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A study of non-steady groundwater flow with special reference to a reservoir-coefficient

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Summary: The discharge from a relatively flat alluvial or diluvial area where no surface run off occurs can be attributed to the outflow of groundwater into the channels of a drainage system. This outflow decreases the reservoir of phreatic groundwater which is replenished by vertical percolation following precipitation. For a deep homogeneous soil this non-steady flow can be computed with the aid of two new tools:

(1) a reservoir-coefficient, which comprises the soil factors and the distance between the drainage channels; (2) a dimensionless diagram showing the increase of groundwater outflow during a steady vertical percolation into the saturated zone. Similar diagrams can be made for the reaction of the groundwater storage and the height of the groundwater table halfway between the outflow channels.

The results of the method presented demonstrate the errors that may be introduced if groundwater flow is considered as a steady state problem.

Introduction

In 1942 *Hursh* and *Brater* [1] pointed out that a considerable contribution to stormflow may consist of seepage resulting from the build up of the groundwatertable along streamchannels after storm rainfall. They also suggested that contributions to storm-flow from various sources might be related to time in some orderly fashion. They felt that these timefactors might become valuable tools for the study of storm-flow.

Prevailing conditions in the Netherlands seem to indicate that the following up of this clue could lead to a better appreciation of discharge from various outflow-channels. In flat regions with deep alluvial or diluvial soils surface runoff will seldom occur. Precipitation on to the open water surface cannot accumulate into a flood-wave of any importance. Further open channels in most cases only occupy a small percentage of the total area. Therefore channel precipitation may also be left out of consideration and the discharge from the area can be attributed almost exclusively to groundwater outflow into the drainage system.

In hydrological terms this groundwater outflow is to be considered as channel-inflow, occurring simultaneously along all component channels of the drainage system. It is evident that the hydrograph of channel-inflow cannot always be equated to the hydrograph of discharge at the outflow-point of the channel system. The degree of deformation which lies between the former and the latter is determined by the nature of the drainage system. Good judgement is required when the results of channel-inflow studies are to be used for hydrograph analysis. This and other difficulties, inherent to practical situations, will not be discussed in this paper, which will be restricted to the study of the outflow of groundwater.

Groundwater outflow stands for the depletion of the phreatic groundwater reservoir. This depletion will be counteracted by vertical percolation of water into the saturated zone. This percolation consists of the remaining part of precipitation after the soil moisture content has been filled to field capacity. The phreatic reservoir may also be depleted by removal of moisture from the capillary zone either by plant roots or by direct evaporation. This evapo-transpiration may be accounted for as a negative percolation against the positive percolation due to precipitation.

Since the rate of percolation into the saturated zone varies with time, the storage of groundwater will not be constant. Consequently the rate of outflow will deviate from the rate of percolation. Thus the passage of water through the soil is a non-steady process.

In this non-steady flow, the buffering action of the phreatic reservoir between percolation and groundwater outflow will depend on the characteristics of the soil, and the nature and density of the artificial or natural drainage system. The following will show that these factors, which constitute the drainage situation, can be incorporated in one reservoir-coefficient, which is the key to the relation between percolation and outflow.

Mathematical Analysis

For the analysis of the non-steady flow of groundwater in an idealized two-dimensional situation, the following notations will be used:

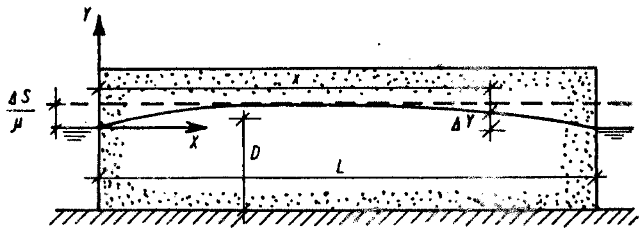

Fig. 1. Two-dimensional flow in a homogeneous soil.

t = time,
b = duration of steady percolation,
x = horizontal distance to origin,
y = elevation of the groundwatertable with respect to the constant level in the outflow-channels,
k = hydraulic conductivity as influenced by the permeability of the soil and the viscosity of the groundwater,
D = mean depth of the impermeable layer below the groundwatertable,
μ = volume-fraction of pores drained at a falling watertable, which is assumed to be equal to the volume-fraction of pores filled at a rising watertable,
L = distance between two outflow-channels,
q_x = rate of groundwater flow per unit length of outflow-channel passing through a vertical plane at a distance x from the origin,
q = rate of groundwater flow from two sides into a unit length of channel,
p = rate of percolation to the saturated zone,
S = total quantity of percolation,
R = storage of phreatic groundwater,
j = reservoir-coefficient,
T_o = beginning of tail-recession,
T_o-b = time interval between the cessation of percolation and the beginning of tail-recession.

Fig. 1 represents an idealized soil profile which is both homogeneous and isotropic. It rests on a horizontal impermeable layer. The profile is bordered by two outflow-channels, which reach down to the impermeable layer.

The calculations are based on the following assumptions:

1. The volume-fraction of pores drained μ and the hydraulic conductivity k are considered as constants.
2. That of Dupuit, stating that vertical flow can be neglected with respect to horizontal flow and the same hydraulic gradient exists over the whole depth of a vertical.
3. The elevation y of the groundwater table is small with respect to the depth of groundwater flow D, so that D can be considered as a constant.

A. Instantaneous addition to the saturated zone

Based on Dupuit-Darcy and the equation of continuity, the following general equation can be derived for the falling watertable if no replenishment by vertical percolation to the saturated zone occurs:

$$\frac{\partial y}{\partial t} = \frac{KD}{\mu}\frac{\partial^2 y}{\partial x^2} \qquad (1)$$

The initial- and boundary conditions are:

$y = \frac{S}{\mu}$ for $0 < x < L$ and $t = 0$

$y = 0$ for $x = 0$ and $t > 0$

$y = 0$ for $x = L$ and $t > 0$

These conditions indicate that the groundwater flow results from an instantaneous addition S to the saturated zone. This addition causes a sudden rise $\frac{S}{\mu}$ of the groundwater table with respect to the watertable in the outflow-channels. The water in the outflow-channels is maintained at the same level.

The solution of equation (1) in accordance with these initial and boundary conditions can be obtained in analogy with one-dimensional heat flow [2]

$$y = \frac{S}{\mu}\frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{n=\infty}\frac{1}{n}e^{-n^2\pi^2\frac{KD}{\mu L^2}t}\sin\frac{n\pi x}{L} \qquad (2)$$

This expression was used by Glover and presented by Dumm in relation to drainage problems [3]. It can be simplified by introducing

$$j = \frac{1}{\pi^2}\frac{\mu L^2}{KD}. \qquad (3)$$

This coefficient will be called the *reservoir-coefficient* because it determines the amount of phreatic groundwater that will be stored if steady percolation continues infinitely, as will be shown later (Fig. 2 — D).

According to Dupuit-Darcy the rate of outflow from two sides into an outflow-channel is:

$$q = -2\,KD\left[\frac{\partial y}{\partial x}\right]_{x=L} = \frac{8\,KD}{\mu L}S\sum_{n=1,3,5,\ldots}^{n=\infty}e^{-n^2\frac{t}{j}} \qquad (4)$$

B. Steady percolation

The next step is to consider vertical percolation into the saturated zone at a constant rate p, and look upon this steady percolation as a succession of infinitesimal instantaneous additions at infinitesimal time intervals. After substitution of S bij pdt' and t by $t - t'$ in eq. (4) the following relation

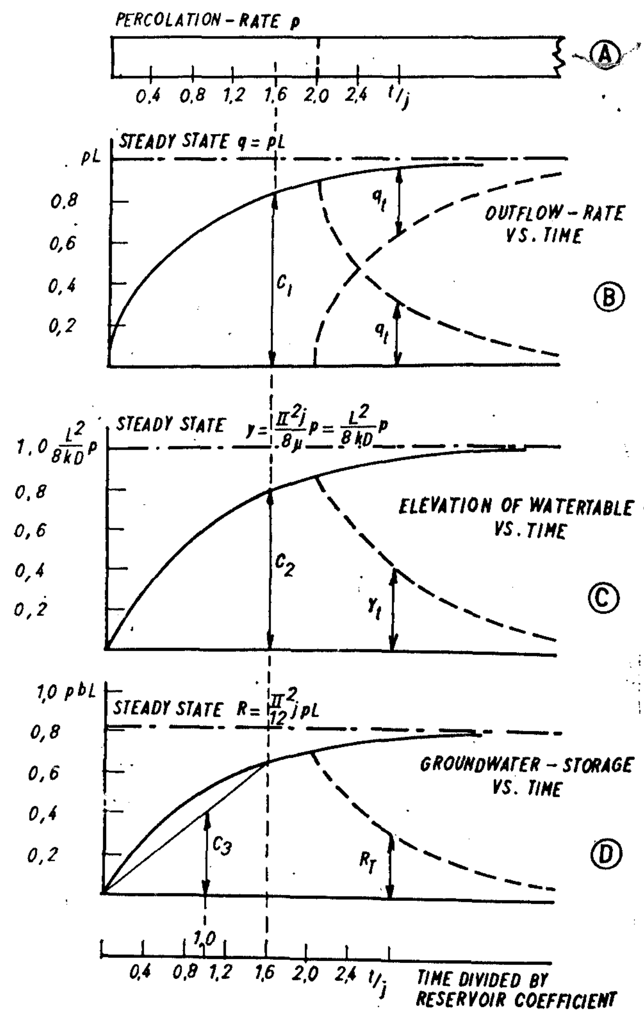

Fig. 2. Groundwater reacting upon steady percolation.

can be found by integration between the limits $t' = 0$ and $t' = b (\leqslant t)$:

$$q = \frac{8}{\pi^2} pL \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^2} \left\{ e^{n^2 \frac{b}{j}} - 1 \right\} e^{-n^2 \frac{t}{j}} \qquad (5)$$

In a similar way the groundwater level halfway between the channels (Fig. 1) is found to be:

$$y = \frac{4}{\pi} \frac{p}{\mu} j \sum_{n=1,-3,5,\ldots}^{n=\infty} \frac{1}{n^3} \left\{ e^{n^2 \frac{b}{j}} - 1 \right\} e^{-n^2 \frac{t}{j}} \qquad (6)$$

At the end of a period of constant percolation, $t = b$ and eq. (5) changes into:

$$q_b = \frac{8}{\pi^2} pL \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^2} \left\{ 1 - e^{-n^2 \frac{b}{j}} \right\},$$

since $\sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$, it follows that:

$$q_b = pL \left\{ 1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^2} e^{-n^2 \frac{b}{j}} \right\}. \qquad (7)$$

In a similar manner eq. (6) changes into:

$$y_b = p \frac{\pi^2}{8} \frac{j}{\mu} \left\{ 1 - \frac{32}{\pi^3} \sum_{n=1,-3,5,\ldots}^{n=\infty} \frac{1}{n^3} e^{-n^2 \frac{b}{j}} \right\} \qquad (8)$$

Each of the values of the expressions between brackets in eq. (1) and eq. (8) will become unity when steady percolation continues infinitely and consequently the steady state prevails (Fig. 2). These expressions may be looked upon as correcting factors for the non-steady state. They are only dependent on the ratio between the duration of steady percolation b and the reservoir-coefficient j. The equations (7) and (8) are consequently reduced to the following expressions:

$$q_b = c_1 pL \qquad (9)$$

and

$$y_b = c_2 p \frac{\pi^2}{8} \frac{j}{\mu} \qquad (10)$$

In order to evaluate the buffering action of the phreatic reservoir between the percolation at a constant rate p, and the rate of outflow q, the total outflow up to a

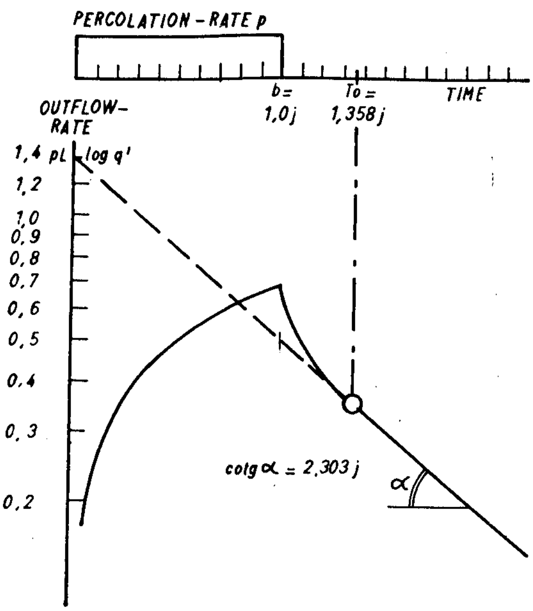

Fig. 3. The depletion curve merges into tail-recession.

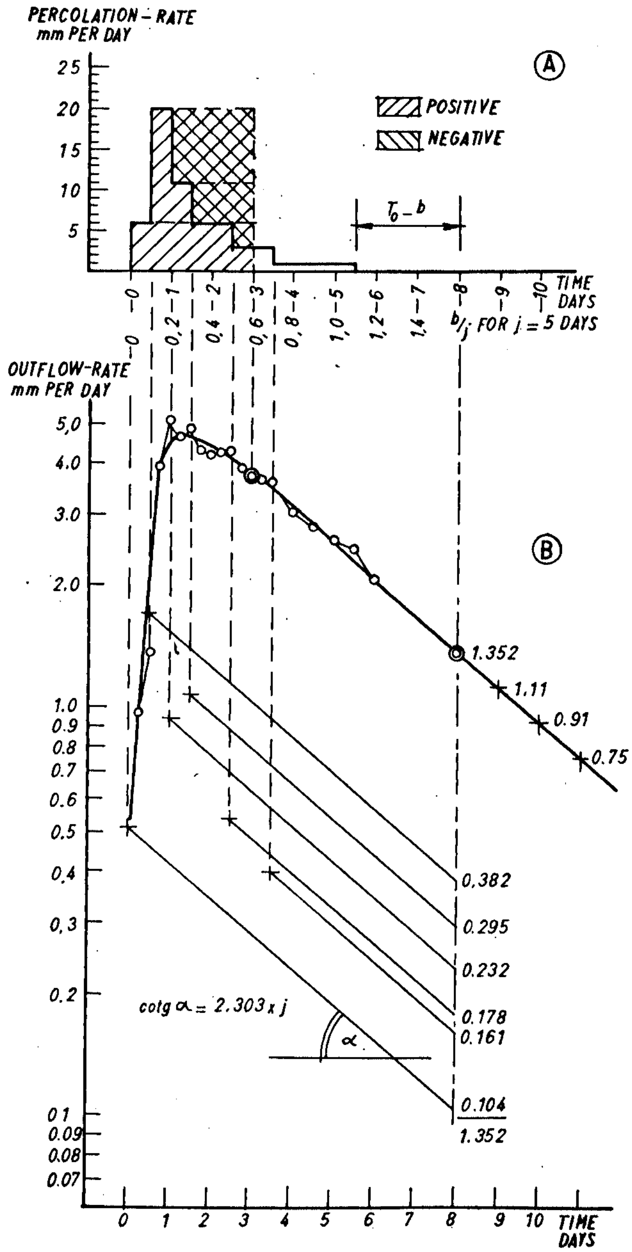

Fig. 4. Computed hydrograph of groundwater-outflow.

time $t = b$ will now be expressed in the rate of percolation p, its duration b and the reservoir-coefficient j:

$$Q = \int_0^b q_b \, db.$$

Substitution of q_b by the right hand member of eq. (7) and integration lead to the following expression:

$$Q_b = pLb \left[1 - \frac{j}{b} \left\{ \frac{\pi^2}{12} - \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^4} e^{-n^2 \frac{b}{j}} \right\} \right].$$

The storage R_b in the phreatic reservoir is the total percolation pLb minus the total outflow Q_b up to $t = b$.

Therefore:

$$R_b = pLb \frac{j}{b} \left\{ \frac{\pi^2}{12} - \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{n=\infty} \frac{1}{n^4} e^{-n^2 \frac{b}{j}} \right\} \qquad (11)$$

or:

$$R_b = c_3 pLb. \qquad (12)$$

The dimensionless diagrams in Fig. 2 represent the values of the factors c_1, c_2 and c_3 in relation to the ratio

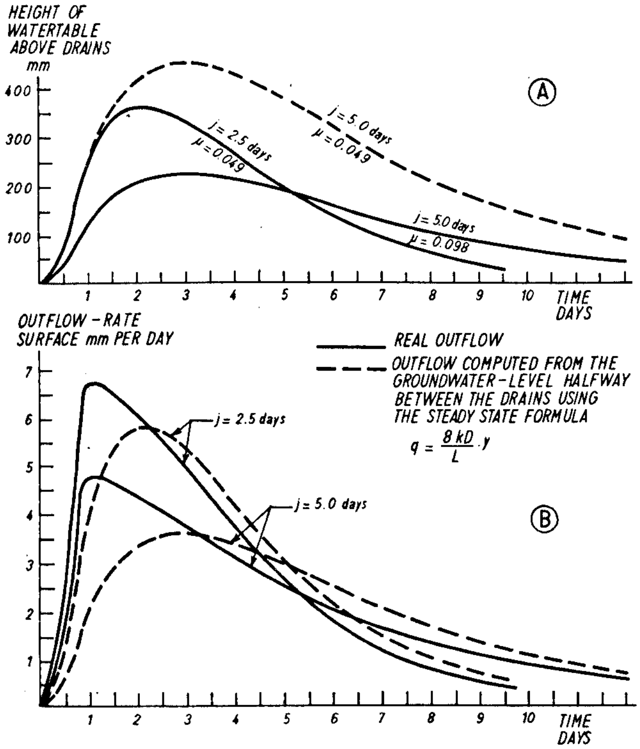

Fig. 5. Error introduced by considering non-steady groundwater-flow as a succession of steady states.

b/j. For practical purposes these relations can also be given in tabular form.

It is now possible to calculate the rate of outflow q and the groundwater storage R at any moment $t = b$ during a continuous percolation with a constant rate p, if only the reservoir-coefficient, which characterizes the drainage situation, is known. It appears that q and R are independent of the internal structure of the reservoir-coefficient

$$j = \frac{1}{\pi^2} \frac{\mu L^2}{KD}.$$

However the waterlevel halfway between the channels depends on the mutual relation of the factors within the reservoir-coefficient (Fig. 5-A). In order to determine this elevation y the soilfactor μ, the volume-fraction of pores drained, must also be known.

C. *Non-steady percolation*

Up to this point the problem of non-steady flow has been solved for the simple and still theoretical case of a steady percolation. In practical situations this percolation will be far from steady since it may follow any time distribution. Edelman [4] and more recently Werner [5] have pointed out that the principle of superposition can be used for breaking down a complicated time distribution of percolation into a number of steady percolations starting at different times. Fig. 2-B shows the application of this method in the simple case, where the steady percolation stops at the end of a duration $b = 2\,j$. Fig. 4-A shows how an arbitrary time distribution has been approximated by a succession of intervals with constant percolation rates, thus forming a block diagram. If one wants to determine the rate of outflow, the groundwater level and the groundwater storage at a time $t = T$, the block-diagram should be considered as the outcome of a number of additions and substractions of periods of constant percolation rates, which continue up to $t = T$. All these component steady percolations may be handled with the equations [9], [10] and [12]. The factors c_1, c_2 and c_3 can be derived from Fig. 2 or from tables giving the relation between these factors and b/j.

Theoretically the problem of non-steady groundwater flow, linking percolation to outflow, has now been solved, considering of course the simplifying assumptions stipulated in the introduction. It is however evident from fig. 4-A that practical difficulties arise as the depletion of the groundwater storage continues. The rate of outflow, the storage, and the groundwater level are then to be calculated as the decreasing difference between increasing quantities. (See Fig. 4-A). As depletion proceeds it will become increasingly difficult to obtain results of sufficient accuracy. For this reason it was found to be necessary to introduce another concept, which will be called "tail-recession".

D. *Tail-recession*

In the series in the right hand member of eq. [5] the sum of all terms will approximate the value of the first term, as t increases, and eq. [5] merges into:

$$q_t = \frac{8}{\pi^2} pL \left(e^{b/j} - 1\right) e^{-t/j} \qquad (13)$$

In a similar manner eq. (6) merges into

$$y_t = \frac{4}{\pi} p \frac{j}{\mu} \left(e^{b/j} - 1\right) e^{-t/j}. \qquad (14$$

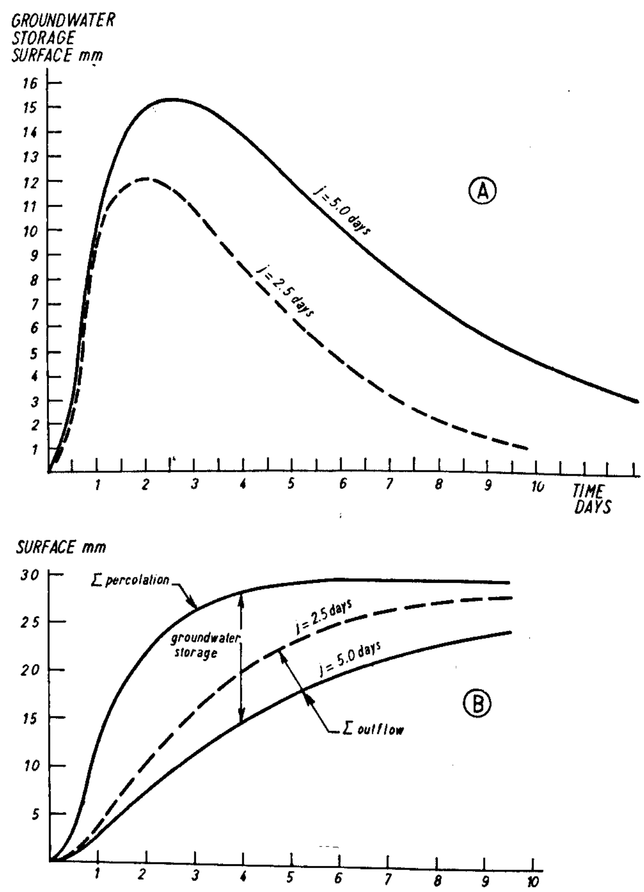

Fig. 6. The reservoir coefficient determines the buffering action of groundwater storage between percolation and outflow.

Through integration of eq. (13) one gets:

$$R_t = -\int_t^{\infty} q_t dt = \frac{8}{\pi^2}\ pL\,(e^{b/j} - 1)\, e^{-t/j} \times j. \qquad (15)$$

The above equations (13), (14) and (15) represent tail-recessions of respectively the q-, y- and R-lines.

For practical purposes it is assumed that tail-recession has started as soon as the second term in a series becomes smaller than one percent of the first term. In other words it is assumed that tail-recession of the q-line has begun as soon as

$$\frac{eq^{b/j} - 1}{e^{b/j} - 1} \cdot \frac{1}{9}\, e^{-8t/j} = 0{,}01 \text{ or } e^{-8t/j} = 0{,}09\, \frac{e^{b/j} - 1}{e^{9b/j} - 1}.$$

The value of t/j which satisfies this condition will be indicated by To/j. This value is determined by the corresponding ratio b/j. For practical purposes it was found to be useful to express the relation between b/j and $\frac{To - b}{j}$ in tabular form.

An interesting feature of tail-recession appears if the logarithms of q, y or R are plotted against corresponding values of the time. It follows from the equations (13), (14) and (15) that these relations will appear as straight lines on semi-log-paper with inclinations typified by:

$$\operatorname{tg}\alpha = \frac{1}{2.303\, j}. \qquad (16)$$

Another interesting feature of tail-recession is that definite ratios exist between R, q and y. From the equations (13) and (15) follows:

$$R_t = j \times q_t \qquad (17)$$

and from eq. (13) and (14) follows:

$$\frac{y}{q} = \frac{\pi}{2}\frac{j}{\mu L} = \frac{1}{2\pi}\frac{L}{KD}. \qquad (18)$$

As soon as one point of the q-tail-recession has been determined, corresponding values of R and y follow immediately if the ratios (17) and (18) are used. The three tail-recessions can be drawn as straight lines on semi-log-paper with the same inclination as given by eq. (16).

This straight line also supplies the key for the determination of one point of the q-tail-recession. If this line is extended towards the log q-axis, the point of intersection will mark $t = 0$ in eq. (13) and:

$$q' = \frac{8}{\pi^2}\,(e^{b/j} - 1)\, pL \text{ or } q' = c_4 pL. \qquad (19)$$

Again this factor c_4 only depends on the ratio between the duration b of steady percolation and the reservoir coefficient j. The relation between c_4 and b/j can be expressed in tabular form.

In a case of steady percolation with a duration b the ratio b/j is determined and c_4 can be found in the appropriate table. Consequently q' is known from eq. (19), giving the point on the log q-axis through which the q-tail-recession can be drawn as a straight line with an inclination according to eq. (16). The real part of this tail-recession begins at a point T_o, which can be determined in the relation between $\frac{T_o - b}{j}$ and b/j.

In case of a non-steady percolation this procedure can be followed for each block of the blockdiagram which approximates the time distribution of percolation. Each block contributes by its own tail-recession to the total tail recession. The latter can be considered to have started after an interval of (T_o-b) has expired since the end of the last block of steady percolation.

E. Determination of the reservoir-coefficient

The results of the above mathematical analysis with respect to the reservoir-coefficient can be summarized as follows:

Steady state ($b = \infty$)	Tail-recession ($t > T_o$)
$pL = q$	$pL = 0$
substituting $b = \infty$ in eq. (8):	
$y = \frac{\pi^2}{8}\frac{j}{\mu} \cdot p = \frac{L}{8\,KD} \cdot q \quad (20)$	$y = \frac{L}{2\pi KD} \cdot q \quad (18)$
substituting $b = \infty$ in eq. (11):	semi log plotting: $\operatorname{tg}\alpha = \frac{1}{2.303\, j} \quad (16)$
$R = \frac{\pi^2}{12} j \cdot q \quad (21)$	$R = j \cdot q \quad (17)$
combination with eq. (20) leads to:	combination with eq. (18) leads to:
$R = \frac{2}{3}\mu y L \quad (22)$	$R = \frac{2}{\pi}\mu y L$
(parabolic watertable)	(sinusoidal watertable)

The significance of the reservoir-coefficient is particularly conspicuous in the relations [17] and [21]. In these relations the reservoir-coefficient solely determines the definite ratio between the groundwater storage R and the groundwater outflow q. It follows that in an actual drainage situation the reservoir-coefficient j can be determined if either the seady state or the tail-recession is sufficiently approximated.

1. Edelman [4] and Werner [5] have called attention to the fact that the steady state may be approximated by the long-term average of groundwater-flow. In an actual situation this approximation will only be practicable if the long-term average is more important than the oscillations caused by non-steady influences. In other words, if the ratio between the groundwater storage and the outflow is sufficiently high, the reservoir-coefficient can be determined with the aid of eq. [21]. Such conditions sometimes prevail in diluvial soils, occurring in many drainage-areas of small rivers in the Netherlands. For example if L = 3000 m, KD = 500 m² per day and μ = 0,20, the reservoir-coefficient is j = 1460 days = 4 years. If percolation caused by winter rains lasts for ½ year, then b/j = 0,125. From the appropriate table follows T_o-b = 0.52 j = 2 years.

It follows that under such conditions tail-recession will never occur, but on the other hand it will be possible to determine the average values for outflow and groundwater storage over a period of several years.

2. If, the ratio between the groundwater storage and the outflow is low, in other words if the reservoir-coefficient is small, the long-term average will be insignificant in comparison with oscillations caused by non-steady influences. Hellinga [6] presented a method of calculating the ratio between the daily discharge from a polder area and the quantity of water yet to be discharged. Later de Zeeuw and Hellinga [7] found that this ratio is not a constant during a rainy period. It appeared to be relatively

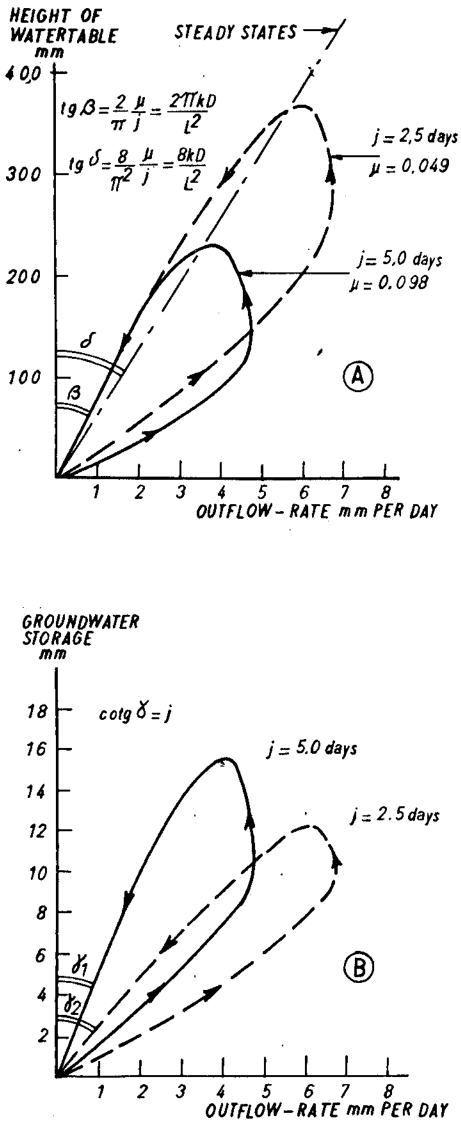

Fig. 7. Only during tail-recession there are fixed relations between the rate of outflow, the height of the watertable and the groundwater storage.

high for high values of the discharge and small for low values of the discharge during periods of depletion.

The present treatment of non-steady flow leads to the same conclusion; a constant ratio between storage and outflow rate will be approximated in a period of depletion (Fig. 7-B).

3. For detail-drainage, involving tile drains and ditches, direct methods may be used: When the hydrograph of outflow is plotted on semi-log paper the depletion curves in dry periods will merge into straight lines. Fig. 4-B shows how the reservoir-coefficient may be derived from the inclination of tail-recession. The same rule applies to the elevation of the watertable halfway between the outflow channels, when this elevation is plotted against time on semi-log paper.

Application of theory

It is felt that the application of the present theory of non-steady flow to a practical drainage situation is the simplest way of showing the insight to be gained by its use in the study of groundwater flow.

The following drainage situation closely approximates the idealized situation treated in the above mathematical analysis:

Tile drains in a soil profile with a permeability increasing with depth.

It is assumed that the drains have been placed at a depth of 1 meter, just below a top layer of clay loam, in the sandy subsoil (a typical soil profile for the Netherlands).

Drain spacing $L = 32$ m. Hydraulic conductivity of subsoil $k = 1$ m per day. Average depth of flow towards the drains $D = 2$ m. It is assumed that this value of D is an equivalent depth derived from Hooghoudt's tables [8] so that radial resistance has been accounted for, and Dupuit's assumption of horizontal flow can be maintained.

In the Netherlands many tile drainage systems in arable land have been developed on the criterium that 7,5 surface-millimeters per day should be removed by the drains when the watertable halfway between the drains is 0,50 m below the surface of the land and the steady state prevails.

Introducing $NL = q$ in eq. (20) it follows that:

$$y = \frac{L^2}{8\,KD} \cdot N = \frac{32^2 \times 0.0075}{8 \times 1 \times 2} = 0.48 \text{ m.}$$

According to the usual standards it may be concluded that the drainage is satisfactory for arable land.

Now the theory of non-steady flow will be applied to find the relation between actual percolation and the resulting flow into the drains, the waterlevel halfway between the drains and the storage of groundwater. Since the storage of groundwater is taken into account, the study of non-steady flow requires one more soil factor, μ the volume fraction of pores drained. In a homogeneous profile this μ, will be strongly correlated to the hydraulic conductivity k. This, however, is not necessarily true for all types of soil profiles. It should be born in mind that the watertable moves up and down in the soil layer above the plane through the drains. The shape of the watertable may be considered as the motive power for the actual flow to the drains, which, in the present example, takes place through the permeable subsoil.

While the data used for the above steady state computation are maintained, μ may have different values without affecting the results of the above computation. It is however evident that the reaction of the groundwater table upon replenishment by vertical percolation will largely depend on the volume fraction of pores drained, which is also called the storage-coefficient. Consequently the storage and the outflow of groundwater caused by the rise of the watertable will also depend on μ. The influence of μ on groundwater flow will be illustrated by determining the q-, y- and R-lines respectively for $\mu_1 =$ 0,098 and $\mu_2 =$ 0,049.

In Fig. 4-A the time distribution of a total of 29.5 millimeters of percolation is approximated by a block diagram. The q-, y- and R-lines in the figures 5 and 6 have been computed from this percolation diagram using the equations (9), (10), (12) and (19) for each percolation increment. The factors c_1, c_2, c_3 and c_4 have been derived from the appropriate tables, which, for the sake of brevity, have not been reproduced in this paper.

It has already been explained that a certain time distribution of percolation causes q- and R-lines that are only dependent upon the value of j, the reservoir coefficient, through the factors c_1 and c_3 in the equations (9) and (12). These values are respectively $j_1 = 5$ days ($\mu_1 = 0{,}098$) and $j_2 = 2{,}5$ days ($\mu_2 = 0{,}049$). The elevation of the groundwater table

$$y = c_2 \frac{8}{\pi^2} \frac{j}{\mu} p$$

moreover depends upon the ratio $\frac{j}{\mu}$. In the present example, where j changes proportionally to μ, this ratio is constant and the elevation of the watertable also depends only upon the value of the reservoir-coefficient. The effect of the ratio $\frac{j}{\mu}$ is illustrated in Fig. 5 — A by the y-line corresponding with $j = 5$ days and $\mu = 0{,}049$.

These figures give rise to the following observations:

1. From his field experiments Wesseling [9] concluded that the rate of outflow from the drains increased rapidly after heavy rainfall, whereas the rise of the watertable halfway between the drains occurred relatively slowly. The figures 2-B and 2-C show that the non-steady charactor of groundwater-flow after heavy rainfall necessarily entails this phenomenon. The figures 5-A and 5-B confirm this statement.

Fig. 7-A shows that the line expressing the relation between the outflow and the height of the watertable halfway between the outflow-channels is not a single line but a loop. Small values of the reservoir-coefficient cause the loop to stretch. High rates of percolation tend to enlarge it; consequently the plotting of corresponding values for q and y will normally result in a group of widely scattered points. The side of this group facing the y-axis will be formed by points representing the depletion of groundwater storage during dry periods. This is caused by the fact that all loops, whatever their shape may be, will merge into one straight line during the period of recession. This line represents the fixed relation (18) between q and y during tail-recession. By drawing a tangent to the cloud of points on the side of the y-axis this relation can be determined. It should be noticed that this relation differs from the relation (20) which is based on the steady state. Since it is independent of μ, the transmissibility of the soil can be computed from

$$KD = \frac{L^2}{2\pi} \operatorname{tg} \beta.$$

2. Since there is no constant ratio between the rate of outflow and the height of the watertable, the former cannot be computed from the latter as long as tail-recession does not occur (Fig. 5). Another exception may be made for those cases in which unsteady influences only cause minor oscillations around a long time average approximating the steady state.

3. The concept of a definite ratio between a depleting groundwater storage and its rate of depletion is rather widespread, but it is only approximate in most cases. Werner and Sundquist [10] have proved that it may apply to artesian aquifers, but it does not hold good under free watertable conditions. It follows from the present study that the ratio between groundwater storage and rate of outflow will only be constant and equal to the reservoir-coefficient as soon as tail recession has begun (Fig. 7-B).

Applicability of the method developed in the present treatment

In the present treatment a mathematical analysis has been applied to a drainage situation typified by the assumptions set forth in the beginning of this paper. Therefore the results of this analysis may be used for determining the groundwater flow in any drainage situation that conforms to these assumptions. Actual conditions may, however, deviate significantly from these basic assumptions. For instance, the impermeable layer may be so shallow that the depth of horizontal flow cannot be considered as a constant. Then the drainage situation will be fundamentally different. In this case it cannot be expected that accurate results will be obtained if the methods developed in this paper are simply applied.

Lindenbergh [11] presented a mathematical solution for the non-steady flow in this homogeneous isotropic profile resting on a shallow impermeable layer. He found that his iteration method was too laborious to be used for the solution of his infiltration problems in the dunes. If indeed a drainage situation is not open to a practicable mathematical analysis, model studies should be applied. It is suggested that the drainage situation treated in this paper should be the starting point for such model studies. By gradual changes of one boundary condition at a time, the effect on the results of the present method could be tested. Empirically determined corrections could be applied to the present method in order to enlarge its field of application.

It has already been pointed out that evapo-transpiration may be considered as negative percolation. In the same manner sub-irrigation may be considered as negative outflow. Therefore the present method is also applicable to problems of sub-irrigation.

Conclusive observations

Summarizing the results of this study on non-steady groundwater flow it may be stated that the problem of transformation of percolation into outflow has been solved for two-dimensional flow in a homogeneous profile on a deep impermeable layer. In the course of the mathematical analysis the following new tools for the study of groundwater flow have been introduced:

1. The reservoir-coefficient. All factors which determine the nature of the soil and the nature and density of the drainage network, are incorporated in this reservoir-coefficient.
   In any practical situation the reservoir-coefficient can be determined from:
   a. The hydrograph of groundwater outflow.
   b. The changes of the watertable.
   c. The ratio between the storage and the outflow of groundwater.

   This reservoir-coefficient determines the groundwater flow.

2. The dimensionless diagrams, which express the increase of outflow, the elevation of the watertable halfway between the outflow channels and the increase of the groundwater storage during a continuous steady percolation. For the sake of accuracy these diagrams can be supplemented by tables.

It is felt that the utilization of these new tools may open new possibilities for the study of hydrological problems varying from tile drainage and polder drainage to discharge from small drainage basins and management of groundwater reservoirs.

Acknowledgement

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KORTE TECHNISCHE BERICHTEN

Aanlegplaats voor schepen in diep water 627.231

Het Amerikaanse leger heeft onlangs in de mond van de Gironde, in Frankrijk, een aanlegplaats voor zeeschepen benevens een verbinding van 1 km lengte naar de oever in 17 dagen gebouwd. Hierbij werd uitsluitend gebruik gemaakt van geprefabriceerde elementen.

De constructie is ontwikkeld uit de eisen, dat een aanlegplaats voor zeeschepen in voldoende diep water moet zijn gelegen, terwijl de verbinding naar de oever op economische wijze tot stand gebracht moet worden. Door de aanlegplaats te ondersteunen door palen van grote diameter, waarvan dan een geringer aantal nodig zijn, wordt het slib- en zandtransport niet verstoord. Dit heeft geleid tot het gebruik van stalen- en betonnenpalen tot 3 m diameter, in buisvorm.

De aanlegplaats kan als bak worden gebouwd, en drijvende ter plaatse in voldoende diep water worden gebracht, waarna de buispalen neergelaten worden en het platform opgevijzeld wordt tot de gewenste hoogte (fig. 1). De bak kan als tijdelijke opslagplaats ingericht worden. De horizontale afmetingen van een sectie zijn 90 tot 120 m lang en 10 tot 45 m breed.

De aanlegplaats kan worden verbonden met de oever door een dijk, steiger of kabelbaan. De kabelbaan, welke de voornoemde aanlegplaats in de Gironde verbindt met de oever, is 1000 m lang en is opgehangen aan torens ter hoogte van 20 m. Het plaatsen van deze torens geschiedt op dezelfde wijze als van de aanlegplaats. De vervoerscapaciteit is 300 ton per uur. De trolley aan de kabelbaan wordt voortbewogen door een benzinemotor. Het platform, dat aan de trolley is opgehangen, is 3 × 6 m en kan aan de oever direct op vrachtauto of trein worden geplaatst.

Op deze wijze kunnen grote hoeveelheden vrachtgoederen in korte tijd worden verladen en de wachttijd van de schepen tot een minimum worden beperkt.

Behalve in de Gironde is deze constructie ook toegepast in Venezuela ten behoeve van ertstransport en in New York voor kolentransport.

T. H.

Civil Engineering, februari 1958, vol. 28 - No. 2.

Waterkracht-stations aan de Zambezi 621.311.2 1

In 1910 bezocht schrijver dezes, tijdens een studiereis naar Zuid-Afrika, de Victoria Watervallen van de Zambezirivier bij Livingstone, ten einde zich op de hoogte te

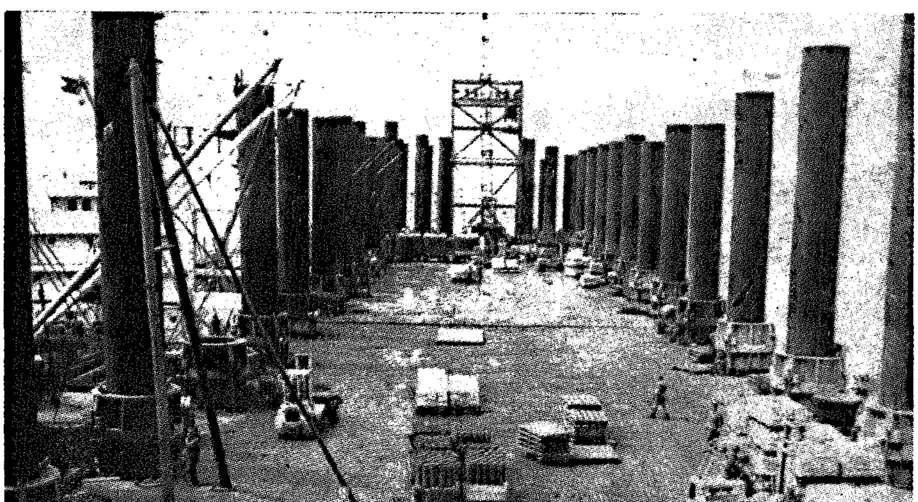

Fig. 1. Aanlegplaats in de Gironde.