Fault-tolerant Wide-area Control of Power Systems

Felix Rafael Segundo Sevilla

Department of Electrical and Electronic Engineering
Imperial College London

A thesis submitted for the degree of

Doctor of Philosophy

March 2013
Declaration of Originality

I, hereby declare that this thesis is the result of my own work, and that any ideas or quotations from the work of other people, published or otherwise, are appropriately referenced.

Felix Rafael Segundo Sevilla
Imperial College London
London, United Kingdom
March 27, 2013
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Abstract

In this thesis, the stability and performance of closed-loop systems following the loss of sensors or feedback signals (sensor faults) are studied. The objective is to guarantee stability in the face of sensor faults while optimising performance under nominal (no sensor fault) condition. One of the main contributions of this work is to deal effectively with the combinatorial binary nature of the problem when the number of sensors is large. Several fault-tolerant controller and observer architectures that are suitable for different applications are proposed and their effectiveness demonstrated. The problems are formulated in terms of the existence of feasible solutions to linear matrix inequalities. The formulations presented in this work are described in a general form and can be applied to a large class of systems. In particular, the use of fault-tolerant architectures for damping inter-area oscillations in power systems using wide-area signals has been demonstrated. As an extension of the proposed formulations, regional pole placement to enhance the damping of inter-area modes has been incorporated. The objective is to achieve specified damping ratios for the inter-area modes and maximise the closed-loop performance under nominal condition while guaranteeing stability for all possible combinations of sensors faults. The performances of the proposed fault-tolerant architectures are validated through extensive nonlinear simulations using a simplified equivalent model of the Nordic power system.
Dedicated to my beloved wife Judith and my dear family.

Dad, you were the best role model I could have,

God bless you.
Acknowledgements

I would like to show my gratitude to my supervisors: Dr Imad Jaimoukha for all his advices and his unconditional support in all aspects, Dr Balarko Chaudhuri for his illustrious guidance and valuable comments and to Dr Petr Korba who gave me the opportunity to work on this project and who has been my friend and guide since we met in 2007.

I would like to acknowledge the financial support provided by ABB under grant EESC P26939 and technical inputs from Dr Ernst Scholtz.

Thanks to all my colleagues in the control and power research group and at the college for the friendly and international atmosphere in which I worked. In particular, to my friends in the Mexican society that made my journey in London, one that I shall never forget. Raul, Diana, Hector, Marcelino and Ruben I am grateful for all the wonderful time we spent together.

I would like to thank specially to my dear wife Judith and express my love to her for all her support and company during the happiest and difficult times of this journey. She left everything she had in Mexico to follow me in London. Together we have shared this adventure.

Thank to all my family for their love and encouragement. Particularly to my loved parents Celia Sevilla and Rafael Segundo, to my brothers Ana and Enrique who have always been there for me and to my parents-in-law, brothers and sisters-in-law, and all my uncles and cousins in Mexico who had supported my career.

Finally, I would like to thank Dr Efrain Alcorta and Dr Rene Galindo from the UANL in Monterrey, who indirectly contributed to make this dream come true.
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CC</td>
<td>Conventional control</td>
</tr>
<tr>
<td>FACTS</td>
<td>Flexible alternating current transmission system</td>
</tr>
<tr>
<td>FT</td>
<td>Fault-tolerant</td>
</tr>
<tr>
<td>FTC</td>
<td>Fault-tolerant control</td>
</tr>
<tr>
<td>FTCit</td>
<td>Iterative fault-tolerant control</td>
</tr>
<tr>
<td>FTCp</td>
<td>Passive fault-tolerant control</td>
</tr>
<tr>
<td>FTMS</td>
<td>Fault-tolerant minimal switching</td>
</tr>
<tr>
<td>FTO</td>
<td>Fault-tolerant observer</td>
</tr>
<tr>
<td>FTP</td>
<td>Fault-tolerant passive</td>
</tr>
<tr>
<td>HVDC</td>
<td>High-voltage direct current</td>
</tr>
<tr>
<td>LMI</td>
<td>Linear matrix inequality</td>
</tr>
<tr>
<td>LPV</td>
<td>Linear parameter varying</td>
</tr>
<tr>
<td>LFR</td>
<td>Linear fractional representation</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear time-invariant</td>
</tr>
<tr>
<td>MC</td>
<td>Modal controllability</td>
</tr>
<tr>
<td>MO</td>
<td>Modal observability</td>
</tr>
<tr>
<td>NMS</td>
<td>Nominal minimal switching</td>
</tr>
<tr>
<td>NP</td>
<td>Nominal passive</td>
</tr>
<tr>
<td>POD</td>
<td>Power oscillation damping</td>
</tr>
<tr>
<td>PSS</td>
<td>Power system stabilizer</td>
</tr>
<tr>
<td>PMU</td>
<td>Phasor measurement unit</td>
</tr>
<tr>
<td>SVC</td>
<td>Static Var compensator</td>
</tr>
<tr>
<td>TCSC</td>
<td>Thyristor controlled series capacitor</td>
</tr>
<tr>
<td>WAMS</td>
<td>Wide-area measurement system</td>
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Chapter 1

Introduction

This chapter provides a motivation for the problem considered in this thesis, the structure of this document and the research questions that have been addressed. Section 1.1 presents the main issues in power systems that motivated this research. It describes how blackouts can suddenly occur, for instance, due to electromechanical oscillations. Then a summary of previous research work in this field is presented, emphasizing the fact that there exist only few effective techniques for dealing with the complete loss of feedback signals in the controllers. The section also highlights the fact that industry is reluctant to use wide-area signals to improve stability in the system due to the lack of effective fault-tolerant strategies. Section 1.2 presents an outline of the structure of the thesis. Finally, Section 1.3 provides a summary of the main research questions addressed in the thesis that will help to contribute to fault-tolerant strategies for the solution of the main problem.

1.1 Motivation

Electricity is an integral part of modern human life and is constantly delivered by the complex power systems of today. These systems occupy vast geographical areas and incorporate thousands of components: generating stations, transformers, transmission lines, power electronic devices, protections and controls, with the purpose of operating every day, year after year. Unfortunately, blackouts
may occur resulting in the disruption of the power supply to entire regions. Electromechanical oscillations between groups of generators limit the transfer capability between regions (along with transient instability and voltage instability) and gradually lead to widespread blackouts [6]. Reference [7] gives an exhaustive report on recent large power system blackouts worldwide.

A large number of works have provided solutions based on the design of power oscillation damping (POD) controllers to deal with the small-signal stability problem in power systems. These works have focused on different aspects such as: (i) the nature of the signal (i.e., wide-area vs. local) [8] and the type of signal [9] used in the controller, (ii) POD control acting through FACTS and HVDC links [10], (iii) signal latency in the remote feedback signals [11] and (iv) emphasis on robust low-order controllers [12]. However, none of these works have investigated in detail the effectiveness of the controllers (fault-tolerance) in the case of loss of feedback signals.

Over the last decade, many authors have argued that wide-area based control could potentially result in a number of benefits concerning system stability [13, 14]. Despite the substantial work at the design stage to illustrate the whole concept [15], no significant progress towards actual implementation has been achieved so far since system operators and reliability regulators are reluctant to implement these schemes [16]. The lack of maturity in the technology and effective fault-tolerant control strategies are the main obstacles to the implementation of stabilizing control strategies based on wide-area measurements [17].

The aim of this work is to propose design strategies that incorporate wide-area measurements and which ensure that POD controllers not only deal with small-signal stability issues but also maintain a minimum level of performance in adverse conditions following the loss of feedback signals in the controller, henceforth referred to as fault-tolerant controllers (FTCs).

1.2 Structure of the Thesis

The thesis is organized as follows.
1. Introduction

Chapter 1 sets the motivation of this work, presents the structure of the thesis and finally summarizes the research questions addressed.

Chapter 2 introduces the basic background to understand the main problem. The necessity of interconnections between different sub-networks in order to satisfy the load needs in a continuously growing power system is explained. It is made clear that stability problems arise due to oscillations caused by these interconnections, an illustrative example of an oscillation in the Nordic power system is presented. Then, the characteristics and benefits of using power systems stabilizers to enhance the damping in the system are discussed, a block diagram of a typical PSS structure is presented. The chapter then describes how the stability of the system can potentially be improved using wide-area signals through static var compensators, also a block diagram of these devices is provided. The use of wide-area signals in power systems is highlighted next. Here, the importance of signal selection in control design is emphasized. Strong reasons are given for the inadequacy of choosing only local signals when inter-area modes exist and the main features of wide-area signals that make them suitable for stabilizing control are explained. The concern of utilities about using these signals is raised and one figure describing the loss of wide-area signals is presented. This chapter also introduces the concept of fault-tolerant schemes and presents some of the approaches reported in this field, describes the use of controller and observer schemes to guard against sensor and actuator faults and summarizes other relevant work in the area. Finally, the notation that will be used is provided and a brief survey of linear matrix inequalities in robust control is presented since these will be the main tools used in the designs.

Chapter 3 describes the models that will be used to represent sensor faults or loss of signals for a family of plants, where it is assumed that one local and at least one remote signal are used. One sensor fault at a time is considered for simplicity but this assumption is generalized in the following chapter. A conventional controller design strategy is next introduced. This conventional design implements regional pole placement (conic sector) based on LMIs and is also designed to limit the control effort, a general block dia-
gram describing the problem is illustrated. The former is designed only for the fault-free (nominal) condition. A passive fault-tolerant control scheme is then proposed. The proposed controller considers the same problems as the conventional approach and, in addition, guarantees a minimum level of performance following the loss of one of the remote signals. The algorithm to calculate the parameters of this passive fault-tolerant controller is outlined at the end of the corresponding section. Due to the computational intractability of the first FT control proposed, an iterative procedure is then presented to reduce the conservatism and potentially improve the control objectives. The iterative algorithm to compute the final controller and the convergence criteria to stop the iterative procedure are presented in detail. The controllers described in this chapter are validated in the Nordic power system through exhaustive nonlinear simulations later in Chapter 5. Finally a summary of the content of this chapter is given.

Chapter 4 defines the concepts of Linear Parameter Varying (LPV) systems, quadratic stability, induced $\mathcal{L}_2$-norm and the bounded real lemma for LPV systems. In this chapter, the description of the system and the observer are presented and a block diagram of both systems is described. Here, the modelling of sensor faults considering all possible combinations is represented using a diagonal matrix, whose entries are either zero or one, at the output of the system and the output of the observer. Two types of observers are proposed: a passive scheme and, as an alternative to active schemes, a ”minimal switching” observer in which only one observer gain is designed, but where a simple switch is incorporated into the observer structure. Next, the derivations of conditions for the solution of both observers design problems for every faulty scenario, are given. When the number of sensors is large, the number of LMIs required to satisfy the conditions is large and therefore finding a solution can potentially be unfeasible due to the combinatorial nature of the problem. In this chapter a general robustness result is derived that provides sufficient conditions in the form of small number of LMIs for the solution of both problems. The final solution and the corresponding proofs are then presented. Also, an illustrative ex-
1. Introduction

ample using a reduced linear version of the Nordic power system is given. In this example, the tracking of the actual states using the proposed observers are compared against a traditional observer design. It is illustrated how the system can become unstable following the loss of wide-area signals. The minimal switching observer described in this chapter is validated in the Nordic power system through exhaustive nonlinear simulations later in Chapter 5. Finally the content of the chapter is summarized.

Chapter 5 presents case studies on a reduced equivalent of the Nordic electric power transmission system. These case studies illustrate the effectiveness of the proposed fault-tolerant control schemes (described in the previous chapters) in the context of electric power systems. Next, a detailed description of the Nordic power system that corresponds to a simplified version (thousand of states) of the original model is given. The reduced system is modelled in Matlab SIMULINK. A single line diagram of the system of the different areas, tie-lines and different devices locations is displayed. There are two poorly damped low frequency inter-area modes of oscillation which are to be damped through a static VAr compensator (SVC) located at Hasle near Oslo. Remote feedback signals were chosen for effective damping control. Tables describing the most appropriate wide-area and local signals are presented and a description of the signals chosen is provided. Section 5.3 validates and compare the performance of the controllers described in Chapter 3: one conventional and two fault-tolerant designs. The design procedures are described and in this section one local and one wide-area signals are chosen to improve the damping of the inter-area modes. The control objectives are defined: to achieve at least 10% of damping ratio in nominal conditions for all the controllers and at least 8% following the loss of the remote signal for the FTCs. Subsection 5.3.1 presents the dynamic performance under the nominal conditions. Here it is shown that although the controllers provide similar performance, both FTCs require more control effort. In Subsection 5.3.2 the dynamic performance following the loss of the remote signal is presented. The deterioration in the performance of the conventional control design is demonstrated, while for both FTCs the
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performance is found to be acceptable. In this chapter, also is validated and compared the performance of the fault-tolerant minimal switching observer described in the previous Chapter including regional pole placement, full details of the observer design are described. Finally, Section 5.6 presents a summary of the chapter.

Chapter 6 summarizes the work and contributions of this thesis and considers future research work.

1.3 Research Questions Addressed

Following the discussion in the previous section. In order to provide effective strategies that incorporate wide-area measurements to design power oscillation damping controls, the key research questions addressed in this thesis are summarized below:

- How to tackle the combinatorial binary problem, if the number of sensors or feedback signals used is large and the number of possible fault scenarios grows exponentially with the number of faulty sensors?

- How to provide linear solutions if the introduction of different constrains in the control design becomes a nonlinear problem?

Alternatively to the main questions, some other issues related to the main problem are:

- Modelling the loss of remote feedback signals or sensor faults in the system.
- POD controllers design that not only ensure the desired damping performance under nominal (no sensor fault) condition but also maintain a minimum level of performance following the loss of feedback signals.
- Including fault-tolerant features in the output feedback controllers and observers.
- Including regional pole placement into the fault-tolerant control and observer design formulation.
Chapter 2

Literature review

2.1 Introduction

This chapter presents the state-of-the-art of the relevant topics addressed in this work such as: the impact of poorly damped oscillations in power systems, the control strategies currently used to deal with this problem, the potential improvement considering wide-area signals instead of only local signals and the general description of the fault-tolerant schemes as well as the linear matrix inequalities in robust control.

Section 2.2 introduces and gives the basic background to understand the main problem. The necessity of interconnections between different sub-networks in order to satisfy the load needs in a continuously growing power system is explained. The section illustrates that stability problems exist due to oscillations caused by these interconnections, an illustrative example of an oscillation in the Nordic power system is presented. Then, the characteristics and benefits of using power systems stabilizers to enhance the damping in the system are discussed, a block diagram of a typical power system stabilizer structure is presented. Finally, it describes how the stability of the system can potentially be improved using wide-area signals through static var compensator. Also a block diagram of this device is provided. Section 2.3 highlights the use of wide-area signals in power systems. Here, the importance of signal selection for control design is emphasized. This section gives reasons for the inadequacy of choosing only local signals when
inter-area modes exist and explains the main features of wide-area signals that make them suitable for stabilizing control. Finally, one figure describing the loss of wide-area signals is presented. Section 2.4 introduces the concept of fault-tolerant scheme, presents some of the approaches reported in this filed, describe the use of controller and observer schemes to guard against sensor and actuator faults and finally summarizes other relevant work in the area. Section 2.5 provides the notation that will be used through the document and finally, Section 2.7 provides a brief survey of linear matrix inequalities in robust control since these will be the main tools used in the designs. Section 2.8 summarizes the content of the chapter.

2.2 Basics and Background

The electric power network is a complex system and consists of multiple components such as, synchronous generators, transmissions lines, transformers, loads, active and reactive compensators switches and relays, just to mention a few. Each of these components contributes towards ensuring supply demand balance of the continuously growing power system.

To match the rapid increase in demand for electricity, synchronous interconnections between power systems are the most reliable way to satisfy all these needs. An interconnection is a group of different sub-networks linked together making possible to share and distribute energy among the sub-networks. However, the increase in the energy demand result in higher loading in the transmission systems and the networks operators may be forced to operate the system closer to its stability limits, requiring more detailed investigations of the global system behaviour in order to maintain system security [18].

The presence of oscillations appears as soon as synchronous generators operate in parallel. As power systems became more interconnected, areas of generation were found to be prone to oscillations [19]. Oscillations are inherent in interconnected power systems but they are acceptable as long as they settle quickly. However, if the oscillations continue for too long, the consequences could be severe [20]. Figure 2.1 shows an example of oscillation in a line voltage of the Nordic power system. The fluctuation was prompted by a tie-line outage at 5 sec and
the variation in the voltage magnitude is large due to the presence of two poorly
damped inter-area modes between 0.2 and 0.6 Hz.

The presence of low frequency oscillations that may result in system instability is an important problem faced by power systems engineers and researchers. Poorly damped oscillations have been reported in several systems worldwide, some examples are: the 0.6 Hz mode in the Quebec’s Canadian system [21], a pair of modes with frequencies between 0.2Hz and 0.6Hz in the Nordic power system [22], another pair of modes in the range of 0.15-0.25 Hz in the Brazilian system [23], and the well known 2003 blackout in Eastern Canada and the US which was also accompanied by severe 0.4 Hz oscillations in several post-contingency stages [24], just to mention some examples.

The stabilization of electromechanical oscillations between interconnected synchronous generators is necessary for secure system operation. The oscillations of one or more generators in an area with respect to the rest of the system are called local modes, while those associated with groups of generators in different areas oscillating against each other are called inter-area modes [25]. Local modes are largely determined and influenced by local area states. Inter-area modes are more difficult to study as they require detailed representation of the entire interconnection system and are influenced by global states of large areas of the power network [8].

The amount of damping and the frequency of oscillations vary with system operating conditions. The operating range of a power system is usually very wide,
2. Literature review

Figure 2.2: Power Systems Stabilizer block diagram

requiring an oscillation damping control strategy that is effective over the whole range. It is necessary to have comprehensive modelling and analysis techniques of all the components that may interact to produce oscillations [19].

In the context of this project, the external events that disrupt the integrity of the system such as circuit outages, loss of generating units, load variations, etc., that produce oscillations as displayed in Figure 2.1 are defined as ‘disturbance’ in the system.

System stability is a major factor that impacts the reliable operation of electric power systems. For the safe operation of a system in the presence of these oscillations, some restrictions often have to be considered on the power transferred across long transmission lines. To allow a better use of the transmission capacity of these lines, specific control procedures and equipment capable of providing damping to the oscillations are necessary [26]. Today, inter-area oscillations is commonly damped through the use of: power systems stabilizers (PSS) and the supplementary control of static Var compensators (SVCs), thyristor controlled series capacitors (TCSCs) and other flexible AC transmission systems [27].

The function of a PSS is to add damping to the electromechanical oscillations. This is achieved by modulating the generator excitation so as to develop components of electrical torque in phase with rotor speed deviations. The PSS thus contributes to the enhancement of small-signal stability of power systems [28]. Figure 2.2 shows the block diagram of a typical PSS structure which is composed of a gain $K_{PSS}$, phase compensation blocks with time constants $T_1$, $T_2$, $T_3$ and $T_4$, a washout filter with a time constant $T_w$ and output limiters $V_{ssMAX}$ and $V_{ssMIN}$.

The PSS parameters should be chosen to enhance the overall performance of the power system. The specific objectives of the control design are [29, 30]

- Maximization of the damping of the local plant mode as well as inter-area mode oscillations without compromising the stability of other mode, such
as the exciter mode.

- Enhancement of system transient stability.

- Prevention of adverse effects on system performance during major system upsets that cause large frequency and voltage excursions.

- Minimization of the consequences of excitation system malfunction due to component failures.

The effectiveness of the PSS in achieving the desired objectives depends on the hardware design, method of deriving the input signal and selection of the control parameters. The choice of the stabilizing signal depends of many factors: the signals should be available and be easily measured. The most widely used signal is the rotor speed, however, bus frequency, electrical power, accelerating power and synthesized rotor speed are also used.

Continuous advances in power electronic technologies have made the application of Flexible AC Transmission Systems (FACTS) devices very popular in power systems. Generally, these devices are installed to perform continuous control over the voltage profile or power flow, to increase the stability margins and minimize losses rather than increasing the damping of low frequencies oscillations [31]. However, a controller may be designed for each FACTS device to increase the damping of certain electromechanical oscillatory modes (inter-area modes), while meeting the primary goal of the device.

Static Var Compensators (SVCs) have been widely used to provide voltage support on long transmission lines. Compared with conventional switched reactors or shunt capacitors, SVCs can provide control actions continuously and rapidly [32]. The high speed response feature of SVCs also provides many opportunities for enhancing the performance of power systems. One of these potential benefits is to improve power system stability by introducing a supplemental signal to the voltage set point, and so SVCs can be used to increase power system damping.

Figure 2.3 presents the block diagram for the SVC, where \( V_T \) is the SVC bus voltage, \( B_{SVC} \) is the susceptance, \( V_{REF} \) is the reference input set to a point to maintain acceptable voltage at the SVC, \( V_{AUX} \) is the supplementary input to
control and damp inter-area oscillations and the rest are the necessary parameters to control the exchange of capacitive or inductive current to maintain or control variables of the power system, typically bus voltage.

Since SVC is usually installed in a transmission line, generator signals, e.g. generator speed, are not available locally. To avoid communication problems, usually, local power system network signals are preferred for input of the damping controller. Possible choices include transmission line active power, line current, SVC bus frequency and SVC bus voltage [31].

In the last years, extensive work has been carried out in the design of supplementary controllers for SVCs to damp inter-area oscillations using local signals and avoiding the use of wide-area signals with respect to the location of the SVC [31, 32, 33, 34]. However, it has been observed that SVCs using these local signals does not necessarily contribute significantly to system damping and, furthermore, that it is possible to achieve significant contribution to this problem by appropriate selection of wide-area signals instead.

### 2.3 Wide-Area Control in Power Systems

The choice of measurement and control signals is a problem regularly faced by engineers. In fact, to obtain the desired performance and robustness, appropriate signals selection has to be made.

Local control should always be used as the first option and was rightly the only option considered when dealing with oversized networks of the past. However, it is less clear that local control alone will suffice to economically and efficiently satisfy the damping needs of the heavily stressed networks of today and the future.

---

![Block diagram of the SVC](image-url)
Local signals cannot always be effective to damp inter-area oscillations and two of the main reasons are explained below.

1. Based on a linearization of a system model in a nominal operating point, conventional local controllers designed by classical control techniques have their validity restricted to a neighbourhood of this operating point. Power systems constantly experience changes in operating conditions due to variations in generation and load as well as changes in transmission networks. In addition, some uncertainty is introduced into the power system model due to inaccurate approximation of the power system parameters, neglected high frequency dynamics and invalid assumptions made in the modelling process.

2. Local signal based controllers could lack adequate observability of inter-area modes. It has been proved that, under certain operating conditions, an inter-area mode may be controllable from one area and be observable from another. In such cases, local controllers are not effective for damping that mode.

With the advent of synchronized phasor measurement units (PMUs), power system controllers can now utilize input signals from many different locations of the grid. This provides a much greater possibility for choosing control loops. A wide-area control loop is inherently more effective than local loop to damp inter-area modes of oscillations. Many works have been directed towards the comparison of local control against wide-area control and all of them agree in the effectiveness of the global control and the substantially advantage of the wide-area signals over local. Some of the main features of these signals are described below.

- Many more signals to select from. Every output signal from all of the PMUs installed in the network.
- Higher observability and controllability factors for inter-area modes.
- Possibility of achieving more ambitious control specifications such as high closed-loop damping ratios and thus lower settling times.
2. Literature review

Figure 2.4: Loss of a wide-area signal $y_{p-1}$ at time $t$ due to sensor or communication problem.

- Reasonably less control effort required to achieve certain control specifications.

With the state-of-the-art wide-area measurement systems (WAMS) infrastructure, power oscillation damping using remote (or wide-area) signals is certainly feasible. This could potentially improve the stability limits and allow operation of the transmission lines closer to their thermal capacity. Utilities, however, are concerned about the consequences of unacceptable delay or complete loss of one or more of the remote feedback signals which could jeopardize the dynamic performance of their system [16].

Several techniques have been reported in the literature to tackle the adverse impact of latency or delay involved in communicating the remote signals. A list of those paper along with a critical review of the different approaches can be found in [38, 11, 39]. Another potential problem could be low data rate/bandwidth availability which is possible to be encountered for networked communication if WAMS infrastructure were to be shared between multiple data intensive services in future. A solution to this problem was presented in [40]. Despite a number of papers on latency and less so on the bandwidth problem, very little has been reported on tackling the situation where one or more remote signals are completely lost (see Fig. 2.4). Impact of loss of signals on stability of interconnected power systems is described in [41] but no technique is presented to resolve the problem.
2. Literature review

2.4 Fault-tolerant Design

The fault-tolerant (FT) theory is an important area of research. Its objective is to minimize the degradation in performance of a system when a fault occurs. With the increment in the requirements of regulation and operation of control systems, the topic of fault-tolerance has attracted the interest of research works in different areas (e.g., [42, 43, 44, 45]). In general, repairing and maintenance services cannot be provided instantly making the FT an important design strategy. The objective of the FT design is to provide an appropriate architecture such that the resulting closed-loop system can tolerate abnormal operations of specific control components and retain the overall system stability with acceptable system performance [46].

Within the FT theory, several approaches have been reported such as: the algebraic Riccatti equation-based approach [47], the coprime factorization approach [48], the Hamilton-Jacobi (HJ)-based approach [49], the sliding-mode control (SMC) approach [50] and the linear matrix inequality (LMI) approach [51]. Among the mentioned studies, the LMI approach is relatively simpler to be implemented making this approach one of the most popular in the field (e.g. [52, 53] and the references therein).

The design of observers and controllers for systems subject to sensor faults [54, 55, 56], actuator faults [57, 58, 59] or both [60] could be critical in several applications. These so called fault-tolerant observers/controllers can be realized using either active or passive schemes. In the case of active schemes some intervention (e.g. switching to another observer gain) is required in order to ensure satisfactory performance under faulty conditions. For passive schemes, performance is maintained under nominal and all possible fault conditions using a fixed time-invariant observer or controller. Both schemes require the solution of multiple equations or inequalities representing the nominal and faulty conditions.

Several papers deal with fault-tolerant controller and observer design for systems with sensor and/or actuator faults. The majority of these papers formulate the design problem using linear matrix inequalities (LMIs). But they differ in terms of the representation of sensor/actuator faults. Some relevant works are summarized below.
In [57] a fault-tolerant tracking controller is investigated for dealing with actuator faults. The controller is designed simultaneously for $2^m$ possible actuator fault combinations, where $m$ is the number of actuators. In [61] a fixed fault-tolerant controller is designed using multiple Lyapunov functions to simultaneously stabilize a set of systems representing possible actuator fault scenarios. Parameter dependent Lyapunov functions are used in the design to reduce conservativeness. In [62] the objective is to improve the closed-loop performance under the nominal condition while ensuring a minimum acceptable performance under all possible fault conditions. Multiple Lyapunov functions are used to reduce conservativeness. However, the proposed formulation leads to matrix inequalities which are not jointly convex and hence an iterative procedure is required for its solution. In [63] it is assumed that at least one actuator is always available allowing the use of open loop unstable systems. Although the proposed approach incorporates regional pole placement in addition to stability, a pre-compensator is required, resulting in an increase in the order of the controller, and $2^m$ inequalities need to be considered. In [55] sensor faults are modelled as variables that take their value in an interval rather than as $(0,1)$ binary variables.

Alongside the above mentioned passive schemes, active schemes have also been proposed in the literature. In [56] observer-based centralized and decentralized controllers are proposed that are sufficient to guarantee $H_\infty$ performance. In a follow-up paper [60] the results are improved for the centralized case. In both [56, 60], the observer needs to adapt to the particular sensor fault situation although the problem of fault detection is not explicitly addressed. In [64] a fault-tolerant control strategy for discrete polytopic systems is presented to deal with actuator faults. Existence of at least one healthy actuator is assumed and a number of controllers corresponding to possible fault conditions are synthesized. The original formulation is bilinear for which the solution is not guaranteed to be convex. The formulation has been transformed from bilinear to linear, however, the proposed approach to controller synthesis is sufficient but not necessary.

[65, 66, 54] propose a fault-tolerant observer design procedure to accommodate sensor faults under the assumption that the corresponding output(s) in the observer are disconnected using estimates of the sensor faults. However, the observer gain is obtained by considering $2^m$ (for $m$ sensors) sensor fault scenarios.
simultaneously. In [58] a switching controller scheme is proposed to deal with actuators faults. An adaptive observer is used to monitor the system and switch to the appropriate controller. This involves solving multiple linear matrix inequalities independently to produce individual controllers. The combinatorial nature of problem is avoided by assuming that not more than one actuator is lost at the time.

2.4.1 Hardware Redundancy

Hardware redundancy is a potent way to improve the reliability of a system. The hardware redundancy fault-tolerant technology stems from the reliability analysis theory, the idea is to place several components with the same function to complete an identical task, if a part of the parallel component has been broken down, the normal operation of the system will be unaffected. This kind of design can effectively improve the reliability of the system, but it can also increase the cost, structure, weight and requisite space of the system [67]. This is the main reason of why in this work, it is not considered this approach as part of the solution to the problem.

2.5 General Notation

The notation used is fairly standard. The set of real and complex $n \times m$ matrices is denoted by $\mathbb{R}^{n \times m}$ and $\mathbb{C}^{n \times m}$, respectively. For $A \in \mathbb{R}^{n \times m}$ the notation $A^T$ to denote the transpose is used. The set of complex numbers is denoted by $\mathbb{C}$. The open left half of the complex plane is denoted by $\mathbb{C}_-$ and the closed right half of the complex plane is denoted by $\mathbb{C}_+$. The $i$th eigenvalue of a $A \in \mathbb{C}^{n \times n}$ is denoted by $\lambda_i(A)$. For a symmetric matrix $A \in \mathbb{R}^{n \times n}$, $A \succ 0$ ($A \prec 0$) denotes that $A$ is positive definite (negative definite), that is, $\lambda_i(A) \succ 0$, $\forall i$ ($\lambda_i(A) \prec 0$, $\forall i$). The notation $A = \text{diag}(a_1, \ldots, a_n)$ denotes that $A$ is a diagonal matrix with diagonal entries $a_1, \ldots, a_n$. The $n \times n$ identity matrix is denoted as $I_n$ and the $n \times m$ null matrix is denoted as $0_{n,m}$ with the scripts dropped if they can be inferred from context.

A state-space description of a linear time-invariant dynamic system is given
2. Literature review

![Block diagram of the state-space representation.](image)

Figure 2.5: Block diagram of the state-space representation.

as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^m\) and \(y(t) \in \mathbb{R}^p\) are the state, input and output vectors, respectively, and where the matrices \(A, B, C\) and \(D\) have compatible dimensions. Figure 2.5 shows the block diagram of the state-space representation. Taking the Laplace transform, assuming zero initial conditions, the input/output description is given by

\[
y(s) = G(s)u(s)
\]

where \(G(s) = D + C(sI - A)^{-1}B\) is the system transfer function. Also, the notation \(G(s) \triangleq (A, B, C, D)\) and

\[
G(s) \triangleq \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]

is used. If \(G_1(s) \triangleq (A, B_1, C, D_1)\) and \(G_2(s) \triangleq (A, B_2, C, D_2)\),

\[
\begin{bmatrix} G_1(s) \\ G_2(s) \end{bmatrix} \triangleq \begin{bmatrix} A & B_1 & B_2 \\ C & D_1 & D_2 \end{bmatrix}
\]

is written. The set \(\mathcal{H}(s)^{m \times p}\) denotes the space of all \(m \times p\) proper, real-rational matrix functions of \(s\). \(\mathcal{L}^{m \times p}_\infty\) denotes the space of \(m \times p\) matrix functions with entries bounded on the extended imaginary axis. The subspace \(\mathcal{H}^{m \times p}_\infty \subset \mathcal{L}^{m \times p}_\infty\).
denotes matrix functions analytic in the closed right-half of the complex plane. For $G(s)$

$$\|G\|_\infty = \sqrt{\sup_{\omega \in \mathbb{R}} \max_i \lambda_i(G(-j\omega)TG(j\omega))}$$

is defined. The matrix $A \in \mathbb{R}^{n \times n}$ is called stable if $\lambda_i(A) \in \mathbb{C}_-$, $\forall i$. The pair $(A,B)$ is called stabilizable if there is a feedback matrix $K$, $u = Kx$, such that $(A+BK)$ is stable. The pair $(A,C)$ is called detectable if there exist a real matrix $L$ such that $A+LC$ is stable. In Chapter 4, the following version of the bounded real lemma [68] is used.

**Lemma 1** Let $G(s) = (A,B,C,D)$ where $A \in \mathbb{R}^{n \times n}$ and let $\gamma > 0$. Then $A$ is stable and $\|G\|_\infty < \gamma$ if and only if there exist $P = P^T \in \mathbb{R}^{n \times n}$ such that $P > 0$ and

$$
\begin{bmatrix}
A^T P + PA & PB & CT \\
B^T P & -\gamma I & DT \\
C & D & -\gamma I
\end{bmatrix} < 0
$$

### 2.6 Linear Parameter Varying Systems

The framework of linear parameter varying (LPV) systems concerns linear dynamical systems whose state-space representations depend on exogenous non-stationary parameters as

$$
\begin{align*}
\dot{x}(t) &= A(\Theta(t))x(t) + B(\Theta(t))u(t) \\
y(t) &= C(\Theta(t))x(t) + D(\Theta(t))u(t)
\end{align*}
$$

(2.1)

where $u(t)$ is an input, $y(t)$ is an output and $\Theta(t)$ is an exogenous parameter that can be time dependent. In Chapter 4 these type of systems are used and therefore introduced in this section. The LPV paradigm was introduced by Shamma in [69] for the analysis of the control design practice of ”gain-scheduling”. In brief, gain-scheduling is a control design approach that constructs a nonlinear controller for a nonlinear plant by patching together a collection of linear controllers. These linear controllers are blended in real-time according to available measurements. The architecture induced by gain-scheduling suggest the LPV framework as a middle ground between linear and nonlinear dynamics. LPV model consist of an indexed
collection of linear systems in which the indexing parameter is independent of the state. The LPV framework eliminate the nonlinear dependency as a difference of gain-scheduling schemes, resulting in linear but non-stationary dynamics \[70\].

2.7 Linear Matrix Inequalities in Robust Control

Control system models must often explicitly incorporate in them uncertainties, which model a number of factors, including: dynamics that are neglected to make the model tractable, as with large scale structures; nonlinearities that are either hard to model or too complicated; and parameters that are not known exactly, either because they are hard to measure or because of varying manufacturing conditions. Robust control deals with the analysis of and design for such control system models. We will consider control system models of the following form.

\[
\begin{align*}
\dot{x}(t) &= f(x, u, d, t) \\
y(t) &= g(x, u, d, t) \\
z(t) &= h(x, u, d, t)
\end{align*}
\] (2.2)

Where \(x(t) \in \mathbb{R}^n\), \(u(t) \in \mathbb{R}^{n_u}\), \(d(t) \in \mathbb{R}^{n_d}\), \(y(t) \in \mathbb{R}^{n_y}\) and \(z(t) \in \mathbb{R}^{n_z}\). The function \(x(t)\) is called the state of the system, while \(u(t)\) and \(d(t)\) are inputs, and \(z(t)\) and \(y(t)\) are outputs. \(d(t)\) consists of exogenous inputs, i.e., those that are not possible to exercise control over, such as noises, reference inputs etc. \(u(t)\) consists of control inputs; it is required to set \(u(t)\) to any desired value, for every \(t\). The outputs \(z(t)\) are those of interest; these may consist, for instance, of components of \(x\) or even those of \(u(t)\). \(y(t)\) consists of outputs that can be measured. In order to accommodate uncertainties, it is assumed that \(f\), \(g\) and \(h\) are not known exactly, but only known to satisfy some properties \[71\]. Equations (2.2) models are so-called continuous-time systems and it is straightforward to extend the discussion to discrete-time systems as well.

We now focus on a special instance of system (2.2), consisting of an interconnection of a linear time-invariant system and an uncertainty or perturbation in the feedback loop. This model has found wide applicability in the analysis and
2. Literature review

![Block diagram of a common robust analysis](image)

Figure 2.6: Block diagram of a common robust analysis

The design of control systems for which only imperfect models are available; see for example, [72]. The model is described below and its block diagram is shown in Figure 2.6

\[
\begin{align*}
\dot{x}(t) &= A(t)x(t) + B_yp(t) + B_u(t)u(t) + B_dt(d(t) \\
y(t) &= C(t)x(t) + D_yp(t) + D_u(t)u(t) + D_dt(d(t) \\
q(t) &= C_q(t)x(t) + D_q(t)p(t) + D_qu(t)u(t) + D_qd(t)d(t) \\
z(t) &= C_z(t)x(t) \\
p(t) &= \Delta(q,t)
\end{align*}
\]

(2.3)

where \( p \in \mathbb{R}^m, q \in \mathbb{R}^m \) and the matrices have appropriate dimensions. \( \Delta \) is in general a nonlinear operator representing the uncertainty in modelling, and is known or assumed to lie in some set \( \Delta \). Often \( \Delta \) contains the origin, i.e. \( \Delta = 0 \), and the linear time-invariant system that results is called the nominal model.

Many commonly encountered systems with structured and/or parametric uncertainties can be represented by the Figure 2.6 [73]. Usually, additional information about the size of the uncertainty, its structure (i.e., diagonal or block-diagonal), and nature (for instance, sector-bounded memoryless, linear time-invariant (LTI) or parametric, etc) is available. A very general framework for Linear Fractional Representation (LFR) systems is provided by Integral Quadratic Constraints; see for example [74].

Polytopic systems form a special class of LFR systems. For these systems, there exists an extensive body of work on analysis and synthesis using quadratic
Lyapunov functions [75]. These systems are described by

\[
\Sigma(t) = \begin{bmatrix}
A(t) & B_u(t) & B_d(t) \\
C(t) & D_u(t) & D_d(t) \\
C_z(t) & 0 & 0
\end{bmatrix} \in \Xi \quad (2.4)
\]

where

\[
\Xi = \begin{bmatrix}
A_i & B_{u,i} & B_{d,i} \\
C_i & D_{u,i} & D_{d,i} \\
C_{z,i} & 0 & 0
\end{bmatrix}, \quad i = 1, \cdots, m \quad \text{are given.} \quad (2.5)
\]

In LMI problems the notion of \textit{quadratic stability} is required. A system is said to be quadratically stable if there exists a positive-definite quadratic Lyapunov function \( \dot{V} = \dot{V} P V^T \) that decreases along every trajectory of the system. For a system such as (2.4), a necessary and sufficient condition for quadratic stability can be directly formulated in terms of a finite number of linear matrix inequalities [75]. For system (2.2), in general, only sufficient conditions for quadratic stability are known; these are stated in terms of a finite number of LMIs. The underlying quadratic Lyapunov functions can be used to derive bounds on robust performance measures; see for example [75].

A system can be robustly stable without being quadratically stable, and more general Lyapunov functions can be employed to derive weaker sufficient conditions for robust stability. For instance, when the state-space matrices of the polytopic system (2.4) vary slowly with time, stability analysis using parameter-dependent Lyapunov functions for example, \( \dot{V} = V P(\theta)V^T \), where \( \theta \) denotes the vector of measurable parameters, and \( P(\cdot) \) is some specified function, usually leads to less conservative robust stability conditions than the analysis based on quadratic Lyapunov functions [76]. For the LFR system (2.2), the framework of integral quadratic constraints (IQCs) provides a systematic method for deriving sufficient conditions for robust stability that are weaker than quadratic stability. These sufficient conditions can be reduced to LMIs either exactly or approximately. In some cases, the framework can be interpreted as searching for more general Lyapunov functionals [77]. In these cases, bounds on robust performance measures can be derived, and computed using LMI optimization.

Addressing the problem of controller synthesis, there are several possibilities
for generating the control input $u(t)$. Perhaps the simplest control law is that of constant state-feedback, $u(t) = Kx(t)$, where $K$ is a real matrix. Of course, in order to implement a state-feedback scheme, the state $x(t)$ has to be measurable at every time $t$. If only the measured output $y$ is available for generating $u$, output feedback control laws of the form $u = \mathcal{K}(y, t)$ can be envisioned. If in addition to the measured output, the uncertainty $\Sigma$ in a polytopic system (or $\Delta$ in an LFR system) is measurable in real time [78, 79, 80, 81, 82, 83], a control law $u = \mathcal{K}(y, \Sigma, t)$ (or $u = \mathcal{K}(y, \Delta, t)$) that explicitly depends on the uncertainty can be implemented. This is the so-called gain-scheduled controller.

The problem of synthesizing robustly stabilizing constant state-feedback for both polytopic and LFR systems, using quadratic Lyapunov functions, can be formulated as LMI feasibility problems [75]. However, no convex reformulation is known for the problem of even constant output feedback synthesis for even polytopic systems. It is worthy of note that a number of results are available for the LMI-based synthesis of LTI controllers for LTI systems (i.e., a model with no uncertainties); a sampling is provided by [68, 84].

It is fair to say the advent of LMI optimization has significantly influenced the direction of research in robust control. A widely-accepted technique for solving robust control problems now is to simply reduce them to LMI problems. While it true in principle that the reduction of a robust control problem to an LMI problem provides a solution, it is also now recognized that in many practical applications, the resulting LMI problems are so large as to test the limits of currently available software. Thus, much remains to be gained with the development of special purpose LMI solvers that take advantage of the underlying problem structure and information [71].

### 2.8 Summary

In this chapter a review of the state-of-the-art in the various issues that form the background and that motivates the problems that will be addressed in this thesis have been presented. The aim of this part of the thesis is to present the reader with the problems discussed here and comment on relevant literature related to this research as well as the background for the mathematical tools that
will be used to develop the design procedures. In particular, the importance, as well as the challenges associated with the use of wide-area measurements for the stabilization of power systems has been highlighted and will form the main theme of the thesis.
Chapter 3

Passive Fault-tolerant Control Design

3.1 Introduction

In this chapter the concepts of conventional and passive fault-tolerant control for linear continuous time-invariant systems are introduced. The problem is formulated in the form of linear matrix inequalities constraints. Lyapunov stability theory is used to define the control objectives and application of the Bounded Real Lemma is required to reduce the control effort. As a first step towards solving the general problem the representation of sensor faults is considered in the simplest form: one sensor loss at the time. Combination of local and wide-area signals is assumed to ensure presence of at least one healthy feedback signal.

Section 3.2 describes the models that will be used to represent sensor faults for a family of plants. It is assumed that one local and at least one remote signal are used. Also, one sensor fault at a time is considered for simplicity but this assumption is relaxed with a generalized formulation in the following chapter. In Section 3.3 a conventional controller design strategy is introduced. This conventional design implements regional pole-placement (conic sector) based on LMIs and is also designed to limit the control effort, a general block diagram describing the problem is displayed in this section. The conventional controller is designed only for the fault-free (nominal) condition. In Section 3.4 a passive
fault-tolerant control scheme is proposed. The proposed controller considers the same constraints as the conventional approach and, in addition, guarantees a minimum level of performance following the loss of one of the remote signals. The algorithm used to calculate the parameters of this fault-tolerant controller is outlined at the end of this section. Due to the computational intractability of the passive controller proposed in Section 3.4, an iterative procedure is proposed in Section 3.5 to reduce the conservatism and potentially achieve stricter control specifications. Subsection 3.5.1 introduces in detail the iterative algorithm to compute final controller and the convergence criteria to terminate the iterative procedure. In Chapter 5, the controllers described in this chapter are validated in the Nordic power system through exhaustive nonlinear simulations. Section 3.7 summarized the content of this chapter.

3.2 Formulation of the Control Design

Consider the following state space representation of a linear time-invariant (LTI) system $G(s)$

$$
\dot{x}(t) = Ax(t) + Bu(t) \\
y(t) = Cx(t)
$$

$$
G(s) = \left[ \begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right]
$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$ are the state, input and output vectors, respectively. $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times q}$ and $C \in \mathbb{R}^{p \times n}$ are the state, input and output matrices of the system, respectively.\footnote{Note that, to simplify the solutions, in this work we are considering system models that do not have a direct feedthrough matrix ($D = 0$).} The output matrix $C$ is described as follows

$$
C = [ c_1^T \ c_2^T \ \ldots \ c_{p-1}^T \ c_p^T ]^T
$$

where $c_j \in \mathbb{R}^{1 \times p}$ represents the $j$th output of the system. In this work $p \geq 2$, as one local and at least one remote signal are used. The sensor faults or loss of

1
3. Passive FTC design

Signals can be represented by a family of plants $G_i(s)$

$$
\dot{x}(t) = Ax(t) + Bu(t) \\
y_i(t) = C_i x(t)
$$

(3.1)

where

$$
G_i(s) = \begin{bmatrix}
A & B \\
C_i & 0
\end{bmatrix}
$$

and

$$
c_i^j = \begin{cases}
0 & \text{if } i = j \\
c_j & \text{if } i \neq j
\end{cases}, \text{ for } i = 0, 1, \ldots, p-1, j = 1, 2, \ldots, p-1
$$

Note that $c_p^T$ denotes a local signal and it will be assumed throughout that it is always available. In nominal conditions where all signals are available, $i = 0$ and $C_0 = [c_1^T \ c_2^T \ \cdots \ c_{p-1}^T \ c_p^T]^T$. Each of the measurements $y_i$ is the output of a sensor that can potentially fail, i.e. loss of sensor $y_2$ is represented as $i = 2$ and is described as $C_2 = [c_1^T \ 0 \ \cdots \ c_{p-1}^T \ c_p^T]^T$. One remote signal loss at a time is considered for simplicity but can be generalized for more than one.

3.3 Conventional Control (CC)

A conventional controller (CC) is designed to satisfy a desired level of dynamic performance when both the local and the remote signals are available. The performance with this controller, however, can deteriorate significantly following sudden loss of the remote signals. A regional pole-placement approach using Linear Matrix Inequalities (LMIs) is adopted for the design of CC. Contrary classical pole-placement approaches such as tuning of lead-lag blocks or state feedback, the LMI approach allows the imposition of additional constraints e.g. on control effort, which is relevant to the present application.

The same methodology is used for the fault-tolerant control (FTC) design to ensure a fair comparison between FTC and CC. For the case of the CC the design formulation is linear and simpler as described below.
The CC, $K_c(s)$, is represented in state space and transfer function formats as follows

$$\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) + B_c y(t) \\
u(t) &= C_c x_c(t)
\end{align*}$$

(3.2)

with $A_c \in \mathbb{R}^{n \times n}$, $B_c \in \mathbb{R}^{n \times p}$, $C_c \in \mathbb{R}^{q \times n}$. With this controller the closed-loop state dynamics is described as $\dot{\tilde{x}} = \tilde{A} \tilde{x}$ where

$$\tilde{A} = \begin{bmatrix}
A & B C_c \\
B_c C & A_c
\end{bmatrix}.$$  

(3.3)

The control objective is to place the eigenvalues of (3.3) within a desired region of the complex plane. The following theorem, taken from [85], describes the objectives.

**Theorem 1** The matrix $\tilde{A}$ is stable and all its eigenvalues lie within the conic sector of the complex plane, shown in Fig. 3.1, if and only if there exist a symmetric matrix $\tilde{P}$ such that

$$\tilde{P} > 0$$

(3.4)

$$\begin{bmatrix}
\sin \theta (\tilde{P} \tilde{A} + \tilde{A}^T \tilde{P}) & \cos \theta (\tilde{P} \tilde{A} - \tilde{A}^T \tilde{P}) \\
\cos \theta (\tilde{A}^T \tilde{P} - \tilde{P} \tilde{A}) & \sin \theta (\tilde{P} \tilde{A} + \tilde{A}^T \tilde{P})
\end{bmatrix} < 0$$

(3.5)

where $\theta$ is the inner angle of the cone (more details to place the poles in different LMI regions can be found in Appendix A.3).

In addition to regional pole-placement within the conic sector, another objective is to limit the control effort. This is achieved by minimizing the infinity norm of the transfer function between the output disturbance $d$ and the input $u$ of the system, see Figure 3.2. Thus the objective is to minimize $\gamma_c$ such that

$$\| K_c(I - GK_c)^{-1} \|_\infty < \gamma_c$$

(3.6)
Applying the Bounded Real Lemma [86] to (3.6) this constraint can be formulated in the form of the matrix inequality

\[
\begin{bmatrix}
\tilde{P} \tilde{A} + \tilde{A}^T \tilde{P} & \tilde{P} \tilde{B} & \tilde{C}^T \\
\tilde{B}^T \tilde{P} & -\gamma_c I & 0 \\
\tilde{C} & 0 & -\gamma_c I \\
\end{bmatrix} < 0.
\]  

(3.9)

The formulation of this problem is bilinear but the nonlinearities can be eliminated by some appropriate change of controller variables [85]. These changes
are implicitly defined in terms of the partition of the Lyapunov matrix $\tilde{P}$ and its inverse

$$\begin{bmatrix} X & U \\ U^T & X_c \end{bmatrix} \quad \tilde{P}^{-1} = \begin{bmatrix} Y & V \\ V^T & Y_c \end{bmatrix}$$

with $X, Y, U$ and $V \in \mathbb{R}^{n \times n}$. Since $\tilde{P}\tilde{P}^{-1} = I$,

$$UV^T = I - XY.$$  

It can be verified that $\tilde{P}$ satisfies the identity

$$\tilde{P}\Pi_2 = \Pi_1$$

with

$$\Pi_1 = \begin{bmatrix} X & I \\ U^T & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & Y \\ 0 & V^T \end{bmatrix}. $$

Pre- and post-multiplying (3.4), (3.5) and (3.9) by the matrices

$$\Pi_2^T$$ and $\Pi_2$,

$$\begin{bmatrix} \Pi_2^T & 0 \\ 0 & \Pi_2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \Pi_2 & 0 \\ 0 & \Pi_2 \end{bmatrix},$$

$$\begin{bmatrix} \Pi_2^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \Pi_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

respectively, carrying out the matrix products and performing the following change of variables [85]

$$\hat{C}_c = C_cV^T,$$

$$\hat{B}_c = UB_c,$$

$$\hat{A}_c = XAY + XB\hat{C}_c + \hat{B}_cCY + UA_cV^T$$

the constraints in (3.4), (3.5) and (3.9) become linear. This can be solved easily using LMI based optimization to calculate the variables of (3.2), which are
3. Passive FTC design

described below
\[
\begin{bmatrix}
X & I \\
I & Y
\end{bmatrix} \succ 0 \tag{3.18}
\]
\[
\begin{bmatrix}
\sin \theta_{L_{11}} & \cos \theta L_{12} \\
\cos \theta L_{12}^T & \sin \theta_{L_{11}}
\end{bmatrix} \prec 0 \tag{3.19}
\]
with
\[
L_{11} = \begin{bmatrix}
XA + A^T X + C^T \hat{B}_c^T + \hat{B}_c C & \hat{A}_c + A^T \\
\hat{A}_c^T + A & AY + Y A^T + B \hat{C}_c + \hat{C}_c^T B^T
\end{bmatrix} \tag{3.20}
\]
\[
L_{12} = \begin{bmatrix}
XA - A^T X + \hat{B}_c C - C^T \hat{B}_c^T & \hat{A}_c - A^T \\
A - \hat{A}_c^T & AY - Y A^T + B \hat{C}_c - \hat{C}_c^T B^T
\end{bmatrix} \tag{3.21}
\]
\[
\begin{bmatrix}
L_{11} & \hat{B}_c & 0 \\
0 & \hat{C}_c^T & -\gamma_c I \\
\hat{B}_c^T & 0 & 0 \\
0 & \hat{C}^T & 0
\end{bmatrix} \prec 0 \cdot \tag{3.22}
\]

The final algorithm to calculate $K_c(s)$ is described below

- Define $Z = I - XY$
- Calculate the singular value decomposition $[U_Z, \Sigma, V_Z] = \text{svd}(Z)$
- Calculate $U = U_Z \sqrt{\Sigma}$ and $V = V_Z \sqrt{\Sigma}$
- Calculate $C_c$, $B_c$ and $A_c$ using (3.17)
- Finally, $K_c(s)$ is given by (3.2).

It is possible to extend the pole-placement design to include more sophisticated criteria such as $H_\infty$, however this will not be included in this thesis to simplify the presentation. Also important to notice is that these more complex methodologies are used much less than pole-placement in the power industry.
3. Passive FTC design

3.4 Passive Fault-tolerant Control (FTCp)

A fault-tolerant controller (FTC) is designed to not only achieve a desired level of dynamic performance when both local and remote signals are available but also guarantee a minimum level of performance following sudden loss of remote signals. The problem of passive FTC design consists of synthesizing a single controller, if such a controller exists, that satisfies the design specifications for the family of plants as described in (3.1). The state space of representation of the fault-tolerant controller, $K_f(s)$, is as follows

\[
\dot{x}_f(t) = A_f x_f(t) + B_f y_i(t) \\
u(t) = C_f x_f(t)
\]

with $A_f \in \mathbb{R}^{n \times n}$, $B_f \in \mathbb{R}^{n \times p}$ and $C_f \in \mathbb{R}^{q \times n}$. This controller is designed for the family of plants (3.1) such that

\[
u(s) = K_f(s)y_0(s), \quad u(s) = K_f(s)y_1(s), \quad \cdots, \quad u(s) = K_f(s)y_p(s)
\]

and the closed-loop state dynamics matrices are given by

\[
\tilde{A}_i = \begin{bmatrix} A & BC_f \\ B_f C_i & A_f \end{bmatrix}, \quad i = 0, 1, \ldots, p.
\]

The requirement is that the eigenvalues of all $\tilde{A}_i$ lie in the conic region described in Fig. 3.1. The constraints are the same as in CC and Theorem 1 is used to formulate the problem. The objective in this case is to find a symmetric $\tilde{P}$ such that

\[
\tilde{P} \succ 0
\]
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\[
\begin{bmatrix}
\sin \theta_i(\tilde{A}_i^T \tilde{P} + \tilde{P} \tilde{A}_i) & \cos \theta_i(\tilde{A}_i^T \tilde{P} - \tilde{P} \tilde{A}_i) \\
\cos \theta_i(\tilde{P} \tilde{A}_i - \tilde{A}_i^T \tilde{P}) & \sin \theta_i(\tilde{A}_i^T \tilde{P} + \tilde{P} \tilde{A}_i)
\end{bmatrix} < 0, \quad i = 0, 1, \ldots, p
\] (3.26)

In this case, \( \theta_i \) is the required inner angle for the \( i \)th system. Similar to CC, another objective is to limit the control effort which is included in the formulation below

\[
\| K_f (I - G_i K_f)^{-1} \|_\infty < \gamma_{if}, \quad i = 0, 1, \ldots, p.
\] (3.27)

We interpret \( \gamma_{if}, \quad i = 1, 2, \ldots, p, \) as a priori constraints on the control effort for the ‘fault’ scenarios (when the remote signals are not available) and the objective is to minimize \( \gamma_{0f} \), which corresponds to the control effort for the normal (both local and remote signals available) scenario. The Bounded Real Lemma is used again to express (3.27) for each \( i \) in a matrix inequality form

\[
\begin{bmatrix}
\tilde{A}_i^T \tilde{P} + \tilde{P} \tilde{A}_i & \tilde{P} \tilde{B} & \tilde{C}^T \\
\tilde{B}^T \tilde{P} & -\gamma_{if} I & 0 \\
\tilde{C} & 0 & -\gamma_{if} I
\end{bmatrix} < 0.
\] (3.28)

Applying the same transformations to (3.25), (3.26) and (3.28) as before and carrying out the corresponding matrix products, it is not possible to perform the same change of variables as in (3.17) to linearize the inequalities due to the problem of having multiple systems. In order to linearize the bilinearities, after applying the transformations (3.14), (3.15) and (3.16) to (3.25), (3.26) and (3.28), respectively, apply the following change of variables [87]

\[
\begin{align*}
L &= C_f V^T, & F &= U B_f, & M^T &= U A_f V^T \\
P &= Y^{-1}, & S^T &= M^T Y^{-1},
\end{align*}
\] (3.29)

which have dimensions determined by the transformations. Defining \( \tilde{A} = A + B C_f \), the final formulation of the problem is represented by the following inequalities

\[
\begin{bmatrix}
P & P \\
P & X
\end{bmatrix} > 0
\] (3.31)
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\[
\begin{bmatrix}
\sin \theta_i L_{11}^i & \cos \theta_i L_{12}^i \\
\cos \theta_i L_{12}^{iT} & \sin \theta_i L_{11}^i
\end{bmatrix} < 0 \quad (3.32)
\]

where

\[
L_{11}^i = \begin{bmatrix}
\hat{A}^T P + P \hat{A} & PA + \hat{A}^T X + C_i^T F^T + S \\
A^T P + X \hat{A} + FC_i + S^T & A^T X + XA + C_i^T F^T + FC_i
\end{bmatrix} \quad (3.33)
\]

\[
L_{12}^i = \begin{bmatrix}
\hat{A}^T P - P \hat{A} & PA - \hat{A}^T X - C_i^T F^T - S \\
A^T P - X \hat{A} - FC_i - S^T & A^T X - XA + C_i^T F^T - FC_i
\end{bmatrix} \quad (3.34)
\]

and

\[
\begin{bmatrix}
L_{11}^i & 0 & C_f^T \\
F & 0 & \gamma_{if} I \\
C_f & 0 & \gamma_{if} I
\end{bmatrix} \prec 0. \quad (3.35)
\]

For further details see [87]. The algorithm to calculate the parameters of the FTC in (3.23) is outlined below

- Define the desired damping required \( \zeta_i \) and calculate \( \theta_i = \cos^{-1}(\zeta_i) \).
- Define the desired control effort levels for \( \gamma_{1f}, \ldots, \gamma_{pf} \)
- Build and solve \( Y \succ 0 \)

\[
\begin{bmatrix}
\sin \theta_i (A^T Y + YA + BL + L^T B^T) & \cos \theta_i (YA - \hat{A}^T Y + L^T B^T - BL) \\
\cos \theta_i (A^T Y - YA + BL - L^T B^T) & \sin \theta_i (A^T Y + YA + BL + L^T B^T)
\end{bmatrix} < 0
\]

\( (3.36) \)

- Obtain \( Y \) and \( L \) from the solution of (3.36) and calculate \( C_f = LY^{-1} \).
- Define \( \hat{A} = A + BC_f \).
- Minimize \( \gamma_{0f} \) subject to (3.31), (3.32) and (3.35) to obtain \( P, X, F \) and \( S \).
- Using \( V = V^T = Y \), (3.11), (3.29) and (3.30) calculate \( B_f \) and \( A_f \).
- Define \( K_f(s) \equiv (A_f, B_f, C_f, 0) \).
3. Passive FTC design

3.5 Iterative Passive Fault-tolerant Control (FTCit)

The passive algorithm to design a fault-tolerant controller is linear and easy to implement but has a drawback: the value of the matrix $C_f$ in the controller is calculated and fixed to the value of $LY^{-1}$ before calculating the other control matrices. Consequently, the values of $A_f$ and $B_f$ depend of the matrix $C_f$. As a result, it may be possible that there is no solution for a specific value of $C_f$ but one may exist for other values. An iterative procedure, described next, can avoid this problem and can potentially improve the damping of the system.

Again pre- and post-multiplying inequalities (3.26) by (3.15), respectively. In order to linearize the bilinear inequalities is required to perform the same change of variables as in (3.29) but in this case $\hat{A}$ has to be defined as

$$\hat{A} = XAY + XBL + M^T.$$  \hfill (3.37)

At this point, the problem can be expressed as follows

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succ 0 \quad (3.38)$$

$$L_i^b = \begin{bmatrix} \sin \theta_i L_{b11}^i & \cos \theta_i L_{b12}^i \\ \cos \theta_i L_{b12}^{iT} & \sin \theta_i L_{b11}^i \end{bmatrix} \prec 0 \quad (3.39)$$

where

$$L_{b11}^i = \begin{bmatrix} A^T X + XA + C_i^T F^T + FC_i & A^T + \hat{A} + FC_i & Y \\ A + \hat{A}^T + YC_i^T & Y A^T + BL + L^T B^T \end{bmatrix}$$ \hfill (3.40)

$$L_{b12}^i = \begin{bmatrix} A^T X - XA + C_i^T F^T - FC_i & A^T - \hat{A} - FC_i & Y \\ A - \hat{A}^T - YC_i^T & Y A^T - Y + L^T B^T - BL \end{bmatrix}$$ \hfill (3.41)

Note that the inequality in (3.39) is nonlinear due to the term $FC_iY$ in the off-diagonal blocks of (3.40) and (3.41). However, since this term is bilinear, there are many excellent methods for dealing with this case [88]. Since the open-loop is stable, it is possible to alternately fix the value of one of the two variables $F$ or
Y to improve the damping in an iterative algorithm described next. Let $k$ denote
the iteration number so that $k = 1, 2, \ldots$ and also note that for a variable $Z$, the
notation $Z(k)$ will be used to denote its value at iteration $k$.

3.5.1 Iterative Algorithm for Passive Design

(i) Set $k = 1$, $F(k) = 0$, so that (3.39) is linear and feasible since the open-loop
is stable.

(ii) Find the minimum value of $\theta_i(k)$ (reduce the angle of the conic sector to
shift the modes to the left hand side of the complex plain) for which there
exist feasible solutions to (3.38) and (3.39). Calculate $Y(k)$, $X(k)$, $L(k)$,
$\hat{A}_c(k)$ and set $Y(k + 1) = Y(k)$.

(iii) Set $k = k + 1$

(iv) Find the minimum value of $\theta_i(k)$ (reduce the angle of the conic sector to
shift the modes to the left hand side of the complex plain) for which there
exist feasible solutions to (3.38) and (3.39). Calculate $L(k)$, $X(k)$, $F(k)$,
$\hat{A}_c(k)$ and set $F(k + 1) = F(k)$.

(v) Set $k = k + 1$ and go to (ii).

Note that $\theta_i(k + 1) \leq \theta_i(k)$. The iterative procedure terminates when

$$
\theta_i(k) - \theta_i(k + 1) \leq \epsilon
$$

(3.42)

where $\epsilon$ can be adjusted to a small value i.e. $10^{-3}$. When the iterative algorithm
has converged, (3.42) is satisfied; to retrieve the elements of $K_f$ ($A_f$, $B_f$ and
$C_f$) use (3.29) and (3.37). Since a linear approximation have been used for the
bilinear inequalities, there is no guarantee that the solution will converge to the
optimal solution. However, in our experience, it does give improvement to the
damping ratio as illustrated in Chapter 5.
3. Passive FTC design

Figure 3.3: Comparison of the input magnitudes after applying a step at the output of the closed loop system, as described in Figure 3.2. These correspond to the values of the different $\gamma$'s: $\gamma_{CC} = 0.0568$, $\gamma_{FTC_p} = 0.1802$, $\gamma_{FTC_{it}} = 0.1480$

3.6 Illustrative Example

Although any small order system can be utilized to illustrate the concepts described before, to continue in the context of power systems, an example from electric power transmission application is considered here to illustrate the proposed methodology. A 4th order reduced equivalent of the Nordic power transmission system, that will be used in full order in Chapter 5, is chosen as the plant. The state space representation of this reduce order model is described in Appendix B.4.

In this example, three different controllers were designed: a conventional control (CC), a fault-tolerant passive (FTCp) and an iterative passive fault-tolerant (FTC_{it}) control following the procedures described in Sections 3.3, 3.4 and 3.5, respectively. The value of the parameters in the controllers are described in Appendix B.6.

Each of the controllers were designed to reject a disturbance at the output of the plant, as described in Figure 3.2. Figure 3.3 shows the input $u(t)$ of the closed loop system between the different controllers and the nordic power system,
3. Passive FTC design

after applying a step at 1sec as disturbance. The values of the corresponding $\gamma$'s are also shown. From this figure, it can be seen that the smallest magnitude of the input signals, corresponds to the case where the conventional control (CC) is used and the largest magnitude corresponds to the case where the fault-tolerant passive (FTCp) control is used. Note that the value of the $\gamma$'s agree with this. Also note that the magnitude of the input when using the iterative passive fault-tolerant (FTCit) is less than using the passive fault-tolerant control (FTCp), this can be attributed to the reduction of conservativeness when using the iterative procedure.

3.7 Summary

This chapter presented the first design scheme to address the issue of fault tolerance in the design of controllers for the stabilization of power systems. It described the representation of sensor faults considering one local and at least one remote signal. Only the loss of one signal at a time was considered, although this restriction will be relaxed in the next chapter. One CC to limit the control effort and place the closed-loop poles in a desired location for nominal conditions was presented. Finally, two passive FTCs designs were proposed, where one is based on an iterative algorithm. Both FTCs have been designed to guarantee a minimum level of performance following the loss of one remote signal.
Chapter 4

A Semidefinite Relaxation Procedure for Fault-tolerant Observer Design

4.1 Introduction

In this chapter the control design problem is reformulated into a state estimation problem, which involves design of an observer (observer based controllers). First, all the necessary concepts to tackle the problem are presented. The definition of the system considers a disturbance at the input and the output of the system. After defining the issues to be solved, a well known robustness result used for systems with structured norm-bounded uncertainty is extended to deal with binary variables. This is because in the case of fault-tolerant design, sensor faults can be represented by an uncertainty matrix whose diagonal entries take binary values (0 or 1 depending on the presence or absence of the sensor/feedback signal) rather than being norm bounded. Using this extended robustness result, it is possible to reformulate the original problem, which requires the solution of an LMI for every fault scenario, to provide a general solution that gives sufficient conditions for stability against any combination of sensor faults in the form of a single LMI (sufficient and necessary for the case where there is only one sensor). Our main result is then applied to the state estimation problem to reduce the number of
4. A Semidefinite Relaxation Procedure for FTO Design

LMIs to be solved.

Section 4.2 defines the concepts of Linear Parameter Varying (LPV) system, quadratic stability, induced $\mathcal{L}_2$-norm and the bounded real lemma for LPV systems. In Section 4.3 the description of the system and the observer are presented and a block diagram of each system is displayed. Here, the modelling of sensor faults considering all possible combinations is presented using a diagonal matrix at the output of the system and the output of the observer. Two types of observers are proposed: a passive scheme and, as an alternative to active schemes, an approach for the design of a "minimal switching" observer in which only one observer gain is designed, but where a simple switch is incorporated into the observer structure. Next, conditions for the solution of both observer design problems for every faulty scenario, are derived. When the number of sensors is large, the number of LMIs required to satisfy the conditions increases exponentially with the number of potentially faulty sensor and finding a solution can potentially be unfeasible due to the combinatorial nature of the problem. Section 4.4 derives a general robustness result to provide sufficient conditions, in the form of small number of LMIs, for the solution of both problems. Section 4.5 derives the final solution and presents the corresponding proofs. Section 4.6 presents an illustrative example using a reduced linear version of the Nordic power system. In this example, the tracking of the actual states using the proposed observers are compared against a traditional observer design. It is demonstrated that the system can become unstable following the loss of wide-area signals. Finally Section 4.7 summarizes the content of the chapter.

4.2 Notation and Preliminaries

Linear parameter varying (LPV) systems are a special class of linear time varying (LTV) systems where the time dependence enters the state equation through exogenous parameters [89, 90]. A state space description of an LPV system can be represented as

\[ \dot{x}(t) = A(\Delta(t))x(t) + B(\Delta(t))u(t) \]
\[ y(t) = C(\Delta(t))x(t) + D(\Delta(t))u(t) \]  

(4.1)
We recall the definition of quadratic stability ($Q$-stability) and $L_2$-induced $Q$-performance and give sufficient conditions for $G_{\Delta}$ to be $Q$-stable and to have performance level $\gamma$ [89, 91].

**Definition 1** The system $G_{\Delta}$ is $Q$-stable if there exist $\tilde{P} = \tilde{P}^T \succ 0$ such that $A(\Delta)^T \tilde{P} + \tilde{P} A(\Delta) \prec 0 \forall \Delta \in \mathcal{F}_{\Delta}$.

**Definition 2** For a $Q$-stable LPV system $G_{\Delta}$ with zero initial conditions, the induced $L_2$-norm is defined as

$$\|G_{\Delta}\|_{i,2} := \sup_{\Delta \in \mathcal{F}_{\Delta}} \sup_{u \in L_2^+} \frac{\|G_{\Delta} u\|_2}{\|u\|_2}. \quad (4.3)$$

**Lemma 2** The LPV system $G_{\Delta}$ is $Q$-stable and $\|G_{\Delta}\|_{i,2} \prec \gamma$ if there exist $\tilde{P} = \tilde{P}^T \succ 0$ such that for all $\Delta \in \mathcal{F}_{\Delta}$

$$
\begin{bmatrix}
\mathcal{H}(\tilde{P} A(\Delta(t))) & \tilde{P} B(\Delta(t)) & C(\Delta(t))^T \\
B(\Delta(t))^T \tilde{P} & -\gamma I & D(\Delta(t))^T \\
C(\Delta(t)) & D(\Delta(t)) & -\gamma I
\end{bmatrix} \prec 0.
$$

(4.4)

Here and elsewhere in this work $\mathcal{H}(X) = X + X^T$ and for symmetric $P$, $P \prec 0$ ($P \succ 0$) denotes that $P$ is negative (positive) definite. Note that the requirement for $Q$-stability is sufficiently strong to ensure stability even for rapidly changing parameters.
4.3 System Description and Fault Tolerant Observer Problem Formulation

Consider a linear time-invariant system described by the equations of the form

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_u u(t) + B_d d(t) \\
y(t) &= \Delta(t)Cx(t) + \Delta(t)D_d d(t) \\
z(t) &= C_z x(t)
\end{align*}
\] (4.5)

where \(x(t) \in \mathbb{R}^n\) is the state of the system, \(u(t) \in \mathbb{R}^{n_u}\) is the control input, \(y(t) \in \mathbb{R}^{n_y}\) are the outputs of the system and \(d(t) \in \mathbb{R}^m\) is the disturbance to be attenuated. \(z(t) \in \mathbb{R}^n\) is the variable to be estimated. The matrices \(A, B_u, B_d, C, D_d\) and \(C_z\) are constant matrix of appropriate dimensions. The matrix \(\Delta(t)\) is a diagonal matrix and is used to model sensor faults with \(\Delta(t) \in \Delta\) where

\[
\Delta := \{\Delta(t) = \text{diag}(\delta_1(t), \ldots, \delta_p(t)) : \delta_i(t) \in \{0, 1\}\}.
\] (4.6)

Note that \(\Delta(t) = I_{n_y}\) if all sensors are working normally and \(\Delta(t) = 0_{n_y}\) if all sensors fail. If a fault occurs, the loss of the \(i\)th sensor can be modelled by setting the \(i\)th element of \(\Delta(t)\) equal to zero, i.e. \(\delta_i(t) = 0\). It should be noted that there are \(2^{n_y}\) possible combinations of sensor failures so that the set has \(2^{n_y}\) elements.

We consider a state observer of the following form

\[
\begin{align*}
\dot{\hat{x}}(t) &= A\hat{x}(t) + B_u u(t) - L(y(t) - \hat{y}(t)) \\
\dot{\hat{y}}(t) &= \hat{\Delta}(t)C\hat{x}(t) \\
\hat{z}(t) &= C_z \hat{x}(t)
\end{align*}
\] (4.7)

where \(\hat{x}(t) \in \mathbb{R}^n\) is the state of the observer, \(\hat{y}(t) \in \mathbb{R}^{n_y}\) is the output of the observer and \(L \in \mathbb{R}^{n \times n_y}\) is the observer gain to be designed while \(\hat{z}(t)\) is the estimate of \(z(t)\). The matrix \(\hat{\Delta}(t)\) is used to define the type of observer required. When \(\hat{\Delta}(t) = I_{n_y}\) we call this a passive observer, while if \(\hat{\Delta}(t) = \Delta(t)\) we call it a minimal switching observer, with the word minimal indicating that the switching is between the observer outputs using one gain matrix \(L\) rather than between
4. A Semidefinite Relaxation Procedure for FTO Design

different observer gain matrices. The minimal switching observer it is within the
class of active and reconfigurable schemes, however the reconfiguration consist in
simply applying the switching that corresponds to the scenario. The system (4.5)
and observer (4.7) are shown in Figure 4.1. A third possibility is that \( \hat{\Delta}(t) \) is an
estimate of \( \Delta(t) \) [55]. Since the focus in this thesis is on the combinatorial nature
of the problem, it is assumed that we have full information about the faults so
that \( \Delta(t) \) is known. For the minimal switching observer (\( \hat{\Delta}(t) = \Delta(t) \)), the state
and signal estimate errors are defined as \( \tilde{x}(t) = x(t) - \hat{x}(t) \) and \( \tilde{z}(t) = z(t) - \hat{z}(t) \),
respectively. Then

\[
\begin{align*}
\dot{x}(t) &= (A + L\Delta(t)C)\tilde{x}(t) + (B_d + L\Delta(t)D_d)d(t) \\
\dot{z}(t) &= C_z\tilde{x}(t)
\end{align*}
\tag{4.8}
\]

and therefore the transfer function from \( d(t) \) to \( \tilde{z}(t) \) is

\[
T_{\tilde{z}d}(\Delta) = \begin{bmatrix}
A + L\Delta(t)C & B_d + L\Delta(t)D_d \\
C_z & 0
\end{bmatrix}.
\tag{4.9}
\]

When \( \hat{\Delta}(t) = I_{n_y} \), the estimated state satisfies the dynamics

\[
\dot{x}(t) = (A + LC)\tilde{x}(t) - L\Delta(t)Cx(t) + B_u u(t) - L\Delta(t)D_d d(t)
\tag{4.10}
\]

and the estimated error \( \tilde{z}(t) \) is given by \( \tilde{z}(t) = C_z x(t) - C_z \tilde{x}(t) \). Defining the
augmented state as \( [x(t)^T, \tilde{x}(t)^T]^T \), and \( w(t) = [u(t)^T, d(t)^T]^T \) as the external
input, the transfer function from \( w(t) \) to \( \tilde{z}(t) \) is given as

\[
T_{\tilde{z}w}(\Delta) = \begin{bmatrix}
A & 0 & B_u & B_d \\
-L\Delta(t)C & A + LC & B_u & -L\Delta(t)D_d \\
C_z & -C_z & 0 & 0
\end{bmatrix}
=: \begin{bmatrix}
\tilde{A}(\Delta(t)) & \tilde{B}(\Delta(t)) \\
\tilde{C} & \tilde{D}
\end{bmatrix}.
\tag{4.11}
\]
Figure 4.1: Fault-tolerant Observer Structure: when \( \hat{\Delta}(t) = I \) we call this a passive observer, while if \( \hat{\Delta}(t) = \Delta(t) \) we call it minimal switching observer.
Note that, in the minimal switching case described by equation (4.9), the performance is unaffected by $u(t)$. This is because when $\hat{\Delta}(t) = \Delta(t)$, the input dynamics do not affect $\hat{z}(t)$.

The fault-tolerant observer (FTO) problem considered in this chapter is to design an observer which is stable and achieves a minimum level of performance under all fault scenarios. Since the expectation is that the observer will mostly operate under the nominal (fault-free) condition, it is therefore required, in addition, to optimize the fault-free performance. In order to formally capture these requirements, the following two problems are considered, the first for the minimal switching FTO and the second for the passive FTO.

**Problem 1 (Fault Tolerant Minimal Switching Observer Problem)** Let $\gamma > 0$ and $\gamma_F > 0$ be given and let all other variables be as defined above. Find $L \in \mathbb{R}^{n \times n_y}$ such that

- $T_{\hat{z}d}(\Delta)$ is $Q$-stable
- $\|T_{\hat{z}d}(\Delta)\|_{i,2} < \gamma_F$
- $\|T_{\hat{z}d}(\Delta = I_{n_y})\|_{\infty} < \gamma$.

Such an $L$ will be called fault-tolerant minimal switching (FTMS) observer gain.

**Problem 2 (Fault Tolerant Passive Observer Problem)** Let $\gamma > 0$ and $\gamma_F > 0$ be given and let all other variables be as defined above. Find $L \in \mathbb{R}^{n \times n_y}$ such that

- $T_{\hat{z}w}(\Delta)$ is $Q$-stable
- $\|T_{\hat{z}w}(\Delta)\|_{i,2} < \gamma_F$
- $\|T_{\hat{z}w}(\Delta = I_{n_y})\|_{\infty} < \gamma$.

Such an $L$ will be called fault-tolerant passive (FTP) observer gain.

**Remark 1** Since it is assumed that all switching combinations are allowed, and in particular, all sensors may be out, it is therefore assumed that $A$ is stable. Of course, an unstable $A$ can be allowed provided it is assumed that at least one sensor is immune to faults.
4. A Semidefinite Relaxation Procedure for FTO Design

Remark 2 Although $H_{\infty}$ design methods to provide robustness against uncertainty is expected, it is not explicitly considered this issue in detail. However, in order to verify the robustness of our design, we carry out simulations using the nonlinear full order model in Chapter 5.

Lemma 2 can be used to give conditions for the solution of Problems 1 and 2.

Theorem 2 Consider the LPV systems $T_{zd}(\Delta)$ and $T_{zw}(\Delta)$ defined in (4.9) and (4.11), respectively.

1. Let all variables as defined in Problem 1. Then $L$ is an FTMS observer gain if there exist a $\tilde{P} = \tilde{P}^T > 0$, such that

\[
\begin{bmatrix}
\mathcal{H}(\tilde{P}(A + LC)) & \tilde{P}B_d + \tilde{P}LD_d & C_z^T \\
B_d^T \tilde{P} + D_d^T L^T \tilde{P} & -\gamma I & 0 \\
C_z & 0 & -\gamma I
\end{bmatrix} < 0
\] (4.12)

\[
\begin{bmatrix}
\mathcal{H}(\tilde{P}(A + L\Delta(t)C)) & \tilde{P}B_d + \tilde{P}L\Delta(t)D_d & C_z^T \\
B_d^T \tilde{P} + D_d^T \Delta(t)^T L^T \tilde{P} & -\gamma_F I & 0 \\
C_z & 0 & -\gamma_F I
\end{bmatrix} < 0, \ \forall \Delta(t) \in \Delta
\] (4.13)

2. Let all variables as defined in Problem 2. Then $L$ is an FTP observer gain if there exist a $\tilde{P} = \tilde{P}^T > 0$, such that

\[
\begin{bmatrix}
\mathcal{H}(\tilde{P}(A + LC)) & \tilde{P}B_d + \tilde{P}LD_d & C_z^T \\
B_d^T \tilde{P} + D_d^T L^T \tilde{P} & -\gamma I & 0 \\
C_z & 0 & -\gamma I
\end{bmatrix} < 0
\] (4.14)

\[
\begin{bmatrix}
\mathcal{H}(\tilde{P}A(\Delta(t))) & \tilde{P}\tilde{B}(\Delta(t)) & \tilde{C}_z^T \\
\tilde{B}(\Delta(t))^T \tilde{P} & -\gamma_F I & 0 \\
\tilde{C}_z & 0 & -\gamma_F I
\end{bmatrix} < 0, \ \forall \Delta(t) \in \Delta
\] (4.15)

Note that (4.12)-(4.15) are nonlinear. Linearization of (4.12)-(4.14) is straightforward by defining a matrix variable $F = \tilde{P}L$ while the linearization of (4.15) is less so, see Theorem 4 below. Note also that (4.13) and (4.15) need to be satisfied for every $\Delta(t) \in \Delta$ and, since $\Delta$ has $2^{ny}$ elements, evaluating $L$ becomes intractable for large $ny$. Note also that (4.14) is the same as (4.12) because, in
nominal conditions, the realizations of $T_{zw}(\Delta = I_{n_y})$ and $T_{zd}(\Delta = I_{n_y})$ are the same.

4.4 A Robustness Result for Binary-type Uncertainty

Note that the inequalities in (4.13) and (4.15) involving $\Delta(t)$ can both be written in the form $T_1 + T_2 \Delta T_3 + T_3^T \Delta T_2^T < 0$ for appropriate $T_1$, $T_2$ and $T_3$ (see Theorems 3 and 4 below). This is a general and widely used form for representing uncertainty in the control literature, although normally $\Delta$ represents a structured norm-bounded uncertainty while in this case $\Delta$ represents a structured binary uncertainty. In order to tackle the combinatorial nature of these problems, a general procedure is developed for representing a general class of uncertainties involving all combinations of binary variables and then use an elimination lemma and an extension of a semidefinite relaxation procedure for binary $(0,1)$ variables \cite{92,93,94}, to derive conditions for their solution. That is, a combinatorial analogue to the robustness results in \cite{95,96} is developed.

Consider the following inequality

$$T(\Delta) := T_1 + \mathcal{H} \left( T_2 \Delta (I - T_4 \Delta)^{-1} T_3 \right) < 0$$  \hspace{1cm} (4.16)$$

where $T_1 = T_1^T, T_2, T_3, T_4$ are given matrices of appropriate dimensions. It is required to find conditions such that $\text{det}(I - T_4 \Delta) \neq 0$ and (4.16) is satisfied for all $\Delta$ belonging to a set, $\Delta_c$ of the form

$$\Delta_c = \{ \text{diag}(\Delta_1, \ldots, \Delta_p) : \Delta_i \in \{ \underline{\Delta}_i, \overline{\Delta}_i \} \subset \mathbb{R}^{N_i \times N_i} \}, \hspace{1cm} i = 1, \ldots, p \}$$  \hspace{1cm} (4.17)$$

where $\underline{\Delta}_i, \overline{\Delta}_i$ for $i = 1, \ldots, p$ are given. Define

$$\underline{\Delta} = \{ \text{diag}(\underline{\Delta}_1, \ldots, \underline{\Delta}_p) : \underline{\Delta} \in \mathbb{R}^{N_i \times N_i} \} \in \Delta_c$$  \hspace{1cm} (4.18)$$

$$\overline{\Delta} = \{ \text{diag}(\overline{\Delta}_1, \ldots, \overline{\Delta}_p) : \overline{\Delta} \in \mathbb{R}^{N_i \times N_i} \} \in \Delta_c$$  \hspace{1cm} (4.19)$$
We will use the following version of the elimination lemma which can be found, e.g. in \cite{96}.

**Lemma 3 (Elimination)** Given real matrices $W = W^T$, $U$ and $V$ of appropriate size, there exists a real matrix $X$ such that

$$W + U X V^T + V X^T U^T < 0$$

if and only if

$$\tilde{U}^T W \tilde{U} < 0 \quad \text{and} \quad \tilde{V}^T W \tilde{V} < 0,$$

where $\tilde{U}, \tilde{V}$ are orthogonal complements of $U, V$.

Next, the elimination lemma is used to give necessary and sufficient conditions for \((4.16)\) in the case $p = 1$.

**Lemma 4** Let $T_1 = T_1^T \in \mathbb{R}^{n \times n}, T_2 \in \mathbb{R}^{n \times N}, T_3 \in \mathbb{R}^{N \times n}, T_4, \Delta, \overline{\Delta} \in \mathbb{R}^{N \times N}$. Define $T_4 := I - T_4 \overline{\Delta}$ and $\overline{T}_4 := I - T_4 \Delta$ and assume that $\det(T_4) \neq 0$ and $\det(\overline{T}_4) \neq 0$. Then \((4.16)\) is satisfied for $\Delta \in \{\Delta, \overline{\Delta}\}$ if and only if there exists $S \in \mathbb{R}^{N \times N}$ such that

$$\begin{bmatrix} T_1 - T_2 T_3^T T_2 + T_3^T \left( T_3 T_4 \Delta T_4^T + T_4 \Delta S T_4^T \right) \\ T_3 + (T_3 \Delta T_4^T + T_4 \Delta S T_4^T) T_2^T \end{bmatrix} < 0.$$  \hfill (4.22)

**Proof** A manipulation demonstrates that \((4.22)\) can be rewritten as \((4.20)\) with

$$W := \begin{bmatrix} T_1 & T_3^T \\ T_3 & 0 \end{bmatrix}, \quad X := -S$$

$$U := \begin{bmatrix} T_2 \Delta \\ -\overline{T}_4 \end{bmatrix}, \quad V := \begin{bmatrix} T_2 \overline{\Delta} \\ -\overline{T}_4 \end{bmatrix}$$

Furthermore, it can be verified that

$$\tilde{U} := \begin{bmatrix} I & T_2 \Delta \overline{T}_4^{-1} \end{bmatrix}^T, \quad \tilde{V} := \begin{bmatrix} I & T_2 \overline{\Delta} T_4^{-1} \end{bmatrix}^T$$  \hfill (4.23)
are orthogonal complements of $U$ and $V$, respectively. The result then follows from Lemma 3 by noting that $\tilde{U}^TW\tilde{U} = T(\Delta)$ and $\tilde{V}^TW\tilde{V} = T(\overline{\Delta})$.□

The following result is a structured version of the above, and allows us to derive sufficient conditions for (4.16) when $p > 1$.

**Lemma 5** Let $T_1 = T_1^T, T_2, T_3, T_4, T_5, T_6$ be as defined in Lemma 4 and let $\Delta_c, \Delta$ and $\overline{\Delta}$ be as defined in (4.17-4.19). Define

$$S = \{\text{diag}(S_1, \ldots, S_p) : S_i \in \mathbb{R}^{N_i \times N_i}\}$$

$$\mathcal{G} = \{G \in \mathbb{R}^{N \times N} : \Delta^G + G^T\Delta^T = 0 \ \forall \Delta \in \Delta_c\}.$$

Then $\det(I - T_4\Delta) \neq 0$ and (4.16) is satisfied for every $\Delta \in \Delta_c$, if there exist $S \in S$ and $G \in \mathcal{G}$ such that

$$\begin{bmatrix}
T_1 - T_2 \mathcal{H}(\Delta S\Delta^T)T_2^T & T_5^T + T_2(\Delta S\Delta^T + \overline{\Delta} \Delta^T)T_2^T - T_2G^T \\
T_3 + (T_1 S\Delta^T + T_1 S\Delta^T)^T T_2^T - G T_2^T & -\mathcal{H}(\Delta S\Delta^T + \overline{\Delta} \Delta^T)
\end{bmatrix} < 0. \quad (4.24)$$

If $p = 1$ the condition is necessary and sufficient.

**Proof** It can be verified from the definitions of $S$, $\mathcal{G}$, $\Delta_c$, $\Delta$ and $\overline{\Delta}$ that

$$\mathcal{H} ((\Delta - \Delta)S(\Delta - \overline{\Delta})^T + \Delta G) = 0, \quad \forall S \in S, \forall G \in \mathcal{G}, \forall \Delta \in \Delta_c. \quad (4.25)$$

Next, is proved that (4.24) implies that $\det(I - T_4\Delta) \neq 0 \ \forall \Delta \in \Delta_c$. Assume, for contradiction, that $\det(I - T_4\Delta) = 0$ for some $\Delta \in \Delta_c$ so that

$$z^T(I - T_4\Delta) = 0 \quad (4.26)$$

for some $z \neq 0$. Pre-and-post multiplying the $(2,2)$-block in (4.24) by $z^T$ and $z$, respectively, and using (4.26) and (4.25)

$$-z^TT_4^T \mathcal{H}((\Delta - \Delta)S(\Delta - \overline{\Delta})^T + \Delta G)T_4^Tz = 0 \quad (4.27)$$

This contradicts the negative definite property in (4.24) since $z \neq 0$.

Next, the sufficiency of (4.24) is provided. A manipulation verify that (4.24)
4. A Semidefinite Relaxation Procedure for FTO Design

can be written as

\[ W + U X V^T + V X^T U^T - J G T_{24}^T - T_{24} G^T J^T < 0 \]  (4.28)

where

\[ T_{24} := \begin{bmatrix} T_2^T & T_4^T \end{bmatrix}^T, \]

and

\[ J = \begin{bmatrix} 0 & I \end{bmatrix}^T, \]

and where \( W, X, U \) and \( V \) are defined in Lemma 4. Let \( Y = \begin{bmatrix} I & T_2 \Delta (I - T_4 \Delta)^{-1} \end{bmatrix}^T \). Then pre- and post-multiplying (4.28) by \( Y^T \) and \( Y \), respectively gives after a manipulation

\[ T(\Delta) + T_2 (I - \Delta T_4)^{-1} \mathcal{H} \{ (\Delta - \Delta) S (\Delta - \overline{\Delta})^T + \Delta G \} (I - \Delta T_4)^{-T} T_2^T < 0 \]  (4.29)

and the result follows from (4.25). Necessity when \( p = 1 \) follows from Lemma 4. Note that in this case, \( \mathcal{G} \) is not required. \( \square \)

**Remark 3** Note that, while \( S \) is easy to characterize since it requires only the size of the blocks of \( \Delta_c \), characterizing \( \mathcal{G} \) may not be straightforward, however, it need only be carried out for the elements of \( \Delta \) and \( \overline{\Delta} \). Note also that the results remain valid if \( \Delta_i = \overline{\Delta}_i \) for some \( i \).

4.5 Fault Tolerant Minimal Switching and Passive Observer Design

The next two results use Lemma 5 to provide a tractable solution to Problems 1 and 2.

**Theorem 3** Let all variables as defined in Problem 1. Then \( L \) is an FTMS observer gain if there exist a \( \tilde{P} = \tilde{P}^T \succ 0, F \in \mathbb{R}^{n \times n_y} \) and a diagonal \( S \in \mathbb{R}^{n_y \times n_y} \)...
such that
\[
\begin{bmatrix}
\mathcal{H}(\hat{P}A + FC') & \hat{PB}_d + F D_d & C^T_z \\
B^T_d \hat{P} + D^T_d F^T & -\gamma I & 0 \\
C_z & 0 & -\gamma I
\end{bmatrix} < 0 \tag{4.30}
\]
\[
\begin{bmatrix}
\mathcal{H}(\hat{P}A) & \hat{PB}_d & C^T_z & F + C^T S \\
B^T_d \hat{P} & -\gamma F I & 0 & D^T_d S \\
C_z & 0 & -\gamma F I & 0 \\
F^T + SC & SD_d & 0 & -2 S
\end{bmatrix} < 0 \tag{4.31}
\]
in which case \( L = \hat{P}^{-1}F \).

**Proof** \( T_{\hat{z}d}(\Delta) \) is \( Q \)-stable and \( \|T_{\hat{z}d}(\Delta)\|_{i,2} < \gamma_F \) if there exist \( \hat{P} = \hat{P}^T > 0 \) such that (4.13) is satisfied from Theorem 2. Defining \( F = \hat{P}L \), this can be written as
\[
\begin{bmatrix}
\mathcal{H}(\hat{P}A) & * & * \\
B^T_d \hat{P} & -\gamma F & * \\
C_z & 0 & -\gamma F
\end{bmatrix} + \mathcal{H}\left(\begin{bmatrix}
C^T \\
D^T_d \\
0
\end{bmatrix} \Delta(t) \begin{bmatrix} F^T & 0 \end{bmatrix}\right) < 0
\]
\[
=: T_1 + \mathcal{H}(T_2 \Delta^T T_3)
\]

since \( \Delta(t) = \Delta^T(t) \) in this case, (4.31) follows from Lemma 5 by noting that \( \Delta = 0, \Sigma = I, S = \{S \in \mathbb{R}^{n_y \times n_y} : S \text{ is diagonal}\}, G = \{0\} \). Finally, \( \|T_{\hat{z}d}(\Delta = I_{n_y})\|_{\infty} < \gamma \) if (4.30) is satisfied from Theorem 2. \( \square \)

**Theorem 4** Let all variables as defined in Problem 2. Then \( L \) is an FTP observer gain if there exist \( X = X^T, \hat{Y} = \hat{Y}^T \in \mathbb{R}^{n \times n}, M \in \mathbb{R}^{n \times n_y} \) and diagonal \( S \in \mathbb{R}^{n_y \times n_y} \) such that
\[
\begin{bmatrix}
X & \hat{Y} \\
\hat{Y} & \hat{Y}
\end{bmatrix} > 0 \tag{4.33}
\]
\[
\begin{bmatrix}
\mathcal{H}\{(X - \hat{Y})A - MC\} & (X - \hat{Y})B_d - MD_d & C^T_z \\
B^T_d (X - \hat{Y}) + D^T_d M^T & -\gamma I & 0 \\
C_z & 0 & -\gamma I
\end{bmatrix} < 0 \tag{4.34}
\]
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\[ \begin{bmatrix}
    \mathcal{H}(XA) & \mathcal{H}(\hat{Y}A) + MC & \hat{Y}B_u & XB_d & C_z^T & -M + C^T S \\
    \mathcal{H}(\hat{Y}A) + C^T M^T & \mathcal{H}(\hat{Y}A) & \hat{Y}B_u & \hat{Y}B_d & 0 & C^T S \\
    B_u^T \hat{Y} & B_u^T \hat{Y} & -\gamma_F I & 0 & 0 & 0 \\
    B_d^T X & B_d^T \hat{Y} & 0 & -\gamma_F I & 0 & D_d^T S \\
    C_z & 0 & 0 & 0 & -\gamma_F I & 0 \\
    -M^T + SC & SC & 0 & SD_d & 0 & -2S \\
\end{bmatrix} \preceq 0 \quad (4.35) \]

in which case \( L = (\hat{Y} - X)^{-1}M \).

**Proof** \( T_{zw}(\Delta) \) is \( Q \)-stable and \( \| T_{zw}(\Delta) \|_{i,2} \prec \gamma_F \) if (4.15) is satisfied from Theorem 2. Next, linearize (4.15), for a given \( \Delta(t) \), by using a procedure similar to that in [97]. Partition \( \hat{P} \) and \( \hat{P}^{-1} \) as

\[
\hat{P} = \begin{bmatrix} X & U \\ U^T & X_c \end{bmatrix}, \quad \hat{P}^{-1} = \begin{bmatrix} Y & V \\ V^T & Y_c \end{bmatrix} \quad (4.36)
\]

with \( X, Y, U \) and \( V \in \mathbb{R}^{n \times n} \). Since \( \hat{P} \hat{P}^{-1} = I \),

\[
UV^T = I - XY. \quad (4.37)
\]

Then \( \hat{P} \Pi_2 = \Pi_1 \) where

\[
\Pi_1 = \begin{bmatrix} X & I \\ U^T & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & Y \\ 0 & V^T \end{bmatrix}. \quad (4.38)
\]

Pre- and post-multiply (4.15) by \( \text{diag}(\Pi_2^T, I, I) \) and \( \text{diag}(\Pi_2, I, I) \), respectively and set \( V = V^T = Y \)

\[
\begin{bmatrix}
    \mathcal{H}(XA - M\Delta(t)C) & \Gamma_1(\Delta(t))^T & XB_u + UB_u & XB_d - M\Delta(t)D_d & C_z^T \\
    \Gamma_2(\Delta(t)) & \mathcal{H}(AY) & B_u & B_d & 0 \\
    B_u^T X + B_u^T U^T & B_u^T & -\gamma_F I & 0 & 0 \\
    B_d^T X - D_d^T \Delta(t) M^T & B_d^T & 0 & -\gamma_F I & 0 \\
    C_z & 0 & 0 & 0 & -\gamma_F I \\
\end{bmatrix} \prec 0 \quad (4.39)
\]
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where $\Gamma_2(\Delta(t)) = A + YA^T X - YC^T \Delta(t)^T M^T + YA^T U^T + YC^T M^T$ and $M = UL$. Note that there is no loss of generality in setting $V = Y$. Then, in order to linearize bilinear terms pre- and post-multiply by $\text{diag}(I, Y^{-1}, I, I, I)$,

$$
\begin{bmatrix}
\mathcal{H}(XA - M\Delta(t)C) & \hat{\Gamma}_2(\Delta(t))^T & XB_u + UB_u & XB_d - M\Delta(t)D_d & C_z^T \\
\hat{\Gamma}_2(\Delta(t)) & \mathcal{H}(\hat{Y}A) & \hat{Y}B_u & \hat{Y}B_d & 0 \\
B_u^T X & B_u^T \hat{Y} & -\gamma_F I & 0 & 0 \\
B_d^T X - D_d^T \Delta(t)^T M^T & B_d^T \hat{Y} & 0 & -\gamma_F I & 0 \\
C_z & 0 & 0 & 0 & -\gamma_F I \\
\end{bmatrix} < 0
$$

with $\hat{\Gamma}_2(\Delta(t)) = \hat{Y}A + A^T X - C^T \Delta(t)^T M^T + A^T U^T + C^T M^T$ and $\hat{Y} = Y^{-1}$. Because $V = V^T = Y$ has been set, using (4.37) obtain $U = \hat{Y} - X$, applying this in (4.40) and separating the terms depending on $\Delta(t)$ the following is obtained

$$
\begin{bmatrix}
\mathcal{H}(XA) & \mathcal{H}(\hat{Y}A) + MC & \hat{Y}B_u & XB_d & C_z^T \\
\mathcal{H}(\hat{Y}A) + C^T M^T & \mathcal{H}(\hat{Y}A) & \hat{Y}B_u & \hat{Y}B_d & 0 \\
B_u^T \hat{Y} & B_u^T \hat{Y} & -\gamma_F I & 0 & 0 \\
B_d^T X & B_d^T \hat{Y} & 0 & -\gamma_F I & 0 \\
C_z & 0 & 0 & 0 & -\gamma_F I \\
\end{bmatrix} + 
\begin{bmatrix}
C^T \\
C^T \\
0 \\
D_d^T \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\Delta^T(t) [ -MT & 0 & 0 & 0 ] \\
\end{bmatrix} < 0
$$

$$
=: T_1 + \mathcal{H} \left( T_2 \Delta^T T_3 \right)
$$

since $\Delta(t) = \Delta^T(t)$ in this case, (4.35) follows from Lemma 5 by noting that $\Delta = 0$, $\overline{\Delta} = I$, $S = \{ S \in \mathbb{R}^{n_u \times n_y} : S \text{ is diagonal} \}$, $\mathcal{G} = \{ 0 \}$. Notice that $T_{\overline{\zeta}}(\Delta) = T_{2\Delta}(\Delta)$ for $\Delta(t) = I_{n_y}$ and therefore, $\| T_{\overline{\zeta}}(\Delta = I_{n_y}) \|_\infty < \gamma$ if (4.34) is satisfied from Theorem 2. Finally, (4.33) is equivalent to $\Pi_2^T \hat{P} \Pi_2 > 0$. □

Remark 4 Compared with the corresponding solutions provided by Theorem 2, those in Theorems 3 and 4 have the advantage of many fewer LMIs to solve. Fur-
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Moreover, the number of extra variables (in $S$) is $n_y$ since $S$ is diagonal. However, the conditions provided by Lemma 5 are only sufficient (except when $p = 1$) although from the numerical experience, reported in the next section, indicates that they are sufficiently tight for practical systems.

4.6 Illustrative Example

An example from electric power transmission application is considered here to illustrate the proposed methodology. A 4th order reduced equivalent of the Nordic power transmission system is chosen as the plant. Six outputs have been selected to illustrate the concepts described before. The state space representation of this reduce order model is described in Appendix B.4.

Further details about this system can be found in [98, 99]. The dynamic response of this system is characterized by a pair of eigenvalues $\lambda_{1,2} = -0.08 \pm j1.82$ and $\lambda_{3,4} = -0.16 \pm j3.46$. Physically, these modes represent low frequency (less than 1 Hz) oscillations where electric power generators in one geographical area swing against the others in different locations. If not adequately damped, these oscillations could threaten the secure operation of the power systems [100].

To improve the damping of these modes, supplementary control loops through appropriate actuators (e.g. excitation systems of generators, static VAr compensators, etc.) are usually employed. The use of multiple feedback signals - both locally measured as well as remotely sensed and communicated - is often more effective due to better observability [35]. With several sensors distributed along the power transmission networks, the potential number of feedback signals available is large. However, there is a risk of loss of one or more of these feedback signals due to sensor failure or communications problems (collectively referred to as ‘sensor faults’ henceforth) which could adversely affect the closed-loop dynamic response.

In this example six feedback signals have been selected from different locations and, although the open loop system is stable, it will be illustrated that sensor faults could lead to bad tracking of the actual state or even closed-loop instability. In this context, the performance of the three types of observers was compared: an observer designed only for the nominal condition, and two fault-tolerant observers
designed using the procedures described in Section 4.5 where all possible sensor fault combinations are considered (a total of $2^6 = 64$ combinations). The values of the performance levels were $\gamma_N = 4 \times 10^{-10}$, $\gamma_{FTMS} = 1.1775$ and $\gamma_{FTP} = 2.5395$ while the corresponding observer gains were

$$L_N = \begin{bmatrix} -20.51 & -1.30 & 0.78 & 8.66 & -4.22 & -4.22 \\ 11.50 & 0.66 & 0.06 & -4.26 & 2.50 & 2.50 \\ -9.21 & -0.36 & -0.23 & 3.20 & -2.01 & -2.01 \\ -3.19 & -0.66 & 0.29 & 1.83 & -0.50 & -0.50 \end{bmatrix}$$

$$L_{FTMS} = \begin{bmatrix} 0.35 & -0.87 & 0 & 3.68 & 0 & -0.01 \\ 1.06 & -0.94 & 0 & 3.84 & 0 & -0.03 \\ -1.87 & 2.07 & 0 & -8.43 & 0 & 0.08 \\ 1.84 & -3.79 & 0 & 11.48 & 0 & 0.01 \end{bmatrix}$$

$$L_{FTP} = \begin{bmatrix} -8.18 & 4.47 & 0.53 & 0 & -2.75 & 0 \\ 4.56 & 1.27 & 1.33 & 0 & 27.50 & 0 \\ -4.54 & -1.22 & -1.30 & 0 & -27.09 & 0 \\ -12.41 & 2.09 & -1.22 & 0 & -36.55 & 0 \end{bmatrix} \times 10^{-3}$$

The cost function for the nominal observer ($\gamma_N$) is close to zero because only one (fault-free) scenario is considered, while the worst cost function corresponds to the passive approach since it makes no use of any information about the faults. Note also that the exact values (obtained by solving $2^6$ LMIs corresponding to each fault scenario) of $\gamma_{FTMS}$ and $\gamma_{FTP}$ are 1.1770 and 2.5373, respectively, so that the approximation is quite accurate.

Table 4.1 lists whether the closed-loop system (described by $A + L\Delta(t)C$) is stable (‘s’) or unstable (‘u’) using the three different types of observers for the first 20 combinations (out of the possible 64) of $\Delta(t)$. It is evident that with the nominal observer (using the minimal switching structure $\hat{\Delta}(t) = \Delta(t)$) the closed-loop system is unstable for certain sensor fault combinations while it always remains stable using either form of the fault-tolerant observers. Figure 4.2 compares the
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<table>
<thead>
<tr>
<th>$\Delta(t)$</th>
<th>Combinations $[1,2,\ldots,20]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1(t)$</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$\delta_2(t)$</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1</td>
</tr>
<tr>
<td>$\delta_3(t)$</td>
<td>0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0</td>
</tr>
<tr>
<td>$\delta_4(t)$</td>
<td>0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0</td>
</tr>
<tr>
<td>$\delta_5(t)$</td>
<td>0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1</td>
</tr>
<tr>
<td>$\delta_6(t)$</td>
<td>0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1</td>
</tr>
</tbody>
</table>

$L_N$ s s u s u u u s u u u s u u u
$L_{FTMS}$ s s s s s s s s s s s s s s s
$L_{FTP}$ s s s s s s s s s s s s s s s

Table 4.1: Closed-loop stability for the first 20 fault combinations using the nominal ($L_N$), fault-tolerant minimal switching ($L_{FTMS}$) and fault-tolerant passive ($L_{FTP}$) observer gains, where “s” stands for stable and “u” for unstable.

time variation of the state $x_1(t)$ (only the first state variable is presented to avoid cluttering) in black, the estimated states $\hat{x}_1(t)$ using the nominal observer (gain $L_N$) both in minimal switching (blue, solid) and passive (blue, dotted) modes, the fault-tolerant minimal switching approach (observer gain $L_{FTMS}$) in red and the fault-tolerant passive approach (gain $L_{FTP}$) in green. The plots represent one particular situation where sensors 1, 2, 3 and 4 have failed ($\Delta(t) = I_6, t < 10$; $\Delta(t) = diag(0,0,0,0,1,1)$, $t \geq 10$). This faulty situation corresponds to column 4 of Table 4.1 where the closed-loop is stable for all the observer gains. It can be seen from the plot that before the fault occurs, all the observer gains track the actual state well (with $L_N$ best, followed by $L_{FTMS}$ then $L_{FTP}$), however, following the fault at 10sec the nominal observer gain ($L_N$) diverge significantly from the actual state with $L_{FTMS}$ performing best.

Figure 4.3 exhibits a similar comparison; here only sensors 3, 4 and 6 are considered to be operating while all the others fail at $t = 10sec$ ($\Delta(t) = I_6, t < 10$; $\Delta(t) = diag(0,0,1,1,0,1)$, $t \geq 10$). This corresponds to column 14 of Table 4.1. Similar conclusions can be drawn as in the previous case, the nominal observer connected in the minimal switching mode (blue trace) becomes unstable.
4. A Semidefinite Relaxation Procedure for FTO Design

\[
\Delta(t) = 0 \quad \text{for } t < 10 \quad \text{and} \quad \Delta(t) = \text{diag}[\delta_1 = 0, \delta_2 = 0, \delta_3 = 1, \delta_4 = 1, \delta_5 = 0, \delta_6 = 1] \quad \text{for } t \geq 10
\]

Figure 4.2: State estimate comparison for nominal observer in switching mode (NMS) and passive mode (NP), fault-tolerant minimal switching (FTMS) and fault-tolerant passive mode (FTP) following a fault in sensors 1, 2, 3 and 4 at 10 sec.

\[
\Delta(t) = 0 \quad \text{for } t < 10 \quad \text{and} \quad \Delta(t) = \text{diag}[\delta_1 = 0, \delta_2 = 0, \delta_3 = 1, \delta_4 = 1, \delta_5 = 0, \delta_6 = 1] \quad \text{for } t \geq 10
\]

Figure 4.3: State estimate comparison for nominal observer in switching mode (NMS) and passive mode (NP), fault-tolerant minimal switching (FTMS) and fault-tolerant passive mode (FTP) following a fault in sensors 1, 2 and 5 at 10 sec.
4.7 Summary

In this chapter, fault-tolerant passive and active observer design methods have been proposed and it has been proved that they guarantee a minimum level of closed-loop performance under all possible sensor fault combinations while optimizing performance under the fault-free condition. The performance is measured by the $H_\infty$-norm of the transfer matrix from the external signals to the state estimation error. The problem was first recast in a more general robust design setting where the uncertainty set is composed of all combinations of a set of binary variables. Sufficient conditions (which are also necessary for the case of one binary variable) for the solution of the problem were derived which resulted in a significant reduction in the number of matrix inequalities needed to solve the problem. Although the chapter considered a fault-tolerant observer design problem against sensor faults, the presented results are general and apply to other problems involving combinations of sensor, actuator and process faults as well as observer/state feedback design.
Chapter 5

Application to Power Systems

5.1 Introduction

In this chapter, case studies on a reduced equivalent of the Nordic electric power transmission system are presented. These case studies illustrate the effectiveness of the proposed fault-tolerant control schemes (described in the previous chapters) in the context of electric power systems.

Section 5.2 presents a detailed description of the Nordic power system that corresponds to a simplified version (thousand of states) of the original model. The reduced system was modelled in Matlab SIMULINK. A single line diagram of the system displaying the different areas, tie-lines and different device locations is presented in Figure 5.1. There are two poorly damped low frequency inter-area modes of oscillation which are to be damped through a static VAr compensator (SVC) located at Hasle near Oslo. Remote feedback signals were chosen for effective damping control. Tables showing the most appropriate wide-area and local signals are listed as well as a description of the signals chosen. Section 5.3 validates and compares the performance of the controllers described in Chapter 3: one conventional and two passive fault-tolerant designs. The design procedures are described and in this section one local and one wide-area signal are chosen to improve the damping of the inter-area modes. The control objectives are defined as follows: to achieve at least 10% of damping ratio in nominal conditions for all the controllers and at least 8% following the loss of the remote signal for the
fault-tolerant controllers (FTCs), to ensure acceptable settling time in the oscillations and avoid activation of line protections [101]. Subsection 5.3.1 presents the dynamic performance under the nominal conditions. Here it is demonstrated that although the controllers provide similar performance, both FTCs require more control effort. In Subsection 5.3.2 the dynamic performance following the loss of the remote signal is presented. It can be seen how the performance of the conventional control design is deteriorated, while for both FTCs the performance is acceptable. Subsection 5.3.3 describes a summary of the comparison. Section 5.4 provides an extension procedure to design a fault tolerant minimal switching observer, as described in Chapter 4, but including regional pole placement for power systems applications. Section 5.5 validates and compares the performance of the fault-tolerant minimal switching (FTMS) observer including regional pole placement. Finally, Section 5.6 presents a summary of the chapter.

5.2 Test System

A equivalent of the Nordic system is used for the case study. The detailed model with approximately 3000 buses, 4000 branches and 1100 generators [22, 102] was reduced down to a 20 generators and 36 bus equivalent system, shown in Fig. 5.1. The aim was to retain the modal behaviour of the two critical poorly damped inter-area modes. A static VAr compensator (SVC) is included in the model at Hasle substation (5101) in south-east Norway as in practice. The total number of state variables for the linearized version of the reduced equivalent system is 296. There are two critical inter-area modes, 0.29 and 0.55 Hz with 4.8% and 5.4% damping, respectively, under nominal condition, see Table 5.1. The first mode (0.29 Hz) comprises the Finnish generators swinging against the rest whereas the second mode (0.55 Hz) involves the generators in the north of Finland, Sweden and Norway swinging against those in the southern parts of these countries.

Following critical contingencies such as an outage of one of the parallel lines connecting (i) 6500-6700 (Norwegian coast line), (ii) 7100-7000 (Finnish line) or (iii) 3359-5101 (Hasle line), the damping of the inter-area modes could be as low as 2-3% requiring improvement through supplementary control. Here the challenge is to ensure satisfactory damping (at least 10%) of both the modes by
designing a power oscillation damping (POD) for the SVC at Hasle.

With a number of phasor measurement units (PMUs) (only four in Norway and two in Finland are considered for this study) installed throughout the Nordic system remote feedback signals were also considered in addition to the local signals. The difference between voltage angles available from the PMUs were chosen as potential candidates [22] for remote signals. The magnitude and phase angle of the residues for each candidate is displayed in Table 5.2. The method for calculating the residues can be found in Appendix A.1. Voltage angle differences between the PMU at 6100 (at Nedre Rossaga) and 7000 (in southern Finland) and the PMU at 5603 (Kristiansand) and 7000 have the highest residue magnitudes for both modes. Their phases are also aligned contrary to the case of 6700-7000 which is in phase opposition. Thus based on both magnitude and phase angle of the residues [103], 6100-7000 and 5603-7000 (presented in boldface in Table 5.2) were selected as the most appropriate signals for the POD.

Out of the available local signals, magnitude and phase angle of the line
5. Application to Power Systems

Table 5.1: Damping and Frequencies of the Inter-Area Modes

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio ζ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.29</td>
<td>4.9</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.55</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 5.2: Remote signal selection: Magnitude and phase angle of residues for difference between relevant voltage angles

<table>
<thead>
<tr>
<th>Signal</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>5101 – 5603</td>
<td>0.03</td>
<td>0.68</td>
</tr>
<tr>
<td>5101 – 7000</td>
<td>2.28</td>
<td>1.55</td>
</tr>
<tr>
<td>5101 – 7100</td>
<td>1.26</td>
<td>1.97</td>
</tr>
<tr>
<td>5603 – 7000</td>
<td>2.32</td>
<td>2.22</td>
</tr>
<tr>
<td>5603 – 6700</td>
<td>0.34</td>
<td>3.41</td>
</tr>
<tr>
<td>5603 – 7100</td>
<td>1.29</td>
<td>2.65</td>
</tr>
<tr>
<td>6100 – 7000</td>
<td>2.31</td>
<td>2.44</td>
</tr>
<tr>
<td>6700 – 7000</td>
<td>1.97</td>
<td>1.26</td>
</tr>
<tr>
<td>7000 – 7100</td>
<td>1.02</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 5.3: Local Signals: Magnitude and phase angle of residues for Currents

<table>
<thead>
<tr>
<th>Signal</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>3359 – 5101</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>5101 – 5501</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>5101 – 5501</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>5100 – 5100</td>
</tr>
<tr>
<td></td>
<td>I</td>
<td>5100 – 5100</td>
</tr>
</tbody>
</table>

currents were found to be the most effective candidates. The magnitude and phase angle of the residues associated with the local current signals are shown in Table 5.3. Phase angle of current in line 5101-5501 (presented in boldface in Table 5.3) was selected as the most appropriate local signal based on both the magnitude and phase angle of the residue.
5. Application to Power Systems

Figure 5.2: Frequency response of the original and the reduced model. The frequency response has been measured from the 3 outputs highlighted in boldface on Tables 5.2 and 5.3 to the single input.

5.3 Case Study 1: Evaluation of the Passive FTC

The control design formulation presented in Chapter 3 produces controllers that are of the same size (dimension) as the plant. To get around this problem, the linearized model of the test system (described in Section 5.2) with 296 states was first reduced to a 12th order equivalent using balanced truncation model reduction [104]. More details of the model reduction technique can be found in Appendix A.2. The similarity between the frequency responses of the original and reduced systems were used to validate the model reduction, Figure 5.2 confirms the validity of this approach.

Case study one (CS 1) compares the performance of three different controllers (each of order 12) using one local (phase angle of the current in line 5101 − 5501) and one remote (voltage angle difference between the PMUs 5603 − 7000) signals: conventional (CC), passive (FTC\textsubscript{p}) and iterative passive (FTC\textsubscript{it}) control. The
controllers were designed using the approaches described in Sections 3.3, 3.4 and 3.5, respectively. Like CC, both of the FTCs also improve the damping of the two critical modes up to 10% under nominal conditions. In addition, both FTCs maintain the damping above 8% without the remote signal. However, with CC the damping of mode 1 reduces to 4.9% in absence of the remote signal.

In this section the dynamic performance with the passive fault-tolerant controller (FTC_p) and the iterative passive fault-tolerant controller (FTC_it) are compared against a conventional controller (CC) under both nominal (when remote signal is present) and ‘fault’ (when remote signal is lost) conditions. Both of the FTCs and CC were designed using the same local signal (boldface in Table 5.3) and the same remote feedback signal (difference between voltage angles 6100-7000) displayed in Table 5.2. The design specification in both cases was to achieve a minimum 10% damping. However, for both of the FTCs, the target damping was reduced to 8% for the ‘fault’ condition to obtain a feasible solution. A short circuit followed by outage of tie-line 6500-6700 was created to compare the dynamic performance with CC, FTC_p and FTC_it.

### 5.3.1 CS 1: Nominal Condition

Under nominal conditions, the dynamic performance with FTC_p and CC as seen from the power flows through the lines 3359-5101 (Fig. 5.3(a)) and 7000-7100 (Fig. 5.3(b)) are similar. The dynamic response with FTC_it as seen from the power flow through the lines 3359-5101 (Fig. 5.3(a)) is better. In addition, as seen from Fig. 5.3(b), the best dynamic response is obtained with FTC_p. This information is substantiate by looking at the closed-loop pole locations in Table 5.4. However, higher control effort is required by the FTCs as indicated by larger excursion in $B_{SVC}$ (Fig. 5.3(c)) and also the bus voltage (Fig. 5.3(d)). Nevertheless, the control effort required by the iterative passive FTC_it is less than the required by the passive FTC_p. Thus under nominal condition, CC is preferable from control effort point of view even though the dynamic variation of SVC output (susceptance) for both FTCs are within the specified limits.
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Figure 5.3: Comparison of dynamic performance with CC, FTC\textsubscript{p} and FTC\textsubscript{it} under nominal (remote signal present) condition

Table 5.4: Closed loop damping ratios and frequencies

<table>
<thead>
<tr>
<th>Nominal Condition</th>
<th>Frequency (Hz)</th>
<th>Damping Ratios (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ζ\textsubscript{CC}</td>
<td>ζ\textsubscript{FTC\textsubscript{p}}</td>
</tr>
<tr>
<td>Mode 1</td>
<td>0.29</td>
<td>11.53</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.55</td>
<td>12.06</td>
</tr>
</tbody>
</table>

5.3.2 CS 1: Loss of Remote Signal 5603 – 7000

The performance with CC is compared against FTC\textsubscript{p} and FTC\textsubscript{it} in Fig. 5.4 in the face of loss of remote signal. The dynamic variation of the power flow through line 3359-5101 (Fig. 5.4(a)) is not affected much by the loss of the remote signal for any of the three controllers. As mentioned before, this power flow predominantly reflects mode 2 which has adequate observability (residue) in the local signal which explains the above trend, see Table 5.5. On the other hand, power flow through 7000-7100 has prominent presence of mode 1 which is not adequately observable in the local signal. Hence, with CC, loss of the remote signal results in significant deterioration in the dynamic response (Fig. 5.4(b)). However, with FTC\textsubscript{p} and FTC\textsubscript{it}, satisfactory performance is maintained for both power flows with only the local signal. Similar to the nominal condition, the control effort
5. Application to Power Systems

![Figure 5.4: Comparison of dynamic performance with CC, FTC\textsubscript{p} and FTC\textsubscript{it} following loss of remote signal](image)

Table 5.5: Closed loop damping ratios and frequencies

<table>
<thead>
<tr>
<th>Faulty Condition</th>
<th>Mode No</th>
<th>Frequency (Hz)</th>
<th>Damping Ratios (%)</th>
<th>FTC\textsubscript{p}</th>
<th>FTC\textsubscript{it}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.29</td>
<td>4.93 8.61</td>
<td>Mode 2</td>
<td>0.55</td>
<td>10.83 13.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.36</td>
</tr>
</tbody>
</table>

required by the FTC\textsubscript{p} is higher than FTC\textsubscript{it} and CC as evident through larger excursions of $B_{SVC}$ (Fig. 5.4(c)).

Although the FTCs require larger control effort than CC to achieve a desired damping under nominal condition, the formers can guarantee an acceptable performance level when the remote signal is lost contrary the CC which leads to significant deterioration in dynamic behavior. The FTC\textsubscript{it} improves the dynamic performance of the system and requires less control effort than the passive FTC\textsubscript{p}.

5.3.3 Summary of Section 5.3

The case study on the Nordic equivalent system determines that the closed-loop response could deteriorate drastically if the remote signal is lost. The passive fault-tolerant control (FTC\textsubscript{p}) design methodology is presented to solve this prob-
lem and ensure an acceptable performance level even in case of loss of remote signals and an iterative passive (FTC\textsubscript{p}) is proposed to reduce conservativeness and improve the dynamic response achieved using the passive procedure. Under nominal condition (when both local and remote signals are present) both fault-tolerant controllers requires more control effort as compared to a conventional controller (CC) in order to achieve similar performance. However, based on the case studies on the Nordic equivalent system confirm that the proposed (FTC\textsubscript{u}) is able to produce acceptable performance in case of loss of the remote signals, improve the dynamic response and reduce the control effort compared with the passive FTC\textsubscript{p} while the response with a CC is unacceptable if a fault occurs.

### 5.4 Extension: FTMS Observer Design Including Regional Pole Placement

In this section an extension procedure to design a fault tolerant minimal switching observer is presented, as described in Chapter 4, including regional pole placement for power systems applications. As mentioned in Section 2.2, power systems typically have low frequency modes that cause oscillations. To improve the stability of these systems, placing the oscillatory modes in a desired location of the complex plain is desirable to reduce settling times and improve damping ratios.

First, the state feedback control law \( u(t) = Kx(t) \) is designed. The feedback matrix \( K \) is chosen such that \( A + BuK \) is stable and has all its eigenvalues in a desired location, based on new settling time or damping ratio. Since the states of an electric power system are not available for feedback, an observer for estimating \( K\hat{x}(t) \) is then designed, as shown in Figure 5.5. The observer gain \( L \) is calculated so that the following design criteria are satisfied:

1. The nominal closed-loop matrix \( A + LC \) for the observer is stable and all the eigenvalues lie within the conic region with inner angle \( \theta \), as displayed in Figure 3.1.
5. Application to Power Systems

2. The performance for the transfer matrix

\[ T_{zd}(\Delta) = \begin{bmatrix} A + L\Delta(t)C & B_d + L\Delta(t)D_d \\ K & 0 \end{bmatrix} \]

from the disturbance \( d(t) \) to the estimation error \( \tilde{z}(t) = z(t) - \hat{z}(t) \) is maximized in the nominal case (by minimizing \( \|T_{zd}(\Delta = I)\|_\infty < \gamma \)).

3. The closed-loop matrix \( A + L\Delta C \) for the observer is stable for all faulty scenarios to guarantee fault tolerance.

The following result uses Lemma 5 to give an LMI procedure for calculating such an \( L \).

**Theorem 5** Let all variables be as defined above. The matrix \( L \) is an FTMS observer gain that places the closed-loop poles of \( A + L\Delta(t)C \) for \( \Delta(t) = I \) within the conic region with inner angle \( \theta \), maximizes the performance for \( \Delta(t) = I \) and guarantees stability for all \( \Delta(t) \in \Delta_c \), if there exist a \( \tilde{P} > 0 \) and a diagonal \( S \in \mathbb{R}^{n_y \times n_y} \) such that

\[
\begin{bmatrix}
\sin \theta(\tilde{P}A + AT\tilde{P} + \tilde{P}LC + CTLT\tilde{P}) & \cos \theta(\tilde{P}A - AT\tilde{P} + \tilde{P}LC - CTLT\tilde{P}) \\
\cos \theta(\tilde{P}A - AT\tilde{P} + \tilde{P}LC - CTLT\tilde{P})^T & \sin \theta(\tilde{P}A + AT\tilde{P} + \tilde{P}LC + CTLT\tilde{P})
\end{bmatrix} < 0
\] (5.1)

\[
\begin{bmatrix}
\tilde{P}A + AT\tilde{P} + \tilde{P}LC + CTLT\tilde{P} & \tilde{P}B_d + \tilde{P}LD_d & K^T \\
B_d^T\tilde{P} + D_d^TLT\tilde{P} & -\gamma I & 0 \\
K & 0 & -\gamma I
\end{bmatrix} < 0
\] (5.2)

\[
\begin{bmatrix}
\tilde{P}A + AT\tilde{P} & \tilde{P}L + CT\tilde{S} \\
LT\tilde{P} + STC & -S - ST
\end{bmatrix} < 0
\] (5.3)

Note that (5.1), (5.2) and (5.3) are nonlinear and their linearization is straightforward by defining a matrix variable \( F = \tilde{P}L \). Also note that from Theorem 1, (5.1) ensures pole placement and from Lemma 2, (5.2) maximizes performance (by minimizing \( \gamma \)), both for nominal conditions (when \( \Delta(t) = I \)). Finally note that (5.3) follows from Lemma 5 and guarantees stability for all faulty conditions. The closed-loop block diagram it is presented in Figure 5.5. An example illustrating the use of Theorem 5 is given in Section 5.5 below.
Remark 5 A nominal observer gain $L$ can be designed excluding condition (5.3). However, this nominal observer does not guarantee stability following any sensor loss, although it may have better performance for the nominal conditions.

5.5 Case Study 2: Evaluation of the FTMS Observer

In this section is presented a second case study (CS 2), to validate the performance of the fault-tolerant minimal switching observer described in the previous section. Although in the previous section it has been confirmed that use of local signal guarantees stability of the closed-loop system following the loss of the remote signal, it should be noted that this is specific to this particular set up but need not necessarily be the case always. Recognizing this fact, in this section is presented the following approach that guarantees stability following the loss of any signal. Because high magnitude of the residue is essential for effective control design, the voltage angle differences between the PMUs 5603 – 7000 and 6100 – 7000 have been chosen as the most appropriate remote signals in the following case study, see Table 5.2 and Figure 5.1.

The simulations presented in this section are the result of applying the state feedback control law $u(t) = Kx(t)$ to the test system. The feedback matrix $K$ has been chosen to improve the two critical inter-area modes of the system $(A + B_uK)$ up to $\zeta = 10\%$ under nominal conditions (when remote signals are present), as seen in Table 5.6.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>Damping Ratios (%) $\lambda(A + B_uK)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.29</td>
<td>10</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.55</td>
<td>10</td>
</tr>
</tbody>
</table>

Since the states of an electric power system are not available for feedback, two different observer gains for estimating the states $K\dot{x}(t)$ have been designed and compared. One observer ($L_N$) was designed only for the nominal condition
5. Application to Power Systems

Figure 5.5: Implementation of the fault-tolerant minimal switching observer ($\Delta(t) = \Delta(t)$), including regional pole placement.
while the fault-tolerant minimal switching observer ($L_{FTMS}$) was designed for both nominal and faulty conditions; the full design details for both observers are described in Section 5.4. Table 5.7 displays the closed-loop stability condition for all possible scenarios. A short circuit followed by outage of tie-line 6500-6700 at 5sec was created to compare the dynamic performance with $L_N$ and $L_{FTMS}$.

Table 5.7: Closed-loop stability condition for all different scenarios using the nominal ($L_N$) and fault-tolerant minimal switching ($L_{FTMS}$) observer gains.

<table>
<thead>
<tr>
<th></th>
<th>Nominal $\Delta(t) = diag(1,1)$</th>
<th>Loss of 5603–7000 $\Delta(t) = diag(0,1)$</th>
<th>Loss of 6100–7000 $\Delta(t) = diag(1,0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_N$</td>
<td>stable</td>
<td>unstable</td>
<td>stable</td>
</tr>
<tr>
<td>$L_{FTMS}$</td>
<td>stable</td>
<td>stable</td>
<td>stable</td>
</tr>
</tbody>
</table>

5.5.1 CS 2: Nominal Condition

To ensure fast state estimation, both observers were designed to place the closed-loop poles in nominal conditions ($A + LC$) inside a cone of at least $\zeta = 18\%$ of damping ratio. The selected region ensure that the observer pole locations are further to the left of the complex plane than using the gain $K$. This damping ensures a conic region with inner angle $\theta = 79.6^\circ$. In addition, the performance for the nominal case is maximized, by minimizing some disturbance rejection measures $\gamma_N$ and $\gamma_{FTMS}$. The simulation results are presented in Figure 5.6.

Under nominal condition, the dynamic performance with $L_N$ and $L_{FTMS}$ is similar; as seen from the power flows through the lines 3359–5101 (Fig. 5.6(a)) and 7000-7100 (Fig. 5.6(b)). Higher control effort is required for $L_{FTMS}$ as indicated by the larger excursion in $B_{SVC}$ (Fig. 5.6(c)) and the bus voltage (Fig. 5.6(d)). This is also substantiated by the values of $\gamma$ ($\gamma_N = 1.46e - 10$ and $\gamma_{FTMS} = 2.99$). This situation corresponds to the first column of Table 5.7 where the closed-loop is stable for all the observer gains, and is recalled Remark 5 from Section 5.4, where it is indicated that better performance and lower value of $\gamma_N$ is expected using the nominal observer gain $L_N$. 

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Figure 5.6: Comparison of dynamic performance with Nominal ($L_N$) and Fault-Tolerant Minimal Switching ($L_{FTMS}$) observer gains under nominal (both remote signals present) condition. Where $\gamma_N = 1.46e - 10$ and $\gamma_{FTMS} = 2.99$

5.5.2 CS 2: Loss of Remote Signal 5603 – 7000

This faulty situation corresponds to the second column of Table 5.7 where the closed-loop is unstable for the nominal observer gain $L_N$ but is stable for the fault-tolerant minimal switching $L_{FTMS}$. The nonlinear simulation results, presented in Figure 5.7, substantiates the results of Table 5.7, where power flows, control output and bus voltage are unstable when using the nominal observer $L_N$. Using the FTMS observer $L_{FTMS}$ not only ensures stability but also maintains the level of performance due to accurate estimation of the actual states.

5.5.3 CS 2: Loss of Remote Signal 6100 – 7000

For this case, the closed-loop system is stable for both observers (as expected from the third column of Table 5.7). However, the dynamic responses shown in Fig. 5.8 confirm that the performance when using the nominal observer ($L_N$) deteriorates. This can be attributed to inaccurate tracking of the actual states in a faulty situation with $L_N$. The simulation results confirm the effectiveness of the FTMS observer ($L_{FTMS}$) and also illustrates that ensuring good state estimation
Figure 5.7: Comparison of dynamic performance with Nominal ($L_N$) and Fault-Tolerant Minimal Switching ($L_{FTMS}$) observer gains following the loss of signal 5603 – 7000.

in faulty conditions guarantees: 1) stability in the system and 2) maintenance of an acceptable level of performance, as seen in Figure 5.8.

### 5.5.4 Summary of Section 5.5

The case study determines that, when the nominal observer gain $L_N$ is used, the closed-loop performance can deteriorate, and even become unstable, if some of the remote signals are lost. The fault-tolerant minimal switching observer gain ($L_{FTMS}$) is presented to solve this problem and provide stability in case of loss of remote signals. Under nominal conditions, the FTMS observer requires more control effort compared to a nominal observer ($L_N$) in order to achieve similar performance. However, based in the case study, the proposed observer scheme ($L_{FTMS}$) is able to maintain stability and performance in case of loss of remote signals.
5. Application to Power Systems

Figure 5.8: Comparison of dynamic performance with Nominal ($L_N$) and Fault-Tolerant Minimal Switching ($L_{FTMS}$) observer gains following the loss of signal 6100 – 700.

5.6 Summary

In this chapter have been presented the simulation results to validate the architectures described in Chapters 3 and 4. First, in Figure 5.1, the diagram of the nonlinear model used in the simulation, along with the main characteristics of the model, has been illustrated. Then, a full description of the remote and local signals considered for control design were presented and summarized. Two case studies were presented: first to illustrate the effectiveness of the passive fault-tolerant controllers proposed in Sections 3.4 and 3.5 and then to evaluate the performance of the fault-tolerant minimal switching observer described in Section 5.4. Results in case study 1 illustrate how passive fault-tolerant controllers guarantee performance in faulty conditions. Case study 1 also illustrates how an iterative procedure in the passive design improves the closed-loop damping ratios and reduces the control effort. Case study 2 confirms that controllers based on minimal switching fault-tolerant observers guarantee accurate state estimation, reliability and stability for all possible conditions.
Chapter 6
Conclusions

6.1 Conclusions

In this work, the stability and the performance of the closed-loop system following the loss of sensors (sensor faults) was studied. Stability and performance optimization for the fault-free conditions was provided in addition to guaranteeing stability as well as a minimum level of performance in the case of sensor loss. One of the main challenges addressed in this work was to deal with the combinatorial binary problem when the number of sensors is large.

First, an extensive review of the state-of-the-art was presented. The review of the background that motivates the problems that were addressed in this thesis was given. The aim of the first part of the thesis was to present the problems discussed and comment on relevant literature related to this research. The background for the mathematical tools that were used to develop the design procedures was also introduced. In particular, the importance, as well as the challenges associated with the use of wide-area measurements for the stabilization of power systems were highlighted and were shown to be the main illustrative case study in this thesis.

The first design scheme to address the issue of fault-tolerance in the design of controllers for the stabilization of power systems was presented. The representation of sensor faults considering one local and at least one remote signal was described. Only the loss of one signal at the time was considered, although this restriction was relaxed in the following chapter. First, one conventional controller
6. Conclusions

(CC) was designed to limit the control effort and place the closed-loop poles in a desired location for nominal conditions. Then, two different passive fault-tolerant controllers (FTCs) designs were proposed; one based on an iterative approach. Both passive FTCs guarantee a minimum level of performance following the loss of one remote signal.

A fault-tolerant passive and active observer design methods were presented. It was proved that they guarantee a minimum level of closed-loop performance under all possible sensor fault combinations while optimizing performance under the fault-free condition. The performance was measured by the $H_\infty$-norm of the transfer matrix from the external signals to the state estimation error. The problem was first recast in a more general robust design setting where the uncertainty set is composed of all combinations of a set of binary variables. Sufficient conditions (which are also necessary for the case of one binary variable) for the solution of the problem were derived which resulted in a significant reduction in the number of matrix inequalities needed to solve the problem. Although a fault-tolerant observer design problem against sensor faults was considered, the results presented are general and apply to other problems. Sensor, actuator and process faults are some examples involving combinations of faults.

Simulation results to validate the architectures described in previous chapters were presented. The diagram of the nonlinear model used in the simulation, along with the main characteristics of the model, were presented. A full description of the wide-area and local signals considered for control design were presented and summarized in different tables. Two case studies were described: case study 1 to illustrate the effectiveness of the passive fault-tolerant controllers and case study 2 to evaluate the performance of the fault-tolerant minimal switching observer. A summary of the results obtained for each study case was described at the end of each section.
6.2 Contributions

The main contributions of this research are summarized as follows.

- **Solution to a combinatorial binary problem.** This was one of the main challenges in this work since the number of fault scenarios grows exponentially with the number of potentially faulty sensors. In Section 4.4, an elimination lemma and an extension of a semidefinite relaxation procedure for binary variables were presented to derive sufficient conditions, in the form of a small number of LMIs, for the solution of the fault tolerant observer design problem.

- **Minimal switching observer approach.** A reconfigurable architecture that lies within the class of active schemes has been proposed. In this approach one gain matrix $L$ is designed and the word minimal switching indicates that the switching is between the observer outputs rather than between different observer gain matrices. It is important to emphasize that the proposed approach requires instant information about the fault.

- **Linearization of Nonlinear Constrains.** All the design algorithms presented in this thesis are linear. Although the analyzed model is linear, due to the incorporation of many constraints in the control/observer design, all of the optimization problems become nonlinear, as seen in Sections 3.4, 3.5 and 4.3. In some of these problems, the linearization was straightforward since it consisted in defining a new variable, however, in some cases the linearization it was not obvious and several mathematical manipulations were needed to provide a linear design algorithm.

- **Publications.** The contributions listed in this section have been disseminated in different international conferences and journals. A list of the publications produced from this project is summarized at the beginning of this thesis.
6.3 Future work

The topic of fault-tolerant wide-area control of power systems is an area that is foreseen to grow in interest. The following topics are research directions that can be investigated as a natural continuation to the problems solved and tackled in this project.

- **Actuator and process faults**: In this work only the loss of sensors has been investigated. An obvious generalization of the problem would be to consider actuator and process faults in addition to sensor faults. This can be represented using a diagonal matrix at the input of the system, similar to the sensor fault representation described in Chapter 4.

- **FT design for a given number of healthy sensors**: In Chapter 3 was presented a formulation to design FT schemes considering one sensor lost at a time. Then, in chapter 4 this limitation was removed and the results were extended to consider all possible sensor faults. However, recognising that all the sensors are highly unlikely to fail at the same time, a natural research direction of this work is to present a fault-tolerant formulation to specify a minimum number of sensors which would always remain in operation. The incorporation of this constraint in the design is not straightforward but introducing extra variables in the design it might be possible.

- **Multiple Lyapunov functions**: The formulation presented in Chapters 3 and 4 require the simultaneous solution of multiple inequalities. Although in Chapter 4 the formulation is relaxed to a few number of LMIs, in all the cases they require the use of the same Lyapunov function. This limitation can be improved by introducing extra slack variables in the formulation of the problem and hence, can reduce conservativeness in the design. In order to apply this, it might be required to work in discrete-time framework.
A. Residues, Model Reduction and LMI Regions

A.1 Residues and Signal Selection

In this section is presented the procedure applied to select the feedback signals. Consider a linear system described in state space form as described in Chapters 3 and 4, where the eigenvalues are defined as \( \lambda_i = \alpha_i \pm j\omega_i \). The modal controllability index

\[
MC_{ij} = v_i B_j
\]

indicates the controllability of the \( i \)th mode using the \( j \)th input, \( v_i \) is the left eigenvector of the system for the corresponding mode \( \lambda_i \). The modal observability index

\[
MO_{ik} = C_k w_i
\]

indicates the observability of the \( i \)th mode of the \( k \)th output, where \( w_i \) is the right eigenvector of the system for the corresponding mode \( \lambda_i \). Thus, the signal having highest observability index is chosen as the feedback signal. The higher the value of these indices, the better the effectiveness of the controller. These measures are normally expressed in the form of residues \( R_i = (MC_{ij})(MO_{ik}) \) can be written as

\[
R_i = |R_i| \angle \delta_i
\]
where $|R_i|$ and $\delta_i$ are the absolute value and the angle of the residue, respectively, for a particular $i$th mode. A high value $|R_i|$ is essential for an effective controller. The angle $\delta_i$ is normally used to calculate phase compensation, more details can be found in [20].

### A.2 Balanced Truncation Model Reduction

Let $G(s) \triangleq (A, B, C, D)$ be an $n$th order stable, but not necessarily minimal state-space realization of a transfer function $G(s) \triangleq D + C(sI - A)^{-1}B$.

$$G(s) \overset{\triangleq}{=} \begin{bmatrix} A & B \\ C & D \end{bmatrix}.$$ 

It is known [105, 106] that there exist an invertible state-space transformation $T \in \mathbb{R}^{n \times n}$ such that the transformed system

$$\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix} := \begin{bmatrix} T^{-1}AT & T^{-1}B \\ CT & D \end{bmatrix}$$

has controllability and observability grammians of the form

$$P = T^{-1}PT^{-T} = \text{diag} (\Sigma_1, \Sigma_2, 0, 0) \in \mathbb{R}^{n \times n}$$

$$Q = T^{T}QT = \text{diag} (\Sigma_1, 0, \Sigma_3, 0) \in \mathbb{R}^{n \times n}$$

where $\Sigma_1, \Sigma_2, \Sigma_3$ are positive definite diagonal matrices and

$$\Sigma_1 = \text{diag} (\sigma_1, \ldots, \sigma_m)$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \succ \sigma_{m+1} = \cdots = \sigma_n = 0$$

and the $\sigma_i$ are determined by

$$\sigma_i := \sqrt{\lambda_i(PQ)}$$
where \( \lambda_i(PQ) \) denotes the \( i \)th eigenvalue of \( PQ \) and \( P, Q \) are determined by the Lyapunov equations

\[
PA^T + AP + BB^T = 0 \\
QA + A^TQ + C^TC = 0.
\]

To obtain a \( k \)th order \((k \leq m)\) model, it has been proposed in [105] to partition the realization \((\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})\) as

\[
\begin{bmatrix}
\tilde{A} & \tilde{B} \\
\tilde{C} & \tilde{D}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & B_1 \\
A_{21} & A_{22} & B_2 \\
C_1 & C_2 & D
\end{bmatrix}.
\]

where \( A_{11} \in \mathbb{R}^{k \times k} \), and \( A_{22} \in \mathbb{R}^{(n-k) \times (n-k)} \), then, truncating the \( n-k \) least reachable-observable states to obtain the reduced model

\[
\hat{G}(s) := C_1(Is - A_{11})^{-1}B_1 + D.
\]

### A.3 LMI Regions for Regional Pole Placement

Let \( \mathcal{D} \) be a subregion of the open left-half complex plane. Then a dynamic system \( \dot{x} = Ax \) is called \( \mathcal{D} \)-stable if all its poles lie in \( \mathcal{D} \) (that is, all eigenvalues of the matrix \( A \) lie in \( \mathcal{D} \)). By extension, \( A \) is then called \( \mathcal{D} \)-stable. When \( \mathcal{D} \) is the entire open left-half plane, this notion reduces to asymptotic stability, which is characterized in LMI terms by the Lyapunov theorem. Specifically, \( A \) is stable if and only if there exists a symmetric matrix \( X \) satisfying

\[
AX +XA^T < 0, \quad X > 0
\]

This Lyapunov characterization of stability has been extended to a variety of regions.
Theorem 6 The matrix $A$ is $\mathcal{D}$-stable if and only if there exists a symmetric matrix $X$ such that

$$M(A, X) \prec 0, \quad X \succ 0$$

where $M(A, X)$ is a block matrix representing different LMI regions, see Figure 1, such as

- The $\alpha$-stability region

$$M(A, X) = AX + XA^T + 2\alpha X$$

- The disk region

$$M(A, X) = \begin{pmatrix} -rX & AX \\ XA^T & -rX \end{pmatrix}$$
• The conic sector

\[ M(A, X) = \begin{pmatrix}
\sin \theta(AX + XA^T) & \cos \theta(AX - XA^T) \\
\cos \theta(XA^T - AX) & \sin \theta(AX + XA^T)
\end{pmatrix} \]

**Proof** The proof can be found in [97].
B. Software Implementation

B.4 4th order equivalent of the Nordic System

The following matrices describe the state space representation of the system used in the illustrative examples in Chapters 3 and 4.

\[
A = \begin{bmatrix}
-0.096 & 1.931 & -0.082 & -0.420 \\
-1.975 & -0.104 & -0.237 & -0.826 \\
0.230 & 0.375 & -0.097 & 3.232 \\
0.526 & 0.874 & -3.241 & -0.207
\end{bmatrix}, \quad B_u = \begin{bmatrix}
-1.774 \\
-1.772 \\
1.544 \\
2.166
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1.161 & -1.431 & 0.104 & -0.777 \\
-0.574 & 0.618 & -0.147 & 0.287 \\
-0.796 & -0.346 & 1.086 & -1.364 \\
-0.802 & -0.341 & 1.073 & -1.381 \\
-0.119 & 0.156 & 0.100 & 0.188 \\
0.421 & -0.671 & 0.114 & -0.447
\end{bmatrix}, \quad D_d = \begin{bmatrix}
0.666 \\
-1.392 \\
-1.300 \\
-0.605 \\
-1.488 \\
0.558
\end{bmatrix}
\]

\[
B_d = \begin{bmatrix}
-0.330 & 0.795 & -0.784 & -1.263
\end{bmatrix}^T, \quad \begin{bmatrix}
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

B.5 Matlab Code for Conventional Controller

The following Matlab function was used to calculate the conventional control (CC) described in Chapter 3, Section 3.3. The controllers described in Sec-
tions 3.4 and 3.5, and the observers described in Chapter 4 were implemented in the same form and can be reproduced following this example.

```matlab
function [Kc,g]=conventional_CC(sys,damp0)
% [K,g] = conventional_CC(G,DAMP0)

%Generates a controller K to place the poles of the system G
%within the CONIC-SECTOR with minimum damping ratio DAMP0.
%Minimizing (g) the infinity norm of the transfer function
%between the output disturbance and the input (Bounded-Real Lemma)

n = size(sys.A,1); % order of the system
ny = size(sys.C,1); % number of outputs
nu = size(sys.B,2); % number of inputs
A = sys.A; B = sys.B; C = sys.C; D = sys.D;

% Defining the Angle of the conic sector
theta = acos(damp0);
sintheta = sin(theta);
costheta = cos(theta);

% Initialize the creation of a system of LMIs
setlmis([]);

% Define the variables
 g = lmivar(1,[1,1]);
 X = lmivar(1,[n,1]);
 Y = lmivar(1,[n,1]);
 Ah = lmivar(2,[n,n]);
```
Bh = lmivar(2,[n,ny]);
Ch = lmivar(2,[nu,n]);

%Inequality (3.18)
E0=newlmi;
lmiterm([-E0,1,1,X],0.5,1,'s');
lmiterm([-E0,1,2,0],1);
lmiterm([-E0,2,2,Y],0.5,1,'s');

%Inequality (3.19)
E1=newlmi;
lmiterm([E1,1,1,X],sint,theta*A,1,'s');
lmiterm([E1,1,1,Ch],sint,theta*B,1,'s');
lmiterm([E1,1,2,0],sint,theta*A);
lmiterm([E1,1,2,-Ah],sint,theta,1);
lmiterm([E1,2,2,Y],sint,theta*A',1,'s');
lmiterm([E1,2,2,Bh],sint,theta,C,'s');

lmiterm([E1,1,3,X],costh,theta*A,1);
lmiterm([E1,1,3,X],-costh,theta*A');
lmiterm([E1,1,3,Ch],costh,theta*B,1);
lmiterm([E1,1,3,-Ch],-costh,theta*B');
lmiterm([E1,1,4,0],costh,theta*A);
lmiterm([E1,1,4,-Ah],-costh,theta,1);
lmiterm([E1,2,3,Ah],costh,theta,1);
lmiterm([E1,2,3,0],-costh,theta*A');
lmiterm([E1,2,4,Y],costh,theta,A);
lmiterm([E1,2,4,Y],-costh,theta*A',1);
lmiterm([E1,2,4,Bh],costh,theta,C);
lmiterm([E1,2,4,-Bh],-costh,theta*C',1);
57  lmterm([E1,3,3,X],sintheta*A,1,'s');
58  lmterm([E1,3,Ch],sintheta*B,1,'s');
59  lmterm([E1,3,4,0],sintheta*A);
60  lmterm([E1,3,4,-Ah],sintheta,1);
61  lmterm([E1,4,4,Y],sintheta,A,'s');
62  lmterm([E1,4,4,Bh],sintheta,C,'s');
63
64  \% Bounded Real Lemma, Inequality (3.22)
65  E2=newlmi;
66  lmterm([E2,1,1,X],A,1,’s’);
67  lmterm([E2,1,1,Ch],B,1,’s’);
68  lmterm([E2,1,2,0],A);
69  lmterm([E2,1,2,-Ah],1,1);
70  lmterm([E2,1,3,-Ch],1,1);
71  lmterm([E2,2,2,Y],A’,1,’s’);
72  lmterm([E2,2,2,Bh],1,C,’s’);
73  lmterm([E2,2,4,Bh],1,1);
74  lmterm([E2,2,4,Bh],1,1);
75  lmterm([E2,3,3,g],-1,1);
76  lmterm([E2,4,4,g],-1,1);
77
78  LMI1 = getlmis;
79  options1 = zeros(1,5);
80
81  \% Minimize respect to gamma
82  [g,x] = mincx(LMI1,eye(decnbr(LMI1),1),options1);
83
84  \%Retrieve the variables
85  X = dec2mat(LMI1,x,X);
86  Y = dec2mat(LMI1,x,Y);
Ah = dec2mat(LMI1,x,Ah);
Bh = dec2mat(LMI1,x,Bh);
Ch = dec2mat(LMI1,x,Ch);
gamma=g;

In = eye(n);
Z = In-X*Y;
[u,q,v] = svd(Z);
U = u*sqrt(q);
V = v*sqrt(q);
Cc = Ch/(U');
Bc = V\Bh;
Ac = V (Ah- Y*A*X - Y*B*Ch- Bh*C*X)/(U');

%Finally Kc is given by
Kc = ss(Ac,Bc,Cc,0);

B.6 Control Parameters

The following parameters describe the conventional control (CC) designed using the procedure described in Section 3.3 and utilized in the illustrative example of Section 3.6, in Chapter 3. The parameters of $K_c(s) = (A_c, B_c, C_c, 0)$ are shown below:
The following parameters describe the passive fault-tolerant control (FTCp) designed using the procedure described in Section 3.4 and utilized in the illustrative example of Section 3.6, in Chapter 3. The parameters of $K_f(s) = (A_f, B_f, C_f, 0)$ are shown below:

$$A_f = \begin{bmatrix} -2.1834 & -3.3219 & 2.0149 & -0.5123 \\ -1.4701 & -2.2369 & 1.3565 & -0.3450 \\ 0.6637 & 1.0099 & -0.6125 & 0.1560 \\ -0.3024 & -0.4601 & 0.2788 & -0.0712 \end{bmatrix} \times 10^4,$$


$$C_f = \begin{bmatrix} -2.4811 & 3.9249 & -2.3963 & 0.0367 \end{bmatrix}$$

The following parameters describe the iterative passive fault-tolerant control
(FTCit) designed using the procedure described in Section 3.5 and utilized in the illustrative example of Section 3.6, in Chapter 3. The parameters of $K_{fit}(s) = (A_{fit}, B_{fit}, C_{fit}, 0)$ are shown below:

$$A_{fit} = \begin{bmatrix} -0.9809 & -1.8263 & 1.1309 & -0.2013 \\ -0.1881 & -0.3506 & 0.2169 & -0.0386 \\ 0.1678 & 0.3128 & -0.1936 & 0.0347 \\ -0.1911 & -0.3558 & 0.2201 & -0.0394 \end{bmatrix} \times 10^4,$$


$$C_{fit} = \begin{bmatrix} 1.0909 & 2.1375 & -1.2560 & -0.2573 \end{bmatrix}.$$
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REFERENCES


REFERENCES


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