Modelling uniformly porous façades to predict dwelling infiltration rates

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ABSTRACT (125 words)

It is important to limit dwelling infiltration to save energy and meet national climate change commitments while concurrently providing adequate ventilation to preserve occupant health. DOMVENT3D is a model of infiltration and exfiltration that assumes a linear pressure distribution over any number of uniformly porous façades and integrates the airflow rate in the vertical plane to predict the theoretically correct airflow rate through them. DOMVENT3D is a development of an existing two-dimensional model of infiltration that provides new opportunities for investigating a greater number of dwellings than was previously possible. Initial testing suggests that DOMVENT3D is mathematically robust and is suitable for modelling a wide variety of dwelling types and geometries to assist engineers and policy makers.

PRACTICAL APPLICATION (125 words)

The modern building services engineer may be required to model airflow networks in a building to balance the conflicting needs of energy consumption reduction and occupant health. Limiting exfiltration is one method of reducing heat losses from a building, and so there is a need to model it accurately. This technical note presents a new model of infiltration and exfiltration through a uniformly porous façade that can be incorporated within advanced complex airflow network tools or applied using a simple spread sheet.

TECHNICAL NOTE (3000 words)

1.0 INTRODUCTION

The infiltration of cold air and the concurrent exfiltration of conditioned air through adventitious openings in the thermal envelope of a dwelling can be a significant component of its heating load. Measuring infiltration is technically difficult, invasive, and expensive, and so it is often inferred from a measurement of air permeability, the rate of airflow through the fabric of a building measured at a steady high pressure difference, normally 50 Pascals (Pa), when the effects of wind and buoyancy forces are effectively eliminated[1]. This inference is also problematic[2] and so there is a clear need to predict dwelling infiltration theoretically, which is both cheap and quick. There are two approaches commonly used to model infiltration. The first approach relies on knowledge of the location of adventitious openings, known as air leakage paths (ALPs), their geometry, or expected losses across them. Each ALP is specified explicitly, and appropriate leakage characteristics are derived either from measurement or from appropriate sources in the literature for specific building components[3].

However, it is suggested that "there is insufficient data available in the literature to justify [anything] other than a uniform distribution"[4] of porosity. The second approach to modelling infiltration uses an appropriate number of ALPs, equally spaced in the vertical plane to account for buoyancy driven flow, and sized according to the dwelling's permeability. This is known as the *multiple element* approach[5], and it is advised that 11 equally spaced ALPs is an adequate number[2].

An elegant development of this differential approach, when the number of ALPs is large, is the method proposed by Lyberg[6] and Lowe[7]. The basic equations proposed by Lyberg are also used to model airflow through large openings[5], but his formulation also handles airflow through envelopes with a wide range of properties. Lowe's two-dimensional infiltration model, known as DOMVENT, assumes a linear pressure distribution over a uniformly porous façade and integrates the airflow rate in the vertical plane to predict the theoretically correct airflow rate through that façade. The simplicity of the DOMVENT model, and its implementation using bespoke MATLAB[8] code, means that the calculation and post processing time is significantly less than that for conventional airflow analysis tools, such as CONTAM[9] and AIDA[3], two independent validated airflow analysis tools. These tools do not have an airflow path that specifically characterizes infiltration and so must follow the multi-element approach described here. The predictions of DOMVENT have been compared against those of established envelope flow models[2] and used to investigate energy use and CO_2 emissions in dwellings[7] and the relationship between permeability and infiltration in conjoined dwellings[2]. Thus, DOMVENT is a useful tool for undertaking the simulations necessary to investigate the infiltration one might expect to find in a dwelling subjected to varying weather conditions. However, the current formulation of DOMVENT described in the literature is exclusively for a cuboid dwelling with two identical exposed facades when internal and external temperatures are unequal. This constrains its application to the modelling of mid-terrace houses and some apartments. Mid-terrace houses account for only 19% of the English housing stock[10], whereas end-terrace, semi-detached, and detached houses account for 53% of the stock[10]. Accordingly, if one is also to have confidence in the predictions of infiltration in dwelling types that comprise the majority of the English stock, a more versatile form of DOMVENT is needed that is able to consider any number of vertical façades with differing geometries.

This technical note addresses the limitation of DOMVENT by developing a three-dimensional model of infiltration and exfiltration known as DOMVENT3D. In Section 2 the model is derived from first principles so that it can predict the infiltration rate of any dwelling with cuboid geometry. Uncertainties and limitations are discussed and the model is corroborated against the predictions of CONTAM. In Section 3, suitable applications of DOMVENT3D are discussed.

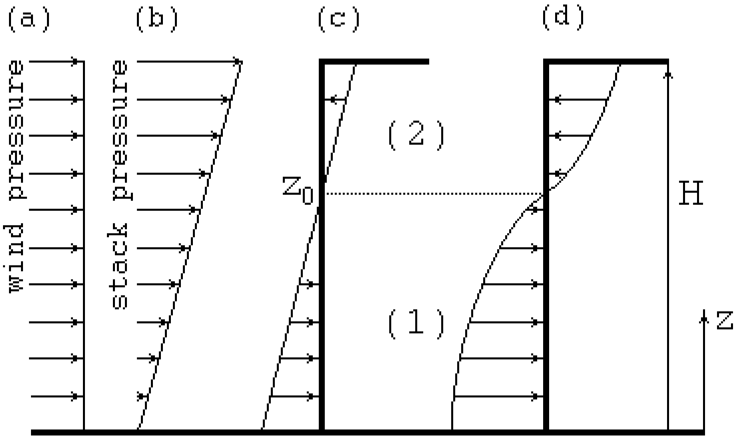

Figure 1: Vertical cross section through a façade of height *H*, under pressure from: (a) action of the wind; and (b) stack pressure. The resulting linear pressure distribution (c) gives a parabolic airflow distribution (d) with areas of (1) infiltration and (2) exfiltration, separated by a neutral point z_0.

2 MODELLING INFILTRATION

Consider a dwelling of height *H* (m), and width *W* (m), with one or more exposed façades; see Figure 1, which shows a cross section through one of any number of façades. When a building contains no mechanical ventilation system or a perfectly balanced mechanical ventilation system, mean infiltration and exfiltration rates are equal in magnitude and opposite in sign. They are a function of the geometry of the building, its local environment, and the prevailing meteorological conditions. When an unbalanced mechanical system is present, the rates of infiltration and exfiltration are also generally unbalanced.

2.1 A GENERAL MODEL

Most models of infiltration and exfiltration use a power law relationship between the pressure difference Δp (Pa), across an adventitious opening, known as an air leakage path (ALP), and the volume flow rate $\dot{Q}$ (m^3/s) of air where

$$\dot{Q} = C|\Delta p|^b \varepsilon(\Delta p) \tag{1}$$

Here, b is the flow exponent and C is a flow coefficient ($m^3/s/Pa^b$). The flow direction function $\varepsilon(x) = 1$ if $x > 0$, $\varepsilon(x) = -1$ if $x < 0$, or $\varepsilon(x) = 0$ if $x = 0$. Airflow into the building is positive in sign whereas airflow out is negative. Therefore, the net flow through a system of ALPs in the thermal envelope of a building is zero and is described by the continuity equation

$$\dot{Q}_m + \sum_{i=1}^{j} \dot{Q}_i = 0 \tag{2}$$

where $\dot{Q}_i$ is the airflow rate through the i^{th} ALP of a total of j, and $\dot{Q}_m$ is the total airflow through a mechanical system, such as an extractor fan.

The modelling of specific ALPs is appropriate if their locations are known, but in most cases they are not. In the absence of a priori knowledge on their locations, it is common to assume that a vertical wall or façade is uniformly porous[2,4]. The vertical pressure distribution over the façade of a building is a function of the action of the wind, the difference between internal and external air densities, known as the *stack pressure*, and a change in internal pressure that occurs to balance mass through all openings in the building (see (a)–(c) in Figure 1). Accordingly, the pressure difference across a point on the façade at a height z (m) above floor level is given by[1]

$$\Delta p(z) = \frac{1}{2}\rho_E u^2 c_p - (\rho_E - \rho_I)gz - p_I \tag{3}$$

where p_I is the internal air pressure relative to atmospheric pressure (gauge), u is the wind velocity at height H, ρ_E is the external air density (kg/m^3), ρ_I is the internal air density (kg/m^3), g is the gravitation acceleration (m/s^2), and c_p is the dimensionless façade pressure coefficient. The three terms on the right hand side of the equation are depicted in Figure 1 by gradients (a), (b), and (c), where z varies between z=0 and z=H and all other terms are constant.

A number of ALPs are defined in the vertical plane to model a uniform distribution of porosity of any number of façades. The airflow rate through and pressure difference across each ALP is defined by Equations (1) and (3), respectively. The two equations are solved by varying p_I so that Equation (2) is satisfied. Accordingly, for j ALPs, j+1 equations are required.

2.2 DOMVENT3D: AN INTEGRATING INFILTRATION MODEL

When z is a variable and all other parameters are constant, the pressure difference across the façade varies linearly with z, and Equation (3) has one root or equilibrium point. The height at which the root occurs is known as the neutral height z_0 (m). When $\Delta p(z_0) = 0$ an expression for z_0 is given by[5]

$$z_0 = \frac{\frac{1}{2}\rho u^2 c_p - p_I}{(\rho_E - \rho_I)g}. \tag{4}$$

Equation (4) shows that as p_I increases, z_0 decreases, and vice versa. Accordingly, an extract fan can reduce p_I below atmospheric pressure and increase z_0 above the height of the building so that all exposed façades provide infiltration.

The pressure difference over a façade can also be described with reference to z_0 thus

$$\Delta p(z) = -(\rho_E - \rho_I)g(z - z_0). \qquad (5)$$

When $0<z_0<H$, both infiltration and exfiltration occur simultaneously through a façade, otherwise when $0>z_0>H$ only infiltration or exfiltration occurs.

If Equation (1) is now assumed to be the flow rate through an infinitesimal section *dz* (m) of a façade in the vertical plane due to a pressure difference across it, it can be rewritten to describe the total volume flow rate of air through the façade, $\dot{Q}_f$ (m^3/s).

$$\dot{Q}_f = C\varepsilon(\Delta p)\int_0^H (|\Delta p|)^b \, dz \qquad (6)$$

Now, *C* can be expressed as

$$C = EaW \qquad (7)$$

where *E* is the dimensionless relative leakage area and *W* (m) is the façade width. The flow exponent *b*, normally has a value[3] in the range of 0.6–0.7, although it is often taken as 0.5 to simplify the analysis, *a* corresponds to $(2/\bar{\rho})^b$. By adopting the Boussinesq approximation[1] $\bar{\rho}$ is the mean of the internal and external air densities. To calculate the total mass flow rate of air through the façade, $\dot{M}_f$ (kg/s) then $a = \bar{\rho}(2/\bar{\rho})^b$, so that when *b*=0.5 then $a = (2\bar{\rho})^{0.5}$. This corrects a mathematical error in Lowe's[7] paper, but it does not affect its predictions and conclusions.

Equations (5), (6) and (7) are combined so that $\dot{Q}_f$ is now given by

$$\dot{Q}_f = EaW\varepsilon(\rho_E - \rho_I)\{|(\rho_E - \rho_I)|g\}^b \begin{bmatrix} +\int_0^{min(z_0,H)} (z - z_0)^b dz \\ -\int_{max(z_0,0)}^{H} (z - z_0)^b dz \end{bmatrix} \quad (8)$$

Note that for simplicity, the flow function input is reduced to the difference between the air densities because the difference between these parameters governs the airflow direction. Each of the integral limits of Equation (8) are taken to be zero if the lower limit of integration exceeds the upper. The integration of Equation (8) describes both infiltration and exfiltration (see Figure 1) and can be split into two separate equations whose sum is equal to $\dot{Q}_f$:

$$\dot{Q}_1 = \frac{EaW\varepsilon(\rho_E - \rho_I)}{b+1}\{|(\rho_E - \rho_I)|g\}^b \begin{bmatrix} +z_0^{b+1}|_{z_0>0} \\ -(z_0 - H)^{b+1}|_{z_0>H} \end{bmatrix} \quad (9)$$

$$\dot{Q}_2 = \frac{EaW\varepsilon(\rho_I - \rho_E)}{b+1}\{|(\rho_E - \rho_I)|g\}^b \begin{bmatrix} +(H - z_0)^{b+1}|_{z_0<H} \\ -(-z_0)^{b+1}|_{z_0<0} \end{bmatrix} \quad (10)$$

When $\rho_I < \rho_E$ then $\dot{Q}_1$ and $\dot{Q}_2$ describe infiltration and exfiltration, respectively. When $\rho_I > \rho_E$ then $\dot{Q}_1$ and $\dot{Q}_2$ describe exfiltration and infiltration, respectively. When $\rho_I = \rho_E$ then Equations (9) and (10) equal zero and must be replaced by a single ALP using Equation (1). It is now possible to model airflow through multiple vertical façades of varying geometries (where *H*, *D*, and *W* are not equal) by stating Equations (1), (9) and (10) for each, thus making the model fully three-dimensional.

Ordinarily, Equations (1), (2), (9) and (10) are solved numerically, but there are three occasions when an explicit solution is possible for a naturally ventilated cuboid dwelling whose external façades are of equal height. Firstly, when u=0 m/s and infiltration is solely attributable to buoyancy forces, z_0=H/2 m. Secondly, when a building has two exposed façades, the mean of the neutral heights on the windward and leeward facades equal H/2 m.

Finally, a single sided dwelling has a neutral height of $z_0=H/2$ m for all environmental conditions. These are true because (i) we use an average value of density in Equation (7) and (ii) the permeability of the exposed façades is considered to be uniformly distributed and so the area of exposed façades that provide infiltration must equal the area of exposed façades that provide exfiltration.

DOMVENT3D only requires three equations to model the airflow rate through a uniformly porous façade, and a maximum of two equations are required at one time. This represents a considerable simplification of the multiple element approach and a development of the original DOMVENT model.

2.3 MODEL LIMITATIONS

The application of Equation (1) to airflow through a single ALP and to airflow through a whole building, characterised by a number of ALPs, is an approximation. One consequence of this approximation is the fact that the coefficients that describe the airflow through such a building are not always constant but it is shown not to be a significant obstacle to the use of this fundamental equation in the way proposed by Lyberg[6]. Furthermore, we note that the power law relationship described by Equation (1) is considered less accurate than the quadratic relationship at operational pressure differences[1], but it is the most widely used method of interpolating between measurements of air leakage rates[2] and so it is employed here.

Equation (3) assumes that p_I and ρ_E are uniform, and u and c_p are not a function of z when $z \leq H$. These assumptions restrict Equation (2) to low-rise buildings. Here, Liddament[11] suggests that mean pressure coefficients are appropriate for low-rise buildings of up to 3 storeys and so this limit is adhered to here. The authors are unaware of any empirical evidence of temperature distributions in dwellings that could be used to add another density term to Equation (3) to accurately describe stratification. Moreover, the consideration of stratification increases the computational complexity of the model. Thus, we acknowledge that although ignoring the effects of stratification introduces uncertainty into the model, it is nevertheless considered to be an acceptable trade-off between model complexity and prediction accuracy[5].

2.4 INITIAL TESTS

The theory that underpins this paper can handle envelopes in which flow varies continuously from turbulent (b=0.5) to laminar (b=1). It can therefore be configured for comparison against the two-way single-opening ventilation element (TWSO) that is used to model airflow through large doors and windows by CONTAM[9], a validated multi-zone ventilation and pollutant transport model. The TWSO requires input of the width of the opening, height, and discharge coefficient, C_d (akin to E, the relative leakage area). The flow exponent is fixed at b=0.5 and the minimum value of C_d is 10^{-3}. These restrictions make the TWSO unsuitable for modelling a porous façade because b is too small— for adventitious cracks[3] b is between 0.6 and 0.7— and C_d is too big; for example[2], E=1.64×10^{-4} for an archetypal apartment with a permeability of 10$m^3/h/m^2$. Nevertheless, a mathematical corroboration of DOMVENT3D against the TWSO element is possible, and so a single DOMVENT façade and TWSO element are modelled when C_d, E, W, and H are set to unity, b=0.5, g=9.81m/s^2, and u=0m/s. The internal and external air temperatures are T_I=292.15K and T_E=282.15K, respectively. The air density is given by $\rho = P/RT$ where atmospheric pressure P=101325Pa and the gas constant, R=287.055J/(kg.K), so that ρ_E=1.251kg/m^3 and ρ_I=1.208kg/m^3. These calculations of air density are identical to those of CONTAM and the mass flow rate (kg/s) of air predicted by DOMVENT3D, using Equation (2), (5), (7), (9) and (10), is 0.07% above that of CONTAM. Next, two façades are considered when T_I=292.15K and T_E=282.15K and a wind pressure is applied to each façade that is equal in magnitude and opposite in sign so that $0.5\rho_E c_p u^2 = \pm 1$Pa. The airflow rate predicted by DOMVENT3D is 0.02% above that of CONTAM.

The differences between the predictions of CONTAM and DOMVENT3D for both buoyancy driven flow and combined wind and buoyancy driven flow are negligible and so can be said to be in agreement.

3.0 APPLICATIONS

When comparing DOMVENT3D to most models of infiltration, its consideration of the physics is relatively complex because it assumes a linear pressure distribution over a uniformly porous façade and integrates the airflow rate in the vertical plane to predict the theoretically correct airflow rate through that façade, yet its application is simple. Thus, DOMVENT3D can be used to make many predictions quickly, making it an ideal tool for predicting the infiltration rates one might expect in a stock of buildings in reasonable computational time. DOMVENT3D's limitations (see Section 2.2) constrain its application to the evaluation of low-rise buildings, such as houses. In England, there are some 22.3m dwellings (DCLG, 2011), yet the number of measurements of dwelling air permeability made in the existing stock is limited[12,13]. Although the air leakage testing of all new dwelling developments is now mandatory, 88% of the stock was built before 1990[10] when tests were not required. Therefore, the government formulates its policy on the retrofitting of energy efficiency measures designed to meet climate change mitigation commitments using a limited quantity of data. A forthcoming paper by the authors uses DOMVENT3D to investigate infiltration rates in English dwellings following a study of infiltration rates in the U.S. housing stock using CONTAM[4]. The latter study[4] assumes uniform porosity, which means that it applied the multiple element approach, although this is not stated. Accordingly, the accuracy of their predictions could be improved if the TWSO element is amended to allow $0.5 \geq b \leq 1$ and a C_d or E value that is at least an order of magnitude smaller than is currently acceptable.

When a simple single zone airflow model is required, Equations (9–10) can be easily incorporated within AIDA[3] or placed into an Excel spread sheet and solved using its “Goal Seek” command.

4.0 CONCLUSIONS

This paper presents an analysis of approaches used to model infiltration in low-rise buildings, such as dwellings, and describes a model of infiltration, known as DOMVENT3D, which assumes that all façades of a building are uniformly porous. The theory that underpins this paper can handle envelopes in which airflow varies continuously from turbulent to laminar, and so the model is configured for corroboration with CONTAM’s TWSO ventilation element. We show that two simple modifications to CONTAM’s TWSO ventilation element would allow it to make predictions of infiltration rates through uniformly porous façades within a multi-zone airflow framework. Finally, with increased confidence in it predictions, it is proposed to use DOMVENT3D to investigate the infiltration rates one might expect to find in English houses and thus to help policy makers make informed decisions on the installation of energy efficiency measures in houses.

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