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Market microstructure, information aggregation and equilibrium uniqueness in a global game[☆]

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ABSTRACT

Speculators contemplating an attack (e.g., on a currency peg) must guess the beliefs of other speculators, which they can do by looking at the stock market. As shown in earlier work, this information-gathering process may be destabilising by creating multiple equilibria. This paper studies the role played by the microstructure of the asset market in the emergence of multiple equilibria driven by information aggregation. To do so, we study the outcome of a two-stage global game wherein an asset price determined at the trading stage of the game provides an endogenous public signal about the fundamental that affects traders' decision to attack in the coordination stage of the game. In the trading stage, placing a full demand schedule (i.e., a continuum of limit orders) is costly, but traders may use riskier (and cheaper) *market orders*, i.e., order to sell or buy a fixed quantity of assets unconditional on the execution price. Price execution risk reduces traders aggressiveness and hence slows down information aggregation, which ultimately makes multiple equilibria in the coordination stage less likely. In this sense, microstructure frictions that lead to greater individual exposure (to price execution risk) may reduce aggregate uncertainty (by pinning down a unique equilibrium outcome).

1. Introduction

Consider a situation where a country may be subject to a speculative attack – on its currency, or its public debt, its banking sector, etc. – that involves an element of strategic complementarity: the attack is all the more likely to be successful, and hence the prevailing state of affairs to collapse, when the number of speculators who challenge it (i.e., bond traders, carry traders, short term lenders etc.) is large. In this situation, a speculator who is contemplating the option of attacking the prevailing regime must not only evaluate how strong the economy is but also, and even more importantly, how strong it is *perceived to be* by the other speculators. Making such an inference on the beliefs – and likely actions – of others is inherently more challenging than merely forecasting the economy's fundamentals. Crucially, it requires one to rely not only on one's own idiosyncratic assessment of the economic outlook, but also on the kind of public information that is visible by all and may guide their actions. The stock market is one the first sources of public information that speculators scrutinise, for

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the very reason that it encodes information about how the market as a whole perceives the economy's soundness. Does this source of information contribute to stabilise the market, by helping anchor speculators' expectations, or does it destabilise it, by easing their coordination on *a priori* indeterminate, but ultimately self-fulfilling, outcomes?

To answer this question, we study the equilibrium of a two-stage global game wherein a market-based asset price determined at the trading stage of the game provides an endogenous public signal about the fundamental that affects traders' decision in the coordination stage of the game. One motivation for doing so is to examine the concern, first raised by Atkeson (2001) and then made formal by Angeletos and Werning (2006) and Hellwig et al. (2006), that a publicly observed market price may aggregate dispersed information so effectively as to crowd out private signals in traders' assessment of the fundamental, and in so doing facilitate their coordination on a self-fulfilling outcome. As illustrated by Angeletos and Werning (2006) and Hellwig et al. (2006), this may precisely occur as the noise in the private signal vanishes, a result that directly challenges Carlsson and van Damme's (1993)) and Morris and Shin's (1998)) argument that a small perturbation of the full-information coordination game restores equilibrium uniqueness.[1]

The possibility that a small amount of private noise lead to multiplicity rather than uniqueness of equilibrium outcomes arises when the precision of the endogenous public signal grows faster than that of the underlying exogenous private signals at high levels of precision. However, we show that this property crucially depends on the type of market microstructure and what this microstructure implies for the amount of private information that is aggregated into the asset price. We substantiate this point by considering a market microstructure for the trading stage wherein informed traders may place either full demand schedules or more basic *market orders*, i.e., order to sell or buy a fixed quantity of assets unconditional on the execution price.[2] All orders (from informed and noise traders) are then aggregated into an asset price by a competitive market-making sector. This price provides the endogenous public signal that informed traders may use to coordinate a speculative attack in the second stage of the game. Our approach builds on Challe and Chrétien (2015), who study the functioning of an asset market with both market orders and full demand schedules under a fairly general information structure. The purpose of the present contribution is to study the implications of the information aggregation properties of this microstructure for the analysis of global games.

To summarise, our results are as follows. In a pure market-order market (see Vives, 1995), the precision of the endogenous public signal provided by the asset price is bounded above, even when the precision of the underlying private signals is very (arbitrarily) large. This is due to the competition of two forces. On the one hand, greater precision leads informed traders to trade more aggressively on their private information by opening the possibility of reaping large payoffs from trading. On the other hand, this very aggressiveness renders the asset price very volatile ex post (after all market orders have irreversibly been aggregated), which raises the conditional volatility of the net payoff, i.e., the terminal dividend minus the trading price of the asset. The first effect makes the informativeness of the price an increasing function of the precision of private signals. The second effect, however, runs counter the first effect: it deters market-order traders, which are exposed to price execution risk, from placing large orders. As the precision of private information increases the strength of the second effect gradually catches up with that of the first effect and the precision of the price signal increases more and more slowly. This boundedness of the information conveyed by the price overturns the result in Angeletos and Werning (2006), because (endogenous) public information can no longer crowd out (exogenous) private information in traders' Bayesian learning of the fundamental. As a consequence, a high level of precision of private information can again uniquely pin down the outcome of the coordination game – and we are back to Morris and Shin (1998). When the share of market-order traders is still exogenous but not necessarily equal to one, our result must be qualified in the following sense. While it is again true that as private information becomes infinitely precise then so does public information, just as in the pure demand schedule/Walrasian auctioneer case of Angeletos and Werning (2006), it is nevertheless the case that for large degrees of precision the uniqueness region can be greatly expanded relative to pure demand schedule case.

We finally examine the case where informed traders can choose their order type ex ante, where the tradeoff is between placing expensive demand schedules or cheap market orders.[3] We notably study the impact of this choice on the equilibrium share of market-order traders and, by way of consequence, on the outcome of the coordination stage. We show that as private noise vanishes the equilibrium is always interior (i.e., market-order and demand-schedule traders are both in positive measure), but market-order traders ultimately overwhelm the market (i.e., their measure tends to one). As a result, the rate of convergence of the precision of the public signal under endogenous order type is half that under exogenous order types. This implies that the endogenous adjustment of the share of market-order traders further reduces the multiplicity region as private noise decreases, relative to the case where this share is exogenous.

The rest of the paper is organised as follows. Section 2 presents two stages of the game. Section 3 analyses the outcome of the game when the shares of market-order and demand-schedule traders are exogenous. Section 4 studies the endogenous

[1] Hellwig (2002) emphasised the role of the relative precision of public versus private information in determining the outcome of the game. Angeletos et al. (2006) study global games wherein endogenous public information comes from policy choices rather than an asset price.

[2] See Brown and Zhang (1997), Wald and Horrigan (2005) and Vives (2008) for further discussion of the importance of market orders in actual asset markets.

[3] There are several reasons why placing a full demand schedule is inherently more costly than placing a market order. First and foremost, a full demand schedule requires a large number of underlying limit orders (in fact a *continuum* of such orders in our model) so as to allow full conditionality of trades on the realised trading price. Aside from the fees, the mere complexity of demand schedules relative to market orders induces additional costs, such as the cognitive cost of handling this complexity, or the wage paid to the skilled traders to whom this task is delegated.

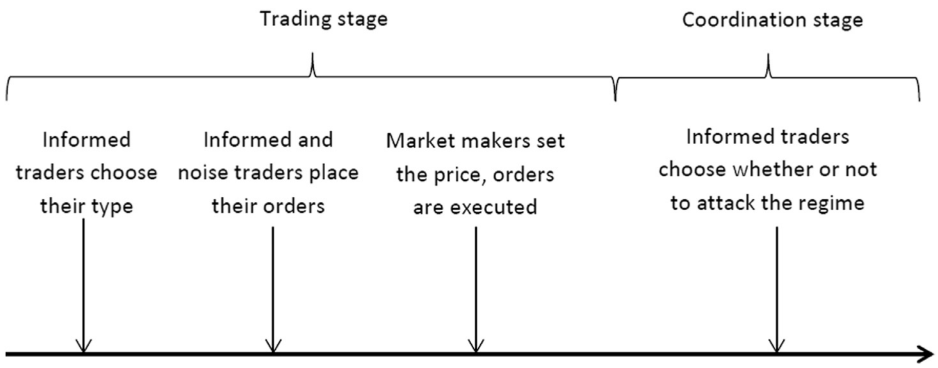

Fig. 1. Sequence of events.

determination of those shares, and how this affects the size of the multiplicity versus uniqueness regions. Section 5 concludes the paper. All the proofs appear in the Appendix.

2. The model

Following Angeletos and Werning (2006), we consider a two-stage global game wherein a continuum of informed traders $i \in I = [0, 1]$ trades an asset in a *trading stage* before deciding whether to attack the regime in the *coordination stage* – see Fig. 1. Before the game starts, an unobserved fundamental θ is drawn from the distribution $\mathcal{N}\left(\bar{\theta}, \alpha_\theta^{-1}\right)$ (which is also the common prior of informed traders) and affects both asset payoffs in the trading stage and the ability of the government to withstand a speculative attack in the coordination stage. Every informed trader gets two noisy signals about θ. First, it gets an exogenous private signal

$$x_i = \theta + \alpha_x^{-1/2} \xi_i,$$

where

$$\alpha_x > 0,\ \xi_i \sim \mathcal{N}(0, 1) \text{ and } \mathrm{cov}(\theta, \xi) = \mathrm{cov}\left(\xi_i, \xi_{j \neq i}\right) = 0.$$

Second, it gets a public signal

$$z = \theta + \alpha_z^{-1/2} \tilde{\varepsilon},$$

which satisfies

$$\tilde{\varepsilon} \sim \mathcal{N}(0, 1) \text{ and } \mathrm{cov}(\tilde{\varepsilon}, \theta) = \mathrm{cov}(\tilde{\varepsilon}, \xi) = 0.$$

The public signal is taken as given by informed traders in the coordination stage but is endogenously determined in the trading stage of the game (as we describe in the next section).

2.1. Coordination stage

In the coordination stage informed trader i chooses action $a_i \in \{0, 1\}$, with $a_i = 1$ ($= 0$) if the trader is attacking (not attacking) the regime.[4] The mass of attacking traders is thus $A = \int_0^1 a_i \mathrm{d}i$, and it is assumed that the regime collapses whenever $A > \theta$. Trader i's payoff at that stage is $U(a_i, A, \theta) = a_i(\mathbf{1}_{A>\theta} - c)$, where $c \in (0, 1)$ is the cost of attacking the regime. Hence, the payoff for a trader who successfully (unsuccessfully) attacks the regime is $1 - c > 0$ ($-c < 0$), while one who does not attack earns 0 for sure. In equilibrium A only depends on the aggregates (θ, z), i.e., $A = A(\theta, z)$. Trader i's policy function is $a(x_i, z) = \arg\max_{a \in \{0,1\}} \mathbb{E}[U(a, A(\theta, z), \theta) | x_i, z]$, with $A(\theta, z) = \int_{\mathbb{R}} a(x_i, z) f(x_i|\theta) \mathrm{d}x_i$, where $f(x|\theta)$ is the density of $.x|\theta$ ($\sim \mathcal{N}(\theta, \alpha_x^{-1})$).

We can restrict our attention to monotone equilibria, in which informed trader i chooses $a_i = 1$ (i.e., to attack) if and only if $x_i < x^*(z)$ (i.e., the trader is sufficiently pessimistic about θ, given (x_i, z)), where $x^*(z)$ is a strategy threshold common to all traders, to be determined as part of the equilibrium.[5] In such equilibria the mass of traders attacking the regime is $A(\theta, z) = \Pr(x_i < x^*(z) | \theta) = \Phi(\sqrt{\alpha_x}(x^*(z) - \theta))$, where $\Phi(.)$ is the c.d.f. of the standard normal. The regime is abandoned whenever $A(\theta, z) > \theta$, or equivalently whenever $\theta < \theta^*(z)$, where $\theta^*(z)$ solves

$$\Phi\left(\sqrt{\alpha_x}(x^*(z) - \theta^*(z))\right) = \theta^*(z). \tag{1}$$

[4] This section parallels Angeletos and Werning (2006), except for the fact that we consider a nondiffuse prior, as is required for the asset demands of market-order traders to be well defined. For the sake of comparability we keep the same notations as theirs whenever this is possible.

[5] See, e.g., Morris and Shin (2004, Lemma 1).

It directly follows from the properties of $\Phi(.)$ that the latter equation has a unique solution $\theta^*(z)\in(0,1)$ for all $x^*(z)\in\mathbb{R}$, and that $\theta^*(z)$ is continuous and strictly increasing in $x^*(z)$. This has the following interpretation. The threshold $x^*(z)$ summarises traders' *aggressiveness*, in that for any (θ, z) a greater value of $x^*(z)$ increases the attacking mass A. $\theta^*(z)$ represents the regime's *fragility*, in that for any z a greater value of $\theta^*(z)$ widens the range of realisations of θ leading to the regime's collapse. Hence Eq. (1) summarises the way in which a greater level of aggressiveness on the part of traders raises the fragility of the regime.

Since the regime collapses if and only if $\theta\le\theta^*(z)$, trader i's expected payoff from attacking the regime is $\Pr(\theta\le\theta^*(z)|x_i,z)-c$. In monotone equilibrium the threshold $x^*(z)$ corresponds to the signal received by the marginal trader (i.e. that indifferent between attacking or not) and hence must satisfy $\Pr(\theta\le\theta^*(z)|x^*(z),z)=c$. Given the assumed information structure, $.\theta|z$, x is normally distributed with variance $\alpha^{-1}\equiv(\alpha_x+\alpha_z+\alpha_\theta)^{-1}$ and mean $\alpha^{-1}\left(\alpha_x x+\alpha_z z+\alpha_\theta\bar{\theta}\right)$. Hence, indifference of the marginal trader requires:

$$\Phi\left(\sqrt{\alpha_x+\alpha_z+\alpha_\theta}\left(\frac{\alpha_x x^*(z)+\alpha_z z+\alpha_\theta\bar{\theta}}{\alpha_x+\alpha_z+\alpha_\theta}-\theta^*(z)\right)\right)=1-c \tag{2}$$

The latter equality implicitly defines traders' aggressiveness $x^*(z)\in\mathbb{R}$ as a continuous, strictly increasing function of the regimes' fragility $\theta^*(z)\in(0,1)$ – i.e., a fragile regime makes it safer to bet on its collapse, thereby inducing a rightward shift in $x^*(z)$. Solving both (1) and (2) for $x^*(z)$ and equating the two gives the equation $G(\theta^*)=\Gamma(z)$, where

$$G(\theta^*)\equiv\Phi^{-1}(\theta^*)-\frac{\alpha_z+\alpha_\theta}{\sqrt{\alpha_x}}\theta^*,\ \ \Gamma(z)=\sqrt{1+\frac{\alpha_z+\alpha_\theta}{\alpha_x}}\Phi^{-1}(1-c)-\frac{\alpha_\theta}{\sqrt{\alpha_x}}\overline{\theta}-\frac{\alpha_z}{\sqrt{\alpha_x}}z,$$

so we have $\theta^*(z)\in G^{-1}(\Gamma(z))$. When $G:(0,1)\to\mathbb{R}$ is monotonically increasing, it necessarily crosses the $\Gamma(z)$ line exactly once whatever the value of z. When $G(.)$ is non-monotonic there are values of z such that $G(.)$ crosses the $\Gamma(z)$ more than once. It then follows from the minimal value of $\partial G/\partial\theta$ that there exists a unique Bayesian Nash equilibrium for all $z\in\mathbb{R}$ if and only if:

$$\sqrt{2\pi\alpha_x}\ge\alpha_z+\alpha_\theta. \tag{3}$$

2.2. Trading stage

Let us now turn to the trading stage, which will determine both the distribution (ex ante) and the realisation (ex post) of the public signal z. We assume that informed traders have access to two assets: (i) a riskless bond in perfectly elastic supply and paying out a constant interest rate (with gross value normalised to one); and (ii) a risky asset with trading price p and payoff θ. All informed traders have zero initial wealth (this is without generality), but may freely borrow at the riskless rate to purchase risky assets. It follows that the terminal wealth of informed trader $i\in I$, is given by:

$$w_i=(\theta-p)k_i,$$

where k_i is the number of units of risky assets the trader has purchased.

We consider a market microstructure wherein informed traders may place two types of orders. The first is a full *demand schedule*, i.e., a continuum of limit orders allowing full conditionality of the amount of trade on the trading price (as in, e.g., Grossman and Stiglitz, 1976, and much of the subsequent literature on information aggregation in asset markets). The second type of orders that traders may use are *market orders*, which are unconditional on the trading price: they are transmitted to the market maker-making sector before the actual trading price is known, and hence entail some price execution risk (as in, e.g., Vives, 1995, and Medrano, 1996). Aside from informed traders, who use demand schedules or market orders, *noise traders* place a net asset demand for the risky asset of $\varepsilon\sim\mathcal{N}\left(0,\alpha_\varepsilon^{-1}\right)$; this will prevent full revelation of dispersed information through asset prices. All orders are gathered by competitive, risk neutral market-makers. Because they are risk neutral and competitive, all market makers set p to the expected value of θ conditional on the information that they get, which is the total demand for risky assets or *order book L*. In other words we have:

$$p=\mathbb{E}(\theta|L),\ \text{with}\ L=\int_{i\in I}k_i\mathrm{d}i+\varepsilon,$$

so that the expected profit of a market marker ($\mathbb{E}(p-\theta|L)$) is zero.

Let us call $M\subset I$ the set of informed traders who submit market orders and $I\backslash M$ the set of informed traders who use full demand schedules. We also define $1-\nu=\int_M\mathrm{d}i\in[0,1]$ and $\nu=\int_{I\backslash M}\mathrm{d}i$ as the measures of those two sets. All informed traders have CARA preferences, i.e. $V(w_i;\gamma_i)=-\mathrm{e}^{-\gamma_i w_i}$, where γ_i is trader i's risk aversion coefficient. Finally, private signals are assumed to be independent of risk tolerance, i.e.,

$$\forall J\subset I,\ \int_J(\xi_i/\gamma_i)\mathrm{d}\ i=0.$$

An *equilibrium* of the trading stage is a pair of investment functions for demand-schedule ($k_{I\backslash M}(x_i,\ p;\ \gamma_i)$) and market-order ($k_M(x_i;\ \gamma_i)$) traders and a price function $p(\theta,\ \varepsilon)$ such that:

- $k_{I\backslash M}(.)$ and $k_M(.)$ maximise traders' expected utility:

$$\forall i \in I\backslash M,\ \ k_{I\backslash M}(x_i, p; \gamma_i) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}[V((\theta - p)k; \gamma_i)|x_i, p], \tag{4}$$

$$\forall i \in M,\ \ k_M(x_i; \gamma_i) \in \arg\max_{k\in\mathbb{R}} \mathbb{E}[V((\theta - p)k; \gamma_i)|x_i]; \tag{5}$$

- The market-making sector sets $p = \mathbb{E}[\theta|L(\cdot)]$, where

$$L(p) = \int_{I\backslash M} k_{I\backslash M}(x_i, p; \gamma_i)\mathrm{d}i + \int_M k_M(x_i; \gamma_i)\mathrm{d}i + \varepsilon. \tag{6}$$

We then have the following lemma.

Lemma 1. *The trading stage has a unique linear Bayesian equilibrium, which is characterised by:*

- *the investment functions*

$$k_{I\backslash M}(x_i, p; \gamma_i) = \frac{\alpha_x}{\gamma_i}(x_i - p) \quad \text{and} \quad k_M(x_i; \gamma_i) = \frac{\beta}{\gamma_i}(x_i - \overline{\theta}), \tag{7}$$

with

$$\beta = \frac{1}{\alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-1}}$$

- *the price function*

$$p(\theta, \varepsilon) = (1 - \lambda B)\overline{\theta} + \lambda B\left(\theta + B^{-1}\varepsilon\right), \text{ with } \lambda = \frac{B\alpha_\varepsilon}{B^2\alpha_\varepsilon + \alpha_\theta}, \tag{8}$$

where the constant $B > 0$ in (7) and (8) is the unique (real) solution to:

$$B = \alpha_x \frac{\nu}{\gamma_{I\backslash M}} + \frac{1 - \nu}{\gamma_M}\left(\frac{1}{\alpha_x} + \frac{1}{\alpha_\theta} - \frac{1}{\alpha_\theta + \alpha_\varepsilon B^2}\right)^{-1}, \tag{9}$$

while $\gamma_{I\backslash M}^{-1}$ and γ_M^{-1} are the average risk-tolerance coefficients of demand-schedule and market-order traders:

$$\gamma_{I\backslash M}^{-1} = \frac{1}{\nu}\int_{I\backslash M} \gamma_i^{-1}\mathrm{d}i,\ \ \gamma_M^{-1} = \frac{1}{1 - \nu}\int_M \gamma_i^{-1}\mathrm{d}i.$$

Eq. (8) implies that observing p is equivalent to observing $\theta + B^{-1}\varepsilon$. Thus, the endogenous public signal z about θ, defined in the preceding section, is $z = \theta + B^{-1}\varepsilon$ (i.e., $\tilde{\varepsilon} = B^{-1}\varepsilon$) and it has precision $\alpha_z = B^2\alpha_\varepsilon$. We then infer from (3) that equilibrium uniqueness in the coordination stage requires

$$\sqrt{2\pi\alpha_x} \geq B^2\alpha_\varepsilon + \alpha_\theta. \tag{10}$$

Note that when $\alpha_\theta \to 0$ (i.e., the prior is diffuse), $\nu = 1$ and $\gamma_i = \gamma\ \ \forall i \in [0, 1]$ (i.e., all informed traders share the same preferences and place demand schedules), then Eq. (9) gives $B = \gamma^{-1}\alpha_x$, so that $p = \theta + \gamma\sigma_x^2\varepsilon$. Condition (10) then becomes $\sqrt{2\pi\alpha_x} \geq \gamma^{-2}\alpha_x^2\alpha_\varepsilon$, which is identical to that in Angeletos and Werning (2006).

3. Equilibrium uniqueness versus multiplicity

3.1. Markets with a single type

We first consider the case where all informed traders place market orders in the trading stage (as in Vives, 1995) and that where they all place full demand schedules. For the sake of simplicity, but without loss of generality, we assume here that traders have homogenous risk aversion (so that $\gamma_{I\backslash M} = \gamma_M \equiv \gamma > 0$). We then have the following proposition.

Proposition 1.

(i) *If all informed traders place market orders (i.e., $\nu = 0$), then the outcome of the coordination stage of the game is unique provided that α_x is sufficiently large. In particular, the outcome is unique provided that $\alpha_x \geq \left(B_0^2\alpha_\varepsilon + \alpha_\theta\right)^2/2\pi$, where B_0 is the unique positive real root of the polynomial $P(X) = X^3 - (\alpha_\theta/\gamma)X^2 - \alpha_\theta^2/\gamma\alpha_\varepsilon$.*

(ii) *If all informed traders place demand schedules (i.e., $\nu = 1$), then there are multiple equilibrium outcomes in the coordination stage provided that α_x is sufficiently large. In particular, there are multiple equilibrium outcomes whenever $\alpha_x \geq B_1^2$, where B_1 is the larger real root of the polynomial $Q(X) = X^4 - (\gamma^2\sqrt{2\pi}/\alpha_\varepsilon)X + \gamma^2\alpha_\theta/\alpha_\varepsilon$, if this root exists, and 0 otherwise.*

Proposition 1 implies that when the market microstructure of the trading stage is such that traders place market orders and market makers set the price, then one recovers the original property in Morris and Shin (1998), according to which the outcome of the coordination stage is unique as the noise in the private signal vanishes. In contrast, in a pure demand-schedule market one recovers the basic result in Angeletos and Werning (2006), in which a Walrasian auctioneer (rather than a market-making sector) sets the price. The intuition for this difference is as follows. In a pure *demand-schedule* market ($\nu = 1$), informed traders are able to condition their trades on the trading price, so the only source of risk they face concerns the true value of the fundamental. As the precision of the private signals increases, traders collectively trade more aggressively against any discrepancy between the observed price p and the fundamental θ. Formally, from Lemma 1 the total asset demand by informed traders in a pure demand-schedule market is given by:

$$\int_{I\backslash M} \frac{\alpha_x}{\gamma}(\theta + \alpha_x^{-1/2}\xi_i - p)\mathrm{d}i = \frac{\alpha_x}{\gamma}(\theta - p),$$

which implies that $B = \gamma^{-1}\alpha_x \to +\infty$, and thus $p \to \theta$, as $\alpha_x \to +\infty$. In the limit p becomes perfectly informative of θ (i.e. $\alpha_z \to +\infty$); this eventually causes every trader to choose a_i based exclusively on p (rather than x_i) in the second stage and thereby facilitates coordination on a self-fulfilling outcome. In contrast, in a pure *market-order* market ($\nu = 0$) informed traders do *not* condition their trades on p and hence face a residual payoff risk even as the x_is get more and more informative of θ. This payoff risk leads market-order traders to trade less aggressively on the basis of their private signal, which limits the amount of information that is aggregated into the price. Formally, from Lemma 1 again the total asset demand by informed traders in a pure market-order market is:

$$\int_{M} \frac{\beta}{\gamma}(\theta + \alpha_x^{-1/2}\xi_i - \overline{\theta})\mathrm{d}i = \frac{\beta}{\gamma}(\theta - \overline{\theta}),$$

In the limit as $\alpha_x \to +\infty$ we have $\alpha_z = B^2\alpha_\varepsilon < +\infty$, i.e., the precision of the public signal is bounded above. In this case *private* signals ultimately determine actions in the second stage of the game, which hinders coordination on a self-fulfilling outcome.

3.2. Market with both types

We now consider the case where both M and $I\backslash M$ have positive measure. We first note that for all $\nu \in [0, 1]$ it is necessarily the case that $0 < B \le \alpha_x/\gamma$, with $B = \alpha_x/\gamma$ when $\nu = 1$ and $B < \alpha_x/\gamma$ when $\nu < 1$.[6] Moreover, the uniqueness condition (10) implies that, for $(\alpha_x, \alpha_\varepsilon, \alpha_\theta, \gamma)$ given, the uniqueness region expands as B falls. Thus, if for a given set of parameters we are in the uniqueness region when $\nu = 1$ (i.e., the pure demand-schedule case), then we are also in the uniqueness region when $\nu < 1$ (and both types coexist). Total differencing (9) and using the fact that $0 < B \le \alpha_x/\gamma$, we find that, for any $(\alpha_x, \alpha_\varepsilon, \alpha_\theta)$ given and for all $\nu \in [0, 1]$ we have

$$\frac{\partial B}{\partial \nu} = \frac{\alpha_x - \beta}{\gamma + 2(1-\nu)B\alpha_\varepsilon\left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-2}\beta^2} > 0. \tag{11}$$

In short, the greater the fraction of market-order traders, the larger the uniqueness region. Again, this is because market order traders face price risk and hence trade less aggressively on their private information than demand-schedule traders do. This reduces the amount of private information that is aggregated into p, thereby reducing its weight in traders' assessment of θ and impeding traders' coordination.[7] We summarise these results in the following proposition:

Proposition 2.

(i) *As $\alpha_x \to +\infty$, and if $\nu \in (0, 1)$, then there are* multiple equilibrium outcomes *in the coordination stage;*
(ii) *At any given α_x, a greater share of market-order traders (i.e., a smaller value of ν) expands the uniqueness region.*

The role of ν in affecting the multiplicity region is illustrated in Fig. 2. From the analysis of Section 2.2 we know that $\alpha_z = B^2\alpha_\varepsilon$. Total differencing equation (9), we find that $\partial B/\partial\alpha_x > 0$, implying that a greater precision of the private signal tends to raise α_z. The dotted and dashed lines shows the monotone response of $\sigma_p \equiv \alpha_z^{-1/2}$ (i.e., the noise in the public price signal) to changes in $\sigma_x = \alpha_x^{-1/2}$ (i.e., the noise in the price signal) for different values of ν. The bold line represents the multiplicity versus uniqueness boundary (3), i.e. the $\sqrt{2\pi\sigma_x^{-2}} = \sigma_z^{-2} + \alpha_\theta$ line. A smaller value of ν is associated with a smaller uniqueness region as $\sigma_x \to 0$.

[6] Since $\alpha_\theta + B^2\alpha_\varepsilon \ge \alpha_\theta$, we have $\alpha_x^{-1} + \alpha_\theta^{-1} - \left(\alpha_\theta + B^2\alpha_\varepsilon\right)^{-1} \ge \alpha_x^{-1}$ and hence $B \le \alpha_x/\gamma$.

[7] Note the total effect of ν on B aggregates two effects. First, as ν increases, traders on average trade more aggressively and hence prices become more informative. Second, the aggressiveness of demand-schedule traders tends to increase the price risk faced by market-order traders, thereby pushing them to trade *less* aggressively on their private information as ν increases. The direct effect always dominate, implying that $\partial B/\partial\nu > 0$.

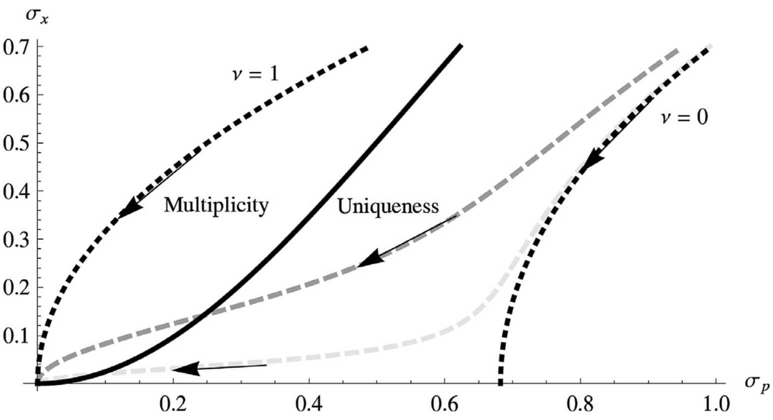

Fig. 2. Multiplicity and uniqueness regions under exogenous order types. Note: $\sigma_p \equiv \alpha_z^{-1/2}$ and $\sigma_x \equiv \alpha_x^{-1/2}$ denote the noise in the public and private signals, respectively. The bold line is the uniqueness frontier, while the dotted lines shows how σ_p depends on σ_x for different values of ν.

3.3. Discussion

3.3.1. Comparative statics

The analysis above shows that the share of market-order traders crucially affects the size of the multiplicity region, due to the impact of their trades on the informational content of the asset price. However, examining how the uniqueness region is affected by ν is not the only comparative-statics result of interest here. One notably wonders how the size of the uniqueness region is modified when the other parameters change.

Again, we know from condition (10) that, at any α_x given, the uniqueness region expands as $H \equiv B^2\alpha_\varepsilon + \alpha_\theta$ falls. Thus, is is sufficient to check how H varies as the other deep parameters of the model (namely, α_ε, α_θ and γ) change. Using (9) to compute those derivatives we obtain, after a few intermediate steps (available upon request):

$$\frac{\partial H}{\partial \alpha_\varepsilon} > 0, \quad \frac{\partial H}{\partial \alpha_\theta} > 0 \quad \text{and} \quad \frac{\partial H}{\partial \gamma} < 0.$$

The impact of those three parameters on the size of the uniqueness region (at ν given) are all intuitive. First, the reduction in noise trading (i.e., the rise in α_ε) causes H to rise and hence the uniqueness region to shrink: all else equal, less noise trading makes the asset price more informative and thus favours traders' coordination on a self-fulfilling outcome. Second, a more precise prior (i.e., a rise in α_θ) also reduces the size of the uniqueness region. This is because the more precise the prior the greater its weight in traders' Bayesian assessment of the fundamental and the smaller the role of dispersed private information; this reinforces the importance of common beliefs that ultimately lead to self-fulfilling equilibria (in the limit where $\alpha_\theta \to \infty$ we have equilibrium multiplicity whenever $\bar{\theta} \in (0, 1)$). Finally, an increase in risk aversion γ causes H to fall and hence the uniqueness region to widen. The reason for this is that as γ rises informed traders trade less and less aggressively on the basis of their private information, hence the informational content of the price – that is, the precision of the public signal – is reduced.

Perhaps a more direct – but computationally more challenging – way of looking at the size of the uniqueness region is to consider the threshold level of precision of private information $\bar{\alpha}_x$ above which equilibrium multiplicity is ensured, and then to look at how this threshold is modified by the deep parameters of the model (including ν). We show in Appendix E that, if neither α_ε nor α_θ are too large and under a minor technical condition, this threshold uniquely exists and satisfies:

$$\frac{\partial \overline{\alpha}_x}{\partial \nu} < 0, \quad \frac{\partial \overline{\alpha}_x}{\partial \alpha_\varepsilon} < 0, \quad \frac{\partial \overline{\alpha}_x}{\partial \alpha_\theta} < 0 \ , \quad \frac{\partial \overline{\alpha}_x}{\partial \gamma} > 0.$$

These results confirm in a more direct way all those discussed above about how price informativeness affects the uniqueness region. All parameter variations that make the asset price more informative (that is, a rise in ν, α_ε, α_θ, or a fall in γ) tend to lower the threshold level of precision $\overline{\alpha}_x$ below which the outcome of the coordination stage is unique – and in this sense they make equilibrium uniqueness less likely. We also derive in Appendix E an approximate closed-form solution for $\overline{\alpha}_x$ in the special case where α_θ is close to 0^+ (i.e., when the prior becomes diffuse). We obtain:

$$\overline{\alpha}_x \underset{\alpha_\theta \to 0^+}{\to} \left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}} \left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}},$$

and we can explicitly compute, still close to $\alpha_\theta = 0^+$, the derivatives of $\overline{\alpha}_x$ with respect to ν, α_ε, α_θ and γ.

3.3.2. Market orders versus noisy dividend

We conclude this section by stressing that, while we examine in this paper a particular way in which information aggregation is impeded (and possibly bounded above), there are of course others mechanisms that may generate the same result. One of them is discussed in Angeletos and Werning (2006, Appendix) and deserves particular attention.[8] To be more specific, they show that when the dividend contains some residual uncertainty that cannot be learned through the aggregation of private information – formally, the dividend is no longer equal to the fundamental θ but to the fundamental *plus* some noise η), then the precision of the price signal is bounded above whatever the precision of traders' information – and so the equilibrium in the coordination stage of the game is unique as $\alpha_x \to \infty$. This property resembles the one that we obtain under full market orders (see Section 3.1 above), so it is important to stress what the differences between the two approaches are. First and foremost, in our framework the boundedness of the precision of the public signal is an *endogenous* feature of the market that arises from the optimal strategies of the traders, namely, the willingness of market-order traders to avoid price risk. In contrast, in Angeletos and Werning this boundedness merely follows from the *assumption* that the dividend contains an unlearnable part. In this sense, our approach to limiting price informativeness is closer to the literature that takes a particular market microstructure friction as a primitive (e.g., imperfect competition, incomplete participation, short-selling constraints etc.) and then *derives* its implications for information aggregation. Regarding the substance, a second difference is how the economy behaves in the limit when $\alpha_x \to \infty$. In our economy there is *always* a region of multiplicity, however small, away from the pure market-order case (i.e., when $\nu \in (0, 1]$), while with residual payoff uncertainty this needs not be the case: a sufficiently high uncertainty may lead to equilibrium uniqueness for any value of the precision of private information.

4. Endogenous sorting

The analysis above shows that the presence of market-order traders tends to reduce the indeterminacy region by limiting the impact of the price signal on ex post beliefs about the fundamental. We now analyse the equilibrium when traders sort themselves into demand-schedule and market-order traders, so that the two sets are endogenous. What determines the choice of order type by a particular informed trader? The key tradeoff a trader faces is as follows. On the one hand, placing a demand schedule insulates the expected net payoff of a trader from price risk (since effective trades are conditional on the price). On the other hand, it is more costly than a market order, as it requires to place a large (in fact, infinite) number of limit orders in order to generate a complete conditionality of the quantity traded on the execution price. Following Vives (2008), we normalise the cost of a market order to zero and set that of a full demand schedule to $c > 0$. We work out the solution to this problem under the maintained assumption that the choice of order type must be made before the traders observe their private signal and place their orders – see Fig. 1 again.[9]

We rank informed traders in terms of their risk aversion as follows: define the function γ: $[0, 1] \to \mathbb{R}_+$, and assume further that $\gamma_i \equiv \gamma(i)$ is continuously increasing and such that $\gamma(0) > 0$. We solve for traders' choice of order backwards. First, we compute the expected utility of a trader of each type conditional on its information set (i.e. (x_i, p) $\forall i \in I\backslash M$, and x_i $\forall i \in M$). Second, we compute the unconditional ex ante utility of each type; and third, we compare the two ex ante utilities for a given risk aversion coefficient.

We know from the CARA-Normal framework that the value function associated with the information set G_i is:

$$W_j(G_i;\gamma_i) \equiv \max_{k_j} \mathbb{E}[V(w_i - \kappa c)|G_i;\gamma_i] = -\exp\left[-\frac{\mathbb{E}[\theta - p|G_i]^2}{2\mathbb{V}[\theta - p|G_i]} + \kappa c \gamma_i\right], \tag{12}$$

where $\kappa = 1$ and $j = I\backslash M$ if $G_i = (x_i, p)$ (i.e., the trader places a full demand schedule) or $\kappa = 0$ and $j = M$ if $G_i = x_i$ (i.e., the trader places a market order). Using the conditional distributions of θ and $\theta - p$ for demand-schedule and market-order traders (see Eqs. (A.1) and (A.2) in Appendix A for details), we find the corresponding value functions to be:

$$W_{I\backslash M}(x_i, p;\gamma_i) = -\exp\left[-\frac{C}{2}(x_i - p)^2 + c\gamma_i\right],\ C \equiv \frac{\alpha_x^2}{\alpha_x + \alpha_\theta + B^2\alpha_\varepsilon}, \tag{13}$$

$$W_M(x_i;\gamma_i) = -\exp\left[-\frac{D}{2}(x_i - \bar{\theta})^2\right],\ D \equiv \beta^2\left(\frac{(1-\lambda B)^2}{\alpha_x + \alpha_\theta} + \frac{\lambda^2}{\alpha_\varepsilon}\right), \tag{14}$$

where β and B are defined in Lemma 1. Note that the risk-aversion coefficient γ_i only affects the ex ante utility of a demand-schedule traders – but has no effect on the ex ante utility of a market-order trader. This is because under the CARA-Normal specification risk aversion does not affect the ex ante utility from trading assets: while greater risk aversion directly lowers the ex ante utility associated with a given amount of risk, it indirectly leads a trader to lower its risk exposure, and the indirect effect exactly offsets the direct effect. However, risk aversion affects the way in which the cost of demand schedule is translated into an ex ante utility loss (more risk aversion raises the impact of the cost on utility).

Let $f(x)$ denote the ex ante (i.e., unconditional) density of the signal x. From the distributions of θ and ξ we have $x \sim \mathcal{N}\left(\theta, \alpha_\theta^{-1} + \alpha_x^{-1}\right)$. Hence, using (14) and rearranging the ex ante utility from being a market-order trader

[8] We discuss other possible impediments to information aggregation in our concluding remarks.

[9] This follows Medrano (1996) and Brown and Zhang (1997).

is found to be

$$\mathbb{E}[W_M(x_i;\gamma_i)] = \int_{\mathbb{R}} W_M(x_i;\gamma_i) f(x_i)\mathrm{d}x_i = -\sqrt{\frac{\frac{\alpha_\theta^2}{\alpha_\theta+\alpha_x}+B^2\alpha_\varepsilon}{\alpha_\theta+B^2\alpha_\varepsilon}}. \tag{15}$$

The ex ante utility of demand-schedule traders is computed in a similar way, except that we must first condition their information set (x_i, p) on x_i before computing the unconditional expectation of $W_{I\setminus M}(x_i, p; \gamma_i)$.[10] Applying the law of iterated expectations and rearranging we get:

$$\begin{aligned}\mathbb{E}\big[\mathbb{E}\big[W_{I\setminus M}(x_i;\gamma_i)\big|x_i\big]\big] &= \int_{\mathbb{R}} \mathbb{E}\big[W_{I\setminus M}(x_i;\gamma_i)\big|x_i\big] f(x_i)\mathrm{d}x_i \\ &= -\mathrm{e}^{c\gamma(i)}\sqrt{\frac{\alpha_\theta+B^2\alpha_\varepsilon}{\alpha_\theta+\alpha_x+B^2\alpha_\varepsilon}}\end{aligned} \tag{16}$$

Trader i chooses to place a full demand schedule if and only if $\mathbb{E}\big[\mathbb{E}\big[W_{I\setminus M}(x_i;\gamma_i)\big|x_i\big]\big] \geq \mathbb{E}[W_M(x_i;\gamma_i)]$, i.e., if and only if

$$\gamma_i \leq \bar{\gamma} = \frac{1}{c}\ln\left(\frac{\sqrt{\left(\frac{\alpha_\theta^2}{\alpha_\theta+\alpha_x}+B^2\alpha_\varepsilon\right)\left(\alpha_\theta+B^2\alpha_\varepsilon+\alpha_x\right)}}{\alpha_\theta+B^2\alpha_\varepsilon}\right), \tag{17}$$

where, from Lemma 1,

$$B = \alpha_x\int_0^{\gamma^{-1}(\bar{\gamma})}\gamma_i^{-1}\mathrm{d}i + \left(\frac{1}{\alpha_x}+\frac{1}{\alpha_\theta}-\frac{1}{\alpha_\theta+\alpha_\varepsilon B^2}\right)^{-1}\int_{\gamma^{-1}(\bar{\gamma})}^1 \gamma_i^{-1}\ \mathrm{d}i, \tag{18}$$

with $\gamma^{-1}(\bar{\gamma}) = 0$ if $\bar{\gamma} < \gamma_0$ and $\gamma^{-1}(\bar{\gamma}) = 1$ if $\bar{\gamma} > \gamma_1$. For a given triplet $(\alpha_x, \alpha_\theta, \alpha_\varepsilon) \in \mathbb{R}_+^3$ given, the properties of the $\gamma(.)$ function imply that the solution $(\bar{\gamma}, B)$ to (17) and (18), if it exists, can be of three types. More specifically, it is either such that $\bar{\gamma} \in [\gamma_0, \gamma_1]$, in which case the solution is interior (i.e., both M and $I\setminus M$ are nonempty); or $\bar{\gamma} < \gamma_0$, so that the solution is corner and all traders placing market orders (i.e., $(M, I\setminus M) = (I, \varnothing)$); or $\bar{\gamma} > \gamma_1$ and all traders place full demand schedules (i.e., $(M, I\setminus M) = (\varnothing, I)$). The intuition for this sorting of informed traders according to their degree of risk aversion is that a greater risk aversion lowers trading aggressiveness, and hence the expected benefit from expanding the information set from x_i to (x_i, p).[11]

As before we are interested in the outcome of the coordination stage of the game as α_x becomes large (holding (α_θ, α_ε, c) fixed), especially with regard to the way market-order traders alter the size of the uniqueness region. This is summarised in the Proposition 2 below.

Proposition 3. *For any $(\alpha_\theta, \alpha_\varepsilon, c) \in \mathbb{R}_+^3$, and as $\alpha_x \to +\infty$,*

(i) *both M and $I\setminus M$ have strictly positive measure (i.e., the equilibrium is interior);*
(ii) *$\bar{\gamma} \to \gamma(0)$ (i.e., market-order traders eventually overwhelm the market);*
(iii) *$\alpha_z \underset{\alpha_x\to\infty}{\sim} (e^{2\gamma(0)c}-1)^{-1}\alpha_x$, so that α_z goes to infinity as the same rate α_x (while it does at the same rate as α_x^2 when $I\setminus M$ has exogenous, positive measure).*

Proposition 3 emphasises several key properties of the equilibrium when α_x is large. Note that the property that the equilibrium is interior as $\alpha_x \to +\infty$ (point (ii)) is valid for any value of the cost c; in contrast, when α_x is small one can easily construct examples of corner solutions with a pure market-order (demand-schedule) market when c is sufficiently high (low). Points (ii) and (iii) are closely related. As discussed in Section 3, market-order traders tend to slow down information aggregation. It is precisely because they crowd out demand-schedule traders as $\alpha_x \to +\infty$ (point (ii)) that the precision of the endogenous public signal grows at the same rate as α_x, instead of α_x^2 when market shares are exogenous (point (iii)). To see how this may expand the uniqueness region, note that under exogenous shares from (9) we have $\alpha_z \underset{\alpha_x\to\infty}{\sim} (\nu/\gamma_{I\setminus M})\alpha_x^2$. Hence for $\nu > 0$ and α_x large enough, since $\left[(e^{2\gamma(0)c}-1)^{-1}\alpha_x\right]/(\nu/\gamma_{I\setminus M})\alpha_x^2 \underset{\alpha_x\to\infty}{\to} 0$ it is necessarily the case that the precision of the price signal is greater under endogenous orders than under exogenous orders. Hence, whenever the uniqueness condition (3) is satisfied under exogenous shares, it is also so under endogenous shares, but the converse is not true. Fig. 3

[10] Here the intermediate step is the computation of $\mathbb{E}\big[W_{I\setminus M}(x_i, p;\gamma_i)\big|x_i\big]$ which derives from the computation of $\sqrt{\frac{C}{2}}(x_i-p)|x_i$. Using the price function (8) and the fact that $\theta|x_i \sim \mathcal{N}\left(\frac{\alpha_x x_i+\alpha_\theta\bar{\theta}}{\alpha_x+\alpha_\theta}, \frac{1}{\alpha_x+\alpha_\theta}\right)$ we find that

$$\sqrt{\frac{C}{2}}(x_i-p)|x_i \sim \mathcal{N}\left(\frac{\alpha_x[\alpha_x(1-\lambda B)+\alpha_\theta](x_i-\bar{\theta})}{\sqrt{2(\alpha_x+\alpha_\theta+B^2\alpha_\varepsilon)}(\alpha_x+\alpha_\theta)}, \left(\frac{\alpha_x\sqrt{(\lambda B)^2(\alpha_x+\alpha_\theta)^{-1}+\lambda^2\alpha_\varepsilon^{-1}}}{\sqrt{2(\alpha_x+\alpha_\theta+B^2\alpha_\varepsilon)}}\right)^2\right).$$

[11] See Medrano (1996) and Vives (2008) for further discussion.

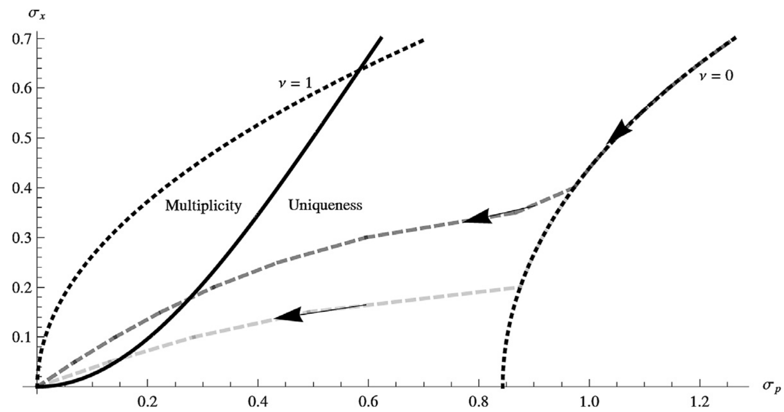

Fig. 3. Multiplicity and uniqueness regions under endogenous order types. Note: $\sigma_p \equiv \alpha_z^{-1/2}$ and $\sigma_x \equiv \alpha_x^{-1/2}$ denote the noise in the public and private signals, respectively. The bold line is the uniqueness frontier, while the dotted lines shows how σ_p depends on σ_x for different values of c (the relative cost of demand schedules), taking into account the endogenous adjustment of ν.

illustrates the relationship between α_z and α_x when α_x is large (i.e., $\sigma_x = \alpha_x^{-1/2}$ is small) and the shares of market-order and demand-schedules traders are endogenous.[12] We conclude this section by stressing the ambiguous effects of the cost c on welfare in our economy. All else equal, a greater value of c affects the equilibrium and traders' welfare in at least three ways. First and directly, a greater value of c negatively impacts the welfare of demand-schedule traders, as is apparent from Eq. (16). Second and indirectly, this greater value of c shifts downwards the threshold level of risk aversion below which traders choose to place demand schedules – see Eq. (17). This raises the share of market-order traders, reduces the informativeness of the price, and ultimately lowers the ex ante welfare of *all* traders. This can be seen as follows. At any value of α_x, a greater share of market-order traders reduces B (see Eq. (11)). And from (15) and (16), this lower value of B reduces the welfare of both market-order and demand-schedule traders. Third and finally, the lower informativeness of the price due to the greater share of market-order traders widens the uniqueness region, which tends to stabilise outcomes in the coordination stage of the game. While traders have been assumed, for simplicity, to have utility linear in payoffs in that stage, the volatility associated with multiple equilibria would be detrimental under risk aversion. Moreover, they may be other social costs associated with self-fulfilling crisis that are not directly captured by private agents' utility, nor fully internalised by them. Due to these contrasting welfare effects, it is not clear that reducing fees or other costs on complex trading orders (such as demand schedules) is always beneficial.

5. Concluding remarks

In this paper, we have analysed a two-stage global game wherein a market-based asset price determined at the trading stage of the game provides an endogenous public signal affecting traders' decisions in the coordination stage of the game. As we have shown, in this context the multiplicity region can be small even when private information is very precise – and especially so when traders optimise over their type of order (in addition to their amount of trade). The reason for this is that the presence of market-order traders limits information aggregation and hence the precision of the endogenous public signal that may serve as a coordination device when deciding whether or not to attack the regime. In this sense, a lower degree of informational efficiency (in the trading stage) may ultimately be stabilising (in the coordination stage).

While our conclusions about equilibrium uniqueness versus multiplicity in the coordination stage of the game were derived under a specific barrier to full informational efficiency in the trading stage of the game – namely, market-order traders' willingness to avoid price risk –, we conjecture that they would obtain in a variety of other contexts where information aggregation is impeded. To take one example, Pagano (1989a, 1989b) has argued that fixed entry cost into financial markets may endogenously generate limited market participation and market fragmentation, ultimately leading to lower price informativeness; we illustrate this point formally in Appendix D and show that limited asset-market participation indeed tends to widen the uniqueness region in a global game similar to that used above.

We conjecture that the same would be true of imperfect competition in asset markets, which is known since Kyle (1989) to slow down information revelation by prices – basically because traders can keep some of their own information private by reducing the extent of their trades and implied price impact. This feature is probably important in practice, since many real-world financial markets do not conform to the ideal world of perfect competition that we have hypothesised. Over-the-Counter markets are one example of this: they are typically characterised by the search of a trading counterpart

[12] Note from (16) that heterogeneity in the cost c is formally equivalent to heterogeneity in risk aversion. To encompass both cases, rank traders in nondecreasing orders of $c(i)\gamma(i)$, assume that the function $g(i) = c(i)\gamma(i)$ is continuous, strictly increasing, that its reciprocal is continuous, and that $0 < g(0) < g(1) < +\infty$, and solve for the marginal trader exactly in the same way as in the case where $c(i) = c \; \forall i \in I$.

followed by a bilateral matching, which de facto places the parties in a situation of bilateral monopoly. Similarly, alternative trading systems such as "dark pools" gather a small number of large trading partners, so that the ability of the market to absorb large trades without large (hence highly informative) price movements is limited. In all these cases, informed traders may choose to strategically limit their trades or offers in order to retain some of their private information, which ultimately leads to lower informational efficiency. As asset prices are more noisy in this situation, we expect such market institutions to favour equilibrium uniqueness in the context of a coordination game of regime change. We leave these themes for future research.

Appendix A. Proof of Lemma 1

We restrict our attention to equilibrium price functions $p(\theta,\varepsilon)$ that are linear in (θ,ε), which implies that p is normally distributed. A trader i with risk aversion coefficient γ_i and information set G_i has a demand for assets $k_i(G_i)=\gamma_i^{-1}\mathbb{E}[\theta-p|G_i]/\mathbb{V}[\theta-p|G_i]$. We may thus write the demands by limit- and market-order traders as follows:

$$\forall i\in I\backslash M,\ \ k^i_{I\backslash M}(x_i,p)=\gamma_i^{-1}f_{I\backslash M}(x_i,p),\ \text{with}\ f_{I\backslash M}(x_i,p)=\frac{\mathbb{E}[\theta|x_i,p]-p}{\mathbb{V}[\theta|x_i,p]},$$
$$\forall i\in M,\ \ k^i_M(x_i)=\gamma_i^{-1}f_M(x_i)\ ,\ \text{with}\ f_M(x_i)=\frac{\mathbb{E}[\theta-p|x_i]}{\mathbb{V}[\theta-p|x_i]},$$

i.e., within each group asset demands are identical up to a risk tolerance correction γ_i^{-1}. Now conjecture that $f_{I\backslash M}(.)$ and $f_M(.)$ have the form $f_{I\backslash M}(x_i,p)=a(x_i-\overline{\theta})+\zeta(p)$ and $f_M(x_i)=c(x_i-\overline{\theta})$, where a and b are normalised trading intensities (for a trader with $\gamma_i=1$) and $\zeta(.)$ is linear. Using the convention that the average signal equals θ a.s., and recalling that γ_i is independent from ξ_i, the limit order book is given by

$$L(p)=\int_{I\backslash M}k^i_{I\backslash M}(x_i,p)\mathrm{d}i+\int_M k^i_M(x_i)\mathrm{d}i+\varepsilon=B\left[\theta+B^{-1}\varepsilon\right]-B\overline{\theta}+\zeta(p)\int_{I\backslash M}\gamma_i^{-1}\mathrm{d}i,$$

where $B=a\nu/\gamma_{I\backslash M}+c(1-\nu)/\gamma_M$. The market making sector observes $L(.)$, a linear function of p, and sets $p=\mathbb{E}[\theta|L(.)]=\mathbb{E}[\theta|z]$, where $z=\theta+B^{-1}\varepsilon$ is the public signal. From standard normal theory we infer that p is indeed linear, normal and given by Eq. (8).

We now need to identify a and c. From the joint distribution of (p,x_i,θ) we get:

$$\forall i\in I\backslash M,\ \begin{cases}\mathbb{E}[\theta|p,x_i]=\dfrac{B^2\alpha_\varepsilon z+\alpha_\theta\overline{\theta}+\alpha_x x_i}{B^2\alpha_\varepsilon+\alpha_\theta+\alpha_x}=\dfrac{(B^2\alpha_\varepsilon+\alpha_\theta)p+\alpha_x x_i}{B^2\alpha_\varepsilon+\alpha_\theta+\alpha_x},\\ \mathbb{V}[\theta|p,x_i]=(B^2\alpha_\varepsilon+\alpha_\theta+\alpha_x)^{-1}.\end{cases}\tag{A.1}$$

$$\forall i\in M,\ \begin{cases}\mathbb{E}[\theta-p|x_i]=\dfrac{(1-\lambda B)\alpha_x}{\alpha_x+\alpha_\theta}(x_i-\overline{\theta}),\\ \mathbb{V}[\theta-p|x_i]=(1-\lambda B)^2\mathbb{V}[\theta|x_i]+\dfrac{\lambda^2}{\alpha_\varepsilon}=\dfrac{(1-\lambda B)^2}{\alpha_x+\alpha_\theta}+\dfrac{\lambda^2}{\alpha_\varepsilon}.\end{cases}\tag{A.2}$$

Hence, we obtain

$$k^i_{I\backslash M}(x_i,p;\gamma_i)=\frac{\mathbb{E}[\theta|p,x_i]-p}{\gamma_i\mathbb{V}[\theta|p,x_i]}=\frac{\alpha_x}{\gamma_i}(x_i-p),\ \ k^i_M(x_i;\gamma_i)=\frac{\mathbb{E}[\theta-p|x_i]}{\gamma_i\mathbb{V}[\theta-p|x_i]}=\frac{\beta}{\gamma_i}\left(x_i-\overline{\theta}\right),\tag{A.3}$$

where $\beta=(\alpha_x^{-1}+\alpha_\theta^{-1}-\left(\alpha_\theta+B^2\alpha_\varepsilon\right)^{-1})^{-1}$. In the special case where $\gamma_i=\gamma\ \forall i\in[0,1]$, we have $k^i_{I\backslash M}(x_i,p)=\gamma^{-1}\alpha_x(x_i-p)$, $k_M(x_i)=\gamma^{-1}\beta(x_i-\overline{\theta})$ and $p=(1-\lambda B)\overline{\theta}+\lambda Bz$, where B solves $B=\nu\gamma^{-1}\alpha_x+(1-\nu)\gamma^{-1}\beta$ and $\lambda=\frac{B\alpha_\varepsilon}{B^2\alpha_\varepsilon+\alpha_\theta}$.

Let us now turn to the parameter B. To establish that B is unique, positive and finite, define the function $f:\mathbb{R}\to\mathbb{R}$ as follows:

$$f:B\to B-\frac{\nu\alpha_x}{\gamma_{I\backslash M}}-\frac{1-\nu}{\gamma_M\left(\alpha_x^{-1}+\alpha_\theta^{-1}-\left(\alpha_\theta+B^2\alpha_\varepsilon\right)^{-1}\right)},$$

so that a root of $f(B)$ is a solution to (9). f is continuous and strictly increasing over $[0,+\infty)$ and such that $f(0)=-\alpha_x\left(\frac{\nu}{\gamma_{I\backslash M}}+\frac{(1-\nu)}{\gamma_M}\right)<0$ and $\lim_{B\to+\infty}f(B)=+\infty$. Hence f is a bijection that admits a unique root $B_0>0$ over $[0,+\infty)$. Moreover, as $B\to\ \alpha_x^{-1}+\alpha_\theta^{-1}-\left(\alpha_\theta+B^2\alpha_\varepsilon\right)^{-1}>0$ on $\mathbb{R}$, $f(.)$ is strictly negative on $\mathbb{R}_-$. Hence B_0 is the unique root of f in $\mathbb{R}$. In the numerical implementation of the model we use the exact solution for B, which is found using Cardano's method and gives:

$$B=\sqrt[3]{\frac{1}{2}\left(-\left(2\frac{a_2^3}{27}-\frac{a_1a_2}{3}+a_0\right)+\sqrt{\frac{4a_1^3+4a_0a_2^3-(a_1a_2)^2}{27}-\frac{2}{3}a_0a_1a_2+a_0^2}\right)}$$
$$+\sqrt[3]{\frac{1}{2}\left(-\left(2\frac{a_2^3}{27}-\frac{a_1a_2}{3}+a_0\right)-\sqrt{\frac{4a_1^3+4a_0a_2^3-(a_1a_2)^2}{27}-\frac{2}{3}a_0a_1a_2+a_0^2}\right)}-\frac{a_2}{3},$$

where

$$a_0=-\frac{\alpha_\theta}{\left(\alpha_x^{-1}+\alpha_\theta^{-1}\right)\alpha_\varepsilon}\left(\frac{\nu}{\gamma_{I\backslash M}}+\frac{(1-\nu)}{\gamma_M}\right),\quad a_1=\frac{\alpha_x^{-1}\alpha_\theta}{(\alpha_x^{-1}+\alpha_\theta^{-1})\alpha_\varepsilon},$$
$$\text{and}\quad a_2=-\frac{\left[\frac{\nu}{\gamma_{I\backslash M}}+\frac{(1-\nu)}{\gamma_M}+\frac{\nu}{\gamma_{I\backslash M}}\alpha_x\alpha_\theta^{-1}\right]}{(\alpha_x^{-1}+\alpha_\theta^{-1})}.$$

Appendix B. Proof of Proposition 1

(i) We know from Lemma 1 that $B\in\mathbb{R}_+^*$ uniquely solves (9). When $\nu=0$, B is the only positive solution to $B=\gamma^{-1}(\alpha_x^{-1}+\alpha_\theta^{-1}-\left(\alpha_\theta+\alpha_\varepsilon B^2\right)^{-1})^{-1}$. In particular, it is such that:

$$B\leq\frac{1}{\gamma}\left(\frac{1}{\alpha_\theta}-\frac{1}{\alpha_\theta+\alpha_\varepsilon B^2}\right)^{-1}\leq\frac{1}{\gamma}\left(\frac{\alpha_\varepsilon B^2}{\alpha_\theta\left(\alpha_\theta+\alpha_\varepsilon B^2\right)}\right)^{-1}\leq\frac{1}{\gamma}\left(\frac{\alpha_\theta\left(\alpha_\theta+\alpha_\varepsilon B^2\right)}{\alpha_\varepsilon B^2}\right),$$

or alternatively:

$$B^3-\frac{\alpha_\theta}{\gamma}B^2-\frac{1}{\gamma}\frac{\alpha_\theta^2}{\alpha_\varepsilon}<0.$$

This implies that $B\leq B_0$ where B_0 is the only real root of $P(X)$. Since uniqueness is ensured when $B^2\alpha_\varepsilon+\alpha_\theta\leq\sqrt{2\pi\alpha_x}$, it is also so when $B_0^2\alpha_\varepsilon+\alpha_\theta\leq\sqrt{2\pi\alpha_x}$. (ii) When $\nu=1$ multiplicity arises whenever $\alpha_x^2\alpha_\varepsilon/\gamma^2+\alpha_\theta>\sqrt{2\pi\alpha_x}$. Defining $Q(X)=X^4-(\gamma^2\sqrt{2\pi}/\alpha_\varepsilon)X+\gamma^2\alpha_\theta/\alpha_\varepsilon$, multiplicity arises as soon as $Q(\sqrt{\alpha_x})>0$. $Q(X)$ has either zero, one or two positive real roots, and is also strictly positive outside the real roots. Hence, either $Q(X)$ has no real roots (which occurs when $(\gamma\pi)^{2/3}<\alpha_\theta\alpha_\varepsilon^{1/3}$) and multiplicity always arises, or $Q(X)$ has real roots (which occurs when $(\gamma\pi)^{2/3}\geq\alpha_\theta\alpha_\varepsilon^{1/3}$) and multiplicity is ensured when α_x is larger than the square of the largest root.

Appendix C. Proof of Proposition 3

(i) We show that $k\equiv\gamma^{-1}(\bar{\gamma})\in[0;1]$ for α_x sufficiently high, and that k is unique. Let us first define the function

$$\tilde{f}:\alpha_x,k\rightarrow e^{2\gamma(k)c}-\left(1-\frac{\alpha_\theta}{\alpha_x+\alpha_\theta}\frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon+\alpha_\theta}\right)\left(1+\frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon+\alpha_\theta}\right),\tag{C.1}$$

where $B(k,\,\alpha_x)$ is the unique solution to

$$B(k,\alpha_x)=\alpha_x\int_0^k\gamma(i)^{-1}\mathrm{d}i+\left(\frac{1}{\alpha_x}+\frac{1}{\alpha_\theta}-\frac{1}{\alpha_\theta+B(k,\alpha_x)^2\alpha_\varepsilon}\right)^{-1}\int_k^1\gamma(i)^{-1}\mathrm{d}i.\tag{C.2}$$

We have $\tilde{f}(\alpha_x,1)\underset{\alpha_x\to\infty}{\rightarrow}e^{2\gamma(1)c}-1>0$ while $\tilde{f}(\alpha_x,0)\underset{\alpha_x\to\infty}{\rightarrow}-\infty<0$. Hence, by the intermediate value theorem there exists $\underline{\alpha}\in\mathbb{R}_+^*$,such that for all $\alpha_x\geq\underline{\alpha}$, $0\in[\tilde{f}(\alpha_x,0),\tilde{f}(\alpha_x,1)]$. By continuity, $\forall\alpha_x\geq\underline{\alpha}$, $\exists k(\alpha_x)\in[0,1]$ such that $\tilde{f}(\alpha_x,k(\alpha_x))=0$. In this range of parameter, there exists an interior equilibrium allocation, and the corner solutions are ruled out (otherwise the polar traders would be better off switching positions).

To establish uniqueness, define $\tilde{\alpha}\equiv\alpha_\theta/(\alpha_x+\alpha_\theta)$ and $X\equiv\alpha_x/\left(B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon+\alpha_\theta\right)$, and rewrite $\tilde{f}(\alpha_x,k(\alpha_x))=0$ as $\mathcal{P}(X)=\tilde{\alpha}X^2-(1-\tilde{\alpha})X+e^{2\gamma(k(\alpha_x))c}-1=0$. This polynomial has the following two real roots:

$$s^-,s^+=\frac{1}{2\tilde{\alpha}}\left[(1-\tilde{\alpha})\mp\sqrt{(1-\tilde{\alpha})^2-4\tilde{\alpha}\left(e^{2\gamma(k(\alpha_x))c}-1\right)}\right].$$

We prove by contradiction that $X=s^-$ is the only possible root of $\mathcal{P}(X)=0$ when α_x becomes large enough. Formally,

$$\exists \underline{\alpha}^1 \geq \underline{\alpha}, \forall \alpha_x \in \mathbb{R}_+^*, \forall k \in [0;1], \quad \left.\begin{matrix} \alpha_x \geq \underline{\alpha}^1 \\ \tilde{f}(\alpha_x, k) = 0 \end{matrix}\right\} \Rightarrow \frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon + \alpha_\theta} = s^-. \tag{C.3}$$

To see this, suppose that $\forall \underline{\alpha}^1 \geq \underline{\alpha}, \exists \alpha_x \in \mathbb{R}_+^*, \exists k \in [0;1]$ such that

$$\left(\alpha_x \geq \underline{\alpha}^1\right) \wedge (\tilde{f}(\alpha_x, k(\alpha_x)) = 0) \wedge \left(\frac{\alpha_x}{B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon + \alpha_\theta} = s^+\right).$$

In particular, for $n \in \mathbb{N}$ large enough (say larger than $n_0 = \lceil \underline{\alpha} \rceil$),

$$\exists \alpha_x, \exists k, (\alpha_x \geq n) \wedge (\tilde{f}(\alpha_x, k) = 0) \wedge (\alpha_x / \left(B(k,\alpha_x)^2\alpha_\varepsilon + \alpha_\theta\right) = s^+)$$

For every $n \geq n_0$ we pick an α_x, and an associated $k(\alpha_x)$, satisfying $(\alpha_x \geq n) \wedge (\tilde{f}(\alpha_x, k(\alpha_x)) = 0) \wedge (\alpha_x / \left(B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon + \alpha_\theta\right))$ and denote it α_n (resp. $k(\alpha_n)$), thereby constructing the series $(\alpha_n)_{n\geq n_0}$ (resp. $(k(\alpha_n))_{n\geq n_0}$). As $\alpha_n \underset{n\to\infty}{\to} \infty$, and since $k(\alpha_n)$ must belong to [0; 1] we have

$$\frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(e^{2\gamma(k(\alpha_n))c} - 1\right) = \frac{4\alpha_\theta}{\alpha_n + \alpha_\theta}\left(\frac{\alpha_n + \alpha_\theta}{\alpha_n}\right)^2\left(e^{2\gamma(k(\alpha_n))c} - 1\right) \underset{n\to\infty}{\to} 0, \tag{C.4}$$

and hence

$$\frac{X\tilde{\alpha}}{1-\tilde{\alpha}} = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(e^{2\gamma(k(\alpha_n))c} - 1\right)}\right] \underset{n\to\infty}{\to} 1.$$

This in turn implies that

$$\frac{\alpha_n}{B(k(\alpha_n),\alpha_n)^2\alpha_\varepsilon + \alpha_\theta} \underset{n\to\infty}{\sim} \frac{\alpha_n}{\alpha_\theta},$$

that is, $B(k(\alpha_n),\alpha_n)^2\alpha_\varepsilon \underset{n\to\infty}{\to} 0$. Since for each $n \geq n_0$, $B(k(\alpha_n),\alpha_n)^2 \geq B(0,\alpha_n)^2$ while $\left(B(0,\alpha_n)^2\right)_{n\in\mathbb{N}}$ admits a finite, non-zero limit as $n \to \infty$, we have a contradiction that proves (C.3). To summarise, for every given set of parameters, the function

$$k \to \frac{\alpha_x}{B(k,\alpha_x)^2\alpha_\varepsilon + \alpha_\theta} - \frac{1}{2\alpha}\left[(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\alpha\left(e^{2\gamma(k)c} - 1\right)}\right] \tag{C.5}$$

is strictly decreasing and thus has a unique root. For every $\alpha_x \geq \underline{\alpha}^1$, we also have that $\alpha_x \geq \underline{\alpha}$, hence we know that there exists $k \in [0;1]$ such that $\tilde{f}(\alpha_x, k) = 0$. Moreover, as $\alpha_x \geq \underline{\alpha}^1$, we know that k is the unique root of (C.4). Hence, $k \in [0;1]$ exists and is unique. We will denote it $k(\alpha_x)$.

(ii) From above, $\forall \alpha_x \geq \underline{\alpha}^1$ we have $\alpha_x / \left(B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon + \alpha_\theta\right) = s^-$. Since $k(\alpha_x) \in [0;1]$ we have

$$\frac{\alpha_x}{B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon + \alpha_\theta} \geq \frac{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(e^{2\gamma(0)c} - 1\right)}}{2\tilde{\alpha}},$$

or, rearranging,

$$B(k(\alpha_x),\alpha_x)^2 \leq \frac{2\tilde{\alpha}}{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(e^{2\gamma(0)c} - 1\right)}}\frac{\alpha_x}{\alpha_\varepsilon}.$$

Moreover, from (18) we know that

$$\left(\frac{k(\alpha_x)}{\gamma(1)}\alpha_x\right)^2 \leq \alpha_x^2\left(\int_0^{k(\alpha_x)} \gamma(i)^{-1}\mathrm{d}i\right)^2 \leq B(k(\alpha_x),\alpha_x)^2$$

Hence, $\forall \alpha_x \geq \underline{\alpha}^1$,

$$0 \leq k(\alpha_x) \leq \frac{\gamma(1)}{\alpha_x}\sqrt{\frac{\alpha_x}{\alpha_\varepsilon}\frac{2\tilde{\alpha}}{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(e^{\,2\gamma(0)c} - 1\right)}}}$$

We know from (C.4) that $\frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(e^{2\gamma(0)c} - 1\right) \underset{\alpha_x\to\infty}{\to} 0$. It follows that

$$\frac{(1-\tilde{\alpha}) - \sqrt{(1-\tilde{\alpha})^2 - 4\tilde{\alpha}\left(e^{2\gamma(0)c} - 1\right)}}{2\tilde{\alpha}} = \frac{(1-\tilde{\alpha}) - (1-\tilde{\alpha})\sqrt{1 - \frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(e^{2\gamma(0)c} - 1\right)}}{2\alpha}$$
$$= \frac{(1-\tilde{\alpha})}{4\tilde{\alpha}}\frac{4\tilde{\alpha}}{(1-\tilde{\alpha})^2}\left(e^{2\gamma(0)c} - 1\right) + \underset{\alpha_x\to\infty}{o}(1) = \frac{e^{2\gamma(0)c} - 1}{1-\tilde{\alpha}} + \underset{\alpha_x\to\infty}{o}(1) \underset{\alpha_x\to\infty}{\to} e^{2\gamma(0)c} - 1,$$

We infer that

$$\frac{\gamma(1)}{\alpha_x}\sqrt{\frac{\alpha_x}{\alpha_\varepsilon}\frac{2\tilde{\alpha}}{(1-\tilde{\alpha})-\sqrt{(1-\tilde{\alpha})^2-4\tilde{\alpha}\left(e^{2\gamma(0)c}-1\right)}}} \underset{\alpha_x\to\infty}{\sim} \frac{\gamma}{\alpha_x}\sqrt{\frac{\alpha_x/\alpha_\varepsilon}{e^{2\gamma(0)c}-1}} \underset{\alpha_x\to\infty}{\to} 0,$$

Hence $k(\alpha_x) \underset{\alpha_x\to\infty}{\to} 0$, and (by the continuity of $\gamma(.)$) $\bar{\gamma} = \gamma(k(\alpha_x)) \underset{\alpha_x\to\infty}{\to} \gamma(0)$.

(iii) Using (C.3) above we get

$$\frac{\alpha_x}{B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon+\alpha_\theta} = s^- = \frac{e^{2\gamma(k(\alpha_x))c}-1}{1-\tilde{\alpha}} + \underset{\alpha_x\to\infty}{o}\left(\frac{e^{2\gamma(k(\alpha_x))c}-1}{1-\tilde{\alpha}}\right) \underset{\alpha_x\to\infty}{\to} e^{2\gamma(0)c}-1.$$

We infer that

$$\frac{B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon}{\alpha_x} = \frac{B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon+\alpha_\theta}{\alpha_x} - \frac{\alpha_\theta}{\alpha_x} \underset{\alpha_x\to\infty}{\to} \frac{1}{e^{2\gamma(0)c}-1}.$$

We conclude that $\alpha_z = B(k(\alpha_x),\alpha_x)^2\alpha_\varepsilon \underset{\alpha_x\to\infty}{\sim} (e^{2\gamma(0)c}-1)^{-1}\alpha_x$.

Appendix D. Equilibrium versus uniqueness under limited participation

In the concluding remarks we hinted that alternative microstructure frictions that slow down information aggregation through market prices would similarly expand the uniqueness region in the coordination stage of the game. We illustrate this point here by considering incomplete market participation due to entry costs, in the tradition of Pagano (1989a, 1989b). We thus replace the binary choice of order type (i.e., market order vs. full demand schedule) by the binary choice of staying out (at zero cost) or entering the asset market (at a fixed cost $\tilde{c}$). For the sake of simplicity we assume here that (i) participating informed traders can only submit full demand schedules, (ii) they all share the same risk aversion coefficient γ, and (iii) the prior is uninformative ($\alpha_\theta = 0$). The other assumptions of the model are unchanged.

We conjecture and verify the existence of an equilibrium where $\tilde{\nu} > 0$ traders enter the market and wherein the equilibrium price function is linear: $p(\theta,\varepsilon) = \theta + \tilde{B}^{-1}\varepsilon$, so that p provides a noisy public signal about θ with precision $\tilde{B}^2\alpha_\varepsilon$. Applying the same reasoning as in Appendix A we get the conditional distribution $\theta|p, x_i$:

$$\theta|p,x_i \sim \mathcal{N}\left(\frac{\alpha_x x_i+\tilde{B}^2\alpha_\varepsilon p}{\alpha_x+\tilde{B}^2\alpha_\varepsilon}, \frac{1}{\alpha_x+\tilde{B}^2\alpha_\varepsilon}\right),$$

while the asset demand of a participating trader is is $k(x_i, p) = \frac{\alpha_x}{\gamma}(x_i - p)$ and the coefficient to be identified is $\tilde{B} = \tilde{\nu}\alpha_x/\gamma$. From (Eqs. 13) and (16), the value of trading knowing (x_i, p) is:

$$W^P(x_i,p;\gamma_i) = -\exp\left[-\frac{\alpha_x^2}{\alpha_x+\tilde{B}^2\alpha_\varepsilon}\frac{(x_i-p)^2}{2}+\tilde{c}\gamma\right],$$

while the ex ante value from participating (expecting a participation level $\tilde{\nu}$) is:

$$\mathbb{E}\left[\mathbb{E}\left[W^P(x_i;\gamma_i)\middle|x_i\right]\right] = -e^{\tilde{c}\gamma}\sqrt{\frac{\alpha_\varepsilon}{\alpha_\varepsilon+\frac{\gamma^2}{\tilde{\nu}^2\alpha_x}}}.$$

Since initial wealth is zero, the expected utility from not participating is $-e^0 = -1$. And since all informed traders are ex ante homogeneous, in the free-entry equilibrium informed traders must all be indifferent between entering and staying out. This gives:

$$\tilde{\nu} = \frac{\gamma}{\sqrt{\alpha_x\alpha_\varepsilon\left(e^{2\tilde{c}\gamma}-1\right)}} \quad (>0),$$

where we are implicitly setting ourselves in a parametric restriction such that $\tilde{\nu} \le 1$. The latter expression states that, at any such $(\alpha_x, \alpha_\varepsilon)$ given a higher participating cost $\tilde{c}$ lowers the share of participating agents; this in turn reduces the precision of the price signal (i.e., $\tilde{B}^2\alpha_\varepsilon = (\tilde{\nu}\alpha_x/\gamma)^2\alpha_\varepsilon$ here) and thereby widens the uniqueness region in the coordination stage.

Appendix E. Comparative statics

We summarise our formal results by means of the following proposition:

Proposition 4. *(a) For any* $(\nu,\gamma,\alpha_\theta,\alpha_\varepsilon)\in(0;1)\times\left(\mathbb{R}_+^*\right)^3$ *satisfying* $\sqrt{\alpha_\varepsilon}\le\frac{9\sqrt{3}}{2048\sqrt{2\pi}}\frac{\gamma}{1-\nu}$ *and* $\alpha_\theta\in(0;\overline{\alpha}_\theta(\nu,\gamma,\alpha_\varepsilon))$, *where* $\overline{\alpha}_\theta(\nu,\gamma,\alpha_\varepsilon)\in\left(0,\sqrt{2\pi}\left(\sqrt{\frac{\sqrt{2\pi}}{\alpha_\varepsilon}\frac{1}{4}\frac{\gamma}{\nu}}\right)^{4/3}\right)$ *solves*

$$\varphi(\left(\sqrt{\frac{\sqrt{2\pi}}{\alpha_\varepsilon}\frac{1}{4}\frac{\gamma}{\nu}}\right)^{4/3},\alpha_\varepsilon,\overline{\alpha}_\theta(\nu,\gamma,\alpha_\varepsilon),\gamma,\nu)=0,$$

with φ *defined as:*

$$\varphi(.,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu):\alpha_x\to\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}-\nu\frac{\alpha_x}{\gamma}-\frac{1-\nu}{\gamma}\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta},$$

then the uniqueness region is reduced to a single, non-empty, interval:

$$[\underline{\alpha}_x,\overline{\alpha}_x]\subset\left[\alpha_\theta^2/2\pi;\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}\right]$$

(b) Moreover, we have:

$$\frac{\partial\overline{\alpha}_x}{\partial\alpha_\theta}<0,\ \frac{\partial\overline{\alpha}_x}{\partial\alpha_\varepsilon}<0,\ \frac{\partial\overline{\alpha}_x}{\partial\nu}<0,\ \frac{\partial\overline{\alpha}_x}{\partial\gamma}>0.$$

Finally, when the prior is close to diffuse we have $\overline{\alpha}_x\underset{\alpha_\theta\to0^+}{\to}\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}$, *and*

$$\frac{\partial\overline{\alpha}_x}{\partial\alpha_\theta}\underset{\alpha_\theta\to0^+}{\to}-\frac{2}{3}\frac{1}{(2\pi)^{1/3}\alpha_\varepsilon^{1/3}\left(\frac{\gamma}{\nu}\right)^{\frac{-2}{3}}}-\frac{4}{3}\frac{1-\nu}{\nu},\quad\frac{\partial\overline{\alpha}_x}{\partial\nu}\underset{\alpha_\theta\to0^+}{\to}-\frac{4}{3\nu}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}},$$

$$\frac{\partial\overline{\alpha}_x}{\partial\gamma}\underset{\alpha_\theta\to0^+}{\to}\frac{4}{3\gamma}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}}\ \text{and}\ \frac{\partial\overline{\alpha}_x}{\partial\alpha_\varepsilon}\underset{\alpha_\theta\to0^+}{\to}\frac{-2}{3\alpha_\varepsilon}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}}.$$

Proof. (a) We know that there is a unique equilibrium in the coordination game when $B^2\alpha_\varepsilon+\alpha_\theta\le\sqrt{2\pi\alpha_x}$ where B solves

$$B=\nu\frac{\alpha_x}{\gamma}+\frac{1-\nu}{\gamma}\left(\frac{1}{\alpha_x}+\frac{1}{\alpha_\theta}-\frac{1}{B^2\alpha_\varepsilon+\alpha_\theta}\right)^{-1}.$$

We are interested in the range of values of α_x such that uniqueness is ensured, holding all the other parameters fixed. The first thing to notice is that there necessary are multiple equilibrium outcomes in the coordination game whenever $\alpha_x\le\alpha_\theta^2/2\pi$ (intuitively, traders then tend to focus on the – common – prior, which favours coordination on a self-fulfilling outcome). We thus restrict our attention, from now on, to the case where $\alpha_x>\alpha_\theta^2/2\pi$.

Defining, for any $(\nu,\gamma,\alpha_\theta,\alpha_\varepsilon)\in(0;1)\times\left(\mathbb{R}_+^*\right)^3$,

$$\varphi(.,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu):\alpha_x\to\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}-\nu\frac{\alpha_x}{\gamma}-\frac{1-\nu}{\gamma}\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta}$$

which is properly defined over $(\frac{\alpha_\theta^2}{2\pi},+\infty)$, the unicity of equilibrium in the coordination game on this parameter range is ensured if and only if $\varphi(\alpha_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)\ge0$: the set of values of $\alpha_x>\frac{\alpha_\theta^2}{2\pi}$ such that $B^2\alpha_\varepsilon+\alpha_\theta\le\sqrt{2\pi\alpha_x}$ where $B=\nu\frac{\alpha_x}{\gamma}+\frac{1-\nu}{\gamma}\left(\frac{1}{\alpha_x}+\frac{1}{\alpha_\theta}-\frac{1}{B^2\alpha_\varepsilon+\alpha_\theta}\right)^{-1}$ is the same set of values of $\alpha_x>\frac{\alpha_\theta^2}{2\pi}$ such that $\varphi(\alpha_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)\ge0$.

When $\alpha_x>\alpha_\theta^2/2\pi$, we have:

$$\frac{\partial^2\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}}{\partial\alpha_x^2}=-\frac{1}{8}\left(\frac{\sqrt{2\pi}}{\alpha_\varepsilon}\right)\left(\frac{1}{\sqrt{\alpha_x}}\frac{1}{\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}}\right)^3\left(\left(\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}\right)+\frac{\sqrt{2\pi\alpha_x}}{2\alpha_\varepsilon}\right)<0,$$

which implies the strict concavity of the function $\alpha_x\to\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}$ on $(\alpha_\theta^2/2\pi,+\infty)$ for any $(\nu,\gamma,\alpha_\theta,\alpha_\varepsilon)\in(0;1)\times\left(\mathbb{R}_+^*\right)^3$. Moreover:

$$\frac{\partial^2\left(\nu\frac{\alpha_x}{\gamma}+\frac{1-\nu}{\gamma}\left(\frac{1}{\alpha_x}+\frac{1}{\alpha_\theta}-\frac{1}{\sqrt{2\pi\alpha_x}}\right)^{-1}\right)}{\partial\alpha_x^2}=\frac{1-\nu}{\gamma}\left(2\alpha_\theta^3\frac{\left(-\frac{1}{\alpha_x^2}+\frac{1}{\sqrt{2\pi}}\frac{1}{2}\frac{1}{\alpha_x^{3/2}}\right)^2}{\left(\frac{\alpha_\theta}{\alpha_x}+1-\frac{\alpha_\theta}{\sqrt{2\pi\alpha_x}}\right)^3}+\frac{\frac{\alpha_\theta^2}{\alpha_x^3}\left(\alpha_x^{1/2}-\frac{8\sqrt{2\pi}}{3}\right)}{\left(\frac{\alpha_\theta}{\alpha_x}+1-\frac{\alpha_\theta}{\sqrt{2\pi\alpha_x}}\right)^2}\right),$$

so that $\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ is strictly concave over $[(8\sqrt{2\pi}/3)^2; \infty)$.

If $\alpha_\theta^2/2\pi \geq (8\sqrt{2\pi}/3)^2$ then $\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ is strictly concave over $[\alpha_\theta^2/2\pi, +\infty)$. Otherwise, we also have, on $[\alpha_\theta^2/2\pi, (8\sqrt{2\pi}/3)^2]$:

$$\frac{\partial^2\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)}{\partial\alpha_x^2}<\frac{1}{\alpha_x}\left[-\frac{1}{8}\left(\sqrt{\frac{2\pi}{\alpha_\varepsilon}}\right)\left(\frac{1}{\sqrt{\alpha_x}}\frac{1}{\sqrt{\sqrt{2\pi\alpha_x}-\alpha_\theta}}\right)+\frac{1-\nu}{\gamma}\left(\frac{8\sqrt{2\pi}}{3}\right)\right].$$

With $\sqrt{\alpha_\varepsilon}\leq\frac{3\gamma}{64(1-\nu)}\left(\sqrt{\left(\frac{8\sqrt{2\pi}}{3}\right)^2\sqrt{2\pi\left(\frac{8\sqrt{2\pi}}{3}\right)^2}}\right)^{-1}$, strict concavity of $\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ is then always ensured over $(\alpha_\theta^2/2\pi, +\infty)$.

Let us now consider the function $\alpha_\theta\to\ \varphi(\left(\sqrt{\frac{\sqrt{2\pi}}{\alpha_\varepsilon}}\frac{1}{4}\frac{\gamma}{\nu}\right)^{4/3}, \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ for a given set $(\nu, \gamma, \alpha_\varepsilon)\in(0; 1)\times\left(\mathbb{R}_+^*\right)^2$. This function is defined, and strictly decreasing over, the interval $(0, \sqrt{2\pi}\left(\sqrt{\frac{\gamma}{4\nu}\sqrt{2\pi}/\alpha_\varepsilon}\right)^{2/3}]$. We also have:

$$\varphi(\left(\frac{\gamma}{4\nu}\sqrt{\sqrt{2\pi}/\alpha_\varepsilon}\right)^{4/3}, \alpha_\varepsilon, 0, \gamma, \nu)>0\ \text{ and }\ \varphi(\left(\frac{\gamma}{4\nu}\sqrt{\sqrt{2\pi}/\alpha_\varepsilon}\right)^{4/3}, \alpha_\varepsilon, \sqrt{2\pi}\left(\frac{\gamma}{4\nu}\sqrt{\sqrt{2\pi}/\alpha_\varepsilon}\right)^{2/3}, \gamma, \nu)<0.$$

This implies that there is a unique $\overline{\alpha}_\theta(\nu, \gamma, \alpha_\varepsilon)\in(0; \sqrt{2\pi}\left(\frac{\gamma}{4\nu}\sqrt{\sqrt{2\pi}/\alpha_\varepsilon}\right)^{2/3})$ such that, for all $\alpha_\theta\in(0, \overline{\alpha}_\theta(\nu, \gamma, \alpha_\varepsilon))$, $\varphi(\left(\frac{\gamma}{4\nu}\sqrt{\sqrt{2\pi}/\alpha_\varepsilon}\right)^{4/3}, \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)>0$.

Hence, for any $(\nu, \gamma, \alpha_\varepsilon)\in(0; 1)\times\left(\mathbb{R}_+^*\right)^2$ such that $\sqrt{\alpha_\varepsilon}\frac{1-\nu}{\gamma}\leq\frac{3}{64\sqrt{\frac{8\sqrt{2\pi}}{3}}}\frac{1}{\sqrt{\sqrt{2\pi\frac{8\sqrt{2\pi}}{3}}}}$ and $\alpha_\theta\in(0, \overline{\alpha}_\theta(\nu, \gamma, \alpha_\varepsilon))$, $\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ is strictly concave and changes signs twice over $(\alpha_\theta^2/2\pi, +\infty)$. This implies that $\varphi(., \alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ cancels exactly twice on this interval, namely at:

$$\underline{\alpha}_x\in\left(\frac{\alpha_\theta^2}{2\pi}, \left(\frac{\gamma}{4\nu}\sqrt{\frac{\sqrt{2\pi}}{\alpha_\varepsilon}}\right)^{4/3}\right)\ \text{and}\ \overline{\alpha}_x\in\left(\left(\frac{\gamma}{4\nu}\sqrt{\frac{\sqrt{2\pi}}{\alpha_\varepsilon}}\right)^{4/3}, +\infty\right).$$

This defines the interval $[\underline{\alpha}_x, \overline{\alpha}_x]$.

(b) We are interested in the way $\overline{\alpha}_x(\alpha_\varepsilon, \alpha_\theta, \gamma, \nu)$ changes with the underlying parameters, assuming that $\sqrt{\alpha_\varepsilon}\frac{1-\nu}{\gamma}\leq\frac{3}{64\sqrt{\frac{8\sqrt{2\pi}}{3}}}\frac{1}{\sqrt{\sqrt{2\pi\frac{8\sqrt{2\pi}}{3}}}}$ and $\alpha_\theta\in(0, \overline{\alpha}_\theta(\nu, \gamma, \alpha_\varepsilon))$. We know that for any $\alpha_\theta<\overline{\alpha}_\theta(\nu, \gamma, \alpha_\varepsilon)$, the uniqueness region is an interval $[\underline{\alpha}_x, \overline{\alpha}_x]$. We will first focus on the way $\overline{\alpha}_x$ varies when $\alpha_\theta\to 0^+$. We first not that $\overline{\alpha}_x\in[(1/4)^{4/3}\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}, \left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}]$. Indeed, we know that $\overline{\alpha}_x\geq(1/4)^{4/3}\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}$, while

$$\frac{\nu\overline{\alpha}_x}{\gamma}\leq\frac{\nu\overline{\alpha}_x}{\gamma}+\frac{1-\nu}{\gamma}\frac{\sqrt{2\pi}\overline{\alpha}_x\alpha_\theta}{\sqrt{2\pi}(\overline{\alpha}_x+\alpha_\theta)-\sqrt{\overline{\alpha}_x}\alpha_\theta}=\sqrt{\frac{\sqrt{2\pi\overline{\alpha}_x}-\alpha_\theta}{\alpha_\varepsilon}}\leq\sqrt{\frac{\sqrt{2\pi\overline{\alpha}_x}}{\alpha_\varepsilon}},$$

which implies that $\overline{\alpha}_x\leq\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}$. Hence, $\overline{\alpha}_x$ is both well defined and bounded above and below by bounds which are stricly positive and independent of α_θ. Some algebraic manipulations then show that $\overline{\alpha}_x$ has a limit when $\alpha_\theta\to 0^+$ and that this limit is $\left(2\pi/\alpha_\varepsilon^2\right)^{\frac{1}{3}}(\gamma/\nu)^{\frac{4}{3}}$.

Finally, the use of the implicit function theorem gives us the signs of $\frac{\partial\overline{\alpha}_x}{\partial\alpha_\theta}$, $\frac{\partial\overline{\alpha}_x}{\partial\alpha_\varepsilon}$, $\frac{\partial\overline{\alpha}_x}{\partial\nu}$ and $\frac{\partial\overline{\alpha}_x}{\partial\gamma}>0$. The first thing to note is that:

$$\frac{\partial\varphi}{\partial\alpha_\theta}=-\frac{1}{2}\frac{1}{\alpha_\varepsilon}\left(\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}\right)^{-1/2}-\frac{1-\nu}{\gamma}\frac{2\pi\alpha_x^2}{\left(\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta\right)^2}<0,$$

$$\frac{\partial\varphi}{\partial\nu}=\frac{1}{\gamma}\left(\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta}-\alpha_x\right)<0,$$

$$\frac{\partial\varphi}{\partial\gamma}=\nu\frac{\alpha_x}{\gamma^2}+\frac{1-\nu}{\gamma^2}\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta}>0 \text{ and } \frac{\partial\varphi}{\partial\alpha_\varepsilon}=\frac{-1}{2\alpha_\varepsilon}\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}<0.$$

Moreover, $\alpha_x\to\frac{1}{2}\left(\frac{\sqrt{2\pi}}{\alpha_\varepsilon}\frac{1}{2\sqrt{\alpha_x}}\right)\left(\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}\right)^{-1/2}-\nu/\gamma-\frac{1-\nu}{\gamma}\frac{2\pi\alpha_\theta^2-\frac{\sqrt{2\pi}\sqrt{\alpha_x}\alpha_\theta^2}{2}}{\left(\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta\right)^2}$ is continuous, differentiable and strictly concave on $[\alpha_\theta^2/2\pi,+\infty)$ and cancels exactly twice on this interval, at $\underline{\alpha}_x$ and $\overline{\alpha}_x$. Rolle's theorem implies that $\partial\varphi/\partial\alpha_x$ cancels exactly once on $(\underline{\alpha}_x,\overline{\alpha}_x)$, while strict concavity of φ ensures that $\frac{\partial\varphi}{\partial\alpha_x}(\underline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)>0>\frac{\partial\varphi}{\partial\alpha_x}(\overline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)$. Hence,

$$\frac{\partial\overline{\alpha}_x}{\partial\alpha_\theta}=-\frac{\frac{\partial\varphi}{\partial\alpha_\theta}}{\frac{\partial\varphi}{\partial\alpha_x}}(\underline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)<0,\quad \frac{\partial\overline{\alpha}_x}{\partial\alpha_\varepsilon}=-\frac{\frac{\partial\varphi}{\partial\alpha_\varepsilon}}{\frac{\partial\varphi}{\partial\alpha_x}}(\underline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)<0,$$

$$\frac{\partial\overline{\alpha}_x}{\partial\nu}=-\frac{\frac{\partial\varphi}{\partial\nu}}{\frac{\partial\varphi}{\partial\alpha_x}}(\underline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)<0,\quad \frac{\partial\overline{\alpha}_x}{\partial\gamma}=-\frac{\frac{\partial\varphi}{\partial\gamma}}{\frac{\partial\varphi}{\partial\alpha_x}}(\underline{\alpha}_x,\alpha_\varepsilon,\alpha_\theta,\gamma,\nu)>0,$$

while the opposite relationships hold true for $\underline{\alpha}_x$.

We can be more specific about these derivatives as $\alpha_\theta\to0^+$. Indeed, holding the other parameters fixed, we have:

$$\frac{\partial\varphi}{\partial\alpha_x}=\frac{1}{2}\left(\frac{\sqrt{2\pi}}{\alpha_\varepsilon}\frac{1}{2\sqrt{\alpha_x}}\right)\left(\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}\right)^{-1/2}-\nu\frac{1}{\gamma}-\frac{1-\nu}{\gamma}\frac{2\pi\alpha_\theta^2-\frac{\sqrt{2\pi}\sqrt{\alpha_x}\alpha_\theta^2}{2}}{\left(\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta\right)^2}\underset{\alpha_\theta\to0^+}{\to}-\frac{3}{4}\frac{\nu}{\gamma},$$

$$\frac{\partial\varphi}{\partial\alpha_\theta}=-\frac{1}{2}\frac{1}{\alpha_\varepsilon}\left(\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}\right)^{-1/2}-\frac{1-\nu}{\gamma}\frac{2\pi\alpha_x^2}{\left(\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta\right)^2}\underset{\alpha_\theta\to0^+}{\to}-\frac{1}{2}\frac{1}{(2\pi)^{1/3}\alpha_\varepsilon^{1/3}\left(\frac{\gamma}{\nu}\right)^{\frac{1}{3}}}-\frac{1-\nu}{\gamma},$$

$$\frac{\partial\varphi}{\partial\nu}=\frac{1}{\gamma}\left(\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta}-\alpha_x\right)\underset{\alpha_\theta\to0^+}{\to}-\frac{\alpha_x}{\gamma},\quad \frac{\partial\varphi}{\partial\alpha_\varepsilon}=\frac{-1}{2\alpha_\varepsilon}\sqrt{\frac{\sqrt{2\pi\alpha_x}-\alpha_\theta}{\alpha_\varepsilon}}\underset{\alpha_\theta\to0^+}{\to}\frac{-1}{2\alpha_\varepsilon}\sqrt{\frac{\sqrt{2\pi\alpha_x}}{\alpha_\varepsilon}},$$

and $\frac{\partial\varphi}{\partial\gamma}=\nu\frac{\alpha_x}{\gamma^2}+\frac{1-\nu}{\gamma^2}\frac{\sqrt{2\pi}\alpha_x\alpha_\theta}{\sqrt{2\pi}(\alpha_x+\alpha_\theta)-\sqrt{\alpha_x}\alpha_\theta}\underset{\alpha_\theta\to0^+}{\to}\nu\frac{\alpha_x}{\gamma^2}.$

This in turn implies:

$$\frac{\partial\overline{\alpha}_x}{\partial\alpha_\theta}=-\frac{\partial\varphi}{\partial\alpha_\theta}/\frac{\partial\varphi}{\partial\alpha_x}\underset{\alpha_\theta\to0^+}{\to}-\frac{2}{3}\frac{1}{(2\pi)^{1/3}\alpha_\varepsilon^{1/3}\left(\frac{\gamma}{\nu}\right)^{\frac{-2}{3}}}-\frac{4}{3}\frac{1-\nu}{\nu};$$

$$\frac{\partial\overline{\alpha}_x}{\partial\nu}=-\frac{\partial\varphi}{\partial\nu}/\frac{\partial\varphi}{\partial\alpha_x}\underset{\alpha_\theta\to0^+}{\to}-\frac{4}{3\nu}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}};$$

$$\frac{\partial\overline{\alpha}_x}{\partial\gamma}=-\frac{\partial\varphi}{\partial\gamma}\frac{\partial\varphi}{\partial\alpha_x}/\underset{\alpha_\theta\to0^+}{\to}\frac{4}{3\gamma}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}};$$

$$\frac{\partial\overline{\alpha}_x}{\partial\alpha_\varepsilon}=-\frac{\partial\varphi}{\partial\nu}/\frac{\partial\varphi}{\partial\alpha_x}\underset{\alpha_\theta\to0^+}{\to}\frac{-2}{3\alpha_\varepsilon}\left(\frac{2\pi}{\alpha_\varepsilon^2}\right)^{\frac{1}{3}}\left(\frac{\gamma}{\nu}\right)^{\frac{4}{3}}.\quad\square$$

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