1	Combined Winds and Turbulence Prediction System for
2	Automated Air-Traffic Management Applications
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1 Abstract

2 A time-lagged ensemble of Energy Dissipation Rate (EDR)-scale turbulence 3 metrics is evaluated against in situ EDR observations from commercial aircraft over the 4 contiguous United States and applied to Air-Traffic Management (ATM) route planning. 5 This method uses the Graphic Turbulence Guidance forecast methodology with three 6 modifications. First, it uses a convection-permitting scale ($\Delta x = 3$ km) Weather and 7 Research Forecast (WRF) model to capture cloud-resolving scale weather phenomena. 8 Second, turbulence metrics are computed for multiple WRF forecasts that are combined 9 at the same forecast valid time, resulting in a time-lagged ensemble of multiple 10 turbulence metrics. Third, probabilistic turbulence forecasts are provided based on the 11 ensemble results, which are applied to the ATM route planning. Results show that the 12 WRF forecasts match well with observed weather patterns and the overall performance 13 skill of the ensemble turbulence forecast compared with the observed data is superior to 14 any single turbulence metric. An example Wind-Optimal Route (WOR) is computed 15 using areas experiencing $\geq 10\%$ probability of encountering severe-or-greater 16 turbulence. Using these turbulence data, lateral turbulence avoidance routes starting 17 from three different waypoints along the WOR from Los Angeles international airport to 18 John F. Kennedy international airport are calculated. The examples illustrate the tradeoff 19 between flight time/fuel used and turbulence avoidance maneuvers.

1 1. Introduction

Previous studies of Wind-Optimal Routes (WORs) and turbulence impacts to the
National Airspace System (NAS) have been conducted separately. This work aims to
develop turbulence forecasts that can be used to evaluate how turbulence information
affects WORs. Previous work has not explicitly accounted for turbulence when
developing those routes though researchers have separately examined how pilots avoid
areas of turbulence.

8 Several researchers have developed strategies for using WORs for Air-Traffic 9 Management (ATM). Ng et al. (2012) developed optimal flight trajectories that 10 minimized flight time and fuel burn by computing minimum-time routes in winds on 11 multiple flight levels. Palopo et al. (2010) conducted a simulation of WORs and the 12 impact on sector loading, conflicts, and airport arrival rates using a method developed 13 by Jardin and Bryson (2001). Jardin and Bryson (2012) continued their research in this 14 area by computing minimum-time flight trajectories using analytical neighboring WOR 15 in the presence of a strong jet stream with winds of up to 80 m s⁻¹.

16 Prior research shows pilots seek to avoid areas of turbulence, and the impact of 17 those maneuvers to ATM has been documented. Krozel et al. (2011) studied the 18 maneuvers pilots made when they encountered Clear-Air Turbulence (CAT). They 19 showed the pilot's response to CAT depended on factors such as aircraft type and 20 company policies. In that study, they looked at turbulence maneuvers for the next 50 21 miles of flight and found that descending to a smooth flight level to be the usual tactical 22 solution. Ignoring CAT near a jet stream of strong winds to achieve minimum-time 23 routes may result in flight and fuel savings that cannot be fully realized due to a pilot's 24 unwillingness to traverse turbulent areas to reach the maximum tail winds. Research

1 shows two-thirds of all severe CAT occurs near the jet stream (Lester 1994). Turbulence 2 information can also aid in the development of routes around convective systems. Ng et 3 al. (2009) calculated convective weather avoidance routes considering the probability of 4 pilot deviation using a model based on radar data. The model used by them and others 5 to predict pilot behavior around convective systems, the Convective Weather Avoidance 6 Model (CWAM), uses ground-based radar information to determine areas of convection 7 where pilots will likely avoid (Delaura and Evans 2006). CWAM is currently used by 8 NASA's Dynamic Weather Routing tool to create in-flight routing around convective 9 weather and has been evaluated in field studies in collaboration with American Airlines 10 (McNally et al. 2012). However, such a model can miss regions of Convectively 11 Induced Turbulence (CIT) outside the convective clouds. 12 To address the lack of turbulence information in WOR applications, a predictive 13 model of aviation-scale turbulence, such as the Graphical Turbulence Guidance (GTG) 14 product (Sharman et al. 2006; Kim et al. 2011) in which an ensemble of turbulence 15 diagnostics are computed can be used to modify the WOR solution. The turbulence 16 diagnostics in turn are based on forecasts from a numerical weather prediction (NWP) 17 model or ensemble of NWP models. Steiner et al. (2013) reviewed ensemble-based 18 forecasting techniques and state that ensemble forecasting can be applied to turbulence. 19 They also point out that probabilistic forecasts are appropriate for ATM strategic 20 planning as they may provide guidance about the uncertainty associated with weather-21 related phenomena. Here, time-lagged ensemble NWP forecasts are used to drive 22 ensembles of turbulence diagnostics to provide probabilistic information about 23 turbulence likelihood. And in order to better predict the effects of convection as well as 24 provide better representation of mountain wave and clear-air turbulence sources, a high-

1	resolution (3 km horizontal grid spacing) NWP model is implemented. Further, each
2	computed turbulence diagnostic is scaled to energy dissipation rate (EDR = $\epsilon^{1/3}$ m ^{2/3} s ⁻¹)
3	as an aircraft-independent atmospheric turbulence metric. EDR is defined as the rate of
4	the turbulent kinetic energy (TKE) transfer from large-scale to small-scale eddies. The
5	large-scale eddies in atmosphere are inherently unstable. These large eddies break up
6	and cascade down to smaller-scale eddies until the viscous dissipation becomes
7	dominant and the TKE is converted to heat. The model-derived EDR metric is
8	consistent with in situ EDR estimates currently available from several fleets of
9	commercial airliners including B767s, B757s and B737s (Cornman et al. 1995;
10	Sharman et al. 2014), which is convenient for forecast verification. The in situ EDR
11	metric can be related to traditional turbulence intensity based on pilot-reported
12	categories of "light (LGT)", "moderate (MOD)", and "severe (SEV)" by appropriate
13	considerations of aircraft type and flight conditions (Sharman et al. 2014). For reasons
14	discussed in Sharman et al. (2014), EDR is the preferred atmospheric turbulence unit for
15	aviation-scale observations and forecasts.
16	The following sections describe the methodology and procedures for creating new
17	turbulence forecasts. A comparison of these new forecasts with observed radar
18	reflectivity and automated in situ EDR data is presented to assess the reliability and
19	accuracy of the forecasts. Finally, as an example of turbulence application to ATM,
20	WORs are computed from Los Angeles International Airport (LAX) to John F.
21	Kennedy International Airport (JFK) with and without a turbulence forecast.
22	
23	2. Methodology and procedures of the turbulence forecasts
24	From a meteorological perspective, small-scale turbulent eddies that directly affect

1	commercial aircraft at cruising altitudes are generated by a number of possible sources.
2	For example, strong vertical shears above and below a jet stream core, inertial
3	instability due to anticyclonic shear and curvature flow, and the gravity wave emissions
4	via geostrophic adjustment in the jet stream exit region are well-known turbulence
5	generation mechanisms near an upper-level jet/frontal system (e.g., Lane et al. 2004;
6	Kim and Chun 2010, 2011; Knox et al. 2008). Mountain wave breaking frequently
7	causes aviation turbulence over complex topographic regions (e.g., Lane et al. 2009;
8	Sharman et al. 2011, 2012). Flow deformation, gravity wave breaking, and thermal-
9	shear instability near the various convective systems are also important sources for
10	aviation turbulence (e.g., Lane et al. 2003; Lane and Sharman 2008, 2014; Kim and
11	Chun 2012; Kim et al. 2014; Trier and Sharman 2009; Trier et al. 2010). To take into
12	account these many turbulence generation mechanisms as well as uncertainties in the
13	NWP model forecasts, a combination of several turbulence metrics due to different
14	mechanisms and from different forecasts is essential, and is more reliable than using a
15	single diagnostic or simple rule-of-thumb predictor (e.g., Sharman et al. 2006; Kim et
16	al. 2011; Gill 2014; Gill and Stirling 2013). In addition, a convection-permitting high-
17	resolution numerical weather prediction model is more useful to capture small-scale
18	turbulent eddies induced by convective activity or other turbulence sources.
19	This new turbulence forecast method is a sequence of four different processes,
20	which is summarized below.
21	1) A high-resolution NWP forecast model is used to produce 3D meteorological
22	data such as u , v , and w wind components, potential temperature (θ), pressure (p),
23	humidity, and cloud mixing ratios at a given valid time. Time-lagged ensembles are
24	constructed from the forecast fields for different lead-times but valid at the same time.

2) Ten aviation turbulence metrics, each based on combinations of horizontal
 and/or vertical gradients of 3D meteorological variables from the NWP model, are
 calculated.

3) The ten metrics from the different time-lagged forecasts are mapped into a
common atmospheric turbulence-scale (EDR-scale) based on the assumed log-normal
(random) distributions.

All EDR-scale metrics are combined to produce both deterministic and
probabilistic turbulence forecasts using different weights as a function of turbulence
forecasting skill of each metric and is used to modify the WORs.

10

11 *a. Weather model*

12 In the first step, the WRF-ARW model version 3.5, is used as the weather forecast 13 model in this study. This model uses a finite-difference method for non-hydrostatic and 14 fully compressible prognostic equations on an Arakawa-C grid and terrain-following 15 vertical sigma levels (Skamarock and Klemp 2007). The WRF-ARW model has been 16 successfully applied to understand possible generation mechanisms of severe turbulence 17 cases under different environmental weather conditions (e.g., Trier and Sharman 2009; 18 Trier et al. 2010; Kim and Chun 2010; 2012; Kim et al. 2014). Design of the WRF-19 ARW model is the same as National Oceanic and Atmospheric Administration (NOAA) 20 high-resolution rapid refresh HRRR (http://ruc.noaa.gov/hrrr/) operational system. The 21 horizontal domain covers the entire CONUS (Figs. 1 and 2). The horizontal grid spacing 22 is 3 km, and model top is at 20 hPa with 50 vertical layers, which leads to be about 500 23 m vertical grid spacing in the Upper-Troposphere and Lower-Stratosphere (UTLS). 24 Rayleigh damping for the *w*-wind component is applied in a sponge layer of uppermost

1	5-km from the model top. Subgrid-scale microphysical processes are parameterized
2	using the Thompson et al. (2004) scheme. Longwave and shortwave radiation
3	parameterizations use the Rapid Radiative Transfer Model (RRTM; Mlawer et al.
4	1997). Land Surface Model (LSM) providing upward fluxes at surface for the Planetary
5	Boundary Layer (PBL) scheme is parameterized by Rapid Update Cycle (RUC) LSM
6	(Smirnova et al. 2000). Subgrid-scale vertical mixing is parameterized by the Mellor-
7	Yamada-Janjić (MYJ) scheme (Janjić 2002) not only in the PBL but also in free
8	atmosphere by solving the 1.5 order Turbulent Kinetic Energy (TKE) equation (Mellor
9	and Yamada 1982).
10	The longest forecast time of each model run is six hours, and the frequency of
11	model output is 15 minutes, which is also the same as the HRRR. Initial and boundary
12	conditions use hourly reanalyses data over the CONUS from the 13-km Rapid Refresh
13	(RAP) model domain. The spin-up time of the model is about 30 minutes, which is
14	somewhat faster than other regional models, likely because the hydrometeors are
15	already enhanced in the initial condition of the RAP 13-km domain by assimilating the
16	ground-based radar observation (http://ruc.noaa.gov/hrrr/). The model was run using the
17	Pleiades supercomputer at the NASA Ames Research Center
18	(http://www.nas.nasa.gov/hecc/). The wall-clock run-time using 500 cores took an hour
19	to complete one model run with 15-minute forecast outputs up to six hours. The run-
20	time could be decreased by using more computer resources, but the one-hr run time
21	should be adequate for most operational purposes.
22	The following are example comparisons of the WRF-ARW model forecasts against
23	the observed meteorology for two selected cases. The first case is for the 36 hour period
24	from 0600 UTC 7 to 1800 UTC 8 September 2012 when several turbulence encounters

1	were observed near convective systems over the CONUS (Figs. 1a and b). At 1730
2	UTC 7 September, several convective clouds begin to develop along a surface cold from
3	elongated from the Great Lakes to Kansas. Locally isolated convective clouds also
4	developed ahead of the cold front along a squall line over Illinois and Indiana (Fig. 1a).
5	Several turbulence events with EDRs $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ scattered in the Northeastern
6	CONUS, reported by the in situ measurements from commercial aircraft, were probably
7	due to the convection well to the west (Fig. 1). An EDR of 0.22 corresponds to
8	moderate turbulence for large commercial aircraft by Sharman et al. (2014). Note this
9	value is lower than the current ICAO standard value form "moderate" of 0.40 $m^{2/3}\ s^{\text{-1}}.$
10	Some of these EDR reports are located within convective cloud, while others are
11	outside of visible deep convection as confirmed by the radar data in Figs. 1a and b. As
12	the upper-level trough deepened, clusters of thunderstorms along the eastward-moving
13	cold front shown in Fig. 1b swept out the entire eastern and southern CONUS regions
14	on 7-8 September 2012.
15	These radar observations are reasonably well captured by the WRF-ARW model. In
16	particular, forecasted echo tops along an elongated front from the Great Lakes to Kansas
17	in Figs. 1c and d are qualitatively similar to the observed radar data in Figs. 1a and b.

18 This gives confidence that the large-scale flow-generated convective clouds responsible

19 for aircraft-scale turbulence are well reproduced by the ARW-WRF model in this study.

20 Considering that the upper-level westerly jet stream is dominant during this period over

21 the northeastern CONUS (see Figs. 8 and 9 later), turbulence scattered in this area

22 during this period is likely to be generated by interactions between a deep convection-

23 induced disturbance and jet stream-related instabilities.

24 The second example case is the 12 hours period (0600-1800 UTC) on 31 December

1 2011 when strong mountain wave activity is dominant due to the passage of a strong 2 northwesterly jet stream over the Rocky mountain region. As shown in Fig. 2a, there are 3 no well-developed convective systems over the CONUS, which is also reliably 4 simulated by the ARW-WRF model (Fig. 2b). Some clouds with echo tops lower than 5 20,000 ft (hereafter FL200) appear over Nebraska, but those are far away from the 6 observed turbulence encounters over the mountain regions in both Colorado and Utah 7 (Fig. 2b). Strong northwesterly jet flow embbedde on a planetary short wave is passing 8 over the complex mountain ranges of the western US (Fig. 2c). This in turn generates 9 mountain waves that propagate vertically up to the tropopause, as evidenced by the 10 complicated wave patterns of vertical velocity over the western mountains (Fig. 2d). 11 During this period, turbulence encounters $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ observed by in situ EDR occur 12 not near the convective system but over the Rocky mountain regions, which is mainly 13 due to the interactions between the mountain waves and the background wind (Figs. 2c 14 and d). Some of elevated in situ reports are far downstream of the mountains near the 15 border between Colorado and Nebraska, which may be related to the downstream 16 propagation of the lee wave and/or jet-stream related gravity waves that trigger 17 instabilities.

Due to the multiple turbulence-causing mechanisms in these and other cases, combinations of turbulence metrics based on various turbulence generation mechanisms are essential to accurately forecast turbulence events. In all, a total of 270 turbulence encounters $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ EDR value were observed over the CONUS by the in situ EDR measurements during two selected periods, and these are available for verification of the turbulence forecasts.

24

1 b. Turbulence diagnostics

2 For the second step, ten different turbulence diagnostics are computed. Although 3 the horizontal grid spacing of 3 km was used in the WRF-ARW model, the horizontal 4 size of aircraft-scale turbulence (normally 10-1,000 m) is still much smaller (i.e., 5 subgrid-scale). However, aircraft-scale turbulence can be diagnosed by assuming that 6 small-scale turbulent eddies directly affecting commercial aircraft cascade down from 7 large-scale (resolved scale) disturbances and are revealed as high values of the 8 turbulence diagnostics (e.g., Sharman et al. 2006; Kim et al. 2011; Williams and Josh 9 2013). In this study, three different time-lagged ensemble members of weather forecasts 10 (e.g., 1.5, 2.5, and 3.5-hr) were used to calculate the turbulence diagnostics for each 11 valid time. The upper-level turbulence diagnostics selected have relatively high 12 performance skill in previous and current operational upper-level GTG systems (e.g., 13 Sharman et al. 2006). The ten turbulence metrics used are the WRF-produced subgridscale turbulent kinetic energy (SGS TKE), Frehlich and Sharman's (2004) EDR (FS 14 15 EDR), square of total deformation (DEFSQ), absolute value of horizontal divergence 16 (ADIV), square of vertical component of relative vorticity (VORTSQ), absolute value of 17 vertical velocity (ABW), two-dimensional frontogenesis function on pressure 18 coordinates (F2D), Brown turbulence index 1 (Brown1), nested grid model turbulence 19 index (NGM), and the horizontal temperature gradient (HTG). These diagnostics were 20 then divided by the gradient Richardson number (Ri_g) (Sharman 2013). Detailed 21 formulations of the diagnostics are provided in Appendix A. 22

23 c. EDR mapping technique

1	This third step maps each turbulence diagnostic to a common atmospheric
2	turbulence scale. The previously described turbulence diagnostics have different
3	numerical formulations and units. However, a final turbulence forecast should be on a
4	common scale such as the EDR. EDR is independent of aircraft type or size and
5	mapping turbulence diagnostics into the EDR scale allows them to be compared with
6	observed in situ EDR measurements. So, all of the turbulence diagnostics calculated
7	were mapped to the EDR metric. In this study, we assumed that each model-derived
8	turbulence diagnostic has a log-normal distribution that can be derived from the best fit
9	function of the log-scale Probability Density Function (PDF) especially for larger values
10	of turbulence diagnostics for longer period of time (Sharman et al. 2014).
11	Figures 3 and 4 show an example of nine EDR-scale metrics from a 2.5-hr forecast
12	product averaged over three different flight levels of FL300, FL350, and FL400 valid at
13	1730 UTC 7 September 2012 and at 1830 UTC December 2011, respectively. In
14	general, most of the EDR-scale metrics for relatively larger values (orange shading;
15	EDR $\geq 0.22~m^{2/3}~s^{1})$ are consistent with the turbulence encounters $\geq 0.22~m^{2/3}~s^{1}$ values
16	in the observed in situ EDR measurements in commercial flights both near the
17	convective system for the first case (Fig. 3) and over the Rocky mountain regions for
18	the second case (Fig. 4). And, relatively lower values of EDR-scale metrics also capture
19	well the smooth areas of the in-flight bumpiness $\leq 0.01 \text{ m}^{2/3} \text{ s}^{-1}$ values depicted as gray-
20	dotted lines over the CONUS. But, there are some places where some EDR-scale
21	metrics over estimate some smooth regions of in situ EDR reports of bumpy areas,
22	which increases the false alarm ratio (FAR), and therefore should be considered as a
23	score function in the ensemble of metrics.

1 *d. Ensemble of EDR-scale turbulence metrics*

The final step combines all EDR-scale metrics into deterministic and probabilistic turbulence forecasts. At a given forecast time, we used a total 30 of EDR-scale metrics [i.e., ten different turbulence metrics from three different NWP forecasts (e.g., 1.5, 2.5, and 3.5 hr forecast data)] for the ensemble EDR forecasts. For the deterministic ensemble EDR, 30 EDR-scale metrics are combined into a weighted ensemble mean (e.g., Figs. 5a and c) using different weighting functions of each metric (*W_i*), as follows.

8 Ensemble
$$EDR(x, y, z) = \sum_{i=1}^{N} W_i EDR_i(x, y, z), \quad i = 1, 2, 3, ..., N = 30.$$
 (1)

$$W_i = \frac{(AUC_i/RMSE_i)^2}{\sum_{i=1}^N (AUC_i/RMSE_i)^2}, \quad i = 1, 2, 3, \dots, N = 30.$$
(2)

Here, the weighting function in Eq. (2) is as a combination of the Root Mean Square Error (RMSE) and Area Under Curve (AUC) of the Probability Of Detection "yes" for the EDR value $\geq 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ and "no" for the EDR value smaller than 0.01 $\text{m}^{2/3} \text{ s}^{-1}$ (PODY and PODN) statistics for each EDR-scale turbulence metric. Details of the AUC metric will be presented in the next section.

15 An attribute of a probabilistic forecast product is that it takes into account the 16 uncertainties in the underlying NWP forecast model. In this study, at the given valid 17 time a 3D probabilistic ensemble for Severe-Or-Greater (SOG)-level turbulence areas 18 are calculated by counting how many EDR-scale individual turbulence metrics out of the total 30 metrics have EDR values $\ge 0.47 \text{ m}^{2/3} \text{ s}^{-1}$ at each grid point in the model, 19 20 which is depicted in Figs. 5b and d. Here, the threshold is adapted from the median 21 value of in situ EDR-severe PIREP pairs for longer period over the CONUS (Sharman 22 et al. 2014).

1 Figure 5 shows a snapshot of (a and c) a deterministic ensemble EDR using Eq. (1) 2 and (b and d) a probabilistic forecast for SOG-level turbulence for the two cases. These 3 are averaged over flight levels FL300, FL350, and FL400 using three time-lagged 4 ensemble members of forecast data (1.5-3.5 hr) valid on 1730 UTC September 2012 5 (upper) and on 1830 UTC December 2011 (lower). The results show the deterministic ensemble EDR for larger values (orange shading; EDR $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$) mostly agrees 6 well with the observed in situ EDR measurements $> 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ (blue asterisks) in Fig. 7 8 5 (left). For the probabilistic forecast (Fig. 5 right), the 10% SOG-level turbulence 9 probability is also well correlated with the observations (blue asterisks) especially over 10 western Michigan and northern Ohio on 7 September 2012 (Fig. 5b) and over the 11 western mountains in Utah and Colorado on 31 December 2011 (Fig. 5d). Considering 12 that the background (natural) probability for SOG-level turbulence encounters in UTLS 13 is less than 0.1% (Sharman et al. 2006; 2014), the forecasted 10% SOG-level turbulence 14 probability (orange color shading) in Fig. 5 (right) is regarded as significantly higher 15 than the background SOG-level turbulence potential in UTLS. The choice of the 10% 16 SOG probability threshold is arbitrary, but has similar features to the 50% MOG 17 probability in this study. But, the reason we emphasize the 10% SOG turbulence 18 probability in this figure is because in the aviation community avoiding SOG turbulence 19 is regarded as a hard constraint that should be always avoided, while any MOG 20 threshold is a soft constraint that aircraft may penetrate by employing the fasten seatbelt 21 sign.

22

23 **3. Evaluation of deterministic EDR metrics**

1 In this section, the forecasted EDR-scaled turbulence diagnostics shown in Figs. 3 2 and 4 and the deterministic ensemble EDR shown in Fig. 5 (left) are compared with in 3 situ EDR reports to objectively obtain their statistical skill. The forecasting performance skills are calculated using the probability-of-detection "yes" for the EDR $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$ 4 5 (PODY) versus "no" for the EDR $\leq 0.01 \text{ m}^{2/3} \text{ s}^{-1}$ (PODN). This technique has been used 6 for the verification of various turbulence forecasts (e.g., Sharman et al. 2006; Kim et al. 7 2011). If the forecasted value of each EDR-scaled turbulence metric at the nearest grid 8 point to the observed MOG location around ± 30 minutes (30 minute time window) of 9 the valid time is higher (lower) than the in situ EDR, the Y_{for}Y_{obs} (N_{for}Y_{obs}) was counted 10 as shown in Table 1 and Eq. (3);

11
$$PODY = \frac{Y_{for}Y_{obs}}{Y_{for}Y_{obs} + N_{for}Y_{obs}}, and PODN = \frac{N_{for}N_{obs}}{Y_{for}N_{obs} + N_{for}N_{obs}}.$$
 (3)

12 and if the forecasted EDR value near the null observation is smaller (higher) than the 13 observed in situ EDR, the NforNobs (YforNobs) was counted. These procedures were applied to a total of 270 turbulence events $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1} \text{ EDR}$ value [hereafter 14 15 moderate-or-greater (MOG) EDR] and 55,150 smooth events with EDR $< 0.01 \text{ m}^{2/3} \text{ s}^{-1}$ 16 on both 7-8 September 2012 and 31 December 2011. This process was repeated through 17 20 different thresholds that ranged from EDR values of 0 to 1, resulting in 20 PODY 18 and PODN statistics for both the ensemble EDR and EDR-scale turbulence metrics. 19 Figure 6 (left) shows example PODY-PODN plots constructed from these 20 20 threshold values for the DEFSQ /Ri diagnostic for (a) 7-8 September 2012, (b) 31 21 December 2011, and (c) both cases, for various forecast lead times (1.5-5.5 hr). Values 22 of area under these curves (AUC) are a measure of the forecast performance skill (e.g., 23 Sharman et al. 2006; Kim et al. 2011). An AUC = 1 is a perfect forecast [i.e., a

1	turbulence metric can perfectly discriminate all MOG EDR and smooth events and/or a
2	turbulence metric has a perfect forecast for MOG EDR without any false alarm ratios
3	(1-PODN)]. Figure 6 (d)-(f) show PODY-PODN plots for five other turbulence metrics
4	(SGS TKE/Ri, FS EDR/Ri, DEFSQ/Ri, ADIV/Ri, and VORTSQ/Ri) from 2.5-hr
5	forecast data, for the two cases as in (a)-(c). Also shown in Fig. 6 are PODY-PODN
6	curves for ensemble EDRs defined from eqn. (3) using three different forecast lead
7	times (1; using 1.5-3.5 hrs data, 2; using 2.5-4.5 hrs data, and 3; using 3.5-5.5 hrs data).
8	In Fig. 6, the ensemble EDRs have generally higher forecasting performance skill
9	than any of single EDR-scale turbulence metric. Especially for the first case on 7-8
10	September 2012 (Figs. 6a and b), the ensemble EDRs have AUC values around 0.83-
11	0.84, which is superior to the AUC values of single EDR-scale metrics of between 0.69-
12	0.81. This result is the same as in the second case on 31 December 2011 (Figs. 6c and d).
13	This is consistent with the previous results of turbulence forecasts that the integrated
14	turbulence metrics provide superior forecasting skill than any single turbulence metric
15	at least in terms of the AUC performance metric (e.g., Sharman et al. 2006; Kim et al.
16	2011; Gill 2014; Gill and Stirling 2013). To assess the stability of these results we
17	randomly picked half fractions of the 270 turbulence and 55,150 smooth events and re-
18	evaluated them 100 times. The minimum and maximum AUC values of the final
19	deterministic EDR forecast are 0.77 and 0.9, respectively, which is about a -9% to $+6\%$
20	difference around the obtained performance (0.85) in Fig. 6e and f. Although this does
21	not imply that the performance would be better or not in other cases, the variability of
22	the obtained skill is statistically stable at least in these cases, and this performance is
23	consistent with the previous studies (e.g., Sharman et al. 2006, Kim et al. 2011).

1 Overall, the forecast skill for the second case is generally higher than those in the 2 first case. This is somewhat expected since the nature of the turbulence events for the 3 first case are associated with convective systems (Fig. 1) which are not as reliably 4 forecasted as those in the second case near the mountain areas (Fig. 2). This implies that 5 the fidelity of the cloud-resolving scale WRF model is higher in the case of mountain 6 wave-induced turbulence than in the convectively driven turbulence case.

7 All of the AUC and RMSE values as well as an example of the weighting scores 8 derived from the Eq. (2) for the ten turbulence metrics are shown in Table 2. In general, 9 as we expected, the RMSE values increase in all EDR-scale metrics, as the forecast-lead 10 time increases from 1.5-hr to 5.5-hr according to the decreasing fidelity of the weather 11 forecast. The combination of the AUC and RMSE values according to the Eq. (1) gives 12 the weighting scores such that the FS EDR/Ri metric has the largest contribution for the 13 ensemble EDR with the highest AUC and lowest RMSE values, while the F2D/Ri 14 metric is the smallest contribution. The SGS TKE metric also has a small contribution 15 because it is from the Planetary Boundary Layer scheme in the WRF-ARW model, 16 which cannot be expected to perform well in stably-stratified shear-flow turbulence 17 characteristic of the UTLS.

18

4. Example of turbulence application to ATM

In this section, an example WOR based on lateral deviations only using turbulence information for ATM planning is described. With a correct choice of initial heading angle, the minimum-time path in the presence of wind (i.e., WOR) can be obtained by applying Pontryagin's Minimum Principle (Bryson and Ho 1975) to determine the analytic solution for the control parameter (here, the heading angle of cruising aircraft; 1 ψ) in the governing equations of the simplified horizontal aircraft motions over a

2 spherical Earth, as follows.

3
$$\frac{d\phi(t)}{dt} = \frac{V_a \cos\psi(t) + U(\phi, \theta, z)}{R \cos\theta(t)}, \qquad (4)$$

4
$$\frac{d\theta(t)}{dt} = \frac{V_a \sin\psi(t) + V(\phi, \theta, z)}{R},$$
 (5)

Here, φ, θ, and ψ are longitude, latitude, and heading angle of the aircraft, and U
and V are wind components, respectively. R is the Earth's radius that Earth is assumed
to be a sphere and R >> z, and V_a is the airspeed of aircraft, assumed to be 250 m s⁻¹.
The analytic solution for the control parameter (ψ) that minimizes the total travel time
from the departure to arrival is

10
$$\frac{d\psi(t)}{dt} = -\frac{F_{wind}(t)}{R\cos\theta(t)}.$$
 (6)

11 A full derivation of the analytic solution including $F_{wind}(t)$ in Eq. (6) is described in 12 Appendix B.

13 A shooting method is used to find the initial heading angle. First, the great circle 14 heading angle (ψ_{GC}) between the departure and arrival points is used as the first guess of 15 the initial heading angle $[\psi(t_0)]$. And then, we solved the Eqs. (4), (5), and (6) using an explicit Euler forward integration scheme, $\left[\alpha(t+1) = \alpha(t) + \Delta t \frac{d\alpha(t)}{dt}\right]$, where $\alpha = \alpha(t) + \Delta t \frac{d\alpha(t)}{dt}$, where $\alpha = \alpha(t) + \Delta t \frac{d\alpha(t)}{dt}$, where $\alpha = \alpha(t) + \Delta t \frac{d\alpha(t)}{dt}$. 16 17 ϕ , θ , and ψ .], from LAX [$\phi(t_0)$, $\theta(t_0)$, $\psi(t_0)$] to JFK [$\phi(t_f)$, $\theta(t_f)$, $\psi(t_f)$] with $\Delta t = 60$ 18 seconds (1 min). This process is iterated with different initial heading angles $[\psi(t_0)]$ 19 within two boundary values ($\psi_{GC} - 22.5^{\circ}$ and $\psi_{GC} + 22.5^{\circ}$) until the distance between a 20 waypoint of the WOR trajectory and JFK is minimized. 21 Figure 7 shows example WORs without consideration of turbulence effects from 22 LAX to JFK along with plots for horizontal winds and probabilistic ensemble EDR

1	for SOG at FL350 and their cross-sections valid at 1730 UTC 7 September 2012 and
2	1830 UTC 31 December 2011. In the first case (Fig. 7a), a flight cruising at FL350
3	along the WOR would take 238 minutes (3 hr 58 min), which is 2 minutes less than the
4	elapsed time along the great circle route with wind (sky-blue line). In Fig. 7a, the WOR
5	would experience a total of 52 minutes of areas $\geq 10\%$ probability of encountering
6	SOG-level turbulence over northern Indiana, Ohio, and western Pennsylvania. In the
7	vertical cross-section along this WOR (Fig. 7c), the 10% SOG-level turbulence areas
8	seem to block all possible flight levels from FL260 to FL450 over these regions. This
9	indicates that Lateral Avoidance Turbulence Routes (LTARs) would be better suited to
10	avoid turbulence than vertical changes of the flight level in this case.
11	In Fig. 7b, the second case shows the same situation as in Fig. 7a with the WOR,
12	taking 238 minutes and experiencing 20 minutes in the SOG-level turbulence areas
13	from LAX to JFK, which is 2 minutes less elapsed time than the great circle route with
14	wind (sky-blue line). And, in Fig. 7d, the turbulence potential areas are vertical from
15	FL230 to FL450 near southern Colorado, which again implies that the LTARs would be
16	a better choice than vertical avoidance. Fortunately, in the second case (Fig. 7b), only a
17	small lateral deviation from the WOR (blue line) can entirely avoid the turbulence
18	region.
19	To demonstrate quantitatively the effects of turbulence avoidance on the WOR
20	routes, we use a probabilistic ensemble $EDR \ge 10\%$ probability of encountering SOG-
21	level turbulence (i.e., LTAR). The SOG-level is selected because it is considered a hard
22	constraint that pilots should avoid due to aviation safety concerns. And, a 10%
23	probability is chosen because it correlates well with observed in situ EDR reports
24	greater than 0.22 $m^{2/3} s^{-1}$ already shown in Figs. 5b and d.

The LTAR can be determined by following the same approach for the optimization
 of the WOR with a different minimization condition or cost function (*J*), as follows.

3
$$J = \int_{t_0}^{t_f} \{C_t + C_r r(\phi, \theta, z)\} dt.$$
 (7)

Here, Ct and Cr are the cost coefficients of travel time and penalty areas along the
LTAR, respectively. In this study, r(φ, θ, z) = 1 when the probabilistic ensemble EDR for
SOG-level ≥ 10%, and elsewhere r(φ, θ, z) = 0 (e.g., Ng et al. 2011; Sridhar et al. 2010).

7
$$\frac{d\psi(t)}{dt} = -\frac{\{F_{wind}(t) + F_{turb}(t)\}}{R\cos\theta(t)\{C_t + C_r r(\phi, \theta, z)\}}.$$
 (8)

8 Equation (8) is the solution for the control parameter (ψ) that minimizes the cost 9 function by Eq. (7) from departure to arrival. A full derivation of the analytic solution 10 including $F_{turb}(t)$ is given in Appendix B. Solving Eqs. (4), (5), and (8) using the same shooting method used for the WOR described earlier, gives us the LTAR (red and green 11 12 lines in Fig. 8) from LAX to JFK for the first case. This LTAR can be initiated at the 13 departure time, however, it would be preferable to delay such a maneuver until closer to 14 the forecasted turbulence constraint, because the maneuver decision needs to consider 15 several factors like confidence of the weather forecast.

In Fig. 8 (upper), the LTAR trajectory for the 10% SOG-level turbulence potential
using 3.5-5.5 hr forecasts initiated from the departure (LAX) is depicted as a red line.
The LTAR (red line) takes a total of 254 minutes flying time and used 6.7% extra time
to entirely avoid the forecasted 10% SOG-level turbulence areas. Two other alternative
LTARs were initiated 1.5 hrs (middle) and 2.5 hrs (lower) after departing LAX along the
WOR (blue lines) with more recently updated forecasts data. An aircraft that follows the
LTAR 1.5 hrs after departing LAX (middle in Fig. 8) has a flying time of 244 minutes,

1 which saves 10 minutes more than that if it were to follow the LTAR initiated from the 2 LAX (red line in upper in Fig. 8). However, if an aircraft follows an LTAR 2.5 hrs after 3 departing LAX, when it is closer to more recently forecasted turbulence regions (lower 4 in Fig. 8), it takes a total of 256 minutes of flying time. This is 2 minutes longer than the 5 LTAR initiated from the departure (red line in upper figure in Fig. 8). Therefore, in this 6 case, the most efficient LTAR is the one that begins its lateral detour 1.5 hrs after the 7 departure (middle figure in Fig. 8). This takes 244 minutes from LAX to JFK, avoiding 8 entirely all areas of SOG probability > 10%. Note that the example of LTARs shown in 9 Fig. 8 may not be the most efficient maneuver, because there are several other ways to 10 avoid the potential constraints of turbulence, such as tactical change of flight altitude 11 and route just ahead of turbulence areas.

12

13 5. Summary and conclusions

14 In this paper, the time-lagged ensembles of the EDR-scaled turbulence diagnostics 15 computed from the high-resolution WRF-ARW model are used in automated ATM route 16 planning and three example applications for re-routing around turbulence are given. The 17 new turbulence forecasting techniques can create both deterministic and probabilistic 18 turbulence information using a sequence of four procedures. These include high-19 resolution weather modeling using time-lagged ensembles, calculation of reliable 20 turbulence diagnostics on these grids, mapping of these metrics to an EDR-scale, and 21 combining the predictions into a turbulence product. In the two cases presented here, 10 22 turbulence diagnostics derived from three time-lagged ensemble members are used to 23 provide a total of 30 different turbulence forecasts. This system uses the operational 24 GTG methodology with three modifications, which include the use of (1) a finer

1 horizontal grid, (2) a time-lagged ensemble forecast with an ensemble of various 2 turbulence metrics, and (3) probabilistic for ATM as opposed to deterministic turbulence 3 information. Using a convection-permitting scale weather model with time-lagged 4 ensemble members would be beneficial for the improved turbulence forecasts related to 5 smaller-scale sources like convective system and mountain waves. Providing 6 probabilistic ensemble EDRs is useful for ATM route planning and decision making. 7 The developed turbulence forecast was created and evaluated both for 7-8 8 September 2012 when several convective clouds developed along a surface frontal 9 system that swept across the mid and eastern CONUS and for 31 December 2011 when 10 a strong northwesterly jet stream generated mountain waves and disturbances over the 11 Rocky Mountain region. The deterministic version of the EDR-scale turbulence forecast 12 was verified against observed in situ EDR-scale turbulence estimates from several 13 commercial aircraft. The new method was observed to have a higher forecasting skill 14 than other single EDR-scale turbulence metrics. 15 A simple WOR and three LTAR applications were developed to show the utility of 16 this forecast product for route planning applications. The results shown in Figs. 7 and 8 17 are summarized in Table 3. Using the WOR with ignoring turbulence maneuvers, a 18 minimum-time path experiences areas $\geq 10\%$ probability of encountering SOG-level 19 turbulence for 52 minutes. Since in both example cases considered, the potential 20 turbulence areas along the WOR are vertically deep, laterally deviating around the 21 turbulence areas seems the best option to avoid turbulence in this case. It is found that to 22 laterally detour around these potential areas of the turbulence from the departure airport 23 (LAX) an aircraft would incur 16 minutes (6.7%) more travel time to fly to its 24 destination (JFK) [LTAR 1 in Table 3 and red line in Fig. 8 (upper)]. Delaying the

1 horizontal maneuver would result in either a savings of 10 minutes if the maneuver 2 were delayed 1.5 hrs after leaving LAX [LTAR 2 in Table 3 and green line in Fig. 8 3 (middle)] or an extra 2 minutes if the maneuver was delayed by 2.5 hrs until the aircraft 4 would get close to the turbulence potential regions [LTAR 3 in Table 3 and green line in 5 Fig. 8 (lower)]. 10 minutes time saving in LTAR2 can be very significant because this 6 reduction roughly equals to about 160 km less distance of flying and about 760 kg of 7 fuel savings, which is a benefit for commercial airline operations. 8 Future work will use different thresholds instead of the 10% SOG probability to 9 explore the tradeoffs between time/fuel used and penetrating certain portions of the 10 turbulence area. In addition, when the fuel consumption model will be included in the 11 cost function of Eq. (10), the current 2-D lateral turbulence avoidance route (LTAR) will 12 be extended to 3-D maneuvers that minimize the fuel consumption and potentials of 13 turbulence encounters during the total flight time. The strategic avoidance methodology 14 suggested for turbulence herein can be also applied to other types of weather constraints 15 such as icing, volcano ash, wind gust, and potential of contrail formation. Reducing the 16 run-time would make the new method useful for tactical decisions such as near-term 17 routing around convective weather as well. This can be accomplished by using data 18 from a nowcast version of the GTG or output from a faster-running numerical model. 19

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4

5 Appendix A

1) SGS TKE: Subgrid scale turbulent kinetic energy (SGS TKE) is a turbulencerelated variable that is directly produced by the weather forecast model. In the WRF
model used, the Mellor-Yamada-Janjić planetary boundary layer parameterization
(Janjić 2002) predicts local vertical turbulent mixing not only in the PBL but also in the
free atmosphere through the Mellor-Yamada level 2.5 turbulence closure model:

11
$$\frac{\partial q^2/2(x, y, z)}{\partial t} = -\overline{u'w}(x, y, z) \frac{\partial U(x, y, z)}{\partial z} - \overline{u'w}(x, y, z) \frac{\partial V(x, y, z)}{\partial z}$$

12
$$+\beta g \overline{\theta'_{v} w}(x, y, z) + \frac{\partial}{\partial z} \left(0.2\ell q \, \frac{\partial q^2/2}{\partial z} \right) - \varepsilon. \tag{A1}$$

13 where $q^2/2$, u', w, U, V, β , g, θ_v , l, and ε are the subgrid-scale TKE, u and w the subgrid 14 wind components, U and V the grid-resolved wind components, $\beta = 1/273$, the gravity 15 acceleration (9.8 m s⁻²), virtual potential temperature, mixing length, and energy 16 dissipation rate as a function of TKE and mixing length (l), respectively. Variables under 17 the bar are subgrid-scale vertical momentum and heat fluxes that are parameterized in 18 the ARW-WRF model.

19 2) FS EDR: The EDR ($\varepsilon^{1/3}$) at given grid point is estimated from second-order 20 structure functions for the resolved scale U and V wind components along horizontal 21 directions by assuming the sensitivity to the structure functions in different NWP 22 models is negligible at small-scales (Frehlich and Sharman 2004):

1
$$\varepsilon^{\frac{2}{3}}(x, y, z) = \frac{\langle \{q(x, y, z) - q(x, y, z + s)\}^2 \rangle}{C_q(s)D_{REF}(s)}.$$
 (A2)

2 where q is U and V wind components, and s, $C_q(s)$, and $D_{REF}(s)$ are separation distance,

3 correction function that takes into account NWP model spatial filter, and the reference

4 structure function given by Lindborg (1999). <> bracket is the ensemble mean.

3) DEFSQ: Square of total deformation (*DEF*) that is sum of shear deformation and
stretching deformation (e.g., Bluestein 1992).

7
$$DEF(x, y, z) = \left[\left\{ \frac{\partial V(x, y, z)}{\partial x} + \frac{\partial U(x, y, z)}{\partial y} \right\}^2 + \left\{ \frac{\partial U(x, y, z)}{\partial x} - \frac{\partial V(x, y, z)}{\partial y} \right\}^2 \right]^{\frac{1}{2}}.$$
 (A3)

8 4) ADIV: Absolute value of horizontal divergence (*DIV*).

9
$$DIV(x, y, z) = \frac{\partial U(x, y, z)}{\partial x} + \frac{\partial V(x, y, z)}{\partial y}.$$
 (A4)

10 5) VORTSQ: Square of vertical component of relative vorticity (*VORT*).

11
$$VORT(x, y, z) = \frac{\partial V(x, y, z)}{\partial x} - \frac{\partial U(x, y, z)}{\partial y}.$$
 (A5)

12 6) ABW: Absolute value of vertical velocity.

13
$$ABW(x, y, z) = |w(x, y, z)|.$$
 (A6)

14 7) F2D: Full 3-dimensional frontogenesis function simplified to two dimensions

15 (F2D) in pressure coordinates using the thermal-wind relation (Bluestein 1992).

1
$$F2D(x, y, z) = \left\{ \nabla_{p} \theta(x, y, z) \right\}^{-1} \left[-\left\{ \frac{\partial \theta(x, y, z)}{\partial x} \right\}^{2} \left\{ \frac{\partial U(x, y, z)}{\partial x} \right\}$$

2
$$-\left\{\frac{\partial\theta(x,y,z)}{\partial y}\right\}\left\{\frac{\partial\theta(x,y,z)}{\partial x}\right\}\left\{\frac{\partial V(x,y,z)}{\partial x}\right\}$$

3
$$-\left\{\frac{\partial\theta(x,y,z)}{\partial x}\right\}\left\{\frac{\partial\theta(x,y,z)}{\partial y}\right\}\left\{\frac{\partial U(x,y,z)}{\partial y}\right\}$$

4
$$-\left\{\frac{\partial\theta(x,y,z)}{\partial y}\right\}^{2}\left\{\frac{\partial V(x,y,z)}{\partial y}\right\}\right]. \quad (A7)$$

5 Here, θ is potential temperature (K).

6 8) Brown1: Brown's index by Brown (1973) is a simplification of the original

Richardson number tendency equation by Roach (1970) using the thermal wind relationand assuming the wind is approximately in gradient wind balance.

9
$$Brown1(x, y, z) = [0.3\{VORT(x, y, z) + f(x, y)\}^2 + DEF(x, y, z)^2]^{\frac{1}{2}}.$$
 (A8)

9) NGM1: Multiplication of horizontal wind speed and total deformation, similar to
Ellrod's index (Reap 1996).

12
$$NGM1(x, y, z) = \{U(x, y, z)^2 + V(x, y, z)^2\}^{1/2} \times DEF(x, y, z).$$
 (A9)

13 10) HTG: Horizontal temperature gradient (*HTG*) provides inferences of the

14 deformation and vertical wind shear via the thermal-wind relation (e.g., Buldovskii et al.15 1976).

16
$$HTG(x, y, z) = \left[\left\{ \frac{\partial T(x, y, z)}{\partial x} \right\}^2 + \left\{ \frac{\partial T(x, y, z)}{\partial y} \right\}^2 \right]^{\frac{1}{2}}.$$
 (A10)

17 Here, *T* is temperature ($^{\circ}$ C).

18

19 Appendix B

Pontryagin's Minimum Principle (Bryson and Ho 1975) is applied to the governing
 Eqs. (4) and (5) of the aircraft motion to determine the control parameter (heading angle
 of aircraft) that minimizes the cost function defined by Eq. (7) from the departure to
 arrival along the trajectory. The necessary condition for the control parameter and the
 optimal trajectory is that there exist continuously differentiable Lagrange multipliers (λ_Φ,
 λ_θ). Using these the Hamiltonian is then,

7
$$H = C_t + C_r r(x, y, z) + \left(\frac{\partial \lambda}{\partial \phi}\right) \left\{ \frac{V_a \cos\psi(t) + U(\phi, \theta, z)}{R \cos\theta(t)} \right\}$$

8
$$+ \left(\frac{\partial \lambda}{\partial \theta}\right) \left\{\frac{V_a \sin\psi + V(\phi, \theta, z)}{R}\right\}. (B1)$$

9 Therefore, the Euler-Lagrange equations are, as follows.

10
$$-\frac{d}{dt}\left(\frac{\partial\lambda}{\partial\phi}\right) = \frac{\partial H}{\partial\phi}$$

11
$$= \frac{\partial}{\partial \phi} \{ C_r r(\phi, \theta, z) \} + \frac{1}{R \cos \theta(t)} \left(\frac{\partial \lambda}{\partial \phi} \right) \left\{ \frac{\partial}{\partial \phi} U(\phi, \theta, z) \right\}$$

12
$$+ \frac{1}{R} \left(\frac{\partial \lambda}{\partial \theta} \right) \left\{ \frac{\partial}{\partial \phi} V(\phi, \theta, z) \right\}. \quad (B2)$$

13
$$-\frac{d}{dt}\left(\frac{\partial\lambda}{\partial\theta}\right) = \frac{\partial H}{\partial\theta}$$

14
$$= \frac{\partial}{\partial \theta} \{ C_r r(\phi, \theta, z) \} + \frac{1}{R \cos \theta(t)} \left(\frac{\partial \lambda}{\partial \phi} \right) \left\{ \frac{\partial}{\partial \theta} U(\phi, \theta, z) \right\}$$

15
$$+\left(\frac{\partial\lambda}{\partial\phi}\right)\frac{\tan\theta(t)\left\{V_a\cos\psi(t)+U(\phi,\theta,z)\right\}}{R\cos\theta(t)}$$

16
$$+ \frac{1}{R} \left(\frac{\partial \lambda}{\partial \theta} \right) \left\{ \frac{\partial}{\partial \theta} V(\phi, \theta, z) \right\}.$$
 (B3)

17 Under the condition that there is extremum for $t_0 \le t \le t_f$, the optimal heading angle 18 should satisfy,

1
$$\frac{\partial H}{\partial \psi} = 0 \rightarrow tan\psi = \frac{\lambda_{\theta} \cos\theta}{\lambda_{\phi}}.$$
 (B4)

2 The necessary condition for optimality is $H(t_f) = 0$, so the Lagrange multipliers are

3 obtained when the Hamiltonian = 0, as follows.

4
$$\frac{\partial \lambda}{\partial \phi} = \frac{-\{C_t + C_r r(\phi, \theta, z)\} R \cos\psi(t) \cos\theta(t)}{V_a + U(\phi, \theta, z) \cos\psi(t) + V(\phi, \theta, z) \sin\psi(t)}.$$
 (B5)

5
$$\frac{\partial \lambda}{\partial \theta} = \frac{-\{C_t + C_r r(\phi, \theta, z)\} R \sin\psi(t)}{V_a + U(\phi, \theta, z) \cos\psi(t) + V(\phi, \theta, z) \sin\psi(t)}.$$
 (B6)

6 Differentiate in right and left hand sides of Eq. (B4) with respect to time, and Eqs.

7 (B2), (B3), (B5), and (B6) are substituted,

8
$$\frac{d\psi(t)}{dt} = -\frac{\{F_{wind}(t) + F_{turb}(t)\}}{R\cos\theta(t)\{C_t + C_r r(\phi, \theta, z)\}}.$$
 (B7)

9
$$F_{wind}(t) = -\sin\psi(t)\cos\psi(t)\frac{\partial U(\phi,\theta,z)}{\partial\phi} + \cos^2\psi(t)\sin\theta(t)U(\phi,\theta,z)$$

10
$$+ \cos^2 \psi(t) \cos\theta(t) \frac{\partial U(\phi, \theta, z)}{\partial \theta} - \frac{\partial V(\phi, \theta, z)}{\partial \phi}$$

11
$$+ \sin\psi(t)\cos\psi(t)\sin\theta(t)V(\phi, \theta, z)$$

12
$$+ \cos\psi(t)\sin\psi(t)\cos\theta(t)\frac{\partial V(\phi,\theta,z)}{\partial \theta} + V_a\cos\psi(t)\sin\theta(t)$$

13
$$+ \cos^2 \psi(t) \frac{\partial V(\phi, \theta, z)}{\partial \phi}$$
. (B8)

1
$$F_{turb}(t) = -\sin\psi(t)\cos\psi(t)\sin\theta(t)V(\phi,\theta,z)C_rr(\phi,\theta,z)$$

2
$$+ \cos\theta(t)\cos\psi(t)\sin\psi(t)\frac{\partial V(\phi,\theta,z)}{\partial\theta}C_r r(\phi,\theta,z)$$

3
$$-\cos\theta(t)\cos\psi(t)\sin\psi(t)V(\phi,\theta,z)C_r\frac{\partial r(\phi,\theta,z)}{\partial\theta}$$

4
$$+ V_a \cos\psi(t)\sin\theta(t)C_r r(\phi,\theta,z) + V_a \sin\psi(t)C_r \frac{\partial r(\phi,\theta,z)}{\partial \phi}$$

5
$$-\frac{\partial V(\phi,\theta,z)}{\partial \phi}C_r r(\phi,\theta,z) + V(\phi,\theta,z)C_r \frac{\partial r(\phi,\theta,z)}{\partial \phi}$$

$$6 \qquad -\sin\psi(t)\cos\psi(t)\frac{\partial U(\phi,\theta,z)}{\partial\phi}C_r r(\phi,\theta,z)$$

7
$$+ \sin\psi(t)\cos\psi(t)U(\phi,\theta,z)C_r \frac{\partial r(\phi,\theta,z)}{\partial \phi}$$

8 +
$$\cos^2 \psi(t) \sin\theta(t) U(\phi, \theta, z) C_r r(\phi, \theta, z)$$

9
$$+ \cos^2 \psi(t) \cos\theta(t) \frac{\partial U(\phi, \theta, z)}{\partial \theta} C_r r(\phi, \theta, z)$$

10
$$-V_a \cos\theta(t) \cos\psi(t) C_r \frac{\partial r(\phi, \theta, z)}{\partial \theta}$$

11
$$-\cos\theta(t)\cos^2\psi(t)U(\phi,\theta,z)C_r\frac{\partial r(\phi,\theta,z)}{\partial\theta}$$

12
$$+ \cos^2 \psi(t) \frac{\partial V(\phi, \theta, z)}{\partial \phi} C_r r(\phi, \theta, z)$$

13
$$-\cos^2\psi(t)V(\phi,\theta,z)C_r\frac{\partial r(\phi,\theta,z)}{\partial\phi}.$$

For the WOR that doesn't take into account the turbulence information,
$$C_t = 1$$
 and
 $C_r = 0$, which changes Eq. (B7) to Eq. (6). On the other hand, for the WOR with
turbulence information (i.e., LTAR), $C_t = 1$, and $C_r = 1$ when the probabilistic ensemble

EDR forecast at given grid point for SOG-level turbulence $\geq 10\%$, while $C_r = 0$ when
the turbulence potential is less than 10%.
References
Bluestein, H. B., 1992: Principles of Kinematics and Dynamics. Vol. I. Synoptic-
Dynamic Meteorology in Midlatitudes. Oxford University Press, 431 pp.
Brown, R., 1973: New indices to locate clear-air turbulence. Meteor. Mag., 102, 347-
360.
Bryson, A. E., and Ho, Y. C., 1975: Applied Optimal Control, Taylor and Fancis,
Levittown, PA, 481 pp.
Buldovskii, G. S., Bortnikov, S. A., and Rubinshtejn, M. V., 1976: Forecasting zones of
intense turbulence in the upper troposphere, Meteor. Gidrol., 2, 9–18.
Cornman, L. B., Morse, C. S., and Cunning, G., 1995: Real-Time Estimation of
Atmospheric Turbulence Severity from In-Situ Aircraft Measurements, J. Aircraft,
32(1) , 171-177.
DeLaura, R., and Evans, J., 2006: An Exploratory Study of Modeling En Route Pilot
Convective Storm Flight Deviation Behavior. Preprints, 12th Conference on
Aviation, Range, and Aerospace Meteorology, Atlanta, GA, Amer. Meteor. Soc.
Frehlich, R., and Sharman, R. D., 2004: Estimates of turbulence from numerical
weather prediction model output with applications to turbulence diagnosis and data
assimilation. Mon. Wea. Rev., 132(10), 2308–2324.
Gill, P. G., 2014: Objective verification of World Area Forecast Centre Clear Air
Turbulence Forecasts. Meteor. Appl., 21, 3-11, DOI: 10.1002/met.1288.
Gill, P. G., and Stirling, A. J., 2013: Introducing Convection to World Area Forecast
Centre Turbulence Forecasts, Meteor. Appl., 20, 107-114, DOI: 10.1002/met.1315.

1	International Civil Aviation Organization (ICAO), 2010: Meteorological service for
2	international air navigation. – Annex 3 to the Convention on International Civil
3	Aviation, 17 th Edition, 206 pp. [Available online at
4	http://store1.icao.int/index.php/publications/annexes/3-meteorological-service-for-
5	international-air-navigation.html].
6	Jardin, M. R., and Bryson, A. E., 2001: Neighboring Optimal Aircraft Guidance in
7	Winds. J. Guid., Control Dynam., 24(4), 710–715.
8	Jardin, M., and Bryson, A., 2012: Methods for Computing Minimum-Time Paths in
9	Strong Winds. J. Aircraft, 35(1) , 165-171.
10	Janjić, Z. I., 2002: Nonsingular implementation of the Mellor-Yamada level 2.5 scheme
11	in the NCEP Meso model. NCEP office note, No. 437, 61 pp.
12	Kim, JH., and HY. Chun, 2010: A numerical study of clear-air turbulence (CAT)
13	encounters over South Korea on 2 April 2007. J. Appl. Meteor. Climatol., 49,
14	2381-2403.
15	Kim, JH., and Chun, HY., 2011: Statistics and Possible Sources of Aviation
16	Turbulence over South Korea. J. Appl. Meteor. Climatol., 50, 311-324.
17	Kim, JH., Chun, HY., Sharman, R. D., and Keller, T. L., 2011: Evaluations of Upper-
18	Level Turbulence Diagnostics Performance Using the Graphical Turbulence
19	Guidance (GTG) System and Pilot Reports (PIREPs) over East Asia. J. Appl.
20	Meteor. Climatol., 50 , 1936-1951.
21	Kim, JH., and Chun, HY., 2012: A Numerical Simulation of Convectively Induced
22	Turbulence above Deep Convection. J. Appl. Meteor. Climatol., 51, 1180-1200.
23	Kim, JH., Chun, HY., Sharman, R. D., and Trier, S. B., 2014: The Role of Vertical
24	Shear on Aviation Turbulence within Cirrus Bands of a Simulated Western Pacific
	71

Ocean. Mon. Wea. Rev., 142, 2794-2813.

2	Knox, J. A., D. W. McCann, and P. D. Williams, 2008: Application of the Lighthill-
3	Ford theory of spontaneous imbalance to clear-air turbulence forecasting. J. Atmos.
4	Sci., 65 , 3292–3304.
5	Krozel, J., Klimenko, V., and Sharman, R. D., 2011: Analysis of Clear-Air Turbulence
6	Avoidance Maneuvers. Air Traffic Control Quart., 4(2), 147-168.
7	Lane, T. P. and R. D. Sharman, 2008: Some influences of background flow conditions
8	on the generation of turbulence due to gravity wave breaking above deep
9	convection. J. Appl. Meteor. Climatol., 47, 2777–2796.
10	Lane, T. P. and R. D. Sharman, 2014: Intensity of thunderstorm-generated turbulence
11	revealed by large-eddy simulation. Geophys. Res. Lett., 41(6), 2221–2227. DOI:
12	10.1002/2014GL059299.
13	Lane, T. P., R. D. Sharman, T. L. Clark, and HM. Hsu, 2003: An investigation of
14	turbulence generation mechanisms above deep convection. J. Atmos. Sci., 60,
15	1297-1321.
16	Lane, T. P., J. D. Doyle, R. Plougonven, M. A. Shapiro, and R. D. Sharman, 2004:
17	Observations and numerical simulations of inertia-gravity waves and shearing
18	instabilities in the vicinity of a jet stream. J. Atmos. Sci., 61, 2692–2706.
19	Lane, T. P., J. D. Doyle, R. D. Sharman, M. A. Shapiro, and C. D. Watson, 2009:
20	Statistics and dynamics of aircraft encounters of turbulence over Greenland. Mon.
21	Wea. Rev., 137 , 2687–2702.
22	Lane, T. P., Sharman, R. D., Trier, S. B., Fovell, R. G., and Williams, J. K., 2012:
23	Recent Advances in the Understanding of Near-Cloud Turbulence. Bull. Amer.
24	<i>Meteor. Soc.</i> , 93(4) , 499-515.

1	Lester, P. F., 1994: Turbulence: A New Perspective for Pilots. Jeppesen Sanderson, 212
2	pp.
3	Lindborg, E., 1999: Can the atmospheric kinetic energy spectrum be explained by two-
4	dimensional turbulence? J. Fluid Mech., 388(6), 259-288.
5	Mlawer, E. J., S. J. Taubman, P. D. Brown, M. J. Iacono, and S. A. Clough, 1997:
6	Radiative transfer for inhomogeneous atmosphere: RRTM, a validated correlated-k
7	model for the longwave. J. Geophys. Res., 102 (D14), 16663–16682.
8	McNally, D., Sheth, K., Gong, C., Love, J., Lee, C. H., Sahlman, S., and Cheng, J., 2012:
9	Dynamic Weather Routes: a Weather Avoidance System for Near-Term Trajectory-
10	Based Operations. 28th International Congress of the Aeronautical Sciences
11	(ICAS), Brisbane, Australia.
12	Mellor, G. L., and T. Yamada, 1982: Development of a turbulence closure model for
13	geophysical fluid problems. Rev. Geophys. Space Phys., 20, 851-875.
14	Ng. H. K., Grabbe, S., and Mukherjee, A., 2009: Design and Evaluation of a Dynamic
15	Programming Flight Routing Algorithm Using the Convective Weather Avoidance
16	Model. AIAA-2009-5862, AIAA Guidance, Navigation, and Control Conference,
17	Chicago, IL.
18	Ng, H. K., Sridhar, B., Grabbe, S., and Chen, N., 2011: Cross-polar aircraft trajectory
19	optimization and the potential climate impact. 30th Digital Avionics Systems
20	Conference (DASC), Seattle, WA.
21	Ng, H. K., Sridhar, B., and Grabbe, S., 2012: A Practical Approach for Optimizing
22	Aircraft Trajectories in Winds. 31st Digital Avionics Systems Conference, Institute
23	of Electrical and Electronics Engineers, Williamsburg, VA.
24	Palopo, K., Windhorst, R. D., Suharwardy, S., and Lee, HT., 2010: Wind Optimal

1	Routing in the National Airspace System. J. Aircraft, 47(5), 1584-1592.
2	Reap, R. M., 1996: Probability Forecasts of Clear-Air Turbulence for the Contiguous
3	U.S. National Weather Service Office of Meteorology Tech., Procedures Bulletin,
4	430, 15 pp.
5	Roach, W. T., 1970: On the influence of synoptic development on the production of high
6	level turbulence. Quart. J. Roy. Meteor. Soc., 96, 413-429.
7	Skamarock, W. C. and Klemp, J. B., 2007: A Time-Split Nonhydrostatic Atmospheric
8	Model for Weather Research and Forecasting Applications. J. Comput. Phys.,
9	227(7) , 3465-3485.
10	Sharman, R. D., Tebaldi, C., Wiener, G., and Wolff, J., 2006: An Integrated Approach to
11	Mid- and Upper-Level Turbulence Forecasting. Wea. Forecasting, 21(3), 268-287.
12	Sharman, R. D., S. B. Trier, T. P. Lane, and J. D. Doyle, 2012: Sources and dynamics of
13	turbulence in the upper troposphere and lower stratosphere: A review. Geophys.
14	<i>Res. Lett.</i> , 39 , L12803.
15	Sharman, R. D., Doyle, J. D., Shapiro, M. A., 2011: An Investigation of a Commercial
16	Aircraft Encounter with Severe Clear-Air Turbulence over Western Greenland. J.
17	Appl. Meteor. Climatol., 51 (1), 311-324.
18	Sharman, R. D., 2013: New Developments in the Graphical Turbulence Guidance
19	Product. Preprint, Workshop on Aviation Turbulence, Boulder, CO.
20	Sharman, R. D., L. B. Cornman, G. Meymaris, J. Pearson, T. Farrar, 2014: Description
21	and Derived Climatologies of Automated In Situ Eddy-Dissipation-Rate Reports of
22	Atmospheric Turbulence. J. Appl. Meteor. Climatol., 53, 1416–1432. doi:
23	http://dx.doi.org/10.1175/JAMC-D-13-0329.1.
24	Smirnova, T. G., J. M. Brown, S. G. Benjamin, and D. Kim, 2000: Parameterization of

1	coldseason processes in the MAPS land-surface scheme. J. Geophys. Res., 105
2	(D3), 4077–4086.
3	Sridhar, B., Ng, H. K., and Chen, N. Y., 2010: Aircraft Trajectory Optimization and
4	Contrails Avoidance in the Presence of Winds. 10th AIAA Aviation Technology,
5	Integration, and Operations (ATIO) conference, Fort Worth, TX.
6	Steiner M., Bateman, R., Megenhardt, D., Liu, Y., Pocernich, M., and Krozel, J., 2010:
7	Translation of Ensemble Weather Forecasts into Probabilistic Air Traffic Capacity
8	Impact. Air Traffic Control Quart., 18(3), 229-254.
9	Thompson, G., R. M. Rasmussen, and K. Manning, 2004: Explicit forecasts of winter
10	precipitation using an improved bulk microphysics scheme. Part I: Description and
11	sensitivity analysis. Mon. Wea. Rev., 132, 519–542
12	Trier, S. B. and Sharman, R. D., 2009: Convection-Permitting Simulations of the
13	Environment Supporting Widespread Turbulence within the Upper-Level Outflow
14	of a Mesoscale Convective System. Mon. Wea. Rev., 137(6), 1972-1990.
15	Trier, S. B., Sharman, R. D., Fovell, R. G., and Frehlich, R. G., 2010: Numerical
16	Simulation of Radial Cloud Bands within the Upper-Level Outflow of an Observed
17	Mesoscale Convective System. J. Atmos. Sci., 67(9), 2990-2999.
18	Williams, P. D. and M. M. Joshi, 2013: Intensification of winter transatlantic aviation
19	turbulence in response to climate change. <i>Nature Clim. Change</i> , 3(7) , 644-648.
20	

1 Table Captions

2	Table 1. 2×2 contingency table for the probability-of-detection (POD) statistics
3	methodology at the given threshold.
4	Table 2. AUC (area under the curve) values of the PODY-PODN statistics/RMSE (Root
5	Mean Square Error) for ten EDR-scale turbulence metrics from different weather
6	forecasts (1.5-5.5 hr forecast) against in situ EDR measurements over the CONUS
7	during two periods (7-8 September 2012 and 31 December 2011) and an example
8	of the weighting values for the time-lagged ensemble EDR 1 using 1.5-3.5 hrs
9	forecasts derived from Eq. (5) is in the rightmost columns.
10	Table 3. Minutes of the total travel time (left column), additional flight time along the
11	LTAR compared to Wind-Optimal Route (middle column), and flight time in areas
12	of SOG probability > 10% along the LTARs from the Los Angeles international
13	airport (LAX) to John F. Kennedy (JFK) international airport. Geographical paths
14	of the LTAR1, LTAR2, and LTAR3 are shown as red line in Fig. 8 (upper), green
15	line in Fig. 8 (middle), and green line in Fig. 8 (lower), respectively.
16	

1 Figure Captions

2 Figure 1. Echo top (×1000 ft) over the Continuous United States (CONUS), obtained 3 from interpolating the raw 1-km Corridor Integrated Weather System (CIWS) 4 analyses data (mosaic of the ground-based WSR-88 radar reflectivity) to 3-km grid 5 at observation times of (a) 1730 UTC and (b) 2230 UTC 7 September 2013, and 6 derived from 2.5-hr forecast data of Weather Research and Forecast (WRF) with 3-7 km horizontal grid spacing valid at (c) 1730 UTC 7 and (d) 2230 UTC 7 8 September 2013. Locations of turbulence encounters measured by in situ Eddy Dissipation Rate (EDR > 0.22 $m^{2/3} s^{-1}$) are depicted as red asterisks in all plots. 9 10 Note that the coverage of WSR-88 radar mosaic is within gray-blue areas in (a) 11 and (b), which is out of range for the storms in northern Mexico and Gulf of 12 Mexico shown in (c) and (d).

Figure 2. (a) and (b) the same as Figs. 1(a) and 1(c) except at 1830 UTC 31 December
2011. (c) Terrain height (shading; km) with horizontal wind vectors (m s⁻¹)
averaged using three layers of FL300, FL350, and FL400 and (d) vertical velocity
(m s⁻¹) at FL350, derived from 2.5-hr WRF forecast valid at 1830 UTC 7
September 2012. As in Fig 1., locations of turbulence encounters measured by in
situ EDR (> 0.22 m^{2/3} s⁻¹) are also depicted as red (a, b, and d) and white (c)
asterisks in all plots.

Figure 3. An example of snapshots of nine EDR-scale turbulence metrics (SGS TKE/Ri,
FS EDR/Ri, DEFSQ/Ri, ADIV/Ri, VRTSQ/Ri, |w|/Ri, F2D/Ri, BR1/Ri, and
NGM/Ri) derived from 2.5-hr forecast data of WRF-ARW model, averaged three
layers of FL300, FL350, and FL400 valid at 1730 UTC 7 September 2012.

2

Observed *in situ* EDR locations are also depicted as gray (EDR > 0.01 m^{2/3} s⁻¹) and blue (EDR > 0.22 m^{2/3} s⁻¹) dots in all plots.

3 Figure 4. The same as Fig. 3 except at 1830 UTC 31 December 2011.

Figure 5. Deterministic ensemble EDR (left) and probabilistic ensemble EDR for
Severe-Or-Greater (SOG)-level turbulence (right), averaged using three layers of
FL300, FL350, and FL400 derived from 1.5-3.5 hr time-lagged weather forecasts
valid at 1730 UTC 7 September 2012 (upper) and 1830 UTC 31 December 2011
(lower). Observed *in situ* EDR measurements (> 0.22 m^{2/3} s⁻¹) (blue asterisks) are
also depicted in all plots. Note that the color shadings in the left and right panels
are different.

Figure 6. X-Y plots for the PODY and PODN statistics of the (left) DEFSQ/Ri metrics 11 12 from 1.5-hr (purple dashed line), 2.5-hr (orange dash-dot-dotted line), 3.5-hr (blue 13 dash-dotted line), 4.5-hr (green dotted line), and 5.5-hr (red long dashed line) 14 forecast data and (right) EDR-scale turbulence metrics (SGS TKE/Ri; purple 15 dashed line, FS EDR/Ri; orange dash-dot-dotted line, DEFSQ/Ri; blue dash-dotted 16 line, ADIV/Ri; green dotted line, VORTSQ/Ri; red long dashed line) from 2.5-hr 17 forecast data, compared with the observed in situ EDR measurements for 7-8 18 September 2012 (upper) and for 31 December 2011 (middle), and for both periods 19 (lower). Those for time-lagged ensemble EDR 1 using 1.5-3.5 hrs data (blue bold-20 solid line), 2 using 2.5-4.5 hrs data (red bold-solid line), and 3 using 3.5-5.5 hrs 21 (black bold-solid line) are also depicted in all plots.

Figure 7. (a and b) Probabilistic ensemble EDR for SOG-level turbulence with
 horizontal wind vectors, Wind-Optimal Route (WOR; blue line), and Great Circle
 Route with wind (sky-blue line) from Los Angeles international airport (LAX) to

John F. Kennedy international airport (JFK) using 2.5-hr forecasted wind from the
WRF-ARW model and (c and d) vertical cross-sections for probabilistic ensemble
EDR for Severe-Or-Greater (SOG)-level turbulence along the WOR valid at (left)
1730 UTC 7 September 2012 and (right) 1830 UTC 31 December 2011. Reference
wind vector in a and b is 30 m s⁻¹. Locations of departure (LAX) and arrival (JFK)
are also depicted as blue dots in a and b.

7 Figure 8. (Upper) Probabilistic ensemble EDR forecast for SOG-level turbulence with 8 horizontal wind vectors and Wind-Optimal Routes (WORs; blue lines) and Lateral 9 Turbulence Avoidance Route (LTAR; red line) at FL350 from Los Angeles 10 international airport (LAX) to John F. Kennedy international airport (JFK) using 11 3.5-5.5 hr forecasts valid at 1730 UTC 9 Sep 2010. Middle and lower panels are 12 the same as upper panel except for the LTARs (green lines) initiated after 1.5-hr 13 (middle) and 2.5-hr (lower) departing from LAX along the WOR (blue lines) 14 between LAX to JFK using 2.5-4.5 hr forecasts (middle) and using 1.5-3.5 hr 15 forecasts (lower) valid at 1730 UTC 9 September 2010.

- Table 1. 2×2 contingency table for the probability-of-detection (POD) statistics
 methodology at the given threshold.

				4
		Observat	ion (obs)	
	Forecast (for) Yes No	Yes Y _{for} Y _{obs} N _{for} Y _{obs}	<u>No</u> Y _{for} N _{obs} NforNobs	6
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Table 2. AUC (area under the curve) values of the PODY-PODN statistics/RMSE (Root Mean Square Error) for ten EDR-scale turbulence metrics from different weather forecasts (1.5-5.5 hr forecast) against in situ EDR measurements over the CONUS during two periods (7-8 September 2012 and 31 December 2011) and an example of the weighting values for the time-lagged ensemble EDR 1 using 1.5-3.5 hrs forecasts derived from Eq. (5) is in the rightmost columns.

						Wt. 1.5-	Wt. 2.5-	Wt. 3.5-
Metrics	1.5-hr	2.5-hr	3.5-hr	4.5-hr	5.5-hr	hr	hr	hr
SGS	0.736 /	0.737 /	0.710 /	0.725 /	0.713 /			
TKE/Ri	0.037	0.038	0.039	0.040	0.041	0.0189	0.0187	0.0175
	0.827 /	0.837 /	0.834 /	0.812 /	0.822 /			
FS EDR/Ri	0.013	0.016	0.017	0.018	0.019	0.1378	0.1160	0.1076
	0.821 /	0.811 /	0.804 /	0.784 /	0.797 /			
DEFSQ/Ri	0.030	0.033	0.035	0.037	0.039	0.0295	0.0256	0.0236
	0.785 /	0.793 /	0.775 /	0.786 /	0.792 /			
ADIV/Ri	0.034	0.036	0.038	0.040	0.042	0.0230	0.0223	0.0202
	0.739 /	0.731 /	0.731 /	0.712 /	0.733 /			
VORTSQ/Ri	0.024	0.026	0.027	0.028	0.030	0.0417	0.0359	0.0344
	0.780 /	0.775 /	0.764 /	0.777 /	0.791 /			
w /Ri	0.034	0.036	0.037	0.039	0.040	0.0237	0.0231	0.0220
	0.755 /	0.777 /	0.758 /	0.741 /	0.739 /			
F2D/Ri	0.051	0.056	0.058	0.060	0.061	0.0096	0.0087	0.0084
	0.819 /	0.820 /	0.804 /	0.782 /	0.786 /			
Brown1/Ri	0.029	0.031	0.032	0.034	0.036	0.0356	0.0304	0.0279
	0.820 /	0.827 /	0.820 /	0.808 /	0.818 /			
NGM/Ri	0.028	0.030	0.031	0.033	0.034	0.0402	0.0349	0.0337
	0.750 /	0.768 /	0.756 /	0.754 /	0.757 /			
HTG/Ri	0.054	0.056	0.058	0.060	0.061	0.0096	0.0089	0.0086
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Table 3. Minutes of the total travel time (left column), additional flight time along the
LTAR compared to Wind-Optimal Route (middle column), and flight time in areas of
SOG probability > 10% along the LTARs from the Los Angeles international airport
(LAX) to John F. Kennedy (JFK) international airport. Geographical paths of the LTAR1,
LTAR2, and LTAR3 are shown as red line in Fig. 8 (upper), green line in Fig. 8 (middle),
and green line in Fig. 8 (lower), respectively.



	Flight time (minutes)				
Truess of the	Total flight	Additional time	Flight time in		
flight routes	from LAX to JFK	WOR	areas of SOG > 10%		
WOR	238	0	52		
LTAR1	254	16	0		
LTAR2	244	6	0		
LTAR3	256	18	0		



Figure 1. Echo top (×1000 ft) over the Continuous United States (CONUS), obtained from interpolating the raw 1-km Corridor Integrated Weather System (CIWS) analyses data (mosaic of the ground-based WSR-88 radar reflectivity) to 3-km grid at observation times of (a) 1730 UTC and (b) 2230 UTC 7 September 2013, and derived from 2.5-hr forecast data of Weather Research and Forecast (WRF) with 3-km horizontal grid spacing valid at (c) 1730 UTC 7 and (d) 2230 UTC 7 September 2013. Locations of turbulence encounters measured by *in situ* Eddy Dissipation Rate (EDR $\ge 0.22 \text{ m}^{2/3} \text{ s}^{-1}$) are depicted as red asterisks in all plots. Note that the coverage of WSR-88 radar mosaic is within gray-blue areas in (a) and (b), which is out of range for the storms in northern Mexico and Gulf of Mexico shown in (c) and (d).



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Figure 3. An example of snapshots of nine EDR-scale turbulence metrics (SGS TKE/Ri, FS EDR/Ri, DEFSQ/Ri, ADIV/Ri, VRTSQ/Ri, |w|/Ri, HTG/Ri, BR1/Ri, and NGM/Ri) derived from 2.5-hr forecast data of WRF-ARW model, averaged three layers of FL300, FL350, and FL400 valid at 1730 UTC 7 September 2012. Observed *in situ* EDR locations are also depicted as gray (EDR $\leq 0.01 \text{ m}^{2/3} \text{ s}^{-1}$) and blue (EDR $\geq 0.22 \text{ m}^{2/3} \text{ s}^{-1}$) dots in all plots.



Figure 4. The same as Fig. 3 except at 1830 UTC 31 December 2011.



Figure 5. Deterministic ensemble EDR (left) and probabilistic ensemble EDR for Severe-Or-Greater (SOG)-level turbulence (right), averaged using three layers of FL300, FL350, and FL400 derived from 1.5-3.5 hr time-lagged weather forecasts valid at 1730 UTC 7 September 2012 (upper) and 1830 UTC 31 December 2011 (lower). Observed *in situ* EDR measurements ($\geq 0.22 \text{ m}^{2/3} \text{ s}^{-1}$) (blue asterisks) are also depicted in all plots. Note that the color shadings in the left and right panels are different.



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Figure 8. (Upper) Probabilistic ensemble EDR forecast for SOG-level turbulence with horizontal wind vectors and Wind-Optimal Routes (WORs; blue lines) and Lateral Turbulence Avoidance Route (LTAR; red line) at FL350 from Los Angeles international airport (LAX) to John F. Kennedy international airport (JFK) using 3.5-5.5 hr forecasts valid at 1730 UTC 9 Sep 2010. Middle and lower panels are the same as upper panel except for the LTARs (green lines) initiated after 1.5-hr (middle) and 2.5-hr (lower) departing from LAX along the WOR (blue lines) between LAX to JFK using 2.5-4.5 hr forecasts (middle) and using 1.5-3.5 hr forecasts (lower) valid at 1730 UTC 9 September 2010.