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Research Article

AGQP-Injective Modules

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Let R be a ring and let M be a right R-module with $S = \mathrm{End}(M_R)$. M is called *almost general quasi-principally injective* (or *AGQP-injective* for short) if, for any $0 \neq s \in S$, there exist a positive integer n and a left ideal X_{s^n} of S such that $s^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n \oplus X_{s^n}$. Some characterizations and properties of AGQP-injective modules are given, and some properties of AGQP-injective modules with additional conditions are studied.

1. Introduction

Throughout R is an associative ring with identity, and all modules are unitary. Recall that a ring R is called *right principally injective* [1] (or *right P-injective* for short) if, every homomorphism from a principal right ideal of R to R can be extended to an endomorphism of R, or equivalently, $\mathbf{lr}(a) = Ra$ for all $a \in R$. The concept of right P-injective rings has been generalized by many authors. For example, in [2, 3], right P-injective rings are generalized in two directions, respectively. Following [2], a ring R is called *right GP-injective* if, for any $0 \neq a \in R$, there exists a positive integer n such that $a^n \neq 0$ and any right R-homomorphism from a^nR to R can be extended to an endomorphism of R. Note that GP-injective rings are also called *YJ-injective* in [4]. From [5], we know that GP-injective rings need not to be P-injective. Following [3], a right R-module M_R with $S = \mathrm{End}(M_R)$ is called *quasiprincipally injective* (or *QP-injective* for short) if, every homomorphism from an M-cyclic submodule of M to M can be extended to an endomorphism of M, or equivalently, $\mathbf{l}_S(\mathrm{Ker}(s)) = Ss$ for all $s \in S$. In 1998, Page and Zhou [6] generalized the concept of GP-injective rings to that of AGP-injective rings. According to [6], a ring R is called *right AGP-injective* if, for any $0 \neq a \in R$, there exist a positive integer n and a left ideal X_{a^n} such that $a^n \neq 0$ and $\mathbf{lr}(a^n) = Ra^n \oplus X_{a^n}$. In [7], the first author introduced the notion of GQP-injective modules which can be regarded as the generalization of GP-injective rings and QP-injective modules. According to [7], a right R-module M with $S = \mathrm{End}(M_R)$ is called *GQP-injective* if, for any

$0 \neq s \in S$, there exists a positive integer n such that $s^n \neq 0$ and any right R-homomorphism from $s^n(M)$ to M can be extended to an endomorphism of M, or equivalently, for any $0 \neq s \in S$, there exists a positive integer n such that $s^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n$. The nice structure of AGP-injective rings and GQP-injective modules draws our attention to define almost GQP-injective modules, in a similar way to AGP-injective rings, and to investigate their properties.

2. Results

Definition 2.1. Let M_R be a right R-module with $S = \mathrm{End}(M_R)$. Then, M is said to be almost general quasiprincipally injective (briefly, AGQP-injective) if, for any $0 \neq s \in S$, there exist a positive integer n and a left ideal X_{s^n} of S such that $s^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n \oplus X_{s^n}$.

Clearly, a ring R is right AGP-injective if and only if R_R is AGQP-injective, GQP-injective modules are AGQP-injective.

Our next result gives the relationship between the AGQP-injectivity of a module and the AGP-injectivity of its endomorphism ring.

Theorem 2.2. *Let M_R be a right R-module with $S = \mathrm{End}(M_R)$. Then,*

(1) *if S is right AGP-injective, then M_R is AGQP-injective;*

(2) *if M_R is AGQP-injective and M generates* $\mathrm{Ker}(s)$ *for each $s \in S$, then S is right AGP-injective.*

Proof. (1) Suppose that S is right AGP-injective then for any $0 \neq s \in S$, there exist a positive integer n and a left ideal I_{s^n} of S such that $s^n \neq 0$ and $\mathbf{l}_S\mathbf{r}_S(s^n) = Ss^n \oplus I_{s^n}$. If $a \in \mathbf{l}_S(\mathrm{Ker}(s^n))$ and $b \in \mathbf{r}_S(s^n)$, then $s^n b = 0$, that is, $b(M) \subseteq \mathrm{Ker}(s^n)$. Hence, $(ab)M = 0$, that is, $ab = 0$. This shows that $\mathbf{l}_S(\mathrm{Ker}(s^n)) \subseteq \mathbf{l}_S\mathbf{r}_S(s^n)$. Therefore, we have $Ss^n \subseteq \mathbf{l}_S(\mathrm{Ker}(s^n)) \subseteq Ss^n \oplus I_{s^n}$, which guarantees that

$$\mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n \oplus (\mathbf{l}_S(\mathrm{Ker}(s^n)) \cap I_{s^n}). \tag{2.1}$$

Thus, (1) is proved.

(2) Suppose that M_R is AGQP-injective then for any $0 \neq s \in S$, there exist a positive integer n and a left ideal X_{s^n} of S such that $s^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n \oplus X_{s^n}$. Assume that $a \in \mathbf{l}_S\mathbf{r}_S(s^n)$ and $\mathrm{Ker}(s^n) = \sum_{t \in T} t(M)$ for some subset T of S. It is easy to see that $at = 0$ for each $t \in T$, so we have $ax = 0$ for each $x \in \mathrm{Ker}(s^n)$. This implies that $\mathbf{l}_S\mathbf{r}_S(s^n) \subseteq \mathbf{l}_S(\mathrm{Ker}(s^n))$, from which we have

$$Ss^n \subseteq \mathbf{l}_S\mathbf{r}_S(s^n) \subseteq \mathbf{l}_S(\mathrm{Ker}(s^n)) = Ss^n \oplus X_{s^n}, \tag{2.2}$$

and hence

$$\mathbf{l}_S\mathbf{r}_S(s^n) = Ss^n \oplus (\mathbf{l}_S\mathbf{r}_S(s^n) \cap X_{s^n}). \tag{2.3}$$

Therefore, S is right AGP-injective. □

Recall that a module N is called *M-cyclic* [3], if it is a homomorphic image of M. Let $S = \mathrm{End}(M_R)$, following [8], we write $W(S) = \{s \in S \mid \mathrm{Ker}(s) \subseteq^{ess} M\}$.

Theorem 2.3. *Let M_R be an AGQP-injective module with $S = \mathrm{End}(M_R)$. Then,*

(1) $W(S) \subseteq J(S)$,

(2) *if every nonzero submodule of M contains a nonzero M-cyclic submodule, then $W(S) = J(S)$.*

Proof. (1) Let $s \in W(S)$. Then, for each $t \in S$, $ts \in W(S)$ and so $1 - ts \neq 0$. Since M_R is AGQP-injective, there exist a positive integer n and a left ideal $X_{(1-ts)^n}$ such that $(1-ts)^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(1-ts)^n) = S(1-ts)^n \oplus X_{(1-ts)^n}$. Note that $(1-ts)^n = 1-u$ for some $u \in W(S)$. Since $\mathrm{Ker}(u) \cap \mathrm{Ker}(1-u) = 0$, we have $\mathrm{Ker}(1-u) = 0$, and then $S = S(1-u) \oplus X_{1-u}$. So $1 = e + x$ for some $e \in S(1-u)$ and $x \in X_{1-u}$, it follows that $e^2 = e$ and $S(1-u) = Se \oplus S(1-e) \cap S(1-u) = Se$. Therefore, $1 - u = ve$ for some $v \in S$, since $\mathrm{Ker}(u)$ is essential in M_R, if $e \neq 1$, then there exists a nonzero element $(1-e)m \in (1-e)M \cap \mathrm{Ker}(u)$, and hence $(1-u)(1-e)m = (1-e)m$. But $(1-u)(1-e)m = ve(1-e)m = 0$, a contradiction. So $e = 1$, and hence $1 - u$ is left invertible, which implies $s \in J(S)$.

(2) We need only to prove that $J(S) \subseteq W(S)$. Let $s \in J(S)$. If $s \notin W(S)$, then there exists $0 \neq t \in S$ such that $\mathrm{Ker}(s) \cap t(M) = 0$ by hypothesis. Clearly, $st \neq 0$ and $\mathrm{Ker}(st) = \mathrm{Ker}(t)$. Since M_R is AGQP-injective, there exist a positive integer n and a left ideal $X_{(st)^n}$ such that $(st)^n \neq 0$ and

$$\mathbf{l}_S(\mathrm{Ker}(st)^n) = S(st)^n \oplus X_{(st)^n}. \tag{2.4}$$

If $m \in \mathrm{Ker}(st)^n$, then $(st)^{n-1}m \in \mathrm{Ker}(st) = \mathrm{Ker}(t)$, and so $m \in \mathrm{Ker}(t(st)^{n-1})$. This shows that $\mathrm{Ker}(st)^n = \mathrm{Ker}(t(st)^{n-1})$. Hence, $t(st)^{n-1} \in S(st)^n \oplus X_{(st)^n}$. Write $t(st)^{n-1} = u(st)^n + v$, where $u \in S$, $v \in X_{(st)^n}$. Then $(1-us)t(st)^{n-1} = v$, which gives that $(st)^n = s(1-us)^{-1}v \in S(st)^n \cap X_{(st)^n} = 0$, a contradiction. □

Corollary 2.4 (see [6, Corollary 2.3]). *If R is a right AGP-injective ring, then $J(R) = Z(R_R)$.*

Following [9], for a set $X \subseteq \mathrm{Hom}\,(N_R, M_R)$, the submodule

$$\mathrm{Ker}\, X = \cap\{\mathrm{Ker}\, g \mid g \in X\} \tag{2.5}$$

of N is called an *M-annihilator submodule* of N. By [7, Lemma 9] and Theorem 2.3, we have the following corollary.

Corollary 2.5. *Let M_R be an AGQP-injective module with $S = \mathrm{End}(M_R)$. If every nonzero submodule of M contains a nonzero M-cyclic submodule, and $M/Soc(M)$ satisfies ACC on M-annihilator submodules, then $J(S)$ is nilpotent.*

Recall that a module M_R is said to be a *GC2 module* [10] if every submodule $N \leq M$ with $N \cong M$ is a direct summand of M. For convenience, we write $N \mid M$ to denote that N is a direct summand of M.

Theorem 2.6. *Let M_R be an AGQP-injective module. Then,*

(1) *if M_1 and M_2 are submodules of M such that $M_1 \subseteq M_2$ and $M_1 \cong M_2 \mid M$, then $M_1 \mid M$. In particular M is a GC2 module;*

(2) *if M_1 and M_2 are simple submodules of M such that $M_1 \cong M_2 \mid M$, then $M_1 \mid M$.*

Proof. (1) Let $S = \text{End}(M_R)$. It is trivial in case $M_1 = 0$. Now suppose that $M_1 \neq 0$ and $M_2 \overset{f}{\cong} M_1$. Then $M_1 = aM$ and $M_2 = eM$, where $e^2 = e \in S$ and $a = fe$. Since M_R is AGQP-injective, there exist a positive integer n and a left ideal X_{a^n} such that $a^n \neq 0$ and $\mathbf{l}_S(\text{Ker}(a^n)) = Sa^n \oplus X_{a^n}$. Let $a^0 = e$, then $f^{-1}(a^{i+1}M) = a^iM$ $(i = 0, 1, \ldots, n-1)$ since $M_1 \subseteq M_2 = eM$. So we have

$$a^iM \mid a^{i-1}M \Longleftrightarrow f^{-1}(a^{i+1}M) \mid f^{-1}(a^iM) \Longleftrightarrow a^{i+1}M \mid a^iM \quad (i = 1, \ldots, n-1). \tag{2.6}$$

Consequently, $aM \mid eM \Leftrightarrow a^2M \mid aM \Leftrightarrow \cdots \Leftrightarrow a^nM \mid a^{n-1}M$. Thus, to show $aM \mid M$, it suffices to show that $a^nM \mid M$. Note that $a|_{eM} : eM \to eM$ is monic and $a^n(m) = a^n(em)$ for every $m \in M$, $eM \overset{a^n}{\cong} a^nM$ and hence $\text{Ker}(a^n) = \text{Ker}(e)$. It follows that $e \in \mathbf{l}_S(\text{Ker}(e)) = \mathbf{l}_S(\text{Ker}(a^n)) = Sa^n \oplus X_{a^n}$. Now, let $e = ba^n + x$ with $b \in S$ and $x \in X_{a^n}$, then $a^n = a^ne = a^nba^n + a^nx = a^nba^n$. Finally, let $g = a^nb$, then $g^2 = g$ and $a^nM = gM$ as required.

(2) Let $M_2 = e_1M$, where $e_1^2 = e_1 \in S$, and let $M_2 \overset{f_1}{\cong} M_1$. Then $M_1 = a_1M$, where $a_1 = f_1e_1$. Since M_R is AGQP-injective, there exist a positive integer n_1 and a left ideal $X_{a_1^{n_1}}$ such that $a_1^{n_1} \neq 0$ and $\mathbf{l}_S(\text{Ker}(a_1^{n_1})) = Sa_1^{n_1} \oplus X_{a_1^{n_1}}$. Note that $0 \neq a_1^{n_1}M \subseteq a_1M$, and a_1M is simple. We have $a_1^{n_1}M = a_1M$. Clearly, $\text{Ker}(e_1) = \text{Ker}(a_1)$ because f_1 is a monomorphism. Since a_1M is simple, $\text{Ker}(a_1)$ is a maximal submodule of M. But $\text{Ker}(a_1) \subseteq \text{Ker}(a_1^{n_1}) \neq M$, so $\text{Ker}(a_1) = \text{Ker}(a_1^{n_1})$ and then $\text{Ker}(e_1) = \text{Ker}(a_1^{n_1})$. It follows that $e_1 \in \mathbf{l}_S(\text{Ker}(e_1)) = \mathbf{l}_S(\text{Ker}(a_1^{n_1})) = Sa_1^{n_1} \oplus X_{a_1^{n_1}}$. Now, let $e_1 = b_1a_1^{n_1} + y$ with $b_1 \in S$ and $y \in X_{a_1^{n_1}}$, then $a_1^{n_1} = a_1^{n_1}e_1 = a_1^{n_1}b_1a_1^{n_1} + a_1^{n_1}y = a_1^{n_1}b_1a_1^{n_1}$. Finally, let $g_1 = a_1^{n_1}b_1$, then $g_1^2 = g_1$ and $M_1 = a_1M = a_1^{n_1}M = g_1M$ as required. □

Recall that a module M is said to be *weakly injective* [11] if, for any finitely generated submodule $N \leq E(M)$, there exists $X \leq E(M)$ such that $N \subseteq X \cong M$.

Corollary 2.7. *Let M be a finitely generated module. Then, M is injective if and only if M is weakly injective and AGQP-injective. In particular, a ring R is right self-injective if and only if R_R is weakly injective and AGP-injective.*

Proof. We need only to prove the sufficiency. Let $x \in E(M)$. Then, there exists $X \subseteq E(M)$ such that $M + xR \subseteq X \cong M$. Hence, X is AGQP-injective and $M \mid X$ follows from Theorem 2.6(1). But M is essential in $E(M)$, so $M = X$ and hence $x \in M$. □

Corollary 2.8. *Let M_R be an AGQP-injective module with $S = \text{End}(M_R)$.*

(1) *If M_R is of finite Goldie dimension, then S is semilocal.*

(2) *If M_R is a noetherian self-generator, then S is semiprimary.*

Proof. (1) Since M_R is AGQP-injective, it satisfies the GC2-condition by Theorem 2.6(1) and then (1) follows immediately by [12, Lemma 1.1].

(2) By (1) and Corollary 2.5. □

Recall that if M and U are two right R-modules, then U is called *M-projective* in case for each epimorphism $g : M_R \to N_R$ and each homomorphism $\gamma : U_R \to N_R$, there is an R-homomorphism $\overline{\gamma} : U_R \to M_R$ such that $\gamma = g\overline{\gamma}$. A module M_R is called *quasiprojective* if it is M-projective.

Let R be a ring. Recall that an element $a \in R$ is called π-regular if there exists a positive integer m such that $a^m = a^m b a^m$ [13] for some $b \in R$. An element $x \in R$ is called *generalized π-regular* if there exists a positive integer n such that $x^n = x^n y x$ for some $y \in R$. A ring R is called π-regular (resp., *generalized π-regular*) if every element in R is π-regular (resp., generalized π-regular). If A is a subset of R, then we say that A is *regular* if every element in A is regular.

Proposition 2.9. *Let M_R be quasiprojective with $S = \mathrm{End}(M_R)$. Then, S is regular if and only if M_R is AGQP-injective and $s(M)$ is M-projective for every $s \in S$.*

Proof. Assume that S is regular. Then, every right ideal of S is a direct summand of S_S, and so every homomorphism from a principal right ideal of S to S can be extended to an endomorphism of S. Hence, S is right P-injective and then right AGP-injective. By Theorem 2.2, M_R is AGQP-injective. The regularity of S also implies that $s(M)$ is a direct summand of M by [14, Theorem 37.7]. But M is quasiprojective, so $s(M)$ is M-projective for every $s \in S$.

Conversely, suppose M_R is AGQP-injective and $s(M)$ is M-projective for every $s \in S$. Then for any $0 \neq a \in S$, by the AGQP-injectivity of M_R, there exist a positive integer n and a left ideal X_{a^n} of S such that $a^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(a^n)) = Sa^n \oplus X_{a^n}$. Since $a^n M$ is M-projective, $\mathrm{Ker}(a^n) = eM$ for some $e^2 = e \in S$. Then, we have $S(1-e) = \mathbf{l}_S(eM) = \mathbf{l}_S(\mathrm{Ker}(a^n)) = Sa^n \oplus X_{a^n}$, and so $1-e = ba^n + x$ for some $b \in S$ and $x \in X_{a^n}$. Thus, $a^n = a^n(1-e) = a^n b a^n + a^n x = a^n b a^n$. This proves that S is π-regular and hence generalized π-regular. Clearly, $N_1(S) = \{0 \neq a \in S \mid a^2 = 0\}$ is regular (in this case, n must be equal to 1). Therefore or, S is regular by [13, Theorem 2.2]. □

Recall that a module M_R is called an *IN-module* [15] if $\mathbf{l}_S(A \cap B) = \mathbf{l}_S(A) + \mathbf{l}_S(B)$ for any submodules A and B of M, where $S = \mathrm{End}(M_R)$.

Proposition 2.10. *Let M_R be an AGQP-injective IN-module with $S = \mathrm{End}(M_R)$. Then, S is regular if and only if $W(S) = 0$.*

Proof. By Theorem 2.3, we need only to prove the sufficiency. Let $0 \neq a \in S$. Since M_R is AGQP-injective, there exist a positive integer n and a left ideal X_{a^n} of S such that $a^n \neq 0$ and $\mathbf{l}_S(\mathrm{Ker}(a^n)) = Sa^n \oplus X_{a^n}$. Since $W(S) = 0$, $\mathrm{Ker}(a^n)$ is not essential in M and then there exists a nonzero submodule K such that $\mathrm{Ker}\,(a^n) \oplus K$ is essential in M. Moveover, we also have

$$\begin{aligned} \mathbf{l}_S(\mathrm{Ker}(a^n)) + \mathbf{l}_S(K) &= \mathbf{l}_S(\mathrm{Ker}(a^n) \cap K) = S, \\ \mathbf{l}_S(\mathrm{Ker}(a^n)) \cap \mathbf{l}_S(K) &\subseteq \mathbf{l}_S(\mathrm{Ker}(a^n) + K) = 0, \end{aligned} \tag{2.7}$$

because M_R is an IN-module and $W(S) = 0$. Thus,

$$S = \mathbf{l}_S(\mathrm{Ker}(a^n)) \oplus \mathbf{l}_S(K) = Sa^n \oplus X_{a^n} \oplus \mathbf{l}_S(K). \tag{2.8}$$

Let $1 = ba^n + x$ with $b \in S$, $x \in X_{a^n} \oplus \mathbf{l}_S(K)$, then $a^n = a^n b a^n$. It follows that S is regular by the last part of the proof of Proposition 2.9. □

Lemma 2.11. *Let M_R be an AGQP-injective module in which every nonzero submodule contains a nonzero M-cyclic submodule and $S = \operatorname{End}(M_R)$. If $s \notin W(S)$, then the inclusion $\operatorname{Ker}(s) \subseteq \operatorname{Ker}(s - sts)$ is strict for some $t \in S$.*

Proof. If $s \notin W(S)$, then $\operatorname{Ker}(s) \cap K = 0$ for some nonzero submodule K of M, and so $\operatorname{Ker}(s) \cap s'(M) = 0$ for some $0 \neq s' \in S$ by hypothesis. Clearly, $ss' \neq 0$. Since M_R is AGQP-injective, there exist a positive integer n and a left ideal $X_{(ss')^n}$ such that $(ss')^n \neq 0$ and $\mathbf{l}_S(\operatorname{Ker}(ss')^n) = S(ss')^n \oplus X_{(ss')^n}$. Thus,

$$s'(ss')^{n-1} \in \mathbf{l}_S(\operatorname{Ker}(s'(ss')^{n-1}) = \mathbf{l}_S(\operatorname{Ker}(ss')^n) = S(ss')^n \oplus X_{(ss')^n}. \tag{2.9}$$

Write $s'(ss')^{n-1} = t(ss')^n + x$, where $t \in S$ and $x \in X_{(ss')^n}$, then $(1 - ts)s'(ss')^{n-1} = x$ and hence

$$(1 - st)(ss')^n = (s - sts)s'(ss')^{n-1} = sx \in S(ss')^n \cap X_{(ss')^n}. \tag{2.10}$$

This means that $(s - sts)s'(ss')^{n-1} = 0$. It is obvious that Ker $(s) \subseteq \operatorname{Ker}(s - sts)$. Note that $s'(ss')^{n-1}M$ is contained in Ker $(s - sts)$ but not contained in $\operatorname{Ker}(s)$, the inclusion Ker $(s) \subseteq$ Ker $(s - sts)$ is strict. □

Theorem 2.12. *Let M_R be AGQP-injective with $S = \operatorname{End}(M_R)$. If every nonzero submodule of M contains a nonzero M-cyclic submodule, then the following conditions are equivalent:*

(1) *S is right perfect;*

(2) *for any sequence $\{s_1, s_2, \ldots\} \subseteq S$, the chain $\operatorname{Ker}(s_1) \subseteq \operatorname{Ker}(s_2 s_1) \subseteq \cdots$ terminates.*

Proof. By Theorem 2.3, Lemma 2.11, and [16, Lemma 2.8], one can complete the proof in a similar way to that of [16, Theorem 2.9]. □

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