the relative costs and benefits between patents and prizes. However, there are some limitations in their industrial organization approach. *First*, they use static partial-equilibrium models. While this approach allows to highlight the role of asymmetric information, it does not allow the dynamic general-equilibrium market mechanism to work, for instance, to effect an innovation's private value and social value as well as the required rate of return that makes households willing to finance business firm's forward-looking investments. Moreover, static models cannot unfold the dynamic nature of innovating activities that invent new products or processes over time. *Second*, these previous studies always posit a prize system that pays prizes, once and for all, for technological innovation. True, in static models, prizes cannot be awarded with any intertemporal arrangement. But in a dynamic real world, the social value of any innovation at a point in time is a discounted sum of the value stream it emanates in the future. This applies to the private value as well. An observed innovation is valuable today, but its value, either social or private, will change over time, and may even risk being destroyed by competing products or processes in the future. Accordingly, prizes are forward-looking and difficult to estimate especially for new innovations. The government may award a prize for some innovation that is worth nothing in the future. *Third*, as Abramowicz (2003) argues, there does not exist any workable formula or algorithm to calculate prizes in the literature.[8] This makes the prize system impractical.

The purpose of the present paper is to resolve all these limitations existing in the patent-prize literature. It represents the first work that provides a dynamic general-equilibrium analysis of replacing finitely-lived patents with a dynamically-amortized prize system that rewards the innovator with periodic bounties. The analytical vehicle uses a non-scale R&D-based growth model based on Jones (1995), Jones and Williams (2000) and Eicher and Turnovsky (2001).[9] This growth model features, among other things, a constant population growth rate and a semi-endogenous innovation rate at which private investment in R&D (research and development) leads to an ongoing flow of innovative blueprints for manufacturing new types of capital goods. In the modeled economy, patents can create monopoly price distortions to weaken demand for capital goods, as commonly seen in the literature. Moreover, these distortions can skew the relative prices of patented and unpatented capital goods. This gives rise to a socially inefficient mix of capital goods in production of final goods and translates into an economy-wide loss in total factor productivity ($TFP$).[10] To

[6]Guell and Fischbaum (1995) also suggest a different buyouts program for the government to exercise its power of eminent domain to purchase pharmaceutical patents at a price determined by judges.

[7]Based on Mansfield et al. (1981), Kremer makes the conservative assumption that the social value of patents is on average twice the private value. He cautions that the government should prevent bidders from colluding in the auction process.

[8]This was the reason that Kremer (1998) proposed an auction mechanism for patent buyouts. As Kremer anticipated, some problems may arise from substitute patents or collusion among bidders. Regardless, it remains difficult for auctioning to determine the private value of an observed innovation at a point in time, given that this value is subject to future innovations.

[9]Romer (1990) is a seminal paper introducing an endogenous R&D-based growth model of expanding variety, Jones (1995) extends by removing the well-know scale effects. Recently, there is an emerging stream of studies examining patent policy using different classes of R&D-based growth models; see, for instance, Helpman (1993), O'Donoghue and Zweimuller (2004), Grinols and Lin (2006), Chu (2009), Iwaisako and Futagami (2013), Lin (2013), etc.

[10]In the endogenous growth literature, little attention was paid to the negative effects of patents on $TFP$. Chu (2010) is one of

better calibrate the model to empirical data, we follow Comin (2004) to allow both R&D and non-R&D elements to drive $TFP$ to change over time, so that R&D can only contribute a plausibly small fraction in $TFP$ growth. The proposed dynamically-amortized prize system can work to remove both capital demand and $TFP$ distortions. Under this system, the reward for a technical innovation (blueprint) is a bounty claim to an infinite stream of ongoing bounties, payable on ex post (observed) market sales at a government-determined bounty rate. These sales cover all those capital goods that embody the same blueprint, irrespective of who these sellers are. That is, instead of a "once-and-for-all" prize paid at a time as in the literature, the prize is dynamically amortized in an infinite time domain as periodic bounty payments, which are equal to the bounty rate times observed market sales in each time period. For brevity, we use ***intertemporal bounty*** to refer to such a dynamically-amortized prize system.[11]

The intertemporal bounty (IB) system as analyzed herein is mandatory. Its designed mechanism is summarized as follows. *First*, any innovative blueprint must be placed in the public domain in exchange for a tradeable bounty claim, thereby creating free access to non-rival innovations (blueprints) and ensuring competitive provision of any bountiable goods. *Second*, bounty payments are financed by lump-sum taxes to sustain an active asset market for bounty claims. The equilibrium price of bounty claims reflects the discounted present value of expected ongoing bounty flows. If desired, the innovator can choose at any moment to cash the entire prize, once and for all, by selling her bounty claim at the free market equilibrium price. *Third*, by raising the bounty rate, the government can increase a bountiable innovation's private value through an increase in the bounty-claim price, ceteris paribus. This can promote more investment in R&D and spur technological innovation. *Fourth*, a valuable flexibility is that the the government can adjust the bounty rate to internalize various externalities such as knowledge spillovers or research congestion, so that the private value of a potential innovation can be navigated to the social value. In addition, the government can always fine-tune the bounty rate as desired if observed economic conditions deviate from policy objectives. In a nutshell, an intertemporal bounty system can function with an adjustable bounty rate and can be Pareto optimal. In contrast, the "once-and-for-all" prize system is lack of such flexibilities, while patents are suboptimal and inflexible to cope with externalities.

An important contribution of the paper is that we derive a formula to calculate a ***long-run Pareto optimal bounty rate*** for the intertemporal bounty system to be Pareto efficient in the long run. This optimal bounty rate enables a patent-regime economy to switch to a bounty-regime economy, and then transition over time to the social planner's long-run optimum.[12] This formula requires information on an economy's steady-state aggregate variables ($TFP$ growth rate, final output growth rate, capital share, and real interest rate), parametrized externalities (knowledge spillovers and research congestion), and price elasticity of bountiable

the very few exceptions.

[11] Grinols and Henderson (2007) proposed to replace pharmaceutical patents with an intertemporal-bounty. They discussed the intertemporal bounty concept in a heuristic way. Later, Grinols and Lin (2011) formulated the intertemporal-bounty system in a dynamic pharmaceutical sectoral-equilibrium model, where the interest rate and the size of the pharmaceutical sector are exogenous, among other things. In stark contrast, the present paper incorporates the intertemporal-bounty system in a non-scale R&D-based growth model of expanding variety, and further relates this system for the first time to a dynamically amortized prize system with tradeable bounty claims, as will be explained later.

[12] In fact, as the economy transitions to its long-rum social optimum, the Pareto optimal bounty rate should vary over time in transition. As an initial inquiry into the IB system for a growing economy, this paper dose not apply the time-varying optimization exercise, but directly sets a time-invariant bounty rate at its long-run optimal level.

goods. Data of this sort is empirically available. In addition, we derive a formula to calculate a ***long-run suboptimal bounty rate***. This bounty rate ensures that in the long run a bounty-regime economy can have the same R&D intensity as does the current (patent-regime) economy. Paying bounties based on either bounty rate can remove patent-created monopoly distortions immediately. But to fully internalize externalities in the long-run, bounties must be based on the long-run Pareto optimal bounty rate. From the perspective of IB implementation, the two formulas can present two benchmark bounty rates.

For the first time in the patent-prize literature, the dynamic trajectory of an aggregate economy that switches from patents to a dynamically amortized prize system is solved numerically in the paper. We simulate the regime switch for a calibrated U.S. economy presumably beginning with a patent-regime steady state. This switch places all innovations in the public domain, forcing the economy to jump to a new trajectory that guides it to transition over time to a bounty-regime balanced growth path. The dynamics in transition is governed by a nonlinear system of four first-order ordinary differential equations (ODE) in two state variables (capital & knowledge stocks) and two control-like variables (final-output consumption & bounty-claim price), where the stock of knowledge is measured by variety of capital goods. This presents a nonlinear boundary value problem (BVP) and is solved numerically with a Python package that wraps a Fortran BVP solver.[13]

Under a set of plausible benchmark parameters, the long-run Pareto optimal bounty rate is 0.30 (30 cents per dollar sales) and the long-run suboptimal bounty rate is 0.14 (14 cents per dollar sales). If the regime switch proceeds with with this long-run Pareto optimal bounty rate, the U.S. economy's steady-state R&D intensity (measured by the R&D/GDP ratio) will rise from 0.024 (patent regime) to 0.053 (bounty regime). From the computed transition paths, the U.S. economy is seen to transition to one much more intensive in both capital and knowledge than under the initial patent regime (20 years patent length). For instance, on the computed bounty-regime balanced growth path, the stock of knowledge is 3.4 *times* as much as on the initial patent-regime balanced growth path, the stock of capital is about 1.7 *times* as much, and per capita consumption is 1.4 *times* as much.

The central market mechanism works as follow. On the one hand, compared to monopolistic provision, competitive provision of capital goods expands market demand for forgone consumption and speeds up capital accumulation. On the other hand, the long-run Pareto bounty rate of 0.30 is strong enough to make the bounty-claim price jump up to match the social value of a potential innovation, thereby expanding R&D demand for foregone consumption and spurring technological innovation. As a result, expanded demand for foregone consumption pushes up the real interest rate in transition to make households willing to save more to finance capital and knowledge accumulation. Households therefore experience short-run consumption sacrifices. However, the long-run Pareto efficiency resulting from the regime switch can generate a net consumption gain of 5.9 percent, after taking into account the short-lived consumption losses and robustness checks. If instead the long-run suboptimal bounty rate of 0.14 is implemented, the net consumption gain is seen to diminish to 3.0 percent for the benchmark scenario. As expected, this suboptimal bounty rate can maintain the initial patent-regime R&D intensity of 0.024, but is not intended to internalize externalities.

[13] Solving a linearized dynamic system around the bounty-regime steady state would not make sense in the present paper, because the patent-bounty regime switch represents a structural change that can lead to a bounty-regime steady state greatly differing from the initial patent-regime steady state. This is especially true if the bounty rate is set at the long-run Pareto optimal level.

In sum, the entire dynamic general-equilibrium analysis delivers the central message that an intertemporal bounty regime, where the prize for innovation is dynamically amortize in an infinite time domain, can be a Pareto-optimal alternative to the world's long-standing patent system.

The rest of the paper is organized as follows. Section 2 lays out a patent-regime R&D-based growth model featuring multiple distortions from patents, knowledge spillovers, and research congestion. Section 3 evaluates the patent regime against the model's social planner optimum. Section 4 introduces an intertemporal bounty regime and the resulting four-dimensional dynamic system while deriving two formulas for the long-run Pareto optimal and suboptimal bounty rates. Section 5 calibrates the model to the initial patent-regime steady state that largely mimics an innovating economy like the United States and solves the dynamic system numerically. Transition paths, welfare gains, and the robustness checks are computed and analyzed. Section 6 concludes.

## 2 The Model for Patent Regime

In this paper we formulated a patent regime for a closed economy using a non-scale R&D-based growth model.[14] This economy is endowed with labor $L$ that grows exogenously. Labor and physical capital are employed to produce final output $Y$, which can be used for consumption or for investment in either durable (capital) goods $X$ or R&D (research and development). Investing in durable goods $X$ contributes to the accumulation of physical capital $K$, while investing in R&D is to innovate technical designs for new types of durable goods. Technical designs are indexed by $i$ in a closed interval $[0,V] \subset \mathbb{R}^+$. For each design, the innovator receives a patent for a finite term denoted by $T \in (0,\infty)$. Patents are tradeable and perfectly enforced. Within the finite patent term, every patented firm has the exclusive power to produce and sell a specific type of durable good $X[i]$. Yet, once a patent expires, the granted monopoly power vanishes and the out-of-patent firm reduces to a perfectly competitive firm immediately. All durables are horizontally differentiated. Variable $V$ represents not only the number of patents that have been issued, but also a measure of the variety of available durable goods. Like $K$ measuring the stock of capital, $V$ is a proxy of the knowledge stock in the model. The growth of $V$ allows for a finer division of capital and leads to an increase in the level of $TFP$ (total factor productivity) for the economy.

The model described below is for an economy that implements a patent system. It has four sectors referred to as household, final good, durable (capital) good, and R&D.

### 2.1 Household

There is a continuum of identical households with measure of one in the economy. Each household has $L$ identical workers, which also represent the economy's population of labor. The population of labor is normalized to one at time $t = 0$ and is assumed to grow at an exogenous rate, $n$. So, the size of a typical

[14] The model of the paper is a modification of Romer (1990) in three dimensions. First, patents are finitely lived. Second, as in Rivera-Batiz and Romer (1991), research input is from forgone consumption rather than from labor (human capital). Third, as in Jones (1995), the R&D (innovation) function displays diminishing marginal returns to the research input, thereby removing the empirically implausible scale effects. With these modifications, this model is closely related to Jones and Williams (2000) and Eicher and Turnovsky (2001).

household at time $t$ is given by $L[t] = e^{nt}$. Define $C[t]$ as the rate of the economy's aggregate consumption of final goods and $c[t] \equiv \frac{C[t]}{L[t]}$ as the rate of per capita (or per worker) consumption at a point in time. For each household, its family lifetime utility $U[t]$ at time $t$ is given by

$$U[t] = \int_t^\infty \left( \frac{c[s-t]^{1-\gamma} - 1}{1-\gamma} \right) e^{-(\rho-n)(s-t)} ds, \ \rho > 0, \ \gamma > 0 \tag{1}$$

where $\rho$ and $\gamma$ are two preference parameters with $\rho$ measuring the rate of time preference and $\frac{1}{\gamma}$ the elasticity of intertemporal substitution. All labor services are used for final goods production. Each household owns assets including physical capital and patents. Its per capita asset stock is $a[t]$ at a point in time and earns the net capital income of $r[t]a[t]$ at the market rate of interest. From each household, every worker earns wage income $w[t]$ at a point in time from a perfectly competitive labor market. Hence, a typical household's flow budget constraint is given by

$$\dot{a}[t] \equiv \frac{da[t]}{dt} = (r[t]-n)a[t] + w[t] - c[t] \tag{2}$$

Households take as given all market prices such as $w[t]$ and $r[t]$. From a household's intertemporal optimization, the familiar Euler condition is

$$\hat{c}[t] \equiv \frac{\dot{c}[t]}{c[t]} = \frac{r[t]-\rho}{\gamma}. \tag{3}$$

From (1) and (3), for the family lifetime utility to be bounded in the steady state, we invoke the parameter constraint that $\rho > n + (1-\gamma)\hat{c}_0$, where $\hat{c}_0$ represents a constant steady-state growth rate of per capita consumption under a patent regime. Throughout the paper, an overdot defines a time derivative of the associated variable and the symbol ^ indicates its instantaneous growth rate. Henceforth, the time variable $t$ will be suppressed in most equations unless otherwise necessary.

## 2.2 Final goods

We chose final good $Y$ as the numeraire. In the final goods sector there are many identical (competitive) producers and the aggregate production function is given by

$$Y = (hL)^{1-\alpha} \left( \int_0^V X[i]^{\sigma\alpha} di \right)^{1/\sigma}, \ 0 < \sigma < \frac{1}{\alpha}, \tag{4}$$

where $h$ is exogenous and can be interpreted as a non-R&D-driven efficiency index, $h^{1-\alpha}$ is a non-R&D-driven productivity term, $Y$ is the economy's aggregate final output or real GDP produced with labor $L$ supplied from households and $V$ types of capital goods $X[i]$ for $i \in [0,V]$ that have been invented. We assume that term $h$ grows at a constant rate $\hat{h} > 0$ so that $h[t] = e^{\hat{h}t}$. Thus, $h[t]L[t] = e^{(n+\hat{h})t}$ grows at an exogenous rate of $n + \hat{h}$. As the functional form implies, production of $Y$ exhibits constant returns in labor and capital goods and these capital goods are imperfectly substitutable. Each capital good, if patented, is priced at a markup equal to $\eta \equiv \frac{1}{\sigma\alpha} > 1$.[15] Profit maximization yields the following first-order conditions:

$$w = (1-\alpha)Y/L \tag{5a}$$

$$p[i] = \alpha (hL)^{1-\alpha} \left( \int_0^V X[i]^{\sigma\alpha} di \right)^{(1-\sigma)/\sigma} X[i]^{\sigma\alpha - 1} \tag{5b}$$

where $p[i]$ is the price of capital good $i$, the fraction of $1-\alpha$ in (5a) represents the labor share, and (5b) is an inverse demand function for any capital good. This demand function implies that the price elasticity of demand for an individual durable is equal to $\varepsilon = \frac{1}{1-\sigma\alpha} > 1$. The markup for any patented durable is therefore given by $\eta = \frac{\varepsilon}{\varepsilon - 1} = \frac{1}{\sigma\alpha} > 1$. As for out-of-patent (or unpatented) durables, they are priced at marginal cost.

Final output $Y$ is allocated for consumption $C$, capital investment $I_K$, and R&D investment $I_V$. As $Y$ represents the economy's real GDP, the national income identity requires

$$Y = C + S = C + I_K + I_V \tag{6}$$

where $S$ is the flow of aggregate saving (or forgone consumption) and must be equal to $I_K + I_V$ in equilibrium. Define $s \equiv \frac{S}{Y} = (Y-C)/Y$ as the saving rate, $s_K \equiv \frac{I_K}{Y}$ as the the rate of capital investment, and $s_V \equiv \frac{I_V}{Y}$ as the rate of R&D investment (or called the economy's R&D intensity). Then the flow equilibrium in saving and investment is given by

$$s = 1 - \frac{C}{Y} = s_K + s_V \tag{7}$$

Investment in capital goods ($I_K$) accumulates the stock of capital $K$, while investment in R&D ($I_V$) creates new varieties of capital goods and raises the stock of knowledge $V$. The two stocks of the economy evolve over time according to

$$\dot{K} \equiv \frac{dK}{dt} = I_K - \delta K = s_K Y - \delta K \tag{8a}$$

$$\dot{V} \equiv \frac{dV}{dt} = \xi I_V = \xi s_V Y \tag{8b}$$

where $\delta$ is the rate of capital depreciation and $\xi$ is an endogenous measure of research productivity. In contrast to physical capital, the stock of knowledge does not depreciate over time, as in most endogenous growth models, whereas Eicher and Turnovsky (2001) introduce knowledge depreciation. The measure of research productivity in (8b) is endogenously determined in terms of $\xi = \mu V^{\phi} I_V^{\lambda - 1}$, with $\mu > 0$, $\phi < \bar{\phi} < 1$, and $0 < \lambda < 1$. The parameter of $\mu$ is an exogenous technology term. To capture the positive externality of knowledge spillovers, the parameter of $\phi$ is restricted be positive, though it might be negative in theory. Also, $\phi$ must be less than an upper bound $\bar{\phi} < 1$ in order to ensure a bounded long-run innovation rate, denoted by $\lim_{t\to\infty} \hat{V}[t] < \infty$, where $\hat{V} \equiv \frac{\dot{V}}{V}$.[16] The parameter of $\lambda$ is positive but less than one, meaning that

[15] If parameter $\sigma$ were set equal to one, production function (4) would reduce to Romer (1990). As such, durables would become neither substitutable nor complementary and the markup $\eta \equiv \frac{1}{\sigma\alpha}$ would be $\frac{1}{\alpha}$. In this paper, $\sigma$ is less than $1/\alpha$ so that the markup $\eta$ is allowed to deviate from the inverse of the capital share ($\alpha$), as in Jones and Williams (2000) and Eicher and Turnovsky (2001).

[16] From (8b) and (18), the instantaneous innovation rate is given by $\hat{V} = \mu V^{\phi - 1}(s_V Y)^{\lambda}$. On a balanced-growth path, it can be

research input $I_V$ displays social diminishing marginal return on research productivity. This captures the effect of research congestion, which may result from too many research firms engaged in similar projects in patent races.

For simplicity, each unit of foregone consumption can presumably produce a unit of each type of capital goods, so that the stock of capital is given by $K = \int_0^V X[i]di$. Foregone consumption is partially invested in R&D, thereby enabling the economy to introduce new types of durables over time. These new capital goods are patentable as soon as they are invented. Active R&D therefore keeps changing the composition of patented and unpatented durable goods under a patent system, as will be analyzed later. What follows describes individual firms' pricing behavior in the durable goods sector.

### 2.3 Durable (capital) goods

Firms of capital goods may engage in monopolistic competition or perfect competition, depending on whether or not they are under patent protection. As each unit of any type of durable goods requires one unit of forgone consumption, $r+\delta$ measures the user cost of capital (interest rate plus capital depreciation). Given $p[i]$ (the rental price of durable $i$), $p[i]-(r+\delta)$ measures a firm's unit profit. The profit function is then given by

$$\pi[i] = (p[i]-r-\delta)X[i], i \in [0,V]. \tag{9}$$

At any point in time, there are $V$ firms (or $V$ varieties of available durables) in the capital goods sector. Of these available durable goods, there are $V_p$ patented durables supplied by monopolistic firms and $V_{np}$ unpatented (out-of-patent) durables by competitive firms. That is, $V = V_p + V_{np}$. Patented durables are priced at the markup of $\eta \equiv \frac{1}{\sigma\alpha} > 1$, while unpatented ones obey marginal cost pricing. These pricing conditions are given by

$$p[i] = \begin{cases} p_{np} = r+\delta & \text{for } i \in [0,V_{np}] \\ p_p = \eta(r+\delta) & \text{for } i \in (V_{np},V] \end{cases} \tag{10}$$

where $p_{np}$ (= marginal cost) represents the competitive price of unpatented durables and $p_p$ the monopolistic price of patented durables. From (10) and (5b), demands for patented and unpatented durables are related in terms of:

$$X_{np} = \eta^{\varepsilon} X_p \tag{11}$$

where $X_{np} = X[i]$ for $i \in [0,V_{np}]$, $X_p = X[i]$ for $i \in (V_{np},V]$, and $X_{np} > X_p$ due to $\eta^{\varepsilon} > 1$. That is, on a per-durable basis, the economy uses more of an unpatented capital good than a patented one, since each patented durable costs more ($p_p > p_{np}$). Define $\zeta \equiv \frac{V_p}{V}$ as the fraction of patented durables and $1-\zeta \equiv \frac{V_{np}}{V}$ as the fraction for unpatented durables. Then one can use (11) to derive the stock of capital,

$$K = \int_0^V X[i]di = VX_{np}(\eta^{-\varepsilon}\zeta + 1 - \zeta) \tag{12}$$

shown that the long-run steady-state innovation rate is determined by $\lim_{t\to\infty} \hat{V}[t] = \frac{\lambda(n+\hat{e})}{1-\phi-\lambda/(\varepsilon\sigma(1-\alpha))}$, which is bounded and positive if and only if $\phi < \bar{\phi} \equiv 1-\lambda/[\varepsilon\sigma(1-\alpha)]$.

where $\eta^{-\varepsilon} = X_p/X_{np}$. Equation (12) actually represents a market clearing condition for capital goods, because $K$ is the stock of forgone consumption borrowed to purchase both patented and unpatented durables.

## 2.4 Innovation under a patent system

This subsection discusses the innovation function (8b) and explains how the R&D-driven innovation rate interacts with the dynamic evolution from patented to unpatented durables over time, given that patents are finitely lived.

### 2.4.1 Innovation, patent value, and R&D intensity

In the research sector there is a stream of identical firms with measure of one. The innovation function (8b) thus applies to the entire economy and any individual research firm as well. Under perfect competition research firms borrow forgone consumption to develop new designs for capital goods. At the firm level, research input $I_V$ and the flow $\dot{V}$ of capital-good designs are linearly related, since the research productivity measure of $\xi$ in (8b) is taken as given in an individual research firm's decision problem. Under a patent system, the flow of new designs $\dot{V}dt$ during an instant means the flow of newly issued patents and each of these patents has a finite patent term denoted by $T > 0$. Each newly issued patent gives birth to a new firm producing a new type of capital goods. Patents are tradeable. The market for patents is perfectly competitive. The price $\upsilon$ of a fresh patent issued at time $t$ must reflect its future profitability during an entire patent life. Hence,

$$\upsilon[t] = \int_t^{t+T} e^{-R[\tau]}\pi[\tau]d\tau \tag{13}$$

where $R[\tau] \equiv \int_t^{\tau} r[\omega]d\omega$ is the cumulative sum of interest rates from $t$ to $\tau$ and $\pi$ measures the profit flow to any firm holding a legally live patent (see (9)). As such, a fresh patent's market price reflects the private value or shadow price of a potential innovation under the patent regime. The vintage of a live patent has no effects on the profit flow. Under non-arbitrage conditions, the equilibrium price of a fresh patent must match the cost of developing a new design. That is,

$$\upsilon[t] = 1/\xi[t] \equiv 1/\left(\mu V[t]^{\phi}(I_V[t])^{\lambda-1}\right) \tag{14}$$

where the inverse of $\xi[t]$ measures the development cost according to (8b).[17] If $\upsilon > 1/\xi$ there would be unbounded R&D demand for foregone consumption. If $\upsilon < 1/\xi$, there would be unbounded demand for patents. With well functioning markets, equilibrium R&D investment $I_V$ must be such that the price of fresh patents equals the cost of capital-good innovation and determines the instantaneous rate of innovation,

$$\hat{V} \equiv \frac{\dot{V}}{V} = \mu V^{\phi-1} I_V^{\lambda}, \tag{15}$$

[17] The research input of $I_V$ generates $\dot{V}$ patents at time $t$. So, the present-time cost of developing a patent (or a new design) is measured by $I_V/\dot{V} = 1/\xi$ in terms of (8b).

using (8b). With the identity of $V = V_p + V_{np}$, the innovation rate $\hat{V}$ is a weighted average of the growth rate $\hat{V}_p$ of patented durables and the growth rate $\hat{V}_{np}$ of unpatented durables. That is, $\hat{V} = \zeta \hat{V}_p + (1-\zeta)\hat{V}_{np}$ with $\hat{V}_p \equiv \frac{\dot{V}_p}{V_p}$ and $\hat{V}_{np} \equiv \frac{\dot{V}_{np}}{V_{np}}$, where the fraction of patented durables $\zeta \equiv V_P/V$ and the fraction of unpatented durables $1-\zeta = V_{np}/V$ evolve over time until the economy is in a steady state. The patent length $T$ plays a role in the motions of $\zeta$, $V_p$, and $V_{np}$. We describe their relationships in what follows.

### 2.4.2 Patent length and the dynamic evolution from patented to unpatented durables

First, with an active R&D sector, there are $\dot{V}[t]$ patents issued at a point in time and these newly issued patents will expire at a future date, $t+T$. That is, it holds that $\dot{V}[t] = \dot{V}_{np}[t+T]$ and this equality relation implies $\dot{V}_{np}[t] = \dot{V}[t-T]$. As such, $\dot{V}_{np}[t]$ is pre-determined by previous innovations at a time of $t-T$. With a finite patent length, the growth rates of unpatented and patented durables are therefore given by[18]

$$\hat{V}_{np}[t] = \left(\frac{1}{1-\zeta[t]}\right)\hat{V}[t-T]e^{-\int_{t-T}^{t^-}\hat{V}[\tau]d\tau} \tag{16a}$$

$$\hat{V}_p[t] = \left(\frac{1}{\zeta[t]}\right)(\hat{V}[t] - \hat{V}[t-T]e^{-\int_{t-T}^{t^-}\hat{V}[\tau]d\tau}) \tag{16b}$$

where $t^-$ is asymptotically close to $t$ and $\int_{t-T}^{t^-}\hat{V}[\tau]d\tau$ represents the cumulative sum of innovation rates between $t-T$ and $t$. Differentiating $\zeta \equiv \frac{V_p}{V}$ with respect to time $t$ yields

$$\dot{\zeta}[t] = (\hat{V}_p[t] - \hat{V}[t])\zeta[t] \tag{17}$$

As indicated, $\dot{\zeta} > 0$ if the growth rate of patented durables, $\hat{V}_p$ , exceeds the innovation rate, $\hat{V}$. If it is the case, there is an *increase* in the fraction of patented durables and a *decrease* in the fraction of unpatented durables. From (17), $\zeta$ is driven by the two motions of $\hat{V}$ and $\hat{V}_p$ in terms of (17). These motions underlie the patent-created innovating mechanism in a dynamic general-equilibrium context. This mechanism contributes to total factor productivity ($TFP$) by introducing new types of capital goods, but it can also induce a technical distortion to offset the $TFP$ enhancement to some extent. The following analysis will unfold this problem.

## 2.5 Innovation and monopoly distortion

In the model patents incentivize innovation of new capital goods by creating monopoly distortions. In this environment, the relative prices of patented and unpatented capital goods are distorted and prevent society from adopting the technically optimal mix of durables, although individual final goods producers always choose their cost-minimizing techniques for whatever relative prices. This problem is missing in almost all endogenous growth models where patents are presumed to be infinitely lived.

[18] (16a) can be obtained using: (i) $\dot{V}_{np}[t] = \dot{V}[t-T]$, (ii) $\hat{V}_{np}[t] \equiv \frac{\dot{V}_{np}[t]}{V_{np}[t]} = \frac{\dot{V}[t-T]}{V[t-T]} \times \frac{V[t-T]}{V[t]} \times \frac{V[t]}{V_{np}[t]}$, (iii) $\frac{V[t]}{V_{np}} = \frac{1}{1-\zeta[t]}$, and (iv) $V[t] = V[t-T]e^{\int_{t-T}^{t^-}\hat{V}[\tau]d\tau}$, where the upper bound $t^-$ is asymptotically close to $t$. Then with (16a) and the identity $\hat{V} = \zeta\hat{V}_p + (1-\zeta)\hat{V}_{np}$, (16b) holds evidently.

### 2.5.1 Monopoly-distorted TFP

Consider symmetries of capital goods permitting to use $X_{np}$ for $X[i]$, $i \in [0, V_{np}]$ and $X_p$ for $X[i]$, $i \in (V_{np}, V]$. From (11) and (12), the final goods production function can reduce to :

$$Y = z(AhL)^{1-\alpha}K^{\alpha} \tag{18}$$

where $A \equiv V^{\frac{1}{\varepsilon\sigma(1-\alpha)}} = V^{\frac{\alpha}{(\varepsilon-1)(1-\alpha)}}$ is an endogenous technology term, $z(Ah)^{1-\alpha}$ ( $= zV^{\frac{\alpha}{\varepsilon-1}}h^{1-\alpha}$) is a measure of $TFP$, and $z$ is the monopoly-induced distortion on R&D-driven $TFP$ defined as

$$z = z[\zeta] = \frac{\left(\eta^{1-\varepsilon}\zeta + 1 - \zeta\right)^{1/\sigma}}{\left(\eta^{-\varepsilon}\zeta + 1 - \zeta\right)^{\alpha}} \tag{19}$$

It can be verified that $z$ bears a convex relationship with $\zeta \in [0,1]$, has a local minimum at $\zeta = \varepsilon([1-(\frac{1}{\eta})^{\varepsilon}]^{-1} - (\frac{1}{\eta})[1-(\frac{1}{\eta})^{\varepsilon-1}]^{-1}) > 0$, and rises to one if $\zeta$ moves to either end of its domain. This relationship is demonstrated in Figure 1, which presumes $\alpha = 1/3$ and $\sigma = 1.75$ or $2.00$.[19]. Note that the socially optimal technique for final goods production ought to involve *symmetric* use of all available capital goods in accordance with (4). But under the patent regime the relative prices of patented and unpatented durables are distorted. This makes use of capital goods biased toward those that are marginal cost priced. This bias gives rise to a socially inefficient technique in production of final goods and translates into an economy-wide drag on the level of $TFP$ (due to $z(Ah)^{1-\alpha} < (Ah)^{1-\alpha}$). Thus, $TFP$ fails to attain its potential level ($= (Ah)^{1-\alpha}$), unless patents are either infinitely lived ($T \to \infty$) or non-existent ($T = 0$). In the latter case, the R&D sector must vanish, however. From (18), the instantaneous growth rate of total factor productivity is determined by

$$T\hat{F}P = T\hat{F}P^{R\&D} + (1-\alpha)\hat{h}; \quad T\hat{F}P^{R\&D} = \hat{z} + (1-\alpha)\hat{A} \tag{20}$$

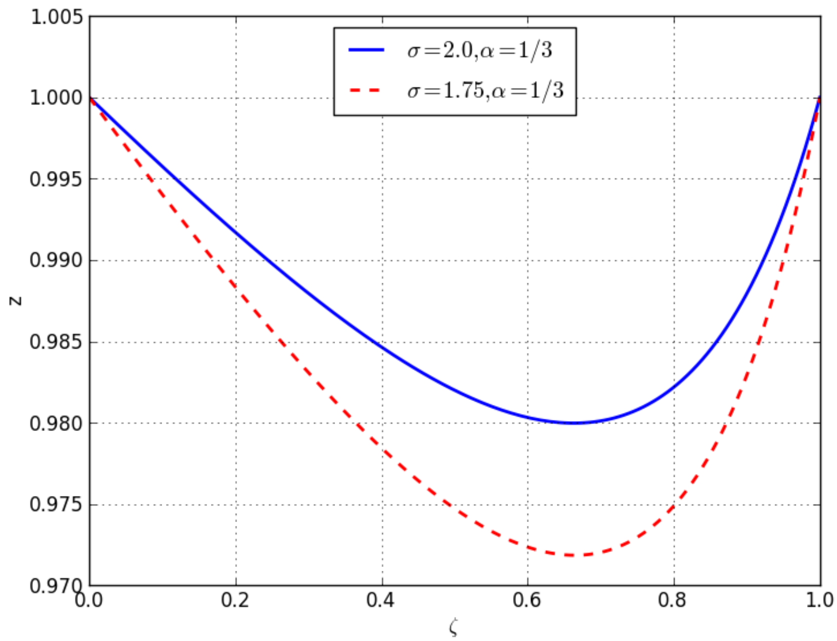
where $(1-\alpha)\hat{A} = \left(\frac{\alpha}{\varepsilon-1}\right)\hat{V}$. As indicated, the overall $TFP$ growth rate is decomposed into $T\hat{F}P^{R\&D}$ (the growth rate of R&D-driven $TFP$) and $(1-\alpha)\hat{h}$ (the exogenous growth rate of non-R&D-driven $TFP$). The R&D-driven $TFP$ growth rate, $T\hat{F}P^{R\&D}$, implies that while invention of new types of capital goods contribute productivity growth, this growth may be weakened by more monopoly distortions if $z$ falls. In this paper, therefore, changes in $TFP$ can result from R&D, or monopoly distortions, or exogenous productivity factors. This feature will prove conductive later in calibrating the model to a real-world economy, as compared to Jones and Williams (2000), for instance.

### 2.5.2 Monopoly-distorted factor income shares

Next, let us look at how factor income shares are influenced by monopoly distortions. Consider the flow of gross capital income $\int_0^V p[i]X[i]di$ that can be decomposed into aggregate (gross) rental income $(r+\delta)K$ and aggregate profit $\Pi = V_p\pi = \zeta V\pi$, where $\zeta V$ is the number of patented firms. The labor share is $\frac{wL}{Y} = 1-\alpha$,

[19] Recall that $0 < \sigma < \frac{1}{\alpha}$, $\varepsilon = \frac{1}{1-\sigma\alpha} > 1$, $\eta = \frac{1}{\sigma\alpha} > 1$, and $0 < \zeta < 1$. Given these parameters and (19), if $\zeta = 1$, $z = \eta^{(1-\varepsilon)/\sigma+\varepsilon\alpha} = 1$ due to $(1-\varepsilon)/\sigma+\varepsilon\alpha = 0$. As well, if $\zeta = 0$, the result of $z = 1$ is self-evident. Certainly, both $z$ and $\zeta$ are determined simultaneously in equilibrium.

Figure 1: The convex relationship between $z$ and $\zeta = V_p/V$




so parameter $\alpha$ represents the gross capital income share and must satisfy the identity relationship that $\alpha = \frac{\Pi}{Y} + \frac{(r+\delta)K}{Y}$, where the shares of profit and gross rental income are given by[20]

$$\frac{\Pi}{Y} = m\alpha, \quad \text{with } m = \frac{\zeta(\eta-1)}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta} \in (0,\,1) \tag{21a}$$

$$\frac{(r+\delta)K}{Y} = (1-m)\alpha \tag{21b}$$

Note that the fraction term $m$ determines how much of the the gross capital income share, $\alpha$, goes to monopolistic profits. Certainly, $(1-m)\alpha$ is the remaining capital income share going for rents and capital depreciation, subject to the relative magnitude of $r$ and $\delta$. Since $\frac{\partial m}{\partial \zeta} = \eta^{\varepsilon} > 0$, the profit share, $\Pi/Y$, rises if patented firms account for a larger fraction in the entire capital goods sector. If this fraction is one ($\zeta = 1$), the profit share reaches its maximum at $\left(\frac{\eta-1}{\eta}\right)\alpha$ and the gross rental share drops to its minimum at $\frac{\alpha}{\eta}$. To the contrary, $\zeta = 0$ (no monopoly) dictates that the entire capital share, $\alpha$, goes for gross rents.

### 2.6 Patent regime's steady-state dynamic system

The patent-regime economy is in the steady state by assumption. Let us close the patent regime model by presenting its balanced-growth dynamic system and its core variables' steady state equilibrium in this

[20] Aggregate profit is $\Pi = \zeta V\pi = (\eta-1)(r+\delta)\zeta VX_p$ based on (9) and (10) and aggregate rental income is $(r+\delta)K = (r+\delta)VX_{np}\left(\eta^{-\varepsilon}\zeta+1-\zeta\right)$ according to (12). Hence, using (11), the ratio of $\Pi/[(r+\delta)K]$ is determined by $\frac{\zeta(\eta-1)}{\zeta+(1-\zeta)\eta^{\varepsilon}}$. Using this ratio, we can solve the identity $\alpha = \frac{\Pi}{Y} + \frac{(r+\delta)K}{Y}$ for (21a) and (21b).

subsection. Later the economy will be perturbed at time $t = 0$ by replacing the patent regime with an intertemporal-bounty (IB) regime. To differentiate one regime from the other other, let us label the initial patent steady state with subscript 0 and a new IB steady state with subscript 1. From (8a), (8b), (3), and (13), the patent-regime dynamic system on a balanced-growth path is represented by four ordinary differential equations:[21]

$$\dot{K}_0[t] = (1 - s_{V0})Y_0[t] - C_0[t] - \delta K_0[t] \tag{22a}$$

$$\dot{V}_0[t] = \mu V_0[t]^{\phi}(s_{V0}Y_0[t])^{\lambda} \tag{22b}$$

$$\dot{C}_0[t] = C_0\left(\frac{r-\rho}{\gamma} + n\right) \tag{22c}$$

$$\dot{\upsilon}_0[t] = r_0\upsilon_0[t] - \pi_0[t](1 - e^{-(r_0 - \hat{\pi}_0)T}) \tag{22d}$$

where $Y_0[t] = z_0(A_0[t]h[t]L[t])^{1-\alpha}K_0^{\alpha}$ and $\pi_0[t] = \alpha m_0 Y_0[t]/(\zeta_0 V_0[t])$ according to (18) and (21a). On the balanced-growth path, the *non-stationary* variables including $K_0[t]$, $V_0[t]$, $Y_0[t]$, $C_0[t]$, $\upsilon_0[t]$, and $\pi_0[t]$ continue to grow over time at their constant growth rates. In contrast, the *stationary* variables such as interest rate $r_0$, R&D investment rate $s_{V0}$, capital investment rate $s_{K0}$, and patented durables share $\zeta_0$ all stay unchanged over time. For those non-stationary variables, their constant steady-state growth rates are given as follows (see Appendix A for derivations):

$$\hat{K}_0 = \hat{Y}_0 = \hat{C}_0 = \theta_K(n + \hat{h}) \tag{23a}$$

$$\hat{V}_0 = \theta_V(n + \hat{h}); \quad \hat{A}_0 = \theta_A(n + \hat{h}); \; T\hat{F}P = (1 - \alpha)(\hat{A} + \hat{h}) \tag{23b}$$

$$\hat{\upsilon}_0 = \hat{\pi}_0 = \theta_{\upsilon}(n + \hat{h}) \equiv (\theta_K - \theta_V)(n + \hat{h}) \tag{23c}$$

where each of terms $\theta_V$, $\theta_A$, $\theta_K$ and $\theta_{\upsilon}$ is a structural composite of growth-relevant parameters for a specific variable or for a set of specific variables, as given below,

$$\theta_V = \frac{\lambda}{1 - \phi - \lambda/[\varepsilon\sigma(1-\alpha)]}; \;\; \theta_A = \frac{\lambda}{(1-\phi)[\varepsilon\sigma(1-\alpha)] - \lambda}; \;\; \theta_K = \theta_A + 1 = (\frac{1-\phi}{\lambda})\theta_V. \tag{24}$$

To facilitate subsequent discussions, let us call these structural composites "*growth kernels*," which readily translate the population growth rate $n + \hat{h}$ into a relevant variable's long-run growth rate. For instance, $\theta_V(n + \hat{h})$ is the long-run innovation rate of $V$, $\theta_A(n + \hat{h})$ the long-run growth rate of $A$, and $\theta_K(n + \hat{h})$ the long-run growth rate of $K$, $Y$, and $C$.[22] Certainly, on per capita terms, $\theta_K(n + \hat{h}) - n$ $(= \theta_A n + \theta_K \hat{h})$ determines the long-run growth rate of $k$, $y$, and $c$ due to $\hat{k} = \hat{K} - n$, $\hat{y} = \hat{Y} - n$ and $\hat{c} = \hat{C} - n$. As for a fresh patent's price $\upsilon$ and a patented firm's profit flow $\pi$, they share the same growth kernel represented by $\theta_{\upsilon}$ (or $\theta_K$ minus $\theta_V$), and thereby their long-run growth rate is given by $\theta_{\upsilon}(n + \hat{h})$ or $(\theta_K - \theta_V)(n + \hat{h})$, as indicated by (23c). This actually makes intuitive sense, because final output growth $(\theta_K(n + \hat{h}))$ expands the market for capital goods, whereas the arrival of newer capital goods $(\theta_V(n + \hat{h}))$ dilutes the market.

Note that the structural parameters ($\alpha$, $\sigma$, $\varepsilon$ $\lambda$, $\phi$) that form the growth kernels are from the production functions of final goods and technical designs of capital goods. The population growth rate $n$ and the

[21] By Leibniz's rule, differentiating (13) on a balanced growth path yields the differential equation of (22d).
[22] In (23b), the long-run $TFP$ growth rate, $T\hat{F}P$, is obtained by setting $\hat{z} = 0$ in (20).

exogenous productivity growth rate $\hat{h}$ are parameters, too. Therefore, the long-run innovation rate $\hat{V}_0$ and all other long-run growth rates mentioned above are exogenous, independent of the size of the economy and the patent length. Also, as noted earlier (see Footnote 16), a bounded, positive long-run innovation rate, $\hat{V}_0$, requires that $\phi$ be less than the upper bound $\bar{\phi}$ $(= 1-\lambda/[\varepsilon\sigma(1-\alpha)] < 1)$. Once $\hat{V}_0$ is bounded, all other non-stationary variables are bounded as well.

In the model ongoing growth makes non-stationary variables unbounded and grow exponentially at different paces in the long run. These features call for multiple normalization factors to transform non-stationary into stationary variables. The "growth-kernel powered labor forces" defined by $(hL)^{\theta_K}$, $(hL)^{\theta_V}$, $(hL)^{\theta_A}$ and $(hL)^{\theta_K-\theta_V}$ provide a systemic way to build such normalization factors. For instance, $K$, $V$, $A$ and $\upsilon$ can be normalized to $\widetilde{K} \equiv K/(hL)^{\theta_K}$, $\widetilde{V} \equiv V/(hL)^{\theta_V}$, $\widetilde{A} \equiv A/(hL)^{\theta_A}$, and $\widetilde{\upsilon} \equiv \upsilon/(hL)^{\theta_\upsilon}$, respectively. In so doing, each variable with tilde is a bounded, measurable scale-adjusted quantity, because $\hat{\widetilde{x}}[t] = \hat{x}[t] - \theta_x(n+\hat{h}) = 0$, $x \in \{K, V, A, \upsilon\}$ as $t$ goes to infinity.[23] We close the patent regime model by deriving the long-run steady-state equilibria of a set of scale-adjusted variables for further analysis (see Appendix A for detailed derivations):

$$\widetilde{K}_0 = \widetilde{A}_0 \left(\frac{z_0 s_{K0}}{\delta + \theta_K(n+\hat{h})}\right)^{1/(1-\alpha)} \tag{25a}$$

$$\widetilde{V}_0 = \left[s_{V0}\left(\frac{z_0 s_{K0}}{\delta + \theta_K(n+\hat{h})}\right)^{\alpha/(1-\alpha)}\left(\frac{\mu}{\theta_V(n+\hat{h})}\right)^{1/\lambda}\right]^{\theta_V} \tag{25b}$$

$$\widetilde{C}_0 = (1-s_0)\widetilde{Y}_0 \tag{25c}$$

$$\widetilde{\upsilon}_0 = \frac{\alpha m_0}{\zeta_0}\left(\frac{\widetilde{Y}_0}{\widetilde{V}_0}\right)\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0}\right) \tag{25d}$$

where

$$s_{K0} = \alpha(1-m_0)\left(\frac{\delta+(n+\hat{h})+\hat{A}_0}{r_0+\delta}\right), \tag{25e}$$

$$s_{V0} = \hat{V}_0 \left(\frac{\alpha m_0}{\zeta_0}\right)\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0}\right), \tag{25f}$$

$$r_0 = \rho + \gamma[\theta_A n + \theta_K \hat{h}], \tag{25g}$$

$$\zeta_0 = 1-e^{-\hat{V}_0 T}, \tag{25h}$$

In the above equations, our earlier definitions suffice to recognize that $\widetilde{A}_0 = \widetilde{V}_0^{1/(\varepsilon\sigma(1-\alpha))}$ is a scale-adjusted technology term, $\widetilde{Y}_0 = z_0\widetilde{A}_0^{1-\alpha}\widetilde{K}_0^{\alpha}$ is a scale-adjusted GDP, $z_0$ is a monopoly-induced technical distortion on $TFP$ (see (19), $s_0 \equiv s_{K0}+s_{V0}$ is the steady-state saving rate, $\alpha(1-m_0)$ is the gross rental income share, and $\alpha m_0$ is the profit share (see (21a).[24] On the initial balanced growth path, a patented firm's scale-adjusted profit flow is $\widetilde{\pi}_0 = \frac{\alpha m_0}{\zeta_0}\left(\frac{\widetilde{Y}_0}{\widetilde{V}_0}\right)$, and with the finite patent length $T$ and the discount factor $\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0}$, $\widetilde{\upsilon}_0$ is a

[23] Similar normalization factors are used in Eicher and Turnovsky (2001).

[24] $A = V^{\frac{1}{\varepsilon\sigma(1-\alpha)}}$ implies $\widetilde{A} = \widetilde{V}^{\frac{1}{\varepsilon\sigma(1-\alpha)}}$ using $A = \widetilde{A}(hL)^{\theta_A}$, $V = \widetilde{V}(hL)^{\theta_V}$, and $\frac{\theta_V}{\varepsilon\sigma(1-\alpha)} - \theta_A = 0$; see (24).

measure of the scale-adjusted private value (shadow price) of a potential innovation according to (25d). This private value represents a discounted sum of the profit stream earned by a newly issued patent. It permits to derive the R&D intensity $s_{V0}$ in (25f) as shown in Appendix A. Equations (25a) - (25h) imply the roles of patent length ($T$) and patent-created monopoly distortions ($z_0$, $m_0$). For instance, if patent length $T$ is set to zero, then: (i) the monopolistically competitive sector vanishes ($\zeta_0 = 0$); (ii) there is no $TFP$ growth ($\hat{V}_0 = \hat{A}_0 = 0$) despite that there is no monopoly distortion on TFP ($z_0 = 1$, see Figure 1); (iii) the profit share is zero due to $m_0 = 0$, so are the R&D intensity ($s_{V0} = 0$), the private value of a potential innovation ($\widetilde{\upsilon}_0 = 0$), and the scale-adjusted knowledge stock ($\widetilde{V}_0 = 0$). In other words, if the patent system is removed and if there is no other decentralized market mechanism to support R&D investment, the model becomes a non-innovative economy can only invest in old capital goods endowed by history.

The paper is aimed at examining whether or not it is socially desirable to replace the entire patent regime with an intertemporal bounty regime that will be laid out later in Section 4. To this end, it is conductive to evaluate the patent regime and the proposed alternative, respectively, against the economy's Pareto optimality. The next section evaluates the patent regime.

# 3 Evaluating Patent Regime against Pareto Optimality

How efficient is a decentralized patent-regime economy in allocating resources? To answer this question, we need look at the economy's Pareto optimal allocation as a benchmark.

## 3.1 Pareto optimality

The Pareto optimal allocation can be obtained by maximizing household's lifetime utility, subject to the economy's initial resource endowments ($K_0$, $V_0$, $L_0 = 1$) and technological constraints. Hence, the social planner optimization problem is given by

$$\max_{c, I_K, I_V} \int_0^\infty \frac{c^{1-\gamma} - 1}{1-\gamma} e^{-(\rho - n)t} dt, \qquad \text{s.t.:} \tag{26a}$$

$$Y = (AhL)^{1-\alpha} K^\alpha = C + I_K + I_V, \;\; C = cL \tag{26b}$$

$$\dot{K} = I_K - \delta K, \;\; K[0] = K_0; \quad \dot{V} = \mu V^\phi I_V^\lambda, \;\; V[0] = V_0 \tag{26c}$$

where $h[t] = e^{\hat{h}t}$ and $L[t] = e^{nt}$ as under the patent regime. With no market imperfections in such a centralized economy, we need to set $\zeta$ at $\zeta^* = 0$ and $z$ at $z^* = 1$.[25] This makes final goods production function (18) reduce to (26b). We can solve the social planner problem for $c$, $I_K$, and $I_V$, which can then translate into $s$, $s_K$, $s_V$, and $\upsilon$ respectively.[26] Using these Pareto-optimal investment rates, we can derive the Pareto-optimal scale-adjusted stocks of capital and knowledge in the same way as we did earlier for the patent regime.

[25] Hereafter, a variable with an asterisk indicates a centralized economy's steady-state equilibrium.

[26] As a standard procedure, one can obtain the socially optimal solution by maximizing the Hamiltonian,

$$\mathscr{H} \equiv \left(\frac{c^{1-\gamma} - 1}{1-\gamma}\right) e^{-(\rho-n)t} + \psi_K \cdot \left(V^{\frac{1}{\varepsilon\sigma}} (hL)^{1-\alpha} K^\alpha - cL - I_V - \delta K\right) + \psi_V \cdot \mu V^\phi I_V^\lambda$$

Also, as we have analyzed, the long-run growth rates of some non-stationary variables are parametrized by their associated growth kernels and the population growth rate (see (23a) - (23c)). Certainly, these long-run growth rates also hold for the centralized economy; that is, $\hat{K}^* = \hat{Y}^* = \hat{C}^* = \theta_K(n+\hat{h})$, $\hat{V}^* = \theta_V(n+\hat{h})$, $\hat{A}^* = \theta_A(n+\hat{h})$, and $\hat{\upsilon}^* = \hat{\pi}^* = (\theta_K - \theta_V)(n+\hat{h})$. However, except for the real interest rate $r$, the centralized economy has a different set of steady-state equilibria given below:

$$\widetilde{K}^* = \widetilde{A}^* \left( \frac{s_K^*}{\delta + n + \hat{h} + \hat{A}^*} \right)^{1/(1-\alpha)} \tag{27a}$$

$$\widetilde{V}^* = \left[ s_V^* \left( \frac{s_K^*}{\delta + \theta_K(n+\hat{h})} \right)^{\alpha/(1-\alpha)} \left( \frac{\mu}{\theta_V(n+\hat{h})} \right)^{1/\lambda} \right]^{\theta_V} \tag{27b}$$

$$\widetilde{C}^* = (1-s^*)\widetilde{Y}^* \tag{27c}$$

$$\widetilde{\upsilon}^* = \frac{1}{\varepsilon\sigma} \left( \frac{\widetilde{Y}^*}{\widetilde{V}^*} \right) \left( \frac{\lambda}{r^* - \hat{Y}^* + (1-\phi)\hat{V}^*} \right) \tag{27d}$$

where

$$s_K^* = \alpha \cdot \left( \frac{\delta + (n+\hat{h}) + \hat{A}^*}{r^* + \delta} \right) \tag{27e}$$

$$s_V^* = \hat{V}^* \left( \frac{1}{\varepsilon\sigma} \right) \left( \frac{\lambda}{r^* - \hat{Y}^* + (1-\phi)\hat{V}^*} \right) \tag{27f}$$

$$r^* = r_0, \quad \zeta^* = 0 \tag{27g}$$

In these socially optimal equations, $\widetilde{Y}^* = \widetilde{A}^{*1-\alpha}\widetilde{K}^{*\alpha}$, $\widetilde{A}^* = \widetilde{V}^{*1/(\varepsilon\sigma(1-\alpha))}$ and $s_K^* + s_V^* = s^*$. Equation (27d) merit special attention.[27] There, the term $\frac{1}{\varepsilon\sigma} = \frac{V}{Y}\frac{\partial Y}{\partial V} = \frac{V}{TFP}\frac{\partial TFP}{\partial V}$ measures the elasticity of final output $Y$ or total factor productivity $TFP$ with respect to the measure of variety $V$ in terms of (18), while the term $\frac{\lambda}{r^* - \hat{Y}^* + (1-\phi)\hat{V}^*}$ is the discount factor that takes into account the externalities of knowledge spillovers $\phi$ and research congestion $\lambda$. Thus in this steady-state centralized economy, $\widetilde{\upsilon}^*$is the scale-adjusted social value of an individual innovation and $\widetilde{\upsilon}^*\widetilde{V}^*$the scale-adjusted social value of aggregate innovations, each in present-value term. Note that the term of $\varepsilon = \frac{1}{1-\sigma\alpha}$ (with $0 < \sigma < 1/\alpha$) is the price elasticity of demand for individual capital goods. This implies that if "innovative" capital goods are very similar (i.e. $\varepsilon$ approaches infinity), then the social value $\widetilde{\upsilon}^*$ of an innovation approaches zero due to $\frac{1}{\varepsilon\sigma} \to 0$. In this polar case, the socially optimal R&D intensity $s_V^*$ drops to zero, so is the scale-adjusted socially optimal knowledge capital.

where $\psi_K$ and $\psi_V$ are Lagrangian multipliers.

[27] In a centralized economy, there are no bounty claims. But in this economy the shadow price $\upsilon^*$ of innovations must satisfy $\upsilon^* = \frac{s_V^* Y^*}{\dot{V}^*} = \frac{s_V^* Y^*}{\hat{V}^* V^*}$, which implies $\widetilde{\upsilon}^* = \frac{s_V^* \widetilde{Y}^*}{\hat{V}^* \widetilde{V}^*}$ using $\upsilon = \widetilde{\upsilon}(hL)^{\theta_\upsilon}$, $Y = \widetilde{Y}(hL)^{\theta_K}$, $V = \widetilde{V}(hL)^{\theta_V}$, and $\theta_\upsilon = \theta_K - \theta_V$ according to (23c). Replacing $s_V^*$ with (27f), we can obtain (27d).

## 3.2 Patent regime is suboptimal

To show that a patent-regime economy is always suboptimal, we use (25a) - (25h) and (27a) - (27g) to derive the following relative ratios:

$$\frac{\widetilde{K}_0}{\widetilde{K}^*} = \left(\frac{\widetilde{V}_0}{\widetilde{V}^*}\right)^{1/[\varepsilon\sigma(1-\alpha)]} \left(\frac{z_0 s_{K0}}{s_K^*}\right)^{1/(1-\alpha)} \tag{28a}$$

$$\frac{\widetilde{V}_0}{\widetilde{V}^*} = \left(\frac{s_{V0}}{s_V^*}\right)^{\theta_V} \left(\frac{z_0 s_{K0}}{s_K^*}\right)^{\alpha\theta_V/(1-\alpha)} \tag{28b}$$

$$\frac{\widetilde{\upsilon}_0\widetilde{V}_0}{\widetilde{\upsilon}^*\widetilde{V}^*} = \left(\frac{s_{V0}}{s_V^*}\right)\left(\frac{\widetilde{K}_0}{\widetilde{K}^*}\right)^{\alpha}\left(\frac{\widetilde{V}_0}{\widetilde{V}^*}\right)^{\frac{1}{\varepsilon\sigma}} \tag{28c}$$

$$\frac{s_{K0}}{s_K^*} = 1 - m_0 \tag{28d}$$

$$\frac{s_{V0}}{s_V^*} = \left(\frac{\alpha m_0/\zeta_0}{1/(\varepsilon\sigma)}\right)\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{1}\right)\left(\frac{1/(r_0-\hat{Y}_0+\hat{V}_0)}{1/[r_0-\hat{Y}_0+(1-\phi)\hat{V}_0]}\right)\left(\frac{1}{\lambda}\right) \tag{28e}$$

It is clear from (28a) that as long as there is a non-zero patent length ($T > 0$), patent-regime agents must underinvest in the capital stock (because $\frac{s_{K0}}{s_K^*} < 1$ due to $0 < m_0 < 1$). The market mechanism is straightforward: patent-created monopolies raise capital-good prices above marginal costs and decrease the market incentive to build the stock of capital. Further, as we had mentioned earlier, there is a monopoly-induced technical distortion, denoted by $z_0$, this can decrease the economy's total factor productivity (see (19)). As a result, through the $s_{K0} - \text{cum} - z_0$ channel, the patent-regime capital and knowledge stocks must fall below their respective long-run Pareto optimal levels, as indicated by (28a) and (28b).

Further, through the $s_{V0}$ channel, the patent-regime capital and knowledge stocks can be affected as well. However, this channel is more complicated. For purposes of exposition, we break the R&D-intensity ratio $s_{V0}/s_V^*$ of equation (28e) into four sources:

(i) From the first parenthesized term, a patent-protected monopolist's valuation of an innovation is always less than social valuation at any point in time; i.e. $\alpha m_0/\zeta_0 < 1/(\varepsilon\sigma)$.[28] This is a *static valuation distortion*, making patent-regime agents **underinvest** in R&D. (ii) From the second term, patent-regime agents **underinvest** in R&D again, because a finite patent length is always too short to reflect an innovative durable's permanent contribution to household's welfare. This is a *dynamic valuation distortion* inherent in any patent system that rewards an innovation only for a finite duration. The static and dynamic valuation distortions combine to create the well-known "social-surplus appropriability" problem. (iii) The third term represents a positive *technical externality*, making patent-regime agents **underinvest** in R&D, because they do not internalize knowledge spillovers ($0 < \phi < 1$) in their private R&D decisions (see (8b)). (iv) Lastly,

[28]The private valuation of a patented capital good is $\alpha m_0/\zeta_0 = \alpha\left(\frac{\eta-1}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta}\right) = \left(\frac{\zeta\eta}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta}\right)\cdot\alpha/\varepsilon$ according to (21a) and $\eta = \varepsilon/(\varepsilon-1) > 1$. Since $\frac{\zeta\eta}{(1-\zeta)\eta^{\varepsilon}+\zeta\eta} < 1$ and $\alpha < \frac{1}{\sigma}$ from (4), the private valuation must be unambiguously less than the social valuation.

from the fourth term, $1/\lambda$ is greater than one, due to the *research congestion* externality ($\lambda < 1$) (see (8b)). For this negative *technical externality*, patent-regime agents **overinvest** in R&D. In contrast to patent-regime agents, the social planner internalizes both positive and negative externalities.

The above analysis indicates multiple distortions that tend to cause underinvestment in R&D in a decentralized patent-regime economy, whereas only the research-congestion distortion tends to the opposite. Empirical estimates seem to suggest that the research-congestion distortion alone should not dominate.[29] Thus, compared to the Pareto optimum, the patent regime should tend to have a smaller R&D intensity ($s_{V0}$) and should tend to build a smaller stock of capital and knowledge, respectively, on account of (28a) and (28b). Another important feature that merits attention is that in general the Pareto-optimal social value $\widetilde{\upsilon}^*$ of an innovation may or may not exceed its patent-regime private $\widetilde{\upsilon}_0$, because each of these two values is tied to the cost of value-created R&D in a general equilibrium framework. This is in contrast to previous partial equilibrium patent-prize studies. But it is clear that if the research congestion externality ($\lambda < 1$) does not dominate, the Pareto social value of aggregate innovations are greater than their corresponding social value according to $\frac{\widetilde{\upsilon}_0 \widetilde{V}_0}{\widetilde{\upsilon}^* \widetilde{V}^*} < 1$ (see (28c)).[30]

Simulation of the model later will lend support to our analysis here. Regardless, a patent system is apparently suboptimal, with no hope to duplicate the social planner's optimal solution.[31]

# 4 Switching to the Intertemporal Bounty Regime

Can a decentralized economy attain the Pareto optimum characterized by (27a) - (27g) if the patent regime is replaced with an alternative that rewards technological innovation with intertemporal bounties payable on the observed marginal-cost sales of bountiable goods? This is the central question of the paper. To facilitate the analysis, I firstly describe institutional arrangements for the intertemporal bounty (IB) regime to be a patent replacement.

## 4.1 Institutional arrangements

A feasible intertemporal bounty regime requires the following institutional arrangements:[32]

First, any bountiable innovation of a differentiated capital good must be *freely* licensed. This creates a free entry condition to establish a perfectly competitive market for bountiable durables that must sell at marginal cost. That is, $p = r + \delta$ with $\eta = 1$ (see (10)).

Second, the innovator is rewarded with a *perpetual bounty claim* to an infinite stream of future bounties, payable on the ex post marginal-cost sales of a specific bountiable capital good at a rate determined by the

[29] For instance, Jones and Williams (1998) find that an innovating economy's socially optimal R&D investment is at least four time greater than actual spending.

[30] Note that at any point in time some there are newly issued patents plus vintage patents that have either expired or not yet expired. But $\widetilde{\upsilon}_0$ is the scale-adjusted private value of a new patent. Therefor, $\widetilde{\upsilon}_0 \widetilde{V}_0$ has overestimated the actual private value of aggregate innovations.

[31] For instance, if the patent length goes to infinity ($T \to \infty$), $z_0 \to 1$, $\zeta_0 \to 1$, $m_0 \to (\eta - 1)/\eta = 1/\varepsilon$, and $\frac{\alpha m_0/\zeta_0}{1/(\varepsilon\sigma)} \to \sigma\alpha < 1$. These changes only mitigate to some extent the monopoly-created distortions. Adjusting the patent breadth (equivalent to changing the markup rate, $\eta$) cannot fix the sub-optimality problem, either.

[32] See Footnote 11 for earlier discussions of the IB system.

government. The number of bountiable goods is the same as the number of bounty claims. Denote by $\beta$ the bounty rate. Then given the ex post market sales $pX$ of the bountiable good, the bounty flow, denoted by $b$, is paid to the holder of a bounty claim at a point in time based on $b = \beta pX$. In this way, the prize for innovation is dynamically amortized in an infinite time domain as periodic bounties $b$, subject to the bounty rate $\beta$ and observed market sales $pX$. The IB system therefore releases the burden of estimating the size of prizes. Of course, the government still needs to estimate the bounty rate $\beta$, which we will show how to calculate using mostly aggregate data. As discussed in the introductory section, the IB system is more practical than those "once and for all" prize systems from the patent-prize economics literature.

Third, the government collects taxes from households to make bounty payments. For simplicity, the paper presumes a non-distortionary lump-sum tax available to fund such intertemporal bounties. The tax-financed bounty payment must be creditable so as to form an active asset market for tradeable bounty claims.[33] The equilibrium price $\upsilon$ of a typical bounty claim must therefore reflect the discounted present value of an infinite stream of future bounties. Thus the equilibrium bounty-claim price, $\upsilon$, informs the government and private agents of the private value of a bountiable innovation (blueprint). Households are the owners of bounty claims. So, the collected lump-sum tax must return to the household sector.

Fourth, the intertemporal-bounty regime grandfathers all those previously innovated durables whose patents are still legally live at the moment of the regime switch, which occurs at $t = 0$ in the modeled economy with live patents amounting to $V_p[0] = \zeta_0 V[0]$. Thus, at $t = 0$, the government needs to issue $\zeta_0 V[0]$ bounty claims immediately, and variable $\zeta$ would henceforth need to be redefined as the fraction of *bountied* (not *patented*) durables. Eliminating the grandfather clause would allow bounties and patents to co-exist before these patents expire legally. This would complicate the model and delay the gain to society from removing monopolies.

Under the above arrangements, all capital goods, either bountied or non-bountied, sell at the same marginal cost. This simplifies calculation of bounties. The government's aggregate bounty payments, denoted by $B$, at a point in time are equal to $B = \beta\alpha Y\zeta = \beta(r+\delta)K\zeta$, where $\alpha Y = (r+\delta)K$ is aggregate capital spending and $\alpha Y\zeta$ is aggregate markets sales of bountiable goods. That is, the government needs to collect from households a lump-sum tax equal to $B$. Certainly, the bounty flow to an individual bounty claim is determined by $b = B/\zeta V = \beta\alpha Y/V$, where $\zeta V$ is the number of bounty claims. In what follows the patent-regime model we have developed earlier will be switched to a bounty-regime model. This can be done by changing some parameters.

## 4.2 Dynamic system under the intertemporal bounty regime

Now, for the modeled economy to enter an intertemporal-bounty regime, we change some parameters that characterize the patent-regime model. To this end, the markup of $\eta > 1$ is changed to $\eta = 1$, the monopoly profit flow $\pi$ is replaced with the bounty flow $b$, and a finite patent length $T < \infty$ is re-parametrized as an infinite bounty length with $T \to \infty$.[34] In so doing, we can parametrize a regime switch from patents

[33] In practice, a bounty claim can split into shares to make it more liquid on the market.

[34] The bounty regime requires $T \to \infty$ because bounty claims are perpetual. For the flow budget constraint (2), per-capita lump-sum tax needs to be included so that $\dot{a} = (r-n)a + w - c - \frac{B}{L}$, where asset stock $a$ (including physical capital and bounty claims) earns net capital income (consisting of the bounty flow) at an interest rate $r$ on risk-free bonds. With no uncertainty, bonds and

to intertemporal bounties. This regime switch pushes the economy off the initial patent-regime balanced-growth path defined by (22a) - (22d) and gives rise to a bounty-regime dynamic system that evolves over time $t \in [0, \infty)$ as given below:

$$\dot{K}[t] = (1 - s_V[t])Y[t] - C[t] - \delta K[t] \tag{29a}$$

$$\dot{V}[t] = \mu V[t]^{\phi}(s_V[t]Y[t])^{\lambda} \tag{29b}$$

$$\dot{C}[t] = C[t](\frac{r[t] - \rho}{\gamma} + n) \tag{29c}$$

$$\dot{\upsilon}[t] = r[t]\upsilon[t] - b[t] \tag{29d}$$

where $Y[t]$, $s_V[t]$, $r[t]$ and $b[t]$ are determined by

$$Y[t] = (A[t]h[t]L[t])^{1-\alpha}K[t]^{\alpha} \tag{30a}$$

$$s_V[t] = \upsilon[t]^{1/(1-\lambda)}(\mu V[t]^{\phi})^{1/(1-\lambda)}/Y[t] \tag{30b}$$

$$r[t] = \frac{\alpha Y[t]}{K[t]} - \delta \tag{30c}$$

$$b[t] = \beta\alpha Y[t]/V[t] \tag{30d}$$

$$\zeta[t] = 1 - (1 - \zeta_0)e^{-\int_0^t \hat{V}[\tau]d\tau}, \ \ \zeta[0] = \zeta_0, \ \ \zeta[\infty] = 1 \tag{30e}$$

This is a nonlinear dynamic system of four first-order ordinary differential equations. This system starts at $t = 0$, driving the economy to transition to a new balanced-growth path in the long run. It contains new elements that warrant attention in some respects:

First, in contrast to (22d), (29d) is a no-arbitrage condition applying to infinitely lived bounty claims, where $\upsilon[t]$ is reinterpreted as the market price of a bounty claim (not a patent) and $b[t]$ is the bounty flow to the holder of a typical bounty claim. Second, the bounty flow $b[t]$ is based on the government-determined bounty rate $\beta$ and the observed market size of a bountied durable, $\alpha Y[t]/V[t]$, according to (30d). Third, the $TFP$ distortion term $z$ jumps to one as $\eta$ is set to one at the moment of the regime switch, thereby raising $TFP$ to the distortion-free level, as implied by (30a). Fourth, using (14) and $s_V Y = I_V$, one can readily derive (30b) to determine the R&D investment rate $s_V[t]$. Fifth, setting $m = 0$ (due to $\eta = 1$) in (21b) yields equation (30c) for the interest rate $r[t]$ at any moment. Lastly, (30e) determines the fraction $\zeta[t]$ of bountied durables. This is a state variable evolving to one in the long run, subject to the instantaneous innovation $\hat{V} = \dot{V}/V$ determined by the the dynamic system. But this state variable does not enter the dynamic system, since bountied and unbountied durables are equally priced.[35]

To examine how the regime switch is to impact the economy, we must solve the dynamic system of

other types of assets are perfect substitutes. Atomistic household treat the lump sum tax $B$ as given in their intertemporal decisions. Also, we need to re-interpret $p_p$ ($p_{np}$) as the price of bountied (unbountied) durables, and $V_p$ ($V_{np}$) as the number of bountied (unbountied) durables.

[35]Perpetual bounty claims implies $\dot{V}_p = \dot{V}$ and $\hat{V}_p = \frac{V}{V_p}\frac{\dot{V}}{V} = \hat{V}/\zeta$. Therefore, (17) needs to be changed to $\dot{\zeta} = (1 - \zeta)\hat{V}$, which solves for (30e).

(29a) - (29d) for a nonlinear stable manifold in the two-dimensional state space of capital and knowledge stocks. This is tantamount to computing the transition paths of $K[t]$, $V[t]$, $C[t]$ and $\upsilon[t]$, $t \in [0, \infty)$. But these variables are unbounded in the long run. Thus as under the initial patent regime, the bounty-regime dynamic system must be transformed into a stationary one.

### 4.3 Normalization of bounty-regime dynamic system

Normalizing the dynamic system of (29a) - (29d) requires four normalization factors, as we did for the patent regime. More details are given in what follows.

#### 4.3.1 Growth kernel-powered labor forces as normalization factors

Note that long-run growth rates are regime-independent in the model. They are parametrized by their associated growth kernels ($\theta_V$, $\theta_A$, $\theta_K$, $\theta_\upsilon$), the exogenous growth rate of efficiency ($\hat{e}$) and the population growth rate ($n$) in accordance with (23a) - (23c) and (24). As under the patent regime, these growth kernel-powered labor forces ($(hL)^{\theta_K}$, $(hL)^{\theta_V}$, $(hL)^{\theta_A}$, $(hL)^{\theta_\upsilon}$) serve as normalization factors such that $\widetilde{K} = K/(hL)^{\theta_K}$, $\widetilde{V} = V/(hL)^{\theta_V}$, $\widetilde{A} = A/(hL)^{\theta_A}$, $\widetilde{C} = C/(hL)^{\theta_K}$, $\widetilde{Y} = Y/(hL)^{\theta_K}$, $\widetilde{\upsilon} = \upsilon/(hL)^{\theta_\upsilon}$, and $\widetilde{b} = b/(hL)^{\theta_\upsilon}$.[36] These scale-adjusted quantities ($\widetilde{K}$, $\widetilde{V}$, $\widetilde{A}$, $\widetilde{C}$, $\widetilde{Y}$, $\widetilde{\upsilon}$, $\widetilde{b}$) are stationary and measurable as $t$ goes to infinity. Using this normalization approach and noting how those growth kernels are interrelated in terms of (24), we can transform (29a) - (29d) into a stationary dynamic system as given below:

$$\dot{\widetilde{K}}[t] = f_1[\vec{\omega}[t]] \equiv s_K[t]\widetilde{Y}[t] - (\delta + \theta_K(n+\hat{h}))\widetilde{K}[t] \tag{31a}$$

$$\dot{\widetilde{V}}[t] = f_2[\vec{\omega}[t]] \equiv \mu\widetilde{V}[t]^{\phi}(s_V[t]\widetilde{Y}[t])^{\lambda} - \theta_V(n+\hat{h})\widetilde{V}[t] \tag{31b}$$

$$\dot{\widetilde{C}}[t] = f_3[\vec{\omega}[t]] \equiv \widetilde{C}[t](\frac{r[t]-\rho}{\gamma} - (\theta_A n + \theta_K\hat{h})) \tag{31c}$$

$$\dot{\widetilde{\upsilon}}[t] = f_4[\vec{\omega}[t]] \equiv r[t]\widetilde{\upsilon}[t] - \widetilde{b}[t] - (\theta_K - \theta_V)(n+\hat{h})\widetilde{\upsilon}[t] \tag{31d}$$

subject to

$$\widetilde{K}[0] = \widetilde{K}_0,\ \widetilde{V}[0] = \widetilde{V}_0;\ \ \widetilde{K}[\infty] = \widetilde{K}_1,\ \widetilde{V}[\infty] = \widetilde{V}_1,\ \widetilde{C}[\infty] = \widetilde{C}_1,\ \widetilde{\upsilon}[\infty] = \widetilde{\upsilon}_1 \tag{31e}$$

where $\vec{\omega}[t] \equiv (\widetilde{K}[t], \widetilde{V}[t], \widetilde{C}[t], \widetilde{\upsilon}[t])$ is a vector of four unknown scale-adjusted quantities. Some other variables that depend on $\vec{\omega}[t]$ are scale-adjusted final output $\widetilde{Y}$[t], scale-adjusted bounty flow $\widetilde{b}[t]$, capital investment rate $s_K[t]$, R&D investment rate $s_V[t]$, interest rate $r[t]$ and bountiable fraction $\zeta[t]$. Their relationships are given by the following static equilibrium conditions:

$$\widetilde{Y}[t] = \widetilde{A}[t]^{1-\alpha}\widetilde{K}[t]^{\alpha},\ \text{with}\ \widetilde{A}[t] = \widetilde{V}^{1/(\varepsilon\sigma(1-\alpha))} \tag{32a}$$

[36] From (30d), the instantaneous growth rate of bounty flow $b$ is $\hat{b}[t] = \hat{Y}[t] - \hat{V}[t]$ and $\hat{b}[\infty] = (\theta_K - \theta_V)(n+\hat{h}) \equiv \theta_\upsilon(n+\hat{h})$ in steady state. Therefore, $(hL)^{\theta_\upsilon}$ serves as a normalization factor for $b$, as in the case of $\upsilon$.

$$\widetilde{b}[t] = \beta\alpha\widetilde{Y}[t]/\widetilde{V}[t] \tag{32b}$$

$$s_K[t] = (1 - s_V[t]) - \frac{\widetilde{C}[t]}{\widetilde{Y}[t]} \tag{32c}$$

$$s_V[t] = (\widetilde{\upsilon}[t] \cdot \mu\widetilde{V}[t]^{\phi})^{1/(1-\lambda)}/\widetilde{Y}[t] \tag{32d}$$

$$r[t] = \frac{\alpha\widetilde{Y}[t]}{\widetilde{K}[t]} - \delta \tag{32e}$$

$$\zeta[t] = 1 - (1-\zeta_0)e^{-\int_0^t(\hat{\widetilde{V}}[\tau]+\theta_V(n+\hat{h}))d\tau}, \;\; \zeta[0] = \zeta_0, \;\; \zeta[\infty] = 1 \tag{32f}$$

Equation (32f) results from (30e) by replacing the innovation rate $\hat{V}$ with $\hat{\widetilde{V}} + \theta_V(n+\hat{h})$ due to $V = \widetilde{V}(hL)^{\theta_V}$, where $\hat{\widetilde{V}} = 0$ in the long run. The normalized system features two state variables ($\widetilde{K}$, $\widetilde{V}$) and two jump variables ($\widetilde{C}$, $\widetilde{\upsilon}$). These two state variables – capital and knowledge stocks – are predetermined at any point in time and must evolve smoothly over time, whereas the two jump variables – aggregate consumption and price of bounty claim – are free to make a discrete change in response to shocks. The normalized system is subject to two initial-value boundary conditions represented by $\widetilde{K}[0] = \widetilde{K}_0$ and $\widetilde{V}[0] = \widetilde{V}_0$ at $t = 0$ as well as four other boundary conditions represented by $\widetilde{K}[\infty] = \widetilde{K}_1$, $\widetilde{V}[\infty] = \widetilde{V}_1$, $\widetilde{C}[\infty] = \widetilde{C}_1$, and $\widetilde{\upsilon}[\infty] = \widetilde{\upsilon}_1$ at $t \to \infty$.[37] This is a nonlinear two-point boundary value problem and can only be solved numerically. However, these boundary conditions are subject to the government-determined bounty rate $\beta$. So, before we can proceed to solve the boundary value problem in Section 5, we need to work out formulas for the government to calculate the bounty rate.

#### 4.3.2 Optimal bounty rate and the new balanced-growth path

At the outset, setting $\dot{\widetilde{K}} = \dot{\widetilde{V}} = \dot{\widetilde{C}} = \dot{\widetilde{\upsilon}} = 0$ in (31a) - (31d) allows us to solve for the steady-state boundary conditions at $t \to \infty$ (see Appendix B for derivations):

$$\widetilde{K}_1 = \left(\frac{s_{K1}}{\delta + \theta_K(n+\hat{h})}\right)^{1/(1-\alpha)} \widetilde{A}_1 \tag{33a}$$

$$\widetilde{V}_1 = \left[s_{V1}\left(\frac{s_{K1}}{\delta + \theta_K(n+\hat{h})}\right)^{\alpha/(1-\alpha)}\left(\frac{\mu}{\theta_V(n+\hat{h})}\right)^{1/\lambda}\right]^{\theta_V} \tag{33b}$$

$$\widetilde{C}_1 = (1 - s_1)\widetilde{Y}_1 \tag{33c}$$

$$\widetilde{\upsilon}_1 = \beta\alpha\left(\frac{\widetilde{Y}_1}{\widetilde{V}_1}\right)\left(\frac{1}{r_1 + \hat{V}_1 - \hat{Y}_1}\right) \tag{33d}$$

where

$$s_{K1} = s_K^*, \qquad \zeta_1^{Bountied} = 1 \tag{33e}$$

[37] Recall that we label a variable with subscript 0 (1) to indicate that variable associated with a balanced-growth patent (bounty) regime.

$$s_{V1} = \beta\alpha\hat{V}_1\left(\frac{1}{r_1+\hat{V}_1-\hat{Y}_1}\right) \quad (33f)$$

Note that $\widetilde{A}_1 = \widetilde{V}_1^{1/(\varepsilon\sigma(1-\alpha))}$ and $\widetilde{Y}_1 = \widetilde{A}_1^{1-\alpha}\widetilde{K}_1^{\alpha}$ due to $z=1$, while $\zeta_1^{Bountied} = \zeta[\infty] = 1$ in terms of (32f). On this bounty-regime balanced-growth path, we have $\hat{K}_1 = \hat{C}_1 = \hat{Y}_1 = \theta_K(n+\hat{h})$, $\hat{V}_1 = \theta_V(n+\hat{h})$, $\hat{A}_1 = \theta_A(n+\hat{h})$, and $\hat{\upsilon}_1 = \hat{b}_1 = (\theta_K-\theta_V)(n+\hat{h})$. As such, these long-run growth rates are regime-independent, so is the long-run interest rate ($r_1 = r_0 = r^*$). As (33e) indicates, on a balanced-growth path, the decentralized bounty regime always has the same physical capital investment rate as the social planner optimum (i.e., $s_{K1} = s_K^*$; see (27e)), irrespective of the magnitude of the bounty rate $\beta$. However, in general, a bounty regime does not have the same R&D investment rate as the Pareto optimal level (i.e., $s_{V1} \neq s_V^*$).[38] Unless the bounty rate, $\beta$, is set rightly to internalize the externalities of knowledge spillovers and research congestion, the bounty regime will not see a R&D investment rate as socially optimal. This problem can be fixed, however. By equalizing $s_{V1} = s_V^*$ based on (27f) and (33f), we can derive a unique ***long-run Pareto optimal bounty rate***, denoted by $\beta^*$. The formula for $\beta^*$ is given by:

$$\beta^* = \frac{\lambda}{\varepsilon-1}\cdot\left(\frac{r+\hat{V}-\hat{Y}}{r+(1-\phi)\hat{V}-\hat{Y}}\right) \quad (34)$$

where $\hat{V} = \left(\frac{\varepsilon-1}{\alpha}\right)(T\hat{F}P-(1-\alpha)\hat{h})$ due to $\hat{z}=0$ in (20). Hence, if $\beta$ is replaced with $\beta^*$ in (33e), in the long-run, the allocation of the bounty-regime economy's foregone consumption as a fraction of final output to R&D will be identical to what would otherwise obtain under social optimum. That is, $s_{K1} = s_K^*$ and $s_{V1} = s_V^*$ must hold together along a balanced-growth path if $\beta = \beta^*$. Given these Pareto optimal allocation of foregone consumption, a decentralized bounty-regime economy can completely mimic the social planner's Pareto optimum in the long run. For instance, if we compare (33a) - (33d) to (27a) - (27d), it is easy to verify that $\widetilde{K}_1 = \widetilde{K}^*$, $\widetilde{V}_1 = \widetilde{V}^*$, $\widetilde{A}_1 = \widetilde{A}^*$, $\widetilde{Y}_1 = \widetilde{Y}^*$, $\widetilde{C}_1 = \widetilde{C}^*$, $\widetilde{\upsilon}_1 = \widetilde{\upsilon}^*$ and $\widetilde{\upsilon}_1\widetilde{V}_1 = \widetilde{\upsilon}^*\widetilde{V}^*$along a balanced growth path. Under this long-run Pareto optimal IB regime, the private value $\widetilde{\upsilon}_1$ of a potential innovation is navigated to its social value $\widetilde{\upsilon}^*$, so is the private value $\widetilde{\upsilon}_1\widetilde{V}_1$ of aggregate innovations to its social value $\widetilde{\upsilon}^*\widetilde{V}^*$. Such a bounty regime can remove monopoly distortions to build a distortion-free capital stock, while also permitting to fully internalize the externalities of knowledge spillovers ($\phi$) and research congestion ($\lambda$) to develop a long-run Pareto optimal knowledge stock.

The formula for $\beta^*$ requires information on an economy's steady-state aggregate variables (innovation rate $\hat{V}$, final output growth rate $\hat{Y}$, and real interest rate $r$), parametrized externalities (knowledge spillovers $\phi$ and research congestion $\lambda$), and price elasticity of bountiable goods ($\varepsilon$). Data of this sort is empirically available. Long-run innovation rate $\hat{V}$ may be not readily available, but this problem can be overcome by relating innovation rate $\hat{V}$ to $TFP$ growth rate.

If the regime switch is only to remove monopoly distortions while maintaining the same R&D intensity at the patent-regime steady state level, we can obtain a ***long-run suboptimal bounty rate***, denoted by $\beta^{**}$,

[38] There is a short cut to deriving either $s_{K1}$ or $s_{V1}$: setting $m_0 = 0$ in (25e) yields $s_{K1}$, while replacing $\alpha m_0\hat{V}_0$ with $\beta\alpha\hat{V}_1$ and letting $T\to\infty$ in (25f) yields $s_{V1}$. In Appendix B, $s_{K1}$ and $s_{V1}$ are derived from steady-state equilibrium conditions.

by equalizing $s_{V0}$ and $s_{V1}$ based on (25f) and (33f). The formula for this suboptimal bounty rate is given by

$$\beta^{**} = \frac{m}{\zeta}\left(1 - e^{-(r-\hat{Y}-\hat{V})T}\right) \tag{35}$$

In addition to the same steady-state aggregate variables as for the calculation of $\beta^*$, the calculation of $\beta^{**}$requires the patent length ($T$), the relative size of the patent-protected noncompetitive sector ($\zeta$), and the monopoly-profit share ($m$) in aggregate capital income. Both $\beta^*$and $\beta^{**}$will be computed in the next section for the model calibrated to an innovating economy like the United States.

# 5 Numerical Analysis: Transition Paths & Welfare

This section presents numerical analysis based on the normalized bounty-regime dynamic system of (31a) - (31d) along with its associated two-point boundary conditions given in (31e).

## 5.1 Calibration of the model and steady-state simulations

At the outset we use a set of benchmark parameters to calibrate the patent-regime model to the U.S. economy. To this end, a systemic calibrating strategy is to solve equations (21a), (21b), (23a) and (25e)-(25g) for parameters $\rho$, $\gamma$, $\sigma$, $\phi$, $\lambda$ and $\hat{h}$, given some empirically more reliable parameter values (such as $n$, $\delta$, $\alpha$, $T$) and some long-run stylized economic statistics.[39] The remaining parameter, $\mu$, in the innovation production function can then be calibrated separately, as described later. Therefore, the calibrated model can deliver a balanced-growth path that closely reflects the U.S. economy's long-run stylized observations. The above model calibration generates a benchmark set of parameters in Table 1. Some of them are identical to or close to those used in previous calibration exercises, such as Jones and Williams (2000), Eicher and Turnovsky (2001) and Grossmann et al. (2013). Descriptions of these benchmark parameters are given below.

Table 1: Benchmark parameters for model calibration

| | | | | | |
|---|---|---|---|---|---|
| Production: | $\alpha = 0.38$ | $\sigma = 1.80$ | $\mu = 0.14$ | $\phi = 0.50$ | $\lambda = 0.50$ |
| Preference: | $\rho = 0.025$ | $\gamma = 2.0$ | | | |
| Patent length: | $T = 20$ | | | | |
| Others parameters: | $\delta = 0.05$ | $n = 0.01$ | $\hat{h} = 0.012$ | | |

**Production and preference parameters** In calibration of the final goods production function, the parameter of $\alpha$ is set equal to 0.38, meaning that the capital share in GDP is 38% and the labor share is 62%. The parameter of $\sigma$ is set equal to 1.80, which reflects the substitutability of capital goods.. These two parameters lead to the price elasticity of demand for a typical capital good equal to 3.16 ($= \varepsilon = 1/(1-\sigma\alpha)$)

[39] For the U.S. economy, parameters $n = 0.01$, $\delta = 0.05$, $\alpha = 0.38$ and $T = 20$ are relatively reliable or accurate. Its long-run stylized statistics include the capital/output ratio ($\frac{K}{Y} \approx 3$), the annual rate of return on capital ($r \approx 0.07$), the annual growth rate of per capital GDP ($\hat{y} = \hat{Y} - n \approx 0.02$), the annual TFP growth rate ($T\hat{F}P \approx 1.25$), the R&D intensity ($s_V \approx 0.025$) and the consumption rate ($1 - s = 1 - s_V - s_K \approx 0.72$).

and the markup rate of about 1.46 ($=\eta=\frac{1}{\sigma\alpha}$) for patented capital goods.[40] For the production function of durable-good designs, the knowledge-spillover parameter, $\phi$, is set equal to 0.50, which is less than the upper bound of $\bar{\phi}=1-\lambda/(\varepsilon\sigma(1-\alpha))\approx 0.86$ (see Footnote 16); the research-congestion parameter, $\lambda$, is also set equal to 0.50; and the research productivity parameter, $\mu$, requires more elaboration and is to be calibrated later. For the preference parameters, the rate of time preference, $\rho$, is constant at 0.025 per year and the elasticity of intertemporal substitution is assumed to be $1/\gamma=0.5$ (that is, $\gamma=2$).

**Patent length and other parameters** The patent length is set at $T=20$ years in accordance with US and WTO (World Trade Organization) patent policy. The rate of physical capital depreciation is set at $\delta=0.05$ per year and the population of labor $L[t]$ is set to grow annually at a constant rate of $n=\frac{dL}{L}=0.01$. As to $h[t]$, a non-R&D-driven efficiency index, is set to grow annually at a constant of $\hat{h}=\frac{dh}{h}=0.012$. While this efficiency index is unobservable, the above-described calibration allows us to determine its value so as to be consistent with the U.S. economy's long-run growth rates of GDP, per GDP, and $TFP$. Note that the time unit in the numerical analysis represents one year, since the chosen parameters such as $\rho$, $n$ and $\hat{h}$ are in annual rates.

**Remarks on R&D-driven TFP growth rate** Note that the above-calibrated production parameters ($\alpha$, $\sigma$, $\phi$, $\lambda$) are sufficient to compute the model's growth kernels in terms of (24). They are $\theta_V=1.40$ , $\theta_A=0.40$, and $\theta_K=1.40$. These growth kernels multiplied by the sum of $n=0.01$ and $\hat{h}=0.012$ can deliver the plausible long-run real interest rate of $r_0=0.066$ (see (25g)) and the following empirically relevant long-run growth rates for the benchmark patent-regime U.S. economy: $\hat{V}_0=0.031$, $\hat{A}_0=0.0087$, $T\hat{F}P_0=(1-\alpha)(\hat{A}_0+\hat{h})=0.0128$, $T\hat{F}P_0^{R\&D}=(1-\alpha)\hat{A}_0=0.0054$, $\hat{K}=\hat{Y}=\hat{C}=0.031$, and $\hat{k}=\hat{y}=\hat{c}=0.02$. These figures say that the annual growth rates of GDP, per capita GDP, and $TFP$ are about 3%, 2%, and 1.28%, respectively. Of the 1.28% growth rate of $TFP$, the R&D-driven portion accounts for 0.58 percent points. That is, in the calibrated model, more than 50% of $TFP$ growth is driven by non-R&D elements such as learning by doing, management, etc., and as an echo of Comin (2004), R&D just contributes a small portion to per capital GDP growth.[41]

**Calibration of research productivity parameter $\mu$** Now let us return to calibration of the research productivity parameter, $\mu$. First, since labor supply $L$ and non-R&D-driven efficiency index $h$ are exogenous, we chose $L[0]=L_0=h[0]=h_0=1$. Second, since there is only one remaining parameter, $\mu$, to be calibrated, we have a degree of freedom to choose either $K_0$ or $V_0$ at $t=0$, and we chose $V_0=1$, which implies $\widetilde{V}_0=V_0/(h_0L_0)^{\theta_V}=1$. Third, with $\widetilde{V}_0=1$ and all the already-calibrated parameters, we can solve the eight equations of (25a) - (25h) for parameter $\mu=0.14$ along with the other seven variables ($\widetilde{K}_0$, $\widetilde{C}_0$, $\widetilde{\upsilon}_0$ $s_{K0}$, $s_{V0}$, $r_0$, $\zeta_0$). Using this procedure, parameter $\mu$ is calibrated at 0.14, as indicated in Table 1.

[40] In contrast, the markup rate for unpatented durables equals one. Given the share of patented durables ($\zeta$) equal to 46% (as in the benchmark case), the industry-wide average markup rate is about 1.21, which falls onto the empirical evidence range (see Norrbin 1993 and Basu 1996).

[41] According to Comin (2004), the contribution of R&D to per capita GDP growth in the U.S. is smaller than $3-5^{th}$ of 1% point.

In calibration of the $\mu$ parameter, we have generated the initial patent-regime steady state at the same time. Now with a complete set of benchmark parameters in Table 1, we can solve (33a) - (33f) and (34) for the bounty-regime steady state, respectively, based on the long-run Pareto-optimal bounty rate ($\beta^*$) and suboptimal bounty rate ($\beta^{**}$). Hence, listed in Table 2 are three sets of steady-state results including (i) patent-regime steady state, (ii) bounty-regime steady state with long-run Pareto optimal bounty rate $\beta^*$, and (iii) bounty-regime steady state with long-run suboptimal bounty rate $\beta^{**}$. The following analysis are in order:

Table 2: Steady states of patent and intertemporal-bounty regimes

| Patent Regime (Benchmark) | | Intertemporal-Bounty Regime | | |
|---|---|---|---|---|
| patent length | $T = 20$ | bounty rate | $\beta^* = 0.30$ | $\beta^{**} = 0.14$ |
| $\widetilde{K}_0$ | 5.67 | $\widetilde{K}_1$ | 9.50 | 6.97 |
| $\widetilde{V}_0$ | 1.00 | $\widetilde{V}_1$ | 3.35 | 1.12 |
| $\widetilde{Y}_0$ | 1.90 | $\widetilde{Y}_1$ | 2.91 | 2.14 |
| $\widetilde{C}_0$ | 1.40 | $\widetilde{C}_1$ | 1.99 | 1.52 |
| $\widetilde{\upsilon}_0$ | 1.49 | $\widetilde{\upsilon}_1$ | 1.49 | 1.49 |
| $\widetilde{\upsilon}_0\widetilde{V}_0$ | 1.49 | $\widetilde{\upsilon}_1\widetilde{V}_1$ | 4.99 | 1.67 |
| $s_{K0}$ | 0.24 | $s_{K1}$ | 0.26 | 0.26 |
| $s_{V0}$ | 0.024 | $s_{V1}$ | 0.053 | 0.024 |
| $\widetilde{C}_0/\widetilde{Y}_0$ | 0.73 | $\widetilde{C}_1/\widetilde{Y}_1$ | 0.68 | 0.71 |
| $\widetilde{K}_0/\widetilde{Y}_0$ | 2.99 | $\widetilde{K}_1/\widetilde{Y}_1$ | 3.27 | 3.27 |
| $\widetilde{V}_0/\widetilde{Y}_0$ | 0.53 | $\widetilde{V}_1/\widetilde{Y}_1$ | 1.15 | 0.53 |
| $z_0$ | 0.98 | $z_1$ | 1.00 | 1.00 |
| $\zeta_0^{Patented}$ | 0.46 | $\zeta_1^{Bountied}$ | 1.00 | 1.00 |
| $\Pi/Y$ | 0.033 | $B/Y\ (=\alpha\beta)$ | 0.11 | 0.052 |

Remarks: $\beta^*$ ($\beta^{**}$) is long-run Pareto-optimal (sub-optimal) bounty rate. Regime-independent are long-run real interest rate $r_0 = r_1 \approx 0.066$ and long-run growth rates: $\hat{V}_0 = \hat{V}_1 \approx 0.031$; $\hat{K}_0 = \hat{Y}_0 = \hat{C}_0 = \hat{K}_1 = \hat{Y}_1 = \hat{C}_1 \approx 0.031$; $\hat{k}_0 = \hat{y}_0 = \hat{c}_0 = \hat{k}_1 = \hat{y}_1 = \hat{c}_1 \approx 0.02$; and $T\hat{F}P_0 = T\hat{F}P_1 \approx 0.0128$, of which $T\hat{F}P_0^{R\&D} = 0.0054$. Figures are computed using benchmark parameters from Table 1.

**Patent-regime steady state (patent length $T = 20$)** First, from Table 2, the steady state of the benchmark patent regime pretty much matches the United States macroeconomic features including the R&D intensity ($s_{V0} = 0.024$), the physical capital investment rate ($s_{K0} = 0.24$), the consumption/GDP ratio ($\widetilde{C}_0/\widetilde{Y}_0 = 1 - s_{K0} - s_{V0} = 0.73$), the capital/output ratio ($\widetilde{K}_0/\widetilde{Y}_0 = 2.99$), in addition to the long-run real interest rate $r$ and long-run growth rates of GDP and $TFP$ (see Remarks of Table 2). The knowledge/output ratio ($\widetilde{V}_0/\widetilde{Y}_0$) is 0.53. Empirically, this ratio is not precisely available. Under the patent regime, 46% of durable goods firms are patented ($\zeta_0 \approx 0.46$) and the resulting monopoly-induced technical distortion makes $TFP$ about 2% below its potential level, as implied by $z_0 \approx 0.98$.

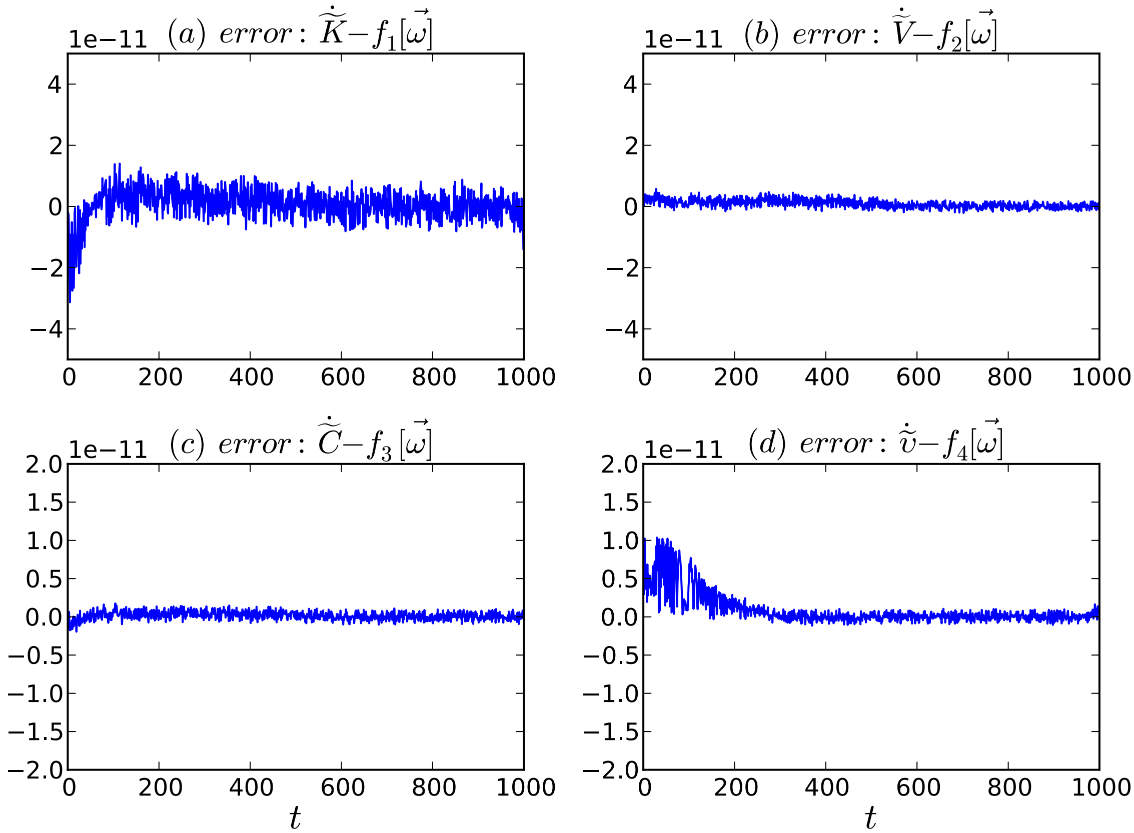
**Bounty-regime steady state (bounty rate $\beta^* = 0.30$)** We assume the calibrated US economy is initially in the patent-regime steady state. If we let this decentralized economy switches to an intertemporal-bounty regime that can fully mimic the social planner's long-run Pareto optimum, as characterized by (27a) - (27g), the required bounty rate needs to be set at $\beta^* = 0.30$ according to (34). This long-run Pareto optimal bounty rate dictates that for each dollar sales of bountied goods, the government pays a bounty flow of 30 cents annually to the bounty-claim holder. The resulting annual aggregate bounty flows ($B$) account for 11% of GDP ($= Y$), which is funded by a lump-sum tax. In contrast, under the patent regime with a markup rate ($\eta$) of 1.46, the patent holder receives a profit margin of 46 cents from each \$1.46 sales of patented goods. Such patent-created monopoly profits ($\Pi$) account for about 3.3% of GDP. Economists call these profits "economic profits," which are profits beyond the usual or standard rate of business profit.

By implementing the long-run Pareto optimal bounty rate of $\beta^* = 0.30$, the regime switch removes monopoly distortions and optimizes the allocation of foregone consumption to fully internalize knowledge-spillover and research-congestion externalities. This allows the economy to attain the Pareto optimum in the long-run, and the associated steady state effects are reported in the $\beta^* = 0.30$ column of Table 2. As indicated, in the long run, the economy becomes much more intensive in capital and knowledge under this bounty regime. The capital investment rate increases from $s_{K0} = 0.24$ to $s_{K1} = 0.26$, while the R&D intensity increases significantly from $s_{V0} = 0.024$ to $s_{V1} = 0.053$. The stock of scale-adjusted capital rises from $\widetilde{K}_0 = 5.67$ to $\widetilde{K}_1 = 9.50$, while even more significant is a remarkable increase in the stock of scale-adjusted knowledge from $\widetilde{V}_0 = 1.00$ to $\widetilde{V}_1 = 3.35$. As a result, under the regime switch, the capital/output ratio ($\widetilde{K}/\widetilde{Y}$) rises from 2.99 to 3.27, the knowledge/output ratio ($\widetilde{V}/\widetilde{Y}$) rises significantly from 0.53 to 1.15, and the shadow price of aggregate innovations is navigated upwards to the social value of $\widetilde{\upsilon}_1 \widetilde{V}_1 = \widetilde{\upsilon}^* \widetilde{V}^* = 4.99$, way above the private value of aggregate innovations under the patent regime.[42] This explains why the regime switch can generate a long-run consumption gain of about 42% (comparing $\widetilde{C}_0 = 1.40$ to $\widetilde{C}_1 = 1.99$). A drop of the consumption/GDP ratio from 0.73 to 0.68 is, in fact, indispensable. Compared to the patent regime, the long-run Pareto-optimal bounty regime requires much more foregone consumption to establish an economy that is more knowledge-intensive and more capital-intensive.

**Bounty-regime steady state (bounty rate $\beta^{**} = 0.14$)** As analyzed above, switching to the long-run Pareto optimal bounty regime requires an annual bounty budget of over 10% of $GDP$ in terms of $B/Y = 0.11$. This could place a noticeable budget pressure on the government in the real world. If instead the government pursues a switch to the aforementioned long-run suboptimal bounty regime, the required long-run suboptimal bounty rate is as low as $\beta^{**} = 0.14$ according to (35) and the resulting annual bounty budget significantly drops to only 5.2% of $GDP$ (that is, $B/Y = 0.052$). The steady-state effects of implementing such a suboptimal regime switch are reported in the $\beta^{**} = 0.14$ column of Table 2. In comparison to the long-run Pareto optimal bounty regime ($\beta^* = 0.30$), we observe that while the capital investment rate still

[42]Note that $\widetilde{\upsilon}_0 = \widetilde{\upsilon}_1 = 1.49$ in Table 2. This is because $\phi = \lambda = 0.5$, which ensure that $\theta_V = \theta_K$ and $\theta_\upsilon = \theta_V - \theta_K = 0$. Thus, $\upsilon = \widetilde{\upsilon}$ and $\hat{\upsilon} = 0$ in the long run according to (23c) and (24).

Figure 2: Absolute errors of solving (31a) -(31d) with Python's **scikits.bvp_solver**




Remarks: $\vec{\omega} \equiv [\widetilde{K}, \widetilde{V}, \widetilde{C}, \widetilde{\upsilon}]$ for $t \in [0, 1000]$ and $-1 \times 10^{-10} < error < 1 \times 10^{-10}$.

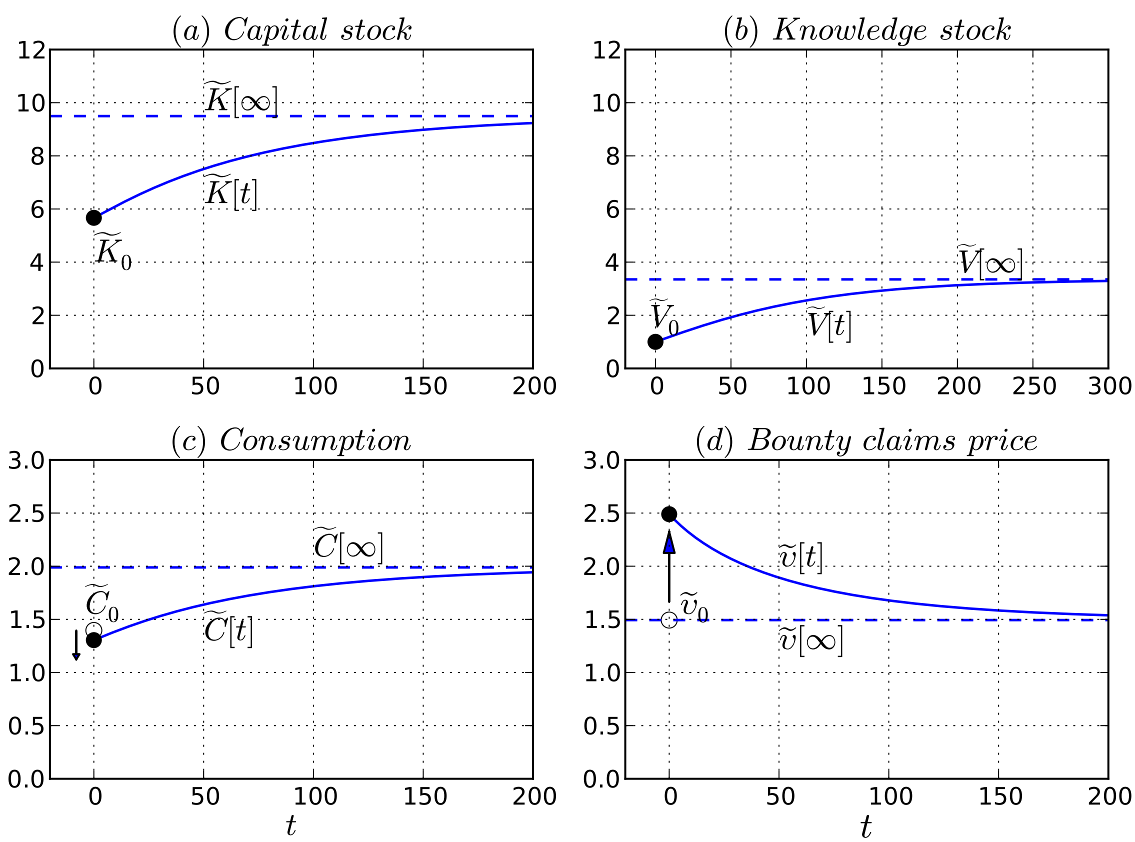
can reach the Pareto optimal level ($s_{K1} = s_K^* = 0.26$), the R&D intensity can only be maintained at the suboptimal patent-regime level ($s_{V1} = s_{V0} = 0.024$). As a consequence, the private value ($\widetilde{\upsilon}_1 \widetilde{V}_1 = 1.67$) of aggregate innovations under the suboptimal bounty regime is way below the Pareto-optimal social value ($\widetilde{\upsilon}_1 \widetilde{V}_1 = \widetilde{\upsilon}^* \widetilde{V}^* = 4.99$). This results in a knowledge stock ($\widetilde{V}_1 = 1.12$) only slightly above the patent-regime level ($\widetilde{V}_0 = 1$) and way below the socially optimal level ($\widetilde{V}_1 = \widetilde{V}^* = 3.35$). In welfare terms, Switching to this suboptimal bounty regime generates a smaller long-run consumption gain of 9% ($\widetilde{C}_1 = 1.52 > \widetilde{C}_0 = 1.40$), compared to the long-run optimal consumption gain of 42% as noted above. This is not a surprise at all, since $\beta^{**}$ only resolves monopoly distortions, whereas $\beta^*$addresses both monopoly distortions and externalities.

The above analysis focuses on the regime switch's steady state effects only. However, a complete welfare analysis must take transitional dynamics into account. To this end, we trap how the bounty-regime dynamic system evolves over time in transition in the next subsection.

### 5.2 Dynamics in transition

**Numerical solution** To compute transitional dynamics, we focus on the regime switch by implementing the long-run Pareto optimal bounty rate $\beta^* = 0.30$. The dynamic system of (31a) - (31d) calibrated by the benchmark parameter set (Table 1) is found to display saddle path stability around its long-run stationary equilibrium, which refers to the bounty-regime steady state, $[\widetilde{K}_1, \widetilde{V}_1, \widetilde{C}_1, \widetilde{\upsilon}_1]$, shown in Table 2. The regime switch from patents to the intertemporal bounty means a structural change taking place at $t = 0$. Computing the transitional dynamics should proceed by trapping the calibrated model's nonlinear stable manifold rather than its linearized model's local saddle path around the bounty-regime steady state. I used a Python boundary value problem solver (**scikits.bvp_solver 1.1**) to solve the calibrated system of (31a) - (31d) for the approximate solution $\vec{\omega} = [\widetilde{K}, \widetilde{V}, \widetilde{C}, \widetilde{\upsilon}]$.[43] The calibrated system is defined on $[0, \infty)$, a semi-infinite time interval. To apply the Python bvp solver, I truncated it into a finite time interval, $[0, t_{max}]$, where $t_{max} = 1000$, and it turns out that one thousand years is long enough for economy to work out all transitional effects.[44] Indeed, the Python bvp solver proved fast in just a few seconds and generated precise enough results. As Figure 2 indicates, the approximate solution is precise to the extent that its absolute errors (or residuals) are all less than $10^{-10}$ in absolute value.

Figure 3: Numerical solution of dynamic system (31a) - (31d)

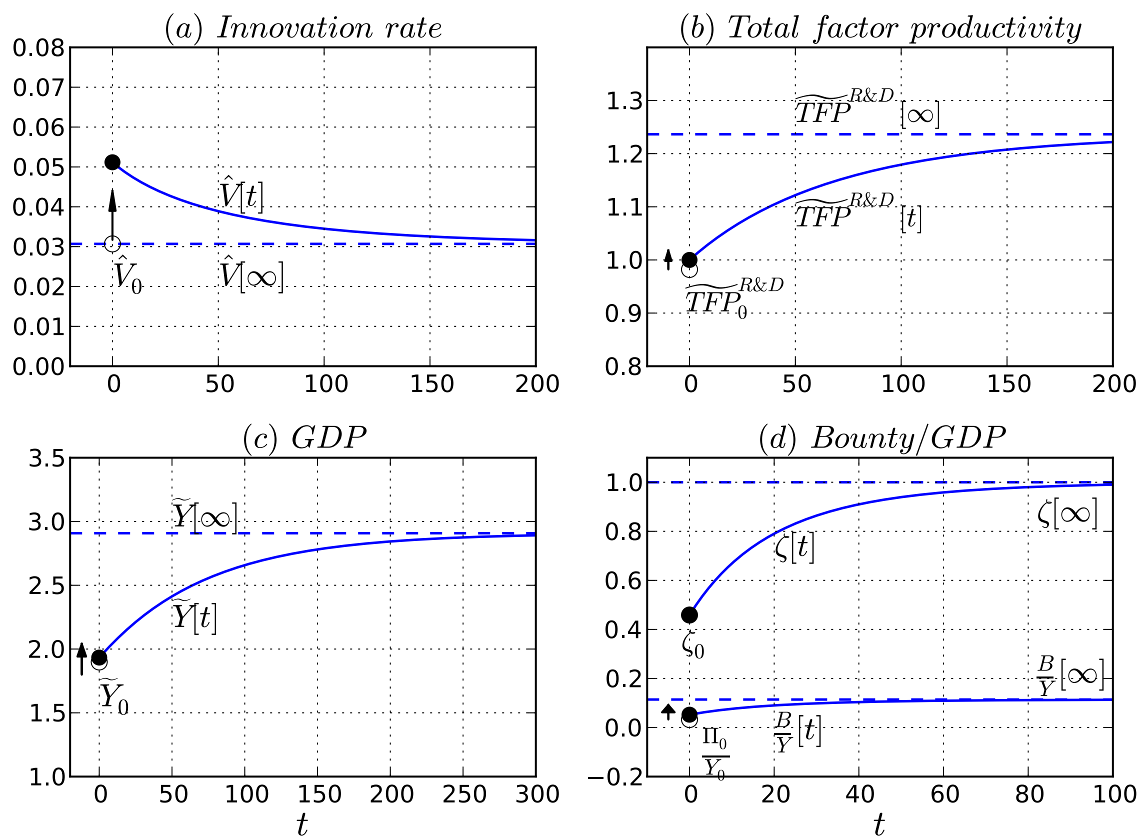



[43] **scikits.bvp_solver** (http://pypi.python.org/pypi/scikits.bvp_solver) is a Python package that wraps a Fortran BVP_SOLVER based on Mono Implicit Runge Kutta methods; see Shampine et al. (2006). All computations for the present paper are implemented using Python 2.7.3 on Mac OS 10.6.8. The Python code is available from the author upon request. A Matlab code using **bvp4c** is also available.

[44] The boundary conditions at $t \to \infty$ need to be approximately satisfied at $t_{max}$. If not, we must increase the value of $t_{max}$ until the errors are negligible.

The approximate solution is shown in Figure 3 including the transition paths of the dynamic system's two state variables ($\widetilde{K}$, $\widetilde{V}$) and two jump variables ($\widetilde{C}$, $\widetilde{\upsilon}$). The stocks of capital $\widetilde{K}[t]$ and knowledge $\widetilde{V}[t]$ have continued to rise in transition until they both reached their long-run steady state equilibrium, respectively. At the moment of the regime switch, the consumption flow $\widetilde{C}$ jumps down from $\widetilde{C}_0$ to $\widetilde{C}[0]$ and then starts to grow over time toward a higher steady-state level. To the contrary, the price of the bounty claim $\widetilde{\upsilon}$ jumps up from $\widetilde{\upsilon}_0$ to $\widetilde{\upsilon}[0]$ and then starts to fall over time toward a lower steady-state level.

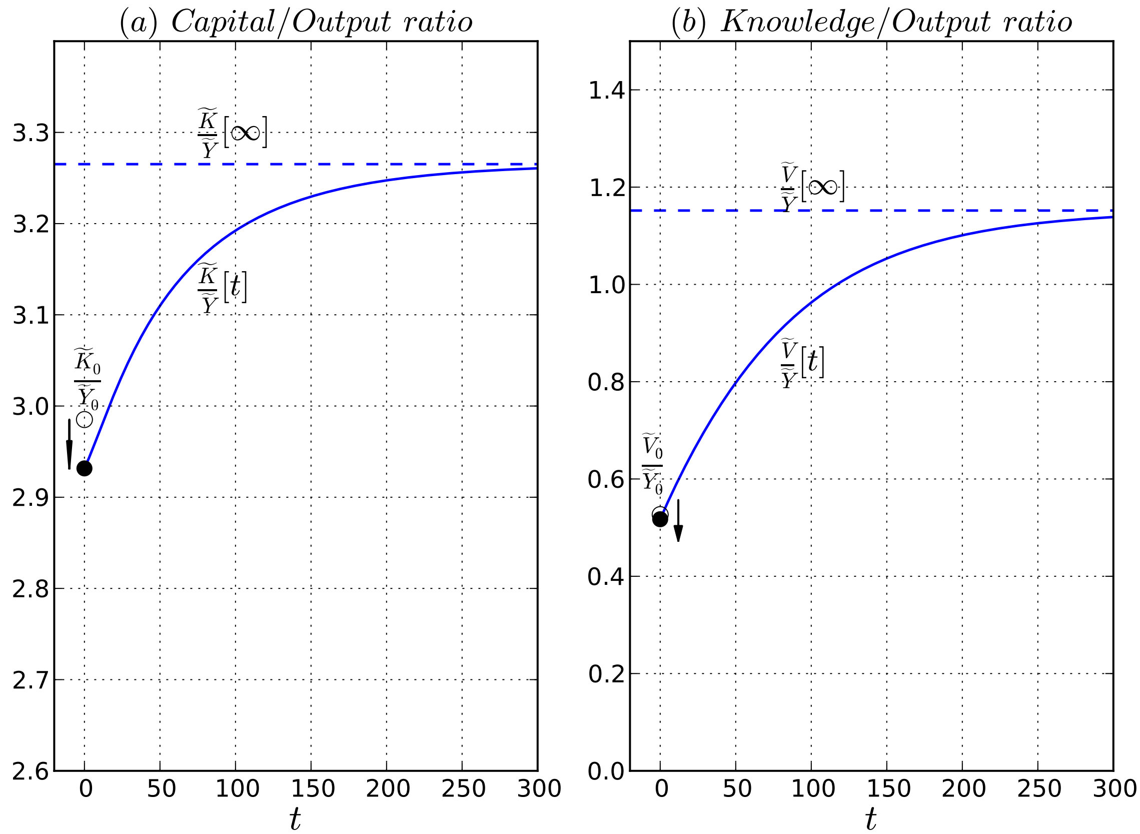
Figure 4: Transition paths: $\hat{V}$, $\widetilde{TFP}$, $\widetilde{Y}$, $\zeta$, and $B/Y$




The above-described solution allows to compute other variables' trajectories. Figure 4 indicates that: (i) at the moment of the regime switch, the innovation rate $\hat{V} \equiv \dot{V}/V$ jumps up remarkably from $\hat{V}_0$ to $\hat{V}[0]$ and then drifts down smoothly in transition to the same steady state level ($\hat{V}_0 = \hat{V}_1 = \theta_V n$) (panel (a)); (ii) to a lesser degree, both total factor productivity $\widetilde{TFP}$ ($= \widetilde{A}^{1-\alpha} = \widetilde{V}^{1/(\varepsilon\sigma)}$) and final output $\widetilde{Y}$ jump up at $t = 0$, but unlike the innovation rate, they both continue to rise in transition to their long-run steady state levels (panels (b) & (c)); and (iii) driven by persistent innovation ($\hat{V}[t] > 0$), the bountied fraction $\zeta$ of capital goods keeps rising all the way toward one (its long-run steady state level), while the bounty/output ratio $B/Y$ also keeps increasing in transition to a higher steady state ratio equal to 0.11 – this ratio is greater than the patent-regime steady state profit/output ratio $\Pi_0/Y_0 = 0.033$ (Table 2 or pane d)

Notably, the regime switch makes the economy transition to one much more intensive in both capital

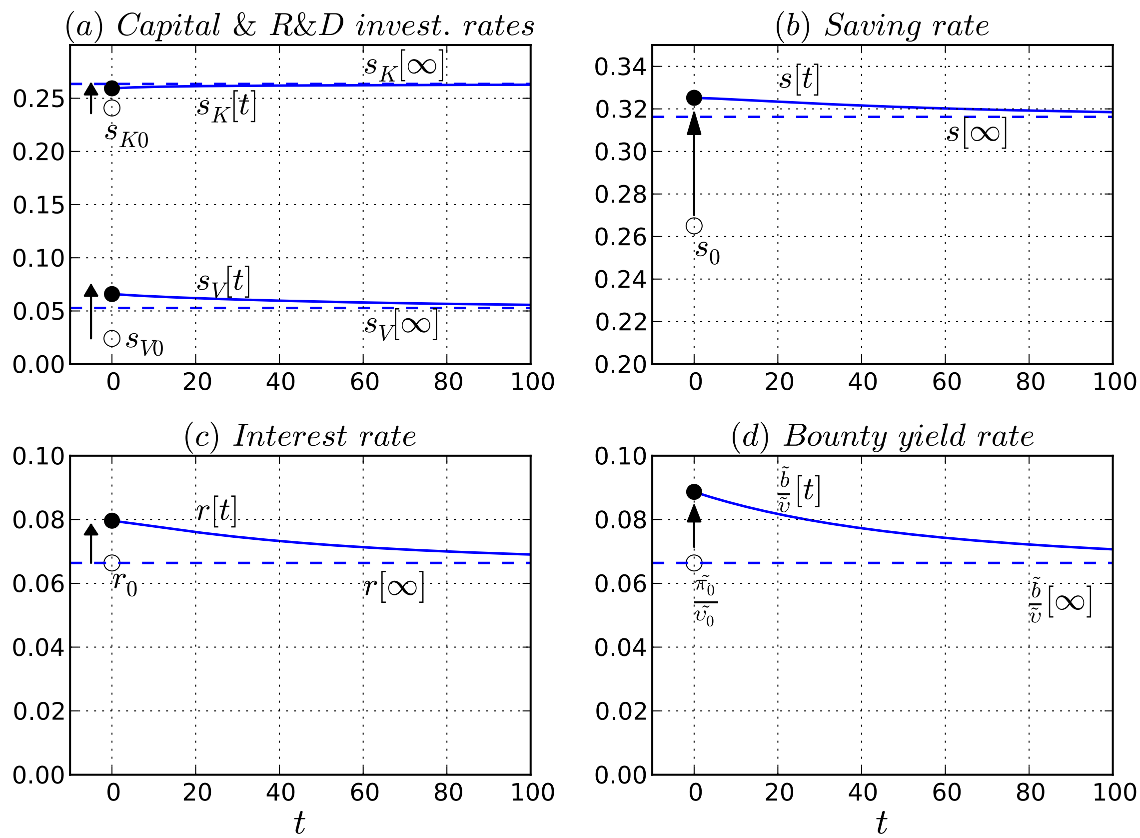
Figure 5: Transition paths: $\frac{\widetilde{K}}{\widetilde{Y}}$, $\frac{\widetilde{V}}{\widetilde{Y}}$




and knowledge. Figure 5 indicates this feature: the capital/output and knowledge/output ratios ($\frac{\widetilde{K}}{\widetilde{Y}}$, $\frac{\widetilde{V}}{\widetilde{Y}}$) have trended upward over time remarkably to their new steady state equilibrium, although these ratios experience a small drop at the moment of the regime switch when final output $Y$ jumps up slightly. All these transition paths described above make intuitive sense: a more capital-intensive, more knowledge-intensive economy must require the dual occurrences of short-run consumption sacrifices and bounty-claim appreciation (compared to the initial patent value $\widetilde{v}_0$) so as to draw more resources into capital accumulation and R&D investment. By implementing the long-run Pareto optimal bounty rate of $\beta^* = 0.30$, this bounty regime can therefore raise the long-run productive capacity to sustain an eventually higher level of consumption than does the initial patent regime.

**Market mechanism** How does the decentralized IB mechanism work to drive the calibrated U.S. economy to transition to a long-run Pareto optimal balanced growth path, as characterized above by Figures 3 - 5? First of all, this involves a removal of all patent-created distortions. Thus, at the moment of the regime switch, the market for capital goods becomes perfectly competitive, selling all durables at marginal cost. This boosts market demand especially for those durables whose patents have not yet expired, while also removing the patent-induced technical distortion on total factor productivity (i.e., $z_0$ jumps to $z[0] = 1$). This

Figure 6: Transition paths: $s_K$, $s_V$, $s$, $r$, $\frac{\tilde{b}}{\tilde{v}}$




market mechanism explains why total factor productivity and GDP ( $\widetilde{TFP}$, $\widetilde{Y}$) jump up together at $t = 0$ (Figure 4 (b) & (c)), and why the capital /output ratio ($\frac{\widetilde{K}}{\widetilde{Y}}$) and knowledge/output ratio ($\frac{\widetilde{V}}{\widetilde{Y}}$) jump down at the same time (Figure 5), given that $\widetilde{K}$ and $\widetilde{V}$ are predetermined at any point in time.

Further, at the moment of the regime switch, the long-run Pareto-optimal bounty rate $\beta^*$ is strong enough to reward technological innovation. It creates bounty claims that are more valuable than patents ($\widetilde{v}$ jumps up at $t = 0$; Figure 3 (d)) in the asset market. It also generates a strong bounty flow relative to bounty-claim price, $\frac{\widetilde{b}}{\widetilde{v}}$, that outweighs the initial monopoly profit flow relative to patent price, $\frac{\widetilde{\pi}_0}{\widetilde{v}_0}$ (Figure 6 (d)). Thus, switching to the bounty regime presents a strong incentive to boost investment in both physical capital and R&D, causing both $s_K$ and $s_V$ to jump up at $t = 0$ (Figure 6 (a)). The resulting strong demands for foregone consumption therefore make the real interest rate $r$ (Figure 6 (c)) and saving rate $s$ (Figure 6 (b)) jump up, respectively, at the moment of the regime switch. All these market forces prompt the calibrated U.S. economy to evolve in transition toward a more capital-intensive and more knowledge-intensive economy.

### 5.3 Welfare change in transition to the bounty-regime steady state

Using the numerical trajectory of consumption shown earlier in Figure 3 (c), we can compute the welfare change for the regime switch from patents to an intertemporal bounty that allows the calibrated U.S. economy to attain Pareto optimality in the long run. The welfare change is the measure of $\Omega[t]$ that obeys

$$U[\{(1+\Omega[t])c[\tau]^{Patent}\}_{\tau=0}^{t}] = U[\{c[\tau]^{Bounty}\}_{\tau=0}^{t}],\ \ 0 \le \tau \le t < \infty \tag{36}$$

where $\{c[\tau]^{Patent}\}_{\tau=0}^{t}$ is a stream of per capita consumption for the time interval of $[0,t]$ under the steady-state patent regime and $\{c[\tau]^{Bounty}\}_{\tau=0}^{t}$ is the stream of per capita consumption for the same time interval under a bounty regime in transition. As such, starting from the moment of the regime switch, $\Omega[t]$ measures a consumption gain for a time interval with the length of duration equal to $t$, whereas $\Omega[\infty]$ is the consumption gain for the entire semi-infinite time interval, $[0,\infty)$. For instance, if $\Omega[t]$ turns out to be positive, it means that in time interval $[0,t]$, the regime switch generates a welfare gain equivalent to a percent ($=\Omega[t]\times 100\%$) increase from the patent-regime steady state.

**Formulae for $\Omega$** Note that per capita consumption $c[t] \equiv C[t]/L[t]$ can be written as $c[t] = \frac{\widetilde{C}[t](h[t]L[t])^{\theta_K}}{L[t]} = \widetilde{C}[t]h[t]^{\theta_K}L[t]^{\theta_A} = \widetilde{C}[t]e^{(\theta_K\hat{h}+\theta_A n)t}$ in terms of (24) and $h[t]=e^{\hat{h}t}$ and $L[t]=e^{nt}$. Now, given the initial steady state consumption of $\widetilde{C}_0$ and the bounty-regime transition path of $\widetilde{C}[t]$, we can compute the welfare measure of $\Omega[t]$, $t\in[0,\infty)$, based on the following formulae:

$$\Omega[t] = \begin{cases} \psi_a[t]^{1/(1-\gamma)}-1,\ \text{ with }\ \psi_a[t] = \frac{\int_0^t \widetilde{C}[\tau]^{1-\gamma}e^{-[\rho-n-(1-\gamma)(\theta_K\hat{h}+\theta_A n)]\tau}d\tau}{\int_0^t \widetilde{C}_0^{1-\gamma}e^{-[\rho-n-(1-\gamma)(\theta_K\hat{h}+\theta_A n)]\tau}d\tau}, & \gamma \neq 1 \\ e^{\psi_b[t]}-1,\ \text{ with }\ \psi_b[t] = \frac{\int_0^t \log[\widetilde{C}[\tau]/\widetilde{C}_0]e^{-(\rho-n)\tau}d\tau}{\int_0^t e^{-(\rho-n)\tau}d\tau}, & \gamma = 1 \end{cases} \tag{37}$$

If one overlooked the transitional impacts by presuming that the economy jumps from the initial patent-regime balanced-growth path directly to the new bounty-regime balanced-growth path, then the bounty-regime scale-adjusted consumption $\widetilde{C}$ would jump to $\widetilde{C}_1$ from $\widetilde{C}_0$ at $t=0$ and formulas (37) would reduce to

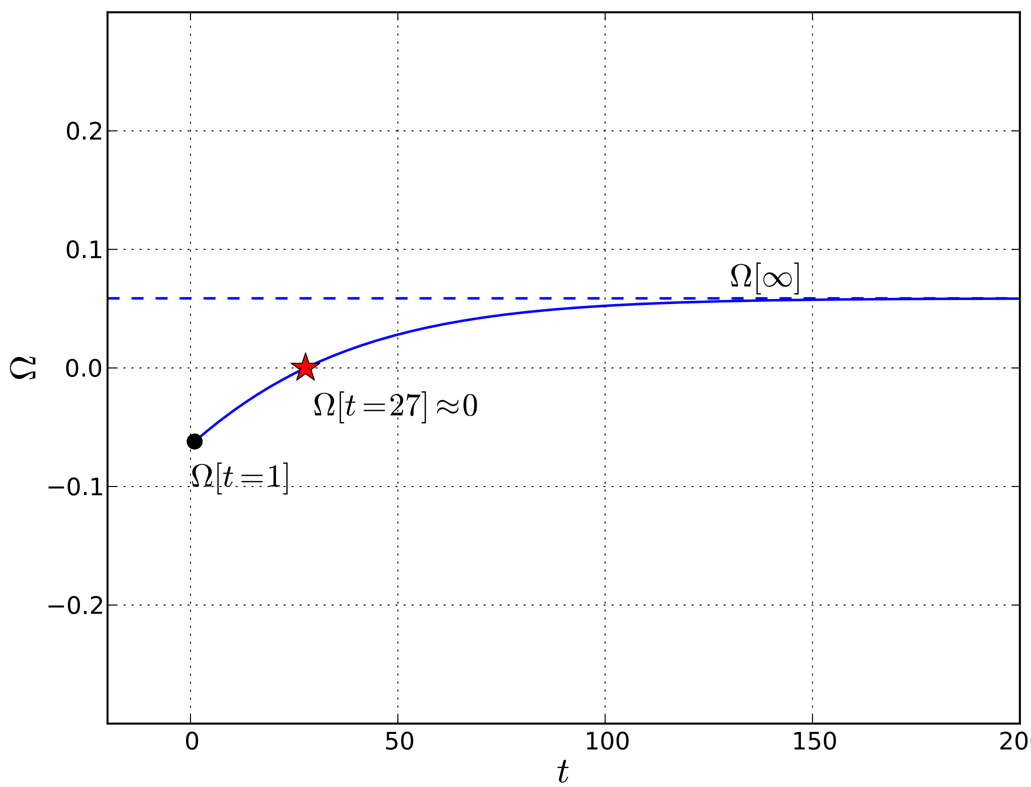
$$\Omega[t] = \frac{\widetilde{C}_1}{\widetilde{C}_0} - 1 \equiv \overline{\Omega},\ t\in[0,\infty), \tag{38}$$

irrespective of whether $\gamma \neq 1$ or $\gamma = 1$.

**Magnitudes of $\Omega[t]$, $\Omega[\infty]$ and $\overline{\Omega}$** From the forgoing transitional analysis, the time profile of the consumption flow (Figure 3 (c)) must imply that that $\Omega[t] \le \Omega[\infty] < \overline{\Omega}$ for $t\in[0,\infty)$. As reported earlier using Table 2, The regime switch can result in a remarkable increase of 42% in steady-state consumption if the bounty rate is set at the long-run Pareto optimal level of $\beta^* = 0.30$. This implies $\overline{\Omega} = 0.42$. The transition path of $\Omega[t]$ is displayed in Figure 7. As indicated, for the first year ($t=1$), there is a consumption loss of 6.2% (due to $\Omega[1] \approx -0.062$), but such a short-run loss keeps diminishing over time. By the time of $t=27$ (years), households have fully recouped short-run consumption sacrifices (due to $\Omega[27]\approx 0$. They would eventually secure a net consumption gain of 5.9% (due to $\Omega[\infty]\approx 0.059$) in the long run, compared to the steady-state consumption gain of 42%.

Certainly, society can instead take a conservative route to a long-run suboptimal bounty regime without correcting underinvestment in R&D by implementing the long-run suboptimal bounty rate of $\beta^{**}=0.14$.

Figure 7: Changing welfare in transition based on $\beta^*$ and benchmark parameters including $\gamma = 2$




From my simulations, the transition paths are qualitatively similar, and as expected, switching to this long-run suboptimal bounty regime incurs a smaller short-run consumption loss. Yet, this is accompanied by a significant drop in net consumption gains (measured by $\Omega[t \to \infty)$ and an even more significant drop in steady-state consumption gains (measured by $\overline{\Omega}$), as summarized in Table 3.

**Robustness checks** We close the welfare analysis by conducting robustness checks on the switch to the long-run Pareto optimal bounty regime. From the transition paths of Figure 3, it is implied that the dynamic system of (31a) - (31d) exhibits saddle-path stability. This feature appears robust to parameter changes from other numerous experiments. It is found that compared to $\overline{\Omega}$, the welfare measure of $\Omega[\infty]$ is sensitive to

Table 3: Steady-state consumption gain and cumulative consumption gains in transition

| Bounty Rate, $\beta$ | $\overline{\Omega}$ | $\Omega[t=1]$ | $\Omega[t \to \infty]$ |
|---|---|---|---|
| $\beta^* = 0.30$ | 0.42 | $-0.062$ | 0.059 |
| $\beta^{**} = 0.14$ | 0.09 | $-0.027$ | 0.030 |

Remarks: $\Omega[t = 27] = 0$ at $\beta^* = 0.30$ and $\Omega[t = 14] = 0$ at $\beta^{**} = 0.14$.

parameters $\gamma$ (intertemporal substitution), $\lambda$ (research congestion) and $\phi$ (knowledge spillovers), as shown in Table 4. For instance, if the elasticity of intertemporal substitution ($1/\gamma$) is increased from 0.5 to 1, the net consumption gain in terms of $\Omega[\infty]$ is seen to increase significant either at $\lambda\,(\phi) = 0.50$ or at $\lambda\,(\phi) = 0.55$. This is actually consistent with our economic intuition: if future consumption is less substitutable for present consumption, the measure of $\Omega[\infty]$ must get smaller. As well, with a smaller extent of research congestion (smaller $\lambda$) or with a larger knowledge spillover (greater $\phi$) , a larger consumption gain is seen to result, irrespective of the elasticity of intertemporal substitution. Regardless, the central message of these numerical experiments maintains that an intertemporal bounty regime can be a welfare-improving Pareto optimal alternative to the world's patent system.

Table 4: Sensitivity analysis of parameters $\gamma$ and $\lambda$ on welfare change

| $\gamma$ | | 1.00 | **2.00** | 2.50 |
|---|---|---|---|---|
| $\lambda = \phi = 0.50$ | $\beta^*$ | 0.35 | **0.30** | 0.29 |
| | $\overline{\Omega}$ | 0.53 | **0.42** | 0.39 |
| | $\Omega[\infty]$ | 0.15 | **0.059** | 0.045 |
| $\lambda = 0.55,\ \phi = 0.50$ | $\beta^*$ | 0.39 | 0.34 | 0.32 |
| | $\overline{\Omega}$ | 0.64 | 0.53 | 0.48 |
| | $\Omega[\infty]$ | 0.19 | 0.072 | 0.054 |
| $\lambda = 0.50,\ \phi = 0.55$ | $\beta^*$ | 0.38 | 0.32 | 0.30 |
| | $\overline{\Omega}$ | 0.63 | 0.50 | 0.45 |
| | $\Omega[\infty]$ | 0.17 | 0.063 | 0.048 |

Remark: (i) The bold figures correspond to benchmark parameters ($\lambda = \phi = 0.5$ and $\gamma = 2$); (ii) $\overline{\Omega}$ ($\Omega[\infty]$) is the rate of consumption gains excluding (including) transitional changes in consumption.

## 6 Concluding Remarks

First-generation endogenous growth models feature the empirically implausible scale effects, implying a faster steady-state growth rate for a larger economy. The progression to non-scale growth models removes the unwanted scale effects, but at the expense of raising the dimensionality. The present paper is no exception. It has to deal with a four-dimensional dynamic system that governs the transitional dynamics when the initial patent regime is switched to an intertemporal bounty regime. This system consists of two state variables – capital & knowledge stocks – evolving at different paces and must be normalized into a stationary one using multiple normalization factors. Python's boundary value problem solver (**scikits.bvp_solver**) proved fast and precise in solving such a normalized dynamic system.

The paper has demonstrated that a decentralized market mechanism via the proposed intertemporal bounty system can duplicate Pareto optimality, provided that the rate of bounty is set rightly to internalize externalities. Under this system, the prize for innovation is not paid once and for all. Instead, the prize is dynamically amortized in an infinitely time domain as periodic bounties paid to the holder of bounty claims. These periodic bounties are calculated using a government-determined bounty rate times observed market sales. Certainly, the intertemporal bounty system allows the innovator choose at any time to cash the entire prize, once and for all, from the bounty claims market. In this paper an important formula is derived to calculate a ***long-run Pareto optimal bounty rate*** that allows the economy to attain Pareto optimum in the long run by correcting both monopoly distortions and externalities such as knowledge spillovers and research congestion. Another important formula obtained is to calculate a ***long-run suboptimal bounty rate*** that can only serve to correct monopoly distortions.

Empirically, the social value of innovation is usually much above the private value of innovation. The government can take the ***long-run Pareto optimal bounty rate*** as an upper bound and the ***long-run suboptimal bounty rate*** as the lower bound when a bounty rate is to be determined to implement the intertemporal bounty system. Empirical data for the two bounty rates are available in the real world. The government does not need to estimate the prize size of a specific innovation, since the importance of each innovation can be revealed over time by the observed market sales of *bountied* goods. If the market sales drop to zero, the associated innovation has no value, despite the bounty claim being perpetual legally. Moreover, the bounty rate can be adjusted as desired to changing economic conditions. This makes the intertemporal bounty system practical either as a Pareto optimal alternative if lump-sum taxes are available or as a suboptimal yet welfare-improving alternative, to the world's patent system.

The tax-financed bounty for innovation is a type of government transfer payments in society. Yet, if lump-sum taxes are not available in the real world, it is necessary to source the least distortionary tax for the IB system to secure a larger welfare gain. Besides, in some cases, there is a need to work out ways to split the bounty among holders of bounty claims. For instance, if a bountied good (like cellphone) is based on multiple innovations, rather than just one single innovation as assumed in the present study, then how to determine their embodied importance will not be an easy task. The present paper represents an initial inquiry in hopes of inspiring further research into the proposed IB system as a decentralized mechanism designed to make innovation viable under perfect competition.

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# Appendix

## A Patent regime's balanced growth paths

Along a balanced-growth path, every variable grows at a constant rate, including zero growth. Such steady-state growth rates ($\hat{V}_0 \equiv \frac{\dot{V}_0}{V_0}$, $\hat{A}_0 \equiv \frac{\dot{A}_0}{A_0}$, $\hat{K}_0 \equiv \frac{\dot{K}_0}{K_0}$ and $\hat{Y}_0 \equiv \frac{\dot{Y}_0}{Y_0}$) can be obtained by solving the four equations system:

$$\hat{Y}_0 - \hat{K}_0 = 0 \tag{A.1a}$$

$$(\phi - 1)\hat{V}_0 + \lambda \hat{Y}_0 = 0 \tag{A.1b}$$

$$\hat{Y}_0 - (1-\alpha)\hat{A}_0 - \alpha \hat{K}_0 = (1-\alpha)(n+\hat{h}) \tag{A.1c}$$

$$\hat{A}_0 - \frac{1}{\varepsilon\sigma(1-\alpha)}\hat{V}_0 = 0 \tag{A.1d}$$

These equations result from (22a), (22b), (18), and the definition of $A$. For instance, (22a) implies $\hat{Y}_0 = \hat{K}_0 = \hat{C}_0$. Solving (A.1a) - (A.1d) yields (23a) - (23c). From (14) and (8b), the patent price equation is given by $\upsilon = s_V Y/\dot{V} = s_V Y/(\hat{V}V)$. Along a balanced-growth path, both $s_V$ and $\hat{V}$ are stationary. Thus, differentiating the patent price equation yields

$$\hat{\upsilon}_0 \equiv \frac{\dot{\upsilon}_0}{\upsilon_0} = \hat{Y}_0 - \hat{V}_0 = (\theta_K - \theta_V)(n+\hat{h}) \tag{A.2}$$

Derivations of the remaining stationary variables are in order. First, from the Euler condition (3) and using growth kernels $\theta_K$ and$\theta_A$, the steady-state interest rate is given by

$$r_0 = \rho + \gamma[n - \theta_K(\hat{h}+n)] \tag{A.3}$$

Next, from (21b), the steady-state capital/output ratio is given below

$$\frac{K_0}{Y_0} = \frac{\alpha(1-m_0)}{r_0+\delta} \tag{A.4}$$

This result and (8a) combine to imply the steady-state capital investment rate,

$$s_{K0} = \alpha(1-m_0)\left(\frac{\delta + \theta_K(n+\hat{h})}{r_0+\delta}\right) = \alpha(1-m_0)\left(\frac{\delta + (n+\hat{h}) + \hat{A}_0}{r_0+\delta}\right) \tag{A.5}$$

Recall that $\Pi$ denotes aggregate profit flows and $\pi = \Pi/(\zeta V)$ measures a patented firm's profit flow. Using (21a) and (13) yields the steady-state price of a newly issued patent,

$$\upsilon_0[t] = \frac{\alpha m_0 Y_0[t]}{\zeta_0 V_0[t]}\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0)}\right), \text{ or } \widetilde{\upsilon}_0 = \frac{\alpha m_0 \widetilde{Y}_0}{\zeta_0 \widetilde{V}_0}\left(\frac{1-e^{-(r_0-\hat{Y}_0+\hat{V}_0)T}}{r_0-\hat{Y}_0+\hat{V}_0)}\right) \tag{A.6}$$

where $\widetilde{\upsilon}_0 = \upsilon_0[t]/(h[t]L[t])^{\theta_K - \theta_V}$, $\widetilde{Y}_0 = Y_0[t]/(h[t]L[t])^{\theta_K}$, and $\widetilde{V}_0 = V_0[t]/(h[t]L[t])^{\theta_V}$ are constant on a balanced-growth path. Also, with $\dot{\zeta}_0 = 0$ and $\hat{V} = \hat{V}_p = \hat{V}_{np} = \theta_V(n+\hat{h})$ in the steady state, either (16a) or (16b) implies the equilibrium patented goods fraction,

$$\zeta_0 = 1 - e^{-\hat{V}_0 T} \tag{A.7}$$

The R&D equilibrium condition (14) requires $\xi[t]\upsilon[t] = 1$ at any instant, which implies $\upsilon[t]\dot{V}[t] = s_V[t]Y[t]$ in terms of (8b). Along a balanced growth, we therefore have $s_{V0} = \upsilon_0 \hat{V}_0 V_0 / Y_0$, which, together with (A.6), implies

$$s_{V0} = \hat{V}_0 \left(\frac{\alpha m_0}{\zeta_0}\right)\left(\frac{1 - e^{-(r_0 - \hat{Y}_0 + \hat{V}_0)T}}{r_0 - \hat{Y}_0 + \hat{V}_0}\right) \tag{A.8}$$

Lastly, the two stock variables ($K$, $V$) are non-stationary. Using these scale-adjusted quantities, $\widetilde{K} \equiv K/(hL)^{\theta_K}$, $\widetilde{Y} \equiv Y/(hL)^{\theta_K}$, and $\widetilde{A} = A/(hL)^{\theta_A}$, we can transform the final goods production function (18) into its intensive form, $\widetilde{Y} = z\widetilde{A}^{1-\alpha}\widetilde{K}^{\alpha}$, and the motion of capital accumulation (12) into $\dot{\widetilde{K}} \equiv \frac{d\widetilde{K}}{dt} = s_K \widetilde{Y} - (\delta + \theta_K(n+\hat{h}))\widetilde{K}$. Setting $\dot{\widetilde{K}} = 0$ then yields the steady state scale-adjusted stock of capital,

$$\widetilde{K}_0 = \widetilde{A}_0 \left(\frac{z_0 s_{K0}}{\delta + \theta_K(n+\hat{h})}\right)^{1/(1-\alpha)}, \tag{A.9}$$

On a balanced-growth path, the stock of knowledge capital $V$ keeps growing at a rate equal to $\theta_V(n+\hat{h})$. But we can normalize $V$ to $\widetilde{V} \equiv V/(hL)^{\theta_V}$. Then dividing each side of the innovation equation (22b) by $V_0$ while replacing $V_0$ and $Y_0$ with $\widetilde{V}_0(hL)^{\theta_V}$ and $\widetilde{Y}_0(hL)^{\theta_K}$ in this equation, we obtain

$$\frac{\dot{V}_0}{V_0} = \theta_V(n+\hat{h}) = \mu \widetilde{V}_0^{\phi-1}(s_{V0}\widetilde{Y}_0)^{\lambda}(hL)^{(\phi-1)\theta_V + \lambda\theta_K} \tag{A.10}$$

where term $(hL)^{(\phi-1)\theta_V+\lambda\theta_K}$ can drop out due to $(\phi-1)\theta_V + \lambda\theta_K = 0$ (see (24)). Now using $\widetilde{Y}_0 = z_0\widetilde{A}_0^{1-\alpha}\widetilde{K}_0^{\alpha}$, $\widetilde{A}_0 = \widetilde{V}_0^{1/(\varepsilon\sigma(1-\alpha))}$, and result (A.9), we can easily solve (A.10) for the steady state scale-adjusted stock of knowledge, which is given in (25b).

# B Steady state of the normalized dynamic system

First, setting $\dot{\widetilde{K}} = 0$ in (29a) and using (32a) yield the stationary equilibrium of $\widetilde{K}$,

$$\widetilde{K}_1 = \left(\frac{s_K}{\delta + \theta_K(n+\hat{h})}\right)^{1/(1-\alpha)} \widetilde{A}_1 \tag{B.1}$$

Second, setting $\dot{\widetilde{K}} = 0$ in (29a) and using (32e) yield the equilibrium capital investment rate,

$$s_{K1} = (\delta + \theta_K(n+\hat{h}))\frac{\widetilde{K}_1}{\widetilde{Y}_1} = (\delta + \theta_K(n+\hat{h}))\left(\frac{\alpha}{r_1 + \delta}\right) \tag{B.2}$$

Third, setting $\dot{\widetilde{V}} = 0$ in (31b) yields $(s_{V1}\widetilde{Y}_1)^{\lambda} = \frac{\theta_V(n+\hat{h})}{\mu \widetilde{V}_1^{\phi-1}}$ and (32d) implies $(s_{V1}\widetilde{Y}_1)^{1-\lambda} = \mu \widetilde{\upsilon}_1 \widetilde{V}_1^{\phi}$. Combining these two equations yields the equilibrium level of R&D,

$$s_{V1}\widetilde{Y}_1 = \widetilde{\upsilon}_1\widetilde{V}_1\theta_V(n+\hat{h}) \tag{B.3}$$

Fourth, replacing $\widetilde{b}$ with (32b) and setting $\dot{\widetilde{\upsilon}} = 0$ in (31d), we obtain

$$r_1 - \beta\alpha\frac{\widetilde{Y}_1}{\widetilde{\upsilon}_1\widetilde{V}_1} = (\theta_K - \theta_V)(n+\hat{h}) \tag{B.4}$$

Then replacing $\widetilde{\upsilon}_1\widetilde{V}_1$ in (B.4) with $s_{V1}\widetilde{Y}_1/(\theta_V(n+\hat{h}))$ based on the result of (B.3), we obtain the equilibrium R&D investment rate,

$$s_{V1} = \frac{\beta\alpha\cdot\theta_V(n+\hat{h})}{r_1-(\theta_K-\theta_V)(n+\hat{h})} \tag{B.5}$$

where $r_1 = r_0 = \rho + \gamma[\theta_A n + \theta_K\hat{h}]$. Substituting $s_{V1}$ into (B.3) yields (33d) in the text.

Fifth, using $\widetilde{Y} = \widetilde{A}^{1-\alpha}\widetilde{K}^{\alpha}$ and (B.1), we can rewrite $(s_{V1}\widetilde{Y}_1)^{\lambda} = \frac{\theta_V(n+\hat{h})}{\mu\widetilde{V}_1^{\phi-1}}$ (from step 3) as

$$s_{V1}^{\lambda}\left(\frac{s_{K1}}{\delta+\theta_K(n+\hat{h})}\right)^{\lambda\alpha/(1-\alpha)}\widetilde{A}_1^{\lambda} = \frac{\theta_V(n+\hat{h})}{\mu\widetilde{V}_1^{\phi-1}} \tag{B.6}$$

Then replacing $\widetilde{A}_1$ with $\widetilde{V}_1^{1/(\varepsilon\sigma(1-\alpha))}$ and collect the $\widetilde{V}_1$ terms in (B.6), we obtain (33b) in the text.

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