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Secular Stagnation and Rational Bubbles

Effects and Nature of the Interplay

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To my mom and my sister.

Contents

Introduction

In this work I focus on the effect of rational asset price bubbles in secular stagnation, an economic environment featuring sluggish economic activity, low inflation and zero or near-zero interest rates.

In the first chapter, I clarify my research question by reviewing the emerging literature on secular stagnation. The secular stagnation theory provides sound theoretical and empirical grounds for interpreting the failure of the prevailing policy framework in the pre-crisis era. In secular stagnation, inflation targeting and contractionary fiscal policies are not effective, because a negative natural interest rate and price stability inhibit standard monetary policy tools and prevent full employment, which can be restored through structural reforms, in the long-run, and through expansionary fiscal policies or higher inflation, in the short-run. As these policy measures are not immediately and necessarily effective or they are unsustainable, low interest rates could foster asset price bubbles and this creates a policy trade-off: the policy makers could temporarily exploit the beneficial effect of financial instability at the cost of a more likely and harmful financial crisis in the future. In order to shed light on the nature of this trade-off, I investigate the effect of rational asset price bubbles in a low interest rates environment.

In chapter 2, I explore the hypothesis that asset price bubbles postponed the low interest rates environment, which has followed the end of the Great Recession, by adding rational bubbles to the theoretical model of Eggertsson et al. (2017)

and by assuming they provide an additional store of value. Rational bubbles play a crucial role, when a permanent demand shock hits the economy. They reallocate and decrease savings, counteracting the downward pressure put on interest rates by the structural shock. This allows the central bank to escape from the zero lower bound.

In chapter 3, the model is further extended by assuming the existence of a bubbly collateral. In this way, I study an additional channel through which rational bubbles affect interest rates and measure the minimum size of the bubble which is necessary to avoid secular stagnation. By affecting saving and borrowing, asset price bubbles redistribute resources between generations and reduce the welfare of the representative agent. The redistribution of resources also reflects in higher interest rates. Even though the "borrowing channel" further raises interest rates by fostering debt accumulation and reducing savings, the "saving channel" mainly contributes to the increase in interest rates. This result has interesting implications for the design of policies aimed at managing asset price bubbles.

Chapter 1

Insights for the Secular Stagnation Theory

Between 2000 and 2009 large fluctuations were observed in the US housing market. The Case-Shiller index, a standard measure of home prices, increased by 57.4% from September 2000 to March 2006, before going back to its 2000 level in March 2009. Fluctuations in house prices were associated with changes in macroeconomic variables. The appreciation of houses promoted irresponsible borrowing, which, in turn, sustained consumption and capital accumulation. Then, the boom turned into a burst and the overborrowed households and firms were forced to cut their spending. The boom-and-bust cycle in the US housing market and the resulting economic crisis have called into question the policy framework of the pre-crisis era. The new secular stagnation hypothesis provides theoretical insights for interpreting the past events and outlining a new policy framework. In this paper, I review the emerging literature on secular stagnation and argue that the presence of asset price bubbles, when interest rates are chronically low, raises important issues for the design of economic policies.

In the pre-crisis era, a broad consensus emerged on the design of monetary and fiscal policies. Conventional wisdom dictated price stability was the best contribution of monetary policy to long-run growth. Building on this intuition, "inflation targeting" was first pioneered by the Reserve Bank of New Zealand and then adopted by most of the central banks worldwide.[1] Inflation targeting was not only the proper framework for maintaining price stability. It could also prevent large fluctuations in asset prices, because these fluctuations transmit to inflation rate and this induces the central bank to change the policy rate, mitigating movements in asset prices (Bernanke and Gertler, 1999, 2001). As regards fiscal policy, a new branch of the economic literature pointed to the expansionary effects of fiscal contractions and to the detrimental effects for economic growth of an excessively high public debt (e.g., Giavazzi and Pagano, 1990; Reinhart and Rogoff, 2010). The policy prescriptions of this new literature, which are referred to as "austerity", were incorporated in the Maastricht Treaty[2] and shaped the design of the European institutions.

The consensus on this policy framework has been undermined by the 2007-2008 crisis. Financial instability was not mitigated by inflation targeting, because the appreciation of houses did not trigger inflationary pressures. Furthermore, the lack of demand deriving from the economic crisis pushed the nominal interest rate against the zero lower bound (ZLB), inhibiting the standard monetary policy tools and forcing the central banks to implement unconventional monetary policies such as quantitative easing and forward guidance. Against this backdrop, the old debate about the design of monetary policy has gained new momentum. At the beginning of the 2000s, Cecchetti et al. (2000), in opposition to Bernanke and Gertler (1999, 2001), provided support for "lean against the wind" (hereafter LAW) monetary policies, which preemptively dampen speculative fluctuations in asset prices

[1] This monetary policy regime implies an explicit inflation target and the short-term nominal interest rate is the main policy instrument.

[2] It was an agreement, signed in 1992 by the Member States of the European Community, to create a single European currency.

by raising the policy rate. In a similar vein, Sverige Riskbank (2013), Bank for International Settlements (2014) and Olsen (2015) have recently made the case for LAW policies. Furthermore, austerity economics has been fiercely criticized. Strict budget rules such as those in the "Stability and Growth Pact"[3] and its amendments have negatively affected the recovery of the Eurozone (Fatás and Summers, 2016), determining unexpected large output losses because of underestimation of fiscal multipliers (Blanchard and Leigh, 2013).[4]

The new theory of secular stagnation provides a unified perspective for interpreting the limits of the policy framework of the pre-crisis era and outlining possible reforms. According to this theory, structural forces have depressed aggregate demand and determined a negative natural interest rate[5] in the last decades, as witnessed by the declining trend of the real interest rates (IMF, 2014; Rachel and Smith, 2015). In this environment, the risk of falling into a liquidity trap due to a negative shock is higher. Hence, though the financial shock, triggered by the Lehman Brothers bankruptcy, is responsible for the recent crisis, it is not responsible for low interest rates and the sluggish recovery following the crisis. Rather, interest rates and aggregate demand were sustained by dot-com and housing bubbles, before the Great Recession, and they are now back to their preexisting low levels. The drivers of the decline in the natural interest rate are still at work and advanced economies may enter in an era of persistently low interest rates and economic activity, which echoes "secular stagnation" of Hansen (1939).

[3] It was an agreement, signed in 1997 by the Member States of the European Union (EU), to maintain sound fiscal policies in the Economic and Monetary Union (EMU).

[4] A possible explanation of the underestimation is provided by Christiano, Eichenbaum and Rebelo (2011), who find fiscal multipliers are larger during ZLB episodes.

[5] It is the real interest rate consistent with output at the potential level. This concept echoes the original intuition of Wicksell (1898, p. 102): "there is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them". The natural interest rate, like potential output, is not observable, but it can be estimated through econometric techniques (e.g., Laubach and Williams, 2003). Recent estimates point to an historical decline in the US natural interest rate (Laubach and Williams, 2016).

The secular stagnation (SecStag henceforth) environment has detrimental effects for monetary policy and financial stability. Not only small adverse shocks are sufficient to push the policy rate against the ZLB, low rates also foster financial instability through three channels: (i) they induce investors to take on more risk, in order to get a higher return; (ii) they stimulate excessive borrowing, because interest payments are low; (iii) they promote Ponzi financial structures. As a result, SecStag creates a trade-off between full employment, low inflation and financial stability. A low inflation target prevents the central bank from keeping demand at the potential level, when the natural interest rate is negative, and, if the central bank maintains price stability, full employment can be restored only by financial bubbles. This trade-off questions the theoretical basis of the prevailing policy framework during the pre-crisis era. The goal of price stability contributes to a persistent negative output gap and the resulting low inflation prevents asset price movements to transmit to other prices. Furthermore, the use of the policy rate is strongly limited by the ZLB and sound fiscal policies further depress aggregate demand. Unlike normal times, high inflation relaxes the ZLB constraint in SecStag and this allows the central bank to restore full employment. At the same time, higher public debt can spur aggregate demand, solving the excess saving problem behind SecStag.

The effects of a higher inflation target and expansionary fiscal policies have been extensively studied by the SecStag literature, whereas less attention has been dedicated to the analysis of asset price bubbles with low interest rates. Yet, asset price bubbles naturally emerge with low rates and are another potential solution, though temporary, to SecStag. In fact, as suggested by the theory of rational bubbles, they absorb saving and stimulate investment, pushing interest rates up. This is especially important, when promising higher inflation is ineffective and expansionary fiscal policies are limited or unsustainable. Of course, managing a bubble is not an easy task. Moreover, bubbles can accelerate the downward trend of interest rates via long-lasting effects on potential output. In any case, all these issues deserve more attention from the SecStag theory, because they are closely related to the design of economic policies.

The rest of the chapter is structured as follows. In section 1.1 and 1.2 I review the economic literatures on secular stagnation and rational bubbles. In section 1.3 I discuss the possible implications of rational bubbles for the analysis of secular stagnation. Section 1.4 concludes.

1.1 The New Secular Stagnation Hypothesis

The SecStag hypothesis dates back to Hansen (1939). In 1939, the Great Depression was coming to an end, but output and employment were still low. According to Hansen, the disappointing US recovery was caused by the structural forces which had driven economic growth in the 19th century. In particular, population growth, which had steadily increased in the previous decades, slowed down, causing low investment and so excess saving. This resulted in a structural lack of demand, which would have prevented full employment for a long time. The coming era of anemic economic growth was termed "secular stagnation". As public spending strongly increased during the Second World War, spurring aggregate demand, and the demographic pattern was reversed by the "baby boom"[6], the US economy never experienced SecStag and the idea of Hansen was no longer investigated by the economic literature.

Recently, Lawrence Summers has recovered the theory of SecStag in order to explain the slow US recovery out of the Great Recession. A strong and fast recovery usually follows a severe recession. Yet, the US economy has underperformed for

[6] A period of pronounced population growth rates between 1945 and 1964.

many years after the 2007-08 crisis, despite the historically high public debt and the extraordinary policy measures implemented by the FED. The recovery has not been undermined by the lasting effects of the financial crisis. Rather, a negative natural interest rate, along with a too low inflation target, prevents the central bank from driving the real interest rate to its natural level.[7] As the declining trend of the natural interest rate is driven by slow-moving forces, the US economy may have been entered in an era of SecStag (Summers, 2013, 2014, 2015). A similar argument has also been extended to the Eurozone (Baldwin and Teulings, 2014).

Several forces have altered the demand for investment and the supply of savings, determining a negative natural interest rate.[8] As regards the investment side, the structure of the US economy is deeply changed with the high-tech revolution of the 1990s. Now, some leading companies are able to create added value without massive investment in capital. Furthermore, the historical decline in the price of capital goods has reduced total spending for any given amount of capital. These two factors have greatly decreased the demand for investment, whereas demographic factors and income inequality have increased the total amount of savings. Higher life expectancy, along with a lower population growth rate, has increased the size of the age cohorts with a high stock of savings,[9] and income inequality has concentrated resources towards people with high propensity to save.

The theory put forward by Summers is referred to as "demand side secular stagnation" (hereafter DSS). Even though it explains the negative gap between current and potential output, it cannot account for the simultaneous slowdown

[7] The natural interest rate is a medium-run concept, whereas the central bank can steer the short-run nominal interest rate. However, as well known, changes in the policy rate affect medium and long-run interest rates by the yield curve.

[8] For a complete list of the sources of SecStag see Summers (2014, 2015).

[9] The reduction in population growth rate has also increased the weight of the old cohorts with a lower propensity to save, but this has not significantly reduced aggregate savings (Carvalho et al., 2016). Many works point to demographic factors as the main source of the decline in interest rates (e.g., Gottfries and Teulings, 2015; Carvalho et al., 2016; Eggertsson et al., 2017).

in the US potential growth. This is the focus of the "supply side secular stagnation" (hereafter SSS), which has been theorized by Gordon (2015).[10] The sluggish potential output growth is caused by labor productivity and hours of work. The growth pace of labor productivity has decreased in the last forty years, because the contribution of total factor productivity (TFP) has been lower compared to the period 1920-1972. The average TFP growth was 2.01% in the period 1920-1972, whereas it was 0.7% between 1972 and 2014. At the same time, the historical decline in labor force participation and in population growth has strongly reduced the growth rate of aggregate hours, which passed from 1.63% in the period 1972-1996 to 0.36% in the period 2004-2014. Both DSS and SSS do not stress the potential role played by "global imbalances" or "global savings glut", which is instead central in Bernanke (2005, 2015). The large amount of savings, accumulated by Asian countries and oil exporters, have flowed into the US capital markets, pushing interest rates down. Therefore, the policy decisions of emerging countries, which started to build foreign reserves after the Asian financial crisis of the late 1990s, and not internal factors are responsible for low interest rates.

Many models try to capture the different views about SecStag. Eggertsson et al. (2017) provide a theoretical framework for the DSS by building an OLG model with three stages of life.[11] As households receive a positive income only in the second stage, they issue debt securities up to an exogenous limit, when young, and save for retirement in the middle age. In this setting, arbitrarily long ZLB episodes are possible, because the real interest rate depends on loan demand and

[10] This theory also reinterprets the causal relation between current and potential output. The DSS postulates the "inverse Say's Law": lack of demand creates lack of supply. In contrast, slow potential output growth determines slow growth of the current output in the SSS.

[11] This is a broad framework, which explores all the possible demand-related sources of SecStag. However, there are also works focusing on specific drivers. For instance, Carvalho et al. (2016) investigate the mechanisms through which demography affects interest rates, whereas Thwaites (2015) studies the connection between the decline in capital goods prices and the downward trend of interest rates.

supply. In contrast, when infinitely-lived agents, along with complete markets, are assumed, the real interest rate is equal to the inverse of the discount factor and it can only temporarily deviate from its steady state value, otherwise the maximization problem of the representative household is not well-behaved.[12] The economy reaches the SecStag equilibrium, because nominal wages are downwardly rigid. Specifically, when the real interest rate is higher than the natural level, the inflation rate turns negative and the nominal rigidity prevents the labor market from clearing.[13]

Benigno and Fornaro (2015) and Anzoategui et al. (2017) relate the anemic recovery of the US economy to the slowdown in long-run growth.[14] Benigno and Fornaro (2015) add nominal wage rigidities to an endogenous growth model with vertical innovation, and allow for the possibility of ZLB episodes. A liquidity trap arises, because negative expectations about future income depress aggregate demand. Pessimistic expectations also affect potential output. The decline in aggregate demand reduces profits and investments in R&D with a consequent fall in long-run growth. The combination of liquidity trap and low growth is called "stagnation trap". Anzoategui et al. (2017) develop a DSGE model with R&D. The drop in demand that followed the financial crisis slowed the adoption of new technologies down, undermining TFP growth. Therefore, the decline in productivity growth is not driven by secular forces, as suggested by Gordon, but it is determined by medium term factors.

Caballero et al. (2008) and Caballero and Farhi (2017) investigate the source and the direction of global imbalances, as well as their impact on interest rates. The

[12] If wealth enters the utility function, SecStag can also arise in infinite horizon models (Michau, 2015).

[13] Deflation is not crucial for this result. The mechanism also works with positive inflation, as long as it is lower than the inflation target and nominal wages are indexed to the inflation target.

[14] The main result of Benigno and Fornaro (2015) also applies to the Japanese "lost decade" and to the slow recovery of the Eurozone in the aftermath of the Great Recession.

model of Caballero et al. (2008) features global imbalances, declining interest rates and a high share of US assets in global portfolios as an equilibrium outcome. Global imbalances arise, because countries have a different capacity of producing financial assets. This intuition has inspired the idea of "safety trap", which is an extreme case of liquidity trap, in which risk premium is positive and extra savings come from the excess demand of safe assets. Emerging countries have a limited capacity of producing safe assets, hence their savings flow into advanced countries, where the supply of riskless securities is larger. After the Lehman Brother bankruptcy, the quantity of safe assets abruptly fell and the resulting excess demand drove interest rates down and risk premium up. The ZLB prevented full adjustment in the safe asset market and a contraction of output was necessary to restore the equilibrium. This has been formalized by Caballero and Farhi (2017) through an OLG model with perpetual youth and nominal rigidities, in which risk neutral agents own Lucas' tree and sell safe assets to risk averse agents.

Finally, Eggertsson et al. (2016) build a two country model with integrated financial markets, which reconciles the DSS with the theory of global imbalances. SecStag spreads through two channels. Firstly, the ZLB transmits via the real exchange rate. When aggregate demand is anemic abroad, exports drop at home. This induces the national central bank to lower the policy rate in order to compensate the fall in demand. Secondly, a country can export excess savings via financial markets, as witnessed by global imbalances. Countries can borrow and lend to each other in global financial markets. When a country experiences SecStag, funds flow to the other countries and this capital inflow pushes interest rates down in the rest of the world.

1.2 The Theory of Rational Bubbles

An economic bubble is "the difference between market price and market fundamental" (Tirole, 1985, p. 1499) and has important consequences for the real economy. Bubble-driven fluctuations in asset or commodity prices affect the allocation of resources in the economy, and change the net worth of firms and households, altering their spending capacity and their ability to repay debt. For these reasons, bubbles have been broadly investigated in economics.

The theory of rational bubbles, which relies on rational expectations and symmetric information, is currently the standard framework for analyzing bubbly episodes and it has been extensively used in macroeconomic applications.[15] In this framework, as shown by Brunnermeier (2009), the price of securities with finite maturity cannot include a bubbly component and the existence of bubbles requires the absence of the transversality condition. The price (value) of an asset, V_t, is given by the expected discounted price and dividends in the next period:

$$V_t = E_t \left[\frac{D_{t+1} + V_{t+1}}{R_{t+1}} \right] \tag{1.1}$$

D_t is dividend payment and $R_t = 1 + r_t$, where r_t is the real interest rate. By assuming a constant return over time ($E_t R_{t+1} = R$) and iterating forward, equation

[15] According to Brunnermeier (2009), the theory of economic bubbles consists of other three classes of models. The second class retains the hypothesis of rational expectations, but relaxes that of symmetric information. Agents are asymmetrically informed and so asset prices aggregate the agents' information, providing a signal to each trader. The third approach relies on the idea that rational traders interact with behavioral traders, whose actions are affected by psychological biases. Rational traders cannot correct mispricing, caused by behavioral traders, due to limited arbitrage (e.g., DeLong et al., 1990; Abreu and Brunnermeier, 2003). Finally, bubbles emerge when agents have heterogeneous beliefs and so different views about the fundamental value of an asset. If this assumption is combined with short sales constraints, optimistic investors drive asset prices up and this upward pressure cannot be counteracted by pessimistic investors (Miller, 1977).

(1.1) can be expressed as:

$$V_t = E_t \left[\sum_{k=1}^{T-t} \frac{D_{t+k}}{R^k} \right] + E_t \left[\frac{V_T}{R^{T-t}} \right] \tag{1.2}$$

Putting differently, the value of an asset at time t depends on the expected discounted stream of dividends in the time interval $[t+1, T]$ and on the expected discounted price in T. If the asset has a finite maturity T, its price is uniquely determined by expected discounted dividends (fundamental value), because the second term in the right-hand side of (1.2) is zero. In contrast, if it has an infinite maturity, its value equals the fundamental component, only if the "transversality" condition holds:

$$\lim_{T \to \infty} E_t \left[\frac{V_T}{R^{T-t}} \right] = 0 \tag{1.3}$$

If it does not hold, the price also includes a bubbly component (B_t),[16] which has to satisfy the condition:

$$B_t = E_t \left(\frac{B_{t+1}}{R} \right) \tag{1.4}$$

This is a no-arbitrage condition. A bubbly asset, which has a zero fundamental value, will be traded only if its expected return equals the real interest rate, which is the return guaranteed by other assets. As the value of bubbles grows at the expected rate R, the real interest rate cannot be greater than the growth rate of the economy, otherwise the value of bubbles would be greater than individual income and agents could not afford bubbly assets.

These intuitions are contained in Samuelson (1958) and formalized by Tirole (1985). Samuelson (1958) develops an OLG model, in which agents live for three periods. During the first two periods they produce one unit of output, in the third

[16] The bubbly component can still have a zero value, but now $V_t = E_t \left[\sum_{k=1}^{T-t} \frac{D_{t+k}}{R^k} \right]$ is only one possible solution of equation (1.2).

period they retire. Output is fully perishable, hence agents cannot carry over it to the old age. Furthermore, they do not subjectively discount future consumption and there is no social security. Given the absence of a store of value, the real interest rate falls below the optimal level, which is the economy's growth rate. Therefore, intrinsically worthless assets, whose market fundamental is zero, are valued. These are bubbles and allow for transferring resources to the old age. In particular, Samuelson (1958) points to money, which, being intrinsically worthless and accepted as medium of exchange, is the most natural bubbly asset. The condition for the existence of rational bubbles, a real return lower than or equal to the economy's growth rate, is a cornerstone of the theory, along with the capacity of bubbles of enabling trading opportunities.

These results also hold in Tirole (1985), which extend the Samuelson's model to a production economy with capital accumulation, along lines suggested by Diamond (1965). In this economy, the condition for the existence of bubbles implies dynamic inefficiency: the steady state level of consumption is not optimal, because less capital can increase the consumption of each generation.[17] Hence, bubbles appear in a dynamically inefficient economy and, by redistributing resources between generations, restore the efficient allocation. Tirole (1985) also proves that the introduction of other assets such as stocks or rents does not prevent bubbly assets to emerge. In Tirole (1985) bubbles last forever, despite in the real world they pop up and burst. This is explicitly modelled by Blanchard and Watson (1982) and Weil (1987), in which bubbles can collapse with a fixed probability. In Weil (1987), stochastic bubbles cannot exist in steady state, when the economy features dynamic efficiency. In contrast, when the economy is dynamically inefficient, bubbles can survive, as long as agents are very confident that bubbly assets will be valued in the future.

[17] The possibility of dynamic inefficiency relates to the absence of the transversality condition in OLG models.

After these early contributions, which laid the foundation for the theory of rational bubbles, this literature lost momentum. This was due to the findings of Abel et al. (1989), who showed that advanced economies are dynamically efficient, questioning the empirical basis of the theory. The challenge, posed by this result, has been recently addressed by a new class of models.[18] Martin and Ventura (2011, 2012) put forward a new theory of rational bubbles, in which financial frictions prevent unproductive investors from lending to productive ones,[19] creating pockets of inefficiency in a dynamically efficient economy. In this setting, bubbles transfer resources from unproductive investments to productive ones, boosting capital accumulation and the average productivity of the economy. This effect goes through two channels. First, the market for bubbles.[20] Bubbles are demanded by inefficient investors, whose return from investment is lower than the return from bubbly assets, and are supplied by efficient investors, whose return from investment exceeds that from selling bubbly assets. Given this structure of the market, bubbles channel funds to efficient investments (Martin and Ventura, 2012). Second, the credit market. Future bubbles raise the net worth of efficient firms. This improves their capacity to repay debt, stimulating borrowing and investment (Martin and Ventura, 2011). The "crowding-in" effect reconciles the theory of rational bubbles with empirical evidence. Bubbly episodes are associated with increase in consumption, capital stock and output. However, the original version of the theory can only capture the expansionary effect on consumption, because bubbles stimulate consumption by crowding out capital and so lowering output

[18] Geerlof (2013) replicates the exercise of Abel et al. (1989) with a different dataset, which have more accurate data on mixed income and land rents. His result points in the opposite direction compared to the original work: none of the advanced economies meets the conditions for dynamic efficiency, whereas Japan and South Korea are dynamically inefficient.

[19] Other important models featuring rational bubbles and financial frictions are Caballero and Krishnamurthy (2006), who investigate the role of financial bubbles in emerging economies, and Fahri and Tirole (2012), who study the effect of bubbles in an economy with limited pledgeability of corporate income.

[20] For real world examples of the market for bubbles see Martin and Ventura (2012).

(Tirole, 1985).[21]

Most of the literature on rational bubbles has been developed within the OLG framework, which does not require the transversality condition, unlike infinite horizon models.[22] Even though OLG models provide clear intuitions about the sources and the implications of economic bubbles, they cannot be easily confronted with data[23] and cannot capture some insights associated with infinitely-lived agents.[24] For these reasons, a new branch of the literature introduces bubbles in infinite horizon models by assuming borrowing constraints. Limits to borrowing play a different role in this setting compared to OLG models. In the OLG framework, they are not a crucial element for the emergence of bubbles, but they make them compatible with a dynamically efficient economy. In contrast, the existence of bubbles crucially depends on the presence of borrowing (or short sales) constraints in infinite horizon models.

This research program builds on the seminal works of Scheinkman and Weiss (1986), Woodford (1990) and Kocherlakota (1992, 2008). The first infinite horizon models with bubbles assumed endowment economies, but models of production economies have recently emerged (e.g., Kiyotaki and Moore, 2008; Kocherlakota, 2009; Miao and Wang, 2011; Hirano and Yanagawa, 2016). The most prominent example is Miao and Wang (2011), who provide a more realistic representation of bubbles and a further solution to the dynamic efficiency problem. In their model, bubbles are not worthless assets, but they are attached to capital, and

[21] Saint-Paul (1992), Grossman and Yanagawa (1993), and King and Ferguson (1993) provide alternative solutions to the dynamic efficiency problem, whereas Oliver (2000) and Ventura (2012) address in a different way the crowding-out criticism.

[22] A thorough analysis of the necessary conditions for rational asset pricing bubbles is contained in Tirole (1982) and Santos and Woodford (1997).

[23] Of course, it is possible to draw quantitative conclusions with OLG models featuring life-cycle structure. However, this comes at the cost of analytical tractability and requires the implementation of numerical methods.

[24] Another important criticism is related to the introduction of altruistic agents. If intergenerational transfers are possible and altruism towards the next generations is assumed, a dynasty coincides with an infinitely-lived agent (Barro, 1974).

affect dividends through a feedback loop mechanism. If agents believe the bubbly component is positive, the market value of firms increases, relaxing the debt limit and fostering investments. This results in higher profits, which validate initial beliefs and represent a "collateral yield" or "liquidity premium". As the return from bubbles also includes the collateral yield, the transversality condition does not rule stock price bubbles out and a bubble can emerge in a dynamically efficient economy.

1.3 Managing Asset Price Bubbles: Motivation and Related Issues

The rational bubbles theory provides clear insights about the salient features of bubbly episodes. Two features stand out. First, bubbles can serve as store of value. Second, bubbles raise the net worth of firms and households, providing them additional collateral to pledge in the credit market. The presence of an alternative investment vehicle and an additional collateral allows to absorb extra savings. As a consequence, bubbles can temporarily alleviate the excess saving problem behind SecStag. This makes the case for asset price bubbles as a possible solution to SecStag, because alternative policy solutions are not always effective or possible.[25]

SecStag results from the combination of a negative natural interest rate and a low inflation target. Therefore, a higher inflation target can relax the ZLB, allowing the central bank to keep demand at the potential level (Ball, 2014). The effectiveness of this policy strongly relies on the capacity of governing inflation expectations, which is different in SecStag compared to normal times. Even if

[25] There are just few works, which explore the interplay between rational bubbles and SecStag. A prominent example is Asriyan et al. (2016), which proves that a long-lasting liquidity trap can originate from a bubble crash.

the central bank is fully credible, in a low rates environment, people expect an inflation rate below the target for a long time, hence promising higher inflation does not alter their expectations about future inflation (Eggertsson et al., 2017). Instead, expansionary fiscal policies can always prevent the occurrence of SecStag. In particular, a debt-financed fiscal stimulus greatly expands aggregate demand, because it implies a larger multiplier than policies financed via taxes. However, fiscal policy could be limited by strict budget rules (e.g., Eurozone) or the necessary fiscal stimulus could undermine the sustainability of public debt, as shown by Eggertsson et al. (2017) for the US economy. Finally, policies aimed at boosting potential output growth (the so-called "structural reforms") can also mitigate or avoid SecStag, but they take a long time to be effective and so cannot immediately sustain economic activity.

Managing asset price bubbles is an ambitious policy goal and it raises several issues, which are particularly important for the SecStag theory. First, the channels through which bubbles affect interest rates have different consequences for the real economy. Although bubbles can both act as store of value and collateral, the way in which these two channels work is different. Bubbly collateral fosters debt accumulation. This poses a serious threat to the economy, because the deleveraging process, triggered by the burst of a leveraged bubble, persistently depresses demand (Jordá et al., 2015). Therefore, the positive effect of leveraged bubbles on interest rates is associated with a potentially high cost. If a leveraged bubble crashes, the resulting recession is severe, hence output gains today come at the cost of high output losses tomorrow. Instead, output losses are low, when bubbles provide an additional store of value, because this does not alter credit growth. Excessive borrowing, promoted by financial instability, can also affect interest rates via potential output. Additional funds are used by firms for financing new investment projects and, in the boom phase, resources are irreversibly allocated in the expanding sectors. When the boom turns into a bust, the high burden of debt slows capital accumulation down, hurting not only demand, but also supply. The potential output growth is further undermined by the misallocation of resources implemented by leveraged bubbles.[26]

Second, the proper policy tools depend on the specific type of bubble. If the effect of bubbles on interest rates strongly depends on the relaxation of borrowing constraints, a leveraged bubble is necessary for alleviating SecStag. In this case, the policy makers could try to replicate the bubble by subsidizing credit (Martin and Ventura, 2016).[27] In contrast, if the additional store of value, provided by bubbles, is crucial for raising interest rates, the policy makers need to manage or replicate an unleveraged bubble. As it originates from the shortage of investment opportunities, the bubble is also "rational". The expected rate of growth of rational bubbles is equal to the real interest rate and so monetary policy can influence its size by steering the nominal interest rate (Galí, 2014). As consequence, if a rational bubble arises in SecStag, the central bank can try to implement the necessary bubble size for restoring full employment via interest rate changes.

1.4 Conclusions

I have reviewed the literature on secular stagnation, which reinterprets the causes of the low interest rates environment, which has followed the Great Recession. Low interest rates and anemic demand were not caused by the Lehman Brothers bankruptcy, rather historically low interest rates were temporarily masked by asset price bubbles and the financial shock only accelerated the fall in interest rates. A negative natural interest rate underlies the secular decline in real interest rates.

[26] The self-reinforcing nature of low interest rates underlies this mechanism. See Borio and Disyatat (2014) for further details.

[27] The potential distortions in the allocation of resources should be taken into account.

When this is combined with a too low inflation target, the central bank cannot maintain output at the full employment level with standard monetary policy tools and secular stagnation arises.

The salient features of secular stagnation, which are low interest rates, a persistent negative output gap and inflation below the target, explain the failure of the prevailing policy framework in the pre-crisis era. When inflation is chronically low, an appreciation of assets does not trigger inflationary pressures and a consequent monetary policy tightening, as happened for the housing bubble of the early 2000s. Furthermore, when output is low, contractionary fiscal policies exacerbate the lack of demand and increase output losses, as witnessed by the slower recovery of the European countries in the aftermath of the 2007-2008 crisis.

Leaving apart structural reforms which take a long time to be effective, the most immediate policy solutions to SecStag are a higher inflation target and an expansionary fiscal policy. These policies are not always effective or possible, hence asset price bubbles could spontaneously emerge and alleviate secular stagnation, by serving as store of value and collateral. As bubbles could naturally appear and the range of possible policies is restricted, the policy makers could try to temporarily exploit the beneficial effect of financial instability. This option, which deserves more attention from the SecStag theory, raises some crucial issues. Specifically, in order to manage asset price bubbles, the policy makers need to know the main mechanisms through which they raise interest rates, because each channel implies a different cost in terms of higher macroeconomic instability and so it has specific policy implications.

Chapter 2

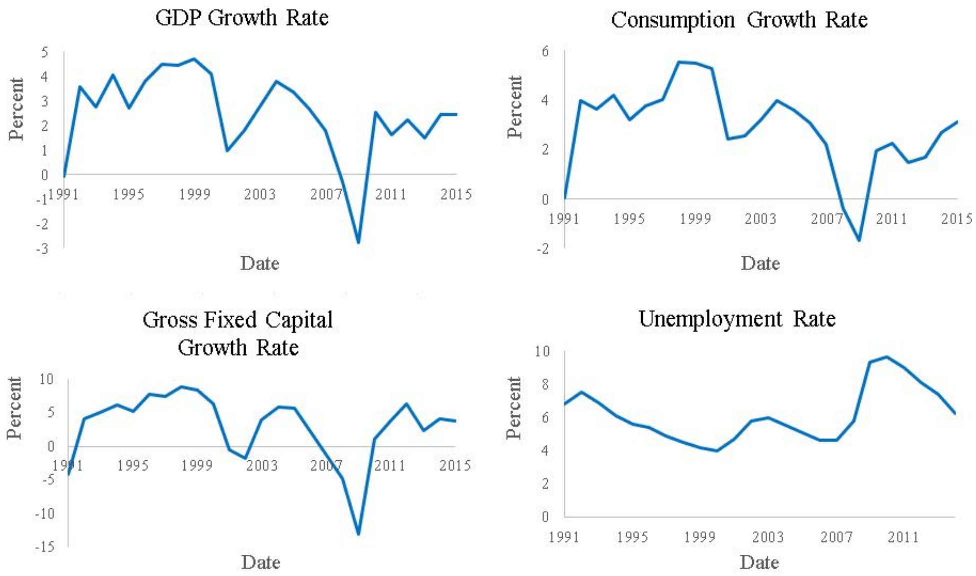
Did Asset Price Bubbles Postpone Low Interest Rates?

Has the US economy entered an era of SecStag characterized by low interest rates? This is the question raised by Lawrence Summers in his 2013 speech at the IMF Forum and later addressed in some related papers (Summers 2014, 2015). The first attempt to formalize the idea of SecStag is due to Eggertsson et al. (2017), who provide a framework to understand several long-lasting slumps such as the Great Recession. In this chapter I augment the theoretical model of Eggertsson et al. (2017) with rational bubbles, as modelled in Galí (2014). This retrospectively helps to explain the US macroeconomic performance before the recent crisis.

The US recovery out of the Great Recession has been weak. As shown by Figure 2.1, the growth pace of output, investment and consumption is very slow relative to the previous decades, whereas unemployment rate is going back to its pre-crisis level, but only after ten years of expansionary monetary and fiscal policies. To explain these trends, Summers has recently resurrected the concept of SecStag, which was originally formulated by Hansen (1939).

We observe a secular downward trend in the US natural interest rate, which

Figure 2.1: Recent US macroeconomic trends



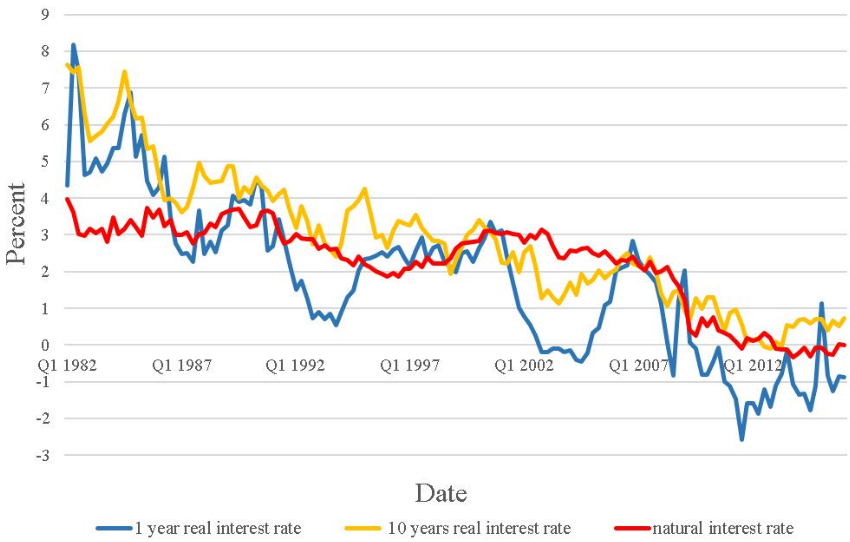
Source: World Bank.

is reflected in the pattern of real interest rates (Figure 2.2) and is mainly driven by the excess saving resulting from demographic factors (Gottfries and Teulings, 2015; Carvalho et al., 2016; Eggertsson et al., 2017).[1] As the demographic pattern is not changed and the declining trend has been further enhanced by the high debt and low investments inherited by the 2007-2008 crisis, the real interest rate may stay low for a long time and this makes the new SecStag hypothesis alarmingly plausible (Baldwin and Teulings, 2014; Eggertsson et al., 2017). The prospect of a persistently low economic activity calls for a new policy framework, because SecStag creates a policy trade-off between full employment, low inflation and financial stability, and an economic environment which is prone to bubbles. Yet, in order to identify policy conclusions, we must first interpret properly the past events.

Demography is a slow-moving force and it was already at work before the recent financial crisis (Figure 2.3), so why did not the US economy experience low

[1] For a complete list of the alternative explanations to SecStag see chapter 1.

Figure 2.2: US natural and real interest rates 1982-2015



Source: Updated estimates from Laubach and Williams (2003), Federal Reserve Bank of Cleveland.

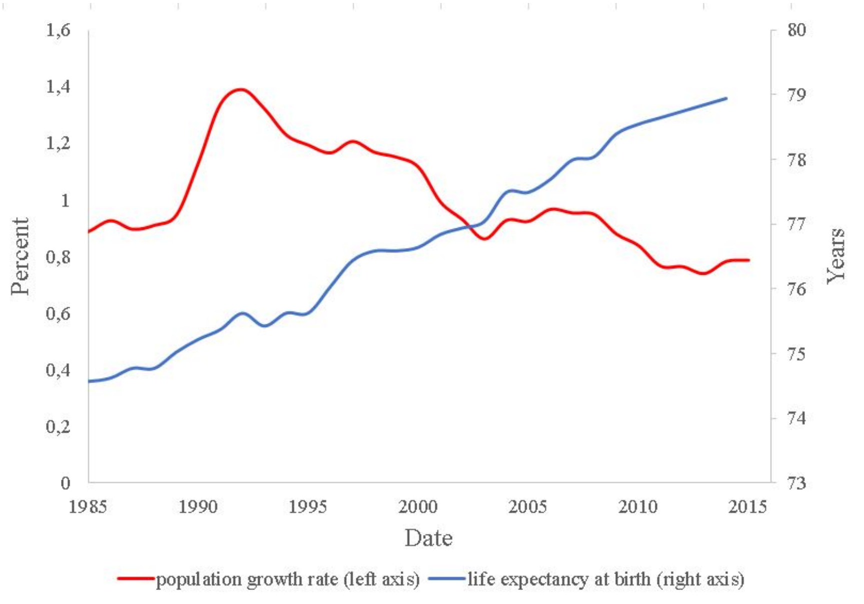
interest rates before 2007?[2] An answer to this question is that the demographic shift was counteracted by dot-com and housing bubbles over the period between the mid-1990s and the mid-2000s (Summers, 2013; Eggertsson et al., 2017).[3]

Here I aim to show that the inclusion of bubbles into the model of Eggertsson et al. (2017) is able to account for the US macroeconomic performance before the crisis, which was characterized by the absence of low interest rates and output at the potential level, despite the declining trend of real interest rates. The intuition is straightforward. In the mid-1990s, the drivers of SecStag were in force and put downward pressure on the natural interest rate, but this pressure was offset by

[2] A similar argument can be extended to other potential sources of SecStag such as technology, income inequality or global imbalances. As illustrated in the previous chapter, Gordon (2015) provides evidence of the historical decline in TFP growth. For further details about the pattern of income inequality in the US see Piketty and Saez (2003), whereas Kraay and Ventura (2007) document the evolution of the US net foreign asset position.

[3] As reported by Leroy (2004), the growth in stock prices between 1995 and 2000 did not reflect changes in fundamentals. A similar conclusion was reached by Shiller (2007) about the extraordinary rise in house prices before the Great Recession.

Figure 2.3: US life expectancy and population growth



Source: World Bank.

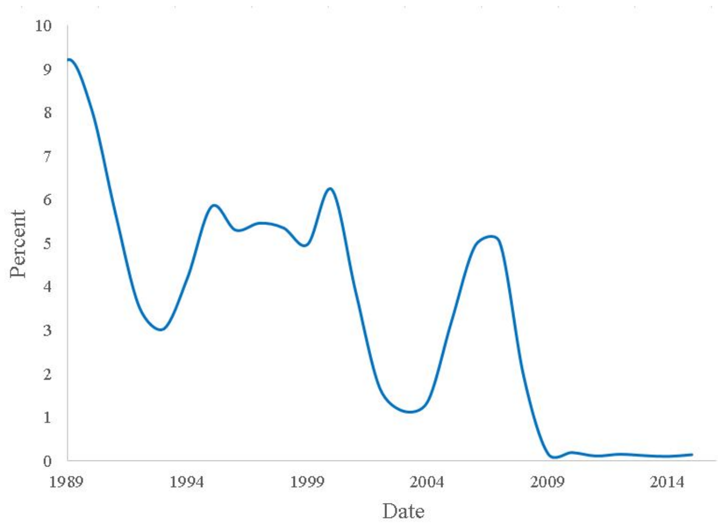
asset price bubbles, which, on the one hand, absorbed savings and, on the other hand, facilitated borrowing by rising the value of collaterals.[4] Both these effects contributed to produce temporary increases of the natural interest rate and this allowed the FED to raise the policy rate escaping from the ZLB in the mid-1990s and especially in the early 2000s (Figure 2.4). Finally, a higher federal funds rate translated into a higher real interest rate, as suggested by Figure 2.5, in which the declining trend of the real interest rate reverses in correspondence of the two bubbly episodes.[5]

To that aim, I modify the model of Eggertsson et al. (2017) to replicate the

[4] Financial deregulation also fed the explosion of borrowing, however the main driver of the credit cycle was the appreciation of collaterals (Justiniano et al., 2015).

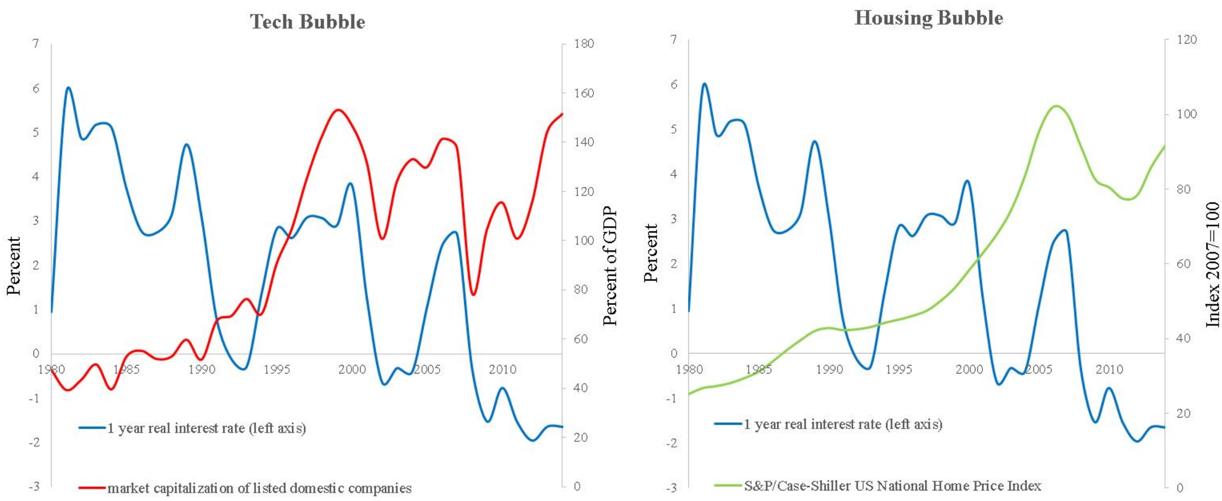
[5] Asset price bubbles are not the only way to absorb excess savings: public debt and pay-as-you-go pension schemes can also play this role. However, the rise in the US government debt was not sufficient to offset the downward pressure on interest rates (Eggertsson et al., 2017) and a similar conclusion can be extended to the public pension system, which is very small in the US compared to other advanced economies. This issue will be addressed in chapter 3.

Figure 2.4: Federal funds rate



Source: FRED.

Figure 2.5: Real interest rate and bubbles



Source: World Bank, Federal Reserve Bank of Cleveland, FRED.

effect of bubbles on savings. In the modified model, households can invest in bubbly assets, but they cannot use them as a collateral in the credit market. Abstracting from the effect of a bubbly collateral keeps the model simple without changing my results. Furthermore, it allows to focus on the way in which bubbles affect interest rates via saving. The introduction of bubbly assets changes the allocation and the amount of savings. On the one hand, bubbles absorb a fraction of savings. On the other hand, they reduce the need to save for retirement, as they provide an additional income in the old age. These effects result in a higher natural interest rate than in a bubbleless economy and this, in turn, avoids the ZLB, despite a permanent fall in the population growth rate. So, the central bank can stabilize the economy by using its standard policy tools and the economy does not suffer neither output losses nor low interest rates.

The rest of the chapter is structured as follows. In section 2.2 and 2.3, I present the extended model with rational bubbles highlighting its innovations compared to the bubbleless case. In section 2.4, I explain the mechanism preventing SecStag in a bubbly economy. Section 2.5 concludes.

2.1 Related Literature

This chapter relates to two different strands of the economic literature. Firstly, it is based on the recent literature on secular stagnation, which includes Summers (2013, 2014, 2015), Baldwin and Teulings (2014), Gordon (2015), Eggertsson et al. (2016) and Eggertsson et al. (2017). My work formalizes the idea that low interest rates and anemic demand were postponed by asset price bubbles, which is contained in Summers (2013) and in Eggertsson et al. (2017).

Secondly, this work builds on the extended literature on rational bubbles. The original theory of rational bubbles dates back to Samuelson (1958) and Tirole (1985), whereas there is a recent stream of models with bubbles, including Caballero and Krishnamurthy (2006), Kraay and Ventura (2007), Kocherlakota (2009), Miao and Wang (2011), Farhi and Tirole (2012), Martin and Ventura (2012), Galí (2014) and Asriyan et al. (2016). I introduce bubble creation and destruction in a fashion to Galí (2014), but I do not analyze the role of bubbles as source of economic instability rather than their capacity to absorb saving and hence prevent a ZLB episode. Similar intuitions are contained in Kraay and Ventura (2007) and Asriyan et al. (2016). Kraay and Ventura (2007) use a two-country model to explain how bubbles can absorb extra savings without exploring the possibility of ZLB episodes and SecStag. In contrast, Asriyan et al. (2016) emphasize the role of bubbles in preventing ZLB episodes, but they do not model explicitly the cause of low rates and ZLB.

2.2 A Model of Secular Stagnation with Rational Bubbles

The economy is represented by an OLG model with three agents: households, firms and a central bank in charge of monetary policy. The size of generation t is N_t and the growth rate of births is $(1+g_t) = \frac{N_t}{N_{t-1}}$.

Generations can borrow and lend to one another and exchange a variety of bubbly (worthless) assets, whose price cannot be negative because of free disposal. Middle-aged households initiate a new variety of bubbly asset and buy the old varieties, which are introduced by the previous cohorts. Each period a fraction of old bubbles is not longer traded. Bubbly asset is a pyramid scheme.[6] It is a claim

[6] "The start of a bubble generates a positive wealth shock which can literally be described as a transfer from the future. This is a central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, entrepreneurs assign themselves and sell the "rights" to the savings of a generation living in the very far future or, to

on future savings, so it entitles the owner to receive a payment, in the old age, from the next (middle) generation. By initiating a new bubble, middle-aged households directly issue this claim. Instead, by purchasing old bubbles, they buy the claims issued by the past generations. As bubbles allow agents to carry over funds to the old age, they represent an investment vehicle. The quantity of bubbly assets grows at the same rate as population. This, along with the process of bubble creation and destruction, makes the total amount of bubbly assets (not their value) equal to the size of the middle generation. Expectations about future value of new and old bubbles, and future inflation are formed rationally.

There is no capital accumulation. Goods and labor markets are perfectly competitive and there exist nominal rigidities. Each middle-aged agent runs a firm, which operates for just one period. As the number of firms is equal to the size of the middle generation, the economy's growth rate is $(1 + g_t)$. Households are reluctant to accept a nominal wage below a minimum level, given by a wage norm.[7] This assumption allows for the non-neutrality of monetary policy, which is specified in terms of a standard Taylor rule.

I will show that this extended model, although highly stylized, is able to explain the absence of low interest rates before the crisis, which was accompanied by stock and house prices at historically high levels due to bubbly episodes. In the model, the bubble counteracts the downward pressure put on the natural rate by a demographic shift, diverting resources away from riskless bonds and decreasing savings.[8]

be more exact, living at infinity. This appropriation of rights is a pure windfall or wealth gain for the entrepreneurs" (Martin and Ventura, 2011, p. 27). For real-world examples of bubbly assets, see Martin and Ventura (2012).

[7] As pointed out by Schmitt-Grohé and Uribe (2016), a downwardly rigid nominal wage does not alter the structure of the labor market, because workers and employers are still wage takers.

[8] The presence of capital would lead to a counterfactual pattern for capital accumulation, as highlighted by Martin and Ventura (2012). This problem, which is not relevant here, where the focus is on the natural and real interest rates, can be solved by introducing financial frictions, along lines suggested by Martin and Ventura (2012).

2.2.1 Households

The representative household goes through three stages of life: youth, middle and old age. New borns can finance their consumption by borrowing up to an exogenous debt limit, D_t.[9] Middle-aged households receive a fraction $\delta \in (0,1)$ of a new bubbly asset whose price is $Q^B_{t|t} \geq 0$, whereas a portion δ of the old bubbly assets loses value. Furthermore, they sell inelastically labor ($L_t = \bar{L}$) for a wage W_t and obtain profits Z_t.[10] A fraction of this income is saved for retirement and invested in two kinds of assets: a one period riskless bond, which is sold by the young generation in the loan market; different varieties of old bubbly assets, which are sold by the old generation in the bubbly assets market. Given these assumptions, the representative household seeks to maximize:

$$\max_{C^y_t, C^m_{t+1}, C^o_{t+2}, Z^B_{t+1|t+1-k}} E_t \left\{ \ln C^y_t + \beta \ln C^m_{t+1} + \beta^2 \ln C^o_{t+2} \right\}$$

s.t.

$$C^y_t = B^y_t \tag{2.1}$$

$$C^m_{t+1} = Y_{t+1} + \delta Q^B_{t+1|t+1} - (1 + r_t) B^y_t - B^m_{t+1} - \sum_{k=0}^{\infty} Q^B_{t+1|t+1-k} Z^B_{t+1|t+1-k} \tag{2.2}$$

$$C^o_{t+2} = (1 + r_{t+1}) B^m_{t+1} + (1 - \delta)(1 + g_t) \sum_{k=0}^{\infty} Q^B_{t+2|t+1-k} Z^B_{t+1|t+1-k} \tag{2.3}$$

$$(1 + r_t) B^i_t \leq D_t \tag{2.4}$$

where C^i_t is the consumption of each generation and B^i_t the real value of risk-free bonds with $i = y, m, o$. $Z^B_{t|t-k}$ is the quantity at time t of bubbly assets introduced in $t - k$ and $Q^B_{t|t-k}$ their price. Equation (2.4) is the exogenous borrowing limit,

[9] This reflects the common view about the safe level of leverage, as in Eggertsson and Krugman (2012).

[10] Note that $Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}$.

which is binding for young households:[11]

$$B_t^y = \frac{D_t}{(1+r_t)} \tag{2.5}$$

The optimality conditions for this problem is the standard Euler equation:

$$\frac{1}{C_t^m} = \beta\left(1+r_t\right)E_t\frac{1}{C_{t+1}^o} \tag{2.6}$$

and:

$$Q_{t|t-k}^B = \left(1-\delta\right)\left(1+g_t\right)\beta E_t\left[\left(\frac{C_t^m}{C_{t+1}^o}\right)Q_{t+1|t-k}^B\right] \tag{2.7}$$

which is the market value of the bubbly asset and depends on its future expected value. Now, we turn to bubbly assets and loan markets. Bubbly assets market clearing requires:

$$Z_{t|t-k}^B = \delta\left(1-\delta\right)^k \tag{2.8}$$

for $k = 0, 1, 2....$

$Z_{t|t-k}^B$ represents the amount of the bubbly asset $t-k$ owned by each middle-aged household, whereas the total amount traded in the market is $N_{t-1}Z_{t|t-k}^B$. I define an economy's bubble index, normalized in terms of the size of the middle generation:

$$Q_t^B = \frac{\tilde{Q}_t^B}{N_{t-1}} = \delta\sum_{k=0}^{\infty}\left(1-\delta\right)^k Q_{t|t-k}^B \tag{2.9}$$

and a corresponding index for the "preexisting" bubbles:

$$B_t = \frac{\tilde{B}_t}{N_{t-1}} = \delta\sum_{k=1}^{\infty}\left(1-\delta\right)^k Q_{t|t-k}^B \tag{2.10}$$

where $\tilde{Q}_t^B = \delta N_{t-1}\sum_{k=0}^{\infty}\left(1-\delta\right)^k Q_{t|t-k}^B$ and $\tilde{B}_t = \delta N_{t-1}\sum_{k=1}^{\infty}\left(1-\delta\right)^k Q_{t|t-k}^B$. Com-

[11] This is the case, if $D_{t-1} < \frac{1}{1+\beta(1+\beta)}\left(Y_t + \delta Q_{t|t}^B\right)$.

bining equations (2.7), (2.9) and (2.10) yields:

$$Q_t^B = U_t + B_t = (1 + g_t)\, \beta E_t \left[\left(\frac{C_t^m}{C_{t+1}^o} \right) B_{t+1} \right] \tag{2.11}$$

where $U_t = \delta Q_{t|t}^B$ is the market value of the new bubble. The aggregate bubble index is driven by two forces: the new and the preexisting bubbles.

The consumption-savings decision of the household affects the equilibrium in the loan market, which requires:

$$N_t B_t^y = N_{t-1} B_t^m$$

or, equivalently:

$$(1 + g_t)\, B_t^y = B_t^m \tag{2.12}$$

Denote the left-hand side of (2.12), the loan demand, with L_t^d and the right-hand side, the loan supply, with L_t^s. Using (2.5) to substitute for B_t^y, the loan demand can be written as:

$$L_t^d = \frac{(1 + g_t)}{(1 + r_t)} D_t \tag{2.13}$$

Combining (2.2), (2.3), (2.5) and (2.6), we obtain the loan supply:[12]

$$L_t^s = \frac{\beta}{1 + \beta} (Y_t - D_{t-1} - B_t) - \frac{(1 + g_t)\, E_t B_{t+1}}{(1 + \beta)(1 + r_t)} \tag{2.14}$$

Finally, combining (2.6) and (2.11), and substituting for B_{t+1}, equation (2.14) can

[12] By using (2.8), (2.9) and (2.10), we get:

$$\sum_{k=0}^{\infty} Q_{t|t-k}^B Z_{t|t-k}^B = U_t + B_t$$

$$(1 - \delta) \sum_{k=0}^{\infty} Q_{t+1|t-k}^B Z_{t|t-k}^B = E_t B_{t+1}$$

be expressed as:

$$L_t^s = \frac{\beta}{1+\beta}(Y_t - D_{t-1} - B_t) - \frac{1}{1+\beta}(B_t + U_t) \tag{2.15}$$

By equating loan demand and supply we derive the equilibrium real interest rate:

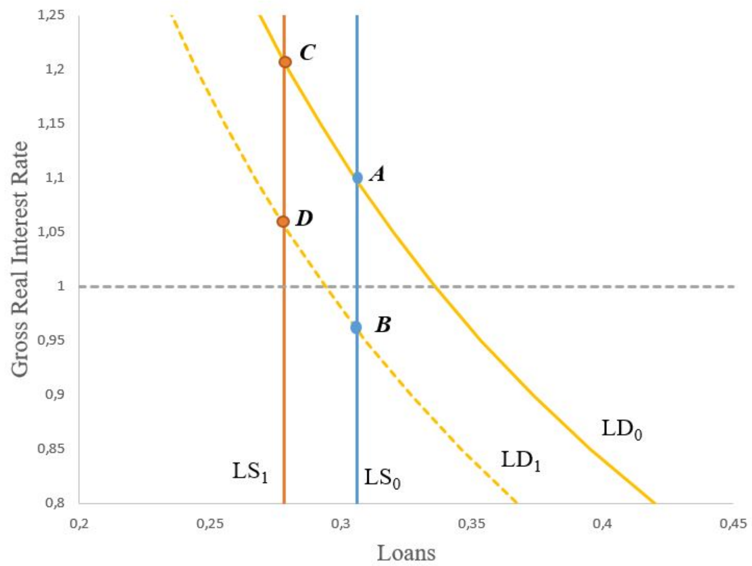
$$(1+r_t) = \frac{(1+g_t)(1+\beta)D_t}{\beta(Y_t - D_{t-1} - B_t) - (B_t + U_t)} \tag{2.16}$$

Equations (2.15) and (2.16) highlight the effect of bubbles on savings and the equilibrium real interest rate. Firstly, the possibility of purchasing bubbly assets decreases the income available for investing in riskless bonds ($(Y_t - D_{t-1} - B_t)$ in equation 2.15). Secondly, investment in bubbly assets reduces the need to save by increasing income in the old age ($-\frac{1}{1+\beta}(B_t + U_t)$ in equation 2.15).

The combination of these effects implies a lower loan supply in a bubbly economy (LS_1 in Figure 2.6) than in a bubbleless one (LS_0), and so a higher equilibrium real interest rate. This is crucial, when the economy is hit by a permanent shock to population growth[13] and the loan demand shifts from LD_0 to LD_1 (Figure 2.6). In a bubbleless world, where $B_t = U_t = 0$, the equilibrium goes from point A to B and the equilibrium interest rate turns negative. The same does not necessarily happen in a bubbly economy, which starts from C and reaches a new equilibrium position (point D), in which the real interest rate is lower, but still positive.

[13] The same argument applies to any other type of permanent shock that leads to a negative real interest rate.

Figure 2.6: Equilibrium in the asset market



2.2.2 Firms

Firms operate in a perfectly competitive market and use labor as the only input. Taking prices as given, they seek to maximize their profit:

$$Z_t = P_t Y_t - W_t L_t \tag{2.17}$$

subject to the production function:

$$Y_t = L_t^{\alpha} \tag{2.18}$$

The resulting labor demand is:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha - 1} \tag{2.19}$$

The nominal wage is downwardly rigid, because workers will never accept a wage lower than:

$$\tilde{W}_t = \gamma W_{t-1} + (1-\gamma)\, P_t \alpha \bar{L}^{\alpha-1} \tag{2.20}$$

where γ is the degree of rigidity. When γ is 0, $\tilde{W}_t$ reduces to the flexible wage; when it is 1, the wage is perfectly downwardly rigid. This assumption implies:

$$W_t = \max\left(\tilde{W}_t, P_t \alpha \bar{L}^{\alpha-1}\right) \tag{2.21}$$

When labor market clearing requires a higher nominal wage than the past one, the wage is given by $P_t \alpha \bar{L}^{\alpha-1}$ with $L_t = \bar{L}$, otherwise it is determined by (2.20) with $L_t < \bar{L}$.

2.2.3 Monetary policy

The central bank sets the nominal interest rate following a standard Taylor rule:

$$1 + i_t = \max\left(1, (1+\bar{i})\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right) \tag{2.22}$$

where $\phi_\pi > 1$. $\bar{i}$ and $\overline{\Pi}$ are the central bank targets for nominal rate and inflation, where $(1+\bar{i}) = \left(1+r^f\right)\overline{\Pi}$. According to (2.22), when the nominal rate is far away from the ZLB, the monetary authority stabilizes inflation around the target $\overline{\Pi}$. Of course, there is also the standard Fisher equation:

$$1 + r_t = (1+i_t)\, E_t \Pi_{t+1}^{-1} \tag{2.23}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation rate. I define a perfect foresight equilibrium as a set of prices $\{P_t, W_t, r_t, i_t\}$ and quantities $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, Y_t, Z_t, L_t, B_t\}$ that solve (2.1), (2.2), (2.3), (2.5), (2.6), (2.11), (2.12), (2.17), (2.18), (2.19), (2.21), (2.22) and (2.23), given $\{D_t, g_t, U_t\}$ and initial values for W_{-1}, B^m_{-1} and B_{-1}

2.3 Equilibrium

2.3.1 Equilibrium dynamics

In the next paragraphs, the focus of my analysis will be the full employment bubbly steady state (hereafter FEB), which replicates the US economy before the financial crisis. The condition for its existence must hence be singled out.

Assume $g_t = g$, $D_t = D$ and $U_t = U$. Nominal rigidities and the ZLB are not binding, so equations (2.21) and (2.22) become:

$$W_t = \alpha P_t \bar{L}^{\alpha-1}$$

$$1 + i_t = \left(1 + r^f\right) \overline{\Pi} \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}$$

It follows from (2.19) that $L_t = \bar{L}$ and, in a sequence, $Y_t = \bar{L}^\alpha = Y^f$ and $\frac{Z_t}{P_t} = (1-\alpha) Y^f$. Combining (2.6) and (2.11), we get:

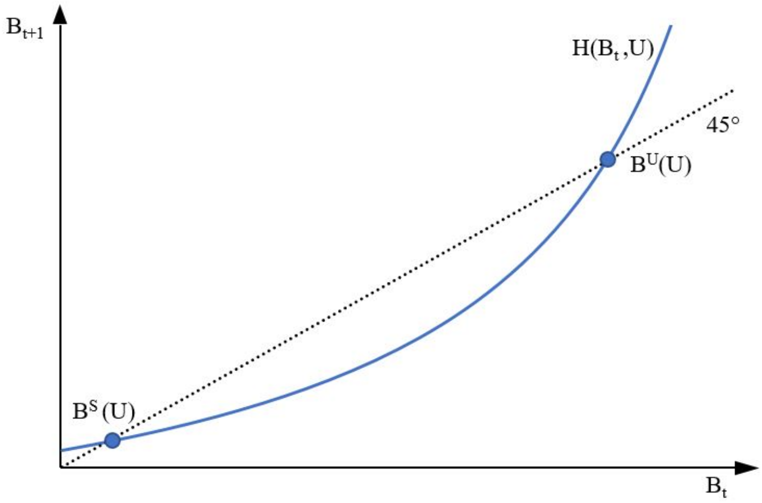
$$B_{t+1} = \left(\frac{1+r_t}{1+g}\right)(U + B_t)$$

and substituting for $(1 + r_t)$ yields:

$$B_{t+1} = \frac{(1+\beta) D (U + B_t)}{\beta (Y^f - D - B_t) - (U + B_t)} = H(B_t, U) \tag{2.24}$$

Given a sequence $\{B_t\}$ with $B_t \in (0, Y^f)$ for all t and g, D, $U \geq 0$, the equilibrium values for all variables can be derived from the remaining conditions. A FEB is a pair (B, U) satisfying $B = H(B, U)$ with $B \in (0, Y^f)$ and g, D, $U \geq 0$. Its stability

Figure 2.7: Bubble dynamics



is guaranteed by the condition $\partial H(B,U)/\partial B < 1$,[14] whereas the necessary and sufficient condition for its existence is:[15]

$$D < \frac{\beta}{(1+\beta)}\left(Y^f - D\right) \tag{2.25}$$

As shown by Figure 2.7, when (2.25) is met, there exists a continuum of stable steady states $\left(B^S(U), U\right)$ and a continuum of unstable ones $\left(B^U(U), U\right)$ for any $U \in [0, \bar{U})$ with $\bar{U} = \left[(1+\beta)(1+2\beta)D + \beta\left(Y^f - D\right)\right] + 2(1+\beta)\sqrt{\beta\left(Y^f + \beta D\right)D}$. In the rest of the chapter, I focus only on the stable FEB. A crucial role for the existence of bubbles is played by the two parameters β and D. When the debt limit is too low, young households cannot raise enough funds to absorb all savings, especially if the discount factor is very high and so people save much during the middle age. With excess savings, the real interest rate falls below the economy's growth rate (the condition above leads to $(1+r) < (1+g)$ in a bubbleless steady

[14] This condition is identical to that in Galí (2014) and it also guarantees the stationarity of the bubble. See appendix A.

[15] The proof for this condition is provided in appendix A.

state), therefore intrinsically worthless assets are valued and bubbles appear in order to absorb inefficient savings. $(1+r) < (1+g)$ still holds in a FEB. The value of old bubbly assets cannot grow at the same rate as that of the economy, because it would otherwise grow unboundedly, violating the definition of steady state. This condition, along with the presence of new bubbly assets, guarantees that the value of the aggregate bubble remains constant over time and imposes an upper bound on B. The upper bound represents the value for which the real interest rate in a FEB equals the growth rate of population, namely $B^U(0) = \frac{\beta}{1+\beta}\left(Y^f - D\right) - D$.

2.3.2 Full employment bubbly equilibrium

Equations (2.21) and (2.22) determine the regime of aggregate supply and aggregate demand, which summarize the steady state of the model. Let me start with aggregate supply (AS), which comprises a vertical and an upward sloping segment with the kink at $\Pi = 1$. With positive inflation ($\Pi \geq 1$), an increase of the nominal wage clears the labor market and so AS is vertical:

$$Y_t = \bar{L}^{\alpha} = Y^f \tag{2.26}$$

When inflation is negative ($\Pi < 1$), the nominal wage cannot fall enough to clear the market and we have unemployment ($L_t < \bar{L}$). Substituting the steady state real wage $\omega = \frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{1-\gamma\Pi^{-1}}$ into (2.19) and using (2.18), we get the upward sloping AS:

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)\left(\frac{Y}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} \tag{2.27}$$

in this case, as inflation goes up, real wages fall and firms produce more.

Similarly, aggregate demand (AD) has two regimes: one with a positive nominal interest rate and another one with binding ZLB. In the first case, combining equations (2.16), (2.22) and (2.23), we obtain a standard downward sloping AD:

$$Y = D + \frac{U}{\beta} + \left(\frac{1+\beta}{\beta}\right) B + \left(\frac{1+\beta}{\beta}\right)(1+g)\frac{\Gamma}{\Pi^{\phi_\pi - 1}} D \tag{2.28}$$

where $\Gamma = \overline{\Pi}^{\phi_\pi - 1}\left(1+r^f\right)^{-1}$. When the lower bound on the nominal rate is binding ($i = 0$), by following the same steps, we get an upward sloping AD:

$$Y = D + \frac{U}{\beta} + \left(\frac{1+\beta}{\beta}\right) B + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D \tag{2.29}$$

In liquidity trap a rise of inflation decreases real interest rates, thus expanding demand and output.

The effect of debt on aggregate demand is twofold: it reduces spending of middle-aged households, for which risk-free assets are liabilities, issued when young; and it provides income to old households, which invested in riskless bonds during the middle age. As the marginal propensity to consume of the elderly (1) is greater than that of middle-aged people ($\frac{1}{1+\beta}$), the net effect on demand is positive, as expressed by the term D in (2.28) and (2.29). Anyway, the negative impact of debt on C^m offsets the multiplicative effect and so D is not multiplied for $\left(\frac{1+\beta}{\beta}\right)$, the keynesian multiplier.[16] B and U express the effect of bubbles on demand. Bubbly assets are a source of income for old households. This contributes to their consumption and so to aggregate demand, as summarized by the term $\left(\frac{1+\beta}{\beta}\right) B$. The presence of this income also increases the consumption of middle-aged households by reducing their need to save. This effect is captured by $\frac{U}{\beta}$. The multiplier is $\frac{1}{\beta}$, because a new bubble worth one dollar raises C^m by $\frac{1}{1+\beta}$. As a consequence, in the FEB,[17] which occurs at the intersection of the vertical AS and the downward

[16] As the marginal propensity to consume of middle-aged households is $\frac{1}{1+\beta}$, the multiplier is $\frac{1}{1-c} = \frac{1}{1-\frac{1}{1+\beta}} = \frac{1+\beta}{\beta}$.

[17] The standard Taylor principle ($\phi_\pi > 1$) guarantees the determinacy of this equilibrium, which is unique for high enough γ and low enough inflation target.

sloping AD, consumption of middle-aged and old households is higher than that in the bubbleless full employment equilibrium (where $U = B = 0$):

$$\begin{aligned} C^m &= Y^f - D - B - \frac{\beta\left(Y^f - D - B\right)}{(1+\beta)} + \frac{1}{(1+\beta)}(U+B) \\ &= \frac{1}{1+\beta}\left(Y^f - D\right) + \frac{1}{(1+\beta)}U \end{aligned} \tag{2.30}$$

$$C^o = (1+g)(D+B) \tag{2.31}$$

Equation (2.30) is derived from (2.2) by using (2.5) and (2.15), while we get (2.31) from (2.3) by using (2.12) and (2.13), and by expressing the summation in terms of B.[18] As clarified by (2.30), the middle generation consumes more, even though investing in bubbly assets reduces the available income ($-B$), because the reduction of saving frees up additional resources for consumption. The positive effect of bubbles on C^m and C^o is fully offset by the decline in C^y:[19]

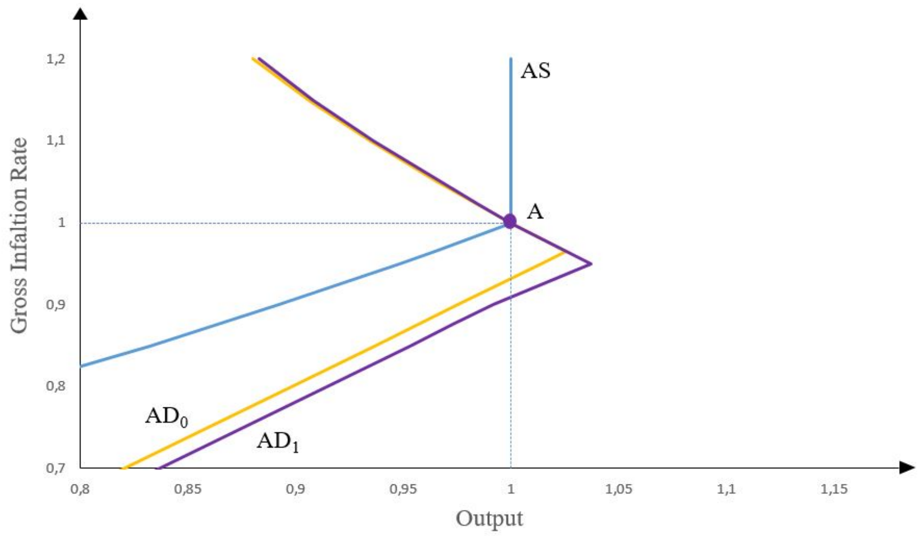
$$C^y = \frac{D}{(1+r)} = \frac{1}{(1+g)}\left[\frac{\beta\left(Y^f - D - B\right)}{(1+\beta)} - \frac{1}{(1+\beta)}(U+B)\right] \tag{2.32}$$

Equation (2.32) is computed by substituting for the steady state value of the real interest rate (which equals the natural level). Bubbles negatively affect the steady state consumption of young people, because they push the real interest rate up. As the net effect on aggregate demand is zero, the FEB is still characterized by inflation at the target level and output at the potential level, like the full employment steady state without bubbles. This is depicted in Figure 2.8. AD_0 and AD_1 represent, respectively, the aggregate demand in a bubbleless and in a

[18] See footnote 12.

[19] The effect of C^y on aggregate demand is represented by $\left(\frac{1+\beta}{\beta}\right)(1+g)\frac{\Gamma}{\Pi^{\phi_\pi - 1}}D$ in (2.28) and by $\left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi D$ in (2.29). $\frac{\Pi^{\phi_\pi - 1}}{\Gamma}$ is the real interest rate, when the ZLB does not bind, and $\frac{1}{\Pi}$ the real interest rate with binding ZLB.

Figure 2.8: AD and AS with rational bubbles



bubbly economy and both cross the vertical AS at point A, where $Y = Y^f$ and $\Pi = \overline{\Pi}$. The only difference between these two curves is the AD kink, which is the inflation level at which monetary policy is constrained by the ZLB and demand becomes upward sloping. This can be derived by equating the two arguments in (2.22):

$$\Pi_{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \overline{\Pi}^{\frac{\phi_\pi - 1}{\phi_\pi}} \tag{2.33}$$

The AD kink is lower in a bubbly economy than in a bubbleless one, because the natural interest rate is higher, as discussed above.[20] Therefore, for any level of inflation, it is less likely for the central bank to hit the ZLB and this has fundamental implications, when a permanent change to aggregate demand occurs.

[20] The real interest rate reaches its natural level in a full employment equilibrium.

2.4 Avoiding Secular Stagnation: the Mechanism

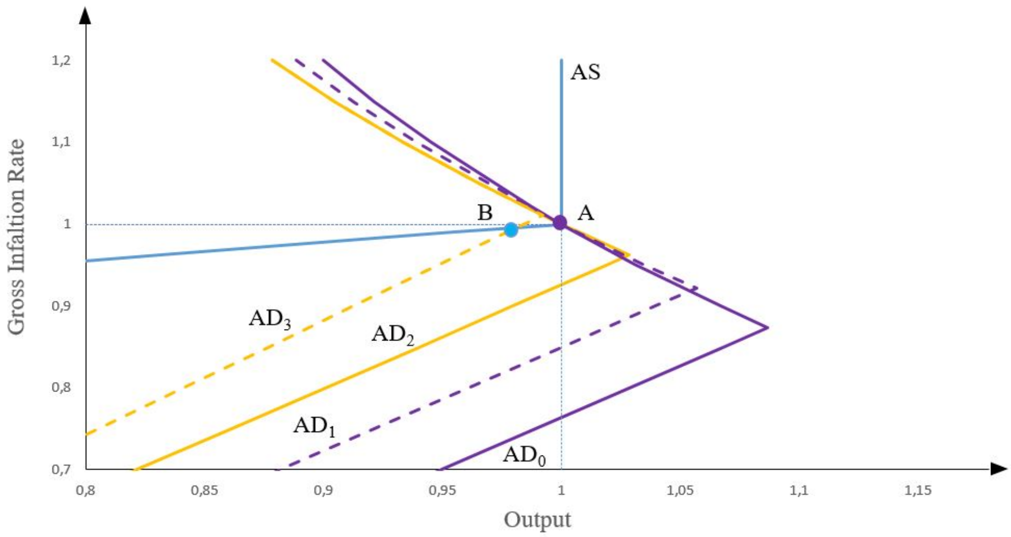
In this section, I describe how bubbles prevent SecStag, when a permanent shock to population hits the economy. The mechanism which I outlines does no rely on the specific shock assumed and, if another permanent change exerting downward pressure on interest rates is considered, it works in the same way.

A permanent change in g decreases the fraction of young households, reducing aggregate expenditure. The central bank cuts the nominal interest rate to stimulate the consumption of middle-aged agents and in this way compensate the drop in demand. If the economy starts from the bubbleless full employment equilibrium, the monetary authority cannot keep demand at the potential level, when the demographic shock leads to a negative natural interest rate and the inflation target is zero, because the ZLB binds. The resulting lack of demand creates deflationary pressures, so nominal rigidities are at work and wages cannot fall to clear the labor market. As a result, we have involuntary unemployment and output below the potential, as depicted in Figure 2.9.[21] The reduction in g raises the AD kink and the upward sloping demand shifts left from AD_2 to AD_3, which intersects supply in its upward sloping segment, not in the vertical one. The new intersection point is the SecStag equilibrium B, where $\Pi < \overline{\Pi}$ and $Y < Y^f$.

The same does not necessarily happen with rational bubbles. In this case, the downward sloping demand is longer. So, even if the upward sloping AD shifts left from AD_0 to AD_1, the equilibrium is still determined at the intersection of the vertical AS and the downward sloping AD (point A), where $\Pi = \overline{\Pi}$ and $Y = Y^f$. This clearly depends on the different location of the AD kink. As the rational bubble pushes the natural rate up, a fall in population growth does not lead to a negative natural interest rate and does not force the central bank to hit the ZLB.

[21] Deflation is not crucial for the existence of a SecStag equilibrium. See footnote 13 in chapter 1.

Figure 2.9: Effect of a demographic shift on bubbleless and bubbly economies



Therefore, the monetary authority can offset the demographic shock via cuts in the policy rate. This result is in accordance with the pattern of the FED funds rate before the Great Recession (Figure 2.4).

2.5 Conclusions

By introducing rational bubbles in the way suggested by Galí (2014) into Eggertsson et al. (2017), I have presented a model consistent with the recent stylized facts represented by the US data: the trend of the real interest rate reversed around the mid-1990s, while stock and house prices skyrocketed, and the US did not experience low interest rates before the 2007-2008 crisis, despite the underlying excess saving. In this way, my model provides theoretical grounds for the intuitions of Summers (2013) and Eggertsson et al. (2017) about the structural nature of the current low interest rates environment and the role of bubbles in postponing it.

Rational bubbles offset the downward pressure on the natural interest rate coming from a lower population growth and the central bank can escape the ZLB. The positive effect on the natural rate is mediated by savings via two channels. First, investment in bubbly assets absorbs a portion of savings. Second, bubbly assets provide an additional income in the old age and this induces agents to save less. In the light of the results here obtained, the next step should be to introduce the effect of bubbles on borrowing. This allows to investigate deeply the channels through which bubbles raise interest rates and to measure the minimum size of the bubble which is necessary to avoid SecStag.

Chapter 3

How Bubbles Allow to Escape from the ZLB

Asset price bubbles are an important source of instability, because their bursting is associated with large fluctuations in key macroeconomic variables. This view has shaped the debate about macroprudential and LAW policies in the last two decades. However, when structural forces depress demand, bubbles also raise the natural interest rate and allow to escape from the ZLB, with substantial gains in terms of output and employment. This additional role of bubbles is crucial in the current macroeconomic scenario. In the last decades interest rates have fallen in most of the advanced countries, forcing the central banks to hit the ZLB, after the Lehman Brothers bankruptcy. The declining trend of the real interest rates, which could make low interest rates very persistent, provides fertile ground for financial bubbles. This, in turn, creates a trade-off for the policy makers, which have a restricted range of policy options. Bubbles can temporarily improve economic conditions at the cost of a more likely and deeper recession in the future. To fully understand the nature of this trade-off, I study how asset price bubbles raise interest rates, with a special reference to the US economy before the Great

Recession.

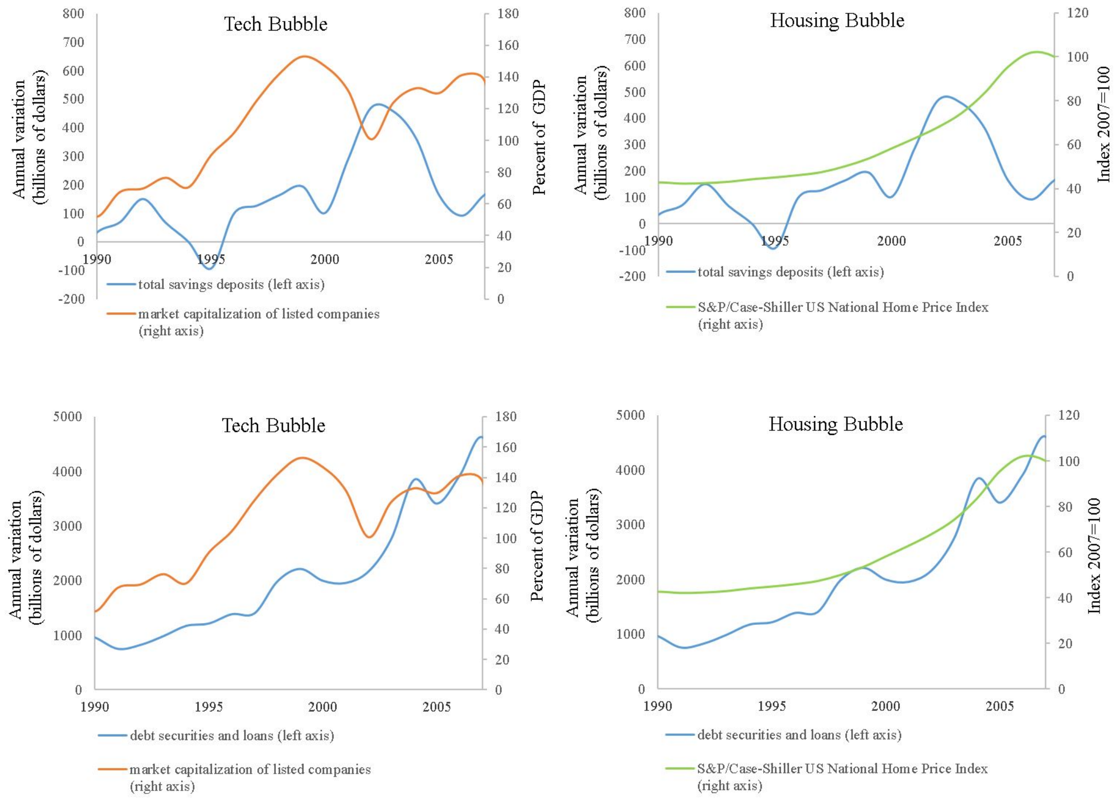
The bursting of the US housing bubble in 2007 triggered one of the most severe and prolonged downturn in economic history, the Great Recession. Even though the resulting anemic demand was reflected in historically low interest rates, these are not a consequence of the recent financial crisis. Rather, secular forces have driven US interest rates down in the last thirty years and low interest rates were masked, between the mid-1990s and the mid-2000s, by speculative movements in asset prices, as illustrated in the previous chapter. This makes the US economy before the Great Recession an illustrative example for the understanding of the channels through which bubbles affect interest rates.

Two bubbly episodes occurred in the US between 1995 and 2007: the tech and housing bubbles. Both were accompanied by substantial changes in the allocation of savings, in the saving behavior and in the amount of borrowing. Figure 3.1 plots the annual variation of the US savings deposits. These data are plotted in the top left panel together with the variation in stock prices and in the top right panel with the variation in house prices. As stock prices inflated in the mid-1990s, households diverted financial resources from savings accounts to stocks. This reallocation was abruptly interrupted in 2001, when the dot-com bubble crashed, but the resulting increase in savings accounts was short-lived, because households started to invest in a new profitable asset: house. In the same period, we observed a huge decline in the personal saving rate, which passed from 6.3% in 1994 to 3% in 2007.[1] Asset price bubbles also fostered borrowing by increasing the value of collaterals. As shown in the bottom left and right panels of Figure 3.1, the annual variation in the amount of the US debt securities closely follows the fluctuations in stock prices between 1995 and 2000, and the movements in house prices between 2002 and 2007.[2]

[1] Data from FRED.

[2] In most recent years asset prices have reached pre-crisis levels, but interest rates have

Figure 3.1: Savings accounts, debt securities and bubbles



Source: World Bank, FRED.

In order to explain how changes in saving and borrowing transmit to interest rates, I extend the model of the previous chapter by allowing for a bubbly collateral. Therefore, in this extended model, bubbles transfer resources between generations via two channels, which are related to their capacity of being simultaneously store of value and collateral. The introduction of an additional store of value causes a reallocation of savings from risk-free assets to bubbly assets and induces to save less, as shown in chapter 2. This is the “saving channel”. On the other

remained low. This evidence highlights the stark difference between the pre-crisis and the post-crisis periods. After the Great Recession, tighter financial regulation may have mitigated the effect of asset prices on interest rates, which have been further compressed by the unconventional monetary policies implemented by the FED.

hand, bubbly collateral relaxes the borrowing constraint increasing the amount of debt. This is the "borrowing channel". Through these channels, asset price bubbles replaced expansionary fiscal policy, which was not able to counteract the downward trend of the US interest rates before the 2007-2008 crisis. After the reallocation operated by bubbles, young agents are worse off and old ones are better off. The redistribution between generations, which matches the US data over the period 1995-2007, makes the bubble welfare reducing and this result relies on the exogenous debt limit in the young age. Furthermore, bubbles push the natural interest rate up. The saving channel is mainly responsible for this result, whereas a small contribution is provided by the borrowing channel, whose relative impact on interest rates decreases with the size of the bubble, unlike the saving channel. A higher natural interest rate than in a bubbleless economy prevents a liquidity trap, despite the downward pressure exerted by a demographic shock. This result holds for a reasonable large bubble size and for a realistic calibration of the demographic shock.

The chapter is organized in the following way. In section 3.2 and 3.3 I outline the model and study its equilibrium. In section 3.4, I calibrate the model. Section 3.5 compares fiscal policy with asset price bubbles and section 3.6 concludes.

3.1 Related Literature

This chapter is related to the recent literature on secular stagnation (Summers, 2013, 2014, 2015; Gordon, 2015; Eggertsson et al., 2016; Eggertsson et al., 2017). This strand of the economic literature originates from the intuitions of Summers (2013), who interprets the slow recovery out of the Great Recession as the beginning of a prolonged period of anemic demand and economic activity in the US. My work is complementary to these papers, because it proves how asset price bubbles postponed the beginning of SecStag.

Furthermore, my work is inspired by the extended literature on rational bubbles (Samuelson, 1958; Tirole, 1985; Caballero and Krishnamurthy, 2006; Farhi and Tirole, 2012; Martin and Ventura, 2012; Galí, 2014; Asriyan et al., 2016). In my model, bubbly assets originate from excess saving like in Samuelson (1958) and Tirole (1985), but the bubble is not welfare enhancing. Bubble creation and destruction are modelled in a fashion to Galí (2014), but I analyze the role of bubbles in preventing low interest rates, neglecting the macroeconomic instability associated with bubbly episodes. Asriyan et al. (2016) also focus on the role of bubbles in preventing liquidity traps, but they do not study explicitly the mechanisms through which bubbles raise interest rates.

3.2 The Model Economy

I consider an OLG economy without capital which consists of households, firms and a central bank. The generation t is composed by N_t agents and the population growth rate is $(1+g_t) = \frac{N_t}{N_{t-1}}$.

The representative agent lives for three periods: youth, middle and old age. The structure of the goods and labor markets is that assumed in the previous chapter. The shortage of investment opportunities allows for the existence of bubbly assets, which are intrinsically worthless and so their price cannot be negative.[3] The total amount of bubbles grows at the same rate as population. This assumption, along with those on bubble creation and destruction, makes the total amount of bubbly assets equal to the number of middle-aged households. Bubbles allow agents to carry over funds to the old age and so represent an investment vehicle. Furthermore, the introduction of new bubbles improves the ability to repay debt

[3] Bubble is still a pyramid scheme. For further details see chapter 2.

and this, in turn, relaxes the borrowing limit faced during youth.[4] Agents form expectations about the future value of bubbly assets and inflation rationally.

3.2.1 Setup

When young, agents borrow to finance their consumption and face an exogenous debt limit, D_t, which can be relaxed by using future bubbly assets as collaterals. Middle-aged households are endowed with a portion $\delta \in (0,1)$ of a new bubbly asset whose price is $Q^B_{t|t} \geq 0$, whereas a fraction δ of the old bubbly assets loses value. They also supply inelastically labor ($L_t = \bar{L}$) for a wage W_t and run a firm, whose profits are Z_t.[5] This income is partially invested in one period riskless bonds and different varieties of old bubbly assets, which are supplied by the young and old generations respectively. All the proceeds from savings are consumed in the old age. The representative household solves the problem:[6]

$$\max_{C^y_t, C^m_{t+1}, C^o_{t+2}, Z^B_{t+1|t+1-k}} E_t \left\{ \ln C^y_t + \beta \ln C^m_{t+1} + \beta^2 \ln C^o_{t+2} \right\}$$

s.t.

$$C^y_t = B^y_t \tag{3.1}$$

$$C^m_{t+1} = Y_{t+1} + \delta Q^B_{t+1|t+1} - (1+r_t) B^y_t - B^m_{t+1} - \sum_{k=0}^{\infty} Q^B_{t+1|t+1-k} Z^B_{t+1|t+1-k} \tag{3.2}$$

$$C^o_{t+2} = (1+r_{t+1}) B^m_{t+1} + (1-\delta)(1+g_t) \sum_{k=0}^{\infty} Q^B_{t+2|t+1-k} Z^B_{t+1|t+1-k} \tag{3.3}$$

$$(1+r_t) B^i_t \leq D_t + \delta E_t Q^B_{t+1|t+1} \tag{3.4}$$

[4] The fact that only new bubbles can be used as collateral makes the model more tractable, without affecting the results.

[5] Recall that $Y_t = \frac{W_t}{P_t} L_t + \frac{Z_t}{P_t}$.

[6] The notation is the same of chapter 2.

Equation (3.4) denotes the exogenous debt limit, which is binding for young households:[7]

$$B_t^y = \frac{D_t + \delta E_t Q^B_{t+1|t+1}}{(1+r_t)} \tag{3.5}$$

The optimality conditions for this problem are the Euler equation and the market value of bubbly assets:

$$\frac{1}{C_t^m} = \beta \left(1+r_t\right) E_t \frac{1}{C_{t+1}^o} \tag{3.6}$$

$$Q^B_{t|t-k} = \left(1-\delta\right)\left(1+g_t\right)\beta E_t \left[\left(\frac{C_t^m}{C_{t+1}^o}\right) Q^B_{t+1|t-k}\right] \tag{3.7}$$

The technology of firms is described by the production function:

$$Y_t = L_t^\alpha \tag{3.8}$$

where L_t is the quantity of labor. Taking prices as given, firms maximize their profit:

$$Z_t = P_t Y_t - W_t L_t \tag{3.9}$$

subject to (3.8). The resulting optimality condition is the labor demand:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \tag{3.10}$$

The nominal wage is downwardly rigid:

$$W_t = \max\left(\tilde{W}_t, P_t \alpha \bar{L}^{\alpha-1}\right) \tag{3.11}$$

[7] For $D_{t-1} < \frac{1}{1+(1+\beta)\beta}\left[Y_t - \beta\left(1+\beta\right)\delta Q^B_{t|t}\right]$. This condition is met in all the simulations presented below.

$\tilde{W}_t$ denotes the minimum wage workers are willing to accept and it takes the shape:

$$\tilde{W}_t = \gamma W_{t-1} + (1-\gamma) P_t \alpha \bar{L}^{\alpha-1} \tag{3.12}$$

where γ is the degree of rigidity. The central bank behaves according to the interest rate rule:

$$1 + i_t = \max\left(1, \left(1 + r^f\right) \overline{\Pi} \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\right) \tag{3.13}$$

where $\phi_\pi > 1$. r^f and $\overline{\Pi}$ are the natural interest rate and the inflation target. Finally, we have the Fisher equation:

$$1 + r_t = (1 + i_t) E_t \Pi_{t+1}^{-1} \tag{3.14}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation rate.

3.2.2 Loan and bubbles markets

Generations exchange financial assets in two markets, which are the bubbly assets and the loan markets. Bubbly assets market clearing requires:

$$Z^B_{t|t-k} = \delta (1-\delta)^k \tag{3.15}$$

for $k = 0, 1, 2....$

The economy's bubble index is:

$$Q^B_t = \frac{\tilde{Q}^B_t}{N_{t-1}} = \delta \sum_{k=0}^{\infty} (1-\delta)^k Q^B_{t|t-k} \tag{3.16}$$

and the index for the old bubbles is:

$$B_t = \frac{\tilde{B}_t}{N_{t-1}} = \delta \sum_{k=1}^{\infty} (1-\delta)^k Q^B_{t|t-k} \tag{3.17}$$

where $\tilde{Q}^B_t = \delta N_{t-1} \sum_{k=0}^{\infty} (1-\delta)^k Q^B_{t|t-k}$ and $\tilde{B}_t = \delta N_{t-1} \sum_{k=1}^{\infty} (1-\delta)^k Q^B_{t|t-k}$. Both are normalized in terms of the size of the middle generation. Combining equations (3.7), (3.16) and (3.17), we get:

$$Q^B_t = U_t + B_t = (1+g_t)\,\beta E_t \left[\left(\frac{C^m_t}{C^o_{t+1}} \right) B_{t+1} \right] \tag{3.18}$$

where $U_t = \delta Q^B_{t|t}$ is the value of new bubbly assets. The equilibrium in the loan market requires:

$$(1+g_t)\, B^y_t = B^m_t \tag{3.19}$$

Denote the loan demand (the left-hand side of (3.19)) with L^d_t and the loan supply (the right-hand side) with L^s_t. Substituting (3.5) into the loan demand, we obtain:

$$L^d_t = \frac{(1+g_t)}{(1+r_t)} (D_t + E_t U_{t+1}) \tag{3.20}$$

Combining (3.2), (3.3), (3.5) and (3.6) yields the loan supply:[8]

$$L^s_t = \frac{\beta}{1+\beta} (Y_t - D_{t-1} - U_t - B_t) - \frac{(1+g_t)\, E_t B_{t+1}}{(1+\beta)(1+r_t)}$$

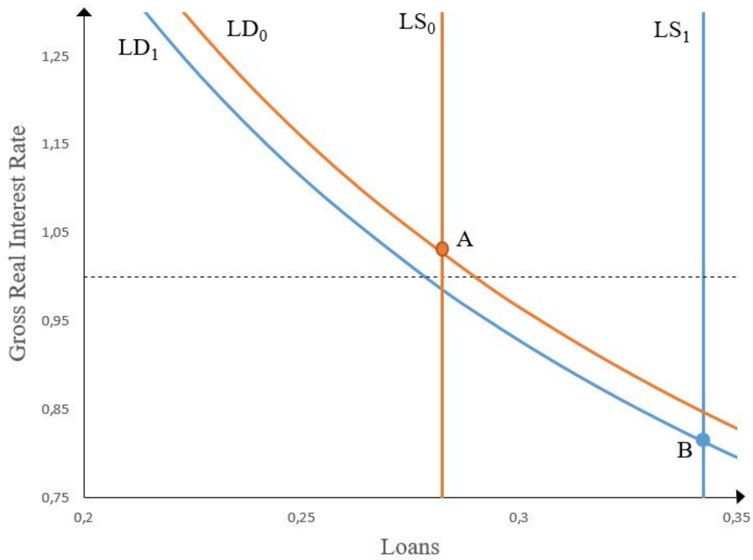
which can be alternatively expressed as:

$$L^s_t = \frac{\beta}{1+\beta} (Y_t - D_{t-1} - U_t - B_t) - \frac{1}{1+\beta} (B_t + U_t) \tag{3.21}$$

by combining (3.6) and (3.18) and substituting the resulting expression for B_{t+1}.

[8] See footnote 12 in chapter 2.

Figure 3.2: Equilibrium in the asset market

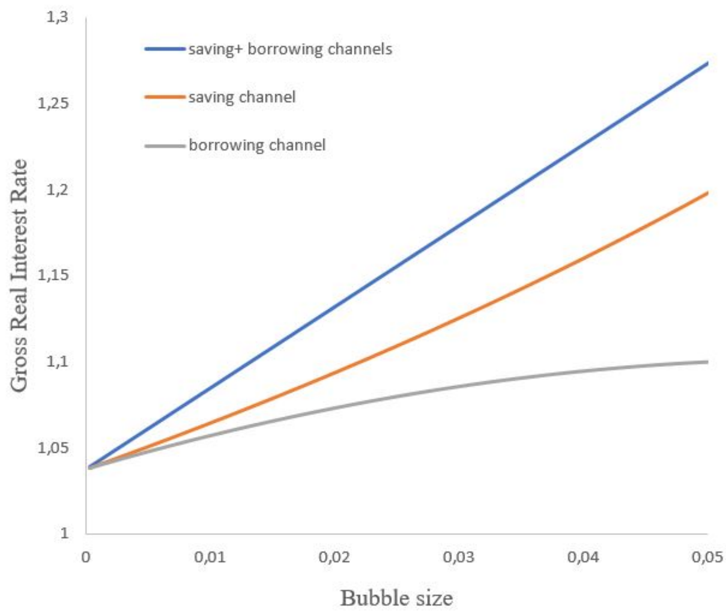


Finally, the equilibrium real interest rate is given by:

$$(1 + r_t) = (1 + g_t) \left[\frac{(1+\beta)(D_t + E_t U_{t+1})}{\beta (Y_t - D_{t-1} - U_t - B_t) - (B_t + U_t)} \right] \tag{3.22}$$

Equations (3.20) and (3.21) show the two channels through which bubbles affect borrowing and saving. These channels reflect the two functions of bubbles. Firstly, they are store of value ("saving channel"). The presence of an alternative investment vehicle diverts resources away from riskless bonds ($-\frac{\beta}{1+\beta}B_t$ in equation (3.21)), and it alters the saving behavior, because an additional income in the old age induces to save less in the middle age ($-\frac{1}{1+\beta}(B_t + U_t)$ in (3.21)). Secondly, bubbly assets serve as collateral ("borrowing channel"). This increases the loan demand by $\frac{(1+g_t)}{(1+r_t)}E_t U_{t+1}$ in equation (3.20). Hence, young households have a higher debt to repay during the middle age and this decreases savings ($-\frac{\beta}{1+\beta}U_t$ in equation (3.21)). Overall, these effects determine a lower loan supply (LS_0 in Figure 3.2) and a greater loan demand (LD_0) in a bubbly economy than in a bubbleless

Figure 3.3: Aggregate bubble and real interest rate



one (LS_1 and LD_1 respectively). Therefore, a bubbly economy has a higher equilibrium real interest rate and a permanent negative force does not necessarily lead it in negative territory, as shown in Figure 3.2.

Furthermore, the real interest rate monotonically increases with the bubble size, as depicted in Figure 3.3.[9] This figure also shows the relative strength of borrowing and saving channels, whose pure effect cannot be derived directly from (3.22) because of their interaction. So, I plot the value of the real interest rate, if one channel operates and the other one does not. The strength of the channels varies according to the size of the aggregate bubble.[10] For small aggregate

[9] In these numerical examples I only consider a (stable) full employment bubbly steady state and I set $\beta = 0.987$, $\gamma = 0.98$, $\alpha = 0.7$, $\overline{\Pi} = 1$, $\phi_\pi = 2$, $D = 0.22$, $g = 0.023$ and $L = 1$. All these values are in annual terms and they have to be converted to 20 years. A similar consideration applies to all calibrations reported in the rest of the chapter. In a bubbly deflationary equilibrium the relation between the aggregate bubble and the real interest rate changes. However, the analysis of this equilibrium goes beyond the scope of this work.

[10] As I have normalized the indexes for old and aggregate bubbles in terms of N_{t-1}, the size of the aggregate bubble only depends on the value of bubbly assets.

bubbles the share of the new ones is higher and this amplifies the impact of the borrowing channel, which only depends on the bubble creation process. As the bubble enlarges, the share of the new bubbles shrinks and that of the old ones increases.[11] This amplifies the saving channel. On the one hand, larger aggregate bubbles strongly decrease savings guaranteeing more proceeds during the old age; on the other hand, larger old bubbles divert more resources away from riskless bonds during the middle age. Independently of the size of the bubble, most of the increase in the real interest rate is due to the saving channel and this holds for different calibrations of β and D.[12]

3.3 Equilibrium

3.3.1 Equilibrium dynamics

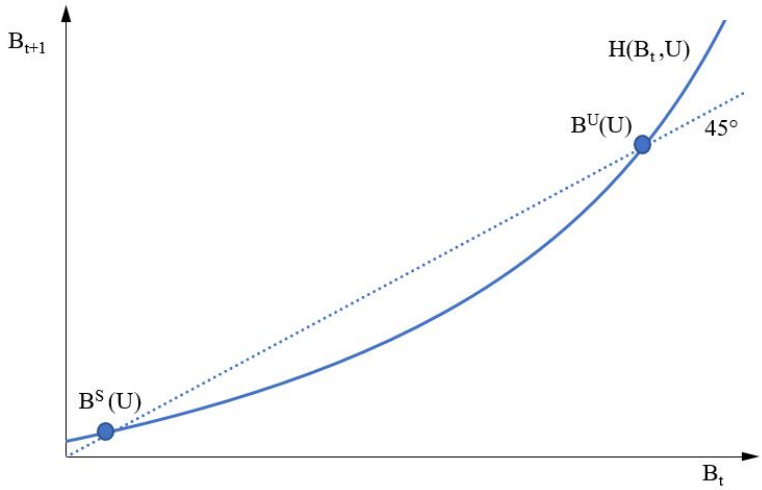
In this section, I derive the condition for the existence of a full employment bubbly steady state (hereafter FEB), whereas in the next sections I analyze its features. Before going on, I define the notion of equilibrium. A perfect foresight equilibrium is a set of prices $\{P_t, W_t, r_t, i_t\}$ and quantities $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, Y_t, Z_t, L_t, B_t\}$ that solve (3.1), (3.2), (3.3), (3.5), (3.6), (3.8), (3.9), (3.10), (3.11), (3.13) (3.14), (3.18) and (3.19), given $\{D_t, g_t, U_t\}$ and initial values for W_{-1}, B_{-1}^m and B_{-1}.

I fix $g_t = g$, $D_t = D$ and $U_t = U$ with g, D, $U \geq 0$. In a FEB nominal rigidities are not binding and the central bank is far away from the ZLB, so we have $L_t = \bar{L}$,

[11] The relation between old and new bubbles and the composition of the aggregate bubble will be more deeply studied in the next section.

[12] In the previous calibration, I set $\beta = 0.987$, a pretty high value, given that most of estimates point to 0.96 for annual data. For lower values of β, closer to 0.96, the relative impact of the saving channel on the real interest rate increases, whereas that of the borrowing channel declines. If agents discount the future greatly, they increase more their consumption because of the additional retirement income provided by bubbles. On the other hand, a lower fraction of income is saved and this dampens the effect of a higher debt on savings. The results are similar for larger values of D, which is initially set to 0.22.

Figure 3.4: Bubble dynamics with saving and borrowing channels



$Y_t = \bar{L}^{\alpha} = Y^f$ and $\frac{Z_t}{P_t} = (1-\alpha) Y^f$. Combining (3.6), (3.18) and (3.22), and expressing the resulting equation in terms of B_{t+1}, we get:

$$B_{t+1} = \frac{(1+\beta)(D+U)(U+B_t)}{\beta(Y^f - D - U - B_t) - (U+B_t)} = H(B_t, U) \tag{3.23}$$

For any sequence $\{B_t\}$ with $B_t \in (0, Y^f)$ for all t, the equilibrium values for the other variables can be computed from the remaining conditions. A FEB is given by a pair (B, U) satisfying $B = H(B, U)$ with $B \in (0, Y^f)$. Its stability depends on the condition $\partial H(B,U)/\partial B < 1$,[13] whereas the necessary and sufficient condition for its existence is:[14]

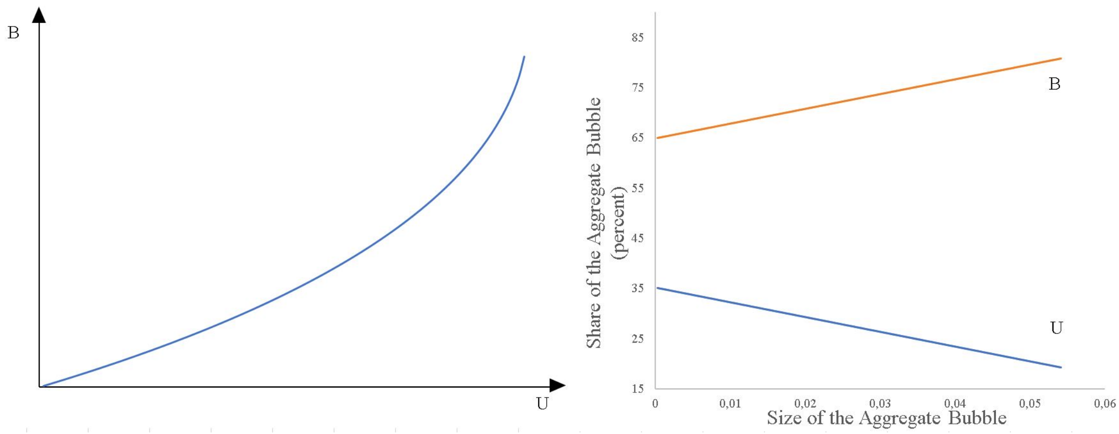
$$D < \frac{\beta}{1+\beta}\left(Y^f - D\right) \tag{3.24}$$

As depicted in Figure 3.4, when this condition is satisfied, there exists a continuum of stable $\left(B^S(U), U\right)$ and unstable $\left(B^U(U), U\right)$ steady states for any $U \in [0, \bar{U})$ with $\bar{U} = \left[(1+2\beta) D - \beta Y^f\right]^2 / \left[4\beta(1+\beta)\left(Y^f - D\right)\right]$. In what follows I restrict

[13] This is also the condition for the stationarity of the bubble. See appendix B.

[14] The proof of this condition is contained in appendix B. The existence, as well as the stationarity, of the bubble is verified in all the simulations reported in the rest of the chapter.

Figure 3.5: New and old bubbles



my attention to the stable FEB. In a bubbleless steady state the condition (3.24) corresponds to $(1+r) < (1+g)$, which is standard in the literature on rational bubbles (e.g., Samuelson (1958) and Tirole (1985)). Furthermore, the condition $(1+r) < (1+g)$ has to be met in a FEB to have a constant aggregate bubble and there is an upper bound on B, namely $B^U(0) = \frac{\beta}{1+\beta}\left(Y^f - D\right) - D$, which is the value for which the real interest rate equals the growth rate of the economy. In a FEB, a positive relation between new and old bubbles underlies equation (3.23), as depicted in the left panel of Figure 3.5. The value of the old bubbles increases more than proportionally with that of the new ones. Hence, the share of the old bubbly assets rises as the overall bubble expands, whereas the share of the new bubbly assets becomes smaller (right panel of Figure 3.5).[15]

3.3.2 Steady state

The steady state of the model can be represented by aggregate demand and supply, which relate production and inflation. Both consist of two regimes, which are determined by equation (3.11) for supply and by equation (3.13) for demand.

[15] These results hold independently of the calibration.

Aggregate supply (AS) is made up of a vertical and an upward sloping segment, with the kink at $\Pi = 1$. The vertical AS is:

$$Y_t = \bar{L}^\alpha = Y^f \tag{3.25}$$

The upward sloping AS is given by:

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)\left(\frac{Y}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} \tag{3.26}$$

which can be derived by following the same steps of the previous chapter.

As regards aggregate demand (AD), when the nominal interest rate is positive, we derive from equations (3.13), (3.14) and 3.22 a downward sloping AD:

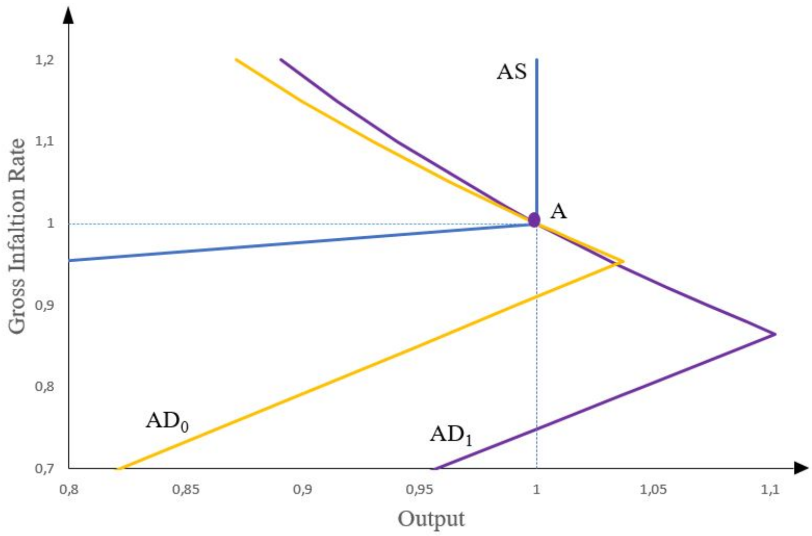
$$Y = D + \left(\frac{1+\beta}{\beta}\right)(U+B) + \left(\frac{1+\beta}{\beta}\right)(1+g)\frac{\Gamma}{\Pi^{\phi_\pi - 1}}(D+U) \tag{3.27}$$

where $\Gamma = \overline{\Pi}^{\phi_\pi - 1}\left(1+r^f\right)^{-1}$. When the lower bound on the nominal rate is binding ($i = 0$), combining equations (3.13), (3.14) and (3.22) yields an upward sloping AD:

$$Y = D + \left(\frac{1+\beta}{\beta}\right)(U+B) + \left(\frac{1+\beta}{\beta}\right)(1+g)\,\Pi\,(D+U) \tag{3.28}$$

The effect of D on demand is the same illustrated in chapter 2, whereas bubbles affect differently AD compared to the model without bubbly collateral. Bubbly assets still serve as a store of value and provide an additional income during the old age, as expressed by $\left(\frac{1+\beta}{\beta}\right)B$. Furthermore, the income of the elderly is increased by the higher debt deriving from a looser borrowing limit. Young households can pledge future new bubbles and so issue more liabilities, which are bought by old households, when middle-aged. On the one hand, this stimulates the consumption

Figure 3.6: AD and AS with borrowing and saving channels



of young households ($(1+\beta/\beta)(1+g)\frac{\Gamma}{\Pi^{\phi_\pi - 1}}U$ in (3.27) and $(1+\beta/\beta)(1+g)\Pi U$ in (3.28)), on the other hand, this directly contributes to the consumption of the elderly (the term $\frac{1+\beta}{\beta}U$). Higher income in the old age also changes the intertemporal allocation of resources, because middle-aged households save less and consume more. However, the effect of the intertemporal reallocation is fully compensated by the higher debt, taken on during youth, and by the funds, invested in old bubbles. For this reason, bubbles do not raise the consumption of the middle generation, when a bubbly collateral is assumed. All these effects deliver a different demand in a bubbly world (AD_1 in Figure 3.6) compared to the bubbleless case (AD_0).[16] In particular, the AD kink, which is expressed as:

$$\Pi_{kink} = \left[\frac{1}{(1+r^f)}\right]^{\frac{1}{\phi_\pi}} \overline{\Pi}^{\frac{\phi_\pi - 1}{\phi_\pi}} \tag{3.29}$$

[16] I set $\beta = 0.987$, $\gamma = 0.98$, $\alpha = 0.7$, $\overline{\Pi} = 1$, $\phi_\pi = 2$, $D = 0.23$, $g = 0.023$ and $U = 0.0082$ to plot Figure 3.6.

is lower, because the natural interest rate is higher than in a bubbleless world.[17]

3.3.3 Redistributive bubbles and welfare

As plotted in Figure 3.6, the presence of bubbly assets alters the shape of AD without changing the nature of the full employment equilibrium, which still occurs at the intersection of the vertical AS and the upward sloping AD, and still features $Y = Y^f$ and $\Pi = \overline{\Pi}$.[18] This reflects the redistributive nature of bubbles assumed in this framework. Bubbles do not affect the production of the economy, but just its allocation and so the natural interest rate. Here I study the allocation of consumption across generations in a FEB and in a full employment bubbleless equilibrium (hereafter FE). Without bubbly assets ($B = U = 0$) the steady state values of the main variables are:

$$\begin{aligned}
(1+r) &= \frac{(1+g)(1+\beta) D}{\beta (Y^f - D)} \\
B^y &= \frac{D}{(1+r)} \\
B^m &= \frac{\beta}{1+\beta}\left(Y^f - D\right) \\
C^y = B^y &= \frac{1}{(1+g)}\left[\frac{\beta}{1+\beta}\left(Y^f - D\right)\right] \\
C^m &= \frac{1}{1+\beta}\left(Y^f - D\right) \\
C^o &= (1+g) D
\end{aligned}$$

The natural implication of the logarithmic utility assumed above is that a fixed fraction of the total income is saved during the middle age, while the remaining fraction is consumed. Savings go to young households in exchange for riskless

[17] Though (3.29) is equal to (2.33), the AD kink is lower in this case, because both saving and borrowing channels operate, unlike the model in chapter 2.

[18] The conditions for determinacy and uniqueness of the FEB are the same of chapter 2.

bonds. These are assets for the middle-aged agent and liabilities (debt) for the young one, so during the middle age the representative agent pays down the total amount of debt (D) to the elderly. The size of generations is different and this affects the consumption of young and old households. In particular, population growth increases the proceeds for each old agent from investing in riskless bonds and reduces the funds raised by each young agent in the loan market. In the FEB, the main variables assume the following values:

$$\begin{aligned}
(1+r) &= \frac{(1+g)(1+\beta)(D+U)}{\beta\left(Y^f - D - U - B\right) - (U+B)} \\
B^y &= \frac{D+U}{(1+r)} \\
B^m &= \frac{\beta}{1+\beta}\left(Y^f - D\right) - (U+B) \\
C^y &= B^y \\
C^m &= \frac{1}{1+\beta}\left(Y^f - D\right) \\
C^o &= (1+g)(D+U+B)
\end{aligned}$$

The real interest rate is higher and the funds supplied in the loan market are lower in a bubbly world, as shown above. Specifically, one dollar increase in aggregate bubble ($Q^B = U + B$) is associated with a one dollar reduction in the supply of riskless bonds.[19] Furthermore, a looser borrowing constraint deriving from bubbly collateral implies a higher debt and so more proceeds for old households (the term U in the last equation), whose consumption increases further because of the investment in bubbly assets (the term B). More debt does not necessarily imply more consumption for young households, because a higher interest rate decreases

[19] This relation depends on the overall effect of saving and borrowing channels. Old bubbles purchases decrease loan supply by $\frac{\beta}{1+\beta}B$. Investing in old bubbles and issuing new ones increases income in the old age and reduces saving by $\frac{1}{1+\beta}(U+B)$, whereas the higher debt to repay in the middle age decreases savings by $\frac{\beta}{1+\beta}U$. The sum of these effects is $\frac{\beta}{1+\beta}B + \frac{1}{1+\beta}(U+B) + \frac{\beta}{1+\beta}U = U + B$.

the amount of funds raised in the credit market. The consumption of the young agent can be rewritten as:

$$C^y = \frac{D+U}{(1+r)} = \frac{1}{(1+g)} \left[\frac{\beta}{1+\beta}(Y^f - D) - (U+B) \right]$$

The negative effect of high interest rates prevails over the positive effect of a looser borrowing constraint and the consumption of young households reduces compared to the bubbleless case. Putting it differently, bubbles decrease loan supply and young households can collect less resources in the loan market. As a consequence, the young households demand more funds and pay down more debt, but they have higher interest payments without raising more funds. As mentioned above, the consumption of the middle-aged household is unchanged, because the positive and negative effects of bubbles cancel out

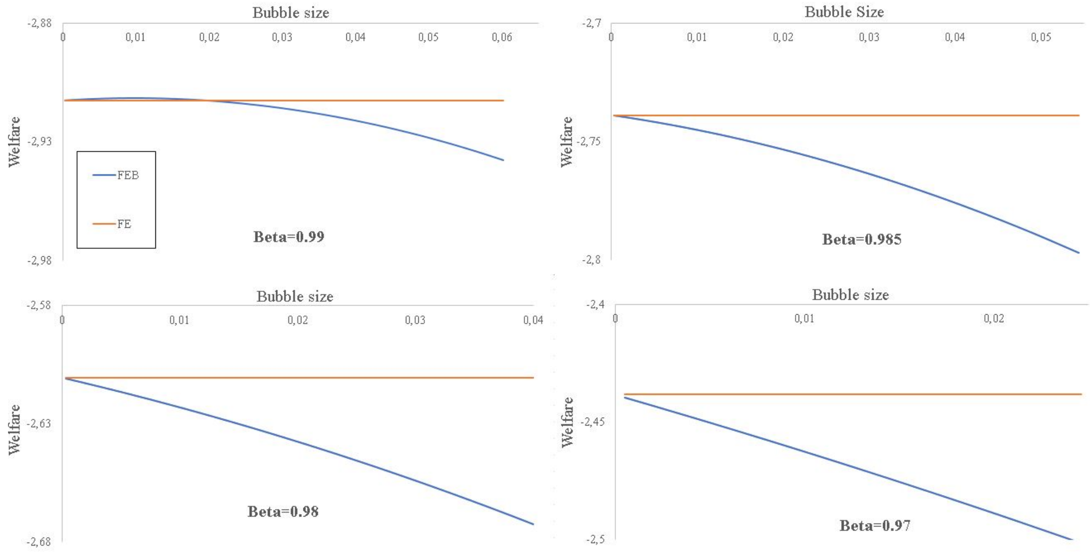
$$C^m = \frac{1}{1+\beta}\left(Y^f - D - U - B\right) + \frac{1}{1+\beta}(U+B) = \frac{1}{1+\beta}\left(Y^f - D\right)$$

The additional income, provided by bubbles in the old age, induces middle-aged households to save less freeing up resources for consumption ($\frac{U+B}{1+\beta}$). On the other hand, the income available for consumption decreases because of the investment in bubbly assets ($-\frac{B}{1+\beta}$) and the higher debt taken on during youth ($-\frac{U}{1+\beta}$).

The new allocation of consumption implies a different level of welfare for the representative agent in a FEB compared to a FE. Let me denote the level of utility in the FEB with U^B and the level in the FE with U^{NB}. Substituting for the consumption of each generation into the utility function and after some manipulations, the difference between the utility in the two steady states is:

$$U^B - U^{NB} = \ln\left[1 - \frac{(1+\beta)}{\beta}\frac{(U+B)}{(Y^f - D)}\right] + \beta^2 \ln\left(1 + \frac{U+B}{D}\right) \tag{3.30}$$

Figure 3.7: Bubble size and welfare for different values of β



As the consumption during the middle age is the same in both steady states, the difference depends on the differential level of consumption during the young and the old age. The first term on the right-hand side reflects the negative impact of bubbles on C^y, and the second term the positive effect on C^o. The relative strength of these effects, and so the overall effect, depends on the parameters β and D, and on the size of the aggregate bubble, $U + B$. I now carry out two numerical exercises. In the first one, I study how the difference between the utility in the two steady states varies according to the size of the bubble, for different values of β.[20] Results are plotted in Figure 3.7. The welfare in the FE is given by an horizontal line, because it is independent of the bubble size. Apart from the case of $\beta = 0.99$ and only for small bubbles, welfare is always higher in the bubbleless steady state. Furthermore, the utility in the FEB reduces as the size of

[20] In this case, I set $\gamma = 0.98$, $\alpha = 0.7$, $\overline{\Pi} = 1$, $\phi_\pi = 2$, $D = 0.22$, $g = 0.023$, $L = 1$. This parameterization implies a positive natural interest rate and so a full employment equilibrium in the bubbleless world. Moreover, changing β alters all the conditions regarding the value of B, of U and their relation. As a consequence, different values of the aggregate bubble are compatible with different values of β.

the bubble increases, widening the gap with the FE. The intuition underlying this result is straightforward. As shown by equation (3.30), the welfare gains from a higher consumption during the old age increases with the size of the bubble, like the losses deriving from less consumption in youth. As $\beta < 1$, the losses are always greater than the gains, except for values of β very close to 1 and for small bubbles, which amplify the positive effect of a higher C^o and dampen the negative effect of a lower C^y respectively.[21]

I go through the same exercise, but I consider different values of D and set $\beta = 0.99$.[22] I choose this value for the discount factor, because this is the only case, in the previous exercise, in which the welfare in the FEB is greater than in the FE. This is a way to check the robustness of the previous result to variations in the borrowing limit. This numerical exercise is plotted in Figure 3.8. For higher values of the borrowing constraint, utility is still greater in the FE. Indeed, the higher is the debt limit the lower is the consumption of young households[23] and the higher is that of old ones. As a result, the marginal effect of consumption losses in youth is amplified and the marginal effect of consumption gains in the old age is dampened. This result not only corroborates the previous one, but reinforce it, because in the previous calibration I have chosen a slightly low value (0.22) for D.[24] Summing up, the redistribution implemented by the bubble, which

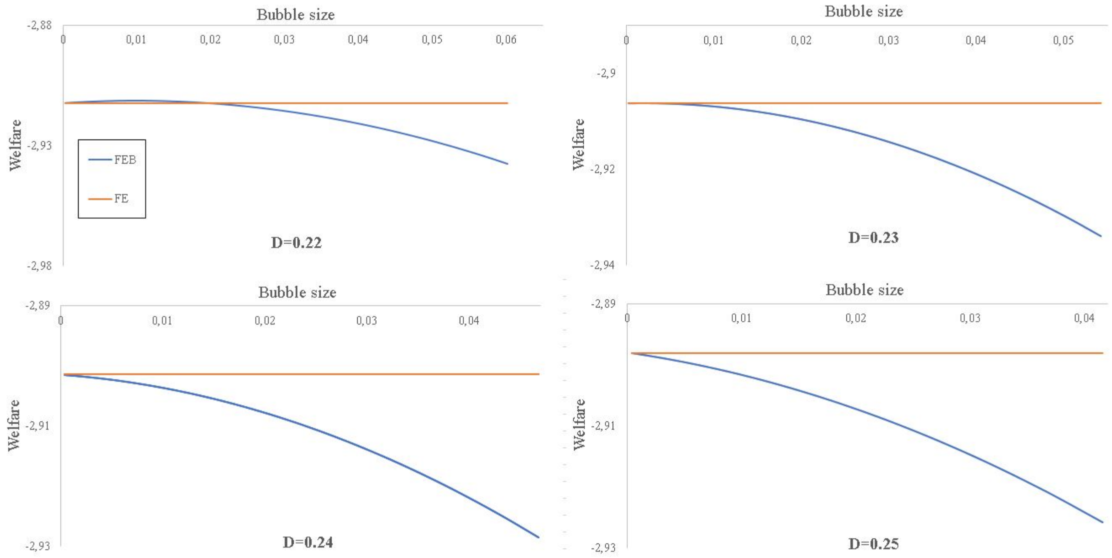
[21] Small bubbles also reduce the welfare gains from a higher C^o, but this is counteracted by a greater evaluation of the consumption during the old age, expressed by a higher β. Furthermore, the higher is β the lower is $\frac{1+\beta}{\beta}$ in (3.30). This reduces the marginal impact of the aggregate bubble on the consumption of young households.

[22] Also in this case, different values of the bubble size are consistent with different calibrations of D.

[23] As young people are borrowing constrained, this seems counterintuitive, but it is not. As shown above, despite a looser borrowing limit, young households consume less in the FEB, because less funds are available in the loan market. Here I refer to this case. If debt is high, loan supply is low and so young households collect few resources by issuing riskless bonds.

[24] Eggertsson et al. (2017) set $D = 0.234$ in order to match some data for the US economy. So, the range of values considered is a plausible calibration of the debt limit. In any case, I cannot study lower values for D, because they do not correspond to a full employment equilibrium without bubbles. Lower values for β do not change the results, because they imply lower

Figure 3.8: Bubble size and welfare for different values of D



transfers resources from the young age to the old one, makes the welfare of the representative agent worse for realistic parameterization of the discount factor[25] and the borrowing limit independently of the size of the aggregate bubble.

3.3.4 Redistributive bubbles and data

Now, I compare the predictions of the model with data. In particular, I show how consumption and income changed across generations in the US in the period 1995-2007. This span embraces the dot-com and housing bubbles.[26] Results are

consumption during the young age, amplifying the marginal effect of a reduction in C^y.

[25] $\beta = 0.99$ is pretty high. Most of estimates show that the discount factor is approximately 0.96 for annual data.

[26] The year 2007 is commonly identified as the ending date of the housing bubble. As regards the beginning of the tech bubble, it's very challenging to identify it, as movements in stock prices were driven by changes in fundamental and bubbly components. I start from the year 1995, because, as reported by Leroy (2004), large increases in the equity value started in that year. Therefore, this is a suitable starting point to represent a bubbleless environment, independently of the beginning of the bubbly episode, whose identification goes beyond the scope of the chapter. I study the percentage change between 1995 and 2007 and not just the change in variables, because age classes have different consumption profiles reflecting the pattern of income over the

Table 3.1: Variation of consumption and income across generations 1995-2007

<table>
<tr><td></td><td colspan="6">Age Cohort</td></tr>
<tr><td></td><td>Average</td><td colspan="2">25-44</td><td colspan="2">45-64</td><td>over 65</td></tr>
<tr><td>Expenditure</td><td>11.5</td><td colspan="2">9.5</td><td colspan="2">7</td><td>19</td></tr>
<tr><td>Income after tax</td><td>31.8</td><td colspan="2">28.6</td><td colspan="2">27.2</td><td>35.3</td></tr>
<tr><td></td><td colspan="6">Age Cohort</td></tr>
<tr><td></td><td>Average</td><td>25-34</td><td>35-44</td><td>45-54</td><td>55-64</td><td>65-74</td></tr>
<tr><td>Expenditure</td><td>11.5</td><td>8.5</td><td>10.6</td><td>0.4</td><td>19.8</td><td>20.4</td></tr>
<tr><td>Income after tax</td><td>31.8</td><td>25.5</td><td>31.3</td><td>20.4</td><td>41.7</td><td>38.6</td></tr>
</table>

Source: Consumer Expenditure Survey, US Bureau of Labor Statistics.
Note: Data in percent. Mortgage interest payments are subtracted from expenditure and income after tax. Price effects are removed by expressing data in real terms by the Consumer Price Index (CPI).

reported in Table 3.1. At the top of the table I report the percentage variation of the consumption and income of the age classes 25-44 years, 45-64 years and over 65 years. The first two age cohorts correspond to young and middle generations in my model, while the remaining one to the old generation. As predicted above, there is a huge increase in the income and the consumption of the elderly, but, inconsistently with my model, the increase in the consumption of the young people is larger than that of the middle-aged ones.

This does not invalidate my results, but it highlights that the steady state behavior is not the right counterpart for the US data. In the real world, even if we interpret the two major bubbly episodes like a unique event, the bubbly environment lasted less than 20 years, which is the length of a generation in my OLG model. As a consequence, not all people in any cohort transitioned from a generation to the next one, when the housing bubble collapsed in 2007. In order to take into account this fact, I study the consumption of each cohort during the

life-cycle. This affects the size of variation in levels. By contrast, the percentage change is a proper measure to detect a redistribution of income and consumption between age cohorts. In particular, a significant difference between generations points to a redistribution implemented by bubbles.

transition to the FEB. Assume that bubbly assets appear in period t ($U_t > 0$) and that for any $s < t$ it is $U_s = B_s = 0$. In this case, the consumption of the generations is:

$$
\begin{aligned}
C_t^y &= \frac{1}{(1+g_t)}\left[\frac{\beta}{1+\beta}\left(Y_t - D_{t-1}\right) - \frac{1}{1+\beta}U_t\right] \\
C_t^m &= \frac{1}{1+\beta}\left(Y_t - D_{t-1}\right) + \frac{1}{1+\beta}U_t \\
C_t^o &= (1+g_t)\, D_{t-1}
\end{aligned}
$$

In the period in which the bubble originates, there is a windfall for the middle-aged household receiving it. New bubbles represent an additional income in the old age and this induces them to consume more and save less. The presence of bubbly assets also relaxes the borrowing constraint faced by young households, but their consumption reduces because of the resulting increase in the real interest rate. The elderly are not affected by the introduction of bubbles. They did not receive them when middle-aged ($U_{t-1} = 0$), so they cannot resell bubbly assets. Furthermore, they receive interest payments from loan contracts signed in the previous period, when interest rates were lower. In period $t+1$, the consumption of the cohorts becomes:

$$
\begin{aligned}
C_{t+1}^y &= \frac{1}{(1+g_t)}\left[\frac{\beta}{1+\beta}\left(Y_{t+1} - D_t\right) - \left(U_{t+1} + B_{t+1}\right)\right] \\
C_{t+1}^m &= \frac{1}{1+\beta}\left(Y_{t+1} - D_t\right) \\
C_{t+1}^o &= (1+g_t)\left(D_t + U_{t+1} + B_{t+1}\right)
\end{aligned}
$$

The new borns pay a higher real interest rate than the young households in t, because the size of the aggregate bubble increases, pushing interest rates up. This further reduces the consumption of young agents. Middle-aged households still receive a windfall from bubble creation, but now it is fully offset by the higher debt taken on in the previous period. This higher debt translates in higher proceeds for the old generation, which also benefits from selling the bubbly assets received in t. The generations in period t are the cohorts which did not transitioned: the classes 35-44 and 55-64 in the data. They always belong to the same generation over the period 1995-2007. By contrast, generations in period $t+1$ represent the cohorts transitioned to the next generation. People between 45 and 54 years of age in 2007 correspond to the cohort 35-44 in 1995, whereas the members of the class 65-74 were in the 55-64 cohort in 1995. Once determined the right counterpart, I can interpret the data through the lens of the model. The age cohort 35-44 had a higher increase in consumption than the cohort 25-34, because a lower interest rate was charged on their debt, e.g., mortgages (bottom part of Table 3.1). Instead, people between 55 and 64 years increased more their consumption than those in 45-54 class, because they took on less debt during the young age.[27] Finally, the age cohort 65-74 benefited from the largest increase in consumption, because bubbles raised the value of their assets and the proceeds from lending.

3.4 Calibration

In this section, I perform a calibration by using US data. The aim of this exercise is not to measure the effect of tech and housing bubbles on the US real interest rate and so to precisely replicate the stylized facts characterizing the US economy before the crisis. Rather, I want to show that the size of the bubble necessary to prevent SecStag is reasonable according to a standard calibration of the model, in

[27] According to the model, the age cohort 45-54 should have a higher increase in consumption than the cohorts 25-34 and 35-44. This does not happen, because, in my model, young households do not raise more funds and so they cannot increase their consumption. This problem can be solved by studying an open economy framework or introducing the financial sector. Indeed, during the two bubbly episodes funds flowed from emerging economies to the US, financing the higher debt of the US households. The higher debt was also financed by the financial system, whose capacity of intermediating funds improved greatly from the 1980s onwards.

Table 3.2: Parameter values

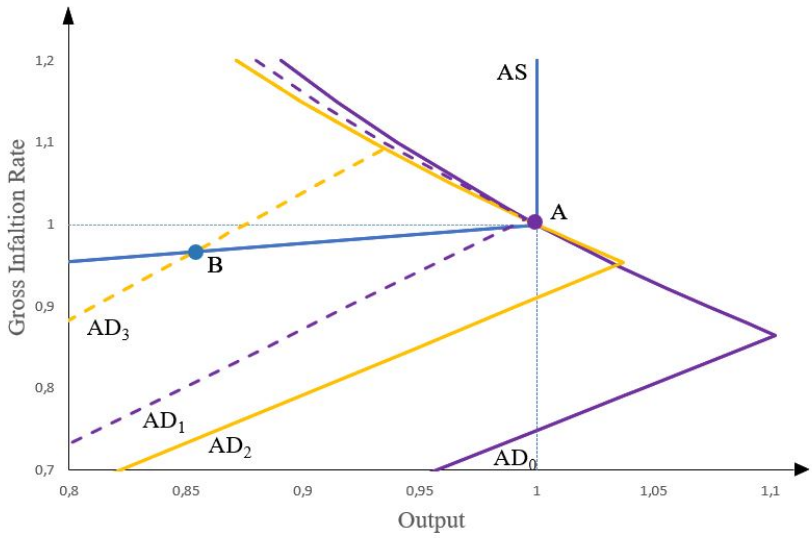
Parameters	**Values**	**Description**
β	0.987	Discount factor
α	0.7	Labor share
D	0.23	Collateral constraint
g	0.023	Population growth
$\overline{\Pi}$	1	Inflation target
ϕ_π	2	Taylor coefficient
γ	0.98	Wage rigidity
L	1	Labor supply

order to corroborate the results of the previous chapter. To this aim, I calibrate a demographic shock consistent with data, which leads the natural interest rate to -1% in a bubbleless economy. Of course, demographic factors are not the only drivers of low interest rates, even though they played a crucial role. This is just a shortcut to simulate the effect of several factors such as demography, technological developments, income inequality and global imbalances.

Table 3.2 contains the values assumed for the parameters of the model. The labor supply is equal to 1 in order to normalize all variables in terms of potential output. $\gamma = 0.98$ falls in the range of values found by Schmitt-Grohé and Uribe (2016), whereas D is set to 0.23, which is approximately the value chosen by Eggertsson et al. (2017) in their quantitative model. The remaining parameters are standard and all kept constant, except for g, which goes from 0.023 to 0.099. I interpret this parameter as the ratio of young (the age cohort 25-44 years) to middle-aged people (the age cohort 45-64 years) in the US; and the two calibrations of g match the average value of this ratio in the period 1977-1997 (1.6) and in the period 1997-2007 (1.22).[28]

[28] Data from World Development Indicators, World Bank.

Figure 3.9: Necessary size of the bubble for avoiding SecStag



The mechanism which prevents the occurrence of SecStag is identical to that outlined in chapter 2. However, the borrowing channel further increases the natural interest rate and so, for any size of the permanent shock, a ZLB episode is less likely in a bubbly world. This is the only difference between the extended model and the basic one, which is contained in the previous chapter.[29]

The necessary size of the aggregate bubble for avoiding SecStag keeps the natural interest rate positive, despite the downward pressure of the demographic shock. In this case, the AD kink is equal to the inflation target, as depicted in Figure 3.9.[30] The minimum size of the bubble which is necessary to keep the natural rate positive is approximately 0.05, which corresponds to 5% of total output ($Y = Y^f = 1$). This is a reasonable value and proves that the existence of bubbly assets avoids a liquidity trap for realistic calibrations of the aggregate bubble and the demographic shock.

[29] This difference is summarized by a lower AD kink in the extended model. See footnote 17.
[30] The meaning and the names of the curves are the same of Figure 2.9 in chapter 2.

3.5 Fiscal policy and Asset Price Bubbles: Substitutes or Complements?

The discussion of the last sections raises a natural question. Bubbles push the natural interest rate up and avoid SecStag, playing the same role of an expansionary fiscal policy in Eggertsson et al. (2017). What are the differences between asset price bubbles and fiscal policy? Did asset price bubbles act as a substitute for fiscal policy, which was not sufficiently expansionary before the crisis, or do they affect interest rates in a different way? In this section, I compare the mechanisms through which fiscal policy and asset price bubbles alter interest rates. I do not focus on the expansionary effects of fiscal policy at the ZLB, like in Eggertsson et al. (2017), rather I first analyze its impact on interest rates in a bubbleless economy by distinguishing a debt-financed increase in public spending from a redistribution of tax burden.[31] Then, I study the interaction between fiscal policy and asset price bubbles.

3.5.1 Fiscal policy in a bubbleless world

As I have not explicitly investigated a bubbleless economy in the previous sections, I write here the loan demand, the loan supply and the equilibrium real interest rate in this case:[32]

$$L_t^d = \frac{(1+g_t)}{(1+r_t)} D_t \tag{3.31}$$

[31] Of course, issuance of government debt could finance tax cuts and tax increases could finance a structural increase in public spending. Though these alternative policies are possible, they do not allow to isolate the effect of fiscal policy on loan demand from that on loan supply. Instead, the chosen fiscal policies do that and this highlights differences and similarities with bubbles. As I assume output is at the potential level, in order to study the period before the crisis, fiscal policy has no effect on demand and production in the model, but it only changes interest rates.

[32] As this economy coincides with that analyzed in the theoretical model of Eggertsson et al. (2017), I refer the reader to this work for further details.

$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1}\right)(3.32)$$

$$\left(1+r_t\right) = \frac{\left(1+\beta\right)\left(1+g_t\right)D_t}{\beta\left(Y_t - D_{t-1}\right)} \tag{3.33}$$

These are the relations affected by fiscal policy.

I start the analysis of fiscal policy in a bubbleless economy by studying a debt-financed fiscal stimulus. A fiscal policy regime corresponds to the choice of specific values for $(T_t^y, T_t^m, T_t^o, G_t, B_t^g)$. T_t^i denotes lump-sum taxes on generations with $i = y, m, o$. B_t^g denotes public debt and G_t is government spending. In my OLG model, Ricardian equivalence does not hold, as long as the increase in government debt is expected to be permanent and, specifically, middle-aged households do not expect higher taxes in the old age. I abstract from taxation ($T_t^y = T_t^m = T_t^o = 0$) and assume government spending is financed by rolling over public debt, which is represented by a one period bond. This assumption, though unrealistic, allows to isolate the impact of debt-financed public spending from that of taxation.[33] The household's problem is:

$$\max_{C_t^y, C_{t+1}^m, C_{t+2}^o} E_t\left\{\ln C_t^y + \beta \ln C_{t+1}^m + \beta^2 \ln C_{t+2}^o\right\}$$

s.t.

$$C_t^y = B_t^y$$

$$C_{t+1}^m = Y_{t+1} - \left(1+r_t\right)B_t^y - B_{t+1}^m$$

$$C_{t+2}^o = \left(1+r_{t+1}\right)B_{t+1}^m$$

$$\left(1+r_t\right)B_t^i \leq D_t$$

The borrowing limit is binding for young households ($B_t^y = \frac{D_t}{1+r_t}$) and the optimal

[33] Being this case purely illustrative, I do not address the sustainability issues related to this fiscal policy.

condition for this maximization problem is (3.6). The production side of the model and monetary policy are the same of section 3.2. Given the assumptions above, the fiscal policy regime reduces to the choice of G_t and B_t^g, and the government budget constraint takes the shape:

$$N_{t-1}B_t^g = N_{t-1}G_t + (1 + r_{t-1})N_{t-2}B_{t-1}^g$$

or, alternatively:

$$B_t^g = G_t + \frac{(1 + r_{t-1})}{(1 + g_t)}B_{t-1}^g \tag{3.34}$$

where B_t^g and G_t are normalized in terms of the size of the middle generation. The presence of government alters the equilibrium condition in the loan market, which here is:

$$N_tB_t^y + N_{t-1}B_t^g = N_{t-1}B_t^m$$

or:

$$(1 + g_t)B_t^y + B_t^g = B_t^m \tag{3.35}$$

Government raises funds in the loan market to finance its spending and this increases loan demand, the left-hand side of (3.35), which becomes:

$$L_t^d = \frac{(1 + g_t)}{(1 + r_t)}D_t + B_t^g \tag{3.36}$$

Instead, loan supply, the right-hand side of (3.35), is unaffected by fiscal policy and it is still given by (3.32).[34] The effect of public debt on loan demand is reflected

[34] I compute loan demand and supply, in any case reported in this section, by following the same steps for the derivation of (3.20) and (3.21), and by also using, in the bubbly case, the expressions contained in Chapter 2 footnote 12.

in the equilibrium real interest rate, which is given by:

$$(1 + r_t) = \frac{(1 + \beta)(1 + g_t) D_t}{\beta (Y_t - D_{t-1}) - (1 + \beta) B_t^g} \tag{3.37}$$

and it is higher than that without fiscal policy (equation (3.33)).

Now, I study a fiscal policy which redistributes resources from middle-aged households to old ones ($T_t^y = G_t = B_t^g = 0$). This can be specified by the equation:

$$N_{t-1} T_t^m + N_{t-2}(1 + g_t) T_t^o = 0$$

Taxes levied on the middle-aged agent, T_t^m, are normalized in terms of the size of the middle generation. This is the same for taxes on the old agent (T_t^o), which are multiplied by $(1 + g_t)$, because, given population growth, 1 dollar increase in T_t^m allows government to reduce T_t^o by $(1 + g_t)$ dollars. After some manipulations, the government budget constraint can be rewritten as:

$$T_t^m = -T_t^o \tag{3.38}$$

To simplify the analysis, I assume taxation does not change over time, hence $T_t^m = T^m$, $T_t^o = T^o$ and $T^m = -T^o$. This fiscal policy, which implies negative taxes on old households and so a subsidy for income in old age, precisely replicates the intergenerational transfers, implemented by a pay-as-you-go (hereafter PAYG) pension scheme, and modifies budget constraints as follows:

$$C_{t+1}^m = Y_{t+1} - (1 + r_t) B_t^y - B_{t+1}^m - T^m$$

$$\begin{aligned} C_{t+2}^o &= (1 + r_{t+1}) B_{t+1}^m - (1 + g_t) T^o \\ &= (1 + r_{t+1}) B_{t+1}^m + (1 + g_t) T^m \end{aligned}$$

The new budget constraints do not affect the loan demand, which is still (3.31), but they alter the loan supply:

$$\begin{aligned} L_t^s &= \frac{\beta}{1+\beta}\left(Y_t - D_{t-1} - T^m\right) + \frac{(1+g_t)\, T^o}{(1+\beta)\,(1+r_t)} \\ &= \frac{\beta}{1+\beta}\left(Y_t - D_{t-1} - T^m\right) - \frac{(1+g_t)\, T^m}{(1+\beta)\,(1+r_t)} \end{aligned} \tag{3.39}$$

and so the equilibrium real interest rate:

$$(1+r_t) = (1+g_t)\left[\frac{(1+\beta)\, D_t + T^m}{\beta\left(Y_t - D_{t-1} - T^m\right)}\right] \tag{3.40}$$

A policy which subsidizes the income of the old households with taxes on the middle-aged ones decreases loan supply, because the higher T^m the lower is the income available for savings, and more income in the old age induces middle-aged people to save less. These two effects determine a higher equilibrium interest rate compared to the case without taxes.

Summing up, a debt-financed increase in public spending replicates the effect of the borrowing channel on the loan demand and the equilibrium real interest rate, whereas a PAYG pension scheme affects loan supply and interest rates in the same way of the borrowing and saving channels. The similarities between asset price bubbles and fiscal policy can be better understood by an extension of the model outlined in section 3.2, which includes government.

3.5.2 Fiscal policy in a bubbly world

In this paragraph, I study the interaction between bubbles and fiscal policy. The model here is identical to that in section 3.2 except for the presence of government. Taxes are levied on middle and old generations.[35] They finance, along with pub-

[35] Taxes on young households are excluded from the analysis, because they do not affect neither loan demand and loan supply. As in the previous paragraph, taxes on old households are

lic debt, government spending and debt service, as expressed by the government budget constraint:

$$N_{t-1}T_t^m + N_{t-2}(1+g_t)T_t^o + N_{t-1}B_t^g = N_{t-1}G_t + (1+r_{t-1})N_{t-2}B_{t-1}^g$$

By assuming $T_t^m = T^m$ and $T_t^o = T^o$ and after some manipulations, this can be rewritten as:

$$T^m + T^o + B_t^g = G_t + \frac{(1+r_{t-1})}{(1+g_t)}B_{t-1}^g \qquad (3.41)$$

This formulation of fiscal policy alters the budget constraint of the representative household in the middle and old age:[36]

$$C_{t+1}^m = Y_{t+1} + \delta Q_{t+1|t+1}^B - T^m - (1+r_t)\,B_t^y - B_{t+1}^m - \sum_{k=0}^{\infty} Q_{t+1|t+1-k}^B Z_{t+1|t+1-k}^B$$

$$C_{t+2}^o = (1+r_{t+1})\,B_{t+1}^m + (1-\delta)\,(1+g_t)\sum_{k=0}^{\infty} Q_{t+2|t+1-k}^B Z_{t+1|t+1-k}^B - (1+g_t)\,T^o$$

As a consequence, the loan demand becomes:

$$L_t^d = \frac{(1+g_t)}{(1+r_t)}\,(D_t + E_t U_{t+1}) + B_t^g \qquad (3.42)$$

and the loan supply is now given by:

$$L_t^s = \frac{\beta}{1+\beta}\,(Y_t - D_{t-1} - T^m - U_t - B_t) - \frac{(1+g_t)}{(1+\beta)\,(1+r_t)}\,(E_t B_{t+1} - T^o) \qquad (3.43)$$

multiplied by $(1+g_t)$ so that revenues from T_t^o equal revenues from T_t^m, given the different size of generations.

[36] Fiscal policy does not change the Euler equation (3.6), but it changes the condition for a binding borrowing limit in the young age, which becomes $D_{t-1} < \frac{1}{1+(1+\beta)\beta}\left[Y_t - T^m - \left(\frac{1+g_t}{1+r_t}\right)T^o - \beta\,(1+\beta)\,\delta Q_{t|t}^B\right]$.

or, alternatively:

$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1} - T^m - U_t - B_t\right) - \frac{1}{1+\beta}(U_t + B_t) + \frac{(1+g_t)}{(1+\beta)(1+r_t)}T^o \tag{3.44}$$

Equations (3.42)-(3.44) confirm the intuitions of the previous paragraph. Public debt and bubbly collateral expand the loan demand.[37] Furthermore, a redistribution of tax burden from old households to middle-aged ones, which, in the extreme case ($T^m = -T^o$), coincides with a PAYG pension system, decreases the loan supply likewise asset price bubbles. Overall, both expansionary fiscal policy and bubbles increase the equilibrium real interest rate:

$$(1+r_t) = (1+g_t)\left[\frac{(1+\beta)(D_t + E_t U_{t+1}) - T^o}{\beta(Y_t - D_{t-1} - T^m - U_t - B_t) - (U_t + B_t) - (1+\beta)B_t^g}\right] \tag{3.45}$$

The perfect substitutability between fiscal policy and bubbles is also reflected in the condition for the existence of a full employment bubbly equilibrium, which is, in this extended model:[38]

$$D < \frac{\beta}{1+\beta}\left(Y^f - D - T^m\right) - B^g + \frac{T^o}{1+\beta} \tag{3.46}$$

A high public debt, a high tax burden in the middle age and a low tax burden in the old age can mitigate or even eliminate excess saving, which is the source of asset price bubbles, preventing any bubbly episode. Putting differently, in my extended model with fiscal policy, the existence of rational bubbles strongly depends on the fiscal policy stance, as well as β and D. If public debt is very large and/or the degree at which PAYG pension scheme redistributes resources between generations is high, bubbles do not arise in order to absorb savings and drive

[37] The role of public debt as private liquidity is investigated in Woodford (1990), whereas an analysis of the effect of bubbly liquidity is contained in Tirole and Fahri (2012).

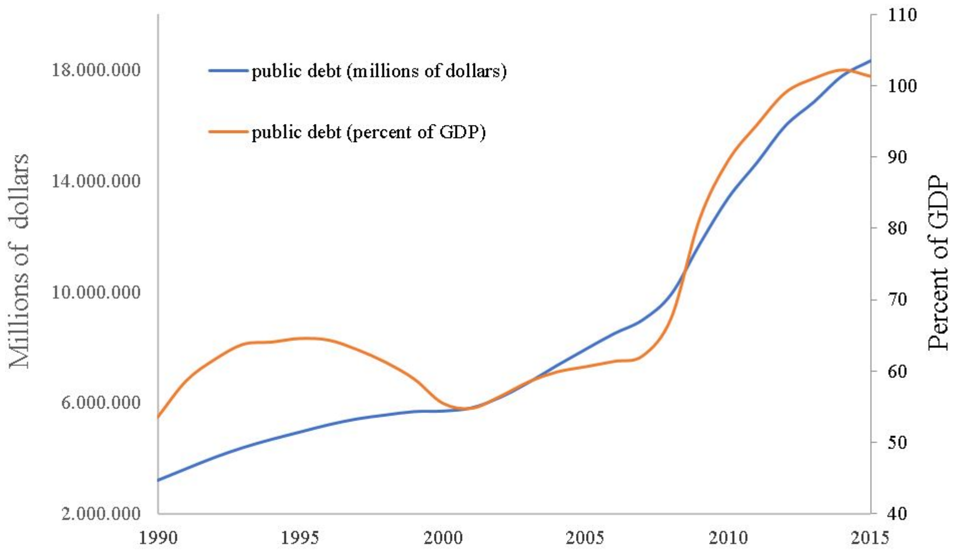
[38] For a proof of (3.46) I refer the reader to appendix B.

interest rates up.[39] Of course, we cannot conclude the US fiscal policy stance before the crisis was one of the main causes of the dot-com and housing bubbles, because the "rational" nature of bubbles, assumed in my model, is not able to capture many aspects of these two bubbly episodes. Rather, the analysis suggests the public debt partially absorbed, between the mid-1990s and the early 2000s, a large amount of savings, which were mainly channelled towards stocks and houses, whose value incorporated a bubbly component for different reasons and under different circumstances. This fits with the quantitative result of Eggertsson et al. (2017). They find the increase in the US public debt over the last forty years, though it counteracted the downward pressure on interest rates, was not sufficient to offset the negative impact of slow-moving forces such as demography and technology. As the US public debt sharply increased only after the recent financial crisis (Figure 3.10), this argument is especially true for the period between the mid-1990s and the early 2000s, and this reinforces the idea that asset price bubbles, in that period, temporarily raised interest rates. Furthermore, my theoretical result complements the analysis of Eggertsson et al. (2017), because it sheds light, not only on the implications of a too low federal debt, but also on the limited impact on interest rates of the US public pension system, which presumably does not divert enough resources towards the old age.[40]

[39] Bubbles and fiscal policy compete for savings like in Kraay and Ventura (2007), but here excess saving is not imported from abroad.

[40] According to OECD data, the US pension spending ranged from 6% to 7% percent of GDP over the period 1990-2013. During the same period, it accounted for more than 10% of GDP in many European countries (Austria, France, Germany, Greece, Italy, Poland, Slovenia) and, in 2013, it was higher than 10% in other advanced European and non-European countries (Belgium, Finland, Japan, Spain).

Figure 3.10: US public debt



Source: FRED.

Note: Public debt is the US federal debt.

3.6 Conclusions

By augmenting the model of chapter 2 with a bubbly collateral, I have shown that bubbles absorb savings and reduce the propensity to save, facilitating the transfer of resources to the old age ("saving channel"); and they also serve as collaterals in the credit market, fostering borrowing ("borrowing channel"). These channels are the same through which public debt and pay-as-you-go pension schemes work, and this suggests fiscal policy has not been enough expansionary to reverse the downward trend of interest rates, as already shown by Eggertsson et al. (2017). In contrast, asset price bubbles redistribute resources from youth to the old age, decreasing the welfare of individuals, and this redistribution increases the natural interest rate, counteracting the downward pressure deriving from structural changes. Specifically, a reasonably large bubble (about the 5% of GDP) avoids secular stagnation for a realistic calibration of the structural shock to the natural interest rate. This corroborates the results of chapter 2, which accounts for the absence of low interest rates before the 2007-08 crisis.

The main driver of the increase in the natural interest rate is the saving channel. This is crucial for the trade-off which arises with low interest rates. As proved by Jordá et al. (2015), leveraged bubbles are more dangerous than unleveraged ones, because they are associated with credit growth and often lead to financial crisis. Hence, policy makers could try to exploit the positive effect of the saving channel, and isolate the potential risk associated with the borrowing channel through macroprudential policies. This policy mix could produce substantial output gains, if low interest rates are persistent because of the declining trend of the natural interest rate, and better policy options are not possible.

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Appendix A

Appendix for Chapter 2

A.1 Stationarity of the Bubble

Taken at t equation (2.24) becomes:

$$B_t = \frac{(1+\beta)\, D_{t-1}\,(U_{t-1}+B_{t-1})}{\beta\,(Y^f - D_{t-2} - B_{t-1}) - (U_{t-1}+B_{t-1})}$$

Denoting log-linearized variables by lowercase letters, the log-linearized version of the equation above is:

$$b_t = \varphi v_b b_{t-1} + \psi v_u u_{t-1} + d_{t-1} + v_d d_{t-2}$$

where $\varphi = \frac{\beta\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]}$, $\psi = \frac{\beta\left(Y^f - D - B\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]}$, $v_u = \frac{U}{U+B}$, $v_b = \frac{B}{U+B}$ and $v_d = \frac{\beta D}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]}$. The condition for the stationarity of the bubble is:

$$\varphi v_b = \frac{\beta\left(Y^f - D + U\right)}{\left[\beta\,(Y^f - D - B) - (U+B)\right]} \frac{B}{U+B} < 1$$

and coincides with that for the stability of the FEB. Indeed:

$$\frac{\partial H(B,U)}{\partial B} = \frac{\beta(1+\beta) D\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^2} < 1$$

or equivalently:

$$\frac{\partial H(B,U)}{\partial B} = \frac{\beta\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]} \frac{(1+\beta) D}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]} < 1$$

In the FEB, equation (2.24) can be alternatively expressed as:

$$\frac{B}{U+B} = \frac{(1+\beta) D}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]}$$

and so:

$$\frac{\partial H(B,U)}{\partial B} = \frac{\beta\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]} \frac{B}{U+B} = \varphi v_b$$

A.2 Existence of a Full Employment Bubbly Equilibrium

The logic of this proof is the same as that in Galí (2014, Appendix 2) and I hence omit many details. Before delivering the proof, I briefly introduce some properties of the H mapping:

1. $H(B,U) \geq 0$ is twice continuously differentiable for $0 \leq B < \bar{B}(U)$, where $\bar{B}(U) = \frac{1}{1+\beta}\left[\beta\left(Y^f - D\right) - U\right]$. If $B > \bar{B}(U)$, $H(B,U) < 0$.

2. The derivatives of $H(B,U)$ with respect to B_t are:

$$\frac{\partial H(B,U)}{\partial B_t} = \frac{\beta(1+\beta) D\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^2} > 0$$

$$\frac{\partial^2 H(B,U)}{\partial B_t^2} = \frac{2\beta(1+\beta)^2 D\left(Y^f - D + U\right)}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^3} > 0$$

Both inequalities hold for $0 \leq B < \bar{B}(U)$ and $\lim_{B\to\bar{B}(U)} H(B,U) = +\infty$.

3. The derivatives of $H(B,U)$ with respect to U are:

$$\frac{\partial H(B,U)}{\partial U} = \frac{\beta(1+\beta)\left(Y^f - D - B\right) D}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^2} > 0$$

$$\frac{\partial^2 H(B,U)}{\partial U^2} = \frac{2\beta(1+\beta)\left(Y^f - D - B\right) D}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^3} > 0$$

Both inequalities hold for $0 \leq B < \bar{B}(U)$ and $\lim_{B\to\bar{B}(U)} H(B,U) = +\infty$.

4. The mixed second derivative is:

$$\frac{\partial H(B,U)}{\partial B_t \partial U} = \frac{\beta(1+\beta) D\left[\beta\left(Y^f - D - B\right) - (U+B) + 2\left(Y^f - D + U\right)\right]}{\left[\beta\left(Y^f - D - B\right) - (U+B)\right]^3} > 0$$

for $0 \leq B < \bar{B}(U)$ and $\lim_{B\to\bar{B}(U)} H(B,U) = +\infty$.

Now, we turn to the proof of (2.25) and study the case $U = 0$. Equation (2.24) reduces to:

$$B_{t+1} = \frac{(1+\beta) D B_t}{\beta\left(Y^f - D - B_t\right) - B_t} = H(B_t, 0) \tag{A.1}$$

$H(0,0) = 0$, so there is a full employment bubbleless steady state. A necessary and sufficient condition for the existence of a FEB ($B^U > 0$) is:

$$\frac{\partial H(0,0)}{\partial B_t} = \frac{(1+\beta) D}{\beta\left(Y^f - D\right)} < 1$$

or, equivalently:

$$D < \frac{\beta}{(1+\beta)} \left(Y^f - D\right) \tag{A.2}$$

As in Galí (2014), $B^U > 0$ is unstable.

Sufficiency: assume that $U > 0$ and condition (A.2) holds; given property 3 and the continuity of H, for any $U \in [0, \bar{U})$ there are two steady states, $B^U(U)$ and $B^S(U)$, where $B^U(U) > B^S(U)$. These have the same stability properties as those in Galí (2014).

Necessity: the proof is identical to that in Galí (2014).

Appendix B

Appendix for Chapter 3

B.1 Stationarity of the Bubble

Denote log-linearized variables by lowercase letters. Equation (3.23) can be expressed in log-linear form as:

$$b_t = \varphi v_b b_{t-1} + \varphi v_u u_{t-1} + \psi_d d_{t-1} + \psi_u u_t + v_d d_{t-2}$$

where $\varphi = \frac{\beta\left(Y^f - D\right)}{\left[\beta\left(Y^f - D - U - B\right) - (U+B)\right]}$, $\psi_d = \frac{D}{D+U}$, $\psi_u = \frac{U}{D+U}$, $v_u = \frac{U}{U+B}$, $v_b = \frac{B}{U+B}$ and $v_d = \frac{\beta D}{\left[\beta\left(Y^f - D - U - B\right) - (U+B)\right]}$. The old bubble is stationary if:

$$\varphi v_b = \frac{\beta\left(Y^f - D\right)}{\left[\beta\left(Y^f - D - U - B\right) - (U+B)\right]} \frac{B}{U+B} < 1$$

This condition is equivalent to that for the stability of the FEB:

$$\frac{\partial H\left(B, U\right)}{\partial B} = \frac{\beta\left(Y^f - D\right)}{\left[\beta\left(Y^f - D - U - B\right) - (U+B)\right]} \frac{(1+\beta)\left(D+U\right)}{\left[\beta\left(Y^f - D - U - B\right) - (U+B)\right]} < 1$$

Indeed, rearranging equation (3.23) yields:

$$\frac{B}{U+B}=\frac{(1+\beta)(D+U)}{\beta\left(Y^f-D-U-B\right)-(U+B)}$$

so:

$$\frac{\partial H(B,U)}{\partial B}=\frac{\beta\left(Y^f-D\right)}{\left[\beta\left(Y^f-D-U-B\right)-(U+B)\right]}\frac{B}{U+B}=\varphi v_b$$

B.2 Existence of a Full Employment Bubbly Equilibrium

H mapping has the following properties:

1. $H(B,U)\geq 0$ is twice continuously differentiable for $0\leq B<\bar{B}(U)$, where $\bar{B}(U)=\frac{\beta}{1+\beta}\left(Y^f-D\right)-U$. If $B>\bar{B}(U)$, then it is $H(B,U)<0$.

2. The derivatives of $H(B,U)$ with respect to B_t are:

$$\frac{\partial H(B,U)}{\partial B_t}=\frac{\beta(1+\beta)(D+U)\left(Y^f-D\right)}{\left[\beta\left(Y^f-D-U-B\right)-(U+B)\right]^2}>0$$

$$\frac{\partial^2 H(B,U)}{\partial B_t^2}=\frac{2\beta(1+\beta)^2(D+U)\left(Y^f-D\right)}{\left[\beta\left(Y^f-D-U-B\right)-(U+B)\right]^3}>0$$

   Both inequalities hold for $0\leq B<\bar{B}(U)$ and $\lim_{B\to\bar{B}(U)}H(B,U)=+\infty$.

3. The first and the second derivative of $H(B,U)$ with respect to U are positive for $0\leq B<\bar{B}(U)$ and $\lim_{B\to\bar{B}(U)}H(B,U)=+\infty$.

4. The mixed second derivative is positive for $0\leq B<\bar{B}(U)$ and $\lim_{B\to\bar{B}(U)}H(B,U)=+\infty$.

Consider first the case $U=0$. Equation (3.23) becomes:

$$B_{t+1} = \frac{(1+\beta)\, DB_t}{\beta\,(Y^f - D - B_t) - B_t} = H\,(B_t, 0) \tag{B.1}$$

$H\,(0,0) = 0$, so there exists a full employment bubbleless steady state. A necessary and sufficient condition for the existence of a FEB ($B^U > 0$) is:

$$\frac{\partial H\,(0,0)}{\partial B_t} = \frac{(1+\beta)\, D}{\beta\,(Y^f - D)} < 1$$

or:

$$D < \frac{\beta}{1+\beta}\left(Y^f - D\right) \tag{B.2}$$

$B^U > 0$ is unstable for the same reasons expressed in Galí (2014).

Sufficiency and **Necessity**: see the proof in appendix A.

B.3 Existence of a Full Employment Bubbly Equilibrium with Fiscal Policy

In a full employment bubbly equilibrium, $g_t = g$, $D_t = D$, $U_t = U$ and $B_t^g = B^g$ with g, D, U,$B^g \geq 0$. Nominal wage behaves as if it is flexible and the ZLB is not binding, hence $L_t = \bar{L}$, $Y_t = \bar{L}^\alpha = Y^f$ and $\frac{Z_t}{P_t} = (1-\alpha)\, Y^f$. The equation describing the dynamics of old bubbles is derived by combining (3.6), (3.18) and (3.45):

$$B_{t+1} = \frac{\left[(1+\beta)\,(D+U) - T^o\right](U + B_t)}{\beta\,(Y^f - D - T^m - U - B_t) - (1+\beta)\, B^g - (U + B_t)} = H\,(B_t, U) \tag{B.3}$$

Before deriving the condition for the existence of a full employment bubbly steady state, which is given by a pair (B, U) such that $B = H\,(B, U)$ with $B \in (0, Y^f)$, I list some relevant properties of $H\,(B_t, U)$:

1. $H(B,U) \geq 0$ is twice continuously differentiable for $0 \leq B < \bar{B}(U)$ with $\bar{B}(U) = \frac{\beta}{1+\beta}\left(Y^f - D - T^m\right) - B^g - U$ and $(1+\beta)(D+U) \geq T^o$.

2. The derivatives of $H(B,U)$ with respect to B_t are:

$$\frac{\partial H(B,U)}{\partial B_t} = \frac{[(1+\beta)(D+U) - T^o]\left[\beta\left(Y^f - D - T^m\right) - (1+\beta)B^g\right]}{\left[\beta\left(Y^f - D - T^m - U - B\right) - (1+\beta)B^g - (U+B)\right]^2} > 0$$

$$\frac{\partial^2 H(B,U)}{\partial B_t^2} = \frac{2(1+\beta)[(1+\beta)(D+U) - T^o]\left[\beta\left(Y^f - D - T^m\right) - (1+\beta)B^g\right]}{\left[\beta\left(Y^f - D - T^m - U - B\right) - (1+\beta)B^g - (U+B)\right]^3} > 0$$

   Both inequalities hold for $0 \leq B < \bar{B}(U)$ and $\lim_{B \to \bar{B}(U)} H(B,U) = +\infty$.

3. The first and the second derivative of $H(B,U)$ with respect to U are positive for $0 \leq B < \bar{B}(U)$ and $\lim_{B \to \bar{B}(U)} H(B,U) = +\infty$.

4. The mixed second derivative of $H(B,U)$ is positive for $0 \leq B < \bar{B}(U)$ and $\lim_{B \to \bar{B}(U)} H(B,U) = +\infty$.

When $U = 0$, equation (B.3) reduces to:

$$B_{t+1} = \frac{[(1+\beta)D - T^o]}{\beta\left(Y^f - D - T^m - B_t\right) - (1+\beta)B^g - B_t} B_t = H(B_t, U) \qquad \text{(B.4)}$$

In this case, a necessary and sufficient condition for the existence of a full employment bubbly steady state ($B^U > 0$) is:

$$\frac{\partial H(0,0)}{\partial B_t} = \frac{[(1+\beta)D - T^o]}{\left[\beta\left(Y^f - D - T^m\right) - (1+\beta)B^g\right]} < 1$$

or:

$$D < \frac{\beta}{1+\beta}\left(Y^f - D - T^m\right) - B^g + \frac{T^o}{1+\beta} \qquad \text{(B.5)}$$

$B^U > 0$ is unstable.

Sufficiency and **Necessity**: see the proof in appendix A.

Let me conclude by noting that, also in this case, if the necessary and sufficient condition is met, there is a continuum of stable and unstable steady states for any $U \in [0, \bar{U})$ where:

$$\bar{U} = \frac{\left[(1+\beta)(D+B^g) - T^o - \beta\left(Y^f - D - T^m\right)\right]^2}{4(1+\beta)\left[\beta\left(Y^f - D - T^m\right) - (1+\beta)B^g\right]}$$

Finally, the upper bound on B is $B^U(0) = \frac{\beta}{1+\beta}\left(Y^f - D - T^m\right) - B^g - D + \frac{1}{(1+\beta)}T^o$.

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