

Consensus in Group Decision Making and Social Networks

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Abstract: The consensus reaching process is the most important step in a group decision making scenario. This step is most frequently identified as a process consisting of some discussion rounds in which several decision makers, which are involved in the problem, discuss their points of view with the purpose of obtaining the maximum agreement before making the decision. Consensus reaching processes have been well studied and a large number of consensus approaches have been developed. In recent years, the researchers in the field of decision making have shown their interest in social networks since they may be successfully used for modelling communication among decision makers. However, a social network presents some features differentiating it from the classical scenarios in which the consensus reaching processes have been applied. The objective of this study is to investigate the main consensus methods proposed in social networks and bring out the new challenges that should be faced in this research field.

Keywords: Consensus, Group decision making, Social networks.

1. Introduction

Group decision making (GDM) is described as the decision process consisting of selecting the best alternative or alternatives from a feasible one considering the opinions verbalized by a group of individuals, usually called decision makers [39].

In a utopian scenario, a unanimous decision is sought, i.e., all the decision makers fully agree with the solution. However, in most of real-world scenarios, full consensus is considered almost unreachable. Due to some differences, inherent to knowledge level and personal interests of the decision makers, they arrive at unanimous agreement on rare occasions. Therefore, consensus has been modelled in a softer way, being it viewed not necessarily as a unanimous and full agreement [24]. In particular, a more flexible and reasonable approach using fuzzy logic has been used to model what is known as soft consensus measures, which show the wide range of possible partial agreements [8], [12]. Based on this assumption, consensus may be defined as an acceptable resolution that a

decision maker can support, even if it is not his/her favourite one.

In practice, a consensus reaching process proceeds in a convergent multistage way, where, at the beginning of the process, the decision makers verbalize their individual opinions and, while the consensus level is not considered enough, they negotiate and bring positions closer by changing their initial points of view [7], [21]. Therefore, this presupposes that a decision maker is willing to accept these opinion changes. Sometimes, there exists a moderator (a person or system) being responsible of controlling all the decision process until the decision makers reach the agreement [21].

Due to the importance of reaching a consensus solution, consensus has been well studied in GDM scenarios and many consensus approaches have been proposed [10], [21].

Web technologies have supported the development

of several services in which users from different countries can join, produce new resources and contents, and interact with other users. Social networks are one of the most recent trends [30], which comprise a collection of design techniques and different web development. In the new virtual environment, social networks allow collaboration, interoperability, information sharing and easy communication [2], [45].

It is therefore clear that the development of more advanced GDM frameworks and consensus reaching processes that may be used in the new social network services is a current necessity.

A social network presents some features differentiating it from the classical scenarios in which the consensus reaching processes have been usually applied [2]. For example, on the one hand, a social network presents thousands of members, but it is possible that many of them do not directly take part in the decision-making process. On the other hand, a frequent issue is that some members might be able to cooperate during a part of the decision process, but not in the whole decision process. Furthermore, there is a real-time discussion among its users and it is common that members exchange opinions through their communication with others. This communication is usually local in the sense that only neighbouring users in the network exchange information, establishing trust relationships among them. In any case, social networks are a dominant and current force in society and the collective beliefs provided in them may determine the paths that society takes.

The objective of this study is to investigate the existing consensus approaches developed to deal with social network services as well as the more relevant challenges that should be faced.

The rest of this study is organized as follows. In Section 2, the classical fuzzy GDM framework is introduced along with a description of a usual consensus reaching process. In addition, the characteristics of the social networks are also presented. In Section 3, the main consensus approaches dealing with social networks are described. Next, the challenges that should be faced are presented in Section 4. Finally, in Section 5, we conclude this paper.

2. Preliminaries

We introduce the classical fuzzy GDM scenario and a typical consensus reaching process in this section. In addition, the main characteristics of the social networks are described.

2.1 Fuzzy GDM Problem

A classical fuzzy GDM scenario is defined as a situation in which there is a problem to solve, a collection of alternatives, $X = \{x_1, \dots, x_n\}$, and a group of decision makers, $E = \{e_1, \dots, e_m\}$, characterized by their own knowledge and background, who convey their preferences or opinions about the collection of alternatives to reach a joint decision [9]. The objective is to order the different alternatives from best to worst by means of the association of some preference degrees expressed in the unit interval [39].

The assessments verbalized by the decision makers have at the beginning been considered equal to some utilities caused by some courses of action, probabilities of them, and in similar manner. Nevertheless, GDM is a process focused on human beings, with their intrinsic imprecision, subjectivity and vagueness in the expression of assessments, and, thus, in this research area, the fuzzy set theory [48] has been utilized for a long time. It is due to the fuzzy set theory is a richer and more general representation of assessments than a subjective probability of the occurrence of an event being considered, which was the origin of the traditional GDM methods [18], [19].

Fuzzy preference relations were used at first in the works proposed by Spillman and Bezdek [35]. A fuzzy preference relation on a collection of alternatives X is a fuzzy set on the Cartesian product $X \times X$, that is, it is characterized by a membership function $\mu_{PR}: X \times X \rightarrow [0,1]$. Then, a fuzzy preference relation PR is usually modelled by the $n \times n$ matrix $PR = (pr_{ij})$, being $pr_{ij} = \mu_{PR}(x_i, x_j)$, ($\forall i, j \in \{1, \dots, n\}$) interpreted as the preference degree of the alternative x_i over x_j . In particular, $pr_{ij} = 0.5$ indicates indifference between x_i and x_j ; $pr_{ij} = 1$ indicates that x_i is absolutely preferred to x_j ; and $pr_{ij} > 0.5$ indicates that x_i is preferred to x_j . Based on this interpretation, we have that $pr_{ii} = 0.5 \forall i \in \{1, \dots, n\}$. In addition, since pr_{ii} 's do not matter, they are usually written as ‘-’ instead of 0.5 [23].

A fuzzy preference relation is the most used representation format of preferences to model the assessments verbalized by the decision makers because of their easiness of use and utility when aggregating the assessments expressed by the decision makers into group ones and their efficiency as an instrument to model decision making processes [23], [37]. Furthermore, other types of preference relations as, for instance, multiplicative preference relations [29], linguistic preference relations [47], or intuitionistic fuzzy preference relations [40] are also used. Nevertheless, a preference relation is not the only representation format of preferences. For instance, other types of preference structures employed to represent the evaluations verbalized by the group of decision makers are:

- Preference orderings. The evaluations given by a decision maker on a collection of possible alternatives are modelled as a preference ordering $O = \{o(1), \dots, o(n)\}$. Here, $o(\cdot)$ is a permutation function over the set of indexes $\{1, \dots, n\}$ [37]. Therefore, using this structure, the decision makers provide an ordered vector of alternatives from best to worst;
- Utility values. A decision maker gives her/his evaluations on a collection of possible alternatives X via a set of n utility values $U = \{u_1, \dots, u_n\}$, $u_i \in [0, 1]$. The higher the utility value given to an alternative, the better it satisfies the objective of the decision maker [22].

It is worthwhile mentioning that for solving GDM problems, a preference relation is the most frequently used preference structure because of its efficiency in modelling decision making processes. One of the reasons is that the attempt to complete pairwise assessments is more practical when contrasted with assigning in a single step membership grades to all the alternatives of the collection. It means that a decision maker must be able to assess each alternative in contrast to all the others as a whole, which may not be an easy task. Pairwise evaluations help a decision maker to focus only on two alternatives at the same time reducing hesitation and uncertainty while leading to a high consistency, that is, information that does not result in contradictions [1], [16].

2.2 Consensus Reaching Process

To solve a GDM problem, a selection process must be performed [11]. The objective of the

selection process is to obtain a solution set of alternatives according to the evaluations verbalized by the group of decision makers. The selection process involves two stages. The first one is the aggregation of the individual assessments verbalized by the decision makers. Here, we obtain a collective opinion via the aggregation of all individual evaluations. The second one is the exploitation of the collective preference. Here, we obtain the solution set of alternatives by transforming the total information about the alternatives into a total ranking of them. The drawback of applying directly the selection process is that it does not consider the consensus reached among the decision makers. Hence, solutions not well accepted by some decision makers could be reached [7]. The cause is that a decision maker could believe that his/her evaluations have not been considered properly to solve the problem and, for that reason, the decision maker might reject the solution obtained. Consequently, before applying the selection process, it is advisable that the decision makers perform a consensus reaching process in which they modify and discuss their evaluations step by step to reach an enough level of agreement. As a result, a GDM problem is usually solved via a consensus process and a selection process [21], [46].

Consensus reaching processes are developed in a multistage setting, where the decision makers modify their first judgments little by little until a sufficient agreement is reached [7]. Here, it is assumed in advance that a decision maker is committed to those changes.

Two approaches have been employed to model a consensus reaching process [21]. Firstly, Markov chains or matrix calculus have been used to model the time evolution of modifications of beliefs toward consensus [18]. Although this approach has made a contribution to the knowledge of the consensus reaching processes and their dynamics, a more promising approach is to perform this process with the help of a moderator that has the responsibility of running the discussion rounds by convincing the decision makers to modify their judgments via persuasion, rational arguments, etc., and keeping the decision making process within a time considered [7]. In this case, it is recommendable to support the moderator with information to be given by consensus support tools and, here, fuzzy logic may come into play. This second approach modelling a consensus reaching

process is more efficient and effective, and, thus, it has been most common in the consensus approaches developed in recent times [21].

Next, the consensus approach based on a moderator is described. In this approach, the consensus process is defined as an iterative process composed by several discussion rounds in which the decision makers accept to change their evaluations according to the advice given by a moderator, who is aware of the agreement level among the decision makers in each step of the consensus process via the calculation of some consensus measures. Hence, a consensus reaching process is composed of the following stages:

1. The problem under consideration is presented to the group of decision makers. In addition, possible alternatives to solve the problem are described;
2. The decision makers exchange their knowledge about the alternatives and the problem with the objective of making easy the process of latterly verbalizing their assessments;
3. The decision makers verbalize their evaluation about the alternatives via some representation format of preferences;
4. All the evaluations provided by the group of decision makers are given to the moderator. Then, he/she calculates some consensus measures allowing him/herto identify if a sufficient consensus has been obtained or not;
5. The consensus process stops and the selection process begins if an enough consensus level has been achieved. On the other hand, a feedback mechanism may be carried out in which the moderator, considering all the information that he/she has, can prepare some advice and guidance for the group of decision makers to more easily achieve consensus. This stage is optional and, therefore, it is not present in all the consensus reaching processes existing in the literature;
6. The decision makers receive the advice and this round of consensus finishes. The decision makers involved in the problem must then discuss their evaluations to approach their beliefs (Stage 2).

Finally, it is worth mentioning that the moderator may introduce some subjectivity in the decision

process. To avoid it, new approaches have been proposed to make more efficient and effective the decision process by providing to the moderator with better analysis tools or by introducing in the decision processes some automatic consensus control mechanism that substitutes the moderator's activity [9], [13], [21], [25], [30].

2.3 Social Networks

A new framework in which social networks may be created to share resources and information, communicate, collaborate, and so on, has been provided by new Web 2.0 technologies. Social networks allow users from different countries to meet other users sharing some of their concerns [41].

Apart from the clear advantage of meeting individuals with related concerns, a social network presents some features making them different from other more common types of organizations. In the following, we analyze some of these features and show how they may influence in the case of a GDM scenario [2]:

- Large user base. A social network usually has a large user base [5]. It implies that the total knowledge is commonly more diverse and greater than in a small organization. This presents an evident advantage: if there is a rich knowledge on the problem under consideration, making a decision is usually better performed. However, handling a diverse and large amount of evaluations to use and extract that knowledge might not be an easy job. For instance, some of the members of the social network could not be familiar with the use of fuzzy preference relations and, thus, linguistic ones should be also implemented.
- Heterogeneous user base. In addition to a large user base, social networks present usually a heterogeneous one. It signifies that we cannot assume that all the members of the social networks can find easy the utilization of the tools developed and introduced in the social network. An evident example is the employment of numerical ratings: some members can find difficult to provide their evaluations on the alternatives using numerical ratings and, consequently, social networks should provide tools dealing with linguistic assessments or natural language.
- Low participation and contribution rates. Despite a social network has usually a large

user base, many of its members do not take part in its activities. In addition, it is a difficult task to encourage them to do so [27]. Many of the members of a social network are just observers making use of the resources produced by other members. However, they do not cooperate with supplementary recourses. It is a serious concern if only a few members contribute to make a decision because it could not reflect the global opinion of the social network.

- Intermittent contributions. In part, due to a different engagement of the members and due to the fast possibilities of communication, it is a frequent issue that some of members of the social network might not be able to cooperate throughout an entire decision process, but only in a part of it. For example, it is common that existing members for a time cease in their contributions or leave the social network and that new users are continuously joined to the social network.
- Real time communication. Social networks are supported by technologies allowing near real-time communication among their users. It allows us the creation of approaches that in traditional situations would be unworkable.
- Difficulty of establishing trust relations. The electronic devices are the main schemes for communication in social networks. Therefore, in most of the cases, the users do not know others in person and it might be difficult to trust in them to, for instance, delegate votes. It signifies that it is important to implement control mechanisms to avoid malevolent members abusing of others.

3. Consensus Approaches in Social Networks

In this section, the main primary investigations on consensus approaches developed to deal with social networks are described by showing their features and performance. We have analysed the following nine consensus approaches based in social networks:

1. Alonso et al. in [2] presented the first consensus approach to deal with this kind of organizations. This approach includes a feedback mechanism to helps users to modify their evaluations about the alternatives and a delegation scheme, in which users may choose to delegate into other users, improving the convergence toward a consensus solution

and the speed of the decision making process. The advantages of this approach are that the problems of social networks (difficulty of establishing trust relations, intermittent and low participation rates, etc.) are minimized while their benefits (real-time communication, diverse and rich knowledge due to many users, and so on) are incorporated.

2. A generalization of the Deffuant-Weisbuch model was proposed in [26]. Here, Li et al. study opinion dynamics in a connected network according to two associated models of interaction: the hard-interaction model and the strategic interaction model. On the one hand, the authors provide a necessary condition guaranteeing convergence of opinion under the hard-interaction model. In addition, they show that this condition does not vary if the communication rates are time-invariant. The trust existing between users with similar opinions is modelled by a trust function. Here, users exchange their judgments with their neighbours and move their judgments closer to each other if they have similar opinions. On the other hand, under the strategic interaction model, the authors show how the process of opinion formation is influenced by individual motivations behind communications. Particularly, two specific utility functions are explored, leading to two distinct asymptotic opinion patterns.
3. In [6], the authors extended the ‘soft’ consensus concept to address the problem of consensus evaluation via the computation of the importance of the users in relation to their influence strength in a social network. To do so, a centrality measure is used and combined with the fuzzy m-ary adjacency relation approach. In this manner, a flexible consensus measure is introduced, which considers the influence strength of the users in line with their eigenvector centrality. In addition, an optimization problem was proposed to determine the maximum number of the most important users sharing a fixed desirable consensus level.
4. Wu et al. presented in [44] a new consensus approach dealing with a networked social group. On the one hand, a novel trust propagation method is included in this approach deriving trust relationships from

- an incomplete connected trust network. On the other hand, the orthopairs of trust/distrust values obtained from distinct trust paths are aggregated by means of a trust score induced order weighted averaging operator. To determine users' weights and to estimate the unknown evaluation values, the approach uses a relative trust score. Graphical representations of the consensus state guaranteeing the convergence of the consensus reaching process are provided to the users via a visual feedback mechanism. This approach also incorporates a recommendation mechanism advising the users on how modify their evaluations.
5. Shang studied in [33] the finite-time cluster consensus behaviour on arbitrary bidirectional graphs. For consensus problems, the bidirectional graphs are essential as they frequently arise in applications where the exchange of information goes in both directions. A finite-time average consensus is shown to be always reached on bidirectional graphs that have cluster-spanning trees, i.e., within each cluster, all users may achieve the mean value of their initial states in a limited number of steps. Distributed linear iterations, which involve a product of stochastic matrices with positive diagonal entries, are only used to achieve it. Shang also presents in detail an algorithm for this finite-time cluster average consensus approach. Compared to other cluster consensus algorithms [20], [34], this approach achieves a much faster consensus.
 6. Dong et al. investigated the question of opinions dynamics in social networks [17]. By studying the structure of a social network where all members may form a consensus, a consensus reaching process is developed in opinion dynamics, based on leadership. In particular, a strategy adding a minimum number of communications in the social network is proposed to form a consensus based on the concept of leader. Then, this strategy is generalized to handle consensus problems with an established objective. As advantage, this approach allows opinion managers to guide and influence the formation of opinions to achieve consensus. For instance, a firm could use this approach to introduce a new product in a market.
 7. Using interval-valued fuzzy reciprocal preference relations to represent the evaluations given by the users, the authors proposed in [42] a social network analysis trust-consensus based group decision making approach. The most important novelty of this approach is the determination of the importance degree of the users by combining both a consensus level and a trust degree. To do so, the authors develop an interval-valued fuzzy social network analysis approach that models and represents trust relationship among users and computes the trust degree of each user. The authors also investigate the property of multiplicative consistency and define the consistency indexes for the three different levels of a preference relation. The level of consensus is obtaining by combining both a similarity index and a consistency index, and it guides a feedback mechanism supporting users in modifying their evaluations to reach a solution of consensus with a high degree of consistency.
 8. The inconsistency problem in GDM caused by different evaluations of multiple users was addressed by Liu et al. in [28]. In this contribution, the authors propose a trust induced recommendation mechanism generating personalized recommendations to the inconsistent users with the aim of achieving a high level of consensus. The uncertainty of users is modelled by an interval-valued trust decision making space. This approach includes the novel concepts of interval-valued knowledge degree, interval-valued trust score and interval-valued trust functions. The concepts of harmony degree between the revised opinion and the first opinion and the consensus degree between a user and the rest of users in the social network are developed for interval-valued trust functions. The authors also propose a more rational policy for group consensus by combining the harmony degree and the consensus degree. The objective is to arrive at the threshold value with the maximum value of consensus and harmony degrees at the same time. In addition, the trust induced recommendation mechanism is focused on modifying inconsistent evaluations utilizing only assessments from the trusted users and not from the distrusted ones. It means that less changes cost are required to reach the

threshold value of consensus in comparison with previous approaches based on the average of the evaluation of all users.

9. Wu et al. presented in [43] a theoretical visual framework to model consensus in social networks. It presents three principal components: a construction of trust relationship, a trust based recommendation mechanism, and a visual adoption mechanism. To connect incomplete trust relationships by trusted third partners in a manner that distrust values increase while trust values decrease, dual propagation is studied. Hence, when compared to previous trust propagation approaches, this new approach addresses and models suitably the information attenuation produced by the trusted thirds partners. This approach also incorporates a trust based recommendation mechanism generating advice following the individual trust relationship. It makes more recommendable recommendations to be implemented by the inconsistent users to reach higher consensus levels. Hence, in comparison with existing interaction approaches, the advantage of the approach proposed in this contribution is that the inconsistent users are not forced to accept recommendations regardless of their trust on the other users. On the other hand, the visual adoption mechanism provides visual information representations allowing users to choose their suitable feedback parameters to reach a balance between individual independence and group consensus. As a result, it adds a needed and real flexibility to guide the consensus reaching process.

4. Challenges and Future Trends

Consensus approaches in social networks have been a productive research field during recent years. In addition to the presented study of the main consensus approaches developed in this research area, in this section, we point out other important challenges identified during this research and that should be considered in future studies.

Firstly, as social networks have both a large and a heterogeneous user base, different types of preference structures should be incorporated to facilitate the expression of evaluations to the users. Therefore, the existing consensus approaches

should be extended to work with new preference structures for representing judgments as, for instance, hesitant fuzzy sets and their extensions [32], [38]. Furthermore, new preference structures should be also developed and their application in consensus approaches dealing with social networks should be investigated.

Other important question is the general supposition about the users' acceptance of the recommendations provided by the feedback mechanism to reach a convergent process and to increase the level of consensus. But sometimes, a user might decide not to accept the advice and maintain his/her own assessments during the decision process. As a result, it is important not only to model this situation but also to persuade the users to accept the recommendations. Some studies have been focused on finding some persuasion principles or psychology concepts to model the influence as a principal component of a consensus reaching process [15], [31]. Among these principles, we can identify the following ones:

- A tendency among individuals is that of giving back a favour;
- When individuals see other people doing something, they usually do the same thing, although they do not comprehend the causes;
- A figure of authority generally imposes some rules obeyed by others with no questions;
- The law of supply and demand causes effect on individuals when they make decisions;
- When some people like an individual, they can be easily persuaded by this particular individual.

Because these principles of persuasion are based on psychological studies about human behaviours, a consensus reaching process should implement them as weapons of influence. In such a way, the possibility of persuading the users to accept the advice and to improve the level of consensus would be higher.

On the other hand, in some real-world situations, we have observed that an important issue is that of explaining the reasons considered by a user to think that his/her beliefs are correct and to convince other users to follow his/her opinions. Therefore, argumentation mechanisms [3], [4] should be incorporated to the current consensus

approaches to develop new argumentative and dynamic consensus models. In the case of introducing arguments in the discussion process, an important issue is to measure their reliability, which could be a new parameter of the consensus reaching process.

5. Conclusions

Since the network model can effectively model interactions between decision makers, social networks have attracted the attention of many researchers in the field of decision making in recent years.

In this paper, we have studied the problem of consensus in social networks. Firstly, we have presented some preliminary concepts to introduce the whole problem environment. Secondly, we have analyzed the main consensus reaching processes proposed to deal with social networks. Finally, we have identified two main research gaps that could be considered as new challenges and that should be addressed by the community. On the one hand, the existing consensus approaches should be adapted to deal with new structures of preference representation. On the other hand, both persuasion models and argumentation models should be developed to deal with non-cooperative decision makers.

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