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Managing non-cooperative behaviors in consensus-based multiple attribute group decision making: An approach based on social network analysis[1]

Hengjie Zhang[a], Iván Palomares[b], Yucheng Dong[c,*], Weiwei Wang[a]

[a] Business School, Hohai University, Nanjing 211100, China
[b] School of Computer Science, Electrical and Electronic Engineering, and Engineering Maths. University of Bristol, Bristol, United Kingdom
[c] Business School, Sichuan University, Chengdu 610065, China

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Abstract: In consensus-based multiple attribute group decision making (MAGDM) problems, it is frequent that some experts exhibit non-cooperative behaviors owing to the different areas to which they may belong and the different (sometimes conflicting) interests they might present. This may adversely affect the overall efficiency of the consensus reaching process, especially when some uncooperative behaviors by experts arise. To this end, this paper develops a novel consensus framework based on Social Network Analysis (SNA) to deal with non-cooperative behaviors. In the proposed SNA-based consensus framework, a trust propagation and aggregation mechanism to yield experts' weights from the social trust network is presented, and the obtained weights of experts are then integrated into the consensus-based MAGDM framework. Meanwhile, a non-cooperative behavior analysis module is designed to analyze the behaviors of experts. Based on the results of such analysis during the consensus process, each expert can express and modify the trust values pertaining other experts in the social trust network. As a result, both the social trust network and the weights of experts derived from it are dynamically updated in parallel. A simulation and comparison study is presented to demonstrate the efficiency of the SNA-based consensus framework for coping with non-cooperative behaviors.

Keywords: Multiple attribute group decision making, consensus reaching process, non-cooperative behaviors, social network analysis

1. Introduction

Group decision making (GDM) is a powerful decision tool to deal with complex decision problems in which a single expert may feel difficult to consider all the aspects of the particular decision problem at hand [30, 36, 59]. Numerous GDM models and approaches have been reported to integrate the knowledge and levels of experience associated with a group of experts

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(e.g., [12, 35, 37]). Conventional GDM models focus on the problem of obtaining a ranking of a group of feasible alternatives to the decision problem without addressing whether or not a reasonable consensus level among experts can be guaranteed. The consensus reaching process (CRP) is an effective way to assist experts improving the consensus level, which has been widely utilized in the GDM literature [3, 13, 20, 21, 28, 33, 40, 44].

Traditionally, the CRP is guided by a "hard" consensus measure that only distinguishes between two possible values or degrees: 0 (no consensus or partial consensus) and 1 (full consensus). However, it is very time-consuming, difficult and unnecessary to achieve a full consensus in many practical GDM problems [31]. As a result, the concept of "soft" consensus [2, 5, 6, 9, 11, 27, 69, 73] has been proposed and used widely in a variety of proposed models for CRPs:

(1) *CRPs with preference representation formats*. For instance, Xu et al. [58] proposed a CRP for GDM with hesitant fuzzy preference relations and discussed its application in water allocation management. Moreover, Xu et al. [61] presented a distance-based CRP for GDM with multiplicative preference relations. Further, Herrera-Viedma et al. [29], and Choudhury et al. [10] presented several CRPs to deal with GDM with heterogeneous preference representation formats. Chen et al. [8] provided a survey for CRPs with heterogeneous preference representation formats.

(2) *Minimum-cost (or adjustments) based CRPs*. Several CRPs based on preserving minimum adjustments or cost have been investigated. Ben-Arieh et al. [4] proposed a CRP model with quadratic cost functions. Moreover, Wu et al. [54] reported a model with a minimum adjustment cost based feedback mechanism for GDM in social networks with distributed linguistic trust information. In addition, a consensus model with minimum cost policy has been investigated by Gong et al. [21-24] and Zhang et al. [70].

(3) *CRPs driven by consistency and consensus measures*. To maintain individual consistency in the consensus building, several approaches for CRPs based on individual consistency and consensus measures have been proposed. For example, Escobar et al. [19] developed a precise consistency consensus matrix-based approach to managing individual consistency and consensus in AHP-GDM. Wu et al. [57] presented an iteration-based approach to address individual consistency and consensus in GDM with multiplicative preference relations. Dong et al. [14] presented an optimization-based CRP model to deal with individual consistency and consensus in GDM under multi-granular unbalanced 2-tuple linguistic preference relations.

(4) CRPs in a dynamic/Web context. Societal and technological trends demand the management of CRPs under dynamic and Web contexts. To address these complex contexts, Pérez et al. [45] proposed a dynamic CRP to manage decision situations in which the set of alternatives

changes dynamically. Alonso et al. [1], Zadrozny and Kacprzyk [68] and Kacprzyk and Zadrozny [32] investigated web-based consensus support systems, while Dong et al. [17] reported a CRP model for complex and dynamic GDM frameworks.

(5) CRPs in multiple attribute group decision making (MAGDM) problems. In some GDM problems, experts evaluate alternatives based on multiple attributes [49], leading to what it has been known as a MAGDM problem. Several CRPs for MAGDM have been reported. For example, Kim et al. [34] suggested a CRP model for the MAGDM problem under contexts of incompleteness. By considering the degrees of confidence of experts' opinions, Guha and Chakraborty [26] developed a consensus model for MAGDM. Wu and Xu [56] presented a CRP model for MAGDM under the hesitant fuzzy linguistic context. Recently, Zhang et al. [70] reported a 2-rank base CRP model in multi-granular linguistic MAGDM problems. Additional CRP approaches for MAGDM settings can be found in [38, 48, 56, 62].

Importantly, real-world GDM situations involve not only mathematical aspects but also psychological behaviors of experts [15]. Several CRPs approaches considering the behaviors/attitudes/trust of experts have been devised [27, 41, 42, 46, 47]. In particular, the existence and management of non-cooperative behaviors in CRPs has been tackled by several researchers (e.g., [43, 60]), demonstrating that it supposes a critical aspect deserving further investigation. Pelta and Yager [43] and Yager [63, 64] studied the non-cooperative behaviors in GDM problems. In their works, the non-cooperative behaviors are referred to strategic preference manipulation behaviors and they are only investigated in the selection process of GDM problems (i.e. problems that do not involve a consensus building phase). However, these non-cooperative behaviors are often accrued during the consensus phase of a GDM problem. Palomares et al. [42] designed a moderator-based approach for coping with non-cooperative behaviors in the CRP of GDM problems involving large groups, in which a moderator compulsively penalizes the weights of the experts who adopt non-cooperative behaviors. Recently, Dong et al. [18] proposed a self-management mechanism for dealing with non-cooperative behaviors in a CRP for GDM with fuzzy preference relations. Moreover, Social Network Analysis (SNA) has emerged as a key technique in GDM problems with the development of information and network technology. On the one hand, a social network provides valuable decision information about the social relationships among the experts and allows information exchange and communication. On the other hand, experts are easy to be influenced by their most trusted experts in the social network, thereby affecting the GDM result. Wu et al. [53, 55] developed two SNA-based approaches for undertaking CRPs in which the trust relationship among experts is considered.

According to the above literature review, we find that consensus reaching has become a hot

topic in the area of GDM, and a considerable number of models for supporting CRPs have been reported. It is worth emphasizing that several SNA-based approaches have been proposed. In the SNA-based CRPs, a lot of non-cooperative behaviors exist, for example, some experts will express their preferences dishonestly to pursue their own interests. This situation may adversely affect the decision efficiency. To guarantee the decision efficiency and improve the SNA-based CRPs results, it becomes paramount to develop some SNA-based consensus approaches to cope with non-cooperative behaviors. Although several models under a social network context have been proposed, they do not take non-cooperative behaviors of participants into account. Moreover, the social network in a SNA-based CRP is assumed unchanged during the consensus reaching. In addition, the existing moderator-based models cannot be directly extended to deal with non-cooperative behaviors in SNA-based CRPs, due to the moderator's task of dealing with these behaviors being sometimes excessively demanding and complicated in practical decision situations. Motivated with the challenge to cope with non-cooperative behaviors in CRPs based on inter-expert trust information, this paper develops an SNA-based consensus framework for dealing with non-cooperative behaviors in CRPs in a dynamic social network and MAGDM context.

In the proposed SNA-based consensus framework, the experts express not only decision matrices regarding a group of alternatives and attributes but also trust values for other experts in their social trust network. Through a trust propagation and aggregation process on the provided trust values, a complete pairwise trust model is determined. Then, an approach is used to generate the weights of experts predicated on this social trust network information. The experts' weights generated from the social trust network are thus embedded into the CRP. During the CRP, the experts not only adjust their decision matrices to achieve the predefined consensus level, but they can also update their initially provided degrees of trust towards other experts in the social network. In particular, we introduce a novel module for non-cooperative behaviors analysis, which has been designed to analyze the behaviors of experts. The analysis results produced by the module are provided for experts, who in turn modify their trust relationships in the social trust network accordingly. In this way, the weights of the experts with non-cooperative behaviors will be decreased as a result of undertaking the overall proposed SNA-based approach. We present a detailed simulation and a comparative experimental study to demonstrate the decision efficiency of the SNA-based consensus framework for coping with non-cooperative behaviors. To the best of our knowledge, this is the first consensus approach for MAGDM capable of meaningfully synergizing inter-user trust information, multiple patterns of non-cooperative behavior and expert weighting over the course of the CRP.

The rest of this paper is organized as follows: Section 2 introduces preliminaries regarding

the SNA. Then, Section 3 describes the consensus-based MAGDM problem with non-cooperative behaviors and proposes a SNA-based consensus framework. Several types of non-cooperative behaviors are analyzed in Section 4. An illustrative example is provided in Section 5. Following this, simulation and comparison experiments are presented in Section 6. Finally, concluding remarks are drawn in Section 7.

2. Social network analysis

SNA (Social network analysis) has emerged as a key technique in modern sociology, and it focuses on the relationships between social entities such as families, corporations or nations [51]. The existing SNA foundations and methodologies to model social trust relationships among a group of individuals (or experts), have proved their usefulness upon their adoption in several GDM approaches [35, 52, 53, 55].

2.1. The structure of the social network

Three elements are included in a social network: the set of actors, the relations themselves, and the actor attributes, which are described in Table 1. The following three representation schemes are often adopted to describe the main elements in a social network:

(1) Graph theoretic: the social network is characterized as a graph in which nodes are connected by directed lines. In the graph, $e_i \rightarrow e_j$ signifies that expert e_i directly trusts expert e_j.

(2) Sociometric: the trust relationships among experts are represented by a matrix $S = (s_{ij})_{m \times m}$ ($s_{ij} \in \{0,1\}$), which is called sociometric. In particular, $s_{ij} = 1$ denotes that there exists a direct trust relationship from expert e_i to expert e_j.

(3) Algebraic: this notation allows to distinguish several distinct relations and represent combinations of relations.

Table 1: Different representation schemes in SNA

Graph	Sociometric	Algebraic
	$S = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$	$e_1 \, R \, e_2, \ e_1 \, R \, e_3, \ e_1 \, R \, e_5$ $e_2 \, R \, e_3, \ e_3 \, R \, e_4, \ e_4 \, R \, e_1$ $e_5 \, R \, e_2, \ e_5 \, R \, e_4$

However, the above sociometric represents a binary relation among social entities (i.e. either there is total trust or no trust at all), which is not be suitable to model uncertainty in trust relationship representation in a social network [55]. To overcome this problem, this paper adopts one type of social networks, namely social trust network in which the users explicitly express their opinion about other users as trust degrees that vary between 0 and 1. In this situation, the

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sociometric in a social trust network is called fuzzy sociometric, which is formally defined below.

Definition 1: A fuzzy sociometric $S = (s_{ij})_{m \times m}$ on E is a relation in $E \times E$ with membership function $u_S: E \times E \rightarrow [0, 1]$, and $u_S(e_i, e_j) = s_{ij}$, where $s_{ij} \in [0, 1]$ denotes the trust degree that expert e_i assigns to expert e_j.

For notation simplicity, *fuzzy sociometric* will be herein referred to as *sociometric* in the paper.

Example 1: The trust relationships across a group of six experts are represented in direct graph form as in Table 1 with the following sociometric $S = (s_{ij})_{m \times m}$:

$$S = \begin{pmatrix} - & 0.7 & 0.8 & 0 & 0.85 \\ 0 & - & 0.78 & 0 & 0 \\ 0 & 0 & - & 0.88 & 0 \\ 0.9 & 0 & 0 & - & 0 \\ 0 & 0.95 & 0 & 1 & - \end{pmatrix}.$$

2.2. Trust propagation in the social trust network

In a social trust network, some experts may not be able to provide a trust value on a specific expert directly. In this case, the sociometric associated with the social trust network is incomplete. This is illustrated in Fig. 1 (a). In this figure, three experts are included, and there is no direct trust value between expert e_1 and e_3. However, some information on whether or not expert e_1 can trust expert e_3 can still be inferred, based on transitivity. Therefore, it is necessary to design a mechanism to analyze whether an unknown expert can be trusted or not. Victor et al. [50] proposed a trust propagation approach based on t-norms to estimate unknown trust values in the sociometric, which will be adopted in this paper.

Before formally presenting the trust propagation method, some concepts regarding triangular norms are introduced.

A function $T: [0, 1]^2 \rightarrow [0, 1]$ is called a triangular norm (t-norm for short) if and only if it is commutative, associative, monotonic and satisfies the following boundary conditions $T(x, 1) = x$, $\forall x$. In the following, we use the Einstein product as the t-norm:

$$T(a_1, a_2) = \frac{a_1 \cdot a_2}{1 + (1 - a_1) \cdot (1 - a_2)}, \tag{1}$$

where $a_1 \in [0, 1]$ and $a_2 \in [0, 1]$ are two real numbers. It is worth mentioning that $T(a_1, a_2) \leq \min\{a_1, a_2\}$, for any T-norm function T.

In Eq. (1), there are only two arguments. By taking n arguments into account, the following t-norm is considered:

$$T(a_1, a_2, ..., a_n) = \frac{2\prod_{i=1}^{n} a_i}{\prod_{i=1}^{n}(2 - a_i) + \prod_{i=1}^{n} a_i} \tag{2}$$

where $a_i \in [0, 1]$ $(i = 1, 2, ..., n)$. We have that $T(a_1, a_2, ..., a_n) \leq \min(a_1, a_2, ..., a_n)$.

(a) No direct trust between e_1 and e_3 (b) Trust propagation between e_1 and e_3 via e_2

Fig. 1. Trust propagation via indirect trust path

Let $e_i \xrightarrow{1} e_{\sigma(1)} \xrightarrow{2} e_{\sigma(2)} \xrightarrow{3} e_{\sigma(3)} \xrightarrow{4} ... \xrightarrow{q} e_{\sigma(q)} \xrightarrow{q+1} e_j$ be a path from expert e_i to expert e_j, where its length is $q+1$. The trust value s_{ij} can be estimated using t-norm:

$$s_{ij} = T(s_{i, \sigma(1)}, s_{\sigma(1), \sigma(2)}, ..., s_{\sigma(q), j})$$
$$= \frac{2 \cdot s_{i, \sigma(1)} \cdot s_{\sigma(q), j} \prod_{k=1}^{q-1} s_{\sigma(k), \sigma(k+1)}}{(2 - s_{i, \sigma(1)})(2 - s_{\sigma(q), j}) \prod_{k=1}^{q-1}(2 - s_{\sigma(k), \sigma(k+1)}) + s_{i, \sigma(1)} \cdot s_{\sigma(q), j} \prod_{k=1}^{q-1} s_{\sigma(k), \sigma(k+1)}} \quad (3)$$

Example 2: In Fig. 1 (b), we consider that $s_{23} = 0.95$ and $s_{12} = 0.9$. Then, the trust degree from expert e_1 to e_3 can be calculated as $s_{13} = T(s_{12}, s_{23}) = T(0.9, 0.95) = 0.851$.

2.3. Trust aggregation in the social trust network

In some situations, there may be multiple trust paths between two experts. This is demonstrated in Fig. 2, in which there exist two trust paths from expert e_1 to e_3: (1) $e_1 \rightarrow e_2 \rightarrow e_3$; and (2) $e_1 \rightarrow e_5 \rightarrow e_4 \rightarrow e_3$.
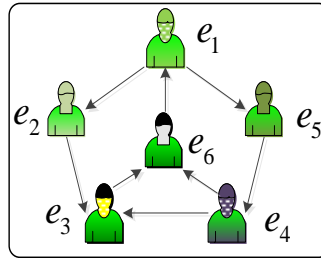
Fig. 2. A social trust network

Suppose that there are N trust paths from expert e_i to expert e_j, and their trust values are $\{s_{ij}^1, s_{ij}^2, ..., s_{ij}^N\}$, a representative trust value s_{ij} is obtained by aggregating the N existing trust degrees between s_i and s_j. The Ordered Weighted Averaging (OWA) operator [65] has been widely adopted in the aggregation processes underlying GDM problems, which allows to flexibly reflect different (Optimistic/Pessimistic) aggregation attitudes. Without loss of generality, the OWA operator is utilized to calculate the aggregated trust value from expert e_i to e_j:

$$s_{ij} = OWA(s_{ij}^1, s_{ij}^2, ..., s_{ij}^N) = \sum_{k=1}^{N} \pi_k \cdot s_{ij}^{\sigma(k)} \quad (4)$$

where $s_{ij}^{\sigma(k)}$ is the kth largest value in $\{s_{ij}^1, s_{ij}^2, ..., s_{ij}^N\}$, and $\pi = (\pi_1, \pi_2, ..., \pi_N)^T$ denotes the weight vector such that $\pi_k \geq 0$ and $\sum_{k=1}^{N} \pi_k = 1$.

In [66], a widely known approach was presented to determine the OWA weights $\pi = (\pi_1, \pi_2, ..., \pi_N)^T$. A quantifier-guided method based on the use of linguistic quantifiers Q and proposed by Yager in [67], is adopted in this work. The weights π_i are computed by the following

formula:

$$\pi_i = Q(\tfrac{i}{N}) - Q(\tfrac{i-1}{N}), \quad i = 1, 2, ..., N,\tag{5}$$

where $Q(c)$ can be represented as

$$Q(c) = \begin{cases} 0, & c < a, \\ \dfrac{c-a}{b-a}, & a \le c \le b, \\ 1, & c > b, \end{cases}\tag{6}$$

with $a,\ b,\ c \in [0,1]$.

The linguistic quantifiers *all*, *most*, *at least half* and *as many as possible,* are often utilized in the literature. Their parameters a are 0, 0.3, 0, and 0.5, respectively; and their parameters b are 1, 0.8, 0.5, and 1, respectively.

Example 3: Suppose that six experts $\{e_1,\ e_2,\ ...,\ e_6\}$ established a number of social trust relationships with each other, as illustrated in Fig. 2, with the following sociometric S :

$$S = \begin{pmatrix} - & 0.7 & - & - & 0.8 & - \\ - & - & 0.9 & - & - & - \\ - & - & - & - & - & 0.8 \\ - & - & 0.6 & - & - & 0.7 \\ - & - & - & 1 & - & - \\ 0.9 & - & - & - & - & - \end{pmatrix}$$

In Fig. 2, there is at least one trust path between every pair of experts. However, some pairs of experts are not directly connected. In other words, the sociometric associated with Fig. 2 is not complete. For example, there is no direct trust link from e_1 to e_3. However, we observe that there are two different paths forming an indirect linkage from e_1 to e_3 : $e_1 \to e_2 \to e_3$ and $e_1 \to e_5 \to e_4 \to e_3$. According to formula (3), we can propagate trust and infer the following trust value from e_1 to e_3.

(1) $s_{13}^1 = T(s_{12},\ s_{23}) = T(0.7,\ 0.9) = 0.612$;

(2) $s_{13}^2 = T(s_{15},\ s_{54},\ s_{43}) = T(0.8,\ 1,\ 0.6) = 0.444$;

In this example, the OWA operator with the linguistic quantifier "most" is used to fuse the trust values in different trust paths. The aggregated trust value from e_1 to e_3 can be computed using Eq. (4):

$$s_{13} = OWA(s_{13}^1,\ s_{13}^2) = OWA(0.612,\ 0.444) = 0.4 \times 0.612 + 0.6 \times 0.444 = 0.511.$$

Similarly, the other unknown trust values can be estimated, which results in a completely defined sociometric denoted by *ES*:

$$ES = \begin{pmatrix} - & 0.7 & \mathbf{0.511} & \mathbf{0.8} & 0.8 & \mathbf{0.42} \\ \mathbf{0.617} & - & 0.9 & \mathbf{0.459} & \mathbf{0.459} & \mathbf{0.706} \\ \mathbf{0.706} & \mathbf{0.454} & - & \mathbf{0.533} & \mathbf{0.533} & 0.8 \\ \mathbf{0.472} & \mathbf{0.288} & 0.6 & - & \mathbf{0.344} & 0.7 \\ \mathbf{0.472} & \mathbf{0.288} & \mathbf{0.31} & 1 & - & \mathbf{0.546} \\ 0.9 & \mathbf{0.612} & \mathbf{0.439} & \mathbf{0.706} & \mathbf{0.706} & - \end{pmatrix}.$$

Based on S and ES, a complete sociometric (denoted as $CS = (cs_{ij})_{m \times m}$) can be provided by experts. When providing $CS = (cs_{ij})_{m \times m}$, we suggest that cs_{ij} should be close to s_{ij} if s_{ij} is unknown in S; otherwise $cs_{ij} = s_{ij}$.

2.4. Obtaining the expert's weights from the SNA

Let $S = (s_{ij})_{m \times m}$ be the complete sociometric associated with a specific social trust network, then the relative node in-degree centrality index, associated with an expert $e_k \in E$, can be computed as below:

$$C(e_k) = \frac{1}{m-1} \sum_{i=1, i \neq k}^{m} s_{ik} \tag{7}$$

Obviously, the larger $C(e_k)$ value indicates the higher importance degree of expert e_k, as a result of a higher overall degree of trust in e_k by the rest of experts in the group. Following this idea, the weight of expert e_k can be defined as below.

Let $\{C(e_1), C(e_2), ..., C(e_m)\}$ be the set of in-degree centrality indexes associated with a group of experts $E = \{e_1, e_2, ..., e_m\}$. The weight of expert e_k, λ_k, can be determined by:

$$\lambda_k = \frac{C(e_k)}{\sum_{i=1}^{m} C(e_k)} \tag{8}$$

3. The consensus reaching framework based on social network analysis

This section presents a model for supporting CRPs in MAGDM problems under the presence of non-cooperative behaviors, along with its integration into a resolution framework based on SNA.

3.1. Consensus reaching problem with non-cooperative behaviors

As noted in Section 1, there may exist a great deal of non-cooperative behaviors in CRPs. Here, we introduce a consensus reaching problem with non-cooperative behaviors in MAGDM context.

In MAGDM, a set of experts $E = \{e_1, e_2, ..., e_m\}$ $(m \geq 2)$ provide their preferences or opinions regarding a set of possible alternatives or solutions $X = \{x_1, x_2, ..., x_n\}$ $(n \geq 2)$ with respect to a group of attributes $A = \{a_1, a_2, ..., a_l\}$ $(l \geq 2)$. Let $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)^T$ be the weight vector over experts E, where $\lambda_k \geq 0$ $(k = 1, 2, ..., m)$ represents the weight of the expert e_k, and $\sum_{k=1}^{m} \lambda_k = 1$. Let $w = (w_1, w_2, ..., w_l)^T$ be the associated weight vector over attributes, where $w_i \geq 0$ $(i = 1, 2, ..., l)$ denotes the weight of the attribute a_i, and $\sum_{i=1}^{l} w_i = 1$. Let $V^{(k)} = (v_{ij}^{(k)})_{n \times l}$ $(k = 1, 2, ..., m)$ be the multiple attribute decision matrix provided by the expert e_k, where $v_{ij}^{(k)} \in [0, 1]$ signifies her/his preference value for the alternative x_i with respect to attribute a_j.

In a CRP, the non-cooperative behaviors may be utilized by some experts to pursue their interests, which will influence the consensus efficiency. Our interest in the MAGDM decision framework considered is, consequently, to assist experts to obtain a consensual collective solution

under the presence (and adverse effects) of such non-cooperative behaviors.

3.2. Proposed consensus reaching framework

Two processes (or models) are frequently utilized for solving GDM problems [10, 29]: *consensus process* and *selection process*. The consensus process is often employed to assist experts to achieve the predefined consensus level, and the selection process is dedicated to produce a collective ranking of the alternatives according to the preferences provided by the experts. Inspired by these two processes, the self-management mechanism for coping with non-cooperative behaviors reported by Dong et al. [18], and the consensus framework based on the trust relationship among experts presented by Wu et al. [55], we propose a novel consensus reaching framework: SNA-based consensus reaching framework, that comprehensively integrates behavior management and social trust information in the decision process. The implementation of the SNA-based consensus reaching framework deals with a four-stage procedure, whose main stages are graphically presented in Fig. 3.

(1) Generating the experts' weights from a social trust network

In this process, the weights of experts are generated from the social trust network. In the initial round (i.e., $z = 0$) of the CRP, the sociometric (i.e., S) associated with the social trust network is often incomplete, and to generate a complete sociometric (i.e., CS) associated with S is critical. In the proposed consensus reaching framework, the trust propagation and aggregation method presented in section 2 is utilized to infer the unknown trust values in the sociometric, and an estimated sociometric (i.e., ES) is then yielded. The estimated sociometric ES can be directly treated as the CS, which will not change the essence of the proposed consensus reaching framework. In order to increase the flexibility of the proposed consensus reaching framework, this study assumes that CS is provided by experts using ES as the reference social information. Then, the approach proposed in section 2 is adopted to generate the experts' weights from CS. In subsequent rounds of the CRP (i.e., $z \geq 1$), the experts update their trust values with respect to other experts (based on the non-cooperative behaviors detected). Thus, both the complete sociometric and weights of experts derived from it change dynamically during the CRP.
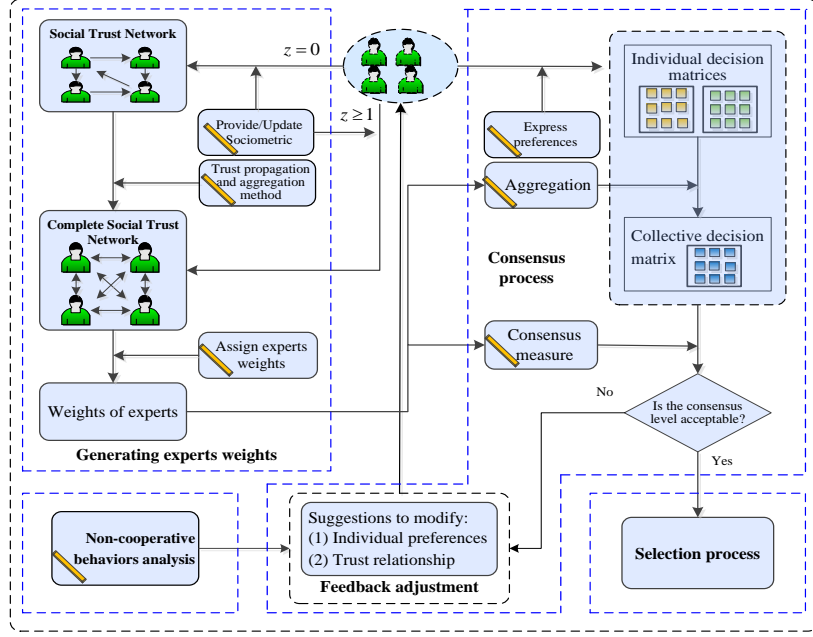
Fig. 3. The SNA-based consensus reaching framework

(2) Consensus process

Two main steps are normally involved in the consensus process: *consensus measure* and *feedback adjustment* [39, 40].

(i) Consensus measure

In this step, a consensus measure method is devised to compute the consensus level among the experts by taking their importance weights into account.

Let λ, $V^{(k)} = (v_{ij}^{(k)})_{n \times l}$ and $V^{(c)} = (v_{ij}^{(c)})_{n \times l}$ be as above. Several consensus measurement methods have been reported (e.g., [9, 25]) to calculate the level of consensus in the CRP. Here, we utilize the consensus measure proposed in [42]. The basic idea of this consensus measure method is that we first compute the similarity levels between each pair of experts regarding their preference values or assessments. Then, a consensus matrix is calculated by aggregating these obtained similarity levels. Based on the consensus matrix, the consensus levels on the preference values, the consensus levels on alternatives, and the group consensus level can be generated.

Let $SM^{(kh)} = (sm_{ij}^{(kh)})_{n \times l}$ ($k = 1, 2, ..., m-1$, $h = k+1, k+2, ..., m$) be the preference similarity matrix, where $sm_{ij}^{(kh)} \in [0,1]$ represents the similarity level between experts e_k and e_h in their preference values $v_{ij}^{(k)}$ and $v_{ij}^{(h)}$, and $sm_{ij}^{(kh)}$ is determined by:

$$sm_{ij}^{(kh)} = 1 - |v_{ij}^{(k)} - v_{ij}^{(h)}|. \tag{9}$$

Let $CM = (cm_{ij})_{n \times l}$ be the consensus matrix, where cm_{ij} is the collective consensus level regarding the preference value v_{ij}, and it is computed by:

$$cm_{ij} = \frac{\sum_{k=1}^{m-1} \sum_{h=k+1}^{m} \lambda_{kh} \cdot sm_{ij}^{(kh)}}{\sum_{k=1}^{m-1} \sum_{h=k+1}^{m} \lambda_{kh}}, \tag{10}$$

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where $\lambda_{kh} \in [0, 1]$ can be determined by $\lambda_{kh} = \min(\lambda_k, \lambda_h)$ (see [42]).

Based on $CM = (cm_{ij})_{n \times n}$, the consensus levels are defined at three levels:

(a) Consensus level on the preference value v_{ij}, $cp_{ij} = cm_{ij}$.

(b) Consensus level on alternative x_i, $ca_i = \dfrac{\sum_{j=1}^{l} cm_{ij}}{l}$.

(c) Group consensus level,

$$cl = \frac{\sum_{i=1}^{n} ca_i}{n}. \tag{11}$$

It is clear that $cl \in [0, 1]$. In particular, $cl = 1$ means that there is a full consensus among experts. Otherwise, a larger cl value indicates a higher consensus level among experts. In the CRP, a consensus threshold $\varepsilon \in [0, 1]$ is often defined. If $cl \geq \varepsilon$, the consensus level among experts is acceptable; otherwise, the feedback adjustment process is utilized to assist experts in modifying their preferences.

Example 4: Let $V^{(1)} = (v_{ij}^{(1)})_{3 \times 3}$, $V^{(2)} = (v_{ij}^{(2)})_{3 \times 3}$, and $V^{(3)} = (v_{ij}^{(3)})_{3 \times 3}$ be three decision matrices provided by experts e_1, e_2 and e_3, respectively. The weight vector over the three experts is $\lambda = (0.2,\ 0.3,\ 0.5)^T$.

Table 2: $V^{(k)}$ $(k = 1,2,3)$ in Example 4

	$V^{(1)}$			$V^{(2)}$			$V^{(3)}$		
	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
x_1	0.3	0.4	0.5	0.5	0.6	0.5	0.5	0.6	0.7
x_2	0.6	0.7	0.5	0.6	0.8	0.7	0.8	0.8	0.8
x_3	0.4	0.5	0.6	0.5	0.6	0.6	0.6	0.6	0.8

The three preference similarity matrices $SM^{(1,2)} = (sm_{ij}^{(1,2)})_{3 \times 3}$, $SM^{(1,3)} = (sm_{ij}^{(1,3)})_{3 \times 3}$, and $SM^{(2,3)} = (sm_{ij}^{(2,3)})_{3 \times 3}$ can be generated based on Eq. (9). For example, $sm_{11}^{(1,2)} = 1 - |v_{11}^{(1)} - v_{11}^{(2)}| = 1 - |0.3 - 0.5| = 0.8$.

Then, the consensus matrix $CM = (cm_{ij})_{3 \times 3}$ can be generated. For example, cm_{11} can be computed by:

$$cm_{11} = \frac{\lambda_{12} \cdot sm_{11}^{(1,2)} + \lambda_{13} \cdot sm_{11}^{(1,3)} + \lambda_{23} \cdot sm_{11}^{(2,3)}}{\lambda_{12} + \lambda_{13} + \lambda_{23}} = \frac{0.2 \cdot 0.8 + 0.2 \cdot 0.8 + 0.3 \cdot 1}{0.2 + 0.2 + 0.3} = 0.886.$$

Similarly, we can obtain $cm_{12} = 0.886$, $cm_{13} = 0.857$, $cm_{21} = 0.857$, $cm_{22} = 0.843$, $cm_{23} = 0.814$, $cm_{31} = 0.871$, $cm_{32} = 0.843$, and $cm_{33} = 0.857$.

Next, cp_{ij} is computed by $cp_{ij} = cm_{ij}$ $(i, j = 1,2,3)$. Based on $ca_i = \sum_{j=1}^{l} cm_{ij} / l$ obtains the consensus levels on the three alternatives, and they are $ca_1 = 0.876$, $ca_2 = 0.871$, and $ca_3 = 0.89$, respectively.

The group consensus level among the three experts is computed by:

$$cl = \frac{\sum_{i=1}^{n} ca_i}{n} = \frac{0.876 + 0.871 + 0.89}{3} = 0.879.$$

(ii) Feedback adjustment

If the predefined consensus level among experts is not achieved, the feedback adjustment process is employed to assist experts to update their decision matrices for increasing the consensus level among experts. The core idea of the feedback adjustment process is to obtain and provide the group decision matrix, which is obtained from the individual decision matrices using a weighted averaging aggregation operator, to experts to reconsider constructing new decision matrices.

Let $V^{(group)} = (v_{ij}^{(group)})_{n \times l}$ be the group decision matrix, where $v_{ij}^{(c)}$ can be yielded as follows:

$$v_{ij}^{(group)} = \sum_{k=1}^{m} \lambda_k \cdot v_{ij}^{(k)} \tag{12}$$

Let $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times l}$ be the adjusted decision matrix associated with $V^{(k)} = (v_{ij}^{(k)})_{n \times l}$. When providing $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times l}$, it is generally advised that the experts adjust their assessments so as to bring them closer to the collective preference, i.e.:

$$\overline{v_{ij}^{(k)}} \in [\min(v_{ij}^{(k)}, v_{ij}^{(group)}), \ \max(v_{ij}^{(k)}, v_{ij}^{(group)})] \tag{13}$$

Example 5: Let $V^{(1)} = (v_{ij}^{(1)})_{3 \times 3}$, $V^{(2)} = (v_{ij}^{(2)})_{3 \times 3}$, $V^{(3)} = (v_{ij}^{(3)})_{3 \times 3}$ and $\lambda = (0.2, 0.3, 0.5)^T$ be as in Example 4.

Using Eq. (12) obtains the group decision matrix, which is provided in Table 3.

Table 3: $V^{(group)}$ in Example 5

	a_1	a_2	a_3
x_1	0.46	0.56	0.6
x_2	0.7	0.78	0.71
x_3	0.53	0.58	0.7

Then, $V^{(group)}$ is provided for the three experts to construct new decision matrices, $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{3 \times 3}$ $(k = 1, 2, 3)$. For example, when providing $\overline{v_{11}^{(1)}}$, it is advised that $\overline{v_{11}^{(1)}} \in [\min(v_{11}^{(1)}, v_{11}^{(group)}), \ \max(v_{11}^{(1)}, v_{11}^{(group)})] = [0.3, \ 0.46]$.

Simultaneously, the results from the non-cooperative behaviors analysis module (described in detail in Section 4) are provided for experts to modify their trust values regarding other experts. In particular, if an expert adopts non-cooperative behavior(s), then other experts will decrease the trust values with respect to this expert.

(3) Non-cooperative behavior analysis

In this stage, the behaviors of experts upon the preference adjustment via feedback are analyzed. The results of the behavior analysis are subsequently provided for experts to modify their trust relationships before a new consensus round is initiated. In particular, if an expert is deemed as adopting non-cooperative behavior(s), other experts are suggested to decrease the trust values of this expert in the social trust network.

The detailed process to analyze non-cooperative behaviors is provided in section 4.

(4) Selection process

Once the predefined consensus level among experts is reached, the selection process is conducted to generate the final collective ranking of alternatives.

Let $V = (v_{ij})_{n \times l}$ be a decision matrix. Applying the weighted averaging (WA) operator to fuse all the preference values in the ith row of $V = (v_{ij})_{n \times l}$, the evaluation value of the alternative x_i,

EV_i, can be generated, that is

$$EV_i = \sum_{j=1}^{l} w_j \cdot v_{ij} \tag{14}$$

where $w_j \in [0, 1]$ is the weight over the attribute a_j.

Further, based on EV_i $(i = 1, 2, ..., n)$, the preference ordering $O = (o_1, o_2, ..., o_n)^T$ can be obtained to rank alternatives from the best to the worst one, where

$$o_i = j \tag{15}$$

if EV_i is the jth largest value in $\{EV_1, EV_2, ..., EV_n\}$.

We can get the collective preference ordering from $V^{(group)}$ applying Eq. (15), which is denoted as $O^{(group)} = (o_1^{(group)}, o_2^{(group)}, ..., o_n^{(group)})^T$. In addition, the preference ordering derived from $V^{(k)}$ is denoted as $O^{(k)} = (o_1^{(k)}, o_2^{(k)}, ..., o_n^{(k)})^T$, which will be used for detecting some types of non-cooperative behaviors.

Example 6: Let the attribute weight vector be $w = (0.3, 0.4, 0.3)^T$. Employing Eq. (14) yields $EV_1 = 0.542$, $EV_2 = 0.735$, and $EV_3 = 0.601$. Further, the preference ordering over the three alternatives is produced, that is $O = (3, 1, 2)^T$.

In the following, we present a flowchart and an algorithm to describe the SNA-based CRP.
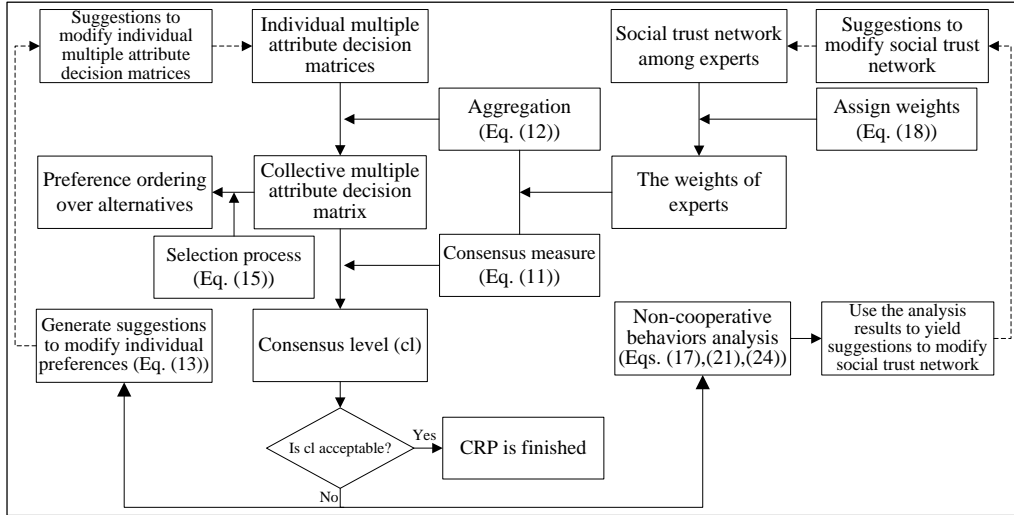
Fig. 4. Flowchart of the proposed SNA-based consensus framework

Table 4: SNA-based consensus reaching algorithm

Input: $V^{(k)} = (v_{ij}^{(k)})_{n \times l}$ $(k = 1, 2, ..., m)$, $S = (s_{ij})_{m \times m}$, $w = (w_1, w_2, ..., w_l)^T$, ε, and $z_{max} \geq 1$.

Output: The adjusted decision matrices $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times l}$ $(k = 1, 2, ..., m)$, the adjusted sociometric $\overline{S} = (\overline{s_{ij}})_{m \times m}$, the number of iterations Z, and the final collective preference ordering $O^{(c)} = (o_1^{(c)}, o_2^{(c)}, ..., o_n^{(c)})^T$.

Step 1: Let $z = 0$, $V^{(k,z)} = V^{(k)}$. Applying the trust propagation and aggregation method, we can estimate the unknown trust values, and they are used as references for constructing complete sociometric. The complete sociometric is denoted as $S^{(z)} = (s_{ij}^{(z)})_{m \times m}$ $(k = 1, 2, ..., m)$.

Step 2: Apply Eq. (8) to generate the experts' weights $\lambda_z = (\lambda_{1,z}, \lambda_{2,z}, ..., \lambda_{m,z})^T$, where

$$\lambda_{k,z} = \frac{\sum_{i=1,\, i \neq k}^{m} s_{ik}^{(z)}}{\sum_{k=1}^{m} \sum_{i=1,\, i \neq k}^{m} s_{ik}^{(z)}}\ .$$

Step 3: Using Eq. (11) provides the consensus level among experts cl_z. If $cl_z \geq \varepsilon$ or $z > z_{\max}$, go to **Step 6**; otherwise, continue with the next step.

Step 4: Employing Eq. (12) yields the collective decision matrix $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times l}$, where $v_{ij}^{(c,z)} = \sum_{k=1}^{m} \lambda_{k,z} v_{ij}^{(k,z)}$. When constructing $V^{(k,z+1)} = (v_{ij}^{(k,z+1)})_{n \times l}$ $(k = 1, 2, ..., m)$, experts are advised to adjust their preferences, such that $v_{ij}^{(k,z+1)} \in [\min(v_{ij}^{(k,z)}, v_{ij}^{(c,z)}), \max(v_{ij}^{(k,z)}, v_{ij}^{(c,z)})]$.

Step 5: The non-cooperative behavior analysis module is conducted to analyze the behaviors of the experts. Based on this, experts update their trust relationships. As a result, a new sociometric is constructed, which is denoted as $S^{(z+1)} = (s_{ij}^{(z+1)})_{m \times m}$ $(k = 1, 2, ..., m)$.

Let $z = z + 1$, then go to **Step 2.**

Step 6: Let $\overline{V^{(k)}} = V^{(k,z)}$, $\overline{S} = S^{(z)}$, and $Z = z$. Based on Eq. (12), the final collective decision matrix $V^{(c)}$ can be generated from $\{\overline{V^{(1)}}, \overline{V^{(2)}}, ..., \overline{V^{(m)}}\}$. Applying selection process offers the final collective preference ordering $O^{(c)}$ from $V^{(c)}$. Output the adjusted decision matrices $\overline{V^{(k)}} = (\overline{v_{ij}^{(k)}})_{n \times l}$, the sociometric $\overline{S} = (\overline{s_{ij}})_{m \times m}$, the number of consensus rounds Z, and the final collective preference ordering $O^{(c)}$.

4. Non-cooperative behavior analysis

In the CRP, it is common that some experts adopt non-cooperative behaviors to further their own interests. In this section, several common non-cooperative behaviors of experts are analyzed in detail.

Let $V^{(k,z)} = (v_{ij}^{(k,z)})_{n \times l}$ be a decision matrix given by the expert e_k in consensus round z, and let $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times l}$ be the collective decision matrix in consensus round z. In the following, we define three types of non-cooperative behaviors: dishonest, disobedient, and divergent behaviors.

(1) *Dishonest behavior*

In the CRP, it is not unusual that some experts will give opinions or preferences regarding alternatives dishonestly. In particular, the evaluation on the alternatives that are preferred by the group may be systematically decreased by an expert in the CRP, which is a common dishonest behavior.

The basic idea to identify a dishonest behavior is below:

The most preferred alternatives of the group are firstly identified. Then, the similarity level between each individual and the group is calculated regarding these identified alternatives. If the similarity level of an expert is small enough, then this expert exhibits a dishonest behavior.

Following this idea, the dishonest behavior is formally defined below.

Definition 2: The preference ordering $O = (o_1, o_2, ..., o_n)^T$ can be equally represented using a matrix $T = (t_{ij})_{n \times n}$, where

$$t_{ij} = \begin{cases} 1, & o_i < o_j \\ 0, & o_i > o_j \\ 0.5, & o_i = o_j \end{cases}.$$

Let $Y_z = (y_{1,z}, y_{2,z}, ..., y_{n,z})^T$ be a vector, where $y_{i,z} = 1$ if x_i is among the most preferred alternatives by the group at round z; otherwise $y_{i,z} = 0$. Without loss of generality, Y_z can be obtained using the following assignment:

$$y_{i,z} = \begin{cases} 1, & if \ o_i^{(group,z)} \le round(\beta \cdot n) \\ 0, & otherwise \end{cases}, \tag{16}$$

where $o_i^{(group,z)}$ is the group ranking position of alternative x_i in consensus round z, and $\beta \in [0, 1]$ is a parameter to distinguish the most preferred alternatives by the group (the lower its value, the more restrictively the parameter behaves) and *round* is the usual rounding operator.

Let $O^{(k,z)} = (o_1^{(k,z)}, o_2^{(k,z)}, ..., o_n^{(k,z)})^T$ be the preference ordering associated with respect to e_k in consensus round z. Let $O^{(group,z)} = (o_1^{(group,z)}, o_2^{(group,z)}, ..., o_n^{(group,z)})^T$ be the group preference ordering in consensus round z, where $o_i^{(group,z)}$ is the group ranking position alternative x_i. Let $T^{(k,z)} = (t_{ij}^{(k,z)})_{n \times n}$ and $T^{(group,z-1)} = (t_{ij}^{(group,z-1)})_{n \times n}$ be two matrices generated from $O^{(k,z)}$ and $O^{(group,z-1)}$, respectively, according to Definition 2.

Let

$$NS_1^{(k,z)} = 1 - \frac{\sum_{i=1}^{n} \sum_{j=1, j \ne i}^{n} y_{i,z-1} \mid t_{ij}^{(k,z)} - t_{ij}^{(group,z-1)} \mid}{(n-1)\sum_{i=1}^{n} y_{i,z-1}}. \tag{17}$$

Clearly, $NS_1^{(k,z)} \in [0,1]$. A smaller $NS_1^{(k,z)}$ value is deemed a stronger indicator that a higher probability of expert e_k exhibits dishonest behavior. Let α_1 ($\alpha_1 \in [0,1]$) be a parameter, which is utilized to ascertain whether an expert features the dishonest behavior. If $NS_1^{(k,z)} \le \alpha_1$, we infer that expert e_k exhibits a dishonest behavior in consensus round z.

Below, an example, i.e., **Example 7**, is presented for better understanding the identification of dishonest behaviors.

Example 7: Let $V^{(1,z-1)} = (v_{ij}^{(1,z-1)})_{6 \times 3}$ be the decision matrix provided by expert e_1 in consensus round $z-1$, and let $V^{(group,z-1)} = (v_{ij}^{(group,z-1)})_{6 \times 3}$ be the collective decision matrix in consensus round $z-1$. $V^{(1,z-1)}$ and $V^{(group,z-1)}$ are provided below:

Table 5: $V^{(1,z-1)}$ and $V^{(group,z-1)}$ in Example 7

	$V^{(1,z-1)}$			$V^{(group,z-1)}$		
	a_1	a_2	a_3	a_1	a_2	a_3
x_1	0.3	0.4	0.5	0.5	0.6	0.7
x_2	0.6	0.75	0.5	0.8	0.85	0.68
x_3	0.5	0.45	0.35	0.7	0.65	0.55
x_4	0.45	0.55	0.6	0.8	0.75	0.82
x_5	0.55	0.4	0.6	0.9	0.65	0.8
x_6	0.45	0.45	0.6	0.65	0.65	0.8

Suppose that expert e_1 expresses his/her adjusted decision matrix $V^{(1,z)}$ as:

Table 6: $V^{(1,z)}$ in Example 7

	a_1	a_2	a_3
x_1	0.48	0.58	0.7
x_2	0.75	0.8	0.6
x_3	0.68	0.65	0.53
x_4	0.47	0.56	0.62
x_5	0.6	0.5	0.65
x_6	0.6	0.65	0.75

Applying Eq. (15) yields the preference ordering $O^{(group,z-1)} = (6,\ 3,\ 5,\ 1,\ 2,\ 4)^T$ from $V^{(group,z-1)}$. When setting $\beta = 0.3$, we have $Y_{z-1} = (0,\ 0,\ 0,\ 1,\ 1,\ 0)^T$. Meanwhile, according to Eq. (15), we have $O^{(1,z)} = (4,\ 1,\ 3,\ 6,\ 5,\ 2)^T$. Further, we can obtain $NS_2^{(1,z)} = 0$, according to Eq. (17).

In this example, if we set $\alpha_1 = 0.5$, we will infer that expert e_1 shows a dishonest behavior because $NS_1^{(1,z)} \leq \alpha_1$, because despite having updated preferences towards the collective opinion, the least preferred alternatives by the group have been adjusted more strongly. This intuitively affects the updated preference ordering $O^{(1,z)}$.

(2) *Disobedient behavior*

To achieve the predefined consensus level among experts, experts are required to change their preferences or opinions according to the suggestions generated from the feedback adjustment process. However, to pursue their interests, some experts may modify their preferences or opinions to a very low extent, or even in the opposite direction as recommended. In this paper, this type of behavior is referred as disobedient behavior.

The basic idea to identify a disobedient behavior is below:

Firstly, the actual adjustment distance of each expert in the CRP is computed. Then, the total adjustment distance of each expert to achieve a full consensus is generated. Next, the adjustment proportion to which each expert changes his/her opinions and shifts them closer to consensus, based on the suggestion received, is produced. If the adjustment proportion of an expert is smaller than an expected minimum value, then this expert exhibits a disobedient behavior.

In the following, we formally define disobedient behavior.

Let

$$
f_{ij}^{(k,z)} = \begin{cases} |\,v_{ij}^{(k,z)} - v_{ij}^{(k,z-1)}\,|, & \text{if } v_{ij}^{(k,z)} \in [\min(v_{ij}^{(k,z-1)}, v_{ij}^{(group,z-1)}),\ \max(v_{ij}^{(k,z-1)}, v_{ij}^{(group,z-1)})] \\ v_{ij}^{(k,z)} - v_{ij}^{(k,z-1)}, & \text{if } v_{ij}^{(k,z-1)} < v_{ij}^{(group,z-1)} \ and \ v_{ij}^{(k,z)} < v_{ij}^{(k,z-1)} \\ v_{ij}^{(k,z-1)} - v_{ij}^{(k,z)}, & \text{if } v_{ij}^{(k,z-1)} > v_{ij}^{(group,z-1)} \ and \ v_{ij}^{(k,z)} > v_{ij}^{(k,z-1)} \\ v_{ij}^{(group,z-1)} - v_{ij}^{(k,z)}, & \text{if } v_{ij}^{(k,z-1)} < v_{ij}^{(group,z-1)} \ and \ v_{ij}^{(k,z)} > v_{ij}^{(group,z-1)} \\ v_{ij}^{(k,z)} - v_{ij}^{(group,z-1)}, & \text{if } v_{ij}^{(k,z-1)} > v_{ij}^{(group,z-1)} \ and \ v_{ij}^{(k,z)} < v_{ij}^{(group,z-1)} \end{cases} \qquad (18)
$$

where $f_{ij}^{(k,z)}$ reflects the situation that the expert e_k modifies his/her preference value regarding the alternative x_i with respect to attribute a_j. In particular, this indicates that the expert modifies his/her preference value in the opposite recommended direction if $f_{ij}^{(k,z)} < 0$.

Let

$$AD^{(k,z)} = \sum_{i=1}^{n} \sum_{j=1}^{l} f_{ij}^{(k,z)} \tag{19}$$

and

$$D^{(k,z)} = \sum_{i=1}^{n} \sum_{j=1}^{l} |v_{ij}^{(k,z-1)} - v_{ij}^{(group,z-1)}| \tag{20}$$

where $AD^{(k,z)}$ signifies the total adjustment distance of expert e_k with respect to all of the elements v_{ij} $(i=1,2,...,n; j=1,2,...,l)$, and $D^{(k,z)}$ represents the total adjustment distance of expert e_k to achieve a full consensus on all of the elements v_{ij} $(i=1,2,...,n; j=1,2,...,l)$.

Let

$$NS_2^{(k,z)} = \frac{AD^{(k,z)}}{D^{(k,z)}}. \tag{21}$$

The value of $NS_2^{(k,z)}$ signifies the adjustment proportion to which expert e_k changes his/her opinions and brings them closer to consensus, based on the suggestions received. Clearly, a smaller $NS_2^{(k,z)}$ value stands for a stronger indicator that expert e_k exhibits disobedient behavior. Let α_2 $(\alpha_2 \in [0,1))$ be a parameter, which is used to check whether an expert features the disobedient behavior. If $NS_2^{(k,z)} \leq \alpha_2$, we infer that expert e_k meets the feature of the disobedient behavior in consensus round z.

Example 8: Let $V^{(1,z-1)} = (v_{ij}^{(1,z-1)})_{3\times3}$ be the decision matrix provided by expert e_1 in consensus round $z-1$, and let $V^{(group,z-1)} = (v_{ij}^{(group,z-1)})_{3\times3}$ be the group decision matrix in consensus round $z-1$. $V^{(1,z-1)}$ and $V^{(group,z-1)}$ are provided below:

Table 7: $V^{(1,z-1)}$ and $V^{(group,z-1)}$ in example 8

	$V^{(k,z-1)}$			$V^{(group,z-1)}$		
	a_1	a_2	a_3	a_1	a_2	a_3
x_1	0.7	0.5	0.55	0.55	0.65	0.65
x_2	0.8	0.65	0.62	0.55	0.75	0.5
x_3	0.85	0.72	0.9	0.65	0.9	0.65

We assume that expert e_1 provides the adjusted decision matrix $V^{(1,z)}$ as follows:

Table 8: $V^{(1,z)}$ in example 8

	a_1	a_2	a_3
x_1	0.6	0.55	0.5
x_2	0.9	0.68	0.52
x_3	0.68	0.88	0.7

Applying Eq. (18) yields $f_{11}^{(1,z)} = 0.1$, $f_{12}^{(1,z)} = 0.05$, $f_{13}^{(1,z)} = -0.05$, $f_{21}^{(1,z)} = -0.1$, $f_{22}^{(1,z)} = 0.03$, $f_{23}^{(1,z)} = 0.1$, $f_{31}^{(1,z)} = 0.17$, $f_{32}^{(1,z)} = 0.16$ and $f_{33}^{(1,z)} = 0.2$. Applying Eqs. (19) and (20) yields $AD^{(1,z)} = 0.66$ and $D^{(1,z)} = 1.5$, respectively. Further, utilizing Eq. (21) leads to $NS_2^{(1,z)} = 0.44$.

Here, if we set $\alpha_2 = 0.5$, we will infer that expert e_1 moderately exhibits a disobedient

behavior pattern, because $NS_2^{(1,z)} < \alpha_2$.

(3) *Divergent behavior*

In the CRP, experts' preferences or opinions will achieve a consensus if they change their decision matrices according to the suggestion of feedback adjustment. However, an expert's decision matrix may diverge from the remainder of the experts. In this paper, this type of behavior is referred as divergent behavior.

The basic idea to define divergent behavior is below:

The distances between all pairs of experts are computed. If the distance between two experts is small enough, then they are considered as neighbors. Following this, the neighbors of each expert can be identified. If an individual has fewer neighbors, that is to say, the proportion of his/her neighbors is too small, then this expert exhibits divergent behavior.

In the following, we formally define divergent behavior.

Let

$$d_{ij}^{(z)} = \begin{cases} 1, & d(V^{(i,z)}, V^{(j,z)}) \ge \gamma \\ 0, & otherwise \end{cases} \tag{22}$$

where $\gamma \in [0,\ 1]$ is a parameter, and $d(V^{(i,z)}, V^{(j,z)})$ is computed by:

$$d(V^{(i,z)}, V^{(j,z)}) = \frac{1}{n \times l} \sum_{p=1}^{n} \sum_{q=1}^{l} | v_{pq}^{(i,z)} - v_{pq}^{(j,z)} |. \tag{23}$$

Clearly, e_i and e_j are neighbors if $d_{ij}^{(z)} = 0$.

Let

$$NS_3^{(k,z)} = 1 - \frac{1}{m-1} \sum_{j=1}^{m} d_{kj}^{(z)} \tag{24}$$

Clearly, $NS_3^{(k,z)} \in [0,\ 1]$ reflects the proportion of neighbors associated with e_k. Let α_3 ($\alpha_3 \in [0,1]$) be the predefined parameter to check divergent behavior. If $NS_3^{(k,z)} \le \alpha_3$, we infer that expert e_k meets the feature of the divergent behavior.

Example 9: Let $V^{(1,\ z)}$ be as in **Example 8**. Let $V^{(2,\ z)}$, $V^{(3,\ z)}$ and $V^{(4,\ z)}$ be three decision matrices associated with experts e_2, e_3, and e_4, respectively, in consensus round z, which are listed in Table 9.

<p align="center">Table 9: Decision matrices $V^{(k,z)}$ ($k = 2,\ 3,\ 4$) in Example 9</p>

	$V^{(2,\ z)}$			$V^{(3,\ z)}$			$V^{(4,\ z)}$		
	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
x_1	0.65	0.6	0.55	0.63	0.59	0.52	0.2	0.25	0.9
x_2	0.85	0.72	0.5	0.88	0.7	0.55	0.45	0.4	0.95
x_3	0.65	0.9	0.68	0.7	0.85	0.72	0.92	0.23	0.4

Based on Eq. (23), we can obtain that $d(V^{(1,\ z)}, V^{(2,\ z)}) = 0.037$, $d(V^{(1,\ z)}, V^{(3,\ z)}) = 0.034$, $d(V^{(1,\ z)}, V^{(4,\ z)}) = 0.383$, $d(V^{(2,\ z)}, V^{(3,\ z)}) = 0.033$, $d(V^{(2,\ z)}, V^{(4,\ z)}) = 0.393$, and $d(V^{(3,\ z)}, V^{(4,\ z)}) = 0.382$. When setting $\gamma = 0.35$, we have $NS_3^{(1,z)} = 2/3$, $NS_3^{(2,z)} = 2/3$,

<p align="center">19</p>

$NS_3^{(3,z)} = 2/3$, and $NS_3^{(4,z)} = 0$, according to Eq. (24). If we set $\alpha_3 = 0.6$, then we will infer that expert e_4 exhibits a divergent behavior because $NS_3^{(4,z)} = 0 < \alpha_3$.

Note 1: The parameter β utilized in Eq. (16) is used to distinguish the most preferred alternatives by the group when analyzing the dishonest behavior, and parameter γ appeared in Eq. (22) is used a threshold to judge whether the diverge degree between two experts is large enough when analyzing the divergent behavior. The parameters α_1, α_2, and α_3 are used as thresholds to deduce whether experts' behaviors satisfy the characteristics of being dishonest, disobedient, and divergent behaviors, respectively. Smaller α_1, α_2, and α_3 values indicate the stricter criteria to deduce dishonest, disobedient, and divergent behaviors, respectively. According to the specific decision problem and situation at hand, the experts can set β, γ, α_1, α_2, and α_3 values. When using the appropriate parameter settings in a given situation, the proposed consensus framework is effective for managing non-cooperative behaviors, as shown in the following simulation experiments and comparison analysis.

Note 2: The proposed consensus reaching framework is a general consensus reaching framework, characterized by its ability for dealing with different types of non-cooperative behaviors. The main reason to define the above different types of non-cooperative behaviors is to provide quantitative indexes to analyze the experts' non-cooperative behaviors, which will provide a decision support for experts to modify their trust values regarding other experts in the social trust network. The three types of non-cooperative behaviors are focused on different aspects: (1) dishonest behavior analysis is focused on whether the evaluation of the alternatives that are preferred by the group is systematically decreased by an expert in the CRP, (2) disobedient behavior analysis is focused on whether an expert modify their preferences or opinions to a very low extent, or even in the opposite recommended direction, (3) divergent behavior analysis is focused on whether an expert's preference diverges from remainder of the experts as the CRP progresses. It should be noted that in some situations, an expert may have multiple types of non-cooperative behaviors, and in this situation our consensus reaching framework is still valid to effectively identify and manage such behaviors.

5. Illustrative example

To show the applicability of the SNA-based consensus reaching framework in a real-life MAGDM problem, an illustrative example is provided in this section. In this example, we assume that a set of eight experts $E = \{e_1, e_2, ..., e_8\}$, a set of four alternatives $X = \{x_1, x_2, x_3, x_4\}$, and a set of four attributes $A = \{a_1, a_2, a_3, a_4\}$ are involved. Here, we assume that all criteria are equally important, i.e. $w = (0.25, 0.25, 0.25, 0.25)^T$. The decision matrices $V^{(k)} = (v_{ij}^{(k)})_{4 \times 4}$ $(k = 1, 2, ..., 8)$ provided by experts e_k are listed in Tables 10-13.

Table 10: Decision matrices $V^{(1)}$ and $V^{(2)}$

	$V^{(1)}$				$V^{(2)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.587	0.447	0.172	0.493	0.375	0.573	0.299	0.296
x_2	0.616	0.557	0.865	0.374	0.1	0.253	0.12	0.394

x_3	0.591	0.951	0.439	0.221	0.346	0.585	0.156	0.078
x_4	0.237	0.689	0.377	0.757	0.832	1	0.699	0.598

Table 11: Decision matrices $V^{(3)}$ and $V^{(4)}$

	$V^{(3)}$				$V^{(4)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.202	0.017	0.396	0.534	0.032	0.693	0.73	0.469
x_2	0.858	0.852	0.498	0.095	0.087	0.929	0.449	0.757
x_3	0.615	0.149	0.67	0.824	0.245	0.116	0.442	0.334
x_4	0.239	0.332	0.661	0.061	0.983	0.141	0.828	0.511

Table 12: Decision matrices $V^{(5)}$ and $V^{(6)}$

	$V^{(5)}$				$V^{(6)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.81	0.937	0.084	0.625	0.58	0.365	0.026	0.074
x_2	0.338	0.507	0.014	0.309	0.478	0.94	0.479	0.402
x_3	0.235	0.061	0.749	0.643	0.091	0.913	0.962	0.105
x_4	0.815	0.424	0.957	0.831	0.784	0.659	0.571	0.396

Table 13: Decision matrices $V^{(7)}$ and $V^{(8)}$

	$V^{(7)}$				$V^{(8)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.837	0.837	0.174	0.597	0.277	0.465	0.116	0.002
x_2	0.837	0.259	0.455	0.149	0.526	0.703	0.978	0.915
x_3	0.039	0.188	0.443	0.275	0.716	0.857	0.848	0.657
x_4	0.56	0.026	0.368	0.902	0.493	0.28	0.094	0.215

Meanwhile, the social trust network among the eight experts is as follows:
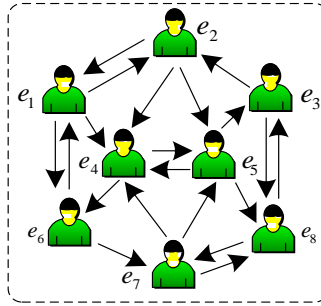
The sociometric, S, associated with the social trust network is provided below:

$$S = \begin{pmatrix} - & 0.92 & - & 0.95 & - & 0.86 & - & - \\ 0.95 & - & - & 0.85 & 0.9 & - & - & - \\ - & 1 & - & - & - & - & - & 1 \\ - & - & - & - & 0.95 & 0.9 & - & - \\ - & - & 0.94 & 0.93 & - & - & - & 0.95 \\ 0.93 & - & - & - & - & - & 0.98 & - \\ - & - & - & 0.96 & 0.87 & - & - & 0.94 \\ - & - & 0.87 & - & - & - & 0.92 & - \end{pmatrix}.$$

In this example, we set $\varepsilon = 0.9$. In the following, we use the proposed consensus reaching

model to help the eight experts achieve a consensus.

(1) In the initial stage of the CRP, the trust propagation and aggregation method presented in section 2 is adopted to help experts produce a complete sociometric, CS, from S. Meanwhile, the weight vector of the eights experts, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_8)^T$, is generated from CS. Further, the consent level among the eight experts, cl, is yielded. If the consensus level is acceptable, the selection process is used to help experts obtain the preference ordering of the four alternatives, otherwise, the first round of the CRP is initiated.

(i) Generating complete sociometric

In the trust aggregation process, the OWA operator with the linguistic quantifier "most" is used to fuse the trust values in different trust paths, and the trust paths with the length larger than or equal to 4 are not taken into account. By using the trust propagation and aggregation method, the unknown trust values in S can be estimated, which are listed in the matrix ES.

$$ES = \begin{pmatrix} - & 0.92 & \mathbf{0.795} & 0.95 & \mathbf{0.763} & 0.86 & \mathbf{0.835} & \mathbf{0.78} \\ 0.95 & - & \mathbf{0.75} & 0.85 & 0.9 & \mathbf{0.771} & \mathbf{0.76} & \mathbf{0.84} \\ \mathbf{0.95} & 1 & - & \mathbf{0.86} & \mathbf{0.81} & \mathbf{0.777} & \mathbf{0.94} & 1 \\ \mathbf{0.831} & \mathbf{0.808} & \mathbf{0.82} & - & 0.95 & 0.9 & \mathbf{0.845} & \mathbf{0.87} \\ \mathbf{0.814} & \mathbf{0.869} & 0.94 & 0.93 & - & \mathbf{0.831} & \mathbf{0.85} & 0.95 \\ 0.93 & \mathbf{0.851} & \mathbf{0.792} & \mathbf{0.814} & \mathbf{0.821} & - & 0.98 & \mathbf{0.92} \\ \mathbf{0.793} & \mathbf{0.811} & \mathbf{0.8} & 0.96 & 0.87 & \mathbf{0.77} & - & 0.94 \\ \mathbf{0.821} & \mathbf{0.87} & 0.87 & \mathbf{0.74} & \mathbf{0.79} & \mathbf{0.783} & 0.92 & - \end{pmatrix}.$$

The matrix ES is used as the reference point for experts to construct a complete sociometric, CS, which is provided below:

$$CS = \begin{pmatrix} - & 0.92 & 0.82 & 0.95 & 0.78 & 0.86 & 0.84 & 0.8 \\ 0.95 & - & 0.75 & 0.85 & 0.9 & 0.8 & 0.75 & 0.8 \\ 0.95 & 1 & - & 0.85 & 0.81 & 0.82 & 0.92 & 1 \\ 0.83 & 0.82 & 0.82 & - & 0.95 & 0.9 & 0.85 & 0.9 \\ 0.82 & 0.88 & 0.94 & 0.93 & - & 0.83 & 0.85 & 0.95 \\ 0.93 & 0.84 & 0.82 & 0.85 & 0.82 & - & 0.98 & 0.92 \\ 0.81 & 0.8 & 0.8 & 0.96 & 0.87 & 0.77 & - & 0.94 \\ 0.83 & 0.87 & 0.87 & 0.82 & 0.75 & 0.81 & 0.92 & - \end{pmatrix}.$$

(ii) Producing the weights of the eight experts

Using the method presented in section 2.4, we can obtain the weights of the eight experts from CS. Specifically, Eq. (7) is first used to generate the node in-degree centrality indexes (i.e., $C(e_1)$,…, $C(e_8)$) of the eight experts. Further, based on Eq. (8), the weights of experts can be yielded from the obtained in-degree centrality indexes of the eight experts, which are $\lambda_1 = 0.127$, $\lambda_2 = 0.127$, $\lambda_3 = 0.12$, $\lambda_4 = 0.128$, $\lambda_5 = 0.122$, $\lambda_6 = 0.12$, $\lambda_7 = 0.126$, and $\lambda_8 = 0.13$.

(iii) Consensus measure

Applying the consensus measure method proposed in section 3.2, the consensus level among the eight experts can be produced. First, the consensus matrix $CM = (cm_{ij})_{n \times n}$ can be produced from $V^{(1)}$ - $V^{(8)}$ based on Eq. (10). Then, the consensus level on the preference value v_{ij} is obtained using formula $cp_{ij} = cm_{ij}$. Following this, the consensus level on alternative x_i can be yielded based on formula $ca_i = \sum_{j=1}^{l} cm_{ij} / l$. Finally, the consensus level among the eight experts

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can be generated based on the formula $cl = \sum_{i=1}^{n} ca_i/n$, which is $cl = 0.66$.

Due to $cl = 0.66 < \varepsilon$, the first consensus round is initiated to assist experts to achieve a consensus.

(2) First consensus round

In the first consensus round, the feedback suggestions for decision matrices modifying are yielded. Based on them, the new decision matrices $V^{(k, 1)}$ associated with $V^{(k)}$ $(k = 1, 2, ..., 8)$ are provided. Moreover, in this consensus round, the sociometric remains unchanged. Following this, the consensus level among the eight experts is measured. If the consensus level is acceptable, the selection process is used to help experts obtain the preference ordering of alternatives, otherwise, the second round of the CRP is activated.

(i) Feedback process for decision matrices modifying

The group decision matrix, $V^{(group)}$, can be generated using Eq. (12), which is listed in Table 14.

Table 14: Decision matrix $V^{(group)}$

	a_1	a_2	a_3	a_4
x_1	0.46	0.544	0.252	0.385
x_2	0.478	0.623	0.486	0.43
x_3	0.362	0.480	0.585	0.391
x_4	0.618	0.442	0.566	0.535

Let $V^{(k,1)} = (v_{ij}^{(k,1)})_{4 \times 4}$ be the updated decision matrix associated with $V^{(k)} = (v_{ij}^{(k)})_{4 \times 4}$. When providing $V^{(k,1)} = (v_{ij}^{(k,1)})_{4 \times 4}$, we advise that: $v_{ij}^{(k,1)} \in [\min\{v_{ij}^{(k)}, v_{ij}^{(group)}\}, \max\{v_{ij}^{(k)}, v_{ij}^{(group)}\}]$.

With the guidance of adjustment suggestions, the experts expressed their adjusted decision matrices $V^{(k,1)} = (v_{ij}^{(k,1)})_{4 \times 4}$ $(k = 1, 2, ..., 8)$, which are shown in Tables 13-16:

Table 15: Decision matrices $V^{(1, 1)}$ and $V^{(2, 1)}$

	$V^{(1, 1)}$				$V^{(2, 1)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.6	0.4	0.4	0.45	0.4	0.55	0.27	0.3
x_2	0.7	0.57	0.7	0.4	0.25	0.4	0.35	0.4
x_3	0.7	0.45	0.44	0.25	0.35	0.5	0.42	0.25
x_4	0.7	0.6	0.5	0.7	0.7	0.7	0.62	0.54

Table 16: Decision matrices $V^{(3, 1)}$ and $V^{(4, 1)}$

	$V^{(3, 1)}$				$V^{(4, 1)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.3	0.25	0.35	0.4	0.25	0.6	0.5	0.4
x_2	0.65	0.7	0.49	0.3	0.3	0.75	0.46	0.65
x_3	0.42	0.25	0.6	0.54	0.3	0.23	0.5	0.35
x_4	0.35	0.35	0.65	0.5	0.85	0.35	0.7	0.52

Table 17: Decision matrices $V^{(5, 1)}$ and $V^{(6, 1)}$

	$V^{(5,\,1)}$				$V^{(5,\,1)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.62	0.65	0.15	0.4	0.48	0.45	0.24	0.25
x_2	0.42	0.55	0.3	0.35	0.478	0.8	0.482	0.41
x_3	0.28	0.38	0.65	0.42	0.25	0.75	0.75	0.12
x_4	0.72	0.43	0.8	0.75	0.7	0.55	0.57	0.45

Table 18: Decision matrices $V^{(7,\,1)}$ and $V^{(8,\,1)}$

	$V^{(7,\,1)}$				$V^{(8,\,1)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.5	0.6	0.23	0.45	0.35	0.49	0.18	0.3
x_2	0.55	0.4	0.46	0.35	0.48	0.63	0.5	0.44
x_3	0.3	0.35	0.55	0.35	0.45	0.55	0.7	0.52
x_4	0.5	0.1	0.39	0.55	0.6	0.42	0.55	0.53

(ii) Feedback process for sociometric modification

Let S^1 be sociometric in this first consensus round. We have that $S^1 = CS$.

(iii) Generating the weights of the eight experts

Eq. (8) is employed to generate the weights of experts from S^1:

$$\lambda_1 = (0.127,\ 0.127,\ 0.12,\ 0.128,\ 0.122,\ 0.12,\ 0.126,\ 0.13)^T.$$

(iv) Consensus measure

Employing Eq. (11) offers the consensus level among experts $cl_1 = 0.844$.

Due to $cl_1 = 0.844 < \varepsilon$, the second consensus round is activated to assist experts to improve the consensus level.

(3) Second consensus round

In this consensus round, the approach for analyzing non-cooperative behaviors is used to analyze the behaviors of the eight experts. Based on the analysis results, a new sociometric, S^2, is provided by the experts. Meanwhile, the feedback process for decision matrices modifying is used to help experts providing new decision matrices $V^{(k,\,2)}$ $(k=1,2,...,8)$. Meanwhile, the consensus level is generated from $V^{(k,\,2)}$ $(k=1,2,...,8)$. If the consensus level is acceptable, the selection process is used to help experts obtain a preference ordering over the alternatives, otherwise, the third round of CRP is activated.

(i) Non-cooperative behaviors analysis

By setting $\beta = 0.3$, $\gamma = 0.18$, the non-cooperative behaviors in the previous consensus rounds are analyzed. The results are listed in Table 19.

Table 19: Non-cooperative behaviors indexes

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
$NS_{1,1}^{(k)}$	1	1	0.8	1	1	1	0	0.8
$NS_{2,1}^{(k)}$	0.011	0.529	0.551	0.481	0.582	0.494	0.615	0.807
$NS_{3,1}^{(k)}$	0.714	0.857	0.571	0.857	1	0.857	1	1

Here, we set $\alpha_1 = 0.5$, $\alpha_2 = 0.45$, and $\alpha_3 = 0.5$. Due to $NS_{1,1}^{(7)} < \alpha_1$ and $NS_{2,1}^{(1)} < \alpha_2$, we infer that expert e_7 has the feature of dishonest behavior, and e_1 has the characteristic of disobedient behavior.

(ii) Feedback process for sociometric modifying

Due to experts e_1 and e_7 having non-cooperative behaviors, the trust values towards e_1 and e_7 in the social trust network are suggested to be decreased. After updating their trust values, the experts provide the adjusted sociometric provided below:

$$S^2 = \begin{pmatrix}
- & 0.92 & 0.82 & 0.95 & 0.78 & 0.86 & 0.8 & 0.8 \\
0.85 & - & 0.75 & 0.85 & 0.9 & 0.8 & 0.7 & 0.8 \\
0.8 & 1 & - & 0.85 & 0.81 & 0.82 & 0.85 & 1 \\
0.75 & 0.82 & 0.82 & - & 0.95 & 0.9 & 0.7 & 0.9 \\
0.76 & 0.88 & 0.94 & 0.93 & - & 0.83 & 0.85 & 0.95 \\
0.87 & 0.84 & 0.82 & 0.85 & 0.82 & - & 0.88 & 0.92 \\
0.75 & 0.8 & 0.8 & 0.96 & 0.87 & 0.77 & - & 0.94 \\
0.76 & 0.87 & 0.87 & 0.82 & 0.75 & 0.81 & 0.8 & -
\end{pmatrix}.$$

(ii) Feedback process for decision matrices modifying

Utilizing Eq. (12) generates the collective decision matrix $V^{(c,1)}$, which is provided in Table 20:

Table 20: Decision matrix $V^{(c,\,1)}$

	a_1	a_2	a_3	a_4
x_1	0.436	0.5	0.291	0.369
x_2	0.477	0.599	0.468	0.414
x_3	0.383	0.432	0.575	0.351
x_4	0.641	0.437	0.596	0.568

Let $V^{(k,\,2)} = (v_{ij}^{(k,\,2)})_{4\times4}$ be the updated decision matrix associated with $V^{(k,1)} = (v_{ij}^{(k,1)})_{4\times4}$. When providing $V^{(k,\,2)}$, we suggest that: $v_{ij}^{(k,2)} \in [\min\{v_{ij}^{(k,1)}, v_{ij}^{(group,1)}\},\ \max\{v_{ij}^{(k,1)}, v_{ij}^{(group,1)}\}]$.

The adjusted decision matrices $V^{(k,\,2)} = (v_{ij}^{(k,\,2)})_{4\times4}$ $(k = 1, 2, ..., 8)$ given by experts are as follows:

Table 21: Decision matrices $V^{(1,\,2)}$ and $V^{(2,\,2)}$

	$V^{(1,\,2)}$				$V^{(2,\,2)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.55	0.42	0.37	0.4	0.42	0.53	0.29	0.345
x_2	0.65	0.58	0.68	0.4	0.4	0.53	0.44	0.41
x_3	0.68	0.44	0.48	0.3	0.37	0.45	0.53	0.32
x_4	0.68	0.62	0.52	0.68	0.66	0.55	0.6	0.55

Table 22: Decision matrices $V^{(3,\,2)}$ and $V^{(4,\,2)}$

	$V^{(3,\,2)}$				$V^{(4,\,2)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.425	0.4	0.3	0.38	0.4	0.55	0.3	0.38
x_2	0.55	0.65	0.47	0.4	0.45	0.62	0.46	0.5

x_3	0.4	0.4	0.59	0.45	0.35	0.39	0.55	0.35
x_4	0.63	0.4	0.62	0.53	0.7	0.4	0.61	0.54

Table 23: Decision matrices $V^{(5,\,2)}$ and $V^{(6,\,2)}$

	$V^{(5,\,2)}$				$V^{(6,\,2)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.45	0.55	0.2	0.38	0.45	0.48	0.27	0.34
x_2	0.45	0.57	0.44	0.4	0.477	0.65	0.47	0.41
x_3	0.32	0.41	0.51	0.39	0.35	0.5	0.62	0.28
x_4	0.7	0.435	0.61	0.68	0.65	0.49	0.58	0.52

Table 24: Decision matrices $V^{(7,\,2)}$ and $V^{(8,\,2)}$

	$V^{(7,\,2)}$				$V^{(8,\,2)}$			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
x_1	0.5	0.52	0.29	0.37	0.4	0.5	0.25	0.34
x_2	0.48	0.45	0.465	0.39	0.48	0.6	0.47	0.42
x_3	0.35	0.4	0.565	0.35	0.39	0.45	0.62	0.42
x_4	0.4	0.25	0.45	0.55	0.62	0.43	0.57	0.55

(iii) Generating the weights of the eight experts

The weights of experts are derived from S^2 using Eq. (8), $\lambda_2 = (0.117,\ 0.13,\ 0.123,\ 0.131,$ $0.124,\ 0.123,\ 0.118,\ 0.134)^T$.

(iv) Consensus measure

By applying Eq. (11), the consensus level in this consensus round can be obtained, $cl_2 = 0.932$. This indicates that the predefined consensus level among the eight experts has been reached.

Finally, the selection process is adopted to find the collective ranking of alternatives, that is $x_4 \text{ f } x_2 \text{ f } x_3 \text{ f } x_1$.

6. Simulation and comparison analysis

This section presents several simulation and comparison experiments to investigate the efficiency of the SNA-based CRP for dealing with the non-cooperative behaviors.

6.1. The design of simulation methods

In simulation methods, the initial multiple attribute decision matrices and sociometric associated with the social trust network are randomly generated. Then, we take them as the input of the proposed CRP, based on which we can obtain the consensus success ratio (P), the consensus rounds (Z), and the adjusted distance of experts' multiple attribute decision matrices (AD). We devise three simulation methods (i.e., simulation methods I-III) in the following. The three simulation methods are based on a natural hypothesis: if an expert is inferred as adopting non-cooperative behavior(s), the trust values towards her/him in the social trust network shall

decreased by other experts.

(1) Simulation experiment I

The basic idea of Simulation method I is provided below:

If the expert e_k is deduced as using the dishonest behavior in the CRP, then based on the above hypothesis, other experts e_h ($h \neq k$) will decrease the trust values to expert e_k. Simulation method I is described in Table 25.

<p align="center">Table 25: Simulation method I</p>

Input: m, n, l, ε, $z_{\max}$, β, α_1, η and g.

Output: P, Z and AD.

Step 1: We randomly generate m $n \times l$ decision matrices $\{V^{(1)}, ..., V^{(m)}\}$ and a sociometric $S = (s_{ij})_{m \times m}$, where $v_{ij}^{(k)}$ is randomly and uniformly generated from the interval $[0, 1]$, and s_{ij} is randomly and uniformly generated from the interval $[0.9, 1]$.

Step 2: Let $z = 0$, $V^{(k,z)} = V^{(k)}$. Using the propagation method presented in section 2.2, we can get a complete sociometric among experts $S^{(z)} = (s_{ij}^{(z)})_{m \times m}$ $(k = 1, 2, ..., m)$.

Step 3: Use Eq. (8) to generate the experts' weights $\lambda_z = (\lambda_{1,z}, \lambda_{2,z}, ..., \lambda_{m,z})^T$, where $\lambda_{k,z} = \dfrac{\sum_{i=1, i \neq k}^{m} s_{ik}^{(z)}}{\sum_{k=1}^{m} \sum_{i=1, i \neq k}^{m} s_{ik}^{(z)}}$.

Step 4: Applying Eq. (11) offers the consensus level, cl_z. If $cl_z \geq \varepsilon$ or $z \geq z_{\max}$, then go to **Step 7**; otherwise, continue with the next step.

Step 5: If $z = 0$, then let $S^{(z+1)} = S^{(z)}$; otherwise, apply Eq. (17) to yield $NS_1^{(i,z)}$ $(i = 1, 2, .., m)$. Based on the above hypothesis, if $NS_1^{(i,z)} \leq \alpha_1$ $(z \geq 1)$, then experts e_k $(k = 1, 2, .., m, k \neq i)$ will reduce the trust values of expert e_i. We assume that the updated sociometric $S^{(z+1)} = (s_{ij}^{(z+1)})_{m \times m}$ $(z \geq 1)$ is given by the following way:

(i) If $NS_1^{(i,z)} > \alpha_1$, then let $s_{ki}^{(z+1)} = s_{ki}^{(z)}$ for $k = 1, 2, ..., m$ and $k \neq i$.

(ii) If $NS_1^{(i,z)} \leq \alpha_1$, then let $s_{ki}^{(z+1)} = \max(s_{ki}^{(z)} - \eta, 0)$ for $k = 1, 2, ..., m$ and $k \neq i$.

Step 6: Employ Eq. (12) to provide the collective decision matrix $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times l}$, where $v_{ij}^{(c,z)} = \sum_{k=1}^{m} \lambda_{k,z} v_{ij}^{(k,z)}$. Meanwhile, Eq. (16) is adopted to obtain Y_z. When constructing $V^{(k,z+1)} = (v_{ij}^{(k,z+1)})_{n \times l}$ $(k = 1, 2, ..., m)$, the following two cases are considered.

Case A: $k \leq g$. Expert e_k gives $V^{(k,z+1)}$ as below:

(i) For $i = 1, 2, ..., n$, $j = 1, 2, ..., l$, and $y_{i,z} = 0$, then let $v_{ij}^{(k,z+1)} = (1-\mu)v_{ij}^{(k,z)} + \mu v_{ij}^{(c,z)}$, where the value of u is selected from $[0.15, 1]$.

(ii) For $y_{i,z} = 1$ and $j = 1, 2, ..., l$, then the value of $v_{ij}^{(k,z+1)}$ is generated from $[0, 1]$;

Utilize Eq. (17) to offer $NS_1^{(k,z+1)}$. Repeat (i) and (ii) until $NS_1^{(k,z+1)} \leq \alpha_1$ $(k \leq g)$.

Case B: $g < k \leq m$. Here, the expert e_k provides $V^{(k,z+1)}$, as follows:

For $i = 1, 2, ..., n$ and $j = 1, 2, ..., l$, then let $v_{ij}^{(k,z+1)} = (1-\mu)v_{ij}^{(k,z)} + \mu v_{ij}^{(c,z)}$, where the value of u is produced from $[0.15, 1]$.

Let $z = z + 1$, then go to **Step 3.**

Step 7: Let $AD = AD + \dfrac{1}{m \times n \times l} \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{l} |v_{ij}^{(k, z+1)} - v_{ij}^{(k, z)}|$.

Step 8: If $cl_z \geq \varepsilon$, then $P = 1$; otherwise $P = 0$. Let $Z = z$. Output P, Z and AD.

(2) Simulation method II

Similar to Simulation method I, Simulation method II is devised. In Simulation method II, if the expert e_k is deduced as adopting the disobedient behavior in the CRP, then other experts e_h ($h \neq k$) will reduce the trust values to expert e_k. In Simulation method I, by replacing Input and Steps 5 and 6 with Input A and Steps 5-A and 6-A below, respectively, we then obtain simulation method II.

Input A: m, n, l, ε, $z_{\max}$, α_2, η and g.

Step 5-A: If $z = 0$, then let $S^{(z+1)} = S^{(z)}$; otherwise, adopt Eq. (21) to yield $NS_2^{(i,z)}$ $(i = 1, 2, .., m)$. If $NS_2^{(i,z)} \leq \alpha_2$ $(z \geq 1)$, then experts e_k $(k = 1, 2, .., m, k \neq i)$ will reduce the trust values of expert e_i. We assume the updated sociometric $S^{(z+1)} = (s_{ij}^{(z+1)})_{m \times m}$ $(z \geq 1)$ is given using the following way:

(i) If $NS_2^{(i,z)} > \alpha_2$, then let $s_{ki}^{(z+1)} = s_{ki}^{(z)}$ for $k = 1, 2, ..., m$ and $k \neq i$.

(ii) If $NS_2^{(i,z)} \leq \alpha_2$, then let $s_{ki}^{(z+1)} = \max(s_{ki}^{(z)} - \eta, 0)$ for $k = 1, ..., m$ and $k \neq i$.

Step 6-A: Employ Eq. (12) to yield the collective decision matrix $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times l}$, where $v_{ij}^{(c,z)} = \sum_{k=1}^{m} \lambda_{k,z} v_{ij}^{(k,z)}$. When constructing $V^{(k,z+1)} = (v_{ij}^{(k,z+1)})_{n \times l}$ $(k = 1, ..., m)$, two cases are taken into account.

Case A: $k \leq g$. Here, the expert e_k provides $V^{(k,z+1)}$ as follows:

For $i = 1, 2, ..., n$ and $j = 1, 2, ..., l$, let a and b be two real numbers and they are randomly and uniformly generated from interval $[0, 1]$, (i) if $a > 0.5$, let $v_{ij}^{(k,z+1)} = (1 - \mu)v_{ij}^{(k,z)} + \mu v_{ij}^{(c,z)}$ with the value of u is uniformly and randomly selected from the interval $[0, \alpha_2]$, (ii) if $a \leq 0.5$ and $b \geq 0.5$, then $v_{ij}^{(k,z+1)}$ is randomly and uniformly generated from interval $[0, \min(v_{ij}^{(k,z)}, v_{ij}^{(c,z)})]$, if $a \leq 0.5$ and $b < 0.5$, then $v_{ij}^{(k,z+1)}$ is randomly and uniformly generated from interval $[\max(v_{ij}^{(k,z)}, v_{ij}^{(c,z)}), 1]$.

Case B: $g < k \leq m$. Here, the expert e_k provides $V^{(k,z+1)}$, as follows:

For $i = 1, 2, ..., n$ and $j = 1, 2, ..., l$, then let $v_{ij}^{(k,z+1)} = (1 - \mu)v_{ij}^{(k,z)} + \mu v_{ij}^{(c,z)}$, where u is produced from $[\alpha_2, 1]$.

Let $z = z + 1$, then go to **Step 3.**

(3) Simulation method III

Simulation method III is designed to investigate whether the SNA-based consensus reaching framework can manage divergent behaviors, and the main idea of this simulation method is also similar to that of Simulation method I. In Simulation method III, if expert e_k is concluded as adopting a divergent behavior in the CRP, the trust values towards expert e_k in the social trust network will be reduced by other experts e_h $(h \neq k)$. Specifically, Simulation method III can be yielded by replacing Input and Steps 5 and 6 with Input B and Steps 5-B and 6-B in Simulation method I, respectively.

Input B: m, n, l, ε, z_{max}, α_3, γ, η and g.

Step 5-B: If $z = 0$, then let $S^{(z+1)} = S^{(z)}$; otherwise, utilize Eq. (24) to produce $NS_3^{(i,z)}$ $(i = 1, 2, .., m)$. If $NS_3^{(i,z)} \leq \alpha_3$ $(z \geq 1)$, then experts e_k $(k \neq i)$ will reduce the trust values associated with the expert e_i. Here, the updated sociometric $S^{(z+1)} = (s_{ij}^{(z+1)})_{m \times m}$ $(z \geq 1)$ is built using the following approach:

(i) If $NS_3^{(i,z)} > \alpha_3$, then let $s_{ki}^{(z+1)} = s_{ki}^{(z)}$ for $k \neq i$.

(ii) If $NS_3^{(i,z)} \leq \alpha_3$, then let $s_{ki}^{(z+1)} = \max(s_{ki}^{(z)} - \eta, 0)$ for $k = 1, 2, ..., m$ and $k \neq i$.

Step 6-B: Applying Eq. (12) produces the collective decision matrix $V^{(c,z)} = (v_{ij}^{(c,z)})_{n \times l}$, where $v_{ij}^{(c,z)} = \sum_{k=1}^{m} \lambda_{k,z} v_{ij}^{(k,z)}$. When building $V^{(k,z+1)} = (v_{ij}^{(k,z+1)})_{n \times l}$ $(k = 1, 2, ..., m)$, cases A and B are considered.

Case A: $g < k \leq m$. Expert e_k provides $V^{(k,z+1)}$, as follows:

For $i = 1, 2, ..., n$ and $j = 1, 2, ..., l$, then $v_{ij}^{(k,z+1)}$ is randomly and uniformly generated from interval $[\min(v_{ij}^{(k,z)}, v_{ij}^{(c,z)}), \max(v_{ij}^{(k,z)}, v_{ij}^{(c,z)})]$.

Case B: $k \leq g$. Here, expert e_k provides $V^{(k, z+1)} = (v_{ij}^{(k, z+1)})_{n \times l}$ as below:

(i) For $i = 1, 2, ..., n$ and $j = 1, 2, ..., l$, then $v_{ij}^{(k,z+1)}$ is generated from $[0, 1]$.

Use Eq. (24) to produce $NS_3^{(k,z+1)}$. Repeat (i) until $NS_3^{(k,z+1)} \leq \alpha_3$ $(k \leq g)$.

Let $z = z + 1$, then go to **Step 3.**

Note 3: In Simulation methods I-III, (1) the parameter η $(\eta \in [0, 1])$ represents the penalty coefficient, and the larger η value represents the larger the penalty strength; (2) the parameter g represents the number of experts who take non-cooperative behaviors in the CRP, and the purpose of Steps 6, 6-A, and 6-B is to ensure that experts $\{e_1, ..., e_g\}$ have dishonest, disobedient, and divergent behaviors, respectively.

6.2. The design of comparison methods

In the proposed SNA-based consensus reaching framework, the experts' weights generated from the social trust network are dynamically updated and embedded into the CRP. However, in traditional CRPs with the social trust network (e.g., [53, 55]), the experts' weights keep unchanged throughout the CRP. In particular, we omit Steps 6, 6-A, and 6-B from Simulation methods I-III, which yields three new methods denominated as Simulation methods I*- III* based on the traditional CRPs with social trust networks, respectively. Based on this, the consensus efficiency of the proposed SNA-based consensus reaching framework and the traditional CRPs under a social trust network will be compared.

6.3. Simulation and comparison results

Here, the simulation and comparison results are presented.

Simulation method I, we fix $z_{max} = 5$, $n = 6$, $l = 5$, $\beta = 0.3$, and $\varepsilon = 0.9$. Then, we set different input parameters m, α_1, η, and g, and we run simulation method I 1000 times to produce the average values of P, Z and AD, respectively. The average P, Z and AD value, respectively, reflect the success ratio, the number of rounds required for achieving the established consensus level, and the amount of preference adjustment required in the simulation experiment. The average values of P, Z and AD, for Simulation method I under different

input parameters, are listed in Table 26.

Table 26: Average values of P, Z and AD in Simulation method I

		g=1						g=2						g=3				
		$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$		
m	α_1	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD
6	0.1	1	2.149	0.219	1	2.043	0.213	1	2.973	0.308	1	2.83	0.292	0.987	4.228	0.463	1	3.017	0.357
	0.3	1	2.146	0.219	1	2.033	0.212	1	2.971	0.306	1	2.813	0.291	0.993	4.218	0.458	1	3.004	0.355
9	0.1	1	2.003	0.209	1	2	0.208	1	2.43	0.252	1	2.229	0.234	1	2.989	0.313	1	2.865	0.3
	0.3	1	2	0.209	1	2	0.209	1	2.408	0.25	1	2.166	0.236	1	2.972	0.313	1	2.861	0.301
12	0.1	1	2	0.208	1	2	0.207	1	2.054	0.225	1	2.006	0.221	1	2.68	0.276	1	2.27	0.25
	0.3	1	2	0.207	1	2	0.208	1	2.059	0.225	1	2.003	0.222	1	2.654	0.273	1	2.259	0.249

In Simulation method II, we set $z_{\max}=5$, $n=6$, $l=5$, and $\varepsilon=0.9$. We then run simulation method II 1000 times under different input parameters m, α_2, η, and g to get the average values of P, Z and AD, which are listed in Table 27.

Table 27: Average values of P, Z and AD in Simulation method II

		g=1						g=2						g=3				
		$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$		
m	α_2	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD
6	0.2	1	3.062	0.269	1	2.875	0.256	0.176	5	0.458	1	3.454	0.341	0	5	0.561	1	4	0.451
	0.4	1	2.377	0.252	1	2.036	0.232	0.72	4.969	0.481	1	3.001	0.329	0	5	0.593	1	4	0.475
9	0.2	1	2.653	0.241	1	2.138	0.218	1	3.948	0.338	1	3	0.285	0.192	5	0.459	1	3.312	0.338
	0.4	1	2.223	0.23	1	2.027	0.229	1	3.115	0.311	1	2.738	0.286	0.875	4.99	0.483	1	3	0.335
12	0.2	1	2.073	0.215	1	2.055	0.211	1	3.01	0.275	1	2.981	0.271	1	4.252	0.368	1	3	0.296
	0.4	1	2.065	0.229	1	2	0.228	1	2.341	0.258	1	2.002	0.241	1	3.672	0.355	1	2.994	0.312

In Simulation method III, the parameters are fixed as $z_{\max}=5$, $n=6$, $l=5$, $\gamma=0.28$, and $\varepsilon=0.9$. When setting different input parameters m, α_3, η, and g for Simulation method III, we run simulation method III 1000 times to obtain the average values of P, Z and AD, which are listed in Table 28.

Table 28: Average values of P, Z and AD in Simulation method III

		g=1						g=2						g=3				
		$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$			$\eta=0.15$			$\eta=0.35$		
m	α_2	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD	P	Z	AD
6	0.2	1	4.009	0.429	1	3.221	0.343	0	5	0.772	1	4	0.617	0	5	1.04	1	4	0.815
	0.4	1	4.008	0.427	1	3.203	0.343	0	5	0.772	1	4	0.618	0	5	1.04	1	4	0.816
9	0.2	1	3.064	0.31	1	3.012	0.302	1	4.946	0.59	1	3.045	0.404	0	5	0.779	1	4	0.629
	0.4	1	3.051	0.307	1	3.031	0.301	1	4.898	0.581	1	3.029	0.399	0	5	0.769	1	4	0.621
12	0.2	1	3.057	0.285	1	3.021	0.282	1	4.251	0.436	1	3.011	0.355	1	5	0.644	1	3.27	0.456
	0.4	1	3.012	0.283	1	3	0.281	1	4.123	0.434	1	3	0.353	1	5	0.638	1	3.194	0.444

Meanwhile, we set $g = 2$, $n = 7$, $l = 6$, $z_{max} = 5$, $\varepsilon = 0.92$, and regarding the threshold values for identifying non-cooperative behaviors, let (1) $\beta = 0.3$ and $\alpha_1 = 0.1$ for simulation method I; (2) $\alpha_2 = 0.2$ for simulation method II; (3) $\gamma = 0.25$ and $\alpha_3 = 0.5$ for simulation method III. When setting different parameters m and η, we run Simulation methods I-III 1000 times, respectively, to produce the average values of P, Z and AD, which are described as Figs. 5-7.
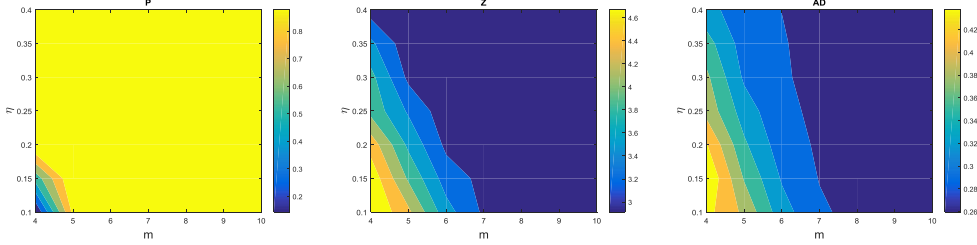
Fig.5. Average values of P, Z and AD under simulation method I
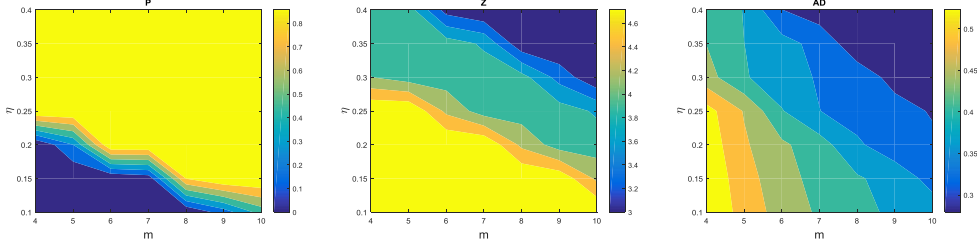
Fig.6. Average values of P, Z and AD under simulation method II
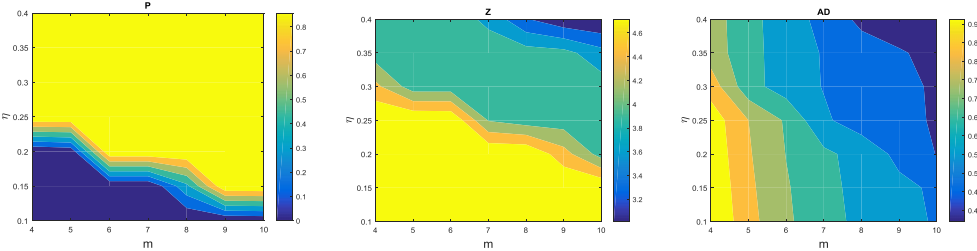
Fig.7. Average values of P, Z and AD under simulation method III

Let $l = 7$, $n = 8$, $z_{max} = 5$, $\alpha_1 = 0.5$, $\alpha_2 = 0.35$, $\alpha_3 = 0.4$, $\beta = 0.25$, $\gamma = 0.28$, $g = 2$, and $\eta = 0.25$. We run Simulation methods I and I*, II and II*, and III and III* 1000 times under different input parameters m and ε, respectively, to generate the average values of P, Z and AD, which are visualized in Figs. 8-10.
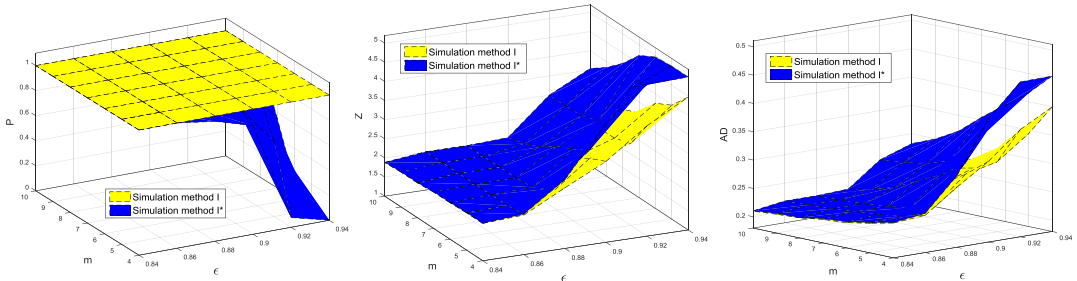
Fig.8. Average values of P, Z and AD under simulation methods I and I*
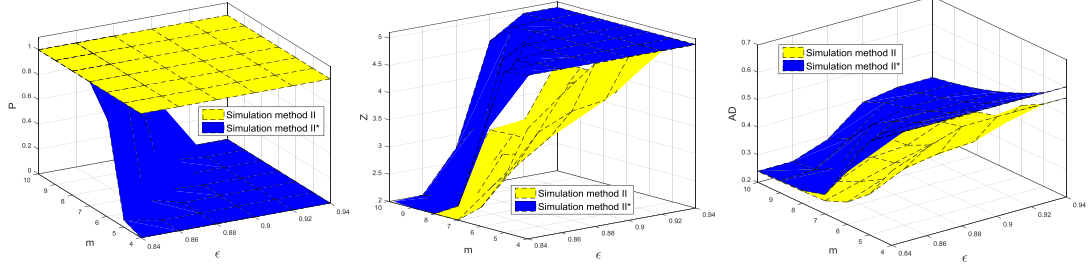
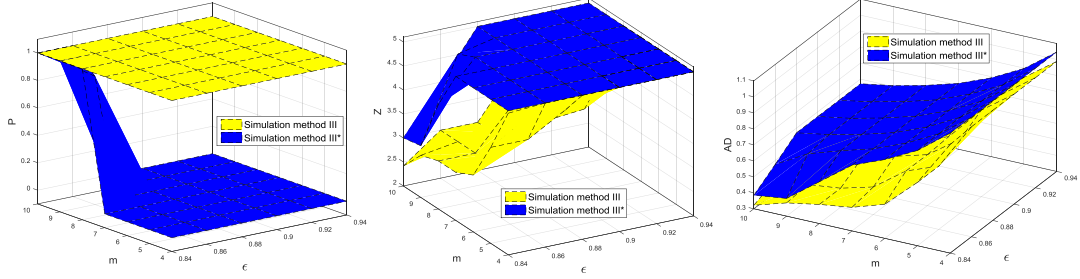Fig.9. Average values of P, Z and AD under simulation methods II and II*

Fig.10. Average values of P, Z and AD under simulation methods III and III*

The results of the above simulation experiments and comparative analysis show that:

(1) The SNA-based consensus reaching framework can manage dishonest, disobedient, and divergent behaviors effectively when setting different parameters. Generally, the consensus success ratios are 1 in most cases, and 2-3 consensus rounds are often required to reach the predefined consensus level. Moreover, the adjustment distances are 0.2-0.4 in most cases.

(2) With the increase of the proportion of the experts who take non-cooperative behaviors, the consensus success ratios have the tendency to decrease, and the consensus rounds increase alongside the preference adjustment distances. This implies that the effectiveness of the proposed consensus reaching framework for coping with non-cooperative behaviors varies, depending on the proportion of the experts who take non-cooperative behaviors in the group.

(3) When the values of α_1, α_2 and α_3 increase or η value increases, the average values of P increase, and the average values of Z decrease. These observations imply that the success ratio of reaching the predefined consensus level will be improved and the speed to reach the predefined consensus level will be accelerated when using a relaxed criterion to infer the non-cooperative behaviors or applying a strong penalty.

(4) There are higher consensus success ratios in the SNA-based consensus reaching with non-cooperative behaviors management framework than in the traditional CRP with social trust network, which indicates that the SNA-based consensus reaching framework can increase the success ratio of achieving the predefined consensus level under the presence of non-cooperative behaviors, by effectively dealing with them.

(5) The average consensus rounds in the SNA-based consensus framework with non-cooperative behaviors management are lower than those in the traditional CRP approaches under social trust networks, which means that the proposed consensus framework can accelerate the speed of convergence to achieve the predefined consensus level.

(6) The adjustment distances in the proposed SNA-based consensus reaching framework with non-cooperative behavior management are lower than those in the traditional CRP with social trust network, which implies that the proposed SNA-based consensus reaching framework can decrease the preference information loss by dealing with the non-cooperative behaviors.

7. Conclusion

In this study, we investigated the non-cooperative behaviors in the CRP in the MAGDM context, and proposed a consensus reaching framework based on SNA to manage different patterns of non-cooperative behaviors exhibited by participants in the process of building consensus. The main motivations and resulting contributions of this study are summarized below.

(1) In the CRP, the trust relationships among experts play a key role, which will influence the decision results. However, the trust relationships are rarely considered by existing CRP models. By taking the trust relationships among experts into account, we developed a SNA-based consensus reaching framework, which can provide a better approximation to real decision situations in which there exist diverse relationships among participants.

(2) The behavior analysis module is designed in the SNA-based consensus framework, and the analysis results are provided for experts to modify their trust values in the social trust network. Meanwhile, a mechanism to dynamically generate experts' weights from the social trust network is presented, and subsequently embedded into the consensus model.

(3) We define several common patterns of non-cooperative behaviors, namely *dishonest*, *disobedient*, and *divergent* behaviors. Likewise, we devise several simulation and comparison experiments to verify the efficiency and validity of the SNA-based framework for coping with diverse non-cooperative behaviors in the CRP, by weighting experts based on social trust information.

Meanwhile, two interesting research directions are pointed out for future work:

(1) In real word CRPs, the preferences of experts are often formed in a complex interpersonal environment where preferences are liable to change due to social influences [7]. We believe that it will be very interesting in future research to incorporate the impact of social influence on the evaluation of experts' preferences in the SNA-based consensus reaching framework.

(2) The large-scale GDM has become a hot research topic along with the development of technology and society [37, 71, 72]. In future research, we plan to design a SNA-based approach to cope with non-cooperative behaviors in a large-scale GDM, in which a larger and more complex sociometric structure would be present.

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