Eur. Phys. J. C (2017) 77:708
DOI 10.1140/epjc/s10052-017-5291-x

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Late-time cosmological approach in mimetic $f(R, T)$ gravity

E. H. Baffou[1,a], **M. J. S. Houndjo**[1,2,b], **M. Hamani-Daouda**[3,c], **F. G. Alvarenga**[4,d]

[1] Institut de Mathématiques et de Sciences Physiques (IMSP), 01 BP 613, Porto-Novo, Benin
[2] Faculté des Sciences et Techniques de Natitingou, Natitingou, Benin
[3] Département de Physique, Université de Niamey, Niamey, Niger
[4] Departamento de Engenharia e Ciências Naturais, CEUNES, Universidade Federal do Espírito Santo, São Mateus, ES CEP 29933-415, Brazil

Received: 15 August 2017 / Accepted: 9 October 2017 / Published online: 25 October 2017

Abstract In this paper, we investigate the late-time cosmic acceleration in mimetic $f(R, T)$ gravity with the Lagrange multiplier and potential in a Universe containing, besides radiation and dark energy, a self-interacting (collisional) matter. We obtain through the modified Friedmann equations the main equation that can describe the cosmological evolution. Then, with several models from $\mathcal{Q}(z)$ and the well-known particular model $f(R, T)$, we perform an analysis of the late-time evolution. We examine the behavior of the Hubble parameter, the dark energy equation of state and the total effective equation of state and in each case we compare the resulting picture with the non-collisional matter (assumed as dust) and also with the collisional matter in mimetic $f(R, T)$ gravity. The results obtained are in good agreement with the observational data and show that in the presence of the collisional matter the dark energy oscillations in mimetic $f(R, T)$ gravity can be damped.

1 Introduction

One of the most important recent scientific discoveries is provided by a set of observational data, as regards on the late-time acceleration expansion of the whole Universe as well as the initial era, the inflationary epoch [1–3]. It is noteworthy that the accelerated expansion of the Universe is created by a mysterious energy called dark energy that is the dominant component. The modified gravity method is one of the methods used to describe the accelerated expansion of the Universe. Modified gravity can be considered a new challenge to cure the shortcomings of the General Relativity at infrared and ultraviolet scales. It is an approach that, by preserving the undoubtedly positive results of Einstein's theory, aims to address the conceptual and experimental problems recently emerged in astrophysics, cosmology and high energy physics. In particular the goal is to encompass in a self-consistent scheme problems like inflation, dark energy, dark matter, large scale structure and, first of all, to give at least an effective description of quantum gravity. The late-time cosmic acceleration can in principle be derived from a modification of gravity rather than an exotic source of matter with a negative pressure. A lot of work on modified gravity has been done to identify the origin of the dark energy [4–6] over the last years. The attractive point in modified gravity models is that they are generally more strongly constrained by cosmological observations and local gravity experiments than the models based on the exotic source of matter. One of the simplest modifications to GR is the $f(R)$ theories of gravity in which the Lagrangian density is supposed to an arbitrary function of R [7,8]. For informative reviews and very important papers on these theories, see [9–24]. In recent years a new theory, named $f(R, T)$ gravity, has been developed: see Ref. [25]. That theory can also be considered as a generalization of the $f(R)$ gravity. In this theory, an arbitrary function of the Ricci scalar R and the trace of the energy-momentum tensor is introduced and is used instead of an arbitrary function of only the Ricci scalar. The main justifications for employing the trace of the energy-momentum tensor may be associated with exotic imperfect fluids or quantum effects (conformal anomaly). Different aspects of such a theory have been investigated in the literature [26–40]. Recently, another kind of alternative gravity model has been proposed in which the metric is not considered a fundamental quantity. Instead, it is taken as a function of an auxiliary metric $\tilde{g}_{\mu\nu}$ and a scalar field ϕ, contemplating so called Mimetic Gravity (MG) [41]. Note that such a dependence on the metric makes the application of the variational principle in the model action yield

[a] e-mails: baffouhet@gmail.com; baffouh.etienne@yahoo.fr
[b] e-mail: sthoundjo@yahoo.fr
[c] e-mail: daoudah8@yahoo.fr
[d] e-mail: f.g.alvarenga@gmail.com

more general equations of motion (EoM) than the Einsteinian relativity theory. To unify the $f(R)$ gravity with this very interesting mimetic theory one has proposed mimetic $f(R)$ gravity [42] as a new class of modified gravities with the same inspiration as mimetic theory. Very recently the author [43] demonstrated how $f(R)$ gravity, the mimetic potential and the Lagrange multiplier affect the late-time cosmological evolution, and in Ref. [44] it was demonstrated that in the context of the mimetic $f(R)$ gravity with the Lagrange multiplier and the mimetic potential it is possible to solve the problem of dark energy oscillations at late times in an elegant way. The review of the main aspects of mimetic gravity, as well as the extensions of the minimal formulation of the model have been addressed in Ref. [45]. For the work on mimetic modified gravity by several authors, see [46–52]. In this paper, we adopt the mimetic $f(R,T)$ gravity approach with the scalar potential $V(\phi)$ and with the Lagrange multiplier $\lambda(\phi)$ to describe the late-time cosmological evolution and the dark energy eras. The specific goal in this paper is to extend the work of authors [53] in the context of mimetic $f(R,T)$ gravity. We shall investigate how the $f(R,T)$ gravity in the presence of the potential and the Lagrange multiplier can offer much freedom in realizing various cosmic evolution scenarios and can be allowed to have compatibility with observational data. The present paper is organized as follows: in Sect. 2 we briefly review mimetic cosmology in $f(R,T)$ gravity. Section 3 is dedicated to the study of the late-time cosmological evolution in mimetic $f(R,T)$ gravity. Our conclusion is presented in the last section.

2 Brief review in mimetic $f(R,T)$ gravity models

We provide in this section a brief review of mimetic $f(R,T)$ gravity with the Lagrange multiplier and potential. The main idea is to study the mimetic approach in modified gravity coming from the general class of mimetic gravities [41,54–61] by parametrizing the metric using new degrees of freedom modified field equations that may admit a wider family of solutions that be obtained. Therefore we express the physical metric $g_{\mu\nu}$ in terms of an auxiliary metric $\tilde{g}_{\mu\nu}$ and of an auxiliary scalar field ϕ, as follows:

$$g_{\mu\nu} = -\tilde{g}^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi\tilde{g}_{\mu\nu}, \tag{1}$$

and thus the gravitational field variation will be performed in terms of both the auxiliary metric $\hat{g}_{\mu\nu}$ and the auxiliary scalar field ϕ. Equation (1) shows

$$g^{\mu\nu}(\tilde{g}_{\mu\nu},\phi)\partial_\mu\phi\partial_\nu\phi = -1. \tag{2}$$

Since we are interested in investigating the cosmological implications of the mimetic $f(R,T)$ gravity we consider in the following the flat Friedmann–Robertson–Walker (FRW) metric of which the line element is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2], \tag{3}$$

where $a(t)$ is the scale factor. We assume in the mimetic $f(R,T)$ gravity with the Lagrange multiplier $\lambda(\phi)$ and the mimetic potential $V(\phi)$ the gravitational action coupled with matter as

$$\begin{aligned} S = \int \sqrt{-g}\mathrm{d}x^4\Big[f\big(R(g_{\mu\nu}),T\big) - V(\phi) \\ +\lambda(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 1) + \mathcal{L}_m\Big], \end{aligned} \tag{4}$$

where we assume $16\pi G = 1$; $R = g^{\mu\nu}R_{\mu\nu}$ and $T = g^{\mu\nu}T_{\mu\nu}$ denotes, respectively, the curvature scalar of the Ricci tensor $R_{\mu\nu}$ and the trace of the energy-momentum tensor $T_{\mu\nu}$; $\mathcal{L}_m$ being the matter Lagrangian density of all fluids present. We defined the energy-momentum tensor of the matter from the Lagrangian density $\mathcal{L}_m$ as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}. \tag{5}$$

Variation of the action (4) with respect to the tensor metric $g_{\mu\nu}$ is given by

$$\begin{aligned} &\frac{1}{2}g_{\mu\nu}f(R,T) - R_{\mu\nu}f_R + \nabla_\mu\nabla_\nu f_R - g_{\mu\nu}\Box f_R \\ &+\frac{1}{2}g_{\mu\nu}\Big(-V(\phi) + \lambda(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + 1)\Big) \\ &-\lambda\partial_\mu\phi\partial_\nu\phi - f_T(T_{\mu\nu} + \Theta_{\mu\nu}) + \frac{1}{2}T_{\mu\nu} = 0, \end{aligned} \tag{6}$$

where

$$\Theta_{\mu\nu} \equiv g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}} = -2T_{\mu\nu} + g_{\mu\nu}\mathcal{L}_m - 2g^{\alpha\beta}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}. \tag{7}$$

∇_i denotes the covariant derivative with respect to the metric $g_{\mu\nu}$ and $\Box = \nabla^i\nabla_i$ the d'Alembertian operator. In Eq. (6), f_R and f_T represent the partial derivation of $f(R,T)$ with respect to the R, T, respectively. Notice that the auxiliary metric $\tilde{g}_{\mu\nu}$ does not appear in these equations by itself but only via the physical metric $g_{\mu\nu}$, while the scalar field ϕ enters the equations explicitly.

Now it is assumed that the matter content of the Universe can be described by a perfect fluid for which the energy-momentum tensor is expressed as

$$T_{\mu\nu} = (\rho_{\mathrm{matt}} + p_{\mathrm{matt}})u_\mu u_\nu + p_{\mathrm{matt}}g_{\mu\nu}, \tag{8}$$

where ρ_{matt} and p_{matt}, respectively, are the energy density and the pressure of the matter and u_μ is the four-velocity. In this way the matter Lagrangian density can be chosen as $\mathcal{L}_m = -p_{\mathrm{matt}}$. In fact, the field equations (6) yield

$$\begin{aligned} &\frac{1}{2}g_{\mu\nu}f(R,T) - R_{\mu\nu}f_R + \nabla_\mu\nabla_\nu f_R - g_{\mu\nu}\Box f_R \\ &+\frac{1}{2}g_{\mu\nu}\Big(-V(\phi) + \lambda(g^{\rho\sigma}\partial_{\rho_m}\phi\partial_\sigma\phi + 1)\Big) \end{aligned}$$

$$-\lambda\partial_\mu\phi\partial_\nu\phi + f_T(T_{\mu\nu} + p_m g_{\mu\nu}) + \frac{1}{2}T_{\mu\nu} = 0. \tag{9}$$

Varying the gravitational action (4) with respect to the auxiliary scalar field ϕ, one gets

$$-2\nabla^\mu(\lambda\partial_\mu\phi) - V'(\phi) = 0, \tag{10}$$

where the prime denotes the derivative of the mimetic potential with respect to the auxiliary scalar ϕ. On the other hand, by the variation with respect to the Lagrange multiplier λ, we obtain

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1, \tag{11}$$

which shows that the scalar field will not be a propagating degree of freedom [46]. This equation expresses the constraint equation (2) in the mimetic gravity, The equation is obtained by varying the gravitational action with respect to the Lagrange multiplier λ.

Considering the FRW space-time (3) and assuming that ϕ depends only on time coordinate t, the field equations (9), (10) and (11) are written

$$\begin{aligned}&-f(R,T) + 6(\dot{H} + H^2)f_R - 6H\frac{\mathrm{d}f_R}{\mathrm{d}t} - \lambda(\dot{\phi}^2 + 1)\\ &\quad + V(\phi) + \rho_{\mathrm{matt}}(2f_T + 1) - 2f_T p_{\mathrm{matt}} = 0,\end{aligned} \tag{12}$$

$$\begin{aligned}&f(R,T) - 2(\dot{H} + 3H^2)f_R + 4H\frac{\mathrm{d}f_R}{\mathrm{d}t} + 2\frac{\mathrm{d}^2 f_R}{\mathrm{d}t^2}\\ &\quad - \lambda(\dot{\phi}^2 - 1) - V(\phi) + (4f_T + 1)p_{\mathrm{matt}} = 0,\end{aligned} \tag{13}$$

$$2\frac{\mathrm{d}(\lambda\dot{\phi})}{\mathrm{d}t} + 6H\lambda\dot{\phi} - V'(\phi) = 0, \tag{14}$$

$$\dot{\phi}^2 - 1 = 0. \tag{15}$$

In these expressions, the dot represents the derivative with respect to the cosmic time t whereas the prime denotes the derivative with respect to the auxiliary scalar ϕ. From Eq. (15) we remark that ϕ can be identified as the time coordinate ($\phi = t$). Then Eq. (12) reduces to

$$\begin{aligned}&f(R,T) - 2(\dot{H} + 3H^2)f_R + 4H\frac{\mathrm{d}f_R}{\mathrm{d}t}\\ &\quad + 2\frac{\mathrm{d}^2 f_R}{\mathrm{d}t^2} - V(t) + (4f_T + 1)p_{\mathrm{matt}} = 0.\end{aligned} \tag{16}$$

In the $f(R,T)$ gravity the mimetic potential $V(t)$ can be expressed by

$$\begin{aligned}V(t) = &\, 2\frac{\mathrm{d}^2 f_R}{\mathrm{d}t^2} + 4H\frac{\mathrm{d}f_R}{\mathrm{d}t} + f(R,T)\\ &- 2(\dot{H} + 3H^2)f_R + (4f_T + 1)p_{\mathrm{matt}}.\end{aligned} \tag{17}$$

Within specific forms of the $f(R,T)$ model and the Hubble parameter the correspondent mimetic potential can be found. Once this expression is known, Eq. (12) can be solved with respect to the Lagrange multiplier $\lambda(t)$,

$$\begin{aligned}\lambda(t) = &-\frac{1}{2}f(R,T) + 3(\dot{H} + H^2)f_R - 3H\frac{\mathrm{d}f_R}{\mathrm{d}t}\\ &+ \rho_{\mathrm{matt}}\left(f_T + \frac{1}{2}\right) - f_T p_{\mathrm{matt}} + \frac{1}{2}V(t).\end{aligned} \tag{18}$$

3 Late-time cosmological evolution in mimetic $f(R,T)$ gravity

In order to establish the main differential equation which governs the dark energy oscillations evolution we recast the FRW equation (12) as follows:

$$\begin{aligned}3H^2 f_R = &\,\rho_{\mathrm{matt}}\left(f_T + \frac{1}{2}\right) - f_T p_{\mathrm{matt}} + \frac{1}{2}(Rf_R - f(R,T))\\ &+ \frac{V(t) - 2\lambda(t)}{2} - 3H\dot{f}_R = 0,\end{aligned} \tag{19}$$

where ρ_{matt} and p_{matt} are the total energy density and the pressure of all fluids present in Universe. In $f(R,T)$ gravity the trace T of the energy-momentum tensor depends on the nature of the matter content. As a novelty, we assume in this paper the matter content of the Universe to be collisional matter and relativistic matter (radiation). The model of collisional matter has been studied in some work before, leading to interesting results [53,62–64]. This approach of considering forms of matter other than cold dark matter can teach us about the choice of the models of modified gravity. Accordingly, we can write the total energy density for the mimetic $f(R,T)$ gravity case thus:

$$\rho_{\mathrm{matt}} = \varepsilon_m + \rho_{r0}a^{-4}, \tag{20}$$

where ε_m is the energy density of collisional matter given by

$$\varepsilon_m = \rho_{m0}a^{-3}\Big(1 + \Pi_0 + 3w\ln(a)\Big), \tag{21}$$

and ρ_{r0} the current energy density of radiation, ρ_{m0} and Π_0 denote present values of the motion invariant mass energy density and of the potential energy, respectively. We can reformulate the total matter energy density in Eq. (20) in terms of the parameter $\mathcal{G}(a)$ describing the nature of the collisional matter (view as perfect fluid) as

$$\rho_{\mathrm{matt}} = \rho_{m0}\left(\mathcal{G}(a) + \chi a^{-4}\right), \tag{22}$$

where the parameter $\mathcal{G}(a)$ is equal to

$$\mathcal{G}(a) = a^{-3}\left(1 + \Pi_0 + 3w\ln(a)\right). \tag{23}$$

Note that for the non-collisional matter (assumed to be dust) for which the parameter $w = 0$, $\mathcal{G}(a) = a^{-3}$ is obtained. Equation (19) can be reformulated:

$$H^2+(1-f_R)\left(H\frac{\mathrm{d}H}{\mathrm{d}\ln a}+H^2\right)+\frac{1}{6}\Big(f(R,T)-R\Big)+H^2 f_{RR}\frac{\mathrm{d}R}{\mathrm{d}\ln a}-\frac{1}{3}f_T(\rho_{\mathrm{matt}}-p_{\mathrm{matt}})-\frac{V(t)-2\lambda(t)}{6}=\frac{\rho_{\mathrm{matt}}}{6}, \tag{24}$$

while the scalar curvature R can be expressed as

$$R=12H^2+6H\frac{\mathrm{d}H}{\mathrm{d}\ln a}. \tag{25}$$

We introduce for reasons of simplicity the following function of the redshift z:

$$\mathcal{Q}(a(z))=V(a(z))-2\lambda(a(z)), \tag{26}$$

which depends on the mimetic potential and on the Lagrange multiplier. In terms of the parameters $\mathcal{G}(a)$ and $Q(a)$, Eq. (24) can be rewritten

$$H^2+(1-f_R)\left(H\frac{\mathrm{d}H}{\mathrm{d}\ln a}+H^2\right)+\frac{1}{6}\Big(f(R,T)-R\Big)+H^2 f_{RR}\frac{\mathrm{d}R}{\mathrm{d}\ln a}-\rho_{m0}\Big(\mathcal{G}(a)+\chi a^{-4}\Big)\left(\frac{1}{6}+\frac{1}{3}f_T(1-w)\right)-\frac{\mathcal{Q}(a)}{6}=0. \tag{27}$$

In an effort to study the late-time cosmological evolution in mimetic $f(R,T)$ gravity better, we introduce the following variable:

$$y_H\equiv\frac{\rho_{DE}}{\rho_{m0}}=\frac{H^2}{\bar{m}^2}-\mathcal{G}(a)-\mathcal{Q}(a)-\chi a^{-4}, \tag{28}$$

$$y_R\equiv\frac{R}{\bar{m}^2}-\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}-\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}, \tag{29}$$

where ρ_{DE} denotes the energy density of the dark energy, $\bar{m}^2$ being the mass scale and χ the ratio defined as $\chi=\rho_{r0}/\rho_{m0}$. The dark energy scale y_H is the new variable that can describe the late-time cosmological evolution. Making use of Eq. (27), the expression $\frac{1}{\bar{m}^2}\frac{\mathrm{d}R}{\mathrm{d}\ln a}$ yields

$$\begin{aligned}\frac{1}{\bar{m}^2}\frac{\mathrm{d}R}{\mathrm{d}\ln a}=\frac{1}{H^2 f_{RR}}\Bigg[&\Big(\mathcal{G}(a)+\chi a^{-4}\Big)\\&\times\left(\frac{\rho_{m0}}{\bar{m}^2}\left(\frac{1}{6}+\frac{1}{3}f_T(1-w)\right)-1\right)\\&-\frac{1}{6\bar{m}^2}\Big(f(R,T)-R\Big)\\&-y_H+\mathcal{Q}(a)\left(\frac{1}{6\bar{m}^2}-1\right)\\&-(1-f_R)\left(\frac{H}{\bar{m}^2}\frac{\mathrm{d}H}{\mathrm{d}\ln a}+\frac{H^2}{\bar{m}^2}\right)\Bigg].\end{aligned} \tag{30}$$

Combining the differentiation of Eq. (29) with respect to $\ln a$ with Eq. (30), we obtain

$$\begin{aligned}\frac{\mathrm{d}y_R}{\mathrm{d}\ln a}=&-\frac{\mathrm{d}^2\mathcal{G}(a)}{\mathrm{d}\ln a^2}-\frac{\mathrm{d}^2\mathcal{Q}(a)}{\mathrm{d}\ln a^2}\\&+\frac{1}{\bar{m}^2\Big(y_H+\mathcal{G}(a)+\mathcal{Q}a(z)+\chi a^{-4}\Big)f_{RR}}\\&\times\Bigg[\Big(\mathcal{G}(a)+\chi a^{-4}\Big)\left(\frac{\rho_{m0}}{\bar{m}^2}\left(\frac{1}{6}+\frac{1}{3}f_T(1-w)\right)-1\right)\\&-\frac{1}{6\bar{m}^2}\Big(f(R,T)-R\Big)+\mathcal{Q}(a)\left(\frac{1}{6\bar{m}^2}-1\right)\\&-y_H-(1-f_R)\left(\frac{1}{2}\frac{\mathrm{d}y_H}{\mathrm{d}\ln a}+\frac{1}{2}\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}\right.\\&\left.+\frac{1}{2}\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}+y_H+\mathcal{G}(a)+\mathcal{Q}(a)-\chi a^{-4}\right)\Bigg].\end{aligned} \tag{31}$$

Moreover, the curvature scalar (25) can be expressed by

$$R=3\bar{m}^2\left[4y_H+4\mathcal{G}(a)+4\mathcal{Q}(a)+\frac{\mathrm{d}y_H}{\mathrm{d}\ln a}+\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}+\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}\right]. \tag{32}$$

Upon differentiation of Eq. (28) with respect to $\ln a$ we obtain

$$\frac{\mathrm{d}y_H}{\mathrm{d}\ln a}=\frac{2H}{\bar{m}^2}\frac{\mathrm{d}H}{\mathrm{d}\ln a}-\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}-\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}+4\chi a^{-4}. \tag{33}$$

Using Eqs. (25), (28) and (29), Eq. (33) becomes

$$\frac{\mathrm{d}y_H}{\mathrm{d}\ln a}=\frac{1}{3}y_R-4y_H-\frac{2}{3}\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}-\frac{2}{3}\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}-4\mathcal{G}(a)-4\mathcal{Q}(a). \tag{34}$$

By differentiation of Eq. (34) with respect to $\ln a$ and also by using Eq. (31), we obtain the following differential equation:

$$\frac{\mathrm{d}^2y_H}{\mathrm{d}\ln a^2}+\left(4+\frac{1-f_R}{6\bar{m}^2 f_{RR}(y_H+\mathcal{G}(a)+\mathcal{Q}(a)+\chi a^{-4})}\right)\frac{\mathrm{d}y_H}{\mathrm{d}\ln a}+\left(\frac{2-f_R}{3\bar{m}^2 f_{RR}(y_H+\mathcal{G}(a)+\mathcal{Q}(a)+\chi a^{-4})}\right)y_H+\mathcal{P}(a)=0, \tag{35}$$

where $\mathcal{P}(a)$ is given by

$$\begin{aligned}\mathcal{P}(a)=&\frac{\mathrm{d}^2\mathcal{G}(a)}{\mathrm{d}\ln a^2}+\frac{\mathrm{d}^2\mathcal{Q}(a)}{\mathrm{d}\ln a^2}+4\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}+4\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}\\&+\frac{1}{18\bar{m}^2\Big(y_H+\mathcal{G}(a)+\mathcal{Q}(a)+\chi a^{-4}\Big)f_{RR}}\\&\times\Bigg[\Big(\mathcal{G}(a)+\chi a^{-4}\Big)\left(\frac{\rho_{m0}}{\bar{m}^2}\Big(-1+2f_T(w-1)\Big)\right)\\&+\frac{1}{\bar{m}^2}\Big(f(R,T)-R\Big)+\Big(\mathcal{Q}(a)+\mathcal{G}(a)\Big)(12-6f_R)\\&-\frac{1}{\bar{m}^2}\mathcal{Q}(a)+6\chi a^{-4}f_R\\&-3(f_R-1)\left(\frac{\mathrm{d}\mathcal{G}(a)}{\mathrm{d}\ln a}+\frac{\mathrm{d}\mathcal{Q}(a)}{\mathrm{d}\ln a}\right)\Bigg].\end{aligned} \tag{36}$$

Taking into account the relations

$$\frac{\mathrm{d}}{\mathrm{d}\ln a}=-(1+z)\frac{\mathrm{d}}{\mathrm{d}z}, \qquad (37)$$

$$\frac{d^2}{d\ln a^2}=(1+z)^2\frac{\mathrm{d}^2}{\mathrm{d}z^2}+(1+z)\frac{\mathrm{d}}{\mathrm{d}z}, \qquad (38)$$

all the physical quantities of (35) can easily be expressed in terms of the redshift z, as follows:

$$\frac{\mathrm{d}^2y_H}{\mathrm{d}z^2}+\frac{1}{1+z}\left(-3+\frac{f_R-1}{6\bar{m}^2f_{RR}(y_H+\mathcal{G}(z)+\mathcal{Q}(z)+\chi(1+z)^4)}\right)\frac{\mathrm{d}y_H}{\mathrm{d}z}+\frac{1}{(1+z)^2}\left[\frac{2-f_R}{3\bar{m}^2f_{RR}(y_H+\mathcal{G}(z)+\mathcal{Q}(z)+\chi(1+z)^4)}\right]y_H+\mathcal{P}(z)=0, \qquad (39)$$

where

$$\mathcal{P}(z)=\frac{\mathrm{d}^2\mathcal{G}(z)}{\mathrm{d}z^2}+\frac{\mathrm{d}^2\mathcal{Q}(z)}{\mathrm{d}z^2}-\frac{3}{1+z}\left(\frac{\mathrm{d}\mathcal{G}(z)}{\mathrm{d}z}+\frac{\mathrm{d}\mathcal{Q}(z)}{\mathrm{d}z}\right)+\frac{1}{18(1+z)^2\bar{m}^2\left(y_H+\mathcal{G}(z)+\mathcal{Q}(z)+\chi(1+z)^4\right)f_{RR}}\times\left[\left(\mathcal{G}(z)+\chi(1+z)^4\right)\left(\frac{\rho_{m0}}{\bar{m}^2}\left(-1+2f_T(w-1)\right)\right)+\frac{1}{\bar{m}^2}\left(f(R,T)-R\right)+\left(\mathcal{Q}(z)+\mathcal{G}(z)\right)(12-6f_R)-\frac{1}{\bar{m}^2}\mathcal{Q}(z)+6\chi(1+z)^4f_R+3(1+z)(f_R-1)\left(\frac{\mathrm{d}\mathcal{G}(z)}{\mathrm{d}z}+\frac{\mathrm{d}\mathcal{Q}(z)}{\mathrm{d}z}\right)\right], \qquad (40)$$

$$g(z)=(1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right). \qquad (41)$$

Inserting Eq. (41) in the differential equation (39) gives

$$\frac{\mathrm{d}^2y_H}{\mathrm{d}z^2}+\frac{1}{1+z}\left[-3+\frac{f_R-1}{6\bar{m}^2f_{RR}\left(y_H+(1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right)+\mathcal{Q}(z)+\chi(1+z)^4\right)}\right]\frac{\mathrm{d}y_H}{\mathrm{d}z}+\frac{1}{(1+z)^2}\left[\frac{2-f_R}{3\bar{m}^2f_{RR}\left(y_H+(1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right)+\mathcal{Q}(z)+\chi(1+z)^4\right)}\right]y_H+\mathcal{P}(z)=0, \qquad (42)$$

and the corresponding function $\mathcal{P}(z)$ reads

$$\mathcal{P}(z)=\frac{\mathrm{d}^2\mathcal{Q}(z)}{\mathrm{d}z^2}-\frac{3}{1+z}\frac{\mathrm{d}\mathcal{Q}(z)}{\mathrm{d}z}-3(1+z)(1+\Pi_0+2w)+\frac{1}{18(1+z)^2\bar{m}^2\left(y_H+(1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right)+\mathcal{Q}(z)+\chi(1+z)^4\right)f_{RR}}\times\left[\left((1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right)+\chi(1+z)^4\right)\left(\frac{\rho_{m0}}{\bar{m}^2}\left(-1+2f_T(w-1)\right)\right)+\frac{1}{\bar{m}^2}\left(f(R,T)-R\right)+\left(\mathcal{Q}(z)+(1+z)^3\left(1+\Pi_0-3w\ln(1+z)\right)\right)(12-6f_R)-\frac{1}{\bar{m}^2}\mathcal{Q}(z)+6\chi(1+z)^4f_R+3(1+z)(f_R-1)\frac{\mathrm{d}\mathcal{Q}(z)}{\mathrm{d}z}+9(1+z)^3(f_R-1)(1+\Pi_0-w-3w\ln(1+z))\right]. \qquad (43)$$

By inspecting the main equation (42) it is obvious that it describes the late-time cosmological evolution of the dark energy in a Universe filled with collisional matter and radiation and is strongly affected by the mimetic potential and the Lagrange multiplier which are contained in the function $\mathcal{Q}(z)$. We now shall specify the exact form of the function $\mathcal{Q}(z)$ and the $f(R,T)$ models to solve numerically the differential equation (42). Once this equation is solved, we will perform the late-time evolution of the cosmological parameters in mimetic $f(R,T)$ gravity.

4 Numerical analysis for variable $\mathcal{Q}(z)$ models

In this section we perform the numerically analysis of Eq. (42) to study the late-time cosmological evolution in mimetic $f(R,T)$ gravity. To do so, we focus attention in particular on viable $f(R,T)=f(R)+f(T)$ models where $f(R)=R-2\Lambda(1-e^{\frac{R}{b\Lambda}})$ [44] and $f(T)=T^{\beta}$ [53]. In the $f(R,T)$ models currently considered, Λ represents the present time cosmological constant, b is a positive free parameter which is assumed to be $\mathcal{O}(1)$ and β a real constant. With variable models of $\mathcal{Q}(z)$ we plot in mimetic $f(R,T)$ gravity the evolution of the Hubble parameter $H(z)$, the total effective equation of state $w_{\rm eff}$, the parameter of the equation of state for dark energy w_{DE} versus z and we compare the results obtained in mimetic $f(R,T)$ gravity in the presence of the collisional matter ($w=0.6$) with those in the presence of the non-collisional matter ($w=0$). Respectively, these parameters can be expressed as follows:

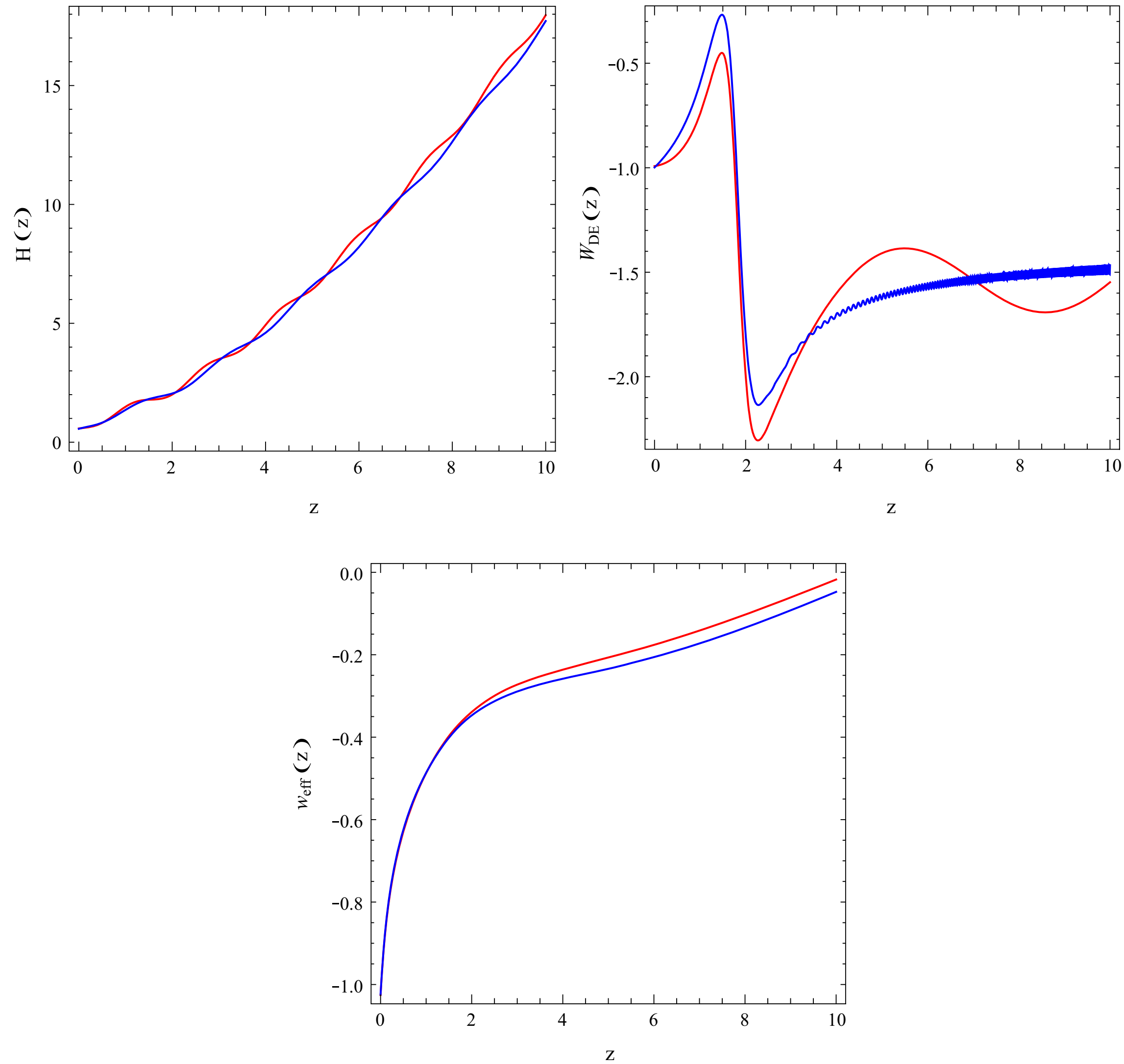

Fig. 1 Comparison of the Hubble parameter $H(z)$ versus z, of the dark energy equation of state parameter w_{DE} versus z and of the effective equation of state parameter $w_{\rm eff}$ versus z. The red curves correspond to the mimetic $f(R,T)$ model in a Universe filled with collisional matter, while the blue curves correspond to the mimetic $f(R,T)$ model in the presence of the non-collisional matter (dust) for the model $\mathcal{Q}(z)=\sqrt{2z+5}$ [44]

$$H(z)=\sqrt{\bar{m}^2\Big(y_H+\mathcal{Q}(z)+(1+z)^3\big(1+\Pi_0-3w\ln(1+z)\big)+\chi(1+z)^4\Big)} \tag{44}$$

$$w_{\rm eff}(z)=-1+\frac{2(1+z)}{3H(z)}\frac{\mathrm{d}H(z)}{\mathrm{d}z}. \tag{45}$$

$$w_{DE}(z)=-1+\frac{1}{3}(1+z)\frac{1}{y_H}\frac{\mathrm{d}y_H}{\mathrm{d}z}. \tag{46}$$

Concerning Fig. 1 we remark that the Hubble parameter, the dark energy equation of the state parameter and the effective equation of the state parameter show practically the same evolution for each type of matter content considered. We note no significant oscillating behavior at the Hubble parameter and the effective equation of the state parameter level, whereas for the dark energy equation of state parameter, it can be seen that, in mimetic $f(R,T)$ gravity and in

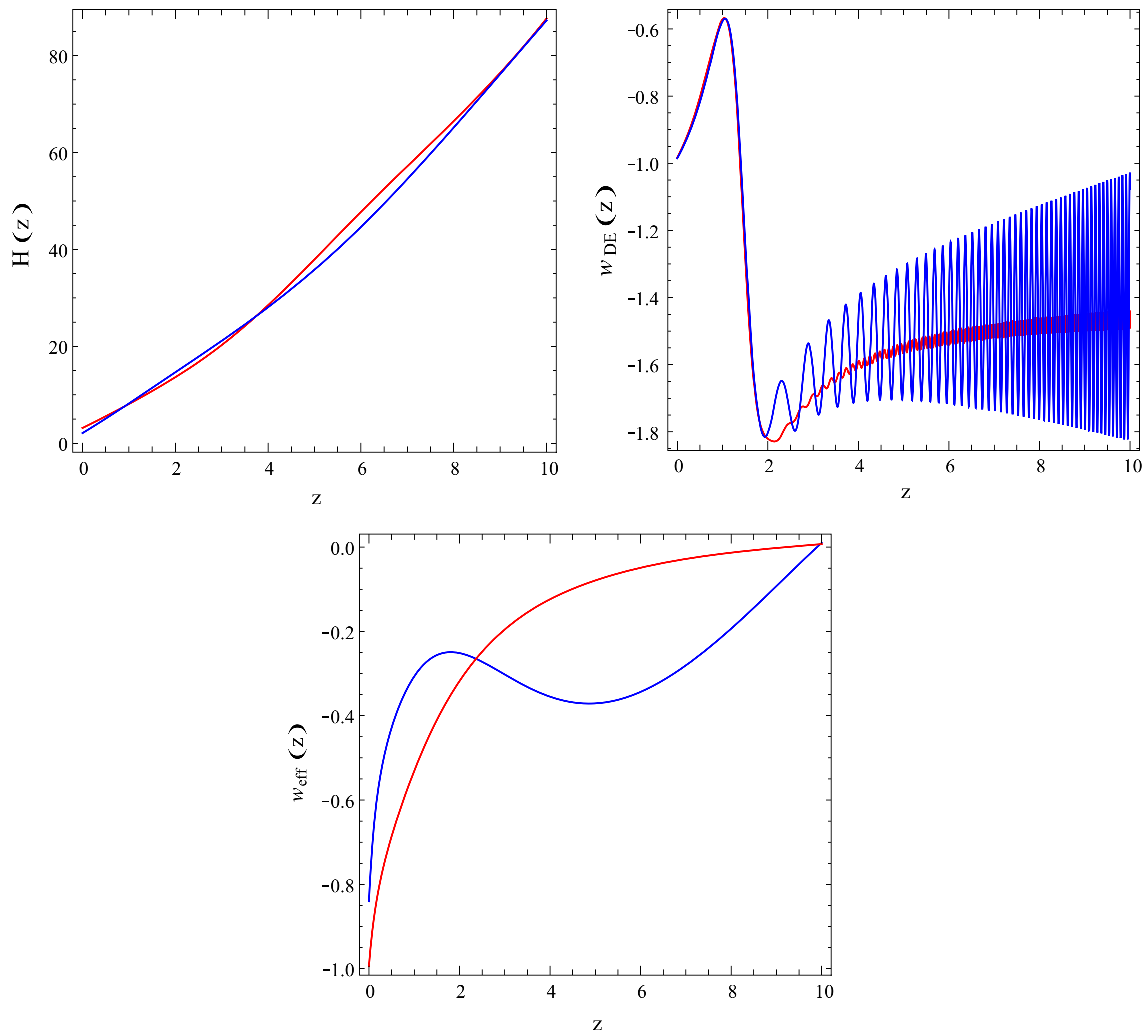

Fig. 2 Comparison of the Hubble parameter $H(z)$ versus z, of the dark energy equation of state parameter w_{DE} versus z and of the effective equation of state parameter w_{eff} versus z. The red curves correspond to the mimetic $f(R,T)$ in Universe filled with collisional matter, while the blue curves correspond to the mimetic $f(R,T)$ in the presence of the non-collisional matter (dust) for the model $\mathcal{Q}(z)=\frac{2z+5}{z+100}$ [44]. The graphs are plotted for $\Pi_0=2.58423$, $\rho_{m0}=3.1\times10^{-4}$, $\beta=\frac{1+3w}{2(1+w)}$, $\chi=1.5$, $b=10^{-5}$ and $\bar{m}^2=1.03\times10^{-4}$

the presence of the non-collisional matter, oscillations occur when the redshift increases and goes toward $z=3.5$. For the second figure, we remark that the Hubble parameter and the dark energy equation state parameter show very similar behavior for each type of matter content. Moreover, at the dark energy equation state level the oscillating behavior is milder in comparison to the mimetic $f(R,T)$ gravity in a Universe filled with non-collisional matter and these oscillations occur until approximately $z\simeq2$. Regarding the effective equation of the state parameter as shown in the bottom plot of Fig. 2, the two curves are very similar in behavior for low values of the redshift but move toward $w_{\text{eff}}=0$ when the redshift z increases.

5 Conclusion

In this paper, we examined the late-time cosmological evolution in the context of mimetic $f(R,T)$ gravity with Lagrange multiplier and mimetic potential. Assuming that the matter contents of the Universe are collisional matter and relativistic matter (radiation), we demonstrated how $f(R,T)$ gravity, the mimetic potential and the Lagrange multiplier affect the late-time cosmological evolution. By numerically solving the main equation that describes the cosmological evolution of the dark energy we focused our attention on the cosmological evolution of the Hubble parameter, the dark energy equation parameter and also the effective equation of state parameter in mimetic $f(R, T)$ gravity in a Universe filled with collisional matter. We compared this with the case that the Universe essentially contains non-collisional matter (dust). The results obtained showed that in both cases and for two $Q(z)$ models the curves correspond to the mimetic $f(R, T)$ gravity in the presence with the collisional matter are in better agreement with the observational data than those obtained in mimetic $f(R, T)$ gravity in the presence of the non-collisional matter (dust) and can be used to reduce the amplitude of the dark energy oscillations. Generally the late-time behavior appears and the contribution of the collisional matter in mimetic $f(R, T)$ gravity may be considered to be an alternative method to damp the dark energy oscillations.

Acknowledgements The authors thank Prof. S. D. Odintsov and Sunny Vagnozzi for useful comments and suggestions.

Funded by SCOAP^3.

References

1. A.G. Riess et al., Astron. J. **116**, 1009 (1998)
2. S. Perlmutter et al., Astrophys. J. **517**, 565 (1999)
3. P. de Bernardis et al., Nature **404**, 955 (2000)
4. E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)
5. S. Nojiri, S.D. Odintsov, Int. J. Geom. Methods Mod. Phys. **4**, 115 (2007)
6. T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451–497 (2010). arXiv:0805.1726 [gr-qc]
7. D.A. Felice, Rev. Rel. **13**, 3 (2010)
8. P.G. Bergmann, Int. J. Theor. Phys. **1**, 2536 (1968)
9. S. Tsujikawa, Lect. Notes Phys. **800**, 99 (2010)
10. S. Nojiri, S.D. Odintsov, Phys. Rep. **505**, 59–144 (2011). arXiv:1011.0544
11. S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rep. **692**, 1–104 (2017). arXiv:1705.11098
12. M.C.B. Abdalla, S. Nojiri, Sergei D. Odintsov, Class. Quantum Gravity **22**, L35 (2005)
13. A. De Felice, S. Tsujikawa, Living Rev. Relativ. **13**, 3 (2010)
14. Thomas P. Sotiriou, Class. Quantum Gravity **26**, 152001 (2009)
15. S. Nojiri, S.D. Odintsov, Phys. Rev. D **70**, 103522 (2004)
16. E. Elizalde, E.O. Pozdeeva, S.Y. Vernov, Phys. Rev. D **85**, 044002 (2012)
17. G. Cognola, E. Elizalde, S. Zerbini, Phys. Rev. D **87**(4), 044027 (2013)
18. K. Bamba, S.D. Odintsov, D. Saez-Gomez, Phys. Rev. D **88**, 084042 (2013). arXiv:1308.5789
19. S. Nojiri, S.D. Odintsov, Int. J. Geom. Methods Mod. Phys. **11**, 1460006 (2014). arXiv:1306.4426
20. L. Sebastiani, D. Momeni, R. Myrzakulov, S.D. Odintsov, Phys. Rev. D **88**(10), 104022 (2013). arXiv:1305.4231
21. K. Bamba, S. Nojiri, S.D. Odintsov, Phys. Lett. B **725**, 368–371 (2013). arXiv:1304.6191
22. Y.F. Cai, E.N. Saridakis, M.R. Setare, J.Q. Xia, Phys. Rep. **493**, 1 (2010). arXiv:0909.2776
23. G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, L. Sebastiani, S. Zerbini, Phys. Rev. D **77**, 046009 (2008)
24. E. Elizalde, S. Nojiri, S.D. Odintsov, L. Sebastiani, S. Zerbini, Phys. Rev. D **83**, 086006 (2011)
25. T. Harko, F.S.N. Lobo, S. Nojiri, S.D. Odintsov, Phys. Rev. D **84**, 024020 (2011). arXiv:1104.2669 [gr-qc]
26. M.J.S. Houndjo, Int. J. Mod. Phys. D **21**, 1250003 (2012)
27. M.J.S. Houndjo, O.F. Piattella, Int. J. Mod. Phys. D **21**, 1250024 (2012)
28. D. Momeni, M. Jamil, R. Myrzakulov, Eur. Phys. J. C **72**, 1999 (2012)
29. F.G. Alvarenga, M.J.S. Houndjo, A.V. Monwanou, J.B. Chabi-Orou, J. Mod. Phys. **4**, 130 (2013)
30. M. Sharif, M. Zubair, J. Cosmol. Astropart. Phys. **03**, 028 (2012)
31. M. Sharif, M. Zubair, J. Cosmol. Astropart. Phys. **05**, E01 (2013)
32. M. Jamil, D. Momeni, R. Myrzakulov, Chin. Phys. Lett. **29**, 109801 (2012)
33. M.J.S. Houndjo, C.E.M. Batista, J.P. Campos, O.F. Piattella, Can. J. Phys. **91**, 548 (2013)
34. E.H. Baffou, A.V. Kpadonou, M.E. Rodrigues, M.J.S. Houndjo, J. Tossa, Space Sci. **355**, 2197 (2014)
35. E.H. Baffou, M.J.S. Houndjo, M.E. Rodrigues, A.V. Kpadonou, J. Tossa, Chin. J. Phys. **55**, 467–477 (2017). arXiv:1509.06997 [gr-qc]
36. E.H. Baffou, M.J.S. Houndjo, I.G. Salako, Int. J. Geom. Meth. Mod. Phys. **14**(4), 1750051 (2017). doi:10.1142/S0219887817500517, arXiv:1606.05265 [gr-qc]
37. M.E.S. Alves, P.H.R.S. Moraes, J.C.N. de Araujo, M. Malheiro, Phys. Rev. D **94**(2), 024032 (2016). arXiv:1604.03874
38. C.P. Singh, P. Kumar, Int. J. Theor. Phys. **56**(10), 3297–3310 (2017). arXiv:1609.01477 [gr-qc]
39. P.H.R.S. Moraes, Eur. Phys. J. C **75**(4), 168 (2015). arXiv:1502.02593 [gr-qc]
40. H. Shabani, M. Farhoudi, Phys. Rev. D **90**(4), 044031 (2014). arXiv:1407.6187v2
41. A.H. Chamseddine, V. Mukhanov, JHEP **1311**, 135 (2013). arXiv:1308.5410 [astro-ph.CO]
42. S. Nojiri, S.D. Odintsov, Mod. Phys. Lett. A **29**(40), 1450211 (2014). arXiv:1408.3561
43. V.K. Oikonomou, Mod. Phys. Lett. A **31**(33), 1650191 (2016). arXiv:1609.03156v1 [gr-qc]
44. S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D **94**(4), 044012 (2016). arXiv:1608.00165v1 [gr-qc]
45. L. Sebastiani, S. Vagnozzi, R. Myrzakulov, Adv. High Energy Phys. **2017**, 3156915 (2017). arXiv:1612.08661v1 [gr-qc]
46. S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Class. Quantum Gravity **33**(12), 125017 (2016). arXiv:1601.07057
47. G. Leona, E.N. Saridakisb, JCAP **1504**(04), 031 (2015). arXiv:1501.00488v3 [gr-qc]
48. A.V. Astashenok, S.D. Odintsov, V.K. Oikonomou, Class Quantum Gravity **32**(18), 185007 (2015). arXiv:1504.04861v2 [gr-qc]
49. M. Shiravand, Z. Haghani, S. Shahidi, arXiv:1507.07726v2 [gr-qc]
50. S.D. Odintsov, V.K. Oikonomou, Ann. Phys. **363**, 503–514 (2015). arXiv:1508.07488v2 [gr-qc]
51. R. Myrzakulov, L. Sebastiani, S. Vagnozzi, Eur. Phys. J. C **75**, 444 (2015). arXiv:1504.07984v3 [gr-qc]
52. S. Nojiri, S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D **94**(10), 104050 (2016). arXiv:1608.07806v2 [gr-qc]
53. E.H. Baffou, M.J.S. Houndjo, M.E. Rodrigues, A.V. Kpadonou, J. Tossa, Phys. Rev. D **92**. doi:10.1103/PhysRevD.92.084043
54. A. Golovnev, Phys. Lett. B **728**, 39 (2014). arXiv:1310.2790
55. A.O. Barvinsky, JCAP **1401**(01), 014 (2014). arXiv:1311.3111
56. A.H. Chamseddine, V. Mukhanov, A. Vikman, JCAP **1406**, 017 (2014). arXiv:1403.3961

57. M. Chaichian, J. Kluson, M. Oksanen, A. Tureanu, JHEP **1412**, 102 (2014). arXiv:1404.4008
58. O. Malaeb, Phys. Rev. D **91**(10), 103526 (2015). arXiv:1404.4195
59. N. Deruelle, J. Rua, JCAP **1409**, 002 (2014). arXiv:1407.0825
60. D. Momeni, A. Altaibayeva, R. Myrzakulov, Int. J. Geom. Meth. Mod. Phys. **11**, 1450091 (2014). arXiv:1407.5662
61. S.D. Odintsov, V.K. Oikonomou, Phys. Rev. D **93**(2), 023517 (2016). arXiv:1511.04559
62. K. Kleidis, N.K. Spyrou, Astron. Astrophys. **529**, A26 (2011)
63. K. Freese, M. Lewis, Phys. Lett. B **540**, 1 (2002)
64. V.K. Oikonomou, N. Karagiannakis, Class. Quantum Gravity **32**, 085001 (2015)