Revealing incentives for compatibility provision in vertically differentiated network industries*

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Abstract

We determine the incentives for compatibility provision of firms that produce network goods with different intrinsic qualities when firms do not have veto power over compatibility. When network effects are strong, there are multiple equilibria in pricing and consumer decisions. We show that in some equilibria, it is the high quality firm that invests in compatibility, whereas in others, the low quality firm triggers compatibility. The socially optimal compatibility degree is zero, except under very strong network effects, where one of the equilibria has all consumers buying the low quality good. In this case, a partial degree of compatibility is optimal.

Keywords: Compatibility, vertical differentiation, network effects.

JEL Classification: L13, L15.

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1 Introduction

Since Rohlfs (1974) first described his theory of the interdependent demand for communication services, economists have studied different aspects of the so-called network industries. These are characterized by the existence of positive externalities in consumption, i.e. the utility of agents is increasing in the number of users of the same good or compatible goods. The consequence of this is that consumers choosing one such good always forego the benefits of interacting with the agents that choose other (incompatible) goods. A possible solution for firms to increase the willingness to pay of the consumers is to establish some degree of compatibility. Indeed, firms decide whether to make their goods compatible with those of their rivals, thus competing *in the market*, or to render them incompatible thus competing *for the market*.[1]

The objective of this paper is to determine the incentives for compatibility provision of vertically differentiated firms operating in a network market. Our paper is the first to concentrate the provision of partial compatibility. We also extend the literature by determining the compatibility incentives under strong network effects, that is when the importance of the network for consumers is so high that it induces multiple equilibria in consumption.

We can think of several examples for which our analysis could be used. These examples have in common the existence of direct or indirect network effects in consumption and differentiated quality of the products offered by distinct firms.[2] As a first example of a good with direct network

[1]As Besen and Farrell (1994) put forward, "there is no general answer to the question of whether firms will prefer competition for the potentially enormous prizes under inter-technology competition, or the more conventional competition that occurs when there are common standards." An example is Microsoft and Sony Team on Digital Entertainment Content Management System: although rivals in the gaming-console market, both companies found mutual gains in working closely to integrate the Sony VAIO XL1 Digital Living System with Microsoft Windows XP Media Center Edition 2005.

[2]It is common to distinguish between direct network effects, stemming from direct interaction of the consumers, and indirect network effects, which stem from other externalities related to the dimension of the consumer base,

effects we can think of Basic LEGO Bricks. This is a network good as the more compatible bricks you and your friends have, the larger your building possibilities. While LEGO remains the most famous bricks-maker and is perceived by consumers as a high quality good (at least for its long history and brand recognition), there are competitors, like Mega Blocks that might offer or not a compatible product.[3] Other examples, mostly characterized by indirect network effects, are: the game console industry, the operating systems industry and the high definition DVD industry. These industries can be classified as *two-sided markets* in the sense that users of one group (game console's, OS's, high definition DVD's users) are better off as the number of users of the other group (game, software, DVD developers) increases.[4] We do not model explicitly the two-sided market and focus on the role of (indirect) network effects. We can say that a consumer's utility for a particular product increases (indirectly) with the number of consumers of the same or compatible products when the availability of components or support services increases with consumption. In these industries, it is frequent to find that one firm is perceived by the consumers as being the high quality firm. For instance in the case of MAC vs Microsoft Windows operating systems, where it is often argued that Apple Macintosh is a higher quality product for example in terms of high resolution graphics, virus detection etc., whereas, historically and also due to large price differences, Microsoft has captured the largest market share. Another example is provided by the high definition DVD industry. HD-DVD was produced by Toshiba and had storage potential around 40 percent lower than the Blu-Ray DVD produced by Sony. Also, we can think about the car industry in which consumers are better off if others buy the same type of vehicle as the supply of customer services will be wider.

Compatibility can be achieved either by *standardization*, or by the introduction of a *converter*,

such as increased quality, services and components, etc.

[3]We borrow this example from Belleflamme and Peitz (2010, p. 579).

[4]For an analysis of competition in two-sided markets see Gabszewicz and Wauthy (2014) and Armstrong (2006).

a device which allows consumers of one product to enjoy (partially or fully) the network of the other product.[5] Often, the converter device represents a compromise of quality, in the sense that compatibility may be imperfect. Likewise, standardization is a costly requirement because it limits product variety. In our model, we assume that compatibility is a feature of the product itself, a characteristic that enhances the network effects, like a converter incorporated in the product.[6]

In general, the degree of compatibility that can be achieved depends on the vetoing power of firms over compatibility and on technical specificities of the products. As pointed out by de Palma et al. (1999), two situations can arise: i) no firm can veto the move towards compatibility, in which case the final degree of compatibility is given by the maximum between the degrees chosen noncooperatively by the firms; ii) both firms have veto power over compatibility, or, in other words, the final degree of compatibility is the minimum between the degrees chosen by the firms.

For the first scenario, we can think of a firm that cannot prevent a rival from reaching compatibility due to the expiration of a patent. Consider again the example of Basic LEGO Bricks. When it was invented in the 1950s, the brick was awarded patent protection, and since its expiration LEGO tried to prevent competitors (like Mega Blocks) from making their building blocks compatible with Basic LEGO Bricks. However, after a sentence of the Canadian Supreme Court in November 2005, Mega Blocks and LEGO bricks may be interchangeable (in spite of LEGO preferences). In this case it is not necessary that both firms contribute to compatibility, that

[5]For instance, in the HD-DVD/Blu-Ray case, producers of DVD discs at some point have come out with a disc which had the HD-DVD version of the movie on one side and the Blu-Ray version on the other. Also, car producers share the service facilities, rendering their products compatible. For instance a buyer of a Nissan sedan can enjoy the indirect network effects of the Renault service facilities. We borrow this example from Alexandrov (2015).

[6]For instance, in the technology transition process from the 5.25 inch to the 3.5 inch floppy disk drives in personal computers (from 1987 to the mid 90s when CD-ROM prevailed), computers where endowed with dual disks. Choi (1997) analyses this process.

is firms' contributions are substitutes.[7] As for the second scenario, we can think of a patented network good, so that a rival firm needs a license to reach compatibility. In this case there must be an agreement between the firms and both have to contribute to compatibility. As such, firms' contributions are complementary.

We here focus on the substitues case, that is we assume that no firm can veto compatibility and we compare the private and social incentives towards compatibility.[8] To this end, we develop a two-stage game where firms first choose the degree of compatibility and then the price of their products.

Our results are as follows: we identify the conditions under which different incentives to provide compatibility arise as a function of the intrinsic quality differentiation and the importance of the network effects. Under weak network effects, i.e., when the weight of the network effects relative to the vertical differentiation is not very strong, we can observe full compatibility at equilibrium. In this case, where both firms may remain active in the market, they are willing to provide compatibility because an increase in the compatibility degree softens competition. However, the low quality firm has higher incentives to provide full compatibility, in order to avoid the possibility of being stranded out of the market. On the other hand, under strong network effects, i.e., when the network effects dominate the vertical differentiation, we observe multiple equilibria for consumers' demands. Namely, as consumers value very highly the network, it is possible that all consumers buy either one of the two goods. In this case consumer expectations play a determinant role and we observe that the consumption-price equilibria are not coalition proof. Indeed, given the

[7]We can find examples also in the network formation literature: Bala and Goyal (2000) refer to the phone call where only the caller has to pay, however information can be exchanged by both parties. Also, Bloch and Dutta (2009) model separable investment implying non complementarity in the link formation.

[8]We examine also the case of complementary contributions in the working paper version Garcia and Vergari (2015).

prices and a demand allocation, we observe that the expectation of a coordinated deviation by consumers, would result in all consumers individually having incentive to deviate from the initial demand allocation. Once we introduce the compatibility decisions of the firms we obtain that, in any subgame perfect equilibrium only the high quality good is consumed, as long as it maintains its overall quality dominance. In order for this to happen, the high quality firm may need to provide compatibility to prevent consumers from buying the low-price low-quality good and remaining stranded out of the market. An example of a situation in which provision of compatibility could have changed the path of adoption is put forth by Postrel (1990).[9] Our results constitute a further rationale for the argument that network effects will not necessarily lead to market failure in the sense of a low quality dominating the market over a higher quality.[10] Also, our results are in line with Spulber (2008a) that argues that firm's interconnection decisions mitigate the lock-in by lower quality goods. To sum up, we show that both firms may have incentives to provide compatibility. In spite of that, as long as the network effects are not high enough to allow a switch in the overall quality differential, the low quality firm is willing to pay more for compatibility. The opposite may hold when the network effects are strong enough for the switch to occur, that is the high quality firm may be willing to offer compatibility for a wider range of costs.

[9]According to Postrel (1990) quadraphonic audio systems failed to replace stereo in the 1970s, despite backing from all the major manufacturers and recording houses and despite a higher quality. Network externalities and consumer expectations played a significant role in this episode. In this paper, Postrel finds that the introduction of competing incompatible quadraphonic systems hindered the development of a viable user base, which paired with a non-sufficient quality increase and high price lead to the non adoption of quadraphonic sound. The provision of at least partial compatibility or a better management of consumer expectations could have led to a different result for the companies supporting the quad sound systems.

[10]This argument has been put forth by Liebowitz and Margolis (1994) and contrasts several theoretical models. Among others, Farrell and Saloner (1985) and (1986) represent two examples of inefficient technology adoption due to the so-called *bandwagon effect*.

Concerning the social optimum, the welfare is maximized at a compatibility degree of zero, as long as, under strong network effects, consumers only consume the high quality good. Comparing the social optimum with the private provision, we find that compatibility is overprovided unless, under very strong network effects, the consumers buy the low quality good. Indeed, in this latter case, the social welfare is maximized by an intermediate degree of compatibility which corresponds to the private optimum.

Related Literature

Several papers examine the issue of compatibility from different points of view, concentrating on the weak network effects case. Baake and Boom (2001) study the decision of full or zero compatibility in a context of vertically differentiated products competing simultaneously in the market, not allowing for partial compatibility, as we do in our paper. Crémer et al. (2000) consider an extension of the seminal paper by Katz and Shapiro (1985) to study compatibility decisions in a Cournot oligopoly with homogeneous goods and heterogeneous consumers, where firms differ in their installed base of consumers. As most of this literature, these papers assume that compatibility requires the consent of both sides (contributions are complementary). The standard result predicts that smaller firms always have higher incentives for product compatibility than bigger firms. In particular, in Crémer et al. (2000), the larger firm (which determines the industry level of connectivity) benefits from a better perceived relative quality when connectivity is imperfect so that it does not take into account the impact of its interconnection decision on its rival, whose profit increases with the level of connectivity, nor on consumers, who also benefit from an improved connectivity. As a result compatibility is underprovided at equilibrium. Baake and Boom (2001) find that the high quality firm prefers incompatibility whereas the low quality firm prefers compatibility; however an adapter is always provided at equilibrium because the low quality firm can successfully prevent the incompatibility equilibrium through its quality choice. The private outcome is also socially optimal because the network externalities consumers enjoy are greater and the vertical differentiation is lower with the adapter than without the adapter. De Palma et al. (1999) focus on the possibility that consumers achieve full compatibility through multi-homing in a context where goods are homogeneous except for the dimension of the network. While most contributions concentrate on compatibility as a mutual decision, De Palma et al. (1999) discuss the impact of different property rights on the provision of compatibility. More recently, Doraszelski et al. (2009) analyze the long run stability of compatible products when differentiation is also derived from the size of the network alone and not from the intrinsic qualities of the products. As for the case of strong network effects, Malueg and Schwartz (2006) extend Crémer et al. (2000) by analyzing the compatibility incentives (as zero-one choice) of a large network facing multiple rivals focusing on the case in which compatibility contributions are complementary. Malueg and Schwartz allow the network effects to be strong enough for tipping to arise (all customers joining the same network). Other papers have addressed the issue of tipping and multiplicity of equilibria when network effects are strong. Under strong network effects, consumers expectations play a determinant role on the equilibrium outcomes. Alexandrov (2015) focuses on the effect of different expectations on the equilibria; also, under strong network effects, equilibria are generally not coalition-proof, in the sense that coordinated actions by players would lead to the market tipping. Ambrus and Argenziano (2009) use the concept of coalitional rationalizability proposed by Ambrus (2006) to select among the possible equilibria. However, in cases in which there is no focal network on which to coordinate, the concept does not necessarily impose successful coordination and multiplicity still arises. Other papers focus on the consumer coordination in network industries. Spulber (2008b) addresses the multiplicity of equilibria and potential coordination failure by allowing for coordination to arise from small to larger groups. A number of articles discuss the alternative path towards compatibility, which, as mentioned before, is standardization. These include Farrell and Saloner (1985, 1992), and more recently Ostrovsky and Schwarz (2005) and Alexandrov (2015).

Finally, it is worth mentioning a strand of the literature which focuses on firms' compatibility strategies towards vertically related firms. Theoretical models distinguish according to whether each component is sold by an independent firm or each firm produces everything necessary to form the final good (system). As an example of the first context, Church and Gandal (1992) study the software provision decision of software firms to hardware firms. As for the case of firms supplying all the necessary components, Matutes and Regibeau (1992) study firms' incentives to standardize components in industries where consumers try to assemble a number of components into a system that meets their specific needs.

The outline of the paper is as follows. Section 2 describes the model. Section 3 provides the results on the price competition and Section 4 analyses the compatibility choice by firms and presents the comparison with the socially optimal compatibility degree. Section 5 concludes the paper with a discussion of the results. The main proofs appear in the Appendix. The more technical proofs can be found in Appendix B of the Working Paper version of the paper (Garcia and Vergari (2015)).

2 Model

We develop a two-stage game involving two firms, A and B who produce competing goods at constant marginal cost set to zero. These goods are vertically differentiated and characterized by network externalities in consumption. In the first stage, the firms choose, simultaneously the compatibility of their products. In the second stage, the firms engage in a Bertrand-Nash pricing game. At the end of the second stage, the consumers make purchases. There is a continuum of consumers indexed by x, uniformly distributed on the interval $[0, 1]$. x represents the consumer's valuation of the quality. Each consumer buys either one unit of good A or one unit of good B. We rule out the possibility of no purchase, that is we concentrate on the situation in which the market is fully covered.[11,12]

Consumer x's utility from the consumption of good i takes the following form:

$$U_i(x) = \beta_i x + \alpha \left[D_i + \tau D_j\right]$$

The first term of the utility function, $\beta_i x$ represents a stand-alone benefit which depends on the intrinsic quality of good i, represented by β_i. We assume throughout that $\beta_B > \beta_A$, i.e., the intrinsic quality of good B is higher than that of good A. The second term in the utility represents the network benefit. The parameter $\alpha > 0$ denotes the intensity of the network effects and D_i and D_j are the demand of good i and j, respectively. The network benefit consists of the externality stemming from the interaction with consumers that buy the same good, (D_i), and the externality resulting from compatibility, which allows consumers to partially benefit from the rival network (τD_j).

The final degree of compatibility is endogenous and given by $\tau \in [0, 1]$ which is a function of the degrees of compatibility chosen by each firm, τ_A and τ_B in the following way: $\tau = \max\{\tau_A, \tau_B\}$.[13] Underlying this formulation is the idea that no firm can prevent a move towards compatibility. As such, the final compatibility is the maximum of the degrees chosen by the firms. In other words, each firm can unilaterally provide compatibility not being necessary that both contribute. The

[11] We also exclude the possibility for consumers to join both networks. This could be an alternative way to achieve compatibility as studied by de Palma et al. (1999).

[12] Covered markets occur in the case of mature and widespread industries, such as the software industry in developed countries versus emerging markets where there are still consumers to be conquered. We discuss this assumption in Section 5.

[13] In Garcia and Vergari (2015) we analyse both the case of $\tau = \max\{\tau_A, \tau_B\}$ and the case of $\tau = \min\{\tau_A, \tau_B\}$.

cost of providing compatibility is linear and given by $c\tau_i$.[14] We assume that firms are equally efficient in providing compatibility and thus face the same cost function.

Let $k \equiv \beta_B - \beta_A$ denote the intrinsic quality differentiation. We use throughout the following definitions:

Definition 1 *There are four categories of network effects: very weak, $k > 3\alpha(1-\tau)$; weak, $2\alpha(1-\tau) < k \leq 3\alpha(1-\tau)$; strong, $\alpha(1-\tau) \leq k < 2\alpha(1-\tau)$; and very strong $k < \alpha(1-\tau)$.*

Definition 2 *The overall quality differential between good B and good A is defined as:*

$$\Delta_{B,A} = k + \alpha(D_B - D_A)(1-\tau). \tag{1}$$

Definition 1 classifies four cases according to the relative importance of the network effects versus the intrinsic quality difference. Definition 2 establishes that there are two sources of quality differentiation between firms. One source is exogenous and is given by k and the other is endogenous and is proportional to the difference in the networks' size and given by $\alpha(D_B - D_A)(1-\tau)$. The endogenous source of differentiation can be manipulated by the firms through the choice of prices and through the choice of the degree of compatibility (τ). We can interpret Definition 2 as follows: when either the two networks have the same size ($D_A = D_B$) or compatibility is perfect ($\tau = 1$), consumers perceive the goods as being identical in terms of the network effects. The overall quality of good B is higher than that of good A if the network effects are not very strong, i.e., $k \geq \alpha(1-\tau)$. This means that even in the extreme case that the network benefit for firm A is the highest ($D_A = 1$ and $D_B = 0$), good B maintains its quality dominance. A switch in the overall differentiation takes place only if the network effects are very strong i.e. for $k < \alpha(1-\tau)$. In that case, the good with low intrinsic quality (good A) becomes the good with high overall quality.

[14]We discuss the role of the linear costs in Section 5

We solve the two-stage game by backward induction to obtain the subgame perfect equilibria. In the remainder of this section we present the consumer's choice.

Consumers choose between the goods in order to maximize their net surplus. In this maximization problem they take the decisions of the others as given and have rational expectations about the size of the networks, as in Katz and Shapiro (1985, 1994) or more recently Amir and Lazzati (2011). As such, in equilibrium, we have that the expected network size coincides with the actual network size.[15] Consumer x buys good A if and only if $U_A(x) - p_A > U_B(x) - p_B$ and $U_A(x) - p_A > 0$. Denote $\hat{x}$ the consumer type that is indifferent between the two goods and assume that the type $x = 0$ has positive net utility from buying product A, i.e. $U_A(0) - p_A = \alpha[D_A + \tau D_B] - p_A$ is nonnegative.[16] Demands are given by:

$$D_B = 1 - \hat{x},$$

$$D_A = \hat{x}.$$

We analyze the situation where both firms face a nonnegative demand, $\hat{x} \in [0, 1]$. There are three possible market configurations.

1. $D_A = 1$ and $D_B = 0$, which is possible for $p_B - p_A \geq k - \alpha(1 - \tau)$.

2. $D_A = 0$ and $D_B = 1$ which is possible for $p_B - p_A \leq \alpha(1 - \tau)$.

3. D_A, $D_B \in (0, 1)$ and $D_A + D_B = 1$. In this market configuration both firms set positive prices and obtain positive profits.[17]

[15] As expectations play an important role in the case of multiple self-fulfilled expectations equilibria, we provide a discussion on expectations in Section 3.2.

[16] The market coverage assumption in this is only possible thanks to the presence of positive network effects. Indeed, with $\alpha > 0$, consumer type zero may prefer buying because even if its valuation of the intrinsic quality is zero he benefits from the network of consumers buying the same good or compatible goods. See Gabszewicz and Garcia (2007) for a discussion.

[17] A complete analysis of the feasible price regions can be found in the Appendix (6.2).

The indifferent consumer is:

$$\hat{x} = \alpha(1-\tau)\frac{D_A - D_B}{k} + \frac{p_B - p_A}{k}, \tag{2}$$

which implies that demands, in the interior solution case (market configuration 3.), are given by:

$$D_A = \frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B - p_A}{k-2\alpha(1-\tau)}, \tag{3}$$

$$D_B = \frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} - \frac{p_B - p_A}{k-2\alpha(1-\tau)}. \tag{4}$$

Observing these expressions, we see that depending on the sign of $k - 2\alpha(1-\tau)$ they are either decreasing or increasing in own price.[18] In what follows we distinguish the two cases.

- Weak and very weak network effects. D_A and D_B are decreasing in own price, as depicted in Figure 1. The demands are:

$$D_B(p_A, p_B) = \begin{cases} 1, & p_B - p_A \leq \alpha(1-\tau) \\ \frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} - \frac{p_B-p_A}{k-2\alpha(1-\tau)}, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha(1-\tau) \\ 0, & p_B - p_A > k - \alpha(1-\tau) \end{cases} \tag{5}$$

$$D_A(p_A, p_B) = \begin{cases} 1, & p_B - p_A > k - \alpha(1-\tau) \\ \frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B-p_A}{k-2\alpha(1-\tau)}, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha(1-\tau) \\ 0, & p_B - p_A \leq \alpha(1-\tau) \end{cases} \tag{6}$$

- Strong and very strong network effects. The network benefit plays a dominant role in the differentiation among products. As such, multiple equilibria in the consumers' choice arise. In particular, as illustrated in Figure 2 for good A, the demands for the network goods are

[18] Note that for $k = 2\alpha(1-\tau)$ the demand in the interior solution case is not defined.

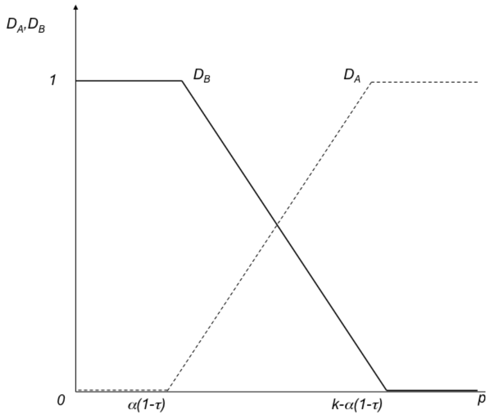

Figure 1: Demand functions: $k > 2\alpha\,(1-\tau)$

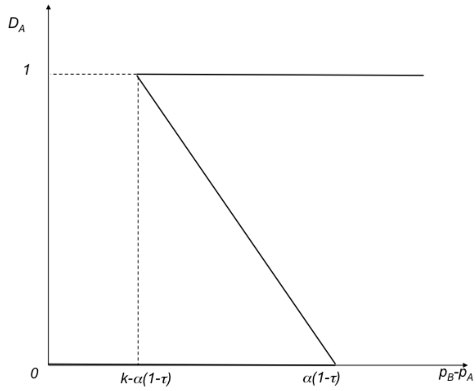

Figure 2: Demand for good A: $k < 2\alpha(1-\tau)$

correspondences:

$$D_B\,(p_A, p_B) = \begin{cases} 1, \quad p_B - p_A \leq \alpha(1-\tau) \\ \frac{-k+\alpha(1-\tau)}{2\alpha(1-\tau)-k} + \frac{p_B - p_A}{2\alpha(1-\tau)-k}, k - \alpha\,(1-\tau) \leq p_B - p_A \leq \alpha(1-\tau) \\ 0, \quad p_B - p_A \geq k - \alpha\,(1-\tau) \end{cases} \tag{7}$$

$$D_A\,(p_A, p_B) = \begin{cases} 1, \quad p_B - p_A \geq k - \alpha\,(1-\tau) \\ \frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} - \frac{p_B - p_A}{2\alpha(1-\tau)-k}, k - \alpha\,(1-\tau) \leq p_B - p_A \leq \alpha(1-\tau) \\ 0, \quad p_B - p_A \leq \alpha\,(1-\tau) \end{cases} \tag{8}$$

We define a consumer partition equilibrium as a set of prices and network sizes such that given prices and consumers expectations about the network size, the consumers choose their optimal product and the network size that they expect must come true. For the range of prices such that $p_B - p_A \in [k - \alpha(1-\tau), \alpha(1-\tau)]$ there are three possible consumer partition equilibria: either all consumers buy good A or they all buy good B or some consumers prefer good A and others prefer good B. Notice that in the last case, demands are increasing in own price. This is due to the fact that when deciding between A and B consumers value mostly the dimension of the network that they will enjoy. Thus, as demand increases, also does the value of the goods and, in turn, the consumers' willingness to pay increases.

In Section 3 and Section 4 we solve the model and compute the subgame perfect Nash equilibrium candidates corresponding to each covered market configuration. Finally we provide the parameter constellations for which the equilibrium candidates effectively yield the corresponding covered market outcomes. The following assumption ensures market coverage and will be used throughout the paper.[19]

Assumption 1. $k \leq 3\alpha$.

3 The price competition stage

3.1 Price competition under weak and very weak network effects

In the second stage of the game, firm i chooses its price p_i so as to maximize its profit Π_i:[20]

$$\Pi_i\left(p_i, p_j\right) = p_i D_i\left(p_i, p_j\right), \text{ with } i \neq j \text{ and } i, j = A, B$$

When $k > 2\alpha(1-\tau)$, the demands for the network goods are well defined functions, in particular they are linear and decreasing in own price. Given demands (5) and (6), the profits are:

$$\Pi_B = \begin{cases} p_B, & p_B - p_A \leq \alpha(1-\tau) \\ \left(\frac{k-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_A - p_B}{k-2\alpha(1-\tau)}\right) p_B, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha\left(1-\tau\right) \\ 0, & p_B - p_A > k - \alpha\left(1-\tau\right) \end{cases}$$

$$\Pi_A = \begin{cases} p_A, & p_B - p_A > k - \alpha\left(1-\tau\right) \\ \left(\frac{-\alpha(1-\tau)}{k-2\alpha(1-\tau)} + \frac{p_B - p_A}{k-2\alpha(1-\tau)}\right) p_A, & \alpha(1-\tau) < p_B - p_A \leq k - \alpha\left(1-\tau\right) \\ 0, & p_B - p_A \leq \alpha\left(1-\tau\right) \end{cases}$$

[19]The intuition behind this condition will become clear in Section 4, when the game is solved and we verify that under Assumption 1, equilibrium candidates are compatible with the market coverage. Note that Assumption 1 is a necessary but not sufficient condition for market coverage to arise.

[20]We forego the compatibility costs which are constant in the price-setting stage. We introduce them in the compatibility choice stage (Section 4).

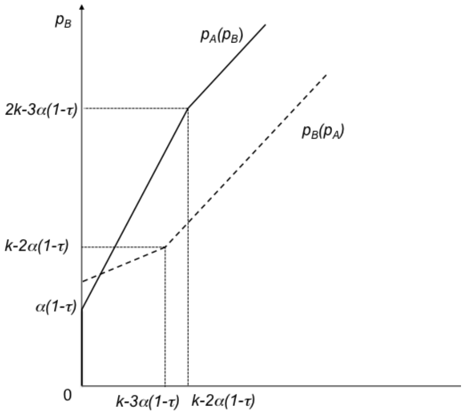

Figure 3: Price reaction functions for very weak network effects

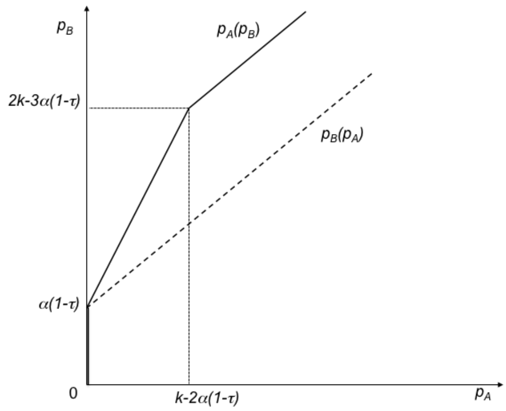

Figure 4: Price reaction functions for weak network effects.

Price competition leads to the following results.

Proposition 1 *1. When the network effects are very weak, there exists a unique Nash equilibrium of the price game, given by:* $p_A = \frac{1}{3}k - \alpha(1-\tau)$ *and* $p_B = \frac{2}{3}k - \alpha(1-\tau)$*. The corresponding equilibrium demands are* $D_A = \frac{1}{3}\frac{k-3\alpha(1-\tau)}{k-2\alpha(1-\tau)}$*, and* $D_B = \frac{1}{3}\frac{2k-3\alpha(1-\tau)}{k-2\alpha(1-\tau)}$*.*

2. When the network effects are weak, there exists a unique Nash equilibrium of the price game, given by $p_A = 0$ *and* $p_B = \alpha(1-\tau)$*, where* $D_A = 0$ *and* $D_B = 1$*.*

Proof. Follows from the observation of the reaction functions derived in the Appendix in Lemma 1. ■

The reaction functions are depicted in Figure 3 for the case of very weak network effects, and in Figure 4 for the case of weak network effects. The computed price equilibrium is the unique intersection of the price reactions functions in the relevant domain.

As in the classical model of vertical product differentiation the firm that produces the high quality good charges a higher price. For very weak network effects, prices are increasing in the degree of compatibility and in the intrinsic vertical differentiation, k. When consumers value highly the network, or in other words, when α is large, firms behave more competitively in order to gain network advantage. This implies that prices are decreasing in α. This effect becomes milder in the presence of compatible goods. Compatibility renders the network size less important for consumers and therefore prices increase with τ.

On the contrary, when the network effects are weak, the high quality firm is the only active firm in the market. In that case a higher valuation of the network, i.e. a higher α, allows the firm to extract a higher consumer surplus by setting a higher price. Also, an increase in τ intensifies price competition. In order to maintain the whole market, firm B needs to set a lower price.

3.2 Price competition under strong and very strong network effects

For the strong and very strong network effects case we need to consider the demands (7) and (8). Profits are given by:

$$\Pi_B = \begin{cases} 0, \quad p_B - p_A \geq k - \alpha\left(1-\tau\right), \\ \left(\frac{\alpha(1-\tau)-k}{2\alpha(1-\tau)-k} + \frac{p_B - p_A}{2\alpha(1-\tau)-k}\right) p_B, k - \alpha\left(1-\tau\right) \leq p_B - p_A \leq \alpha(1-\tau), \\ p_B, \quad p_B - p_A \leq \alpha(1-\tau) \end{cases}$$

$$\Pi_A = \begin{cases} 0, \quad p_B - p_A \leq \alpha\left(1-\tau\right), \\ \left(\frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} - \frac{p_B - p_A}{2\alpha(1-\tau)-k}\right) p_A, k - \alpha\left(1-\tau\right) \leq p_B - p_A \leq \alpha(1-\tau), \\ p_A, \quad p_B - p_A \geq k - \alpha\left(1-\tau\right) \end{cases}$$

Profits are nondecreasing in own price, hence firms have incentive to set prices as high as possible. Given the market coverage assumption, prices are bounded from above. Moreover, the existence of multiple consumer partition equilibria suggests for multiple price equilibria. In order to solve the price competition we follow a reasoning which is similar to that developed by Grilo et al. (2001) and we define an equilibrium of the price subgame as a price pair associated with a consumer partition. Proposition 2 shows that there are multiple equilibria when the network effects are strong or very strong.

Proposition 2 *1. When the network effects are strong, there exist multiple equilibria for the price subgame. Namely, any price pair* (p_A, p_B) *such that* $p_B = \alpha(1-\tau)$ *and* $0 \leq p_A \leq 2\alpha(1-\tau) - k$ *associated with* $D_B = 1$ *is an equilibrium.*

2. When the network effects are very strong there exist multiple equilibria for the price subgame. Namely, there are two sets of corner equilibria:

(a) any price pair (p_A, p_B) *such that* $p_B = \alpha(1-\tau)$ *and* $0 \leq p_A \leq \alpha(1-\tau) - k$ *associated with* $D_B = 1$*;*

(b) any price pair (p_A, p_B) *such that* $p_A = \alpha(1-\tau) - k$ *and* $0 \leq p_B \leq \alpha(1-\tau)$ *associated with* $D_A = 1$.[21]

Proof. See the Appendix (6.3). ■

In both the cases of very strong and strong network effects, consumers exhibit what is known as *strong conformity* (Grilo et al. 2001). This means that consumers would like to be able to coordinate their choices on the same good in order to enjoy the maximum network effects because the difference in intrinsic qualities is not relevant. This is what our results predict. As long as the overall quality of good B ($\Delta_{B,A}$ defined in (1)) is superior, i.e., $k \geq \alpha(1-\tau)$, at equilibrium all consumers buy the high quality good B. In contrast, if we let $0 < k < \alpha(1-\tau)$, a switch in the overall quality occurs when all consumers buy good A (as at the equilibrium $p_A = \alpha(1-\tau) - k$

[21] We also find the following interior equilibrium: the price pair (p_A, p_B) such that $p_A = p_B = \alpha\frac{\alpha(1-\tau^2)-k\tau}{2\alpha(1-\tau)-k}$ associated with $D_A = \frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} \in (\frac{1}{2}, 1)$. However, we exclude this second-stage price equilibrium because it locates in the boundary of the market coverage condition. Hence, it is safe to assume that without the market coverage assumption this equilibrium would not exist.

and $p_B \geq 0$ with $D_A = 1$, $\Delta_{B,A} = k - \alpha(1-\tau) < 0$). Note that prices become decreasing in τ: the compatibility degree is a device for firms competing for the market to be more aggressive.

It is worth noting that the two sets of corner equilibria described in Proposition 2 are not robust to the joint deviations of the consumers to the alternative good (or to the expectations that all consumers buy the alternative good), that is these equilibria are not coalition-proof.[22,23] The strong network effects (which dominate the quality difference in this space of parameters) justify this feature of the equilibria. Under strong network effects, the intrinsic qualities are similar enough that the price and the network effects play a dominant role in the choice of the consumers. This has been recognized in the literature. Despite this, most literature on network externalities to the exception of Ambrus and Argenziano (2009) does not provide a refinement of the equilibria based on coalitional-proofness. Ambrus and Argenziano (2009) use the concept of coalitional rationalizability proposed by Ambrus (2006) to select among the possible equilibria. However, in cases when there is no focal network on which to coordinate, the concept does not necessarily impose successful coordination and multiplicity still arises. In our analysis we assume that there are no deviations by any group of consumers. Likewise, in the very strong network effect case, expectations matter for the characterization of the equilibria. Figure 5 provides further intuition

[22]We thank the anonymous referee that pointed out this observation.

[23]To observe this we start from a price-demand equilibrium pair and test whether it would survive the expectation that all consumers buy the alternative good. Consider first $(p_A, p_B) = (\alpha(1-\tau) - k, 0)$ and $(D_A, D_B) = (1, 0)$. Even though there is no individual incentive for consumers to deviate, note that there would be mutual gain to all consumers to jointly deviate to B when presented with this price pair. Indeed, at this price pair, if instead $D_B = 1$, consumer $x \in [0,1]$ gets the utility $\beta_A x + \alpha\tau - \alpha(1-\tau) + k$ if x buys A and $\beta_B x + \alpha - 0$ if x buys B. Comparing these utilities we obtain that each consumer is better off buying B if and only if: $k(x-1) \geq 2\alpha(\tau - 1)$. This condition is most demanding for $x = 0$, namely: $k \leq 2\alpha(1-\tau)$. We can apply the same argument to the other equilibrium $(p_A, p_B) = (0, \alpha(1-\tau))$ and $(D_A, D_B) = (0, 1)$. Indeed at this price pair, if instead, $D_A = 1$ any consumer $x \in [0,1]$ is better off consuming A if and only if $k \leq 2\alpha(1-\tau)$, which includes the range of parameters that we are analyzing.

for Proposition 2.[24]

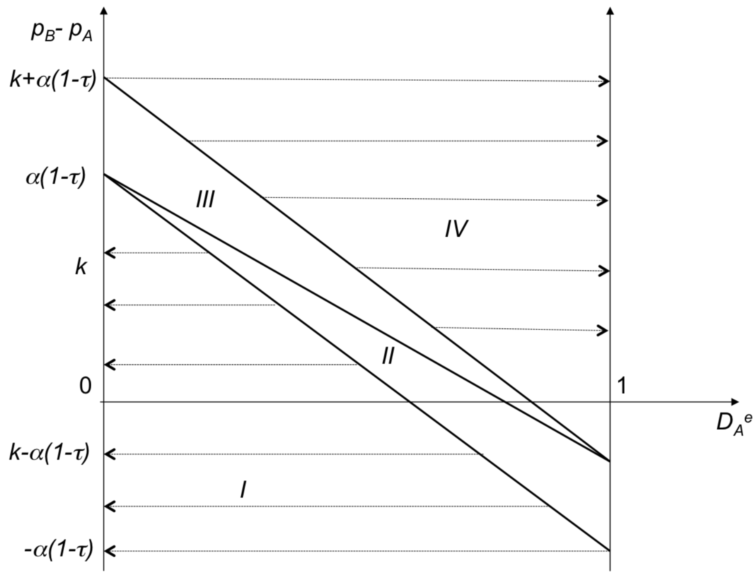

Figure 5: Characterization of potential partition equilibria under very strong network effects.

Figure 5 is obtained from the expression for the indifferent consumer, considering different expected sizes of the network of good A (denoted by D_A^e), either $D_A^e = 0$, or $D_A^e = 1$, or $D_A^e \in (0, 1)$) and the price difference. Four areas in the space $(p_B - p_A)/(D_A^e\ )$ are identified. Area I is not bounded from below and characterized by horizontal arrows pointing to the ordinate: here, no matter what exactly the expectations compatible with this area are, all consumers prefer to choose product B, thus, the only expectation that can come true, given the price differences is $D_A^e = 0$, implying all consumers buying from B. The situation in area IV (not bounded from the top) marked with horizontal arrows to the right is equivalent: all consumers prefer product A and the only expectation that can come true is $D_A^e = 1$, implying that all consumers buy A. In area II and III the situation is different. Consumers are split and part of them prefers A and the other B. However, if consumers expect a slightly smaller market share than which corresponds to this borderline at the same price difference (inside of area II) more consumers would decide in favor of product B than they had originally expected and their original expectations could never come

[24]We thank an anonymous referee for suggesting this figure.

true. Similarly, inside of area III more consumers would finally choose product A than would correspond to the consumers' original expectations. As for the borderline between areas II and III, if the consumers coordinate correctly on either of the three partitions and the firms correctly anticipate the split, then only the two equilibria characterized in Proposition 2 can arise. Indeed, if the firms correctly anticipate such a split, then each firm wants to increase its price, such that a split of consumers cannot be a Nash equilibrium.

In order to solve the compatibility stage, and in turn the full game, in what follows, we need to consider a particular price-stage equilibrium for the strong and very strong network effects case. We analyze both sets of corner equilibria in turn:[25]

(i) we focus on the price-stage equilibrium such that $p_A = 0$ and $p_B = \alpha(1-\tau)$ with $D_A = 0$ and $D_B = 1$;

(ii) we focus on the price-stage equilibrium in (i) when the network effects are strong and the price-stage equilibrium such that $p_A = \alpha(1-\tau) - k$ and $p_B = 0$ associated with $D_A = 1$ when the network effects are very strong.

In the following, we develop fully the solution to the compatibility stage taking in consideration the price-stage equilibrium described in (i). We discuss the outcome of the compatibility choice given the equilibrium described in (ii) at the end of the next section.

4 The Compatibility stage

In this section we determine the subgame perfect Nash Equilibria of the game and study the socially optimal choice of compatibility.

[25]We can exclude the price equilibria in which a firm sets a positive price when its demand is nil.

In the first stage of the game, firms choose their compatibility degrees non-cooperatively. Given the equilibria obtained in the price competition stage, there are different equilibrium profits depending on the strength of the network effects. Accordingly, we have the three following cases for the overall *first-stage profits* of firms.

Case 1 *Very weak network effects: (Unique) Interior solution in prices*

$$\Pi_A^I = \begin{cases} \frac{\left(\alpha(\tau_A-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - c\tau_A \text{ if } \tau_A \geq \tau_B \\ \frac{\left(\alpha(\tau_B-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_B)} - c\tau_A \text{ if } \tau_A < \tau_B \end{cases} \tag{9}$$

$$\Pi_B^I = \begin{cases} \frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \frac{\left(\alpha(\tau_B-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_B)} - c\tau_B \text{ if } \tau_A < \tau_B \end{cases} \tag{10}$$

Case 2 *Weak network effects: (Unique) Corner solution in prices*

$$\Pi_A^C = 0 - c\tau_A \tag{11}$$

$$\Pi_B^C = \begin{cases} \alpha\left(1-\tau_A\right) - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \alpha\left(1-\tau_B\right) - c\tau_B \text{ if } \tau_A < \tau_B \end{cases} \tag{12}$$

Case 3 *Strong and very strong network effects case.* In order to illustrate such a possibility characterized by multiple price equilibria, we focus on the one where $p_A = 0$ and $p_B = \alpha(1-\tau)$ and all consumers buy good B which implies

$$\Pi_A^S = 0 - c\tau_A \text{ , } \Pi_B^S = \begin{cases} \alpha\left(1-\tau_A\right) - c\tau_B \text{ if } \tau_A \geq \tau_B \\ \alpha\left(1-\tau_B\right) - c\tau_B \text{ if } \tau_A < \tau_B \end{cases}$$

To analyze the compatibility stage we must consider three potential regions for the parameters as illustrated in Figure 6.

Notice that focusing on the price-stage equilibrium such that $p_A = 0$ and $p_B = \alpha(1-\tau)$ with $D_A = 0$ and $D_B = 1$ for the range of k such that $0 < k < 2\alpha(1-\tau)$, then the corner solution of the price competition coincides with the strong network effects solution of the price competition,

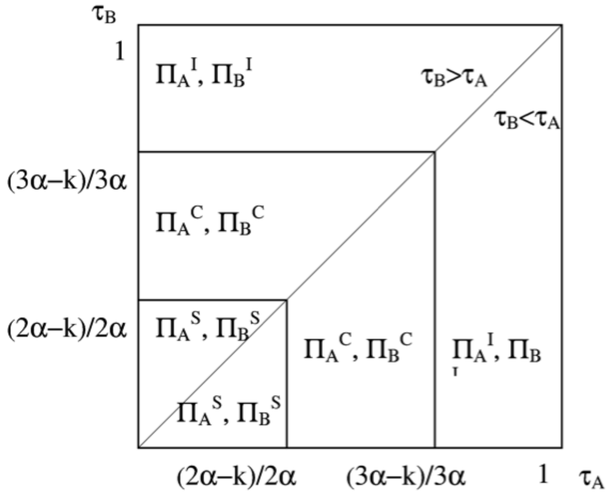

Figure 6: Parameters space for the compatibility

i.e., $\Pi_A^C = \Pi_A^S$ and $\Pi_B^C = \Pi_B^S$, therefore the last two regions can collapse in one. This is what we do to solve the compatibility stage.

The following Proposition presents the results of the compatibility game for each partition of the parameter space defined above.

Proposition 3 *Given $\tau = \max\{\tau_A, \tau_B\}$, full compatibility occurs if and only if $c \leq k/9$, otherwise compatibility is zero.*

Proof. See the Appendix (6.4). ■

Proposition 3 highlights firms' incentives to provide compatibility. It deserves a closer analysis. First, notice that when Assumption 1 does not hold, namely $k > 3\alpha$, firms have the incentive to choose full compatibility, as long as the cost is not too high. This is due to the fact that as the degree of compatibility increases, the price competition softens. However, when the result is either full or no compatibility, low consumer types prefer not to buy anything. In particular, when compatibility is absent, the quality of good A is so low with respect to the quality of good B that low consumer types do not buy it. On the other hand, when compatibility is full, although the overall differentiation decreases, the price increase prevents some consumers from buying. Therefore when $k > 3\alpha$, at equilibrium the market is not covered.

When Assumption 1 holds, we find that firms have incentive to provide full compatibility for small levels of its cost. However, the lower intrinsic quality differentiation allows for a market coverage equilibrium.

Values of c	$0 \leq \frac{3\alpha - k}{3\alpha}$
$c = 0$	$\tau_A = 1, \tau_B \in [0, 1]$ $\tau_B = 1, \tau_A \in \left[\frac{9\alpha - 4k}{9\alpha}, 1\right]$ $\}\ \tau = 1$
$0 < c < \frac{(4k - 9\alpha)}{9}$	$\tau_A = 1, \tau_B = 0$ $\tau_B = 1, \tau_A = 0$ $\}\ \tau = 1$
$\frac{(4k - 9\alpha)}{9} < c < \frac{k}{9}$	$\tau_A = 1, \tau_B = 0\} \ \tau = 1$
$c > \frac{k}{9}$	$\tau_A = 0, \tau_B = 0\} \ \tau = 0$

Table 1: Compatibility and costs

Looking at the particular behavior of each firm for different levels of c, summarized in Table 1, we can see that as long as $0 < c < \frac{(4k-9\alpha)}{9}$, the game has two Nash equilibria: $(\tau_A = 1, \tau_B = 0)$ and $(\tau_A = 0, \tau_B = 1)$. This is due to the fact that reaction functions are discontinuous and have a unique downward jump. Intuitively, when the opponent chooses a degree of compatibility high enough (not necessarily $\tau_i = 1$), the firm prefers to enjoy this degree of compatibility rather than investing in compatibility themselves. Furthermore, when $\frac{(4k-9\alpha)}{9} < c < \frac{k}{9}$, in equilibrium, only the low quality firm has incentive to offer compatibility. This is so, because, otherwise, firm B would be in a position to dominate completely the market. By offering compatibility, firm A attracts consumers and becomes active in the market.

From Table 1, we can also notice that whenever $c > 0$, firms never incur in wasteful duplication of compatibility costs. Indeed, at any equilibrium, there is only one firm providing compatibility.

In the following, we present the subgame perfect equilibrium results for all the relevant variables. When the vertical differentiation is such that $0 < k \leq 3\alpha$, equilibrium prices demands and profits depend on the compatibility cost in the following way. If $c < \frac{k}{9}$, $\tau = 1$ and we have the interior solution in prices given by $D_A^* = \frac{1}{3}$, $D_B^* \frac{2}{3}$, $p_A^* = \frac{k}{3}$ and $p_B^* = \frac{2k}{3}$ Profits are then either, $\Pi_A^* = \frac{k}{9} - c$ and $\Pi_B^* = \frac{4k}{9}$ or $\Pi_A^* = \frac{k}{9}$ and $\Pi_B^* = \frac{4k}{9} - c$. Notice that also for low values of vertical product differentiation both firms are active in the market as long as the compatibility cost is sufficiently low. If $c \geq \frac{k}{9}$, $\tau = 0$ and in we obtain the corner (or the strong network effects) solution in prices. Namely, $D_A^* = 0$, $D_B^* = 1$, p_A^*, and $p_B^* = \alpha$. The profits are $\Pi_A^* = 0$, $\Pi_B^* = \alpha$.

We conclude the characterization of the compatibility equilibria by briefly discussing the possibility of the price-stage equilibrium such that $p_A = \alpha(1-\tau) - k$ and $p_B = 0$, associated with $D_A = 1$ arises for the case of very strong network effects, i.e., $0 < k < \alpha(1-\tau)$. Our results show that in this case, the outcome of the compatibility choice yields positive compatibility for a wider range of costs, namely, for $0 < c < \alpha k/(\alpha - k)$. However, compatibility is always partial $\tau = (\alpha - k)/\alpha$. The intuition is the following. For such strong network effects, it is possible for the low quality firm to dominate completely the market if the compatibility degree, τ, is low. As a result, firm A prefers zero compatibility. In contrast firm B, in order to prevent the solution in which its demand is zero, chooses partial compatibility. In this particular equilibrium the high quality firm has, thus, a higher incentive to provide compatibility than the low quality firm. Notice that the specific equilibrium value of $\tau_B = (\alpha - k)/\alpha$ is the boundary under which demand of B is zero and over which it is one.[26] In particular, equilibrium variables at $\tau = (\alpha - k)/\alpha$ are: $D_A^* = 0$, $D_B^* = 1$, $p_A^* = 0$, $p_B^* = \frac{k\alpha - c(\alpha - k)}{\alpha} > 0$ if and only if $\frac{k\alpha}{(\alpha - k)} > c$. The equilibrium profits are $\Pi_A^* = 0$, $\Pi_B^* = \frac{k\alpha - c(\alpha - k)}{\alpha}$. In other words, under very strong network effects, when firms compete for the market, by providing partial compatibility the high quality firm is able to conquer the market: the

[26] The complete proof of this result follows from the proof presented in the paper and can be obtained from the authors upon request.

compatibility acts as a device to behave more aggressively or, as pointed out in Spulber (2008b) and Farrell and Klemperer (2007), compatibility functions as a ”coordination device”.

Welfare In this section we investigate whether the equilibrium compatibility degree is optimal from a social welfare point of view. That is, we let the social planner choose the compatibility degree, τ at a cost $c\tau$ and firms compete in prices, as before.[27]

Define, as usual, the social welfare by the following expression:

$$SW(\tau) = \int_0^{\widehat{x}} (U_A(x) - p_A)\, dx + \int_{\widehat{x}}^1 (U_B(x) - p_B)\, dx + \Pi_A + \Pi_B - c\tau. \tag{13}$$

We need to distinguish three cases according to the price competition outcome.

- When the network effects are very weak:

$$SW^I(\tau) = \beta_B \frac{1}{2} - \frac{11k^3 - 72\alpha k^2 + 54\alpha\tau k^2 + 153\alpha^2 k - 234\alpha^2 k\tau + 81\alpha^2\tau^2 k - 108\alpha^3 + 252\alpha^3\tau - 180\alpha^3\tau^2 + 36\alpha^3\tau^3}{18(k - 2\alpha + 2\alpha\tau)^2} - c\tau.$$

- When the network effects are weak:

$$SW^C(\tau) = \frac{1}{2}\beta_B + \alpha - c\tau.$$

- When the network effects are strong and very strong, the social welfare depends on the price equilibrium selected in the second stage. Let us treat first the case in which the selected price equilibrium has all consumers buying good B, so that the solution under strong and very strong network effects coincides with the corner solution.

$$SW^S(\tau) = SW^C(\tau) = \frac{1}{2}\beta_B + \alpha - c\tau.$$

  Consider next the possibility that when the network effects are very strong, the price equilibrium selected in the second stage is such that all consumers buy good A. In this case

$$SW^S(\tau) = \frac{1}{2}\beta_A + \alpha - c\tau.$$

[27] This means that we consider a second best situation.

Proposition 4 summarizes the results for the optimal compatibility choice of the social planner.

Proposition 4 *The optimal degree of compatibility from the social point of view is zero, except if, under very strong network effects, the price competition leads all consumers to buy the low quality good. In this specific situation, the social planner only chooses zero compatibility for very high costs, otherwise, the optimum occurs at an intermediate degree of compatibility.*

Proof. See the Appendix (6.5). ■

The welfare maximizing solution differs from the private optimum in that firms overprovide compatibility. Firms are willing to offer full compatibility, whereas from the welfare point of view it would be better to have zero compatibility as long as all consumers buy the high quality good. In fact, this way the network externality is maximized and society does not incur any compatibility cost. This result is altered in case of very strong network effects and assuming that all consumers buy the low quality good (one possible outcome of the price competition stage). In that case, the social planner chooses an intermediate degree of compatibility, namely $\tau = \frac{\alpha - k}{\alpha}$, if the costs are sufficiently low, i.e. $c < \frac{\alpha k}{2(\alpha - k)}$. The reason for this is that by choosing this positive degree of compatibility, the nature of price competition changes and consumers choose the high quality instead. For high cost of compatibility, it would be better from the social point of view that no compatibility is offered at all. This solution matches the private solution.[28]

[28] In case both firms can veto compatibility, i.e., $\tau = \min\{\tau_A, \tau_B\}$, we get similar results under weak and very weak network effects. In these cases both firms benefit from compatibility, that is their preferences towards τ are analogous so that compatibility is again overprovided. In contrast, under strong and very strong network effects, as firms' incentives are opposite, the unique equilibrium is $\tau = 0$ so that the private solution coincides with the social optimum as long as all consumers buy the high quality good B; whereas, if all consumers buy the low quality good A, compatibility can be underprovided. Formal details are in Garcia and Vergari (2015).

5 Conclusion

In this paper, we have analyzed firms' incentives to provide compatibility between two network goods with different intrinsic qualities. We have provided a complete analysis by studying how the relative importance of vertical differentiation with respect to the network effects influences the price competition as well as the compatibility choice. From the private point of view, full compatibility arises at equilibrium for low compatibility costs: both firms gain from compatibility as it attenuates price competition by increasing the value of both goods for consumers. Providing full compatibility makes the two goods identical for consumers in terms of network effects. However, the strength of the network effects matters to asses the differences in the firms' incentives. When network effects are not strong enough to switch the overall quality differentiation, the low quality firm has higher incentives to provide compatibility in order to prevent the rival from dominating the market. In contrast, under very strong network effects, the firm with lower intrinsic quality and low price could conquer the market, and hence the high quality firm has more to gain from compatibility. Thus, we can find conditions under which it is the high quality firm that drives compatibility. Esser and Leruth (1988) provide an example about the compact disc industry in the 1980s. In their words (page 268) "it may have been the case that Philips (the high quality firm) knew it could do better than the Japanese (the low quality firm) in terms of quality but much less so in terms of economies of scale. So, in order to avoid the risk of being defeated by the Japanese in terms of 'actual' quality, Philips may have preferred compatibility." A further example that may support our result is the failure of quadraphonic sound that was introduced as an alternative to stereo sound for playing audio recordings in the early 1970s. As pointed out by Bellaflamme and Peitz (2010, p. 572) "In spite of its higher quality, quadraphonic sound failed to become the new industry standard" and one of the reasons is that "the technology was proposed under several incompatible formats". In this case the high quality good would have probably gained by providing at least some compatibility.

As for the social optimum, the welfare is maximized at a compatibility degree of zero except if, under very strong network effects, all consumers buy the low quality good. Because there is an outcome in the price competition for which all consumers buy the high quality good, then it is possible to maximize the network externality without any compatibility at all.[29] In contrast, compatibility allows both firms to stay in the market so that the average quality of the goods on sale decreases but prices increase as a result of compatibility. However, if the equilibrium in which all consumers buy the low quality good arises, then, from the social point of view, it is beneficial that some compatibility is offered so that consumers can be persuaded, by lower prices and network effects, to buy from the high quality good. Thus, the social planner prefers *de facto* standardization on the high quality rather than competition among the two (potentially compatible) network goods. Indeed, in our model, because market coverage is assumed, as long as both firms stay in the market, increasing degrees of compatibility reduce overall quality differentiation but increase consumers' willingness to pay leading to softer price competition, however it does not affect the consumption base, as all consumers are served in any case. In contrast, when either firm conquers the market, increasing degrees of compatibility strengthen price competition. In other words, when firms compete for the market, the compatibility degree is a device for firms to behave more aggressively. Compatibility is then overprovided, except if, for very strong strong network effects all consumers buy the low quality good. In this case, the social optimal degree of compatibility corresponds to the private optimum. Thus, in vertically differentiated markets, depending on the strength of the network effects, policy intervention could go in the direction of influencing compatibility costs (by taxes or licensing costs for instance). The failure of the market is not however the rule. The high definition DVD industry makes a case study in which private and social incentives matched.

[29]We thank an anonymous referee for pointing out this intuition.

This industry is characterized by very strong network effects and at a certain point some partial compatibility was to be offered (producers of DVD discs had come out with a disc which had the HD-DVD version of the movie on one side and the Blu-Ray version on the other), but finally the standard war has been won by the high quality firm Sony (Blu-Ray DVDs). These social welfare conclusions are obtained for *fully covered markets*, a pervasive and reasonable assumption in models with network effects and they can be applied when analysing mature and widespread markets in contrast with emerging markets.

It is worth acknowledging that these policy conclusions do not necessarily hold for uncovered markets. Compatibility may have a —welfare improving— demand–expansion effect which is absent in our model. Indeed, previous contributions on compatibility strategies suggest that compatibility is welfare improving. In particular, without assuming fully covered markets, Crémer et al. (2000) show that compatibility is underprovided. Also, Baake and Boom (2001) show that welfare is higher in the presence of an adapter. Note, however, that the welfare improving demand effect does not necessarily overcompensate the welfare detrimental price increase associated with higher degrees of compatibility. For instance, de Palma et al. (1999), in the presence of double purchase, that is when consumers can achieve compatibility on their own, find that firms overprovide compatibility with respect to social welfare in spite of the induced larger market coverage.

Another assumption of the model is the *linearity of compatibility costs.* We have assumed that the firms face constant marginal costs of compatibility, whereas in previous work, for instance Katz and Shapiro (1985) and Farrell and Saloner (1992), the cost of compatibility was assumed fixed. More recently, Baake and Boom (2001) consider zero compatibility costs. Linear costs can be justified in the context of a product with several features that need to be adjusted in order to render the product compatible. Each additional feature would bring additional costs. Naturally, other cost functions could be considered, namely, we could envisage a situation of convex compatibility costs. The qualitative results would not change, in the sense that for certain range of parameters, compatibility would be offered, however, we expect that partial compatibility outcomes would be more likely to arise in equilibrium.

Our analysis points out new interesting results about firms' incentives to offer compatibility. Indeed, as Besen and Farrell (1994) describe, firms' horizontal compatibility strategies determine the form of competition in the market. In particular, with two firms, there are three combinations of such strategies: both firms choose incompatibility which results in a standard war; both firms prefer compatibility; and finally, one firm chooses incompatibility whereas the other prefers compatibility. This last case is natural under firm asymmetry. Katz and Shapiro (1985) show how a larger firm is more likely to prefer incompatibility than a smaller firm. Similarly, Baake and Boom (2001) show that the high quality firm, in contrast with the rival firm, is against compatibility. However, we show that this need not be the case. In fact, for weak and very weak network effects both firms have incentives to provide compatibility while for strong and very strong network effects, they may have asymmetric preferences. In the first case, offering compatibility can soften competition. In contrast, in the second case, either the high quality firm or the low quality firm are likely to conquer the market thus having opposite incentives for compatibility. Under very strong network effects, expectations about the network size are crucial in determining the dominant firm in the market.

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6 Appendix

6.1 Lemma 1

Lemma 1 *For $k > 3\alpha(1-\tau)$, the price reaction function of firm B is given by,*

$$p_B(p_A) = \begin{cases} \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}, \text{ if } p_A \leq k - 3\alpha(1-\tau), \\ \alpha(1-\tau) + p_A, \text{ if } p_A > k - 3\alpha(1-\tau). \end{cases}$$

Whereas for $2\alpha(1-\tau) < k \leq 3\alpha(1-\tau)$, the price reaction function is:

$$p_B(p_A) = p_A + \alpha(1-\tau).$$

As for firm A, the price reaction function, for $k > 2\alpha(1-\tau)$, is

$$p_A(p_B) = \begin{cases} p_A = 0, \ p_B \leq \alpha(1-\tau), \\ \frac{p_B - \alpha(1-\tau)}{2}, \text{ if } \alpha(1-\tau) < p_B \leq 2k - 3\alpha(1-\tau), \\ p_B - k + \alpha(1-\tau), \text{ if } p_B > 2k - 3\alpha(1-\tau). \end{cases}$$

Proof. Let us start by solving the maximization problem of firm B. In the first domain of the profit function, the profit is increasing in the price, as such, it attains its maximum in the border of the interval in which this branch of the profit is defined, i.e. $p_B = p_A + \alpha(1-\tau)$. The second branch of the profit function is concave and it attains its maximum at $p_B = \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}$. Whenever this maximum falls outside the relevant domain, i.e. $\frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2} \leq p_A + \alpha(1-\tau)$, or equivalently $p_A \geq k - 3\alpha(1-\tau)$ the optimum is $p_B = p_A + \alpha(1-\tau)$. Whenever $p_A \leq k - 3\alpha(1-\tau)$, the optimum is $p_B = \frac{1}{2}(k - \alpha(1-\tau)) + \frac{p_A}{2}$. Evidently, given that the optimal solution for firm depends on whether p_A is superior or inferior to $k - 3\alpha(1-\tau)$, we must guarantee that this value is positive. In case $k < 3\alpha(1-\tau)$, p_A is always higher than $k - 3\alpha(1-\tau)$ and as such, the only relevant best reply for firm B is $p_B = p_A + \alpha(1-\tau)$.

Let us now solve the maximization problem of firm A. In the first domain of the profit function, the profit is increasing in the price, therefore it attains its maximum in the border of the interval in which this branch of the profit is defined, i.e. $p_A = p_B - (k - \alpha(1-\tau))$. The second branch of the profit function is concave and it attains its maximum at $p_A = \frac{1}{2}(p_B - \alpha(1-\tau))$. When $\frac{1}{2}(p_B - \alpha(1-\tau)) \in (0, p_B - (k - \alpha(1-\tau))]$, or equivalently, $\alpha(1-\tau) < p_B \leq 2k - 3\alpha(1-\tau)$, the optimum obtains at $p_A = p_B - (k - \alpha(1-\tau))$, when $\frac{1}{2}(p_B - \alpha(1-\tau)) < 0$, or equivalently, $p_B \leq \alpha(1-\tau)$ then $p_A = 0$. Finally, when $\frac{1}{2}(p_B - \alpha(1-\tau)) > p_B - (k - \alpha(1-\tau))$, that is, $p_B > 2k - 3\alpha(1-\tau)$, the global maximum obtains at $p_A = \frac{1}{2}(p_B - \alpha(1-\tau))$. ■

6.2 Price regions

The fully covered market configurations described in the model occur when the following conditions hold:

$$p_A - p_B \leq \alpha(1-\tau)(D_A - D_B), \tag{14}$$

$$p_B - p_A \leq \alpha(1-\tau)(D_B - D_A) + k, \tag{15}$$

$$p_A \leq \alpha(D_A + \tau D_B). \tag{16}$$

Given the demands (3) and (4), these conditions can be reduced to:

$$\begin{aligned} p_A - p_B &\leq \alpha(1-\tau)\frac{(2(p_B - p_A) - k)}{(k - 2\alpha + 2\alpha\tau)}, \\ p_B - p_A &\leq -\alpha(1-\tau)\frac{(2(p_B - p_A) - k)}{(k - 2\alpha + 2\alpha\tau)} + k, \\ p_A &\leq \alpha\frac{\left(-\alpha\left(1-\tau^2\right) + k\tau - (p_A - p_B)(1-\tau)\right)}{(k - 2\alpha + 2\alpha\tau)}. \end{aligned}$$

The price regions assume different values depending on the strength of the network effects. The case of weak and very weak is depicted in figure 7.

6.3 Proof of Proposition 2

Consider first the relevant price region depicted in Figure 8 for strong network effects. We divide this price space in 5 regions that we analyze in turn.

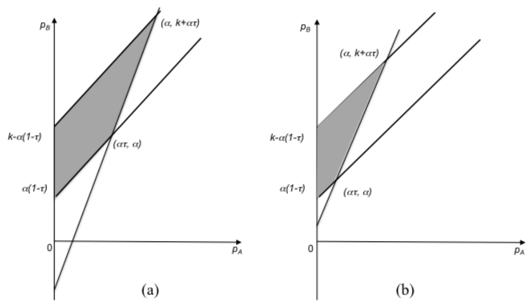

Figure 7: Price region for weak and very weak network effects.

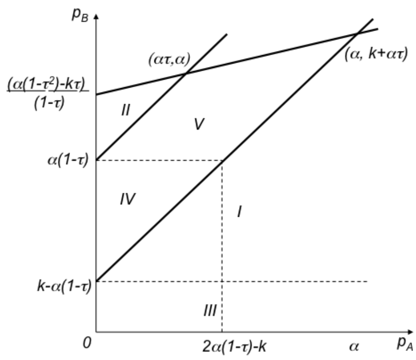

Figure 8: Price space for strong network effects.

In regions I and II, there is a unique consumer partition equilibrium: $(D_A = 0, D_B = 1)$ and $(D_A = 1, D_B = 0)$, respectively. In these regions, no price equilibrium exists: in region I (II), firm A (B) can profitably deviate by reducing its price p_A (p_B).

In regions IV and V, all three consumer partitions $D_A = 1$, $D_A = 0$ and $D_A \in (0, 1)$ are equilibria of the consumers' choice. We proceed by first eliminating the consumer partitions which are not compatible with a price equilibrium in these regions. Concerning region IV, $D_A = 1$ can never be part of the equilibrium of the full game because firm B can always reduce its price p_B and conquer a positive market share (going to region I). In contrast, for $D_B = 1$, given any p_B

belonging to region IV, firm A cannot profitably deviate as its profit is zero in any case. Therefore, in region IV any price pair (p_A, p_B) such that $p_A \geq 0$ and $p_B = \alpha(1-\tau)$ is an equilibrium, whatever the consumer partition which actually realizes. Concerning region V, no price equilibrium can be associated with either $D_A = 1$ or $D_B = 1$ because the firm with no consumers can always reduce its price and obtain positive profit, (p_A can decrease and reach region II and similarly p_B can decrease and reach region I). We conclude by considering the possibility of $D_A \in (0,1)$ in regions IV and V.

Lemma 2 *Let $\alpha(1-\tau) < k < 2\alpha(1-\tau)$, the reaction functions of firms A and B in regions IV and V with $D_A \in (0,1)$, are given by,*

$$p_B(p_A) = \begin{cases} \alpha(1-\tau) + p_A, \ \textit{if } p_A \leq \alpha\tau \\ \frac{\alpha(1-\tau^2)-k\tau}{1-\tau} + p_A \frac{(k-\alpha(1-\tau))}{\alpha(1-\tau)}, \ \textit{if } p_A > \alpha\tau \end{cases}$$

$$p_A(p_B) = \begin{cases} p_B - k + \alpha(1-\tau), \ \textit{if } p_B \geq k - \alpha(1-\tau) \\ 0, \ \textit{if } p_B < k - \alpha(1-\tau) \end{cases}$$

Proof. Consider the situation in which for $p_B - p_A \in [k - \alpha(1-\tau), \alpha(1-\tau)]$ the equilibrium demand is such that $D_A \in (0,1)$ and $D_B = 1 - D_A$. Looking at the profit function of firm B overall it is easy to see that: it is nondecreasing as long as $p_B < p_A + k - \alpha(1-\tau)$, it has a downward jump to zero at $p_B = p_A + k - \alpha(1-\tau)$, after that it starts increasing again as long as $p_B < p_A + \alpha(1-\tau)$ and it is zero otherwise. As such, the maximum is attained at $p_B = p_A + \alpha(1-\tau)$ if this price is lower than the limits imposed by conditions (14)-(16) on the prices. Otherwise, the reaction function of firm B is the upper bound of the price region, which for $k < 2\alpha(1-\tau)$ is defined by $p_B < p_A + \alpha(1-\tau)$, $p_B > p_A + k - \alpha(1-\tau)$ and $p_B < \frac{\alpha(1-\tau^2)-k\tau}{1-\tau} + p_A \frac{k-\alpha(1-\tau)}{\alpha(1-\tau)}$. A similar reasoning applies to firm A. ■

Drawing the price reaction functions in Lemma 2, we can easily see that they intersect only once at $p_A = \alpha$ and $p_B = k + \alpha\tau$. However, such a price pair is incompatible with the consumer partition

$D_A \in (0,1)$ which we can thus exclude. Indeed, as Figure 2 illustrates, at $p_B - p_A = k - \alpha(1-\tau)$, the equilibrium consumers' choice is either $D_A = 0$ or $D_A = 1$.

Finally, in region III, we have a unique equilibrium of the consumers' choice: any price pair (p_A, p_B) such that $p_B \leq k - \alpha(1-\tau)$ and $p_A \geq 0$ is associated with $D_A = 0$, $D_B = 1$. Profits are then $\Pi_A = 0$ for firm A and $\Pi_B = p_B$ for firm B: thus, firm A will be indifferent between any $p_A \geq 0$; however, firm B would always have an incentive to increase p_B so as to move to region IV (as long as p_A is sufficiently low), where it can set a higher price.

2. For the range $0 < k < \alpha(1-\tau)$, we need to distinguish two cases: $k \in (0, \alpha(1-2\tau))$ and $k \in (\alpha(1-2\tau), \alpha(1-\tau))$ depending on: $k + \alpha\tau > \alpha(1-\tau)$ which is equivalent to $k > \alpha(1-2\tau)$. Figure 9 depicts the relevant price region for the case $k \in (\alpha(1-2\tau), \alpha(1-\tau))$. We divide this price space in 6 regions that we analyze in turn.

In regions I and II, there is a unique consumer partition equilibrium: $(D_A = 0, D_B = 1)$ and $(D_A = 1, D_B = 0)$, respectively. In these regions, no price equilibrium exists: in region I (II), firm A (B) can profitably deviate by reducing its price p_A (p_B).

In regions III, IV, V and VI, all three consumer partitions $D_A = 1$, $D_A = 0$ and $D_A \in (0,1)$ are equilibria of the consumers' choice. We proceed by first eliminating the consumer partitions which are not compatible with a price equilibrium in these regions. Concerning region III, $D_A = 1$ can never be part of the equilibrium of the full game because firm B can always reduce its price p_B and conquer a positive market share (going to region I). In contrast, for $D_B = 1$, given any p_B belonging to region III, firm A cannot profitably deviate as also in region VI D_A may be zero. Therefore, in region III any price pair (p_A, p_B) such that $p_A \in III$ (that is $p_A \in [\alpha(1-\tau) - k, 2\alpha(1-\tau) - k]$) and p_B is $\alpha(1-\tau)$ with $D_B = 1$ is an equilibrium. Concerning region IV, $D_B = 1$ can never be part of the equilibrium of the full game because firm A can always reduce its price p_A and conquer a positive market share (going to region II). In contrast, for $D_A = 1$, given any p_A belonging to region IV, firm B cannot profitably deviate as also in region VI D_B may be zero. Therefore, in region IV any price pair (p_A, p_B) such that $p_B \in IV$ (that is $p_B \in [\alpha(1-\tau), 2\alpha(1-\tau)-k]$) and p_A is $\alpha(1-\tau)-k$ with $D_A = 1$ is an equilibrium. Concerning region V, no price equilibrium can be associated with either $D_A = 1$ or $D_B = 1$ because the firm with no consumers can always reduce its price and obtain positive profit, (p_A can decrease and reach region II or IV and similarly p_B can decrease and reach region III or I).

Concerning region VI, both $D_A = 1$ and $D_A = 0$ are compatible with a price equilibrium:

1. $D_A = 1$ is associated with $p_A = \alpha(1-\tau) - k$ and $p_B \in [0, \alpha(1-\tau)]$ (firm B always gets zero profit).

2. $D_B = 1$ is associated with $p_B = \alpha(1-\tau)$ and $p_A \in [0, \alpha(1-\tau) - k]$ (firm A always gets zero profit).

We conclude by considering the possibility of $D_A \in (0, 1)$ in regions III, IV, V and VI.

Lemma 3 *Let $k \in (\alpha(1-2\tau), \alpha(1-\tau))$, the reaction functions of firms A and B in regions III, IV, V and VI with $D_A \in (0, 1)$, are given by*

$$p_B(p_A) = \begin{cases} \alpha(1-\tau) + p_A, \text{ if } p_A \leq \alpha\tau \\ \frac{\alpha(1-\tau^2)-k\tau}{1-\tau} + p_A \frac{(k-\alpha(1-\tau))}{\alpha(1-\tau)}, \text{ if } p_A > \alpha\tau \end{cases}$$

$$p_A(p_B) = \begin{cases} p_B, \text{ if } p_B \leq \frac{\alpha(1-\tau)(2\alpha(1-\tau)-k)}{\alpha(1-\tau)-k} \\ p_B - \alpha(1-\tau), \text{ if } p_B > \frac{\alpha(1-\tau)(2\alpha(1-\tau)-k)}{\alpha(1-\tau)-k} \end{cases}$$

These reaction functions intersect only once at $p_A = p_B = \alpha\frac{\alpha(1-\tau^2)-k\tau}{2\alpha(1-\tau)-k}$. Such a price pair is compatible with the consumer partition $D_A = \frac{\alpha(1-\tau)}{2\alpha(1-\tau)-k} \in (\frac{1}{2}, 1)$. However, we exclude this second-stage price equilibrium because it locates in the boundary of the market coverage condition. Hence, it is safe to assume that without the market coverage assumption this equilibrium would not exist.

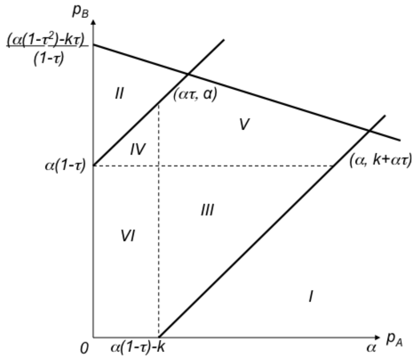

Figure 9: Price space for $\alpha(1-2\tau) < k < \alpha(1-\tau)$.

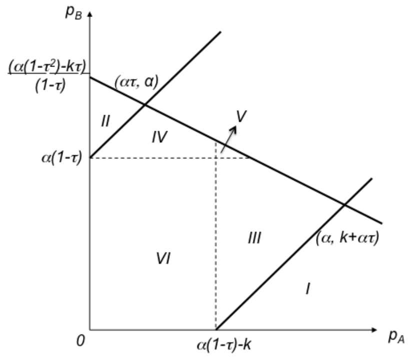

Figure 10: Price space for $k < \alpha(1-2\tau)$

Figure 10 depicts the relevant price region for the case $k \in (0, \alpha(1-2\tau))$. We divide this price space in 6 regions that we analyze in turn.

In regions I and II, there is a unique consumer partition equilibrium: $(D_A = 0, D_B = 1)$ and $(D_A = 1, D_B = 0)$, respectively. In these regions, no price equilibrium exists: in region I (II), firm A (B) can profitably deviate by reducing its price p_A (p_B).

In regions III, IV, V and VI, all three consumer partitions $D_A = 1$, $D_A = 0$ and $D_A \in (0,1)$ are equilibria of the consumers' choice. We proceed by first eliminating the consumer partitions which are not compatible with a price equilibrium in these regions. In region III, $D_A = 1$ can never be part of the equilibrium of the full game because firm B can always reduce its price p_B and conquer a positive market share (going to region I). In contrast, for $D_B = 1$, given any p_B belonging to region III, firm A cannot profitably deviate as also in region VI D_A may be zero. Therefore, in region III any price pair (p_A, p_B) such that $p_A \in III$ (that is $p_A \in [\alpha(1-\tau)-k, \alpha]$) and p_B is $\alpha(1-\tau)$ with $D_B = 1$ is an equilibrium. In region IV, $D_B = 1$ can never be part of the equilibrium of the full game because firm A can always reduce its price p_A and conquer a positive market share (going to region II). In contrast, for $D_A = 1$, given any p_A belonging to region IV, firm B cannot profitably deviate as also in region VI D_B may be zero. Therefore, in region IV any price pair (p_A, p_B) such that $p_B \in IV$ (that is $p_B \in [\alpha(1-\tau), \alpha])$ and p_A is $\alpha(1-\tau)-k$ with $D_A = 1$ is an equilibrium. Concerning region V, no price equilibrium can be associated with either $D_A = 1$ or $D_B = 1$ because the firm with no consumers can always reduce its price and obtain positive profit, (p_A can decrease and reach region II or IV and similarly p_B can decrease and reach region III or I).

Concerning region VI, both $D_A = 1$ and $D_A = 0$ are compatible with a price equilibrium:

1. $D_A = 1$ is associated with $p_A = \alpha(1-\tau)-k$ and $p_B \in [0, \alpha(1-\tau)]$ (firm B always gets zero profit).

2. $D_B = 1$ is associated with $p_B = \alpha(1-\tau)$ and $p_A \in [0, \alpha(1-\tau)-k]$ (firm A always gets zero profit).

We conclude by considering the possibility of $D_A \in (0,1)$ in regions III, IV, V and VI.

Lemma 4 *Let $k \in (0, \alpha(1-2\tau))$, the reaction functions of firms A and B in regions III, IV, V and VI with $D_A \in (0,1)$, are the same as in Lemma 3.*

Proof. Follows the same reasoning as the proof for Lemma 3 ■

6.4 Proof of Proposition 3

i $0 < \frac{3\alpha-k}{3\alpha}$

In this case, for values of $\tau_A, \tau_B \in [0, \frac{3\alpha-k}{3\alpha})$, we have the corner (or the strong network effects) solution in the price competition stage and profits are given by (11) and (12); and for values of $\tau_A, \tau_B \in (\frac{3\alpha-k}{3\alpha}, 1]$ we have the interior solution in the price competition stage, with profits (9) and (10). Let us consider first the revenue function of firm B. If $\tau_A \leq \frac{3\alpha-k}{3\alpha}$, as long as $\tau_B \leq \tau_A$, firm B's revenue is constant and equal to $\alpha(1-\tau_A)$; for $\tau_A < \tau_B < \frac{3\alpha-k}{3\alpha}$, its revenue is decreasing in τ_B and equal to $\alpha(1-\tau_B)$; finally, for $\tau_B > \frac{3\alpha-k}{3\alpha}$, the revenue is convex in τ_B. If $\tau_A > \frac{3\alpha-k}{3\alpha}$, then, the revenue of firm B is constant and equal to $\frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)}$, as long as $\tau_B \leq \tau_A$; and convex increasing otherwise. If $\tau_A = 1$, then the revenue is constant and equal to $\frac{4k}{9}$. Similarly, for firm A, when $\tau_B \leq \frac{3\alpha-k}{3\alpha}$, its revenue is zero as long as $\tau_A \leq \frac{3\alpha-k}{3\alpha}$, and positive and convex otherwise. When $\tau_B > \frac{3\alpha-k}{3\alpha}$, firm A's revenue is constant and equal to $\frac{\left(\alpha(\tau_B-1)+\frac{1}{3}k\right)^2}{k-2\alpha(1-\tau_B)}$, as long as $\tau_A \leq \tau_B$; and convex increasing otherwise. When $\tau_B = 1$, then the revenue is constant and equal to $\frac{k}{9}$.

If $\mathbf{c} = \mathbf{0}$, The reaction functions are:

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k}{9\alpha}\right] \\ 1\text{,if } \tau_A \in [\frac{9\alpha-4k}{9\alpha}, 1) \\ [0,1]\text{ ,if } \tau_A = 1 \end{cases} \quad , \quad \tau_A(\tau_B) = \begin{cases} 1\text{,if } \tau_B \in [0,1) \\ [0,1]\text{ ,if } \tau_B = 1 \end{cases}$$

There are multiple pure strategy Nash equilibria in the compatibility game, namely $\tau_A = 1$ and $\tau_B \in [0,1]$, and $\tau_B = 1$ and $\tau_A \in \left[\frac{9\alpha-4k}{9\alpha}, 1\right]$. The overall compatibility degree is $\tau = 1$. This equilibrium respects the conditions (14)-(16) for the partition of the parameter space in which it arises.[30]

If $\boldsymbol{c} \in (\mathbf{0}, \frac{\boldsymbol{k}}{\mathbf{9}}]$, we must consider three subsets of $(0, \frac{k}{9}]$.[31] Let first $c < \frac{4k-9\alpha}{9}$, then firm B prefers $\tau_B = 1$ to $\tau_B = 0$, if $\tau_A \leq \frac{3\alpha-k}{3\alpha}$; otherwise, for $\tau_A > \frac{3\alpha-k}{3\alpha}$ she prefers $\tau_B = 0$ to $\tau_B = 1$, if $\frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)} > \frac{4k}{9} - c$. This inequality holds if and only if[32]

$$\tau_A > \frac{1}{9\alpha}\left(-9c + (9\alpha - 2k) + \sqrt{(k-9c)(4k-9c)}\right) \equiv \widetilde{\tau}. \tag{17}$$

[30]The market coverage condition is satisfied for $k < 3\alpha$. When $\tau \to 1$, the RHS of condition (16)$\to -\infty$. Therefore, as p_B is finite and positive, the condition always holds.

[31]We assume that the boundaries of these subsets are positive. In the case in which they are negative, only the last subset is valid. Nevertheless, results are not affected.

[32]Define $\Phi(\tau_A) = \frac{\left(\alpha(\tau_A-1)+\frac{2}{3}k\right)^2}{k-2\alpha(1-\tau_A)} - \left(\frac{4k}{9} - c\right)$. $\Phi(\tau_A)$ has two real roots, τ^+ and τ^-. It is straightforward to see that $\tau^- < 0 < \tau^+ < 1$. We denote $\tau^+ = \frac{1}{9\alpha}\left(-9c + (9\alpha-2k) + \sqrt{(k-9c)(4k-9c)}\right) \equiv \widetilde{\tau}$. This is positive for $c < \frac{\alpha(4k-9\alpha)}{9(k-2\alpha)}$.

Likewise, firm A prefers $\tau_A = 1$ to $\tau_A = 0$, if $\tau_B \leq \frac{3\alpha-k}{3\alpha}$; otherwise, if $\tau_B > \frac{3\alpha-k}{3\alpha}$ she prefers $\tau_A = 0$ to $\tau_A = 1$, if and only if $\tau_B > \widetilde{\tau} > \frac{3\alpha-k}{3\alpha}$. The reaction functions are, thus,

$$\tau_B(\tau_A) = \begin{cases} 1 \text{ if } \tau_A \in [0, \widetilde{\tau}] \\ 0, \text{if } \tau_A \in [\widetilde{\tau}, 1] \end{cases}, \qquad \tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in [0, \widetilde{\tau}] \\ 0 \text{ if } \tau_B \in [\widetilde{\tau}, 1] \end{cases}.$$

Therefore, there are two asymmetric pure strategy Nash equilibria in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$ and $(\tau_A, \tau_B) = (0, 1)$. Moreover there exists a unique degree of τ_i such that firms are indifferent between $\tau_i = 1$ and $\tau_i = 0$, that is $\widetilde{\tau}$, defined by (17). Now, let $\boldsymbol{\frac{4k-9\alpha}{9} < c < \frac{\alpha}{9}\left(\frac{4k-9\alpha}{k-2\alpha}\right)}$. If $\tau_A < \frac{3\alpha-k}{3\alpha}$, firm B prefers $\tau_B = 0$ to $\tau_B = 1$, if $\alpha(1-\tau_A) > \frac{4k}{9} - c \iff \tau_A < \frac{9\alpha-4k+9c}{9\alpha} < \frac{3\alpha-k}{3\alpha}$; otherwise, for $\tau_A > \frac{3\alpha-k}{3\alpha}$, firm B prefers $\tau_B = 0$ to $\tau_B = 1$, if $\tau_A > \widetilde{\tau}$. Firm A prefers $\tau_A = 1$ to $\tau_A = 0$, if $\tau_B \leq \frac{3\alpha-k}{3\alpha}$; otherwise, if $\tau_B > \frac{3\alpha-k}{3\alpha}$ she prefers $\tau_A = 0$ to $\tau_A = 1$, if and only if $\tau_B > \widetilde{\tau}$. The reaction functions are,

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k+9c}{9\alpha}\right] \\ 1 \text{ if } \tau_A \in \left[\frac{9\alpha-4k+9c}{9\alpha}, \widetilde{\tau}\right] \\ 0 \text{ if } \tau_A \in [\widetilde{\tau}, 1] \end{cases}, \qquad \tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in [0, \widetilde{\tau}] \\ 0 \text{ if } \tau_B \in [\widetilde{\tau}, 1] \end{cases}.$$

Then there is a unique asymmetric pure strategy Nash equilibrium in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$. Also the reaction function of firm B has two jumps: one upwards at $\tau_A = \frac{9\alpha-4k+9c}{9\alpha}$, and one downwards at $\tau_A = \widetilde{\tau}$. Finally, let $\boldsymbol{\frac{\alpha}{9}\left(\frac{4k-9\alpha}{k-2\alpha}\right) < c < \frac{k}{9}}$, in this case, $\widetilde{\tau} < 0$, and therefore the reaction functions become

$$\tau_B(\tau_A) = \begin{cases} 0 \text{ if } \tau_A \in \left[0, \frac{9\alpha-4k+9c}{9\alpha}\right] \\ 1 \text{ if } \tau_A \in \left[\frac{9\alpha-4k+9c}{9\alpha}, \frac{3\alpha-k}{3\alpha}\right] \\ 0 \text{ if } \tau_A \in \left[\frac{3\alpha-k}{3\alpha}, 1\right] \end{cases}, \qquad \tau_A(\tau_B) = \begin{cases} 1 \text{ if } \tau_B \in \left[0, \frac{3\alpha-k}{3\alpha}\right] \\ 0 \text{ if } \tau_B \in \left[\frac{3\alpha-k}{3\alpha}, 1\right] \end{cases}.$$

Both firms choose zero compatibility for lower degrees of the rival's choice, $(\frac{3\alpha-k}{3\alpha} < \widetilde{\tau})$. Then, there is a unique asymmetric pure strategy Nash equilibrium in the compatibility game, namely, $(\tau_A, \tau_B) = (1, 0)$. Also the reaction function of firm B has two jumps: one upwards at $\tau_A = \frac{9\alpha - 4k + 9c}{9\alpha}$, and one downwards at $\tau_A = \frac{3\alpha - k}{3\alpha}$. Independently of the cost subsets, the overall compatibility is $\tau = 1$. Equilibria, then respect conditions (14)-(16).

If $\boldsymbol{c \in [\frac{k}{9}, \infty)}$, there is a unique symmetric pure strategy Nash equilibrium in the compatibility game, namely $\tau_A = 0$ and $\tau_B = 0$. Both for $\tau_A > \frac{3\alpha - k}{3\alpha}$ and $\tau_A < \frac{3\alpha - k}{3\alpha}$, the best reply of firm B is to choose $\tau_B = 0$. The overall compatibility degree is $\tau = 0$. This equilibrium respects conditions (14)-(16).

ii For completeness we prove that for $\frac{3\alpha - k}{3\alpha} < 0$ the equilibrium candidate computed is not compatible with the market coverage assumption. Remember that for $\frac{3\alpha - k}{3\alpha} \leq 0$, the unique outcome of the price stage is the interior solution. As such, profits are given by (9) and (10). The revenues are either constant in τ_i (when $\tau_i < \tau_j$) or convex in τ_i, (when $\tau_i > \tau_j$). Consequently, the profit is maximized either at $\tau_i = 1$ or $\tau_i = 0$. Therefore overall compatibility is either $\tau = 0$ or $\tau = 1$. Checking the condition for market coverage (16) it is easy to verify that it does not hold whenever $k > 3\alpha$, given the second stage equilibrium prices and demands, for the very weak network effects case as defined in Proposition 1.

6.5 Proof of Proposition 4

Consider the following relevant regions for the parameters:

i) $0 < \frac{3\alpha - k}{3\alpha}$. For this range of parameters, if in the strong and very strong network effects case we focus on the corner solution such that all consumers buy good B, the social welfare (13) is defined by:

$$SW(\tau) = \begin{cases} \frac{1}{2}\beta_B + \alpha - c\tau, \ \tau \leq \frac{3\alpha - k}{3\alpha} \\ \beta_B \frac{1}{2} - \frac{11k^3 - (36(3-\tau)(1-\tau)\alpha + 9k(9\tau - 17))(1-\tau)\alpha^2 + 18\alpha k^2(3\tau - 4)}{18(k - 2\alpha + 2\alpha\tau)^2} - c\tau, \ \tau > \frac{3\alpha - k}{3\alpha} \end{cases}$$

The maximum of $SW(\tau)$ occurs when $\tau = 0$.

Consider now the possibility that when $k \in (0, \alpha(1-\tau)]$ or $\tau \in [0, \frac{\alpha-k}{\alpha})$, the price equilibrium selected in the second stage is such that all consumers buy good A.

ii) $0 < \frac{\alpha-k}{\alpha} < \frac{2\alpha-k}{2\alpha} < \frac{3\alpha-k}{3\alpha}$. For this range of parameters, the social welfare (13) is defined by:

$$SW(\tau) = \begin{cases} \frac{1}{2}\beta_A + \alpha - c\tau, \; \tau \leq \frac{\alpha-k}{\alpha} \\ \frac{1}{2}\beta_B + \alpha - c\tau, \; \tau \in (\frac{\alpha-k}{\alpha}, \frac{3\alpha-k}{3\alpha}] \\ \beta_B\frac{1}{2} - \frac{11k^3-(36(3-\tau)(1-\tau)\alpha+9k(9\tau-17))(1-\tau)\alpha^2+18\alpha k^2(3\tau-4)}{18(k-2\alpha+2\alpha\tau)^2} - c\tau, \; \tau > \frac{3\alpha-k}{3\alpha} \end{cases}$$

This SW function is not continuous: at $\tau = \frac{\alpha-k}{\alpha}$, the left limit of the function is $\frac{1}{2}\beta_A + \alpha - c\frac{\alpha-k}{\alpha}$ and the right limit is $\frac{1}{2}\beta_B + \alpha - c\frac{\alpha-k}{\alpha}$. So, the SW function either attains its maximum at $\tau = 0$ or at $\tau = \frac{\alpha-k}{\alpha}$. If c is high enough, namely $c > \frac{\alpha k}{2(\alpha-k)}$, the social welfare is maximized at $\tau = 0$, however, for lower values of c, the social planner would rather have positive compatibility ($\tau = \frac{\alpha-k}{\alpha}$). To summarize, we have:

$$\tau = 0, \text{ if } c > \frac{\alpha k}{2(\alpha-k)}, \qquad \tau = \frac{\alpha-k}{\alpha} \text{ if } c < \frac{\alpha k}{2(\alpha-k)}.$$