On the Solution of NP-hard Linear Complementarity Problems

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Abstract
In this paper two enumerative algorithms for the Linear Complementarity Problems (LCP) are discussed. These procedures exploit the equivalence of the LCP into a nonconvex quadratic and a bilinear programs. It is shown that these algorithms are efficient for processing NP-hard LCPs associated with reformulations of the Knapsack problem and should be recommended to solve difficult LCPs.

Key Words: mathematical programming, complementarity, global optimization, enumerative algorithms.

AMS subject classification: 90C33, 65K10.

1 Introduction

The Linear Complementarity Problem (LCP) consists of finding vectors \( z \in \mathbb{R}^n \) and \( w \in \mathbb{R}^n \) such that

\[
\begin{align*}
    w &= q + Mz \\
    z &\geq 0, \quad w \geq 0 \\
    z^T w &= 0
\end{align*}
\]

for a given matrix \( M \in \mathbb{R}^{n \times n} \) and a vector \( q \in \mathbb{R}^n \). This problem has originally appeared in the sixties for the solution of bimatrix games and convex quadratic programs. Since then, it has received an increasing interest,

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