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Quenching time of solutions for some nonlinear parabolic equations with Dirichlet boundary condition and a potential

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Abstract

In this paper, we address the following initial-boundary value problem

$$\begin{cases} u_t(x,t) = Lu(x,t) + r(x)(b-u(x,t))^{-p} \quad \text{in} \quad \Omega \times (0,T), \\ u(x,t) = 0 \quad \text{on} \quad \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) \geqslant 0 \quad \text{in } \Omega, \end{cases}$$

where $p > 2$, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, L is an elliptic operator, $b = \text{const} > 0$, $r \in C^1(\overline{\Omega})$, $\sup_{x\in\Omega} r(x) > 0$, $r(x)$ is nonnegative in Ω, $u_0 \in C^1(\overline{\Omega})$, $u_0(x)$ is a nonnegative in Ω, $\sup_{x\in\Omega} u_0(x) < b$. Under some assumptions, we show that the solution of the above problem quenches in a finite time, and its quenching time goes to that of the solution of the following differential equation $\alpha'(t) = r_0(b-\alpha(t))^{-p}$, $t > 0$, $\alpha(0) = M$, as M tends to b, where $M = \sup_{x\in\Omega} u_0(x)$ and $r_0 = \sup_{x\in\Omega} r(x)$. Finally, we give some numerical results to illustrate our analysis.

Keywords: Nonlinear parabolic equation, Dirichlet boundary condition, numerical quenching time, quenching

MSC: 35B40, 35B50, 35K60, 65M06

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. Consider the following initial-boundary value problem for a nonlinear parabolic equation with

Dirichlet boundary condition and a potential of the from

$$u_t(x,t) = Lu(x,t) + r(x)(b - u(x,t))^{-p} \quad \text{in} \quad \Omega \times (0,T), \tag{1.1}$$

$$u(x,t) = 0 \quad \text{on} \quad \partial\Omega \times (0,T), \tag{1.2}$$

$$u(x,0) = u_0(x) \geqslant 0 \quad \text{in} \quad \Omega, \tag{1.3}$$

where $p > 2$, $b = \text{const} > 0$,

$$Lu = \sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u}{\partial x_j}\right),$$

where $a_{ij} : \overline{\Omega} \to \mathbb{R}$, $a_{ij} \in C^1(\overline{\Omega})$, $a_{ij} = a_{ji}$, $1 \leqslant i,j \leqslant N$, and there exists a constant $C > 0$ such that

$$\sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geqslant C\|\xi\|^2 \quad \forall x \in \overline{\Omega} \quad \forall \xi = (\xi_1, \ldots, \xi_N) \in \mathbb{R}^N,$$

where $\|.\|$ stands for the Euclidean norm of $\mathbb{R}^N$.

The initial data $u_0 \in C^1(\overline{\Omega})$, $u_0(x)$ is a nonnegative in Ω, $\sup_{x\in\Omega} u_0(x) < b$, $r \in C^1(\overline{\Omega})$, $r(x)$ is nonnegative in Ω, $\sup_{x\in\Omega} r(x) > 0$. Here, $(0,T)$ is the maximal time interval of existence of the solution u of (1.1)–(1.3), and by a solution, we mean the following.

Definition 1.1. A solution of (1.1)–(1.3) is a function $u(x,t)$ continuous in $\overline{\Omega} \times [0,T)$, $u(x,t) < b$ in $\overline{\Omega} \times [0,T)$, and twice continuously differentiable in x and once in t in $\Omega \times (0,T)$.

The time T may be finite or infinite. When T is infinite, then we say that the solution u exists globally. When T is finite, then the solution u develops a singularity in a finite time, namely,

$$\lim_{t\to T} \|u(\cdot,t)\|_\infty = b,$$

where $\|u(\cdot,t)\|_\infty = \max_{x\in\Omega} |u(x,t)|$. In this last case, we say that the solution u quenches in a finite time, and the time T is called the quenching time of the solution u.

Throughout this paper, we suppose that there exists $a \in \Omega$ such that

$$M = \sup_{x\in\Omega} u_0(x) = u_0(a) \quad \text{and} \quad r_0 = \sup_{x\in\Omega} r(x) = r(a).$$

Solutions of nonlinear parabolic equations which quench in a finite time have been the subject of investigations of many authors (see [3–5, 7, 9–15, 18, 21, 22, 24–26, 28, 29] and the references cited therein). In particular, the above problem has been studied by many authors, and by standard methods based on the maximum principle, local existence, uniqueness, quenching and global existence have been treated (see [7, 23, 24, 29]). In this paper, we are interested in the asymptotic behavior of the quenching time. Our work was motivated by the paper of Friedman and Lacey in [16], where they have considered the following initial-boundary value problem

$$u_t = \epsilon \Delta u + f(u) \ \text{ in } \ \Omega \times (0, T),$$

$$u = 0 \ \text{ on } \ \partial\Omega \times (0, T),$$

$$u(x, 0) = u_0(x) \geqslant 0 \ \text{ in } \ \Omega,$$

where $f(s)$ is positive, increasing, convex function for nonnegative values of s, $\int_0^\infty \frac{ds}{f(s)} < \infty$, ϵ is a positive parameter. The initial data $u_0(x)$ is a continuous function in Ω. Under some additional conditions on the initial data, they have proved that the solution u of the above problem blows up in a finite time, and its blow-up time goes to that of the solution of the following differential equation

$$\alpha'(t) = f(\alpha(t)), \quad \alpha(0) = M,$$

as ϵ goes to zero, where $M = \sup_{x\in\Omega} u_0(x)$ (we say that a solution blows up in a finite time if it attains the value infinity in a finite time). Also in [28], Nabongo and Boni have considered the problem (1.1)–(1.3) in the case where the potential $r(x) = 1$ and the operator L is replaced by ϵL. They have obtained a similar result as that found in [16] by Friedman and Lacey. Let us notice that for this kind of problems, other parameters have been taken such that the norm of the initial data (see, for instance [17]) in the case of blow-up problems. In the present paper, we also take the norm of the initial data as parameter and obtain an analogous result using both a modification of Kaplan's method (see [20]) and a method based on the construction of upper solutions. Our paper is written in the following manner. In the next section, under some conditions, we show that the solution u of (1.1)–(1.3) quenches in a finite time, and its quenching time goes to that of the solution of a certain differential equation as the norm of the initial data goes to b. Finally, in the last section, we give some numerical results to illustrate our analysis.

2. Quenching times

In this section, under some assumptions, we show that the solution u of (1.1)–(1.3) quenches in a finite time, and its quenching time tends to that of the solution of a certain differential equation as M tends to b.

In the introduction of the paper, we have mentioned that there exists $a \in \Omega$ such that $r_0 = \sup_{x\in\Omega} r(x) = r(a)$ and $M = \sup_{x\in\Omega} u_0(x) = u_0(a)$. Consider the following eigenvalue problem

$$-L\psi = \lambda_\delta \psi \ \text{ in } \ B(a, \delta), \tag{2.1}$$

$$\psi = 0 \ \text{ on } \ \partial B(a, \delta),$$

$$\psi > 0 \ \text{ in } \ B(a, \delta),$$

where $\delta > 0$, such that, $B(a,\delta) = \{x \in \mathbb{R}^N; \quad \|x-a\| < \delta\} \subset \Omega$. It is well known that the above eigenvalue problem admits a solution (ψ, λ_δ) such that $0 < \lambda_\delta \leqslant \frac{D}{\delta^2}$, where D is a positive constant which depends only on the upper bound of the coefficients of the operator L and the dimension N. We can normalize ψ so that $\int_{B(a,\delta)} \psi dx = 1$.

Now, we are in a position to state the main result of this paper.

Theorem 2.1. *Let K be an upper bound of the first derivatives of u_0 and r. Suppose that $\sup_{x\in\Omega} u_0(x) = M > 0$ and let $A = (1 + bDK^2 2^p)/r_0$. If*

$$b - M < \min\{1, A^{-3/(p+1)}, (K\mathrm{dist}(a, \partial\Omega))^{3/(p+1)}\},$$

then the solution u of (1.1)–(1.3) quenches in a finite time, and its quenching time T satisfies the following estimates

$$0 \leqslant T - T_M \leqslant \frac{1}{r_0}\left(1 + \frac{A}{p+1}\right)(b-M)^{(4p+1)/3} + o((b-M)^{(4p+1)/3}),$$

where $T_M = \frac{(b-M)^{p+1}}{r_0(p+1)}$ is the quenching time of the solution $\alpha(t)$ of the differential equation defined as follows

$$\alpha'(t) = r_0(b - \alpha(t))^{-p}, \quad t > 0, \quad \alpha(0) = M.$$

Proof. Since $u_0 \in C^1(\overline{\Omega})$ and $r \in C^1(\overline{\Omega})$, invoking the mean value theorem and the triangle inequality, we find that

$$u_0(x) \geqslant M - (b-M)^{(p+1)/3} \quad \text{for} \quad x \in B(a,\delta),$$

$$r(x) \geqslant r_0 - (b-M)^{(p+1)/3} \quad \text{for} \quad x \in B(a,\delta),$$

where $\delta = \frac{(b-M)^{(p+1)/3}}{K}$. Let $w(x,t)$ be the solution of the following initial-boundary value problem

$$w_t(x,t) - Lw(x,t) - r(x)(b - w(x,t))^{-p} = 0 \quad \text{in} \quad B(a,\delta) \times (0, T^*), \tag{2.2}$$

$$w(x,t) = 0 \quad \text{on} \quad \partial B(a,\delta) \times (0, T^*),$$

$$w(x,0) = u_0(x) \quad \text{in} \quad B(a,\delta),$$

where $(0, T^*)$ is the maximal time interval of existence of the solution w. By an application of the maximum principle, we see that w is nonnegative in $B(a,\delta) \times (0, T^*)$, because the initial data is nonnegative in $B(a,\delta)$. Introduce the function $v(t)$ defined as follows

$$v(t) = \int_{B(a,\delta)} w(x,t)\psi(x)dx \quad \text{for} \quad t \in [0, T^*).$$

Take the derivative of v in t and use (2.2) to obtain

$$v'(t) = \int_{B(a,\delta)} \psi Lw dx + \int_{B(a,\delta)} r(x)(b-w)^{-p}\psi dx \quad \text{for} \quad t \in (0, T^*).$$

Applying Green's formula, we arrive at

$$v'(t) = \int_{B(a,\delta)} wL\psi dx + \int_{B(a,\delta)} r(x)(b-w)^{-p}\psi dx \quad \text{for} \quad t \in (0, T^*).$$

Due to the fact that $r(x) \geqslant r_0 - (b-M)^{(p+1)/3} > 0$ for $x \in B(a,\delta)$, using (2.1) and Jensen's inequality, we discover that

$$v'(t) \geqslant -\lambda_\delta v(t) + (r_0 - (b-M)^{(p+1)/3})(b - v(t))^{-p}.$$

Let us notice that $0 \leqslant v(t) \leqslant b$ for $t \in (0, T^*)$, and

$$0 < \lambda_\delta \leqslant \frac{D}{\delta^2} = \frac{DK^2}{(b-M)^{(2p+2)/3}}.$$

We deduce that

$$v'(t) \geqslant r_0(b-v(t))^{-p}\left(1 - \frac{(b-M)^{(p+1)/3}}{r_0} - \frac{bDK^2(b-v(t))^p}{r_0(b-M)^{(2p+2)/3}}\right) \text{ for } t \in (0,T^*).$$

Obviously, we have $(b-M)^{(p+1)/3} \leqslant (b-M)^{(p-2)/3}$ and

$$b - v(0) \leqslant b - M + (b-M)^{(p+1)/3} \leqslant 2(b-M),$$

which implies that

$$v'(0) \geqslant r_0(b - v(0))^{-p}(1 - A(b-M)^{(p-2)/3}) > 0.$$

We claim that

$$v'(t) > 0 \text{ for } t \in (0, T^*).$$

To prove the claim, we argue by contradiction. Indeed, let t_0 be the first $t \in (0, T^*)$ such that $v'(t) > 0$ for $t \in [0, t_0)$ but $v'(t_0) = 0$. Thus, we have $v(t_0) \geqslant v(0)$, which implies that

$$0 = v'(t_0) \geqslant r_0(b - v(0))^{-p}(1 - A(b-M)^{(p-2)/3}) > 0.$$

But, this is a contradiction, and the claim is proved. Consequently, we get

$$b - v(t) \leqslant b - v(0) \leqslant 2(b-M) \text{ for } t \in (0, T^*),$$

and with the help of the above inequalities, we arrive at

$$v'(t) \geqslant r_0(b - v(t))^{-p}(1 - A(b-M)^{(p-2)/3}) \text{ for } t \in (0, T^*).$$

This estimate may be rewritten as follows

$$(b-v)^p dv \geqslant r_0(1 - A(b-M)^{(p-2)/3})\mathrm{d}t \text{ for } t \in (0, T^*).$$

Integrate the above inequality over $(0,T^*)$ to obtain

$$\frac{(b-v(0))^{p+1}}{p+1} \geqslant r_0(1-A(b-M)^{(p-2)/3})T^*,$$

which implies that

$$T^* \leqslant \frac{(b-M+(b-M)^{(p+1)/3})^{p+1}}{r_0(p+1)(1-A(b-M)^{(p-2)/3})}.$$

We conclude that w quenches in a finite time because the quantity on the right hand side of the above inequality is finite. On the other hand, by the maximum principle, we have $u \geqslant 0$ in $\Omega \times (0,T)$. Exploiting this estimate, it is easy to see that

$$u_t - Lu - r(x)(1-u)^{-p} \geqslant w_t - Lw - r(x)(1-w)^{-p} \text{ in } B(a,\delta)\times(0,T_*),$$

$$u \geqslant w \text{ on } \partial B(a,\delta)\times(0,T_*),$$

$$u(x,0) \geqslant w(x,0) \text{ in } B(a,\delta),$$

where $T_* = \min\{T, T^*\}$. It follows from the maximum principle that

$$u(x,t) \geqslant w(x,t) \text{ in } B(a,\delta)\times(0,T_*),$$

which implies that

$$T \leqslant T^* \leqslant \frac{(b-M+(b-M)^{(p+1)/3})^{p+1}}{r_0(p+1)(1-A(b-M)^{(p-2)/3})}. \tag{2.3}$$

Indeed, suppose that $T > T^*$. We have $\|u(\cdot,T^*)\|_\infty \geqslant \|w(\cdot,T^*)\|_\infty = b$. But, this is a contradiction because $(0,T)$ is the maximal time interval of existence of the solution u. Now, setting $z(x,t) = \alpha(t)$ in $\overline{\Omega}\times[0,T_0)$, it is not hard to see that

$$z_t - Lz - r(x)(1-z)^{-p} = 0 \text{ in } \Omega\times(0,T_0),$$

$$z \geqslant 0 \text{ on } \partial\Omega\times(0,T_0),$$

$$z(x,0) \geqslant u_0(x) \text{ in } \Omega.$$

The maximum principle implies that $0 \leqslant u(x,t) \leqslant z(x,t) = \alpha(t)$ in $\Omega\times(0,T^0)$, where $T^0 = \min\{T_0, T\}$. We infer that

$$T \geqslant T_0 = \frac{(b-M)^{p+1}}{r_0(p+1)}. \tag{2.4}$$

Indeed, suppose that $T_0 > T$, which implies that $\alpha(T) \geqslant \|u(\cdot,T)\|_\infty = b$. But, this is a contradiction because $(0,T_0)$ is the maximal time interval of existence of the solution $\alpha(t)$. Apply Taylor's expansion to obtain

$$(b-M+(b-M)^{(p+1)/3})^{p+1} = (b-M)^{p+1}$$

$$+(p+1)(b-M)^{(4p+1)/3}+o((b-M)^{(4p+1)/3}),$$

$$\frac{1}{1-A(b-M)^{(p-2)/3}}=1+A(b-M)^{(p-2)/3}+o((b-M)^{(p-2)/3}).$$

Use (2.3), (2.4) and the above relations to complete the rest of the proof. □

Remark 2.2. Let us notice that the estimates obtained in Theorem 2.1 may be rewritten in the following form

$$0 \leqslant \frac{T}{T_M}-1 \leqslant (p+1+A)(b-M)^{(p-2)/3}+o((b-M)^{(p-2)/3}).$$

We deduce that $\lim_{M\to b}\frac{T}{T_M}=1$.

3. Numerical results

In this section, we give some computational results to confirm the theory established in the previous section. We consider the radial symmetric solution of the initial-boundary value problem below

$$u_t=\Delta u+\frac{1}{\|x\|+1}(1-u)^{-p} \quad \text{in} \quad B\times(0,T),$$

$$u=0 \quad \text{on} \quad S\times(0,T),$$

$$u(x,0)=u_0(x) \quad \text{in} \quad B,$$

where $B=\{x\in\mathbb{R}^N;\|x\|<1\}$, $S=\{x\in\mathbb{R}^N;\|x\|=1\}$ and $u_0(x)=M\cos(\frac{\pi\|x\|}{2})$ with $M\in(0,1)$. The above problem may be rewritten in the following form

$$u_t=u_{rr}+\frac{N-1}{r}u_r+\frac{1}{r+1}(1-u)^{-p}, \quad r\in(0,1), \quad t\in(0,T), \tag{3.1}$$

$$u_r(0,t)=0, \quad u(1,t)=0, \quad t\in(0,T), \tag{3.2}$$

$$u(r,0)=\varphi(r), \quad r\in(0,1), \tag{3.3}$$

where $\varphi(r)=M\cos(\frac{\pi r}{2})$. We start by the construction of some adaptive schemes as follows. Let I be a positive integer and let $h=1/I$. Define the grid $x_i=ih$, $0\leqslant i\leqslant I$, and approximate the solution u of (3.1)–(3.3) by the solution $U_h^{(n)}=(U_0^{(n)},\ldots,U_I^{(n)})^T$ of the following explicit scheme

$$\frac{U_0^{(n+1)}-U_0^{(n)}}{\Delta t_n}=N\frac{2U_1^{(n)}-2U_0^{(n)}}{h^2}+(1-U_0^{(n)})^{-p},$$

$$\frac{U_i^{(n+1)}-U_i^{(n)}}{\Delta t_n}=\frac{U_{i+1}^{(n)}-2U_i^{(n)}+U_{i-1}^{(n)}}{h^2}+\frac{(N-1)}{ih}\frac{U_{i+1}^{(n)}-U_{i-1}^{(n)}}{2h}$$

$$+\frac{1}{ih+1}(1-U_i^{(n)})^{-p}, \quad 1 \leqslant i \leqslant I-1,$$

$$U_I^{(n)} = 0, \quad U_i^{(0)} = M\cos\left(\frac{ih\pi}{2}\right), \quad 0 \leqslant i \leqslant I,$$

where $n \geqslant 0$. In order to permit the discrete solution to reproduce the properties of the continuous one when the time t approaches the quenching time T, we need to adapt the size of the time step so that we take

$$\Delta t_n = \min\left\{\frac{h^2}{2N}, h^2(1-\|U_h^{(n)}\|_\infty)^{p+1}\right\}$$

with $\|U_h^{(n)}\|_\infty = \sup_{0\leqslant i\leqslant I}|U_i^{(n)}|$. Let us notice that the restriction on the time step ensures the nonnegativity of the discrete solution. We also approximate the solution u of (3.1)–(3.3) by the solution $U_h^{(n)}$ of the implicit scheme below

$$\frac{U_0^{(n+1)}-U_0^{(n)}}{\Delta t_n} = N\frac{2U_1^{(n+1)}-2U_0^{(n+1)}}{h^2} + (1-U_0^{(n)})^{-p},$$

$$\frac{U_i^{(n+1)}-U_i^{(n)}}{\Delta t_n} = \frac{U_{i+1}^{(n+1)}-2U_i^{(n+1)}+U_{i-1}^{(n+1)}}{h^2} + \frac{(N-1)}{ih}\frac{U_{i+1}^{(n+1)}-U_{i-1}^{(n+1)}}{2h}$$
$$+\frac{1}{ih+1}(1-U_i^{(n)})^{-p}, \quad 1 \leqslant i \leqslant I-1,$$

$$U_I^{(n+1)} = 0, \quad U_i^{(0)} = M\cos\left(\frac{ih\pi}{2}\right), \quad 0 \leqslant i \leqslant I.$$

As in the case of the explicit scheme, here, we also choose

$$\Delta t_n = h^2(1-\|U_h^{(n)}\|_\infty)^{p+1}.$$

For the above implicit scheme, the existence and nonnegativity of the discrete solution are also guaranteed using standard methods (see, for instance [6]).
We note that

$$\lim_{r\to 0}\frac{u_r(r,t)}{r} = u_{rr}(0,t),$$

which implies that

$$u_t(0,t) = Nu_{rr}(0,t) + (1-u(0,t))^{-p} \quad \text{for} \quad t \in (0,T).$$

This observation has been taken into account in the construction of the above schemes at the first node. We need the following definition.

Definition 3.1. We say that the discrete solution $U_h^{(n)}$ of the explicit scheme or the implicit scheme quenches in a finite time if $\lim_{n\to\infty}\|U_h^{(n)}\|_\infty = 1$, and the series $\sum_{n=0}^{\infty}\Delta t_n$ converges. The quantity $\sum_{n=0}^{\infty}\Delta t_n$ is called the numerical quenching time of the discrete solution $U_h^{(n)}$.

In the following tables, in rows, we present the numerical quenching times, the numbers of iterations, the CPU times and the orders of the approximations corresponding to meshes of 16, 32, 64, 128. We take for the numerical quenching time $t_n = \sum_{j=0}^{n-1} \Delta t_j$ which is computed at the first time when

$$\Delta t_n = |t_{n+1} - t_n| \leqslant 10^{-16}.$$

The order (s) of the method is computed from

$$s = \frac{\log((T_{4h} - T_{2h})/(T_{2h} - T_h))}{\log 2}.$$

Numerical experiments

First case: $p = 3$, $N = 2$, $M = 0.90$

Table 1. Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method.

I	t_n	n	CPU_t	s
16	2.5257 e-5	1361	1	-
32	2.5174 e-5	5100	3	-
64	2.5186 e-5	19007	32	2.79
128	2.5226 e-5	70461	2182	1.74

Table 2. Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method.

I	t_n	n	CPU_t	s
16	2.5258 e-5	1361	1	-
32	2.5174 e-5	5100	6	-
64	2.5186 e-5	19007	155	2.81
128	2.5226 e-5	70461	5534	1.74

Second case: $p = 3$, $N = 2$, $M = 0.95$

Table 3. Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the explicit Euler method.

I	t_n	n	CPU_t	s
16	1.5725 e-6	1183	1	-
32	1.5657 e-6	4384	3	-
64	1.5642 e-6	16124	44	2.18
128	1.5641 e-6	58833	2373	3.91

Table 4. Numerical quenching times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with the implicit Euler method.

I	t_n	n	CPU_t	s
16	1.5725 e-6	1183	1	-
32	1.5657 e-6	4384	4	-
64	1.5642 e-6	16124	103	2.18
128	1.5641 e-6	58833	3366	3.91

Remark 3.2. If we consider the problem (3.1)–(3.3) in the case where the initial data $\varphi(r) = 0.9\cos(\frac{\pi r}{2})$ and $p = 3$, then it is not hard to see that the quenching time of the solution of the differential equation defined in Theorem 2.1 equals 2.5 e-5. We observe from Tables 1-2 that the numerical quenching time is approximately equal 2.5 e-5. This result has been proved in Theorem 2.1. When the initial data $\varphi(r) = 0.95\cos(\frac{\pi r}{2})$ and $p = 3$, then we find that the quenching time of the solution of the differential equation defined in Theorem 2.1 equals 1.5625 e-5. We discover from Tables 3–4 that the numerical quenching time is approximately equal 1.5625 e-6 which is a result proved in Theorem 2.1.

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