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THE IMPLEMENTATION OF MAXIMUM LIKELIHOOD ESTIMATION IN SPACE LAUNCH SYSTEM VEHICLE DESIGN

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The Space Launch System uses a Maximum Likelihood Estimation process in conjunction with Design of Experiments to develop statistically representative vehicles for the Block I configuration. These vehicles are then used to estimate maximum load conditions for simulating stressing cases in other simulations. This paper discusses the modeling process and how SLS captures manufacturing uncertainty in the launch vehicle design. It also provides an overview of the differences between Block I statistical representations. This paper also discusses proper grid choice as well as which uncertainties drive the vehicle design.

INTRODUCTION

Space Launch System (SLS) three-degree-of-freedom (3-DOF) Reference trajectories are critical to establishing vehicle performance metrics and event timings, but there are key limitations to these trajectories when accounting for manufacturing uncertainties. Uncertainties like these are most significant in the early parts of the design cycle, when the vehicle's dry mass and engine performance characteristics are still being designed and tested. If not properly accounted for in the design process, these uncertainties have the potential to lead to a loss of performance.

Trajectory Dispersed (TD) vehicles provide a means to incorporate uncertainties into trajectory simulations. These statistically representative vehicles are produced by combining response surfaces developed using Design of Experiments (DOE) grids in conjunction with the Maximum Likelihood Estimation (MLE) process. The trajectories stress key response parameters such as payload performance, liftoff-thrust-to-weight (LOTW), or dynamic pressure. These are then used in analyses critical to SLS mission design like separation events, propellant calculations, and vehicle loads.

This paper describes the background of the TD Vehicle process as applied to the SLS program. This includes description and results of the TD Vehicle process, but also why this process is utilized over a more common Monte Carlo analysis. An outline of the DOE and MLE methodology is also discussed.

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BACKGROUND

Space Launch System is NASA's next generation of launch vehicles that will carry crews and science safely and reliably to space. The Block 1 vehicle configuration is the first design iteration and is the focus of this paper.

SLS Vehicle

Similar to the Space Shuttle, the SLS vehicle is comprised of a common core liquid stage with two solid rocket boosters (SRB). The common core stage implements four RS-25 LOX/LH2 engines derived from heritage Space Shuttle Main Engine designs, and the two 5-segment PBAN solid motors developed during the Constellation Program. SLS Block 1 utilizes a modified Delta-IV upper stage called an Interim Cryogenic Propulsion Stage (ICPS). This accommodates either a crew module and the launch abort system (LAS), or a payload module for early cis-lunar and deep space applications. This vehicle also provides accommodation for secondary payloads. An expanded view identifying the vehicle components is shown in Figure 1.

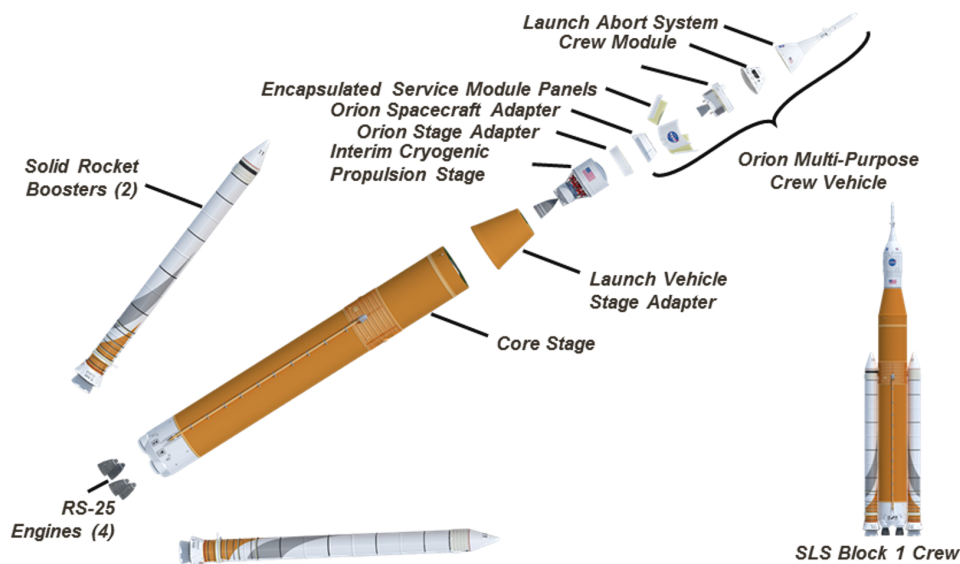


Figure 1. Block 1 Configuration

Mission Design and Analysis

Current SLS mission design and analysis processes stem from work performed during the Constellation program. During Constellation, a need to rapidly assess mission design and flight characteristics led to a multiphase analysis process that alleviated the need for a large number of runs typically used in Monte Carlo analysis by relying on statistical processes to reduce computational requirements and improve turn around time without sacrificing accuracy. This process consists of constructing a reference trajectory that functions as a baseline, from which perturbed manufacturing variation vehicles known, as TD vehicles, are developed. 3-DOF analysis provides the initial comparison and analysis points for follow-on studies such as 6-DOF Monte Carlo analyses, Core Stage propellant inventory analysis, and mission trajectory optimization. 3-DOF modeling for SLS is performed using POST.¹

TRAJECTORY DISPERSED VEHICLE DESIGN

Design and response parameters are key in defining the TD vehicle configurations. Unlike day of launch uncertainties, design parameters are vehicle characteristics, such as dry mass and engine performance, which have a level of uncertainty since they are still being designed and tested. The uncertainties are known variations that can be statistically examined. Response parameters are critical outputs of the trajectory that are affected by manufacturing uncertainty. Combinations of design and response parameters are used to create three vehicles, each representing a different stressing condition.

Design Parameters

The identified design parameters for the SLS Block 1 vehicle are listed below in Table 1. Booster characteristics tend to be the most significant influence on the design parameters listed.

Table 1. Design Parameters

Booster Burn Rate	RS-25 Specific Impulse
Booster Propellant Mass	Core Stage Dry Mass
Booster Stage Mass	LAS Mass
RS-25 Thrust	

The RS-25 thrust and specific impulse uncertainties are incorporated differently. Engine dispersion data are created on a per engine basis. For a given dispersed trajectory (or dispersed vehicle), each engine is independently dispersed and the standard deviations of the individual engine thrust dispersions are equal. Therefore, the standard deviation of the total vehicle thrust T_{veh} , is given through a root-sum-square (RSS) of the n per engine dispersions.²

$$\sigma_{T_{veh}} = \sqrt{\sigma_{\Delta T_1}^2 + \sigma_{\Delta T_2}^2 + \dots + \sigma_{\Delta T_n}^2} = \sqrt{n}\sigma_{\Delta T_{eng}} \quad (1)$$

The standard deviation of the engine cluster specific impulse is derived using the standard deviation of each engine. Therefore, from the relation between thrust and mass flow, the vehicle specific impulse can be determined.

$$I_{sp_{veh}} = \frac{T_{veh}}{g_0 (\dot{m}_1 + \dot{m}_2 + \dots + \dot{m}_n)} \quad (2)$$

A first-order variation is derived next to make a simple linear relationship that calculates a final uncertainty result. This first-order variation in specific impulse with respect to variations in mass flow rates is shown in the below:

$$\delta I_{sp_{veh}} = \frac{T_{veh} (-\delta \dot{m}_1 - \delta \dot{m}_2 - \dots - \delta \dot{m}_n)}{g_0 \dot{m}_{veh}^2} \quad (3)$$

With a variational analysis, there is an assumption that the dispersions are small. For the purposes of this derivation, this is expected to be a very good assumption given the relatively small variations in liquid engine performance. However, the relationship above needs to be in terms of specific impulse variations, not mass rate variations, since specific impulse variations are the specified quantities.

The resulting relation is shown in Equation 4, and reflects how dispersion in specific impulse alone changes the mass flow rate of a given engine.

$$\delta \dot{m} = \frac{-\dot{m} \delta I_{sp}}{I_{sp}} \quad (4)$$

Now, use the relation in Equation 2 and Equation 4 to create the vehicle specific impulse without mass flow rate terms.

$$\delta I_{sp_{veh}} = \frac{\delta I_{sp_1} + \delta I_{sp_2} + \dots + \delta I_{sp_n}}{n} \quad (5)$$

As with the vehicle thrust derivation, a root-sum-square is used to produce the standard deviation of vehicle specific impulse in terms of the per-engine specific impulse standard deviation and the number of engines.

$$\sigma_{I_{sp_{veh}}} = \frac{\sigma_{I_{sp_{eng}}}}{\sqrt{n}} \quad (6)$$

Response Parameters and Vehicle Configurations

The identified vehicle response parameters are listed in Table 2. Each of the three TD vehicles considers a different combination of these parameters at a 10th or 90th percentile level expected value, corresponding to a $\pm 1.28\sigma$ level.

Monthly environmental effects like temperature and wind are also factors in creating TD vehicles. Monthly temperatures have a direct influence on the SRB Burn Rate, which affects the trajectory by changing the overall thrust during boost phase flight. Each TD vehicle is simulated in the appropriate month that would also stress the chosen responses.

Table 2. TD Vehicle Responses and Applications

Month	Response Parameter	Applications
Heavy Slow		
February	Thrust-to-Weight (10th Percentile) Payload (10th Percentile)	Payload Performance Flight Performance Reserve Calculation Lift-Off Clearances
Light Fast		
July	Thrust-to-Weight (90th Percentile) Max Dynamic Pressure (90th Percentile) Max Heat Rate (90th Percentile) Max 1st Stage Acceleration (90th Percentile)	Vehicle Loads
Hybrid		
July	Max Dynamic Pressure (90th Percentile) Max Heat Rate (90th Percentile) Max 1st Stage Acceleration (90th Percentile) Payload (10th Percentile)	Payload Performance Inlet Pressure Clearances

Three TD vehicle configurations are called the Heavy/Slow, Light/Fast, and Hybrid vehicles. The Heavy/Slow configuration reflects statistically derived heavier than normal element masses, and lower performing propulsive elements to produce a given required injected weight at a low thrust to weight ratio at liftoff (LOTW). The Light/Fast is the opposite of that as it reflects lighter than normal element masses, lower drag, and higher performing propulsive elements. Extremes in these focus parameters are provided for analysis of vehicle loads such as Q-alpha total, heating, and acceleration. Finally, the Hybrid vehicle produces high loads, but low performance, with high performing SRBs and a low performing Core Stage. This configuration produces the same performance level as Heavy/Slow but also produces higher loads like the Light/Fast configuration. The higher loads on the Hybrid configuration are a direct result of the low performance of the Core Stage as the vehicle cannot throttle to reduce loads without reducing performance. The vehicle and the corresponding grouped response parameters are outlined in Table 2.

METHODOLOGY

The SLS launch vehicle performance is a complex function of many uncertain design parameters and exploration of the performance can be computationally costly. The Constellation program developed a process to explore a launch vehicle design space without the significant computational burden that comes with the standard Monte Carlo process. If these design parameters are known, the system becomes deterministic and responses easier to analyze. In application, this never occurs and the effects of unknown factors need to be accounted for in the design process. SLS follows a Design of Experiments process similar to the one conceived by Dr. George Box. His success with chemical process improvement transferred over to many aspects of engineering, and became the start of the modern Design of Experiments method.³

The process works by using a defined set of experiments to examine the overall response to a design parameter set. Next, a response surface of appropriate order is fitted to the response parameters using a least squares method. For this analysis, the response surfaces act as an outcome prediction to the design parameters.

Response surfaces can point to process improvements by showing the minimum or maximum within the design parameter space. In this case, there are several response surfaces for a given TD vehicle, one for each response parameter. The Maximum Likelihood Estimation process is used in conjunction with the response surfaces to produce a desired TD vehicle. The MLE process is a constrained optimization process that targets TD vehicle requirements, while maintaining a high probability of a design parameter offset. This provides a vehicle that has a reasonable expectation of occurring and does not include a worst on worst case stacking. This method does not provide necessary conservatism, but provides vehicles that stress SLS performance in different ways, such as Payload and LOTW characteristics.

Process Overview

The design parameters are split into two main groups; manufacturing uncertainties and flight day uncertainties. As stated by Beard and Hanson,⁴ manufacturing uncertainties decrease as the design matures, while flight day uncertainties do not change during the design process. The creation of TD vehicles follows a five step iterative process, shown in Figure 2

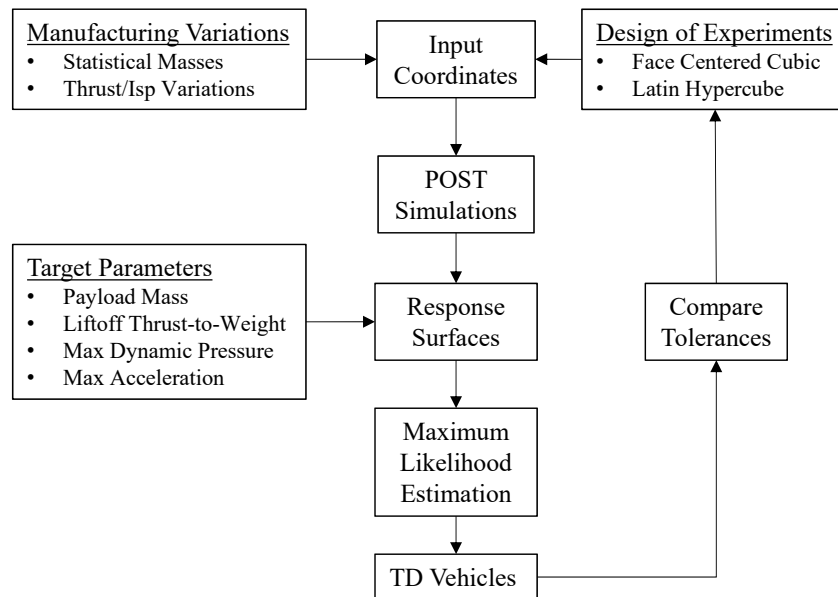


Figure 2. Flowchart of the Maximum Likelihood Estimation Process

- 1 Qualify/Quantify the manufacturing uncertainties.
- 2 Choose a DOE method to produce a set of test cases from the design parameters.
- 3 Fit response surfaces to the outcomes of the test cases
- 4 Use the response surfaces and the MLE optimization process to develop targets for the desired response offsets.
- 5 Run a final simulation that compares its outcome to the response surface, to ensure the system closes.

Our response surface design is second order; this allows us to capture significant interactions between two design parameters and their effect on the response parameter.

Step 1: Design Inputs

The first step is to collect and organize the design parameter uncertainties. SLS incorporates the design parameters presented in Table 1, which represent the influencing factors that impact the overall performance and trajectory. These parameters represent the 3σ deviations from the nominal values and represent the widths of the underlying distributions. The assumed probability density functions are normal distributions for these design parameters. Normality is not a requirement and other distribution types can be used in this process.

Step 2: DOE Grid Generation and Simulations

Step two is the design and construction of the DOE grid used. The DOE grid is used in the setup of the simulations to list the design parameter offsets for each simulation. Our process was based on a two factor DoE grid using a Circumscribed Central Composite (CCC) method outlined in Box.³

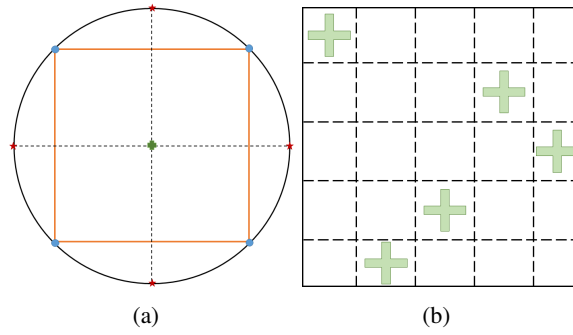


Figure 3. Circumscribed Central Composite design (left) and Latin Hypercube (right).

Notional representation of a CCC is shown in Figure 3. The purpose of the two factor CCC is that the central point in the design parameter space, as well as the corners and faces are examined. The corners of the CCC represent the interactions of the design parameters, while the faces represent the influence of a single design parameter. A two factor grid keeps the number of experiments to a reasonable level, however it does limit the amount of overall interaction between parameters. During the analysis, a limitation to the CCC was found; it does not sample the interior of the design parameter space. It is important to ensure the proper analysis within the design parameter search space. This led to a change in DOE grid to a Latin Hyper Cube (LHC). The LHC is a DOE grid more akin to a Monte Carlo method; it sweeps through the design parameter space and samples it while maintaining a reasonable parameter spacing. Figure 3 right panel, the LHC samples points such that no point is repeated and ensures that design parameters are evenly spaced. The LHC does not look at the minimums and maximums of the search space at each corner, in this way the LHC has better interior and interaction properties, but lacks the effects of an extreme in a parameter.⁵ The grid is then used to setup POST runs. POST optimizes the trajectory based on the design parameters calculated in the DOE grid. The outcome of the simulations are then used in the third step.

Step 3: Response Surface Development

The third step is the production of the response surfaces of order n for each of the responses in a specific TD vehicle. The response surface is a least squares fit of the responses to the desired order curve. A least squares fit is the well known minimization of the error between the fit and the measured data.³ SLS used second order curve fits with interaction terms to ensure there is a proper characterization of the response parameter to design parameter inputs and two parameter interactions. Equation 7 is the generalized second order curve fit for a response that includes interaction terms $x_i x_j$, to get the coefficients β .

$$R(x_i) = \sum_{i=0}^N \beta_i x_i + \sum_{i=0}^N \sum_{j=0}^N \beta_{ij} x_i x_j + \beta_0 \quad (7)$$

The errors given by Equation 8 are minimized using the least squares process

$$e = \sum_{i=1}^N (y_i - R(x_i))^2 \quad (8)$$

Standard methods for testing the goodness of fit for the surfaces were used, such as the coefficient of determination R^2 value. An Analysis of Variance is also performed to test significance and goodness of fit. These two methods ensured a reasonable and consistent fit for the response parameters. The intercept value of the regression is also a test; since these are differences from the reference value, the intercept ideally will be zero. Often, this is not the case and judgment must be used to determine if the intercept offset is acceptable based on the response parameter.

Step 4: Maximum Likelihood Estimation

The purpose of step four is to achieve the design parameter offsets that reach the desired TD vehicle. To accomplish this task, an optimization process is performed. The constraints are the desired percentile offsets for a given TD vehicle type. In the case of the Heavy/Slow vehicle we want a 10th percentile vehicle. The choice of 10th percentile is program heritage reaching back to the Constellation program. The choice is based on the tail of the distribution, because the Heavy/Slow vehicle is intended to stress a lower bound on Payload and LOTW. Heavy/Slow vehicles target the lower 10% of the payload and lift off distribution. Since this is a one tail target, the corresponding 10th percentile is equivalent to a -1.28σ standard deviation value. If a TD vehicle desires a 90th percentile bound, then this corresponds to the 1.28σ value.

The Payload and LOTW targets are the constraints used in the optimization. The objective function for the optimization process is to maximize the sum of the natural log of the probabilities for the design parameters shown as Equation 9 and subject to the constraints in Equation 10.

$$J(p) = \max \left(\sum_i^N \ln P_i \right) \quad (9)$$

$$\begin{aligned} R_{PAYLOAD} &= -1.28\sigma_{PAYLOAD} \\ R_{LOTW} &= -1.28\sigma_{LOTW} \end{aligned} \quad (10)$$

The design parameters are assumed to be normally distributed. Maximizing the probability of occurrence for a design parameter increases its chance of occurrence. By doing this to all the design parameters, the odds that a given TD vehicle occur is higher. This is due to the MLE process maximizing the probabilities within the objective function. This provides a good measure of an extreme vehicle without using worst case stacking. The constraints ensure that we are achieving a vehicle that is not close to the mean of the response parameters.

Step 5: Final Simulation

The final step provides a means of determining if the Payload and LOTW targets were met by the MLE process. A final trajectory is simulated using the design parameters calculated in the previous step. The final response from the simulation is compared to the response surface evaluated at the design parameter values determined in step four. Comparing the two values, a small amount of difference is expected due to the simulation being an optimization process. The desired difference is kept to about 0.5% of the response parameter variation. The next section will provide detailed analysis of the Heavy/Slow vehicle.

DISCUSSION

This section provides a statistical comparison between the DOE grid used in deriving TD vehicles and a sample Monte Carlo set to show the data sets are statistically similar. The DOE grid used for this comparison is based on a CCC design which is a similar design choice used when initially developing the TD vehicles for SLS applications. The seven design parameters are used to define a seven dimensional grid resulting in 142 coordinates including the original reference trajectory at the origin. The hypercube corners correspond to scaled coordinates at $\pm 0.89\sigma$ values in each dimension, and each single-axis value set to $\pm 3\sigma$. The sample Monte Carlo consists of 10,001 runs. Each of the seven design parameters are normally sampled around the origin with a standard deviation of 1σ with the exception of the LAS mass which is uniformly sampled between $\pm 3\sigma$. The Monte Carlo also includes the reference trajectory at the origin for comparisons with the DOE grid. Both the response parameters and the times used in this comparison are scaled by the insertion values of the reference trajectory. Therefore, the reference trajectory progresses from 0% to 100%, but subtle differences in the all the other trajectories will cause the trajectories to end close to but not necessarily at 100%.

A Statistical Comparison of DOE Grid and Monte Carlo Data

The DOE grid and the Monte Carlo set are compared using statistical parameters such as mean and standard deviation in order to describe their relative distributions. Each dataset is also tested for normality in an effort to better understand the underlying distribution type. Determining the distribution of the response parameters is important given that the design parameters are either normally distributed or symmetrically spaced, and therefore, the response parameters are be expected to be normally distributed as well.

Table 3. Comparison of DOE and Monte Carlo Scaled Statistics

DOE CCC Vehicles:			
Scaled Response Parameters	Sample Mean	Standard Deviation	P Normality
Payload	-2.38e-2	1.26	0.31
LOTW	5.69e-3	0.58	0.00
Monte Carlo Vehicles:			
Scaled Response Parameters	Sample Mean	Standard Deviation	P Normality
Payload	-1.50e-3	1.40	0.57
LOTW	1.88e-3	0.63	0.01

Two types of tests are used to describe the normality of the datasets in conjunction with visual inspection using Quartile/Quartile (QQplot) plots. The DOE grid was examined using a Shapiro-Wilk test due to the dataset's small sample size.⁶ The Monte Carlo set was examined using the D'Agostino and Pearson's test,^{7,8} which is more applicable to larger datasets and also combines skew and kurtosis to test normality. Both these methods have similar null hypotheses that states that a particular sample from a dataset comes from a normal distribution. Probabilities above an α significance level reject the null hypothesis and show the data are normally distributed. Both the Payload and the LOTW for the Heavy/Slow vehicle were compared to normal distributions. Payload was shown to be normally distributed for both the DOE grid and the Monte Carlo set, whereas the LOTW was not.

The Payload Response Parameter: To investigate the distributions further, the Payload was also plotted using a QQplot to visually compare the normality of both the DOE and the Monte Carlo data, the results of which are presented as Figure 4. Both QQplots appear to be normally distributed as expected. Similarly, a kernel density estimate plot (kdeplot) was created to compare the overall distribution shape for each dataset. Figure 5 shows the comparison between the DOE/CCC grid distribution for Payload with that of the Monte Carlo set. Both compare well to each other and also appear normally distributed.

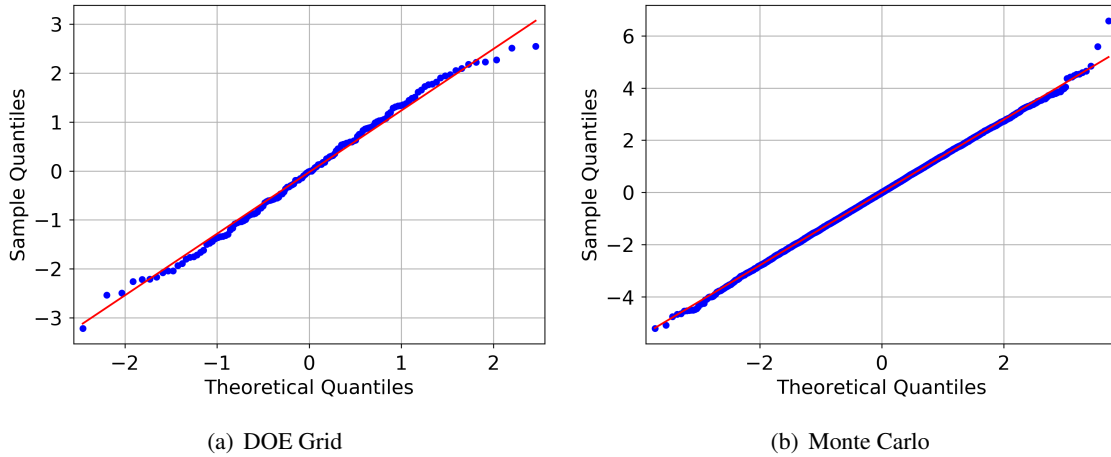


Figure 4. Scaled Payload QQplot Comparison for Normality

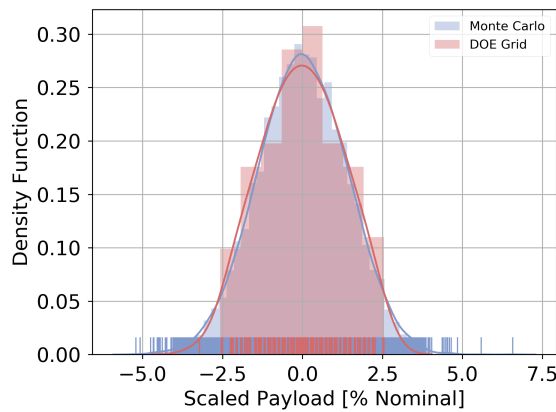


Figure 5. DOE vs Monte Carlo Scaled Payload Distributions

A regression plot of each design parameter vs Payload was used to better understand their relative sensitivity. Regression plots are created by fitting the data represented as dots using a first order least-squares regression which is represented by a line. The translucent bands around the line represent a 95% confidence interval for that regression. Core thrust and SRB Burn Rate influence

the vehicle payload the most and are shown in Figure 6. Vehicle payload is largely driven by RS-25 Thrust and to a lesser extent SRB Burn Rate, with the regression line slope illustrating the relative impact. The dominance of RS-25 Thrust is due to the overall total impulse provided over the duration of the mission and not the magnitude of thrust. Thus, if there is a desire to increase payload, investigations should first start with core thrust and SRB thrust second.

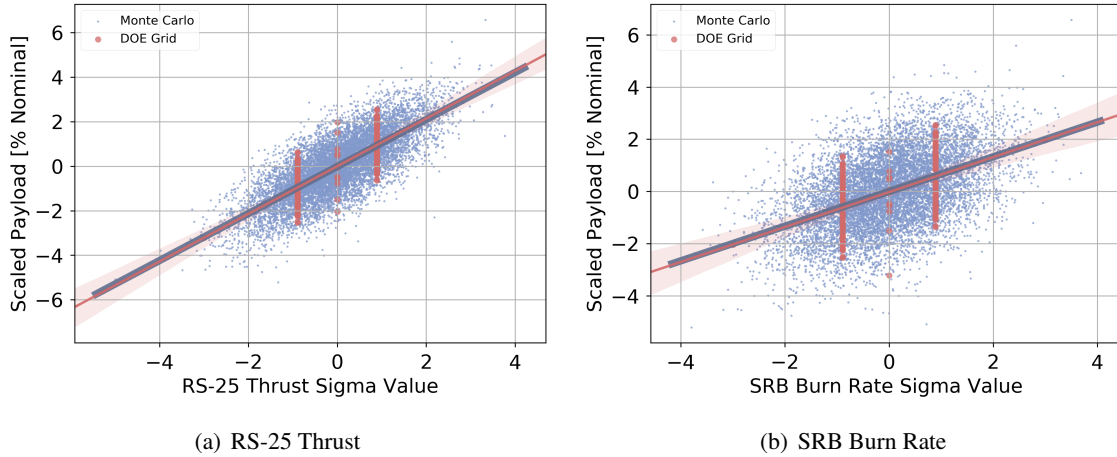


Figure 6. Comparison of Monte Carlo and DOE Grid Regression Plots

The Liftoff Thrust to Weight Response Parameter: Additional analysis was performed on the LOTW, as well, to gain insight regarding the behavior of the response parameter. Figure 7 illustrates the QQplot for the DOE/CCC and Monte Carlo datasets. Figure 7(a) shows the DOE grid exhibits non-normal characteristics, whereas the Monte Carlo data shown in Figure 7(b) appears more normal, with the exception of the data at extreme quartiles. Again, a kdeplot was created to compare the overall distributions of the datasets, and is provided below as Figure 8.

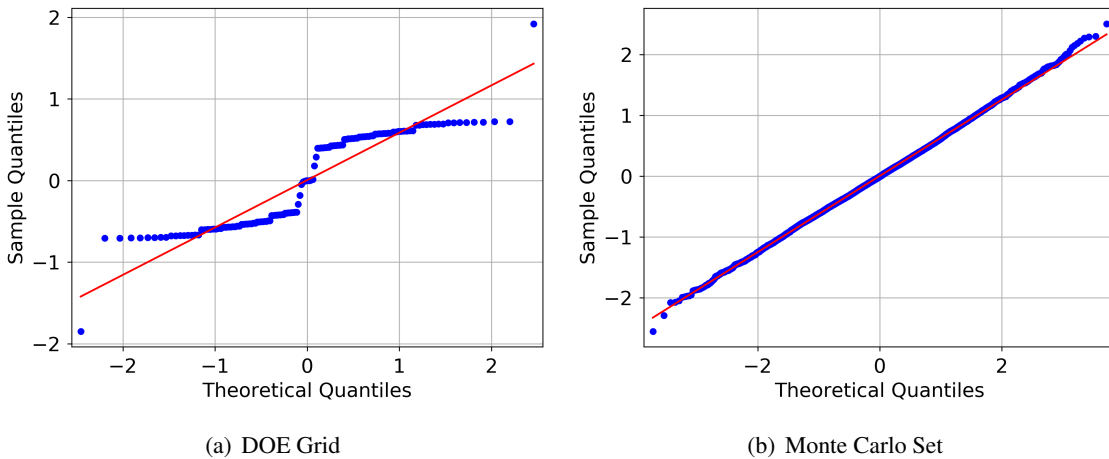


Figure 7. Scaled Payload QQplot Comparison for Normality

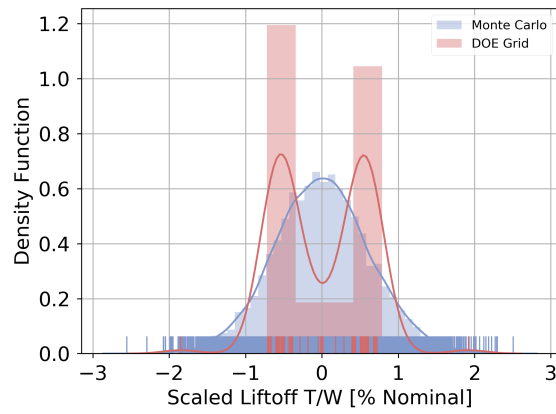


Figure 8. DOE vs Monte Carlo Scaled LiftOff T/W Distributions

Here the DOE LOTW data appear to be binomially distributed and is primarily caused by the sensitivity of LOTW with respect to SRB Burn Rate. SRB thrust is the largest component of the overall vehicle thrust at liftoff and any variance in inert masses are also small at liftoff, given that they are dwarfed by the relatively large propellant mass. Regression plots were again used to visualize the relative influence of each design parameter on LOTW. Figure 9 shows a similar regression plot as shown above in Figure 6, but the sensitivity to SRB Burn Rate is larger than the Payload response.

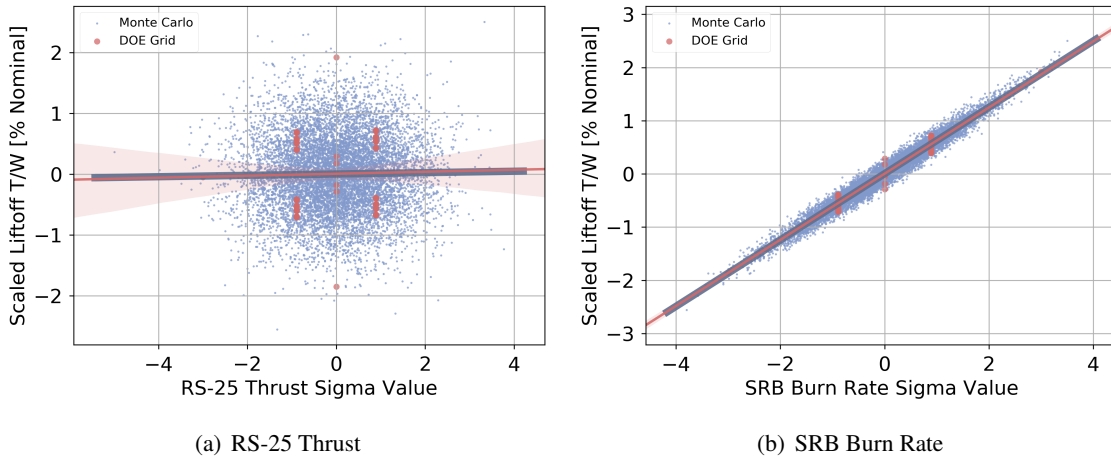


Figure 9. Comparison of Monte Carlo and TD Grid Regression Plots

Alternative grid architectures were also constructed in order to explore the binomial characteristic of the data and determine whether this was an artifact of grid choice or indicative of the underlying physics itself. CCC grids provide coordinates at design parameter extremes without information in the middle. This may exacerbate the response of sensitive parameters like SRB Burn Rate due to the min/max nature of the grid itself.

The alternative grids were created with a higher number of interior coordinates and different dis-

tributions. The first alternative used a multi-level CCC grid construction that mirrored the original CCC grid symmetry. This grid included similar points at 75%, 50%, and 25% of the original CCC levels, providing more cases interior to the original CCC, but keeping the same overall grid symmetry. A second alternative that was developed was a LHC, which provides a random distribution of coordinates internal to that of the CCC hypercube, but provides little explicit symmetry in the grid layout. QQplots for both the alternative DOE grids were created and were still not normally distributed. Kernel Density Plots of the LOTW for the alternative grids show the distributions to be more of a skewed distribution.

Therefore, it can be concluded that the binomial appearance of the DOE data is influenced by the grid coordinates themselves. Conversely, the fact that the DOE data are not normally distributed indicates that the response must be independent of the grid choice. This conclusion also compares well with the initial data provided in Table 3 that shows that the Monte Carlo set is not normally distributed either, and must be heavily influenced by the SRB Burn Rate. The fact that the data are binomially distributed does not impact the underlying MLE process, but illustrates how different design parameters can potentially impact response parameters as well as provide potential sources of errors or biases in the analysis. The influence of each design parameter on a given response parameter must be checked to insure that sufficient information is available along a given design parameter dimension for a given grid choice. Otherwise, poor response surface fits may occur, resulting in less than ideal offsets. Comparing the data at this stage of analysis helps alleviate confusion later in the MLE process, when causes of issues become less obvious.

Statistical Testing: Next, the DOE grid results were compared against the Monte Carlo results to determine if they are statistically representative of each other. The payload responses were compared using a two-sample Welch t-test. The Welch t-test is an adaptation of the Student's t-test for two samples that have unequal variances and unequal sample sizes.⁹

The LOTW was also tested in a similar fashion, but given that it is not normally distributed, another test was used in the place of the Welch t-test. The LOTW used the Mann-Whitney U test¹⁰ which is not dependent on normally distributed data. The null hypothesis for this test states that it is equally likely that a random sample from one distribution will be less than or greater than a randomly selected value from the second sample. Therefore, this test can be used in a similar fashion to the Welch t-test since the null hypotheses test the samples with a similar interpretation. The Payload response had a p-value of 0.84 using the Welch t-test, and the LOTW response had a p-value of 0.47 using the Mann-Whitney U test. Therefore, the DOE data is statistically representative of the Monte Carlo data at a 95% confidence level.

Response Surface Comparisons

Simple first-order linear response surfaces were also created for the sake of comparing the DOE grid and the Monte Carlo set. Typically, second-order linear regression is used during an SLS design cycle to create response surfaces for TD vehicles. For this analysis, two first-order surfaces were created for the sake of brevity and succinctness, since the coefficient list for second order surfaces can get large. The majority of the response parameters are known to be roughly linear, but higher order regression is needed to better capture the slight curvature of the surface for the other TD vehicles not discussed in this paper. The appropriate order of the response surface should be investigated to correctly capture the desired offset accuracy since it is dependent on the data set structure.

Table 4. Comparison of First Order Linear Regression Response Surfaces for Scaled Payload

Dependent Variable	TD Surface Coefficient	Monte Carlo Surface Coefficient	Difference
SRB Propellant Mass	0.1562	0.1552	0.64%
SRB Burn Rate	0.6685	0.6720	-0.52%
RS-25 Specific Impulse	0.1835	0.1829	0.33%
RS-25 Thrust	1.0703	1.0714	-0.10%
Core Dry Mass	-0.2556	-0.2562	-0.23%
SRB Jettison Mass	-0.4999	-0.5030	-0.62%
LAS Mass	-0.0224	-0.0231	-3.03%

Table 4 compares the differences of the design parameter coefficients between the two surfaces for Payload. There is little difference between the two. The coefficients also describe the sensitivity of each response parameter on Payload, with RS-25 Thrust being the largest, followed by SRB Burn Rate and SRB Jettison Mass. Both grids provide good R-squared values with the DOE grid having an R^2 of 1.0 while the Monte Carlo has a R^2 of 0.998, and therefore, illustrating the fairly linear nature of the surfaces for the Heavy/Slow vehicle.

RESULTS

An example Heavy/Slow vehicle was constructed using the MLE process for this paper. The vehicle was created using first order linear response surfaces and optimized via the MLE process, where the resulting offsets for the design parameters are presented in Table 5. Offsets for a second order response surface that was created during an SLS design cycle are presented as well. Both types of surfaces exhibit little difference between the coefficients, indicating that a first order fit may be sufficient to represent a Heavy/Slow vehicle.

There are slight differences in the offsets for Core Dry Mass and LAS Mass, but these are less sensitive parameters in comparison to the SRB Burn Rate, RS-25 Thrust, and SRB Jettison Mass. This is less of a concern since second order response surfaces are typically used in the design of the SLS in order to better capture the response parameters of the other TD vehicles, which would minimize any impact from these two offsets.

Table 5. Offsets That Define Heavy/Slow Vehicles

Dependent Variable	First Order Fit	Second Order Fit	Difference
SRB Propellant Mass	-0.1738	-0.1710	1.64%
SRB Burn Rate	-1.2302	-1.2236	0.54%
RS-25 Specific Impulse	-0.1019	-0.1012	0.69%
RS-25 Thrust	-0.6526	-0.6507	0.29%
Core Dry Mass	0.2214	0.0002	>100.0%
SRB Jettison Mass	0.3487	0.3728	-6.46%
LAS Mass	0.0011	0.0166	-93.37%

A comparison between the second order Heavy/Slow TD vehicle and the Reference vehicle trajectories are also compared. Figure 10(a) shows the geodetic altitude time histories for the Reference vehicle, the Heavy/Slow vehicle, the 3σ bounds for the Monte Carlo set as well as the 3σ bounds

for the DOE grid. Figure 10(b) shows a closer view of the four items. The Heavy/Slow trajectory does not deviate far from the Reference trajectory, and both lie well within the boundary of the DOE grid. The DOE grid is also within the 3σ bounds of the Monte Carlo which is expected given that the DOE is a subset of coordinates within the Monte Carlo set.

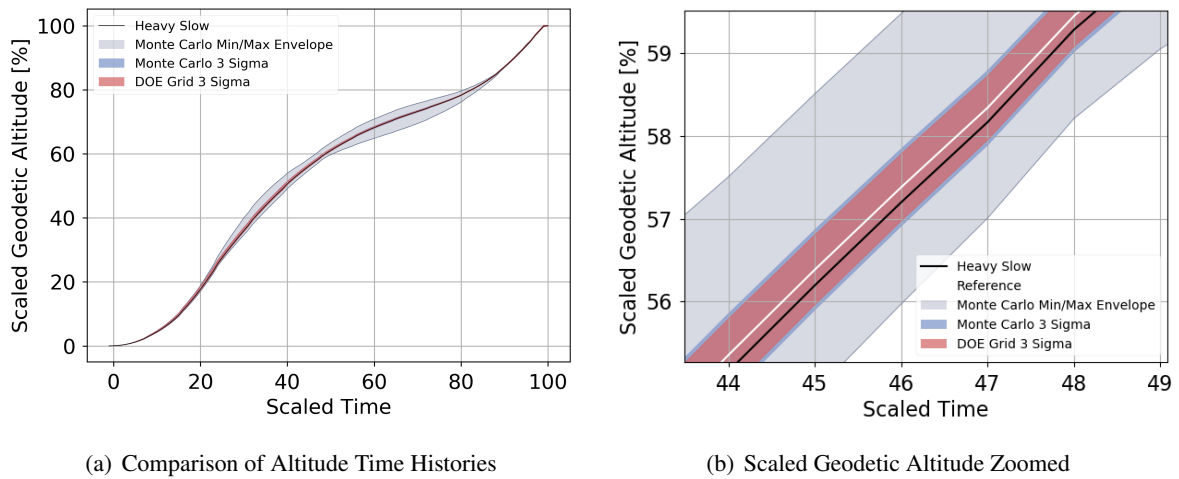


Figure 10. Comparison Reference Trajectory and Heavy/Slow Vehicle Trajectory

CONCLUSIONS

An overview of the Maximum Likelihood Estimation process was discussed as applied to the estimation of uncertainties in launch vehicle design. It has been demonstrated that the using a Design of Experiments grid in conjunction with the MLE process can serve as a statistically representative replacement for performing Monte Carlo simulations in certain situations, thus reducing simulation complexity and run time. Analysis must be performed to understand the sensitivity of each parameter in the DOE grid and how they are distributed, which impacts grid choice and response surface error. Coefficients for notional response surfaces were compared to discuss the relative impact of each design parameter on Payload. While the MLE process has been described in relation to the design process of the SLS, it can be used for any launch vehicle or any other physically uncertain system as a means to reduce computational requirements while still providing statistically accurate results.

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