Unraveling Risk Appetite

Applications of Decision Theory
in the Evaluation of Organizational Risk Appetite

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dissertation

Unraveling Risk Appetite

Applications of decision theory in the
evaluation of organizational risk appetite

Het ontrafelen van risicoacceptatiegraad

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evaluatie van de risicoacceptatiegraad van organisaties

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“Nobody said it was easy”

The Scientist – Coldplay

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1 Introduction

1.1 The economic problem of organizational risk appetite

In our current era of major global challenges and worldwide crises the human race is continuously searching for solutions to the fundamental economic problem of how to determine which aims should be pursued and how limited resources ought to be allocated. Economics as the science that studies "human behavior as a relationship between ends and scarce means" (Robbins 1932 p.16) is poised to prescribe normative antidotes to the calamities and acts of man that plague today's financial and economic markets. Aiming to minimize the adverse effects of risk at minimum cost, organizations are engaged in a balancing act between the *ex ante* allocation of resources for risk reduction and the *ex post* adequacy of resources to absorb losses. The outcome of this trade-off is an expression of the organization's willingness to accept risk, also known as its risk appetite. Scarcity of resources implies that the probability of occurrence and the potential impact of events identified as risks cannot always be reduced beforehand, and thus requires these risks to be ranked in priority by a decision maker in the organization. Risk attitude, a concept from decision theory, allows one to specify the rank order of a set of identified risks. In addition, the concept of risk appetite specifies the subset of this rank ordered set of risky events that requires control measures and its complement that, in contrast, is accepted. Given that organizational risk appetite is not unlimited, this thesis explores how measurements of risk attitude can be applied meaningfully in risk management to the economic problem of scarcity of resources.

In order to survive, human kind has always been engaged actively in the management of risk. As a formal field of study and practice, however, risk management only established itself in the second half of the 20th century. In the 1990s the idea that organizations should manage their risks holistically led to what is now commonly referred to as enterprise risk management (ERM). The COSO (2004a p.4) framework for enterprise risk management provides the following definition for this enterprise-wide view of risk management:

> "Enterprise risk management is a process, effected by an entity's board of directors, management and other personnel, applied in strategy setting and across the enterprise, designed to identify potential events that may affect the entity, and manage risk to be within its risk appetite, to provide reasonable assurance regarding the achievement of entity objectives."

The risk management process is typically composed of the following steps: setting organizational objectives, identifying events that threaten the objectives (risk identification), assessing the probability that these events will occur and their impact on the objectives (risk assessment), determining the acceptability of the events identified (risk evaluation), formulating a suitable response to the events in terms of control measures (risk treatment) and finally monitoring whether the controls are effective. The risk evaluation step requires the organization to formulate its risk appetite. Through its definition of enterprise risk management, the COSO framework formalized a requirement for organizations to become more explicit about their risk appetite and thereby created "a new managerial and regulatory object of attention" (Power 2007 p.78). In the COSO framework, organizational risk appetite is defined as "the broad-based amount of risk a company or entity is willing to accept in pursuit of its mission" (p.110). The large number of internet pages on risk appetite and an abundance of references in professional literature indeed demonstrate that the topic has not been wanting in attention. Despite this large stock of guidelines, advice and good practice examples, practitioners still characterize the topic as "challenging", "tough", and "one of the hardest things to define" (www.LinkedIn.com ERM discussion group).

The *communis opinio* in risk management is that risk appetite should be formally expressed and that it should reflect the risk attitude of the senior management of the organization, which acts in the interests of its stakeholders. In line with the "rituals of verification" that today's "Audit Society" (Power 1997) requires organizations to perform, these Board approved statements of risk appetite act as traces of evidence of good governance. A formal and approved expression of risk appetite is presumed to attest to the active involvement of senior management in the setting of risk appetite so that it supposedly reflects a deliberate and well thought-out attitude towards organizational risk taking. Recent publications by risk management practitioners explicitly call for the need to embed the senior management's risk attitude in the organizational risk appetite.

These demands for the validation of organizational risk appetite could be addressed by models and methods from behavioral economics that facilitate the analysis and the elicitation of a decision maker's risk attitude. Behavioral economics is a branch of economics that incorporates findings from psychology into economic models with a view to understanding human decision making in economic settings. Since the 1990s, behavioral economics has been a mainstream branch in economics and, rather recently, received a lot of media attention due to the publication of popular monographs such as *Thinking Fast and Slow* (Kahneman 2011), *Predictably Irrational* (Ariely 2008), and *Nudge* (Thaler & Sunstein 2008). The discipline in which behavioral economists and cognitive psychologists cooperate in the study of individual decision making under risk and uncertainty is referred to as decision theory. An accessible academic treatment of decision theory recently became available in the form of the textbook *Prospect Theory for Risk and Ambiguity* (Wakker 2010). The application of decision theory to decision problems in practice is referred to as decision analysis. Despite the high regard in which decision theory is held in academic circles, it is not yet commonplace for its concepts to be applied in practice, not even in very important decisions. In relation to risk appetite, decision theory holds the promise that its descriptive models can uncover the professional risk attitudes of key decision makers and that its normative models may offer to improve strategic decisions on organizational risk appetite.

1.2 Decision theory and decision analysis

One of the models in decision theory that is commonly applied in enterprise risk management is expected value (EV, Huygens 1657). This decision model assumes a risk neutral decision maker who determines the priority of risks on the basis of their mathematical expectation. For example, under expected value theory a decision maker is indifferent between accepting a 50/50-gamble with monetary outcomes −100 and 0 and losing the certain amount −50. The shorthand for this indifference between the standard gamble and its certainty equivalent is

$$-50 \sim (.5 : -100, .5 : 0), \tag{1.1}$$

where the symbol ~ represents equivalence. Another model for decision making under risk is expected utility (EU, Bernoulli 1738/1954). Expected utility theory is famous in economics but has found little application in enterprise risk management. In this model the decision maker is assumed to be risk averse. Examples of aversion to risk can be found in the indifferences

$$-60 \sim (.5 : -100, .5 : 0), \text{ and} \tag{1.2a}$$

$$-50 \sim (.4 : -100, .6 : 0). \tag{1.2b}$$

In both cases the decision maker is willing to lose a sure amount that is more negative than the expected value of the gamble, and is thus said to be risk averse. Indifferences similar to those in equations 1.2a and 1.2b can be found by requesting a decision maker to express her preference for either the sure amount or the gamble in equation 1.1. Let us assume that the indifference in this equation does not hold for a particular decision maker. We can then either let this decision maker adjust the sure amount until she is indifferent (*certainty equivalent method*) or we let her adjust the probability of losing until indifference is reached (*probability equivalent method*). Let us assume that as a result of these two choice processes the indifferences in equation 1.2a and 1.2b result. Expected utility theory models these indifferences using a concave utility function. It assigns priority to risks on the basis of a mathematical expectation over outcomes that have been transformed to utility. The expected utility model then represents the two indifferences by the following equations

$$u(-60) = .5 \times u(-100) + .5 \times u(0), \text{ and} \tag{1.3a}$$

$$u(-50) = .4 \times u(-100) + .6 \times u(0). \tag{1.3b}$$

One of the major achievements of expected utility theory is that it enables the measurement of utility by inferring utility from revealed preferences (Samuelson 1938) using representations such as equations 1.3a and 1.3b. For example, when we assign utilities to outcomes and let $u(-100) = -1$ and $u(0) = 0$ then we infer from equation 1.3a that $u(-60) = -.5$

and from equation 1.3b that u(−50) = −.4. By plotting these data points in a graph with outcomes on the horizontal axis and utility on the vertical axis we derive a concave utility function.

Both expected value and expected utility have a normative status as rational models for decision making under risk. The rationality of expected value hinges on the law of large numbers. The rationality of the expected utility model was firmly established by von Neumann and Morgenstern (1944) who formulated four axioms for rational choice and proved that these axiomata and the expected utility representation were equivalent. However, despite their attractiveness from a normative standpoint empirical evidence indicates that expected value and expected utility theory do not accurately describe actual choice behavior under risk.

Expected value was already disregarded as a descriptive theory of choice under risk since Bernoulli published the St. Petersburg paradox (Bernoulli 1738/1954). Expected utility's descriptive validity was rejected when the outcomes of several choice experiments, one of the famous being the Allais paradox (Allais 1953), did not correspond with the predictions of the theory. To provide a descriptively accurate theory of choice under risk that is capable of teasing out the rational from the irrational components of choice behavior, Kahneman and Tversky invented prospect theory (PT, Kahneman & Tversky 1979, Tversky & Kahneman 1992). This theory accommodated several biases from expected utility that had already been reported previously in the literature.

One of these biases was the phenomenon of risk seeking in the domain of losses. Contrary to the indifferences presented in equations 1.2a and 1.2b responses in the domain of losses typically do not express a risk averse but instead risk seeking tendency. Given the choice to adjust either the sure amount or the probability of losing, a risk seeking decision maker could for instance end up with the following indifferences

$$-40 \sim (.5 : -100, .5 : 0), \text{ and} \qquad (1.3c)$$

$$-50 \sim (.6 : -100, .4 : 0). \qquad (1.3d)$$

Decision makers rather consistently prefer the gamble in equation 1.1 over losing the sure amount with same expected value. The inference drawn from equations 1.3c and 1.3d by the decision analyst under expected utility would then be that utility in the domain of losses is convex, with u(−40) = −.5 and u(−50) = −.6.

Empirical evidence additionally suggests that decision makers not only transform outcomes but also transform probabilities. While the utility function u(.) in equation 1.3a and 1.3b is already used to transform outcomes, an additional transformation can be applied to probabilities using a probability weighting function w(.). Under prospect theory the indifferences in equations 1.3c and 1.3d would then be represented by

$$u(-60) = w(.5) \times u(-100) + (1 - w(.5)) \times u(0), \text{ and} \qquad (1.4a)$$

$$u(-50) = w(.4) \times u(-100) + (1 - w(.4)) \times u(0). \tag{1.4b}$$

Under prospect theory the decision analyst would now infer $u(-40) = -w(.5)$ and $u(-50) = -w(.4)$, which leaves both u(.) and w(.) initially undefined. By assuming functional specifications for u(.) and w(.) and employing a statistical optimization technique, such as nonlinear regression analysis, the decision analyst would, however, still be able to infer a shape for both functions.

Another deviation from the expected utility model is loss aversion, the phenomenon that "losses loom larger than corresponding gains" (Tversky & Kahneman 1991 p.1039). A loss averse decision maker is not indifferent between maintaining her *status quo* and accepting a 50/50 "fair" gamble with monetary outcomes +100 and −100. To break even with zero a loss averse decision maker typically requires the magnitude of gains to be much higher than the absolute value of losses such as, for example, in the indifference

$$0 \sim (.5 : 100, .5 : -50). \tag{1.5}$$

Under prospect theory, loss aversion is represented by the loss aversion coefficient λ, with $\lambda > 1$, which is multiplied with utility u(.) in the domain of losses. As a result, graphical plots indicate the slope of the loss utility function to be much steeper than that of the gain utility function and show a distinct kink at zero, the point at which both functions connect. Under prospect theory, the indifference in equation 1.5 is represented by

$$u(0) = w^{+}(.5) \times u(100) + w^{-}(.5) \times \lambda \times u(-50). \tag{1.6}$$

This equation illustrates that prospect theory requires the decision analyst to estimate u(.) for gains as well as losses, probability weighting functions for gains and losses $w^{+}(.)$ and $w^{-}(.)$, and, in addition, the loss aversion parameter λ. In comparison with the expected utility model, prospect theory offers more descriptive accuracy at the expense of an increase in model complexity.

While evidence of discrepancies with expected utility theory offers more descriptive insight into the domain of decision making under risk, the knowledge of these deviations reduce the freedom of the decision analyst to employ her traditional range of instruments prescriptively. Only by dismissing an abundance of empirical evidence on nonlinear probability weighting, can the analyst freely elicit utility by means of the certainty equivalent and probability equivalent methods and continue to believe that the resulting utility function is unbiased. The assumption of concave utility over the entire outcome domain, which is inherent in economic models, can only be maintained by discharging evidence of risk seeking in the domain of losses. By forfeiting empirical findings on loss aversion and its distinctive kink in utility at zero, the decision analyst is able to safeguard the analysis from "sources of embarrassment", given that "few people or companies would want a utility curve with kinks and bumps" (Howard 1988 p.689). However, the mainstream application of decision analysis to corporate decision-making (*e.g.* Spetzler 1968, Walls & Dyer 1996, Stanford

University Strategic Decisions Group 2010) continues to adhere to utility elicitation under the assumptions of expected utility. An alternative to this "classical elicitation assumption" is to elicit utility under the assumptions of prospect theory, while maintaining the "normative assumption" in decision analysis of expected utility's superiority as a rational model (Bleichrodt, Pinto, & Wakker 2001 p.1499). Several utility elicitation methods enable the elicitation of such an unbiased utility function under prospect theory (Wakker & Deneffe 1996, Abdellaoui, Bleichrodt, & Paraschiv 2007).

These utility elicitation methods offer the decision analyst the tools to measure the unbiased utility function of senior managers under prospect theory. These utility functions are assumed to represent the true preferences of the organizational decision makers and can be applied in the rational expected utility model to formulate organizational risk appetite. The application of these unbiased utility functions to the traditional expected utility model can then be referred to as "unbiased expected utility" and its application to the formulation of organizational risk appetite opens a window of opportunity for decision theorists who, despite their impressive academic track record, have not yet been able to "conquer the world" (Kahneman 2007, session six). There are, however, several unresolved issues that need consideration in order to tighten the gap between the "practice and promise" (Howard 1988 p.679) of decision analysis for enterprise risk management.

1.3 Questions

One of the greatest stumbling blocks for the adoption of decision theory in the field of enterprise risk management is its empirical finding of risk seeking in the domain of losses. Risk seeking in the domain of losses implies that a decision maker who faces two risky choices with the same expected value, of which neither promises any prospect of a gain, will prefer the risky alternative, *i.e.* the option with a lower probability of occurrence at the expense of a larger loss. Risk seeking for losses applies to pure risk, a decision context in which losses are the only possible outcome and no gains can be made. Historically the mainstream convention in risk management has been to adopt a neutral attitude to pure risk, whereas several authors, partly in response to the 2007–2009 financial crisis, favor an averse attitude to this risk category. In economics risk aversion is normatively advocated under the law of diminishing marginal utility. Here risk aversion implies that the utility function for losses is concave under expected utility theory. Under expected utility theory, risk seeking for losses is modeled through a convex utility function. If organizational risk appetite is based on a utility function elicited from the ranks of its senior management which, despite being unbiased, is a convex function of losses, then the normative application of the expected utility framework will stimulate the organization to engage in low probability – high impact gambles with negative consequences only.

The measurement of organizational risk appetite requires the organization to define appropriate risk measures. Such risk measures may be based on the utility function of its senior management. Measure theory stipulates whether a suitable risk measure can be obtained from such a function. The question remains how utility functions are used to obtain suitable measures for risk.

In operational risk management it is very popular to prioritize risks using the risk matrix, a grid with on its two axes a scale for the probability of the occurrence of a risk and a scale for its negative impact respectively. The risk matrix expresses risk appetite by indicating which risks are acceptable to the organization and which risks are unacceptable and thus should be controlled. Practitioners find it hard to decide where to draw the line between acceptable and unacceptable risks. While an unbiased utility function of senior management can be used to draw this boundary within the risk matrix, it remains unclear how exactly this should be done.

Relatively few studies in decision theory have empirically elicited utility and probability weighting functions within the context of organizational decision making. Organizational life is full of examples that resemble mixed gambles, *i.e.* gambles that incorporate both gains and losses. In the context of organizational risk appetite, mixed gambles that involve large monetary amounts are particularly relevant. In decision theory, the use of mixed gambles is restricted mainly to the elicitation of the loss aversion parameter. The technicalities of eliciting this parameter require furthermore that the decision analyst use very small monetary amounts in the elicitation process. This line of questioning, which is completely accepted in decision theory, is not compatible with the real conditions of enterprise risk management to which a senior management is exposed in practice.

Organizations usually do not publish their formalized expressions of risk appetite. It is therefore as yet unknown what risk attitude is expressed in organizational policies that formulate organizational risk appetite. *A priori* it is unclear whether to expect a risk seeking tendency, such as predicted under prospect theory in the domain of losses, a risk averse attitude, as assumed in economic models, or a risk neutral, expected value type of reasoning.

While decision theory is very quantitative, enterprise risk management is often very qualitative in nature. Quantification of risk is prominently present in financial risk management. In operational risk management, however, it is common to use verbal expressions of probability and outcome in risk assessments. As a result, risk appetite is often considered as something vague and qualitative. The question is then what the relevance is for operational risk management of a quantitative approach, such as proclaimed in decision theory. The value added of decision theory could lie in its ability to connect qualitative and quantitative risk analysis by determining the utility of verbal outcome expressions and the weight of verbal probability expressions.

The cornerstone of the theory of incentives, or principal-agent theory (Laffont & Martimort 2002), is that economic agents are risk averse. To align the objectives of the agent with those of the principal, incentives can be used. The empirical finding of risk seeking in the domain of losses challenges one of the theory's major assumptions, namely the risk aversion of the agent. While the problem in principal-agent theory is to induce the agent to take risk, in response to the 2007–2009 financial crisis there is an increased call for performance-based incentives to promote prudent behavior by economic agents. What kind of incentives can be designed to induce organizations to adopt an averse appetite to risk remains an unresolved issue.

Based on these issues, five central questions have been formulated:

1. Which models from decision theory and which methods from measure theory ensure that organizational risk appetite both complies with the risk attitude of senior management and a rational model of decision making? What effect does risk attitude have on organizational risk appetite in the risk matrix?
2. Which utility elicitation method is compatible with the elicitation of utility in the context of organizational risk appetite?
3. If the objective is to describe the organizational risk appetite of practitioners, which decision theory (expected value, unbiased expected utility or prospect theory) performs best?
4. If verbal probability and outcome expressions are used in risk evaluation, do the introspective judgments of their value differ from those that can be inferred from the choices of participants under prospect theory?
5. What is the influence of performance-based incentives on expressions of risk appetite in the risk matrix?

1.4 Structure

To answer these five central questions this dissertation reviews relevant literature in risk management, decision theory, measure theory, linguistics and principal-agent theory. To answer the first central question, a literature review of risk management, decision theory and measure theory was undertaken. An attempt was made to integrate the findings from these fields in order to apply measurements of a senior management's risk attitude to organizational risk appetite. Given the lack of archival data on the subject of organizational risk appetite, the remaining four central questions could not be answered by archival-based methodologies. The exploratory nature of these questions, however, did lend itself to the use of individual case-studies as a data-collection method. To answer the third and fourth central questions, research was therefore carried out at two anonymous case companies. Given our aim to unravel organizational risk appetite from the perspective of decision theory, survey and experimental methods from decision theory were applied to these two companies. Surveys were used to assess the correspondence between organizational risk appetite and several decision theories in response to the third central question. To answer the fourth central question, an experimental design was used to compare introspective and choice-based values of verbal expressions under a small versus a large project condition. The survey method was also applied in trial sessions in which the elicitation method was tested that has been designed in response to the second central question. To test the existence of a causal relationship between performance-based incentives and risk appetite, an experimental design was used to answer the fifth central question.

This thesis is structured as follows.

Decision theory, measure theory, and organizational risk appetite (Chapters 2 & 3)
Chapters 2 and 3 are theoretical chapters. Chapter 2 introduces the topic of this thesis, namely organizational risk appetite. Furthermore three decision theories are discussed, in particular expected value, expected utility and prospect theory. The chapter establishes the consistency of valuation methods from decision theory with the definition of a risk measure, in the sense in which this concept is used in measure theory. The chapter provides new definitions for organizational risk appetite, unbiased utility, and unbiased risk attitude. It also introduces a risk measure that is derived from the willingness-to-accept and willingness-to-pay concepts in decision theory: the zero equivalent risk measure. Several decision rules for organizational risk appetite are defined and illustrated by examples.

In chapter 3, the zero equivalent risk measure and these decision rules are applied to the risk matrix. The chapter contains three observations and one proposition that are derived from isocontours in the risk matrix. The chapter concludes with examples that illustrate the different expressions of organizational risk appetite that result in the risk matrix under expected value, exponential expected utility, and expected utility based on the unbiased utility function from prospect theory.

Utility elicitation and organizational risk appetite (Chapter 4)
Chapter 4 identifies several utility elicitation conventions in decision theory that are not compatible with risk as it is experienced by organizations. It defines a new four-step non-parametric elicitation method that extends the trade-off method (Wakker & Deneffe 1996) to the whole domain of outcomes and is more compatible with real-life business conditions. Initial results of trial sessions in which the practical applicability of this method was tested are reported.

Describing organizational risk appetite using decision theory (Chapter 5)
Based on a field study in which the senior management of a large Dutch company was involved, chapter 5 assesses which decision theory performs best in describing organizational risk appetite. In this study utility and probability weighting functions of senior managers were elicited under prospect theory using the elicitation method of Abdellaoui, Bleichrodt and Paraschiv (2007). The introspective judgments of organizational risk appetite of these senior managers were compared with predictions from expected value theory, unbiased expected utility theory, and prospect theory.

The utility and probability weight of verbal probability and outcome expressions (Chapter 6)
Chapter 6 presents a field experiment in which consultants of a Dutch subsidiary of an international consultancy company were engaged. It presents an extensive overview of vagueness studies, in particular with respect to verbal expressions. On the basis of this, two hypotheses are defined regarding the utility and probability weight of verbal probability and outcome expressions. Prospect theory values in this study were derived using the elicitation method of Abdellaoui, Bleichrodt and L'Haridon (2008).

Performance-based incentives and organizational risk appetite (Chapter 7)
Chapter 7 reports the results of a laboratory experiment on the effect of performance-based incentives on risk appetite. It reviews the role of incentives in risk management and principal-agent theory and, based on this theory, suggests an effective performance-based incentive. Three hypotheses are posited about the effect of incentives on risk appetite and the efficiency and effectiveness of risk management. The experimental design is based on the risk matrix. Monte Carlo simulation is used to define optimum choice strategies for risk appetite.

Chapter 8 contains a discussion of the conclusions derived in this thesis and the direction future research in this area may take.

The chapters in this thesis have been written as separate articles for journals and for the purpose of this thesis have been rewritten so as to contain a minimum of overlap between the chapters. The ordering of the chapters follows the five research questions in this introduction and not the chronological order in which they were written. At the time each chapter was written, the findings in preceding chapters may therefore not have been known to the author. While the chapters in this thesis are written by a single author, occasionally the first-person plural form is used for stylistical reasons. When referring to a decision maker in the third-person singular form, the feminine form is adopted.

2 On the Measurement of Risk Appetite and Risk Attitude

Summary

This chapter proposes to measure organizational risk appetite on the basis of an expected utility model in which the utility function is elicited under the assumptions of prospect theory. This utility function is free from the bias of probability distortion and is assumed to reflect the true preferences of the decision maker. The unbiased risk attitude of the decision maker, which is embedded in this function, can subsequently be used to calculate the utility based shortfall risk (UBSR) measure. Three examples illustrate how the zero equivalent risk measure, a member of the UBSR-family that resembles the willingness-to-accept valuation method, can be used to measure organizational risk appetite.

2.1 Introduction

Just as humans who in their pursuit of pleasure need to accept the risk of suffering pain, organizations in their pursuit of gain risk incurring losses. To what degree organizations should be willing to accept risk is currently a lively debate in regulatory and management circles. The common theme in these discussions is that organizational willingness to accept risk, or *risk appetite*, is an important topic but no one seems to agree or even know how it should or could be measured. While the measurement of risk appetite is one challenge, another challenge for organizations is to incorporate the *risk attitude* of its senior management into its formal statements of risk appetite. Practitioner literature and enterprise risk management frameworks voice a clear need for the measurement of organizational risk appetite as well as the consolidation of the Board's attitude to risk therein. It is evident that the ambiguity surrounding organizational willingness to accept risk is frustrating attempts by practitioners to determine the acceptable level of risk for their organization.

Both needs, the measurement of risk as well as the incorporation of risk attitudes, can simultaneously be addressed by expected utility theory (Cramer 1728/1954, Bernoulli 1738/1954). In this theory the sensitivity of a decision maker to monetary outcomes is expressed by a utility function. This function can subsequently be used to measure risk, by calculating the minimum amount of capital that the organization requires to accept risk (Föllmer & Schied 2004, *utility based shortfall risk*). From these risk measures decision rules can be derived that define organizational risk appetite. The utility function specifies, at the same time, the risk attitude of the decision maker. For example, under expected utility theory a risk averse attitude – the decision maker prefers each probability distribution less than its expectation – is expressed by a concave utility function. Under expected utility theory elicitation of the utility function can be biased by phenomena that are considered deviations from this rational theory of choice, such as nonlinear probability weighting. Under prospect theory (Kahneman & Tversky 1979, Tversky & Kahneman 1992) utility elicitation produces a function that is free from these biases. These unbiased utility functions from decision theory, in combination with the risk measures from measure theory and decision rules derived from these, address the current need in enterprise risk management to define the amount of acceptable risk on the basis of senior management's attitude to risk.

This chapter aims to clarify which models in decision theory and which methods from measure theory ensure that organizational risk appetite complies with the risk attitude of senior management as well as a rational model of decision making. Its primary contribution is to respond to the practitioners' call for a clear measurement of organizational risk appetite and the incorporation of the risk attitude of senior management therein. It applies the concept of risk attitude to organizational risk appetite in the vernacular of the risk manager, the decision theorist, and the actuary, which should facilitate communication between these fields.

The chapter proceeds as follows. The next section describes risk appetite and reports how organizational risk appetite is measured in risk management and how risk attitudes have traditionally been incorporated in these measurements. The third section discusses how risk attitude is modeled under three theories for decision making under risk: expected value, expected utility, and prospect theory. It also illustrates that under prospect theory the willingness-to-accept (WTA) and willingness-to-pay (WTP) valuation methods for valuing lotteries are not only different in magnitude, but are dissimilar in terms of risk attitudes as well. The fourth section assesses whether these two valuation methods are risk measures and provides definitions for organizational risk appetite, unbiased utility, and unbiased risk attitude. It furthermore introduces the zero equivalent risk measure, which is based on both the WTA and WTP valuation methods from decision theory and which is congruent with measure theory. The fifth section defines decision rules for organizational risk appetite and provides several illustrative examples. Section six draws conclusions. Appendices are in the final section.

2.2 The measurement of risk appetite in risk management

This section draws upon several risk management frameworks to define risk appetite and discusses the practical measurement of risk appetite in risk management and how this involves risk attitude.

2.2.1 The formal definition and relevance of risk appetite

Risk appetite is a key element in the COSO Enterprise Risk Management – Integrated Framework (2004a). In this 125-page long document the term is mentioned 105 times, is instrumental in defining Enterprise Risk Management itself,[1] and the term is associated with five of the eight components of the framework. In this framework the term applies to appetite for downside risk only because in COSO risk is defined as the possibility that an event will occur and adversely affect the achievement of objectives. COSO defines *risk appetite* as "the broad-based amount of risk an entity is willing to accept in pursuit of its mission/vision." (p.110). This definition clearly states that any appetite for downside risk is contingent on the value that is being pursued. Another, closely related term, *risk tolerance*, is reserved for describing the acceptable variation relative to the achievement of an objective (COSO 2004a p.124). Where risk appetite pertains at a high level to the entity as a whole, risk tolerance relates to specific objectives (COSO 2004a p.116). By requiring organizations to become more explicit about defining and monitoring their risk appetite, COSO creates a new object of attention for management and regulatory bodies and signifies a new "organizational consciousness" of risk appetite (Power 2007 pp.77–78). The COSO (2004a) framework specifically refers to risk appetite as an acceptable "amount of risk" which suggests a purely quantitative interpretation of the term. While this suits many financial service providers that express risk appetite as the maximum allowable probability of ruin before their risk capacity (financial buffer) is depleted, this interpretation does not sit well with organizations with a different "quantitative culture" (Mikes 2009). It appears that organizations tend to interpret risk appetite both qualitatively as a component of a "boundary system" or quantitatively as part of a "diagnostic control system" (Simons 1995).

The FERMA risk management standard briefly stipulates four methods to describe risk appetite and risk tolerance but it does neither define nor clearly distinguish the two terms from each other (2003 table 4.2.1). The standard prescribes that an organization's risk management policy should set out its appetite for risk (p.12). An important premise underlying the standard is that a subset of all downside risks is acceptable to bear and that its unacceptable complement should be treated.

ISO 31000 : 2009 principles and guidelines on risk applies risk appetite both to upside and downside risk. The term *risk appetite* is defined as the "amount and type of risk (..) that an organization is prepared to pursue, retain or take." Even though the principles and guidelines do not make any other explicit reference to risk appetite, its concept is implicitly embedded in the process of risk evaluation in order to determine whether risk is acceptable or not.

Risk appetite plays a crucial role in modern risk management practice and has recently become the object of academic research (Ashby & Diacon 2010, AIRMIC 2009, Power 2009). Textbooks and standards for risk management invariably put risk appetite at

[1] Enterprise Risk Management is "a process, effected by an entity's board of directors, management, and other personnel, applied in strategy setting and across the enterprise, designed to identify potential events that may affect the entity, and manage risk to be within risk appetite, to provide reasonable assurance regarding the achievement of entity objectives." (COSO 2004a p.1)

the heart of good practice (AIRMIC 2009 p.9).[2] The importance of formally stating risk appetite is emphasized by enterprise risk management standards, guidelines (HM Treasury 2004), best practice studies (Collier, Berry, & Burke 2006) and practitioners (EY 2009, Mercer Oliver Wyman 2006). The ability to set risk appetite and manage risks in line with this policy has been cited as one of the least mature competencies of organizations (RIMS 2008) and the lack of competency in this field as one of the causes of the 2008 financial breakdown (McDonald 2009, RIMS 2009). An incentive to articulate risk appetite comes from rating agencies that give positive weight to the articulation of risk appetite (Standard & Poor's 2006). Regulators demand more transparency on risk appetite in financial reporting (Mertens & Blij 2008, Van Beurden & Bos 2007, Monitoring Commissie 2007) and continuously demand that organizations are "in control", which can be rephrased as demanding them to act within risk appetite (Paape 2008 p.23). Practitioner literature suggests that formal statements of risk appetite should articulate the risk attitude of senior management (Semple 2007, Van den Brink 2007, PwC 2006, Barfield 2005, Zurich 2004), that risk preferences should be elicited and integrated in risk acceptability statements (CAS 2005), and mentions the need for independent validation of risk appetite (Pool & Kuijck 2009, EY 2008). The need for measurement of organizational risk appetite is implied by its definition as the acceptable "amount" of risk as well as proclamations by the risk management community that "risk appetite needs to be a measurable concept" (Anderson 2011 p.7).

2.2.2 The practical measurement of risk appetite

In financial risk management risk appetite is expressed using quantile-based risk measures, such as Value-at-Risk (VaR) and Expected Shortfall (Tail-VaR), which focus on the tail of the predicted loss distribution and implicitly assume risk neutrality (Ai & Brockett 2008, Dowd & Blake 2006). Less commonly applied are risk measures that incorporate a weighting function to describe different degrees of risk aversion over quantiles, such as spectral measures (Acerbi 2002, see also Wang 2000, 2002). In economic capital calculations risk appetite is typically expressed by a threshold for the acceptable frequency of ruin and occasionally other "material events", such as dividend cuts and profit warnings (Thomson & Chan 2008). Such an *ex ante* formulation of the desired distribution of payoffs expresses the decision maker's risk attitude, in particular her aversion to variance and negative skewness (Filbeck, Hatfield, & Horvath 2005). In finance measures for expressing risk appetite with an "on-going concern", rather than the aforementioned "solvency concern", focus on the standard deviation or variance of the distribution function and imply risk aversion (Ai & Brockett 2008, Wakker 2010 p.75 *aversion to mean-preserving spreads*). For example, in the risk-return diagram of modern portfolio theory (Markowitz 1952a, COSO 2004b p.18 exhibit 3.7) risk aversion is expressed by the increase of financial returns with variance (see Ashby & Diacon 2010 for a theoretical framework for risk appetite using a risk-return

[2] Opinion expressed by AIRMIC's Technical Director, Paul Hopkin.

diagram). In capital budgeting expected utility has been incorporated in a risk adjusted discounted cash flow analysis approach, which employs a hurdle rate as well as a measure for risk aversion (Cozzolino 1977, Spetzler 1968). This approach has been adopted in the oil and gas exploration industry (Walls 1995, Walls & Dyer 1996) and was recently reintroduced to a mainstream public under the heading of "quantified risk appetite" (Stanford University Strategic Decisions Group 2010).[3]

In operational risk management one of the most widespread tools to express organizational risk appetite is the risk matrix (AIRMIC 2009 p.10). A *risk matrix* is composed of a grid with two axes, one for the probability of occurrence of an event and one for the negative impact of that event on an objective of an organization (Goldberg, Everhart, Stevens, Babbitt, Clemens, & Stout 1994, Clemens, Pfitzer, Simmons, Dwyer, Frost, & Olson 2005, Kwak & LaPlace 2005 p.692). Once risky events have been identified by the organization, they are then plotted within this probability-impact grid. Risk appetite is expressed in the risk matrix by a boundary line that separates the unacceptable risk events from the acceptable ones. Risk matrices originate from military standards for systems safety where both probability and impact are expressed in quantitative terms and where the boundary line for risk appetite is based on expected value calculations (Pfitzer, Hardwick, & Dwyer 2001, MIL-STD-882B 1984). This implies that these standards adopt a neutral attitude to risk to express risk appetite. Apart from the observation that the concept of utility can be applied to the risk matrix (Kwak & LaPlace 2005) and the notion that subjective risk attitudes play an essential "but seldom articulated" role in risk matrices (Cox 2008), risk neutrality has remained the mainstream interpretation of risk attitude in risk matrices. In the absence of precise information on probability and impact these risk characteristics can be assessed subjectively using verbal descriptors and imprecision intervals. Inspired by the introspective, "professional judgment" of decision makers a numerical rating can be assigned to each verbal descriptor and interval. These precise rating scales over the imprecise descriptors and intervals facilitate the use of mathematical operations in the evaluation of risk, not unlike the calculation of expected values (Fine 1971, Kinney & Wiruth 1976). Commonly referred to as *Kinney-values* these numerical indicators of risk have found widespread adoption by the risk management community. These rating scales assign weights to outcomes and probabilities and resemble utility and probability weighting functions from decision theory (see appendix A). The underlying risk attitude that these scales imply is, however, not explicitly stated.

Despite evidence of scale aversion in responses to large-scale incidents, and the use of societal risk aversion indices by Denmark and The Netherlands to determine what level of risk to lives is tolerable, the general stance to risk appetite in public policy making is that risk neutrality should be adopted (Marszal 2001 p.397, Bennett & Murray 2009, Ball & Floyd

[3] Quantified risk appetite is synonymous to the concept of risk tolerance in decision theory, which is discussed in §2.5.5. An Excel-based tool for calculating risk-adjusted values is made available by Stanford at www.sdg.com/sdg-toolbox/risk-adjusted-value-tool.

1998, Fisschoff, Lichtenstein, Slovic, Derby, & Keeney 1981 pp.113–114). Risk appetite is typically expressed in risk matrices that are referred to as FN diagrams with F representing the frequency of N casualties or more. The difficulty in assigning monetary value to such intangible benefits as saved lives is suggested as one of the reasons why expected utility maximization is rarely used in social safety issues (Marszal 2001 p.393).

In summary, measurements of risk appetite by organizations and society typically assume a risk neutral decision maker. This holds in particular for the most popular expressions of risk appetite, such as quantile-based risk measures in financial risk management and risk matrices in operational risk management. A notable exception is the widespread adoption of mean-variance analysis, which implies aversion to risk. While enterprise risk management has for quite some time been exposed to the methods that decision theory has available to measure organizational risk appetite and define the risk attitude of senior management, this has barely resulted in their practical application (*e.g.* Borge 2001 pp.20–24, COSO 2004b pp.51–52, Vaughan 1997 pp.54–62).[4]

2.3 The measurement of risk attitude in decision theory

This section introduces risk attitude as it is defined in decision theory and explains the importance of the use of lotteries to determine risk attitude. It furthermore discusses three theories that allow us to model decision making behavior under risk: expected value, expected utility, and prospect theory (for an overview of decision theories see Starmer 2000 and Schoemaker 1982).

2.3.1 The definition of risk attitude

In decision theory risk is operationalized by probability distributions over monetary or non-monetary outcomes and these distributions are referred to as lotteries, gambles or prospects. Lotteries are the primary tools of decision theory to express risk and application of these tools to risk management by no means implies that organizational decision making resembles sweep staking or gambling. The lottery-concept, which has so much been ingrained in the decision theory literature, is adopted in this chapter because thinking about risk in terms of lotteries brings out the essence of decision making under risk in a tractable way. The com-

[4] In his risk management textbook Vaughan (1997 p.56) explains expected utility theory in the following way: “The initial step in the expected utility model approach is to derive the individual’s utility function. Several approaches have been suggested for this task, but in general they involve asking a subject a series of questions concerning the amount he or she would pay to eliminate the possibility of loss of a given magnitude.” He then voices his opinion about expected utility theory in footnote 6: “We will not discuss the process by which the utility function is derived primarily because we believe that it is an essentially useless tool for risk management decisions.”

plexity that characterizes probability distributions in practice often obscures a plain view on decision making under risk. For this reason lotteries with only two outcomes (*binary lotteries*) are commonly used in decision theory.

A lottery L that returns outcome x with probability p and outcome y with probability $(1-p)$ is expressed as $L = (p : x, (1-p) : y)$ with $x > y$, $0 < p \leq 1$. The preference relationships of a decision maker between two lotteries, L and M, are expressed as $L \succcurlyeq M$ when L is weakly preferred to M and as $L \sim M$ when the decision maker is indifferent between L and M. Decision makers express their risk preferences by choosing between lotteries. Because risk appetite relates to the acceptance of losses in the pursuit of gains, the class of mixed lotteries that combines both a positive outcome ($x > 0$) and a negative outcome ($y < 0$) are in this context of particular interest. In this chapter we assume that all outcomes are monetary.

The term "risk attitude" refers to the attitude of a decision maker towards a particular lottery (Keeney & Raiffa 1976 p.149). This attitude is characterized by the preference of the decision maker either for the certain amount z or for a risky lottery L with expected value z.[5] A person is *risk averse* if she prefers the amount z to any risky lottery L with expected value z, *i.e.* $z \succcurlyeq L$. *Risk seeking* holds if every lottery is preferred to its expected value, *i.e.* $L \succcurlyeq z$, and *risk neutrality* holds if the decision maker is indifferent between every lottery and its expected value, *i.e.* $L \sim z$ (Kahneman & Tversky 1979 p.264, see Wakker 2010 p.52 for an alternative definition). This typology of risk attitudes can also be applied to characterize different kinds of risk appetite or to facilitate the comparison of statements of risk appetite of different decision makers.

Studies on the temporal stability of risk preferences find that some individuals whose risk attitude is elicited at two separate moments in time change their preferences. Temporal instability of risk preferences is in some studies reported to be quite high (Baucells & Villasís 2010 p.193, Zeisberger, Lang, & Vrecko 2012 p.359), with low correlation coefficients (Wehrung, MacCrimmon, & Brothers 1984, Smidts 1997), while other studies report it to be quite stable (Harrison, Johnson, McInnes, & Rutström 2005, Andersen, Harrison, Lau, & Rutström 2008).

2.3.2 Decision theories under risk: EV, EU and PT

Decision theory has produced many theoretical models that either describe how humans choose between lotteries or prescribe what choices they should make under risk. The rational choices that normatively oriented theories prescribe are frequently at odds with actual human choices that are, on the contrary, "predictably irrational" (Ariely 2008). The finding that the rational predictions of the prescriptive expected value and expected utility models failed to deliver, acted as a call to assembly for the academic community

[5] For lottery L its expected value (EV) is calculated as the sure amount $\text{EV}(L) = p \times x + (1-p) \times y$.

to “hunt for a descriptive theory of choice under risk” (Starmer 2000). A theory that captures many irrational biases of choice behavior while separately maintaining the rational elements of choice that emanate from prescriptive theories of choice is prospect theory (Kahneman & Tversky 1979, Tversky & Kahneman 1992). This section describes how risk preferences from paragraph 2.3.1 are modeled on the basis of the aforementioned three theories of decision making under risk.

2.3.2.1 Expected value theory

The origin of expected value (EV), the oldest known theory for decision making under risk, is attributed to a letter correspondence between Blaise Pascal and Pierre de Fermat. Its foundations were first published by Christiaan Huygens (1657). Under expected value theory a preference for lottery L over M ($L \succcurlyeq M$) is represented by the inequality $\mathrm{EV}(L) \geq \mathrm{EV}(M)$. Important features of expected value are the multiplication of probabilities and outcomes (thus $p \times x$) and the additivity of these products, which implies that the outcomes are statistically independent of each other. When the same lottery is played over and over again then, subject to the law of large numbers, the sum of all the outcomes in these trials is approximated by the sum of their mathematical expectations. When this law does not hold, such as in one-shot decisions, maximization of expected value is generally still considered a reasonable decision criterion for lotteries when these involve small stakes (see various annotations in Wakker 2011, Wakker 2010 §8.2, Kahneman & Lovallo 1993 p.21). The theory assumes one single attitude to risk taking, which is risk neutrality, and is ignorant of any preferences for degrees of risk and levels of outcome that may want to reveal themselves in choice behavior.

2.3.2.2 Expected utility theory

A theory that allows for the accommodation of all kinds and degrees of individual risk attitude was proposed by Cramer (1728/1954) and Bernoulli (1738/1954) and has become known as expected utility (EU). Similar to expected value, a decision maker’s preference between lotteries is under expected utility represented by a mathematical expectation that, unlike expected value, is defined over a *utility* transformation of lottery outcomes. Given the transformation of outcomes by a utility function U(.) the EU of lottery L is calculated as

$$\mathrm{EU}(L) = p \times \mathrm{U}(x) + (1 - p) \times \mathrm{U}(y). \tag{2.1}$$

Under expected utility theory the inequality $\mathrm{EU}(L) \geq \mathrm{EU}(M)$ represents a preference for lottery L over M ($L \succcurlyeq M$).

The utility curve U is monotonically increasing and its shape can be either concave, convex, linear or any combination hereof. Concavity implies that when any two points on a curve are connected by a straight line then the curve will be positioned just at or above that line.[6] Comparison of the coordinates of this straight line and the utility curve at any outcome in the domain of the utility curve indicates that the property of concavity can be summarized by the inequality

$$U(\theta x + (1 - \theta)y) \geq \theta U(x) + (1 - \theta)U(y), \tag{2.2}$$

for all $0 < \theta < 1$ and x,y (Wakker 2010 p.72, Chiang 1984 p.342). Replacement of θ by p, allowed under the condition $0 < \theta < 1$, produces the inequality $U(px + (1 - p)y) \geq pU(x) + (1 - p)U(y)$, or in short $EU(EV(L)) \geq EU(L)$, and implies the preference $EV(L) \succcurlyeq L$, which is the definition of risk aversion presented in paragraph 2.3.1.[7] Under expected utility concavity of the utility curve is thus equivalent to risk aversion.[8] Similarly it can be shown that under expected utility convex utility implies risk seeking and linear utility implies risk neutrality. Expected value thus remains nested in the expected utility model. In most economic applications a concave utility function that expresses marginal decreasing utility is assumed, which under expected utility implies risk aversion (Kahneman & Tversky 1979 p.264). For this reason concavity is commonly considered to be an integral assumption of expected utility theory as well.

Expected utility is considered a reasonable theory of choice under risk for one-shot decisions with large stakes (Wakker 2010 p.44). The rationality of expected utility was underpinned by von Neumann and Morgenstern (1944, reformulated by Marschak 1950 and Nash 1950) who defined a set of necessary and intuitively appealing conditions for rational decision making under risk and demonstrated that a decision maker who agrees with this set should behave according to expected utility.[9] In expected utility theory outcomes are defined as final wealth states (*asset integration*). Following Markowitz (1952b), most experimental measurements of utility, however, define utility over gains and losses rather than on final asset positions (Kahneman & Tversky 1979 p.271), and initial wealth, which is often unknown, is usually not expressed in the outcomes (Wakker 2010 p.234). Because initial wealth is the same in all choice options, it follows that outcomes defined to this fixed reference point are uniquely related to a final wealth position, even when the latter remains unspecified (Wakker 2010 p.238). Over the years expected utility has retained its prescriptive status and, just as expected value, can have descriptive validity for subsets of individuals in a

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6 In more precise terms, concavity of the utility curve implies that when any two coordinates $\{x, U(x)\}$ and $\{y, U(y)\}$ with $x < y$ are connected by a straight line that consists of the coordinates $\{\theta x + (1 - \theta)y, \theta U(x) + (1 - \theta)U(y)\}$ with $0 < \theta < 1$, then at any θ the coordinate $\{\theta x + (1 - \theta)y, U(\theta x + (1 - \theta)y)\}$ on the utility curve will be positioned just at or above the corresponding coordinate on the straight line. Under EU strict concavity of the utility function (the second derivative of $U(x)$ is negative, $U''(x) < 0$), explains an unwillingness to accept any mixed lottery with $EV(L) = 0$ (Pratt 1964).

7 The first part in these inequalities represents the utility of the expected value of lottery L and the second part the expected utility of lottery L.

8 That this equivalence also holds for lotteries with more than two outcomes was demonstrated by Wakker (2010, Exercise 3.2.1b).

9 The axioms of decision theory are completeness, transitivity, continuity, and independence.

heterogeneous population. For example, elicitation of the utility functions in organizations under the assumptions of expected utility (Wehrung 1989, Swalm 1966) and interviews with corporate executives (March & Shapira 1987, Spetzler 1968) displayed a significant degree of risk aversion. Numerous falsifications of expected utility in experiments, however, produced evidence of various irrational biases in choice behavior that eventually led to the development of other, nonexpected theories of choice.

2.3.2.3 Prospect theory

An extension to expected utility that sprung from the minds of Daniel Kahneman and Amos Tversky and accommodates various empirical choice anomalies is prospect theory (1979, 1992), "the first rational theory of irrational behavior" (Wakker 2010 p.2). Prospect theory (PT) assumes a fixed reference point, usually 0 or the status quo, and gains and losses relative to this point are evaluated by an *overall utility* function U. The utility function U takes the form $\mathrm{U}(x) = \mathrm{u}(x)$ for gains and $\mathrm{U}(x) = \lambda\mathrm{u}(x)$ for losses, with u(.) measuring *basic utility* in the gain and loss domains separately, $\mathrm{u}(0) = 0$, and the *loss aversion* parameter $\lambda > 0$, measuring sensitivity to losses relative to gains (Wakker 2010 pp.239, 252). Loss aversion refers to the empirical finding that gains bear less weight in evaluations than losses. Under prospect theory loss aversion is modeled by a much steeper slope of the utility function for losses than for gains ($\lambda > 1$) which results in a kink in the utility function at reference point zero (see Tversky & Kahneman 1991 for an application to riskless choice).[10] There is strong empirical evidence that λ exceeds 1 considerably (Novemsky & Kahneman 2005 p.119). Evidence of concave utility in the gain domain is much stronger than the evidence for convexity in the domain of losses where the prevalence of convexity over concavity is "a close call" (Wakker 2010 p.264).[11] Comparable to the transformation of outcomes under expected utility, prospect theory transforms probabilities using a *probability weighting* function for gains $\mathrm{w}^+(.)$ and losses $\mathrm{w}^-(.)$.[12] Its typical shape resembles an inverse-S that overweights small probabilities and underweights large probabilities. Overweighting means that in evaluating a decision under risk an objective probability p receives additional weight so that $\mathrm{w}(p) > p$ and underweighting implies the reverse. In the domain of gains (losses) probability overweighting encourages risk seeking (risk aversion) and underweighting encourages risk aversion (risk seeking). The prospect theory value of a lottery L that is either loss-free or gain-free is calculated by

$$\mathrm{PT}(L) = \mathrm{w}^s(p) \times \mathrm{U}(x) + (1 - \mathrm{w}^s(p)) \times \mathrm{U}(y), \tag{2.3}$$

[10] The opposite of loss aversion is referred to as gain seeking ($0 < \lambda < 1$) which is a rare empirical finding.

[11] In their (1992) study Tversky and Kahneman empirically find a reflection effect for the utility for gains and losses, *i.e.* $\mathrm{u}(x) = -\mathrm{u}(-x)$ for all $x \in \mathbb{R}$. This is, however, not a structural element of prospect theory.

[12] Original prospect theory (1979) contained only one probability weighting functions that applied to both gains and losses. Cumulative prospect theory (1992) incorporated sign-dependent probability weighting functions.

with $s = +,-$, $x > y > 0$ or $x < y < 0$, and $0 < w^s(p) \leq 1$. For a mixed lottery L with $x > 0 > y$ this value is calculated by

$$\mathrm{PT}(L) = w^{+}(p) \times U(x) + w^{-}(1-p)) \times U(y).^{13} \tag{2.4}$$

If $w^s(p) = p$ for all probabilities then prospect theory reduces to expected utility. Under prospect theory the inequality $\mathrm{PT}(L) \geq \mathrm{PT}(M)$ represents a preference for lottery L over M ($L \succcurlyeq M$).

One of the strengths of prospect theory is that the flexibility of its model specifications accommodates empirical findings that were already known before prospect theory was even developed such as concavity for gains and convexity for losses (Fishburn & Kochenberger 1979), overweighting of losses relative to gains (Robertson 1954), and overweighting of small probabilities and underweighting of large probabilities (Preston & Barrata 1948). Another major strength of the model is that it accommodates anomalies to expected utility, such as the Allais paradox (1953) and many phenomena that can be explained by a "fourfold pattern of risk attitudes", such as the coexistence of gambling and insurance (Tversky & Kahneman 1992 p.297).[14] By accommodating deviations from expected utility into the novel parameters of probability weighting and loss aversion while retaining expected utility's utility function, the model allows us to make "descriptive use of prospect theory to improve the prescriptive use of expected utility" (Bleichrodt, Pinto, & Wakker 2001). Where the utility function under expected utility carried the heavy load of representing both the rational and irrational elements of risk attitude, prospect theory allows it to share this burden with parameters that specifically are designed to deal with irrationality and trusts on utility to be able to bear the weight of rationality. Prospect theory can thus be used to elicit utility functions that represent the decision maker's attitude to outcomes only. This unbiased utility can subsequently be applied to determine risk appetite under the assumptions of expected utility. Interestingly the popular COSO (2004a pp.51–52) framework for enterprise risk management does mention prospect theory but fails to show how its concepts can be applied to express risk appetite. To this end we discuss in the next section two measurement methods from decision theory for the valuation of lotteries: willingness-to-accept and willingness-to-pay.

2.3.3 Willingness-to-accept (WTA) and willingness-to-pay (WTP)

Willingness-to-accept and willingness-to-pay are two methods for the valuation of lotteries that are commonly applied in decision theory. Willingness-to-accept and risk appetite, being the amount of risk that an organization is willing to accept, appear to be related seman-

[13] For binary lotteries original prospect theory (1979) and cumulative prospect theory (1992) PT-values are calculated in the same way, apart from the absence of sign-dependent probability weighting in original prospect theory.

[14] The fourfold pattern of risk attitudes: risk aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk aversion for losses of low probability.

tically. This section investigates whether the two terms are also related in terms of the risk attitudes that they portray.

Several studies have demonstrated that the ask-price for a lottery largely exceeds its bid-price, often by a factor two to one.[15] This spread between willingness-to-accept (the *ask*) and willingness to pay (the *bid*) is much larger than economic theory predicts (Willig 1976). Many studies have investigated the causes of this WTA/WTP-disparity and have tested methods that aim to diminish it (for overviews of these studies see Sayman & Öncüler 2005, Horowitz & McConnell 2002, Brown & Gregory 1999, Hoffman & Spitzer 1993). The marked disparity between both valuation methods emphasizes the need to investigate which measurement method is most appropriate in a particular context (Knetsch & Sinden 1984 p.520). Given the need for proper measurements of organizational risk appetite it is important to assess whether these should be based on willingness-to-accept or willingness-to-pay.

Let χ represent the set of all lotteries that an entity faces that are real-valued functions on a finite set of states of nature Ω. A lottery $X \in \chi$ is thus a mapping $X : \Omega \rightarrow \mathbb{R}$ (Wakker 2010 p.13). We assume that explicit probabilities Q are given on Ω and restrict our discussion to binary lotteries of the type $X = (p : x, (1 - p) : y)$ with $x, y \in \mathbb{R}$, $0 < p \leq 1$ and refer to $N = (1 : 0)$ as the neutral lottery. Favorable lotteries are denoted by X^+ and unfavorable lotteries by X^-. For favorable lotteries the preference $X^+ \succcurlyeq N$ holds and for unfavorable lotteries the preference $X^- \preccurlyeq N$. Table 2.1 provides definitions of willingness-to-accept (WTA) and willingness-to-pay (WTP) both for X^+ and X^-.

Table 2.1 Willingness-to-accept and willingness-to-pay for both X^+ and X^-.[16]

Notation	Definition
WTP(X^+)	maximum sure amount $b \geq 0$ that a decision maker is willing to pay to *acquire* X^+
WTA(X^-)	minimum sure amount $a \geq 0$ that a decision maker is willing to accept to *acquire* X^-
WTP(X^-)	maximum sure amount $b \geq 0$ that a decision maker is willing to pay to *avoid* X^-
WTA(X^+)	minimum sure amount $a \geq 0$ that a decision maker is willing to accept to *abandon* X^+

The indifference relationships that can be used to elicit WTA and WTP values directly are presented in table 2.2. These four relationships are generated by either the zero equivalent or the certainty equivalent method. In the zero equivalent method a decision maker is required to state a sure amount to be added to or subtracted from a lottery in order to equate this lottery with zero (Merkhofer 2009 p.95, Walls 1995 figure 2). In the certainty equivalent method the decision maker is required to state equivalence between a lottery and a positive or nonpositive sure amount.

[15] This is according to Kahneman, Knetsch & Thaler (1990) true for goods in general.

[16] Following Eisenberger & Weber (1995 p.224) the following interpretations can be used as well:
WTP(X^+) : maximum willingness-to-pay to participate in a favorable lottery.
WTA(X^-) : minimum willingness-to-accept to organize a favorable lottery once / sell the lottery short.
WTP(X^-) : maximum willingness-to-pay *not* to organize a favorable lottery once / buy the lottery short.
WTA(X^+) : minimum willingness-to-accept *not* to participate in a favorable lottery.

Table 2.2 Zero equivalence and certainty equivalence relationships for direct elicitation of WTA and WTP for X^+ and X^-.

Elicitation method	**Favorable lottery** X^+ (*i.e.* $N \preccurlyeq X^+$)	**Unfavorable lottery** X^- (*i.e.* $N \succcurlyeq X^-$)
zero equivalent (ZE) method	$N \sim X^+ - \mathrm{WTP}(X^+)$ $(1:0) \sim (p: x-b, (1-p): y-b)$	$N \sim X^- + \mathrm{WTA}(X^-)$ $(1:0) \sim (p: x+a, (1-p): y+a)$
certainty equivalent (CE) method	$X^+ \sim \mathrm{WTA}(X^+)$ $(p: x, (1-p): y) \sim a$	$X^- \sim -\mathrm{WTP}(X^-)$ $(p: x, (1-p): y) \sim -b$

WTP(X^+) is the largest, positive sure monetary amount b that must be subtracted from X^+ to enable equivalence with zero. WTA(X^-) is the smallest, positive sure monetary amount a that must be added to X^- to result in zero equivalence.

Using the indifference relationships for unfavorable lotteries from table 2.2, equations 2.5 and 2.6 illustrate how prospect theory predicts a disparity between WTA(X^-) and WTP(X^-) for a gain-free lottery with $x < y < 0$.

$$N \sim X^- + \mathrm{WTA}(X^-) \quad \Leftrightarrow \quad \mathrm{u}(0) = \mathrm{w}^-(p)\lambda\mathrm{u}(x+a) + \mathrm{w}^+(1-p)\mathrm{u}(y+a) \qquad (2.5)$$

$$X^- \sim -\mathrm{WTP}(X^-) \quad \Leftrightarrow \quad \mathrm{w}^-(p)\lambda\mathrm{u}(x) + (1-\mathrm{w}^-(p))\lambda\mathrm{u}(y) = \lambda\mathrm{u}(-b) \qquad (2.6)$$

Comparison of equations 2.5 and 2.6 suggests that under any specification of prospect theory other than expected value, a need not be identical to b. Under prospect theory differences between a and b can occur due to the presence of loss aversion ($\lambda > 1$), differences in gain and loss curvature ($\mathrm{u}(x) \neq -\mathrm{u}(-x)$ for all $x \in \mathbb{R}$), differences in probability weighting ($\mathrm{w}^+(1-p) \neq (1-\mathrm{w}^-(p))$), and absence of additivity in the utility function ($\mathrm{u}(x+a) \neq \mathrm{u}(x) + \mathrm{u}(a)$ for all $x, a \in \mathbb{R}$).[17]

While the focus in the WTA/WTP–disparity literature has been on explaining the difference in magnitude between a and b, the literature is much less voluble on the difference in risk attitudes which both valuation methods portray. Equation 2.5 illustrates that application of WTA(X^-) under prospect theory invokes an averse attitude to risk, mainly generated by the loss aversion coefficient λ.[18] In decision theory direct elicitations of WTA(X^-) are rare. The certainty equivalent method, however, is a very popular method to elicit utility in the domain of losses. The certainty equivalent method that in equation 2.6 is applied to elicit WTP(X^-) typically results a risk seeking attitude.

[17] Schmidt, Starmer and Sugden (2008 p.204) claim that their Third-generation specification of prospect theory explains the WTA/WTP-disparity while the prior versions of prospect theory are not able to this. Equations 2.5 and 2.6 were, however, solely based on the 1992-model of prospect theory.

[18] On the empirical relevance of loss aversion in relation to risk aversion Wakker (2010 p.234) states: "I think that more than half of the risk aversion empirically observed has nothing to do with utility curvature or with probability weighting. Instead, it is generated by loss aversion, the main empirical phenomenon regarding reference dependence."

Given the tradition in risk management to assume a risk neutral attitude to risk combined with the catastrophic events that have recently plagued the global economy as well as individual organizations, an attitude of "risk seeking for losses" is bound to raise eyebrows among prudent practitioners of risk management. For unfavorable lotteries the willingness-to-accept valuation method is therefore both in wording and in terms of risk attitude congruent with organizational risk appetite. The aversion to unfavorable lotteries that is inherent in the willingness-to-accept measure translates into a higher monetary valuation of risk for these lotteries. The disparity in both magnitude and risk attitude between the two valuation methods emphasizes the need to assess which method is most appropriate as a risk measure in the context of organizational risk appetite.

2.4 Risk measures for organizational risk appetite

This section derives a risk measure for organizational risk appetite drawing on economic valuation methods, actuarial methods for measuring risk, and decision theory. This risk measure is conceptually compatible with organizational risk appetite and existing concepts in decision theory and belongs to the family of convex risk measures, in particular the utility based shortfall risk measure. Its adoption by a decision maker requires the elicitation of an unbiased utility function that can be accomplished by utility elicitation under prospect theory. This unbiased utility function is assumed to reflect the true risk attitude of the organization. For a concave exponential functional it results the entropic risk measure.

2.4.1 Risk measures and acceptance sets

A measure of risk ρ is a functional that assigns a real number to lottery X and describes how close or how far a lottery X is from acceptance (Goovaerts, Kaas, & Laeven 2010, Artzner, Delbaen, Eber, & Heath 1999 p.207). Famous examples of risk measures are value at risk and average value at risk (also known as expected shortfall or tail conditional expectation). The acceptance set associated with risk measure ρ is the subset of lotteries denoted by A for which the risk measure is nonpositive and defined by

$$A = \{X \in \chi : \rho(X) \leq 0\}. \qquad (2.7)$$

From this follows the definition of risk measures for positive and nonpositive values of $\rho(X)$.

Definition 2.1a: For $\rho(X) > 0$, $\rho(X)$ specifies the smallest risk-free monetary amount m that must be added to X to enable it join the acceptance set A.
Definition 2.1b: For $\rho(X) \leq 0$, $-\rho(X)$ specifies the largest risk-free monetary amount m that can be subtracted from X without forcing the lottery to leave the acceptance set A.

The mathematical definition of risk measure $\rho(X)$ is

$$\rho(X) = \inf\{m \in \mathbb{R} : X + m \in A\} \tag{2.8}$$

(Giesecke, Schmidt, & Weber 2008 p.5, Föllmer & Schied 2004 pp.155–156).

On the basis of the definition of risk measures in equation 2.8 and the definition of organizational risk appetite in ISO Guide 73:2009 we define organizational risk appetite quantitatively as follows:

Definition 2.2: Organizational risk appetite is the sum of a risk measure $\rho(X)$ for $X \in B$ where B is a subset of χ that the organization is willing to bear, *i.e.* $\Sigma_B \rho(X \in B : B \subseteq \chi)$.

Where subset A specifies the lotteries for which the risk measure is nonpositive, subset B specifies the set of lotteries which an organization accepts to bear.

It is important to realize that the additivity of risk measures in definition 2.2 demands that lotteries are independent.

Assumption 2.3: Lotteries $X \in B$ are independent.

While the assumption of independence is certainly implausible for χ it is considered a plausible assumption for the risks that are selected by the organization to be part of subset B. In risk management objective historical data on risks is often absent or considered useless for forecasting the future (Vaughan 1997 p.61). The probability of occurrence, the impact of risks and their correlation are in risk management frequently derived from subjective estimates. This limited information is usually not sufficient to make elaborate calculations involving correlation. It is often possible, however, to identify groups of risks that have a common cause. These groups of correlated risks can then subsequently be treated as single independent risks. The practical relevance of assumption 2.3 is corroborated by the fact that the risk matrix, one of the most popular tools for risk evaluation in enterprise risk management, requires risks to be uncorrelated as well (Stanford strategic decisions group 2010b p.19).

The next section assesses whether WTA and WTP are risk measures.

2.4.2 The WTA($^-$) and WTP($^+$) risk measure continuum

The definitions of WTA(X^-) and WTP(X^+) in table 2.1 are consistent with risk measure definition 2.1. WTA($^-$) is thus for the subset of unfavorable lotteries a risk measure in the sense of ρ. It can be interpreted as a contingent claim on the entity's assets that is retained to absorb losses when these materialize. WTP($^+$) is a risk measure for favorable lotteries and suggests to what degree the assets of an entity can be safely reduced. Together they define a continuum of measures of risk on the whole of χ. Definition 2.1 clearly precludes WTA($^+$) and WTP($^-$), assessed by the certainty equivalent method, to join the rank of risk measures.

The definition in COSO of risk appetite as "the amount of risk (..) an entity is willing to accept", with risks defined as "events with a negative impact", calls for the application of a risk measure for unfavorable lotteries (COSO 2004 pp.4, 19). In such a context the WTA($^-$) risk measure can be applied on its own.[19] The risk appetite definition of ISO as the "amount (..) of risk that an organization is willing to pursue or retain", in which case the impact of risk can be negative as well as positive, calls for the application of a risk measure for unfavorable as well as favorable lotteries (ISO Guide 73 : 2009 pp.1, 9). In this context the continuum of WTA($^-$) and WTP($^+$) risk measures can be applied. These two risk measures have in common that they can be directly assessed on the basis of the zero equivalent method. For this reason we will refer to them in combination as the *zero equivalent risk measure*.[20]

Table 2.2 indicates that zero equivalent risk measures can be assessed by direct judgment on the part of a decision maker. To directly assess the value of each individual lottery is, however, highly impractical. Instead the functionals of prospect theory can be elicited from the decision maker and used to estimate the value of the zero equivalent risk measure for each lottery. Both a decision maker's direct judgments as well the estimations based on prospect theory can, however, contain inconsistencies with a rational model of choice, such as expected utility. The next section proposes how to unbias zero equivalent risk measures from these deviations.

2.4.3 Unbiased utility

On the basis of its rational foundations set forth by von Neumann & Morgenstern (1944) we accept expected utility as the normative model for decision making under risk (*normative assumption*). However, we do not accept the *classical elicitation assumption* (Wakker, Bleichrodt, & Pinto 2001 p.1499) that proposes that expected utility is a suitable model to be used in the elicitation of utility functions. Section 2.3.3 discusses several deviations from the rational model of expected utility that potentially penetrate the utility function when expected utility is assumed throughout the elicitation process. When such a biased utility function, which contradicts the assumptions of expected utility, is used as input for this rational model of choice, it is hard to argue in favor of the normativity of its output. We propose therefore to elicit the utility function under the assumptions of a descriptively valid nonexpected theory of choice, such as prospect theory. When applying prospect theory this allows one to distinguish between basic utility u(.) and probability transformation w(.), in both the gain and loss domain, as well as loss aversion λ.[21] This leads to the follow-

[19] Artzner et al. (1999 p.207 REMARK 2.2) specifically allow for the application of measures of risk over the whole domain of χ. Risk measures can therefore also be applied in situations which only involve unfavorable lotteries.

[20] Zero equivalent risk measures should not be confused with the zero utility premium principle for deciding on the value of premium H. The zero utility premium principle is defined as $E[u(H - X)] = 0$, with $u(0) = 0$ (Pfeifer & Heidergott 1997).

[21] The use of the λ-coefficient is so widespread in decision theory that it appears to obscure the fact that the phenomenon of loss aversion can be defined independently from λ (see, however, Abdellaoui, Bleichrodt & Paraschiv 2007 for a nonparametric elicitation of loss aversion).

ing definition of unbiased utility and an assumption related to the true preferences of the decision maker.

Definition 2.4: An unbiased utility function u* is a von Neumann-Morgenstern utility function that is not biased by deviations from the rational expected utility decision model.

Assumption 2.5: The true preferences of a decision maker are represented by an unbiased utility function u*.

The application of u* within the framework of expected utility is referred to as *unbiased expected utility* EU* or E[u*]. Because function u* is assumed to reflect the true or unbiased risk attitude of the decision maker it can then be implemented in equation 2.1 to calculate EU*(X).

Definition 2.6: Given an unbiased utility function u* the unbiased risk attitude is under EU* defined as

(a) [Unbiased risk aversion ⇔ u* is concave];
(b) [Unbiased risk neutrality ⇔ u* is linear];
(c) [Unbiased risk seeking ⇔ u* is convex].

When assumption 2.5 is accepted it still needs to be decided which deviations from expected utility are considered biases. For the purpose of this chapter we consider nonlinear probability weighting, but not loss aversion, to be a bias. We oppose the normative status of probability transformation because overweighting or underweighting of objective probabilities generates possibilities for arbitrage (or a Dutch book) against the decision maker, which makes her worse off on the long-run (Wakker 2010 §1.5).[22] While loss aversion is a deviation from the expected utility model we, contrary to most studies in decision theory (Köbberling & Wakker 2005 p.124), do not oppose its normative status in the context or organizational risk appetite. We argue that the fact that losses erode the earnings base of a business[23]

[22] Bleichrodt, Pinto, and Wakker (2001 p.1500) contains a list of proponents of the normative status of probability transformation.

[23] In business decisions monetary gains and losses are frequently expressed as returns on investments (ROI). ROI is the monetary gain or loss that is returned by the invested principal amount and is expressed as a percentage relative to this invested amount. A gain of amount r is relative to an invested principal amount i expressed as r / i and the loss amount $-r$ is expressed as $-r / i$. Once the principal i has because of loss $-r$ been reduced to $i - r$ then a return on investment of $r / (i - r) = {}^{r}\!/\!{}_{i} / (1 - {}^{r}\!/\!{}_{i})$ is required to compensate the initial loss and recapture the original principal again. Thus an initial loss of –10% in terms of ROI can be recouped by subsequent gain of $.1 / .9 \approx +11.1\%$ in terms of ROI. A negative ROI of –50% is recouped by a subsequent gain of $.5 / .5 = +100\%$, thus requiring twice the effort in terms earning ROI percentage points to break even again. A ROI of –90% requires a massive positive response of a ROI of $.9 / .1 = +900\%$. ROI exposes an important feature of losses in business being that losses erode the earnings base of a business. This feature is often lost in translating reality to the laboratory where commonly utility elicitations do not require investment of a principal amount and financially incentivized lottery choices are played out once and immediately. The empirical implication that losses erode the earning potential of a business (while gains ever more increase this potential) justifies a preference intensity for gains relative to losses in a business context.

and damage its capacity to act as a going concern[24] constitute genuine empirical reasons for the adoption of loss aversion relative to a reference point around zero. We therefore consider loss aversion to be part of the von Neumann-Morgenstern utility function.[25]

A risk measure is appropriate if and only if its characterizing axioms are. To this end the next section assesses how measurements of unbiased utility can be used to connect the zero equivalent risk measure with the axiomatizations of the utility based shortfall risk (UBSR) measure.

2.4.4 Utility based shortfall risk (UBSR)

In their seminal article on risk measures Artzner, Delbaen, Eber and Heath (1999) argue that four axioms, in particular monotonicity, translation invariance, subadditivity, and positive homogeneity, should hold for any risk measure. They call measures satisfying these properties *coherent*. In combination, their last two axioms discourage diversification. In *convex* risk measures they are for this reason replaced by the less restrictive axiom of convexity (Deprez & Gerber 1985, Heath 2010, Föllmer and Schied 2002). Within the family of convex risk measures *utility based shortfall risk* (UBSR) has in particular useful properties such as distribution invariance and invariance under randomization (Föllmer & Schied 2004, Giesecke, Schmidt & Weber 2008, Gundel & Weber 2008, Föllmer & Schied 2010). As a result UBSR measures are the only distribution-invariant convex risk measures that should be used for the dynamic measurement of risk over time (Weber 2007).[26]

Under UBSR acceptance sets are determined by the valuation of lottery X in terms of expected utility $\mathrm{E}[\mathrm{u}(X)]$ for a strictly increasing and typically concave function $\mathrm{u} : \mathbb{R} \rightarrow \mathbb{R}$.[27] A requirement is that for lotteries $X \in \chi$ the expectation $\mathrm{E}[\mathrm{u}(X)]$ is well-defined and finite. A lottery belongs to the acceptance set if $\mathrm{E}[\mathrm{u}(X)]$ is bounded from below by a given threshold z within the range of u, *i.e.*

$$A = \{X \in \chi : \mathrm{E}[\mathrm{u}(X)] \geq z\} \tag{2.9}$$

24 In business life monetary losses may result in severe second-order effects such as damaged reputation, liquidity squeeze, and eventually insolvency. Losses act as a negative signal to the suppliers, capital providers and other stakeholders in the business. All these effects disable businesses to act as going concerns and eventually threaten their survival. For these reasons the dispreference for losses is amplified in general and for large losses in particular. Further amplification of the aversion to accept potential losses results from budget-constraints for making expenditures on measures to control risk and from capital-constraints for absorbing the potential impact of losses.

25 For cases where relevance of the reference point is plausible Bleichrodt, Pinto & Wakker (2001 p.1500) state that: "If there are intrinsic reasons why losses with respect to a status quo are more serious than corresponding gains, then we consider this effect as part of the genuine von Neumann-Morgenstern utility function. It belongs to the expected utility model and does not depend on irrelevant reframings."

26 The axiomatic foundations in this section trace back to studies in decision theory such as Gilboa & Schmeidler (1989).

27 By letting $\ell(\mathrm{x}) \equiv -\mathrm{u}(-\mathrm{x})$ we can alternatively use a convex loss function $\ell : \mathbb{R} \rightarrow \mathbb{R}$ with z being a point in the interior of the range of ℓ, so that $A = \{X \in \chi : \mathrm{E}[\ell(-X)] \leq z\}$.

(Giesecke, Schmidt, & Weber 2008).

In the context of organizational risk appetite UBSR measures are in particular relevant because a utility function that is concave is sensitive to large losses. As a result this risk measure is in particular useful to measure risks of a catastrophic nature. It should be noted that the UBSR does not require the utility function to be strictly concave even though several authors adopt the assumption of diminishing marginal utility from economics (Gundel & Weber 2008 p.1128, Föllmer & Schied 2010). UBSR measures are thus capable of accommodating convex utility function and risk seeking attitudes.

In the next section UBSR measure is used to define the zero equivalent risk measure.

2.4.5 The definition of the zero equivalent risk measure

When the utility function u(.) is invertible to $u^{-1}(.)$ equation 2.9 for the UBSR measure can alternatively be expressed as

$$A = \{X \in \chi : u^{-1}(E[u(X)]) \geq u^{-1}(z)\} \tag{2.10}$$

in which utility values are transformed back to monetary amounts. When the inverse utility of the expected utility of X, $u^{-1}(E[u(X)])$,[28] in equation 2.10 exceeds $u^{-1}(z) = 0$ then equation 2.10 returns the acceptance set for the zero equivalent risk measure.[29]

Definition 2.7: The zero equivalent risk measure $\rho^{ze}(X)$ is defined by the acceptance set

$$A = \{X \in \chi : u^{-1}(E[u(X)]) \geq 0\}. \tag{2.11}$$

When utility elicitation is performed parametrically with the exponential utility function or when nonparametric utilities can be approximated by an exponential utility function then the entropic risk measure can be used. The entropic risk measure is discussed in the next section.

2.4.6 Entropic risk

The entropic risk measure is a member of the family of UBSR measures that is based on the assumption of Constant Absolute Risk Aversion (CARA) of the decision maker (Pratt 1964).[30] Under CARA utility depends on the risk tolerance R of the decision maker and is for concave utility described by the exponential utility function

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[28] The term $u^{-1}(E[u(X)])$ is also known as the certainty equivalent of X, CE(X). While Definition 2.1 excludes WTA and WTP values which have been directly elicited by means of the certainty equivalent method, it does not preclude the use of certainty equivalents in general.

[29] Restriction to zero is allowed given that 0 lies in the interior of u.

[30] Relative entropy is a non-symmetric measure in probability theory of the difference between two probability distributions. It can be shown that minimization of relative entropy is equivalent to the maximization of expected utility.

$$u(x) = 1 - e^{-x/R}. \tag{2.12}$$

Risk tolerance is the amount an individual is willing to stake in a 50/50 bet that either returns twice the stake or makes one lose half of it (Merkhofer 2009 p.83, Walls 1995 p.308).[31] This definition clearly differs from the interpretation of risk tolerance as the "acceptable level of variation relative to achievement of a specific objective" (COSO 2004a p.20) or its interpretation as the probability α in confidence level $(1 - \alpha)$ in Economic Capital calculations (RMTF 2004 p.5, Solvency II 2009 article 45 p.34, Basel II 2006 article 730 p.206). Just as linear utility the exponential utility function is additive. Additivity in the utility function is defined by $u(x + y) = u(x) + u(y)$ for all $x, y \in \mathbb{R}$.

On the basis of equation 2.9 the entropic risk measure is defined as

$$\rho^{ent}(X) = \inf\{m \in \mathbb{R} : E[1 - e^{-(m+X)/R}] \geq z\}.^{32} \tag{2.13}$$

The solution of equation 2.13 returns

$$m = R\,(ln\, E[e^{-X/R}] - ln\,(1 - z)). \qquad \text{(2.14, proof in appendix)}$$

(Föllmer & Schied 2004 §3.2, p.174 EXAMPLE 4.33, p.213 EXAMPLE 4.105). This result is consistent with equation 2.10 (see proof in appendix).

In the next section we define several decision rules for organizational risk appetite that are derived from the aforementioned risk measures and provide several examples in which we apply these principles using the zero equivalent risk measure and exponential utility.

2.5 Decision rules for organizational risk appetite

While definition 2.2 provides a precise definition for measuring the "amount" of organizational risk appetite by means of a risk measure, it does not specify the principles on the basis of which an organization decides which subset of risks it is willing to bear. This section specifies two decision rules that can be applied in the context of organizational risk appetite. Subsequently it provides several examples.

2.5.1 Derivation of decision rules for organizational risk appetite

This section derives several decision rules that are are defined over risk measures and that, by a restriction or optimization procedure, aid in the decision making on organizational

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31 In other words: "What amount would you be willing to invest if you can double your investment or lose half of it given 50/50 odds?" See also Wakker 2010 p.82 OBSERVATION 3.5.3 and EXERCISE 3.5.5.

32 Using the loss function $\ell(x) = e^{-\beta x}$ Föllmer and Schied (2004 p.213) define the entropic risk measure as $\rho(X) = \inf\{m \in \mathbb{R} : E[e^{-\beta(m+X)}] \leq x_0\}$. Because the utility function $u(x) = 1 - e^{-\beta x}$ is used the sign in this formula reverses.

risk appetite.[33] A rudimentary set of decision rules can be found in the first textbook on the subject of risk management by Mehr and Hedges (1963 chapter 1, see also Vaughan 1997 pp.62–67). It stipulates that risk management decisions are guided by three rules of risk management:

Rule #1: *Don't risk more than you can afford to lose.*
On the basis of the maximum possible loss this rule identifies risks that cannot be retained and require treatment. The decision to treat or retain risk is made on the basis of the *minimax cost rule*, *i.e.* choose the treatment or retention strategy with the smallest maximum cost.

Rule #2: *Consider the odds.*
For the set of unacceptable risks identified by the first rule the second rule states that only low probability risks are suitable candidates for insurance coverage.

Rule #3: *Don't risk a lot for a little.*
Below the maximum retention level some risks require risk treatment as well. The decision to treat or retain risk is made on the basis of the *marginal benefit-marginal cost rule*.

We use rule #1 and #3 to derive two decision rules. In deriving these decision principles we make four simplifying assumptions. Adoption of these assumptions allows us to negate many complexities related to the timing of risk, such as risk incubation time, risk lifecycles, differences in risk management planning horizons, and the timing of risk reduction expenses. They also ensure that the cost of risk reduction efforts are expressed in the same unit as their benefits in terms of lower damages.

Assumption 2.8: All risks are resolved at the end of a single period.
Assumption 2.9: All probabilities $p \in Q$ relate to a single period.
Assumption 2.10: All costs for risk reduction are expensed at the start of the period.
Assumption 2.11: All outcomes are discounted to present values.

The first decision rule is inspired by rule #1 but not equivalent. It stipulates that the amount of risk should not exceed a risk bearing constraint (Spinard, Faris, Culp, & Nunes 2010).

Decision rule 1: The membership of the subset $B \subseteq \chi$ that the organization is willing to bear is constrained by the risk bearing capacity d which should not be exceeded by the sum $\sum$ of the amount of risk $\rho(X)$ embodied in B, *i.e.*

[33] Decision rules should not be confused with decision principles (see Goovaerts, Kaas and Laeven 2010 p.294–295).

$$B = \{X \in \chi : \textstyle\sum_B \rho(X) \leq d\}. \tag{2.15}$$

While rule #1 considers single risks only, decision rule 1 considers in addition sets of risks that together should not exceed the risk bearing capacity.

The second decision rule is derived from rule #2 and states a risk reduction optimization rule. It is based on the aforementioned *marginal benefit-marginal cost rule* (Vaughan 1997 p.67). Traditionally this rule defines the marginal benefits of risk reduction efforts in terms of decreases in expected loss. Instead we define benefits in terms of the opportunity gains that result from reductions in the sum $\sum$ of risk measure $\rho(X)$. The principle basically states that additional risk reduction efforts should be made only when these increase organizational value.

Decision rule 2: Replace lottery X by Y if and only if the resulting change in risk measure, $\rho(X) - \rho(y)$, does equal or exceed the cost c of this change, *i.e.*

$$\rho(X) - \rho(y) \geq c, \text{ with } c \geq 0.^{34} \tag{2.16}$$

Derivation of these decision rules from the zero equivalence risk measure (Definition 2.7) ensures their compatibility with the concept of organizational risk appetite. Calculation of the zero equivalence risk measure on the basis of an unbiased utility function (Definition 2.4) elicited from the organization's senior management ensures that this risk appetite is a reflection of their true attitude to risk (Definition 2.6) under assumption 2.5.

2.5.2 Examples of the application of the organizational risk appetite rules

On the basis of the definition of risk appetite in the BS 31100: 2008 in which organizational risk appetite is defined as the amount of risk that an organization is prepared to seek, accept or tolerate we consider three examples in which subsequently speculative risk is sought, inherent risk is accepted, and residual risk is tolerated.[35]

[34] If $\rho(X) > \rho(y)$ then risk reduction implies that the capital requirements of the organization are reduced. This capital is freed for investment purposes and given rate of return k the opportunity gains are $k \times (\rho(X) - \rho(y))$. Because we restrict ourselves to a single period (Assumption 2.8) and assume present values only (Assumption 2.11) the reduction in capital $(\rho(X) - \rho(y))$ is directly compared with the increased expenditure of capital c. When we relax the single period assumption equation 2.16 still holds if we define $k \times c$ as the periodic risk expense and c as an infinite expense for risk reduction.

[35] Loosemore Raftery Reilly Higgon (2006 §1.4.5 p.12) define speculative risks as risks that offer a chance of loss and gain and pure risks as those that offer only the prospect of loss. In risk management inherent risk is defined as risk before risk reduction and residual risk as risk after risk reduction.

Example 2.1: Risk pursuit

In *Fooled by randomness* (2004 p.99 table 6.1) Nassim Nicholas Taleb introduces the speculative lottery $X_1 = (.999 : 1, .001 : -10{,}000)$ that offers the prospect of a high probability of generating small profits and a small probability of a catastrophic result (a *Black Swan*, Taleb 2007). He uses this example to illustrate the combined effect of *asymmetric odds* and *asymmetric outcomes*, a feature that is common to stock markets (Taleb 2008 figures 1 and 2, the *classical problem of the turkey*).

We adopt this example to illustrate the difference between linear and exponential utility. We interpret lottery X_1 as the pay-off structure of a department of a firm with the outcomes of X_1 representing after tax net operating profits. We additionally assume a departmental risk bearing capacity of 10, a total shareholder value of 100, a weighted average cost of capital k of .05, a return on investment r of .1, and an initial capital investment i of 10. To assess whether the pursuit of value warrants the acceptance of the risk inherent in lottery X_1, we calculate the department's contribution in Economic Value Added to the firm (EVA, Bennett Stewart III 1991). EVA is an estimate of the firm's economic profit and is based on net operating profits ($r \times i$), weighted average cost of capital (k), and capital employed. Capital employed includes both capital invested (i) and the economic capital that the firm is required to set aside for pursuing risk (m). Its formula is: EVA = $(r \times i) - k \times (i + m)$.

Linear utility. We first apply the zero equivalent risk measure ρ^{ze} to X_1 for a firm who is risk neutral with $u(x) = x$. Risk neutrality of the firm is in the finance literature as well as in decision theory (*e.g.* Smith 2004) frequently assumed to be in the best interests of the shareholder. Under linear utility the minimum additional capital requirement that would be sufficient to include X_1 in the acceptance set is $m = 9$ (*i.e.* −EV).[36] EVA = .05, which is a factor 20 lower than profits but still positive.[37] Given the availability of sufficient risk capacity (decision rule 1) the department may still be granted a license to operate.

Exponential utility. We next apply the zero equivalent risk measure ρ^{ze} to X_1 for a firm who is risk averse with exponential utility $u(x) = 1 - e^{-x/R}$. Empirical surveys indicate that risk tolerances of investors, boards, and CEOs rate R between 10%–25% of the shareholder value of the firm (Stanford University Strategic Decisions Group 2010). Based on this rule of thumb, R is assumed to be 25. We apply these numbers to the formula for the entropic risk measure in the appendix and find that the minimum additional capital requirement that

[36] After addition of the additional capital requirement $m = 9.001 \approx 9$ prospect $X_2 = (.999 : 10.001, .001 : -9{,}991)$ results which EV equals 0.

[37] EVA = $1 - .05\,(10 + 9.001) \approx .05$.

would be sufficient to include X_1 in the acceptance set is $m = 9{,}827$.[38] This amount differs from the additional capital requirement under linear utility by more than a factor 1,000. EVA = −491, which demonstrates that the department is not adding value to the firm but instead is destroying value.[39] On the basis of the immense amount of additional economic capital that needs to be maintained and the negative EVA, the department is unlikely to receive approval for continuing its operations.

In this example the department is under the popular assumption of risk neutrality still considered to contribute value to the firm while under plausible assumptions of risk aversion, the department is destroying value. This is true even though it is expected to be profitable in 999 out of a 1000 periods. Our augmentation of Taleb's original example with corporate risk attitudes adds to the strength of his exclamation that: "The *frequency* or *probability* of the loss, in and by itself, is totally irrelevant; it needs to be judged in connection to the *magnitude* of the outcome." (2004 pp.98–99). When risk managers do indeed "place a greater emphasis on earning profits than they do on avoiding losses" (Taleb, Goldstein, & Spitznagel 2009 p.80), then adopting the zero equivalent risk measure, which is based on a risk tolerance which is accepted by the firm's stakeholders, will aid in the pursuit of real value.

Example 2.2: Risk retention

In their management of operational risk banks retain those risks that are considered too small to be material. In his contribution to *Mastering operational risk* Pezier (2003) illustrates this by means of a log-frequency / log-severity diagram (see figure 2.1). The figure displays a boundary between a zone where risks deserve attention and a zone where they are considered negligible. The axes define decimal logarithm scales for annual frequency (vertically) and relative severity (horizontally). Relative severity is defined as monetary impact as a fraction of a bank's capital.[40] In Pezier (2003) risks of which the expected loss and the standard deviation of losses is less than ten thousandth of capital are considered immaterial. This boundary is in figure 2.1 displayed by the kinked boundary line. This method assumes that all risks within the boundary line fulfill decision rule 1 even though their aggregated amount of risk is not compared directly with risk bearing capacity (Clemens & Swallom 2005).

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[38] Using the formula for m which is derived in the appendix we find $m = 25(ln(.999\ e^{-1/25} + .001\ e^{10{,}000/25})) \approx 9{,}827$.

[39] EVA = 1 − .05 (10 + 9,827) ≈ −491.

[40] Pezier (2003) divides the impact of operational losses reported in Basel's Second Quantitative Impact Study by € 3 billion, the average capital of the sample of banks in the study.

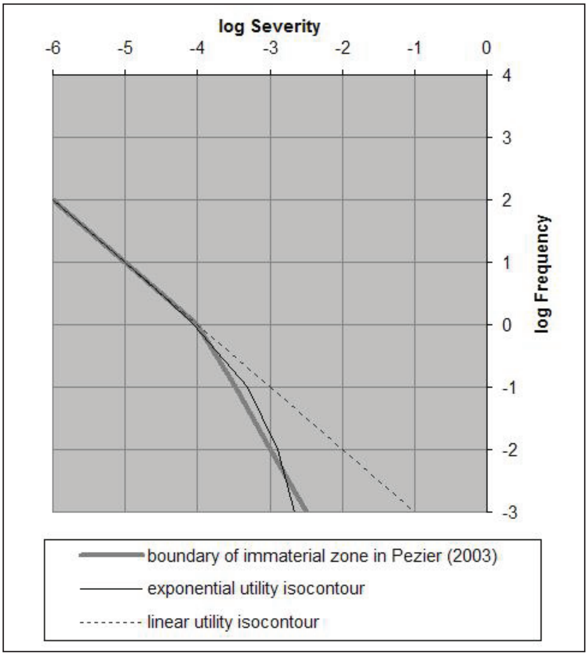

Figure 2.1 Log-frequency / log-severity diagram specifying the boundary of immaterial losses and exponential and linear utility isocontours.[41]

The kinked boundary line can be approximated by an exponential utility function with risk tolerance R = 1 million (see figure 2.1).[42] The rather low tolerance for risk of this exponential function suggests a high aversion to risk. The risk averse nature of the exponential utility isocontour is illustrated by the large difference between exponential and linear utility isocontours

[41] The observant reader acquainted with Pezier's original ISMA (2002) discussion paper which was made in preparation for his chapter in *Mastering operational risk* will notice that the lines in his figure 1 are incorrectly positioned. By elongating in his figure 1 the line section above the kink to the coordinate (–4, 0) and shifting the line section below the kink 0.25 points to the right, his figure becomes consistent with the text in his paper and with our figure 2.1.

[42] The exponential utility isocontour in figure 2.1 was derived using the following steps. In figure 2.1 the coordinate (–6, 2) represents an event with a loss with a magnitude of 10^{-6} of a bank's average capital and a frequency of 10^{2} per year. This annual frequency translates to a daily probability of occurrence of approximately .27 (= 100/365). Multiplication of 10^{-6} with average bank capital results in loss amount of 3.000 ($=10^{-6} \times 3 \times 10^{9}$). The equation for the expected utility of the lottery $X = (.27 : -3000, .73 : 0)$ which corresponds with coordinate (–6, 2) is then $\mathrm{EU}(X) = .27 \times (1 - e^{3.000/\mathrm{R}})$. Along the isocontours in figure 2.1 the value of EU(X) is held constant. For each ordinate y of coordinates in the exponential isocontour in the range [–3, 2] its corresponding abscissa x is derived by the equation $x = log((\mathrm{R}\ ln(1 - (\mathrm{EU}(X) / (10^{y} / 365))) / (3 \times 10^{9}))$. Applying ordinary least squares to this equation results in the optimum R = 1.173.777,073 which for ease of exposure was rounded to 1 million in example 2.2.

in figure 2.1 at an annual frequency of .001. When we adopt the 25% of firm value benchmark for risk tolerance that was used in example 2.1 and consider the average capital of 3 billion to be a conservative estimate of firm value then risk tolerance would be estimated at 750 million. This rather conservative estimate differs by a factor 750 from the exponential approximation of the boundary in figure 2.1.

This example illustrates how utility functions can be used to benchmark the boundaries of risk appetite in risk management tools, such as the log-frequency / log-severity diagram in figure 2.1. If an unbiased utility function of senior management is available then this function can be directly used to express the degree of risk appetite in these tools.

While the previous examples considered risk pursuit and risk retention, the focus in the next two examples will be on risk treatment. Risk treatment stands for the effort the organization takes to reduce the probability of occurrence and/or the impact of a risk. By risk treatment the original, inherent risk is transformed to a tolerable, residual risk. In the first example we consider risk transfer to an insurer. In the subsequent example we focus on control activities that the organization carries out itself to reduce the probability of occurrence or the impact of risk.

Example 2.3.1: Risk treatment by transfer

We adopt the example in Vaughan (1997 p.61) of the insurable risk $X_1 = (.01 : -100{,}000, .99 : 0)$ and a premium of $X_2 = (1 : -1{,}500)$ for full insurance coverage. We assume no other risks beside X_1. On the basis of this example decision rules 1 and 2 are applied for organizations with either low or high risk bearing capacity.

Low risk bearing capacity. We first consider a decision maker with a risk bearing capacity d of 70,000 and assume a risk tolerance of 15,000.[43] Because we assume no other risks we can directly assess whether the zero equivalent risk measure ρ^{ze} applied to X_1 does not exceed d (decision rule 1). On the basis of ρ^{ze} the minimum additional capital that this decision maker would require to include X_1 in the acceptance set is $m = 32{,}703$, which falls below the risk bearing capacity.[44] X_1 is thus acceptable under decision rule 1 even though by choosing to retain X_1 the decision maker clearly risks more than she can afford to lose (rule #1). Because the additional capital requirement for retaining X_1 is much higher than the insurance premium of 1,500 the decision maker still prefers risk transfer by insurance (decision rule 2). This example illustrates rule #3 "don't risk a lot for a little" where the risk averse decision maker considers the amount of additionally required capital to be "a lot" and the insurance premium "a little".

[43] Delquié (2008) in table 1 demonstrates the relationship that under exponential utility exists between maximum acceptable loss at a particular probability level and risk tolerance. According to this table the maximum acceptable loss at $p = .01$ is 4.61 times risk tolerance. Starting with a maximum acceptable loss of 70,000 we estimate risk tolerance at 15,184 (= 70,000 / 4.61) and round this to 15,000.

[44] $m = 32{,}702 = 15{,}000(ln(.01\ e^{100{,}000\,/15{,}000} + (1 - .01)))$.

High risk bearing capacity. Next we consider a decision maker with a risk bearing capacity of 700,000 for which we assume a risk tolerance of 150,000.[45] The minimum additional capital that this decision maker would require to include X_1 in the acceptance set on the basis of ρ^{ze} is m = 1,415.[46] This amount clearly falls below risk bearing capacity and retention of X_1 would thus be acceptable on the basis of decision rule 1. The additional capital requirement for retention of X_1 can be waived by accepting the insurance policy at a cost of 1,500. However, based on the marginal benefit-marginal cost rule (decision rule 2) such a risk treatment is not advisable.

Even when insurance is not advisable for X_1 on the basis of decision rules 1 and 2, X_1 can still be a suitable candidate for risk treatment by probability or impact reduction. This is illustrated in the next example.

Example 2.3.2: Risk treatment by probability or impact reduction

We consider a decision maker with a risk tolerance of 150,000 who requires an additional capital amount of $m = 1{,}415$ in order to retain risk $X_1 = (.01 : -100{,}000, .99 : 0)$. Now assume that at the expense of 750 this decision maker has either the option to halve the probability of occurrence of X_1, which obtains $X_2 = (.005 : -100{,}000, .995 : 0)$ or halve the consequent impact of X_1, which results $X_3 = (.01 : -50{,}000, .99 : 0)$. Notice that both treatments halve the expected value of X_1 at half the cost of full insurance. While the mean values of X_2 and X_3 are identical the variance of X_2 is more than twice as large than the variance of X_3.[47] Variance is a measure of variability. When it is interpreted as an indicator of risk this suggests that the residual risk after probability reduction, X_2, is more riskier than residual risk after impact reduction, X_3.

This difference in riskiness is corroborated by the minimum capital requirement for holding risk X_2 and X_3. For risk X_2 an amount of m = 709 is required while risk X_3 requires a lower amount of m = 592.[48] Compared with the capital requirement that needs to be maintained to retain X_1 the reduction in capital requirement is larger for impact reduction (1,415 − 592 = 823) than for probability reduction (1,415 − 709 = 706). The marginal benefit-marginal cost rule (decision rule 2) suggests that only the benefits of impact reduction warrant the use of the 750 additional expense.

This is an illustration of the how the risk-free monetary amount m, typically the amount of additional capital that makes retention of a risky event acceptable, can alternatively be used to determine whether it is advisable to treat the risk of the event itself. This application of risk measures to risk treatment was already alluded to in Artzner, Delbaen, Eber and Heath (1999 p.205). It also illustrates that when impact and probability reduction accommodate the same reduction in expected losses, impact reduction

45 A risk tolerance of 129,000 would still induce decision makers to insure at a premium of 1,500. At a maximum acceptable loss of 700,000 at probability p = .01 the risk tolerance is in line with Delquié (2008) estimated at 152,003, which for convenience is rounded to 150,000.

46 $m = 1{,}415 = 150{,}000(ln(.01\, e^{100{,}000\,/150{,}000} + (1 - .01)))$.

47 $\mathrm{Var}(X_2) = 49{,}750{,}000$, $\mathrm{Var}(X_3) = 24{,}750{,}000$.

48 $m = 709 = 700{,}000(ln(.005\, e^{100{,}000\,/700{,}000} + (1 - .005)))$.
$m = 592 = 700{,}000(ln(.01\, e^{50{,}000\,/700{,}000} + (1 - .01)))$.

is preferred over probability reduction under risk aversion (Wakker 2010 p.75, *aversion to mean-preserving spreads*).

2.6 Discussion

This chapter combines models and methods from decision theory and measure theory to enable the measurement of organizational risk appetite by means of the risk attitude of its senior management. It demonstrates how the rational, normative choice model of expected utility combined with the convex, utility based shortfall risk measure is capable of measuring the organizational willingness to accept risk in the pursuit, retention, and treatment of risk. Contrary to the mainstream tradition in decision analysis it proposes to elicit the utility function under prospect theory. Under prospect theory utility elicitation produces an unbiased utility function representing the unbiased risk attitude of senior management, which under the assumptions of expected utility theory can be used to measure organizational risk appetite.

The unbiased utility function of senior management need not be concave, as assumed in economics, but can be convex, resulting in an unbiased risk seeking attitude to risk. While risk seeking in the domain of losses is a rather robust descriptive finding in decision theory it is not used normatively in risk management and is considered normatively undesirable in economics. While decision theory offers no normative arguments against such a convex utility function – it is a von Neumann-Morgenstern utility function and fulfills the axioms of rational decision making – the law of marginal diminishing utility in economics does offer such an argument by requiring risk measures need to be derived from concave utility functions. This emphasizes the mere fact that descriptive findings under decision theory need not correspond with normative theory (Howard 1988 p.683). The finding of convex utility requires the decision analyst, in the context of organizational risk appetite, to request the decision maker to reconsider her natural preferences in the light of their undesirable tendency to promote risk seeking behavior.

The robust empirical finding of a disparity in both magnitude and risk attitude between the willingness-to-accept (WTA) and willingness-to-pay (WTP) valuation methods, emphasized the need to assess which method is most appropriate as a risk measure in the context of organizational risk appetite. Based on measure theory this chapter therefore introduced a continuum of risk measures comprising WTA($^-$), the minimum amount that a decision maker demands for accepting an unfavorable lottery, and WTP($^+$), the maximum amount that a decision maker is willing to pay for acquiring a favorable lottery. It is interesting to note that a similar conclusion was formulated in environmental economics, where WTA was proposed as the most appropriate measure for compensating citizens for damage to environmental resources and WTP for measuring the value of extension of environmental resources (Knetsch 2005 p.94, Brown & Gregory 1999). This suggests that WTA and WTP are conceptually symmetrical and that WTA should be used to evaluate losses and WTP to evaluate gains (Knetsch 2000).

The risk measure concept is compatible with a utility elicitation method in which a sure amount is added to or subtracted from a lottery to attain zero-equivalence. In decision theory utility elicitations based on zero equivalence are rare. In contrast, elicitations based on certainty equivalents, which are incongruent with the risk measure concept, are very common. On the premise that utility functions should be applied in the context in which they were elicited (Hey, Morone, & Schmidt 2009), the lack of congruence between certainty equivalents and risk measures suggests that in the context of organizational risk appetite, it is preferable to avoid the use of certainty equivalents.

The illustrative examples in this chapter illustrate that it is rather easy to apply the exponential utility function, on the premise that this represents the unbiased risk attitude of senior management, in combination with a selection of decision rules to facilitate decision making about organizational risk appetite. Practical measurements of risk appetite by organizations and society most of the times assume a risk neutral attitude and only rarely employ expected utility theory. A risk neutral decision does not differentiate between low probability-high impact and high probability-low impact risks, given that expected values are identical. While the means of these two risks are identical, their variances are very different, which suggests that the two risks are characteristically dissimilar in terms of riskiness. Under risk neutrality, whether the risk bearing capacity of the organization is sufficient to recover from the high impact loss is not a consideration. Organizations who deal with one-shot opportunities and threats will in case of default find little consolation in the fact that given more time they would on average have succeeded based on the law of large numbers in statistics. The severe organizational "pain" caused by high impact risks can be expressed by assigning more weight to outcomes that have serious organizational repercussions by means of utility functions. Contrary to risk neutrality, which assumes a linear utility function, this requires outcomes to be weighed nonlinearly by means of a concave utility function.

The lack of adoption of the models and methods of decision theory in risk management may be attributed to the fact that most practitioners in risk management have not received a formal training in decision analysis. Well known tools in risk management do, however, implicitly assume nonlinear weighting of outcomes and probabilities, which is only evident to those who have been exposed to decision modeling (see for example appendix A). It is our hope that this chapter will inspire risk practitioners to consciously apply the concepts of decision theory in their measurement of organizational risk appetite and attract researchers from decision theory to explore this new applied field of research.

2.7 Appendices

2.7.1 Appendix A: Rating scales for financial damages and frequency in Fine (1971)

In Fine's method the priority assigned to a hazard[49] is determined by a risk score (R), which is calculated on the basis of numerical ratings of a hazard's consequences (C), exposure (E), and probability (P) using the formula

[49] Fine (1971 p.2) defines a hazard as "any unsafe condition or potential source of an accident" and hazard-event as "an undesirable occurrence; the combination of a hazard with some activity or person which could start a sequence of events to end in an accident."

$$R = C \times E \times P. \tag{2.17}$$

The *consequences* of a hazard are defined as the most probable results of a potential accident, *exposure* as the frequency of occurrence of the hazard-event that could lead to the accident, and *probability* as the likelihood that the hazard-event will result in a chain of events leading to the accident and its consequences. Probability P is thus a conditional probability, which demonstrates that accidents depend on exposure to a hazard. Considering that the operation $E \times P$ in the formula represents the likelihood of an accident, equation 2.17 thus as a whole directly translates to the traditional mantra in risk management that risk is an accident's "probability times impact" (Goodpasture 2004 p.229, *P * I analysis*), which merely is a restatement of the expected value concept.

For each of the three hazard characteristics in equation 2.17 the method defines six degrees of seriousness described mainly in qualitative terms and mapped to numerical ratings in classification tables. By setting the highest score of consequences to 100 and those of exposure and probability to 10 each, the method assigns equal importance to the accident's impact and likelihood. Figure 2.2 presents the relationship between numerical ratings and a selection of (a) consequences, expressed in financial damages per accident, and (b) exposure, expressed in frequency of occurrence per year.

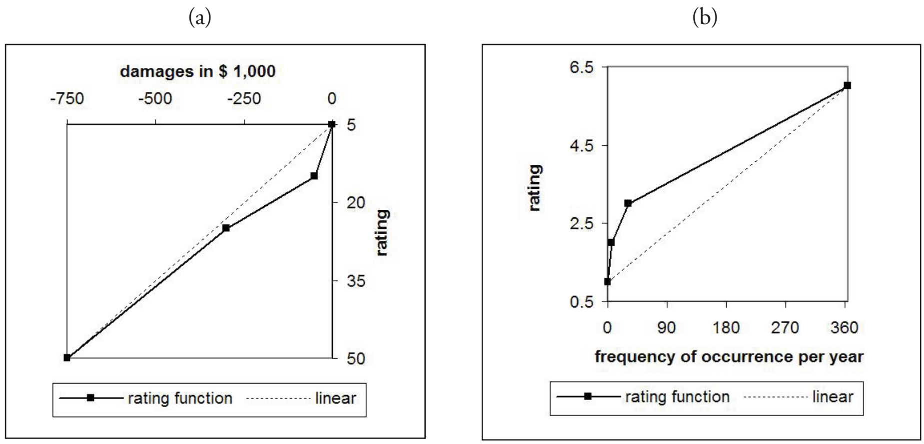

Figure 2.2 Midpoint rating of a selection of (a) financial damages and (b) frequency of occurrence based on Fine (1971 appendix B p.28).[50]

The curves in figure 2.2 display a convex relationship between financial damages and its rating and a concave relationship between frequency of occurrence and its rating.

[50] For four out of the six degrees the classification tables suggest upper and lower limits for both consequences and exposure. Figure 2.2 presents the midpoint rating between these two limits.

2.7.2 *Appendix B: Proofs*

PROOF OF DERIVING EQUATION 2.14 FROM EQUATION 2.13

$$\rho^{ent}(X) = \inf \{m \in \mathbb{R} : E[1 - e^{-(m+X)/R}] \geq z\} \tag{2.13}$$

$E[1 - e^{-(m+X)/R}] = z$ (assess the infimum *i.e.* the greatest lower bound)

$E[1 - e^{-X/R} \cdot e^{-m/R}] = z$

$1 - E[e^{-X/R}]\, e^{-m/R} = z$ (the expectation of a constant is a constant)

$E[e^{-X/R}]\, e^{-m/R} = 1 - z$

$ln\, (E[e^{-X/R}]\, e^{-m/R}) = ln\, (1 - z)$

$ln\, E[e^{-X/R}] - m/R = ln\, (1 - z)$

$-\, m/R = -\, ln\, E[e^{-X/R}] + ln\, (1 - z)$

$$m = R\, (ln\, E[e^{-X/R}] - ln\, (1 - z)).\ \square \tag{2.14}$$

PROOF OF EQUALITY BETWEEN EQUATION 2.14 AND EQUATION 2.10

$$A = \{X \in \chi : u^{-1}(E[u(X)]) \geq u^{-1}(z)\} \tag{2.10}$$

$u^{-1}(E[u\backslash(X)]) = u^{-1}(z)$

Assume exponential utility and apply this to $X + m$.

STEP 1: find the inverse utility $u^{-1}(.)$

$u(x) = 1 - e^{-x/R}$

$1 - u(x) = e^{-x/R}$

$ln(1 - u(x)) = -x/R$

$u^{-1}(u(x)) = x = -R ln(1 - u(x))$

STEP 2: calculate the expected utility $E[u(X + m)]$

$E[u(X + m)] = pu(x + m) + (1 - p)u(y + m)$

$E[u(X + m)] = p(1 - e^{-(x+m)/R}) + (1 - p)\, (1 - e^{-(y+m)/R})$

$E[u(X + m)] = p(1 - e^{-x/R}\, e^{-m/R}) + (1 - p)\, (1 - e^{-y/R}\, e^{-m/R})$

$E[u(X + m)] = p - p\, e^{-x/R}\, e^{-m/R} + (1 - p) - (1 - p)\, e^{-y/R}\, e^{-m/R}$

$E[u(X + m)] = 1 - p\, e^{-x/R}\, e^{-m/R} - (1 - p)\, e^{-y/R}\, e^{-m/R}$

$E[u(X + m)] = 1 - e^{-m/R}\, (p\, e^{-x/R} + (1 - p)\, e^{-y/R})$

Step 3: insert $\mathrm{E}[\mathrm{u}(X + m)]$ into $\mathrm{u}^{-1}(.)$

$$\mathrm{u}^{-1}(\mathrm{E}[\mathrm{u}(X + m)]) = -\mathrm{R}\, ln(1 - (1 - e^{-m/\mathrm{R}}\, (p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}})))$$

$$\mathrm{u}^{-1}(\mathrm{E}[\mathrm{u}(X + m)]) = -\mathrm{R} ln(e^{-m/\mathrm{R}}\, (p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}}))$$

Step 4: equate the result with $\mathrm{u}^{-1}(z)$

Let $\mathrm{u}^{-1}(\mathrm{E}[\mathrm{u}(X + m)]) = \mathrm{u}^{-1}(z)$

$$\mathrm{u}^{-1}(z) = -\mathrm{R} ln(1 - z)$$

$$\mathrm{u}^{-1}(\mathrm{E}[\mathrm{u}(X+m)]) = -\mathrm{R} ln(e^{-m/\mathrm{R}}\, (p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}}))$$

$$-\mathrm{R} ln(e^{-m/\mathrm{R}}\, (p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}})) = -\mathrm{R}\, ln(1 - z)$$

$$ln(e^{-m/\mathrm{R}}\, (p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}})) = ln(1 - z)$$

$$-m + \mathrm{R}\, ln(p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}}) = \mathrm{R}\, ln(1 - z)$$

$$m = \mathrm{R}\, (ln(p\, e^{-x/\mathrm{R}} + (1 - p)\, e^{-y/\mathrm{R}}) - ln(1 - z))$$

$$m = \mathrm{R}\, (ln\, \mathrm{E}[e^{-X/\mathrm{R}}] - ln\, (1 - z)).\ \square \qquad (2.14)$$

3 Articulating Risk Attitude in the Risk Matrix using Decision Theory

Summary

This chapter examines how the unbiased risk attitude of a decision maker can be articulated in the risk matrix on the basis of the zero equivalent risk measure. Using isocontours the shape and location of the unacceptable risk zone in the risk matrix is derived on the basis of marginal benefit – marginal cost analysis. The implication of this analysis is that a concave utility function, which under expected utility theory reflects an averse attitude to risk, is consistent with a preference for an outcome reduction over a probability reduction strategy. Examples illustrate how three different functional specifications under unbiased expected utility theory articulate organizational risk appetite in the risk matrix.

3.1 Introduction

Since its instigation as a formal discipline, risk management has advocated prudence in the face of risk, articulated in statements such as "don't risk more than you can afford to lose" and "don't risk a lot for a little" (Mehr & Hedges 1963 chapter 1). The widespread neglect of the small probability – large impact events that initiated the 2007–2009 financial crisis is indicative for the relevance of these basic principles for organizations and society in the present day. The lack of competency of organizations in managing their risk appetite during this crisis (McDonald 2009, RIMS 2009) pushed their risks into a portion of their loss distribution which is usually associated with unlikely, catastrophic events, resulting in an "unfortunate tail" (Edwards 2010). Because of the widespread use of the risk matrix in defining organizational risk appetite (AIRMIC 2009 p.10), it is important to consider to what degree this popular risk management tool prevents organizations from "stuffing risk into the tails".[51]

The risk matrix, which consists of a plane with an axis for the probability of occurrence of a risk and an axis for its consequent impact, visually expresses organizational risk appetite by segregating risks between an acceptable and unacceptable zone in the matrix. At its own discretion each organization determines the size of each zone and the shape of the boundary line that separates them. Originally risk matrices were based on expected value calculations, which imply a risk neutral attitude to risk. Most of the times, however, the risk attitude implied in the risk matrix remains unarticulated (Cox 2008 p.508).

The aim of this chapter is to investigate what effect different attitudes to risk have on organizational risk appetite in the risk matrix. Drawing upon methods from decision theory

[51] A quote from Marc Groz in Nocera (2009).

it derives under expected utility theory the size of each zone in the matrix as well as the shape of the boundary line. Prior attempts to apply expected utility theory to risk matrices failed to clarify how the risk attitude embodied in a utility function translates to the zoning in the risk matrix (Kwak & LaPlace 2005). The primary contribution of this chapter is that it elucidates how methods from decision theory aid in the consolidation of a prudent attitude to risk taking in one of risk management's most popular risk management tools.

This chapter proceeds as follows. The next section provides a detailed exposition on the risk matrix. The third section discusses risk attitudes under expected value theory (EV), expected utility theory (EU), and prospect theory (PT). It furthermore explains how utility functions from these theories can be used to calculate risk measures. The fourth section explains how isocontours, which are derived from these risk measures, are used to determine the size and the shape of the zones in the risk matrix. It discusses the relationship between risk attitudes and preferences for outcome and probability reduction in risk management. The fifth section presents three examples in which risk measures are used to determine the acceptability of individual cells in the risk matrix. The sixth section contains a discussion. Appendices are in the final section.

3.2 The risk matrix

In enterprise risk management the risk matrix is one of the most popular and easy to use tools for expressing organizational risk appetite. The risk matrix[52] consists of a grid of cells, defined over a probability and impact axis, in which risks are positioned on the basis of their probability of occurrence and negative impact on the organizational objectives. An illustrative example of a risk matrix is presented in figure 3.1.

The aim of the matrix is to classify risks as acceptable or unacceptable to the organization, thereby articulating organizational risk appetite over a set of risky events that threaten the organization. In enterprise risk management it is generally assumed that some amount of pure risk needs to be accepted because the cost of risk reduction outweighs the benefits. To this end a boundary line in the risk matrix separates those risks that are deemed acceptable from those that are not. Risks that occupy the unacceptable zone require risk responses such as control activities that reduce the risk's impact or probability (*risk treatment*), risk transfer to a third party (*risk sharing*), or abandoning the activity which causes the risk (*risk avoidance*). Only in special circumstances can risks in the unacceptable zone be provisionally accepted (COSO 2004 p.3). An important consideration underlying the risk matrix is the assumption that the combined risk of all risky events together is tolerable if it is judged to be acceptable if taken item-by-item (Clemens & Swallom 2005). A shortcoming of the risk matrix is that risks in the acceptable risk zone are typically not mitigated even when the costs of control measures are relatively minor (Clemens & Pfitzer 2006).

[52] Also known as Risk assessment matrix, Risk map, Heat map, and Probability Impact Grid (PIG).

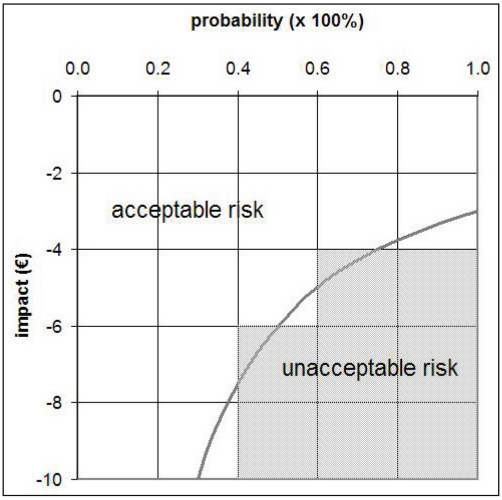

Figure 3.1 Risk matrix in which acceptable and unacceptable risks are segregated on the basis of a boundary line.

In figure 3.1 risks positioned in the white cells are considered acceptable whereas risks in the grey cells are deemed unacceptable. The risk matrix employs cells because assessments of probability and impact in risk management are often subjective and vague, capable only of identifying imprecision intervals and verbal expressions of probability and impact instead of their precise measurements. While the natural upper boundary of the probability scale is 100%, the upper boundary of the impact scale is either the worst conceivable negative impact or is left undefined. Traditionally, and in contrast to the linear scaling in figure 3.1, the impact and probability axes of risk matrices have in risk analysis been scaled logarithmically in order to enhance the resolution of the risk matrix to clearly distinguish minor from major risks (Clemens, Pfitzer, Simmons, Dwyer, Frost, & Olson 2005, Pfitzer, Hardwick, Pfitzer, & Ward 2004, Clemens 1995). The isocontour in figure 3.1 partitions the cells in the risk matrix into an acceptable and unacceptable risk zone. When at least 50% of the area of a cell is located above the isocontour then all risks positioned inside that cell are considered acceptable. If not, then the risks positioned in the cell are considered unacceptable, thus falling outside the organizational risk appetite (see Goldberg, Everhart, Stevens, Babbitt, Clemens, & Stout 1994 p.3–7 figure 3–3 for a detailed explanation of this procedure).

Commonly risk practitioners define risk as the product of probability (p) and negative impact (x), *i.e.* $p \times x$, an approach attributed to Blaise Pascal (Pfitzer, Hardwick, & Dwyer 2001). Following this convention the boundary between acceptable and unacceptable risks has traditionally been determined on the basis of expected value calculations, thus assuming a neutral attitude to risk.[53] The origins of risk matrices can be traced to military standards for system safety that are meant to mitigate mishap risk (Clemens et al. 2005). These standards

[53] The coordinates of the isocontour in figure 3.1 have an expected value of −3.

leave it to their users to determine what level of risk is acceptable (see for an example MIL-STD-882B 1984 p.A-4). Surveys indeed show that users vary in risk acceptability (Clemens 1995). While the use of risk matrices is recommended in the COSO (2004) guidelines, there is a lack of generally agreed standards for their application in enterprise risk management. Under this lack of guidance organizations have been actively engaged in copying existing risk matrix formats leading to a high degree of organizational "mimicry and isomorphism" (Power 2007) in expressions of organizational risk appetite.

3.3 Utility functions and risk measures

This section discusses the basic elements from decision theory and measure theory that are needed to articulate risk attitude in the risk matrix. It starts with discussing three different utility specifications from decision theory and then continues by explaining how these utility functions can be used to calculate a risk measure.

3.3.1 Utility functions

We model risky events, or risks, by means of the binary lottery

$$X = (p : x,\ 1 - p : y) \tag{3.1}$$

with outcomes $x, y \in \mathbb{R}$ and the probability of occurrence of x being $0 < p \leq 1$. Under expected utility theory (Bernoulli 1738/1954, von Neumann & Morgenstern 1944) the preference of a decision maker for lotteries is represented by the probability weighted sum of utilities or, alternatively, the expected utility of a lottery

$$\mathrm{EU}(X) = p \times \mathrm{u}(x) + (1 - p) \times \mathrm{u}(y). \tag{3.2}$$

The utility function $\mathrm{u}(z)$ is a monotonically increasing transformation of outcomes $z \in \mathbb{R}$.

Under expected utility theory it holds that the preference relation $X_1 \succ X_2$, implying a strict preference for X_1 over X_2, is represented by $\mathrm{EU}(X_1) > \mathrm{EU}(X_2)$. Similarly the equivalence $X_1 \sim X_2$ is represented by $\mathrm{EU}(X_1) = \mathrm{EU}(X_2)$ and the weak preference relation $X_1 \succcurlyeq X_2$ by $\mathrm{EU}(X_1) \geq \mathrm{EU}(X_2)$.

Under expected utility theory the utility function models the risk attitude of the decision maker. Under expected utility a concave utility function expresses a risk averse attitude, which means that the decision maker prefers each probability distribution less than its expectation. A risk seeking attitude – the decision maker prefers each probability distribution to its expectation – is under expected utility theory expressed by a convex utility function. Under expected utility theory a risk neutral attitude, the equivalent of expected value theory, is expressed by a linear utility function.

The preferences of a risk neutral decision maker, representative for the original, normative approach in risk management to incorporate risk attitude into organizational risk appetite using expected values, are described by a linear utility function with

$$u(z) = z. \tag{3.3}$$

The normative view of economics, which assumes expected utility and adheres to concave utility, is for the purpose of this chapter represented by a risk averse decision maker with a risk tolerance (R)[54] that is described by the exponential utility function

$$u(z) = 1 - e^{-z/R} \text{ (Pratt 1964, } \textit{constant absolute risk aversion}\text{, CARA).} \tag{3.4}$$

Finally, to provide a descriptive benchmark, the utility function of a decision maker with utility specifications from cumulative prospect theory (Tversky & Kahneman 1992, henceforth TK'92) is presented. In TK'92 utility is described by the power functions

$$u(z) = z^{\alpha}, \text{ for } z \geq 0$$

$$u(z) = -\lambda(-z)^{\beta}, \text{ for } z < 0 \text{ (Pratt 1964, } \textit{constant relative risk aversion}\text{, CRRA)} \tag{3.5}$$

with the powers $\alpha = \beta = .88$ and loss aversion coefficient $\lambda = 2.25$. Under prospect theory the utility function is concave over the gain domain, convex over the loss domain and has a distinctive kink at zero, resulting from the finding that the disutility experienced from losing a monetary amount by far outweighs the utility of gaining that amount (*loss aversion*).

In figure 3.2 the utility curves resulting from equations 3.3, 3.4 and 3.5 are presented with a normalization of u(10) = 1 and u(0) = 0. To avoid extreme cases of risk aversion, we let R = 11 in equation 3.4 to provide a close fit with the utility specifications in TK'92 over the range [−10,10] (see figure 3.2).

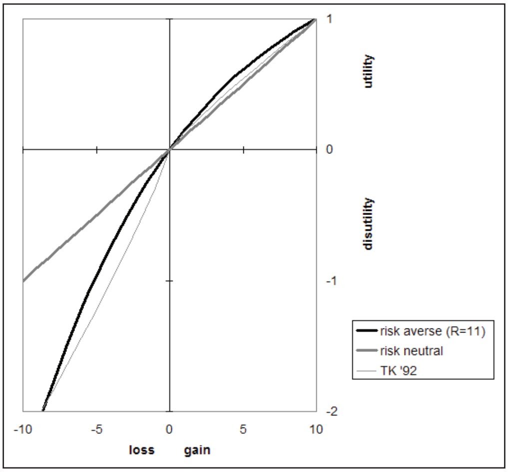

Figure 3.2 Utility functions over the loss and gain domains of a risk neutral decision maker, a risk averse decision maker with risk tolerance R = 11 and a representative agent from TK'92.

[54] Risk tolerance is the amount an individual is willing to stake in a 50/50 bet that either returns twice the stake or makes one lose half of it.

3.3.2 Zero equivalent risk measure

For the purpose of this chapter we assume throughout that utility functions are not biased by nonlinear probability weighting and that, under expected utility theory, these functions represent the unbiased risk attitude of senior management (see DEFINITION 2.6). For each lottery X this unbiased utility function is used to calculate the zero equivalent risk measure $\rho^{ze}(X)$. For an *unfavorable lottery* denoted by X^-, which is a lottery that is less preferred than retaining the status quo, $\rho^{ze}(X^-)$ is the minimum amount m that added to lottery X^- makes the utility of lottery X^- equivalent to the utility of zero (see DEFINITION 2.7). $\rho^{ze}(X^-)$ is thus the minimum sure amount, or *ask* price (a), that a decision maker, under the assumptions of expected utility theory and an unbiased utility function, requires as a compensation for accepting the unfavorable lottery X^-. $\rho^{ze}(X^-)$ can be interpreted as the minimum additional capital requirement that needs to be retained to accept X^-.

For the unfavorable lottery $X^- = (p : x, 1 - p : 0)$ with $x < 0$, $0 < p \leq 1$, the $\rho^{ze}(X^-) = a$, with $a > 0$, is under expected utility calculated by solving the formula in equation 3.6 for a

$$0 = p \times \mathrm{u}(x + a) + (1 - p) \times \mathrm{u}(0 + a). \tag{3.6}$$

Under linear utility, concave exponential utility and the utility specifications of TK'92 the following formulae can be derived from equation 3.6:

$$a = -\mathrm{EV}(X^-) = -px \qquad \text{(linear utility, 3.7a)}$$

$$a = -\mathrm{u}^{-1}(\mathrm{EU}(X^-)) = \mathrm{R}\ ln(p\ e^{-x/\mathrm{R}} - p + 1) \quad \text{(exponential utility, 3.7b)}$$[55]

$$0 = -p(\lambda)(-(-x + a))^{\beta} + (1 - p)a^{\alpha}. \quad \text{(TK'92 utility specifications, 3.7c)}$$[56]

The zero equivalent risk measure $\rho^{ze}(X^-)$ is closely related to the willingness-to-accept valuation method for unfavorable lotteries WTA(X^-). The difference is that WTA(X^-) is an ask-price for an unfavorable lottery which is directly observable and can be biased by nonlinear probability weighting. The zero equivalent risk measure $\rho^{ze}(X^-)$ is derived from a utility function and, when an unbiased utility function is used, is free from a probability distortion bias (see DEFINITION 2.4).

WTP(X^-), which is the maximum sure amount that the decision maker would be willing to pay to avoid the unfavorable lottery X^-, is not compatible with the risk measure con-

[55] $\mathrm{u}^{-1}(\mathrm{u}(x))$, *i.e.* the inverse utility of the utility function is based on the relationship $\mathrm{u}(x) = 1 - e^{-x/\mathrm{R}} \Rightarrow \mathrm{u}^{-1}(\mathrm{u}(x)) = x = -\ ln(1 - \mathrm{u}(x))\mathrm{R}$. Replacing $\mathrm{u}(x)$ with $\mathrm{EU}(X)$ results $x = \mathrm{u}^{-1}(\mathrm{EU}(X)) = -\ ln(1 - \mathrm{EU}(X))\mathrm{R} = -\ ln(1 - p(1 - e^{-x/\mathrm{R}}))\mathrm{R} = -\mathrm{R}\ ln(p\ e^{-x/\mathrm{R}} - p + 1))$, with $x < 0$. Ask price a is under exponential, concave utility defined by $a = -\mathrm{u}^{-1}(\mathrm{EU}(X^-)) = \mathrm{R}\ ln(p\ e^{-x/\mathrm{R}} - p + 1)$, with $a > 0$.

[56] Equation (3.7c) can be rewritten as $(x - a))^{\beta} / a^{\alpha} = (1 - p) / p(\lambda)$. TK '92 find $\beta = \alpha$. Replacing β for α results $(x/a - 1)^{\alpha} = (1/p - 1) / (\lambda) \Rightarrow a = x / (1 + ((1/p - 1) / (\lambda))^{1/\alpha})$. Alternatively equation (3.7c) can be solved for a in MS Excel by the command 'Goal Seek'.

cept. Only under the assumptions of expected utility with linear or exponential utility the equality $\rho^{ze}(X^{-}) = \mathrm{WTP}(X^{-})$ holds.[57] Under TK'92 the size of $\mathrm{WTP}(X^{-})$, which is typically assessed by the indifference between a sure loss and a risky loss, expresses under prospect theory a risk seeking attitude to risk in the domain of losses. Seeking risk when confronted with potential losses suggests an attitude to risk that is incompatible with the prudence proclaimed in risk management. As organizational risk appetite represents the willingness to accept losses in the pursuit of gains it is both semantically and conceptually incompatible with $\mathrm{WTP}(X^{-})$.

3.4 Articulation of risk attitude in the risk matrix

This section discusses first how two decision rules for organizational risk appetite can be applied to the risk matrix. On the basis of the second decision rule, marginal analysis, the section assesses the size and shape of the unacceptable zone in the risk matrix by means of isocontours, assuming either linear utility or concave exponential utility. Several observations are derived from the shape and location of these isocontours.

3.4.1 Decision rule 1: Risk bearing constraint

An important decision rule in relation to organizational risk appetite is that the amount of risk that the organization is willing to bear is constrained by its risk bearing capacity (see Decision rule 1 in chapter 2 and equation 2.15). The risk matrix does not specify risk acceptance for the organization's full exposure to risk but specifies risk acceptance for each risk individually (Clemens & Swallom 2005). Therefore the risk matrix does not facilitate the direct comparison between the organization's risk bearing capacity and its risk exposure. The risk matrix requires that all risks that occupy its unacceptable zone are transferred to the acceptable zone or, alternatively, avoided altogether. While the risk matrix does not explicitly specify a risk bearing constraint it can accommodate an increase or decrease of the organization's risk bearing capacity by an adjustment of the size of its unacceptable zone.[58]

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[57] Proof that for exponential expected utility $\rho^{ze}(X^{-}) = \mathrm{WTP}(X^{-})$. Let $\rho^{ze}(X^{-}) = \mathrm{WTA}(X^{-}) = \text{ask price} = a > 0$ and $\mathrm{WTP}(X^{-}) = \text{bid price} = -b > 0$. Without loss of generality we use the $u(x) = -e^{-x/R}$ member of the CARA-family of utility function. Using this function we can, given prospect $X = (p : x, 0)$ with $0 < p \leq 1$, $x < 0$, calculate a and b in the following way:

$0 = -ln(pe^{-(x+a)/R} + (1-p)e^{-a/R})R \Rightarrow 1 = pe^{-x/R}e^{-a/R} + (1-p)e^{-a/R} \Rightarrow e^{a/R} = pe^{-x/R} + (1-p) \Rightarrow$

$a = ln\,(pe^{-x/R} + (1-p))R$

$b = -ln(pe^{-x/R} + (1-p)\,e^{-0/R})R$

$a = -b = ln\,(pe^{-x/R} + (1-p))R.$ □

[58] The location of the boundary lines in the risk matrices of a bank's business units are derived from the amount of economic capital that is reserved for each business unit (personal communication by a bank's risk officer). This illustrates how a risk bearing constraint influences the location of boundary lines in the risk matrix in practice.

3.4.2 Decision rule 2: Marginal analysis[59]

A second decision rule for organizational risk appetite is derived from the *marginal benefit-marginal cost rule* (Vaughan 1997). It states that the marginal benefits of risk treatment or risk sharing should at least be equal to the marginal cost of these risk responses (see DECISION RULE 2 in chapter 2 and equation 2.16). This marginal analysis enables to express the exact boundary between acceptable and unacceptable risks in the risk matrix by isocontours. This section derives these isocontours under linear and concave exponential utility.

We assume diminishing marginal returns of risk reduction (Li, Pollard, Kendall, Soane, & Davies 2009 figure 2, Cooper, Grey, Raymond, & Walker 2005 figure 6.4 p.81). We describe these diminishing marginal returns by the cost function γ which is defined over logarithmic probability and outcome scales. These probability and outcome scales are described by the functions $p_i = b^{(i-n)}$ and $x_j = x_m\ b^{(j-m)}$ with growth rate $b > 1$, the exponent i (j) indicating the position on the logarithmic probability (outcome) scale and n (m) the maximum position on this scale, with $i,j \in \mathbb{R}^+$, $n,m \in \mathbb{R}_0^+$, $0 \leq i \leq n$, $0 \leq j \leq m$. Cost function γ assumes a fixed cost of risk reduction $c > 0$ for unitary changes[60] along each of the logarithmic scales and defines the cost of reducing probability p_n to p_i or outcome x_m to x_j by

$$\gamma(i,j) = c(n - i + m - j). \tag{3.8}$$[61]

For the purpose of this chapter the benefits of risk reduction are defined as reductions in the ρ^{ze} risk measure that result from risk reduction. Let $\rho(p_i,x_j)$ represent the benefit function for risk reduction measuring the benefit of reducing probability p_n to probability p_i and outcome x_m to outcome x_j with $\rho(p_i,x_j) \geq 0$, $0 \leq p_i \leq 1$, and $x_m \leq x_j \leq 0$ by means of the equation

$$\rho(p_i,x_j) = \rho^{ze}(p_n,x_m) - \rho^{ze}(p_i,x_j). \tag{3.9}$$

Figure 3.3 presents an illustrative example of the total benefits curve (ρ) and the total cost curve (γ) under the assumption of concave exponential utility for a risk reduction strategy in which respectively impact and probability is reduced.

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59 The following assumptions from chapter 2 hold in section 3.4.2 as well. ASSUMPTION 2.3 (independence of lotteries), ASSUMPTION 2.5 (true preferences represented by unbiased utility), ASSUMPTION 2.8 (risk resolved at end of single period), ASSUMPTION 2.9 (probabilities relate to a single period), ASSUMPTION 2.10 (risk reduction costs expensed at start of period) and ASSUMPTION 2.11 (outcomes discounted to present values).

60 Unitary changes along the scales are $\Delta i = 1$ for the logarithmic probability scale and $\Delta j = 1$ for the logarithmic outcome scale.

61 For ease of exposition we assume for that growth rate b and the same fixed cost c apply to both the probability and impact scale. Notice that $p_0 \neq 0$ and $x_0 \neq 0$.

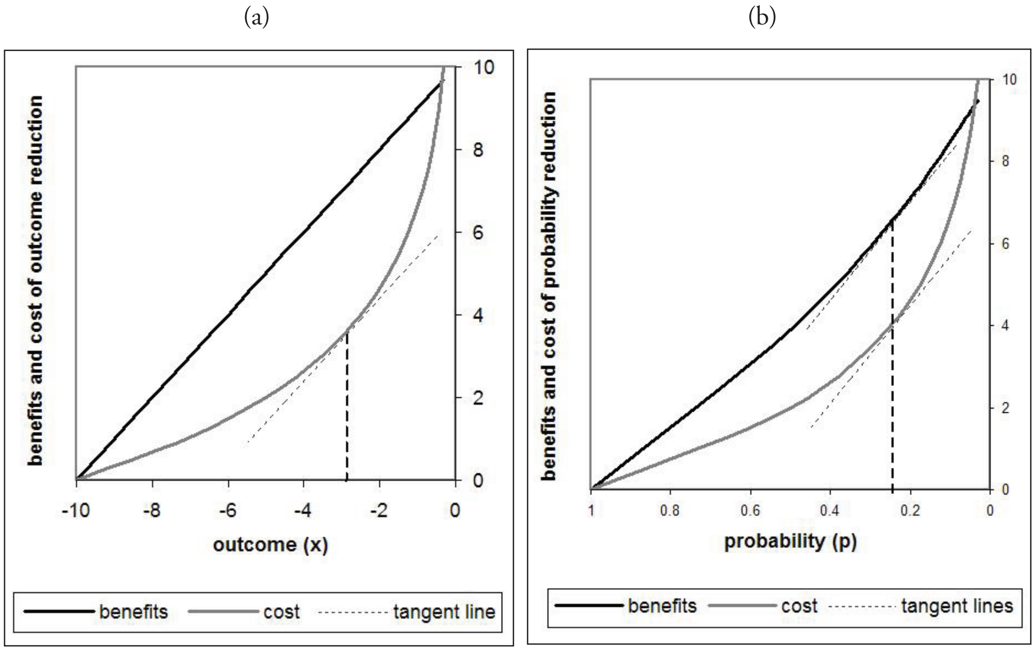

Figure 3.3 Benefits (ρ) under concave exponential utility and cost (γ) under diminishing marginal returns for (a) reducing outcomes from x_m to x_j holding probability constant at $p = 1$ and (b) reducing probabilities from p_n to p_i holding outcome constant at $x = x_m$.

The marginal analysis defines the boundary between acceptable and unacceptable risks in the risk matrix by the condition $\partial\rho/\partial x = \partial\gamma/\partial x$ for outcome reduction and $\partial\rho/\partial p = \partial\gamma/\partial p$ for probability reduction. The example[62] in figure 3.3a illustrates that at $p = 1$ the condition $\partial\rho/\partial x = \partial\gamma/\partial x$ is fulfilled at $x = -2.9$, or in general $-c$ / $ln\ b$, the point at which the tangent line of γ runs exactly parallel to the slope of ρ. The parallel tangent lines in figure 3.3b illustrate that for $x = -10$ the condition $\partial\rho/\partial p = \partial\gamma/\partial p$ is fulfilled at $p = .24$. The isocontours derived from the marginal analysis are presented in figure 3.4 for a risk matrix with linear scaling (a) and logarithmic scaling (b) (the derivation of the equations is for figure 3.4a provided in appendix A and for figure 3.4b in appendix B). The aforementioned coordinates {1,–2.9} and {.24,–10}, derived in figure 3.3 under the condition of concave exponential utility, are presented in figure 3.4a on respectively the right side of the matrix for an outcome reduction strategy and at the bottom of the matrix for the probability reduction strategy.

[62] For the example in figure 3.3 and figure 3.4 it holds that n = m = 5, $c = 2$, $x_m = -10$, $b = 2$, and R = 11.

(a) (b)

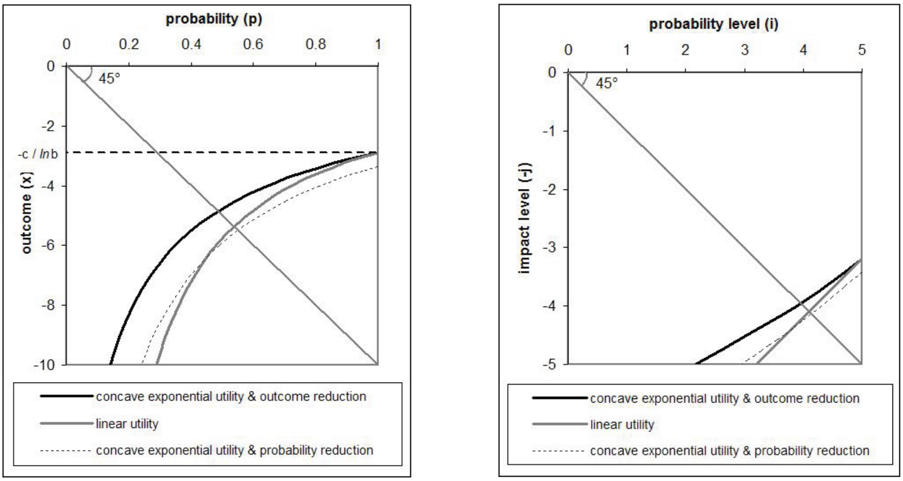

Figure 3.4 Isocontours derived from marginal analysis under a strategy of outcome and probability reduction for linear and concave exponential utility in risk matrices with (a) linear and (b) logarithmic probability and impact scales.

Under linear scaling (see figure 3.4a) the isocontours are curved upward and classify a large area in the risk matrix as unacceptable. The same area appears to be much smaller under logarithmic scaling (see figure 3.4b) both for the straight isocontour under risk neutrality and the curved inward isocontour under concave exponential utility. The 45° straight line downwards from the origin in both figures demonstrates that the risk neutral isocontours above these straight lines are an exact mirror image of their counterparts underneath. This symmetry hold both under linear scaling (figure 3.4a) and logarithmic scaling (figure 3.4b) (see appendix C for an application of the procedure outlined in Goldberg et al. (1994) to partition the cells in the risk matrix to the acceptable or unacceptable zone).

3.4.3 Implications of the isocontours in the risk matrix for risk management

Reflection on the isocontours in figure 3.4 suggests that the shape and location of the isocontours depend on both the risk attitude of the decision maker and the risk reduction strategy that is chosen (*i.e.* outcome versus probability reduction). This section investigates under which conditions these relationships hold.

Observation 3.1: In the risk matrix the isocontour under linear utility is stochastically dominated by the isocontour under concave exponential utility in combination with an outcome reduction strategy for all $x < -c / ln(b)$. (see proof in appendix D)

Under linear utility the lotteries along the isocontour have identical means but increase in variance in the direction of small probability – large impact risks. In comparison with linear utility, the isocontour derived under concave exponential utility combined with outcome reduction designates a larger area in the risk matrix and relatively more small probability – large impact lotteries as unacceptable. In contrast the isocontour under concave exponential utility combined with a probability reduction strategy intersects the risk neutral isocontour. In comparison with the isocontour under linear utility this isocontour designates relatively more small probability – large impact lotteries as unacceptable. Its level of losses at $p = 1$ is, however, higher than that of the isocontour under concave exponential utility and outcome reduction.

Observation 3.2: Under concave exponential utility the x-value on the isocontour at $p = 1$ is higher for an outcome reduction strategy than a probability reduction strategy. (see proof in appendix D)

Under concave exponential utility risk reduction by means of outcome reduction designates a larger area in the risk matrix as unacceptable than risk reduction by means of probability reduction.

Observation 3.3: In the risk matrix the isocontour under concave exponential utility in combination with a probability reduction strategy is stochastically dominated by the isocontour under concave exponential utility in combination with an outcome reduction strategy for all $x \in \mathbb{R}$ except for $x = 0$. (see proof in appendix D)

Under concave exponential utility the difference between total benefits and the total cost of risk reduction is maximized by an outcome reduction strategy. This suggests a preference for outcome reduction over probability reduction under concave exponential utility.

Under expected utility theory concavity in utility implies risk aversion. Risk aversion implies that when the means of lotteries are preserved a sure lottery is preferred to a risky lottery. Holding the lottery means constant, the risk of a unfavorable lottery can be reduced either by reducing the spread of outcomes (*outcome reduction*) or reducing the probability of the worst outcome (*probability reduction*).

Proposition 3.4 [*Preference for mean-preserving outcome reduction*]. Under expected utility theory and concave utility a mean-preserving outcome reduction is weakly preferred to a mean-preserving probability reduction. (see proof in appendix D)

From proposition 3.4 we derive that under expected utility theory risk aversion, represented by a concave utility function, implies a preference for mean preserving outcome reduction over probability reduction.

3.5 Deriving risk appetite for individual cells

This section presents three examples in which the acceptability of individual cells in the risk matrix is determined by means of marginal analysis. The net benefits of risk reduction are defined as reductions in the ρ^{ze} risk measure minus the cost γ of this risk reduction. These marginal benefits are derived under the assumptions of expected utility theory with linear utility, concave exponential utility, and the utility specifications from TK'92.

The risk matrix in table 3.1 contains a portfolio of four risky, probabilistically independent unfavorable lotteries A, B, C, and D with a total expected, present value of –16. Lottery D, with expected value −9, is stochastically dominated by the other three lotteries in the matrix. Lotteries B and C, having expected values of −3, are both stochastically dominated by lottery A. Assume a single period decision horizon in which this portfolio of lotteries (*risk profile*) is owned by an entity which, apart from disruptions originating from its risk profile, earns at the end of a particular period a steady net cash flow of $NCF_{t=1} = 20$. Discounted at a rate of $k = .25$ the present value of the entity's net cash flows at the start of the period is calculated as $PV_{t=0} = 20 / 1.25 = 16$. The entity is allowed to reduce either the probability of occurrence or the impact of lotteries B, C and D by moving any of these lotteries to an adjacent cell to the left or upwards within the risk matrix at a cost of 3 per move. To be considered a viable investment opportunity the present value of the entity ($PV_{t=0}$) should at least equal the sum of risk measures ($\Sigma\rho^{ze}$) that is required by a decision maker in return for accepting the risk profile and the risk reduction expenses made (γ), *i.e.* $PV \geq \Sigma\rho^{ze} + \gamma$. For the purpose of this section we define the internal rate of return (IRR) for a single period by the formula $IRR = NCF_{t=1} / (\Sigma\rho^{ze} + \gamma) - 1$. The IRR is a rate of return used to measure the profitability of the entity in comparison with the discount rate.

Table 3.1 Risk profile with lotteries A, B, C and D and their expected values (EV) in a 2 × 2 risk matrix with probability expressed in the columns with decimal values .25 and .75 and impact expressed in the rows with present values € –4 and € –12.[63]

		probability in decimals	
		.25	**.75**
present value of impact in €	**–4**	A = (.25 : –4, .75: 0) EV(A) = –1	B = (.75 : –4, .25: 0) EV(B) = –3
	–12	C = (.25 : –12, .75: 0) EV(C) = –3	D = (.75 : –12, .25: 0) EV(D) = –9

[63] We assume that in the event that a risk materializes impact occurs at a fixed time-period and is discounted at a fixed discount rate.

3.5.1 Linear utility

A risk neutral decision maker adheres to expected value theory, *i.e.* expected utility theory with linear utility, to determine her risk appetite. Under linear utility the risk measure ρ^{ze} of an unfavorable lottery is equal to the absolute value of its mathematical expectation (see equation 3.7a). From table 3.1 can be discerned that the sum of the expected values of the risk profile is −16 and that the entity's present value of 16 is just sufficient to compensate her for the inherent risk of this portfolio of lotteries. This implies that in this particular instance the condition $PV \geq \Sigma\rho^{ze} + \gamma$ is fulfilled even before any risk reduction effort is considered. At $\Sigma\rho^{ze} = 16$ the discount rate $k = .25$ exactly equals the entity's internal rate of return ($IRR = 20 / (16 + 0) - 1 = .25$). Additional risk reduction increases the internal rate of return of the entity, however. A risk neutral decision maker can reduce the amount of risk measure required to accept the risk profile by reducing the probability of lottery D from .75 to .25, a leftward move, or by reducing the impact of D from −12 to −4, an upward move. A risk neutral decision maker is indifferent between lottery D either joining lottery B or C in the risk matrix. In both cases the net benefits are 3, resulting from reducing $\rho^{ze}(D)$ from 9 to 3 at a cost of 3. As the entity's present value of 16 is larger than the sum of required risk measures of 10 and risk reduction expenses of 3 the investment opportunity is after risk reduction considered more viable than before ($PV - \Sigma\rho^{ze} - \gamma = 16 - 10 - 3 = 3$) and offers a higher internal rate of return ($IRR = 20/(10 + 3) - 1 = .54$).

In this illustrative example the risk appetite of the risk neutral decision maker is expressed in the risk matrix by classifying lottery D as an unacceptable lottery, which risk needs to be reduced. A typical feature of a risk neutral decision maker is that small probability – large impact risks and large probability – small impact risks are considered equally important.

3.5.2 Concave exponential utility

For each lottery in the risk matrix table 3.2 presents the ρ^{ze} risk measure that a risk averse decision maker requires under expected utility and a concave exponential utility function (see equation 3.7b). Given that the entity's present value is 16 and the sum of risk measures of the four lotteries is 18.6, the entity is an unviable investment opportunity for the risk averse decision maker. The risk averse decision maker can reduce the sum of risk measures by an upward risk reduction move in the risk matrix, reducing the impact of lottery D from −12 to −4. The net benefits of this move are 3.9, resulting from reducing $\rho^{ze}(D)$ from 10 to 3.1 at a cost of 3. The net benefits of this upward move are higher than those of a leftward move, which reduces lottery D's probability from .75 to .25 and produces a net benefit of 2.6, by reducing $\rho^{ze}(D)$ from 10 to 4.4 at a cost of 3. A risk averse decision maker is thus <u>not</u> indifferent between lottery D moving to join either lottery B or C and generally prefers impact reduction over probability reduction (see proposition 3.4). An upward impact reduction move of lottery C to join lottery A in the risk matrix creates an additional net benefit of 0.3, by reduction of $\rho^{ze}(C)$ from 4.4 to 1.1 at a cost of 3. The entity's present value of 16 is sufficiently large to accept the residual risk of 14.4 in the risk matrix. This makes the entity a viable investment opportunity ($PV - \Sigma\rho^{ze} - \gamma = 16 - 8.4 - 6 = 1.6$) with an internal rate of return that is higher than the discount rate ($IRR = 20/(8.4 + 6) - 1 = .39$).

Table 3.2 Risk matrix with lotteries A, B, C and D, and their ρ^{ze} risk measure under the assumption of expected utility theory and concave exponential utility ($R = 11$).

		probability in decimals	
		.25	**.75**
present value of impact in €	**−4**	$A = (.25 : -4, .75 : 0)$ $\rho^{ze}(A) = 11\ ln(.25\ e^{4/11} - .25 + 1) = 1.1$	$B = (.75 : -4, .25 : 0)$ $\rho^{ze}(B) = 11\ ln(.75\ e^{4/11} - .75 + 1) = 3.1$
	−12	$C = (.25 : -12, .75 : 0)$ $\rho^{ze}(C) = 11\ ln(.25\ e^{12/11} - .25 + 1) = 4.4$	$D = (.75 : -12, .25 : 0)$ $\rho^{ze}(D) = 11\ ln(.75\ e^{12/11} - .75 + 1) = 10$

The risk appetite of the risk averse decision maker is expressed in the risk matrix by classifying lotteries C and D as unacceptable lotteries, which impact needs to be reduced. Contrary to the risk neutral decision maker a risk averse decision maker is not indifferent between B and C and prefers risk reduction for lottery C over B ($B > C$). Even though lotteries B and C have identical means ($EV = -3$), they differ in variability as measured by the variance measure of dispersion ($Var(B) = 3$, $Var(C) = 27$).[64] Following the mean-variance criterion of risk aversion (Markowitz 1952a) a rational decision maker would, giving equal means, prefer the lottery with the lowest spread around the mean. This can intuitively be recognized by assessing the worst outcome of lottery C (−12) which deviates much more from the mean (−3) than the worst outcome of B (−4). Taking the value of the mean as a benchmark then only 25% of the variability in Var(B) falls below the benchmark whereas for Var(C) a substantial 75% of all variability pertains to downside risk (Estrada 2008).[65]

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[64] $Var(B) = .75\ (-4 - (-3))^2 + .25\ (0 - (-3))^2 = 3.$
$Var(C) = .25\ (-12 - (-3))^2 + .75\ (0 - (-3))^2 = 27.$

[65] Downside Semi-Variance(B) $= .75\ (-4 - (-3))^2 = 0.75.$
Downside Semi-Variance(C) $= .25\ (-12 - (-3))^2 = 20.25.$
Downside Semi-Variance(B)/Var(B) $= 0.75/3 = .25.$
Downside Semi-Variance(C)/Var(C) $= 20.25/27 = .75.$

The number of cells that a risk averse decision maker considers unacceptable is equal or higher than those of a risk neutral decision maker. A typical feature of a risk averse decision maker is an aversion to small probability – large impact cells. Another typical feature is that risk aversion implies that impact reduction is favored over probability reduction.

3.5.3 TK'92 utility specifications

For each lottery in the risk matrix table 3.3 presents the ρ^{ze} risk measure that a decision maker with TK'92 utility specifications requires under expected utility (see equation 3.7c). The entity's present value of 16 falls short of the 21.1 required sum of risk measures required which makes the entity an unviable investment opportunity.

Table 3.3 Risk matrix with lotteries A, B, C and D, and their ρ^{ze} risk measure under the assumption of expected utility theory with utility specified by the power function with TK'92 specifications.[66]

		probability in decimals	
		.25	**.75**
present value of impact in €	**−4**	A = (.25 : −4, .75 : 0) $\rho^{ze}(A) = 4/(1 + ((1/.25 - 1)/2.25)^{1/.88}) = 1.7$	B = (.75 : −4, .25 : 0) $\rho^{ze}(B) = 4/(1 + ((1/.75 - 1)/2.25)^{1/.88}) = 3.6$
	−12	C = (.25 : −12, .75 : 0) $\rho^{ze}(C) = 12/(1 + ((1/.25 - 1)/2.25)^{1/.88}) = 5.0$	D = (.75 : −12, .25 : 0) $\rho^{ze}(D) = 12/(1 + ((1/.75 - 1)/2.25)^{1/.88}) = 10.8$

The total required amount of risk measures can be reduced by reducing the impact of lottery D from −12 to −4. The net benefits of this move are 4.2, resulting from reducing $\rho^{ze}(D)$ from 10.8 to 3.6 at a cost of 3. These net benefits outweigh the effects of reducing the probability of occurrence of lottery D from .75 to .25, which offers a net benefit of 2.8, by reducing $\rho^{ze}(D)$ from 10.8 to 5.0 at a cost of 3. An additional net benefit of 0.3 is created by reducing the impact of lottery C which reduces $\rho^{ze}(C)$ from 5.0 to 1.7 at a cost of 3. Together these two impact reductions still do not make the entity a viable investment opportunity ($PV - \Sigma\rho^{ze} - \gamma = 16 - 10.6 - 6 = -0.6$). The entity can only become a viable investment opportunity by accepting an internal rate of return below the discount rate ($IRR = 20/(10.6 + 6) - 1 = .20$).

[66] The formula used in table 3.3 is discussed in footnote 6.

For each lottery in the risk matrix the maximum sure amount that a decision maker under expected utility theory and TK'92 utility specifications would be willing to pay to avoid these unfavorable lotteries is presented in table 3.4. The willingness-to-pay for lottery $X^- = (p : x, (1-p) : 0)$ with $x < 0$, $0 < p \leq 1$ is under expected utility and a power utility function calculated by

$$\text{WTP}(X^-) = (p(-x)^\beta)^{1/\beta}. \tag{3.10}$$ [67]

Table 3.4 Risk matrix with lotteries A, B, C and D, and their willingness-to-pay (WTP) under the assumption of expected utility theory with utility specifications from TK'92.

		probability in decimals	
		.25	**.75**
present value of impact in €	**−4**	A = (.25 : −4, .75 : 0) $\text{WTP(A)} = (.25(-(-4))^{.88})^{1/.88} = 0.8$	B = (.75 : −4, .25 : 0) $\text{WTP(B)} = (.75(-(-4))^{.88})^{1/.88} = 2.9$
	−12	C = (.25 : −12, .75 : 0) $\text{WTP(C)} = ((.25(-(-12))^{.88})^{1/.88} = 2.5$	D = (.75 : −12, .25 : 0) $\text{WTP(D)} = (.75(-(-12))^{.88})^{1/.88} = 8.7$

Calculations of the willingness-to-pay (WTP) in table 3.4 and the zero equivalent risk measure (ρ^{ze}) in table 3.3 suggest paradoxically contradictory preferences for a decision maker with utility specifications from TK'92. In table 3.3 the value of the ρ^{ze} risk measure is lower for lottery B than for C ($\rho^{ze}(B) < \rho^{ze}(C)$) which suggests a preference for lottery B over C ($B \succ C$). In table 3.4 the decision maker is willing to pay a higher sure amount for discarding lottery B than for lottery C (WTP(B) > WTP(C)), which suggests the opposite preference ($B \prec C$). The rationale for this paradox is that the ρ^{ze} risk measure is to a large degree affected by loss aversion, whereas willingness-to-pay is not. Where loss aversion explains the majority of risk aversion embodied in ρ^{ze}, risk seeking in the domain of losses explains the contradictory value in WTP. Mainly due to loss aversion the risk appetite of the decision maker in this section is comparable with that of a decision maker under concave exponential utility in the previous section.

3.6 Discussion

This chapter demonstrates the role that a prudent, averse attitude to risk can play in the prevention of risks being stuffed into the tail of the loss distribution. Its primary contribu-

[67] Notice the absence of probability weighting in equation 3.10 in comparison with equation 2.6.

tion is that it elucidates how decision theory aids in the consolidation of such an attitude in one of risk management's most popular risk management tools, the risk matrix. Current applications of decision theory to risk management typically assume a risk neutral decision maker who is indifferent between small probability – large impact and large probability – small impact risks. In contrast to this mainstream adoption of risk neutrality in risk management methods, several authors argue that small probability – large impact risks require more priority because of their devastating effects on organizations as well as society (Shaw 2005 p.25, Pickett 2006, Taleb 2007, Ai and Brockett 2008). This practitioner viewpoint resounds the normative view in economics that is adopted in this chapter, which assumes a risk averse decision maker. In comparison with a neutral attitude to risk, articulation of risk aversion in the risk matrix designates more risks as unacceptable. In particular those risks with a small probability of occurrence and a large impact, which typically occupy the tail of the loss distribution, then fall outside the organizational risk appetite. In addition risk aversion promotes a risk reduction strategy of outcome reduction over probability reduction, which directs risks more to the center of the loss distribution than to its tail.

The exact boundary between acceptable and unacceptable risks was in this chapter determined on the basis of marginal cost-benefit analysis. On the assumption of diminishing marginal returns of risk reduction efforts and by defining the benefits hereof in terms of the lower amount of risk measure that is required as a compensation for accepting risk, an optimum boundary for organizational risk appetite was calculated. Because the amount of risk that the organization is willing to bear is constrained by its risk bearing capacity (see Decision rule 1) the "optimum" size of the unacceptable zone may need additional adjustment to bring the risk exposure of the organization in line with its risk bearing capacity. The trade-off between the marginal cost and benefit of risk reduction demonstrates the value added of the risk management function in capital budgeting decisions under risk. Whenever the present value of an investment opportunity is exceeded by a claim of its risk profile on assets, then based on the trade-off between marginal cost and benefits the organization can either abandon the opportunity, seek less costly risk reduction measures, or accept a lower internal rate of return.

The zero equivalent risk measure (ρ^{ze}) calculated on the basis of the descriptively accurate utility function of Tversky and Kahneman (1992) promotes in general a risk averse attitude to risk in the risk matrix. The zero equivalent risk measure calculates the monetary, sure gain that added to a potential loss makes this risk acceptable. This measure is calculated under expected utility theory on the basis of the utility function for both gains and losses. Loss aversion in this utility curve explains why the zero equivalent risk measure promotes a risk averse attitude in the risk matrix. Estimates of the decision maker's willingness-to-pay (WTP) under expected utility theory, which are solely based on the convex utility function in the domain of losses, in contrast promote a risk seeking attitude to risk. Seeking risk when confronted with potential losses suggests an attitude to risk that is incompatible with the prudence proclaimed in risk management.

The exposition in this chapter assumed a one period time span for decision making under risk as well as a fixed discount rate. Given that the cost of risk reduction is certain and spent in the present whereas the potential losses are uncertain and manifest themselves in the future, risk management decisions are not only influenced by attitudes towards risk but by time preferences as well. Decision makers, typically impatient, may therefore decide to accept more risk than is to be expected on the basis of their risk attitude alone due to their proneness to hyperbolic time discounting (Ainslie 1975).

By articulating risk attitude in the risk matrix, a practical tool which is used in the front lines of risk management, this chapter aids in the alignment of organizational risk appetite with risk-taking in practice, where it has been observed that risk attitudes in risk matrices are rarely defined and that in particular the translation of risk appetite into practical risk limits is wanting (Pricewaterhouse Coopers 2008, Horgan 2008 p.57).

3.7 Appendices

3.7.1 Appendix A: Functional specifications of isocontours with linear scaling

Under linear utility the location of the isocontour that separates the unacceptable from the acceptable risky events in the risk matrix specified by the conditions $\partial\rho^{ze}/\partial p = \partial\gamma/\partial p$ and $\partial\rho^{ze}/\partial x = \partial\gamma/\partial x$ is described by the functions

$$p_i = -c \,/\, (x_j \, ln(b)) \tag{3.11a}$$

$$x_j = -c \,/\, (p_i \, ln(b)) \tag{3.11b}$$

Under concave exponential utility the location of the isocontour that separates the unacceptable from the acceptable risky events in the risk matrix specified by the conditions $\partial\rho^{ze}/\partial p = \partial\gamma/\partial p$ and $\partial\rho^{ze}/\partial x = \partial\gamma/\partial x$ is described by the functions

$$p_i = c\, e^{x_j/R} \,/\, ((e^{x_j/R} - 1)\,(c - R\, ln(b))) \tag{3.12}$$

$$p_i = c\, e^{x_j/R} \,/\, (-x_j ln(b) + c\,(e^{x_j/R} - 1)) \tag{3.13}$$

Proof:

Part 1: Find the partial derivatives of the zero equivalent risk measure for p and x

Let $\rho(p_i, x_j)$ represent the benefit function for risk reduction measuring the benefit of reducing probability p_n to probability p_i and outcome x_m to outcome x_j with $\rho(p_i, x_j) \geq 0$, $0 \leq p_i \leq 1$, and $0 \leq x_j \leq x_m$.

$$\rho(p_i, x_j) = \rho^{ze}(p_n, x_m) - \rho^{ze}(p_i, x_j)$$

Define the zero equivalent risk measure under linear utility for $X^- = (p_i : x_j,\ 1 - p_i : 0)$

$$\rho^{ze}(X^-) = -p_i x_j$$

Derive the partial derivatives of $\rho(p_i, x_j)$ for p_i and x_j under linear utility

$\partial\rho/\partial p_i = d/dp_i\ (\rho(p_i, x_j))$

$\partial\rho/\partial p_i = d/dp_i\ (\rho^{ze}(p_n, x_m) - \rho^{ze}(p_i, x_j))$

$\partial\rho/\partial p_i = d/dp_i\ (-p_n x_m - (-p_i x_j))$

$\partial\rho/\partial p_i = d/dp_i\ (p_i x_j)$

$\partial\rho/\partial p_i = x_j$

$\partial\rho/\partial x_j = d/dx_j\ (p_i x_j)$

$\partial\rho/\partial x_j = p_i$

Define the zero equivalent risk measure under concave exponential utility for X^-

$\rho^{ze}(X^-) = \mathrm{R}\ ln(p\ e^{-x/\mathrm{R}} - p + 1)$

Derive the partial derivatives of $\rho(p_i, x_j)$ for p_i under concave exponential utility

$\partial\rho/\partial p_i = d/dp_i\ (\rho(p_i, x_j))$

$\partial\rho/\partial p_i = d/dp_i\ (\rho^{ze}(p_n, x_m) - \rho^{ze}(p_i, x_j))$

$\partial\rho/\partial p_i = d/dp_i\ (\mathrm{R}\ ln(p_n\ e^{-x_m/\mathrm{R}} - p_n + 1) - \mathrm{R}\ ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$

$\partial\rho/\partial p_i = d/dp_i\ (-\mathrm{R}\ ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$

$\partial\rho/\partial p_i = -\mathrm{R}\ d/dp_i\ (ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$

Use the chain rule with $u = p_i\ e^{-x_j/\mathrm{R}} - p_i + 1$ and $d\,ln(u)/du = 1/u$

$\partial\rho/\partial p_i = -\mathrm{R} \times d\,ln(u)/du \times du/dp_i$

$\partial\rho/\partial p_i = -\mathrm{R} \times 1/u \times d/dp_i\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$

$\partial\rho/\partial p_i = -\mathrm{R} \times 1/(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1) \times (e^{-x_j/\mathrm{R}} - 1)$

$\partial\rho/\partial p_i = -\mathrm{R}\ (e^{-x_j/\mathrm{R}} - 1)\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$

Derive the partial derivatives of $\rho(p_i, x_j)$ for x_j under concave exponential utility

$\partial\rho/\partial x_j = d/dx_j\ (\rho(p_i, x_j))$

$\partial\rho/\partial x_j = d/dx_j\ (\rho^{ze}(p_n, x_m) - \rho^{ze}(p_i, x_j))$

$\partial\rho/\partial x_j = d/dx_j\ (\mathrm{R}\ ln(p_n\ e^{-x_m/\mathrm{R}} - p_n + 1) - \mathrm{R}\ ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$

$$\partial\rho/\partial x_j = \mathrm{d}/\mathrm{d}x_j\ (-\mathrm{R}\ ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$$

$$\partial\rho/\partial x_j = -\mathrm{R}\ \mathrm{d}/\mathrm{d}x_j\ (ln(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1))$$

Use the chain rule with $u = p_i\ e^{-x_j/\mathrm{R}} - p_i + 1$ and $\mathrm{d}ln(u)/\mathrm{d}u = 1/u$

$$\partial\rho/\partial x_j = -\mathrm{R} \times \mathrm{d}ln(u)/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}x_j$$

$$\partial\rho/\partial x_j = -\mathrm{R} \times 1/(p_i\ e^{-x_j/\mathrm{R}} - p_i + 1) \times \mathrm{d}/\mathrm{d}x_j\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = -\mathrm{R}\ (p_i\ \mathrm{d}/\mathrm{d}x_j(e^{-x_j/\mathrm{R}}) - \mathrm{d}/\mathrm{d}x_j(p_i) + \mathrm{d}/\mathrm{d}x_j(1))\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = -\mathrm{R}\ (p_i\ \mathrm{d}/\mathrm{d}x_j(e^{-x_j/\mathrm{R}}))\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

Use the chain rule with $u = -x_j/\mathrm{R}$ and $\mathrm{d}e^u/\mathrm{d}u = e^u$

$$\partial\rho/\partial x_j = -\mathrm{R}p_i \times \mathrm{d}e^u/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}x_j\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = -\mathrm{R}p_i \times e^u \times \mathrm{d}/\mathrm{d}x_j\ (-x_j/\mathrm{R})\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = -\mathrm{R}p_i \times e^{-x_j/\mathrm{R}} \times -1/\mathrm{R}\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = p_i\ e^{-x_j/\mathrm{R}}\ /\ (p_i\ e^{-x_j/\mathrm{R}} - p_i + 1)$$

$$\partial\rho/\partial x_j = p_i\ /\ ((1 - p_i)\ e^{-x_j/\mathrm{R}} + p_i)$$

Part 2: Find the partial derivatives of the cost function of risk reduction for *p* and *x*

Define the logarithmic scale for the probability dimension of the risk matrix by $p_i = b^{(i-n)}$ with base b defining the growth rate, exponent i indicating the position on the logarithmic probability scale, and n the total number of cells along this scale, with $b > 1$, $i \in \mathbb{R}^+$, $n \in \mathbb{R}_0^+$, $i \leq n$, and $b^{-n} \leq p_i \leq 1$.

$$p_i = b^{(i-n)}$$

$$i = ln(p_i)\ /\ ln(b) + n$$

Define the logarithmic scale of the outcome dimension of the risk matrix by $x_j = x_m\ b^{(j-m)}$ with base b defining the growth rate, exponent j indicating the position on the logarithmic outcome scale, and m the total number of cells along this scale, with $b > 1$, $j \in \mathbb{R}^+$, $m \in \mathbb{R}^+$, $j \leq m$, and $x_m\ b^{-m} \leq x_j \leq x_m$.

$$x_j = x_m\ b^{(j-m)}$$

$$x_j\ /\ x_m = b^{(j-m)}$$

$$j = ln(x_j\ /\ x_m)\ /\ ln(b) + m$$

Let $\gamma(i,j)$ represent the cost function for risk reduction measuring the cost of reducing position n on the logarithmic probability scale to position i and position m on the logarithmic outcome scale to position j, with $\gamma(i,j) \geq 0$, $c > 0$, $0 \leq i \leq n$ and $0 \leq j \leq m$.

$$\gamma(i,j) = c(n - i + m - j)$$

Let $\gamma(p_i, x_j)$ represent the cost function for risk reduction measuring the cost of reducing probability p_n to probability p_i and outcome x_m to outcome x_j with $\gamma(p_i, x_j) \geq 0$, $b^{-n} \leq p_i \leq 1$, and $x_m\, b^{-m} \leq x_j \leq x_m$.

$$\gamma(p_i, x_j) = -c\,/\,ln(b)\ (ln(p_i) + ln(x_j\,/\,x_m))$$

Derive the derivative of $\gamma(p_i, x_j)$ for p_i

$$\partial\gamma/\partial p_i = \partial/\partial p_i\ (-c\,/\,ln(b)\ (ln(p_i) + ln(x_j\,/\,x_m)))$$

$$\partial\gamma/\partial p_i = -c\,/\,ln(b)\ \mathrm{d}/\mathrm{d}p_i\ (ln(p_i))$$

$$\partial\gamma/\partial p_i = -c\,/\,ln(b) \times 1/p_i$$

$$\partial\gamma/\partial p_i = -c\,/\,(p_i\ ln(b))$$

Derive the derivative of $\gamma(p_i, x_j)$ for x_j

$$\partial\gamma/\partial x_j = \partial/\partial x_j\ (-c\,/\,ln(b)\ (ln(p_i) + ln(x_j\,/\,x_m)))$$

$$\partial\gamma/\partial x_j = -c\,/\,ln(b)\ \mathrm{d}/\mathrm{d}x_j\ (ln(x_j\,/\,x_m))$$

Use the chain rule with $u = x_j\,/\,x_m$ and $\mathrm{d}ln(u)/\mathrm{d}u = 1/u$

$$\partial\gamma/\partial x_j = -c\,/\,ln(b) \times \mathrm{d}ln(u)/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}x_j$$

$$\partial\gamma/\partial x_j = -c\,/\,ln(b) \times x_m\,/\,x_j \times \mathrm{d}/\mathrm{d}x_j\ (x_j\,/\,x_m)$$

$$\partial\gamma/\partial x_j = -c\,/\,ln(b) \times x_m\,/\,x_j \times 1\,/\,x_m\ \mathrm{d}/\mathrm{d}x_j\ (x_j)$$

$$\partial\gamma/\partial x_j = -c\,/\,ln(b) \times 1\,/\,x_j$$

$$\partial\gamma/\partial x_j = -c\,/\,(x_j\ ln(b))$$

Part 3: Equate the partial derivatives for the zero equivalent risk measure with the partial derivatives of the cost functions for probability and outcome reduction.

Marginal analysis for probability reduction under linear utility

$$\partial\rho/\partial p_i = \partial\gamma/\partial p_i$$

$$x_j = -c\,/\,(p_i\ ln(b))$$

Marginal analysis for outcome reduction under linear utility

$\partial\rho/\partial x_j = \partial\gamma/\partial x_j$

$p_i = -c \,/\, (x_j\, ln(b))$

Marginal analysis for probability reduction under exponential utility

$\partial\rho/\partial p_i = \partial\gamma/\partial p_i$

$-R\,(e^{-x_j/R} - 1) \,/\, (p_i\, e^{-x_j/R} - p_i + 1) = -c \,/\, (p_i\, ln(b))$

$R\,(e^{-x_j/R} - 1)\,(p_i\, ln(b)) = c(p_i\, e^{-x_j/R} - p_i + 1)$

$R\,(e^{-x_j/R} - 1)\, ln(b) = c(e^{-x_j/R} - 1) + c/p_i$

$R\,(1 - e^{x_j/R})\, ln(b) = c(1 - e^{x_j/R}) + c\, e^{x_j/R}/p_i$

$c\, e^{x_j/R}/p_i = -c(1 - e^{x_j/R}) + R\,(1 - e^{x_j/R})\, ln(b)$

$p_i \,/\, (c\, e^{x_j/R}) = -1 \,/\, (c(1 - e^{x_j/R}) - R\,(1 - e^{x_j/R})\, ln(b))$

$p_i = -\, c\, e^{x_j/R} \,/\, (c(1 - e^{x_j/R}) - R\,(1 - e^{x_j/R})\, ln(b))$

$p_i = c\, e^{x_j/R} \,/\, (c(e^{x_j/R} - 1) - R\,(e^{x_j/R} - 1)\, ln(b))$

$p_i = c\, e^{x_j/R} \,/\, ((e^{x_j/R} - 1)\,(c - R\, ln(b)))$

Marginal analysis for outcome reduction under exponential utility

$\partial\rho/\partial x_j = \partial\gamma/\partial x_j$

$p_i \,/\, ((1 - p_i)\, e^{x_j/R} + p_i) = -c \,/\, (x_j\, ln(b))$

$p_i x_j\, ln(b) = c p_i\, e^{x_j/R} - c\, e^{x_j/R} - c p_i$

$x_j\, ln(b) = c\, e^{x_j/R} - c\, e^{x_j/R}/p_i - c$

$c\, e^{x_j/R}/p_i = c\, e^{x_j/R} - c - x_j\, ln(b)$

$p_i = c\, e^{x_j/R} \,/\, (-x_j\, ln(b) + c\,(e^{x_j/R} - 1))$. □

3.7.2 Appendix B: Functional specifications of isocontours with logarithmic scaling

Under linear utility the location of the isocontour that separates the unacceptable from the acceptable risky events in the risk matrix with logarithmic scaling specified by the condition $\partial\rho^{ze}/\partial j = \partial\gamma/\partial j$ is described by the functions

$$i = ln(-c/(x_m\, ln(b))) \,/\, ln(b) - j + m + n. \tag{3.14}$$

Under concave exponential utility the location of the isocontour that separates the unacceptable from the acceptable risky events in the risk matrix with logarithmic scaling specified by the condition $\partial\rho^{ze}/\partial j = \partial\gamma/\partial j$ is described by the function

$$i = (n\ ln(b) - ln(-1\ /\ (c\ e^{x_m/R\ b^{(j-m)}})\ (-c\ e^{x_m/R\ b^{(j-m)}} + x_m\ ln(b)\ b^{(j-m)} + c)))\ /\ ln(b). \qquad (3.15)$$

Part 1: Find the partial derivatives of the zero equivalent risk measure for *j*

Let $\rho(i,j)$ represent the benefit function for risk reduction measuring the benefit of reducing position n to position i on the logarithmic probability scale and position m to position j on the logarithmic outcome scale with $\rho(i,j) \geq 0$, $0 \leq i \leq n$, and $0 \leq j \leq m$.

$$\rho(i,j) = \rho^{ze}(n,m) - \rho^{ze}(i,j)$$

Replace p_i by $b^{(i-n)}$ and x_j by $x_m\ b^{(j-m)}$ in the zero equivalent risk measure under linear utility for $X^- = (p_i : x_j,\ 1 - p_i : 0)$

$$\rho^{ze}(X^-) = -p_i x_j$$

$$\rho^{ze}(X^-) = -\ b^{(i+j-n-m)}\ x_m$$

Derive the partial derivative of $\partial\rho(i,j)$ under linear utility for j

$$\partial\rho/\partial j = d/dj\ (\rho(i,j))$$

$$\partial\rho/\partial j = d/dj\ (\rho^{ze}(n,m) - \rho^{ze}(i,j))$$

$$\partial\rho/\partial j = d/dj\ (-b^{(n+m-n-m)}\ x_m - (-b^{(i+j-n-m)}\ x_m))$$

$$\partial\rho/\partial j = d/dj\ (b^{(i+j-n-m)}\ x_m)$$

$$\partial\rho/\partial j = x_m\ d/dj\ (b^{(i+j-n-m)})$$

Use the chain rule with $u = i + j - n - m$ and $db^u/du = b^u\ ln(b)$

$$\partial\rho/\partial j = x_m \times db^u/du \times du/dj$$

$$\partial\rho/\partial j = x_m \times b^u\ ln(b) \times d/dj\ (i + j - n - m)$$

$$\partial\rho/\partial j = x_m \times b^{(i+j-n-m)}\ ln(b) \times 1$$

$$\partial\rho/\partial j = x_m\ ln(b)\ (b^{(i+j-n-m)})$$

Replace p_i by $b^{(i-n)}$ and x_j by $x_m\ b^{(j-m)}$ in the zero equivalent risk measure under concave exponential utility for $X^- = (p_i : x_j,\ 1 - p_i : 0)$

$$\rho^{ze}(X^-) = R\ ln(p_i\ e^{-x_j/R} - p_i + 1)$$

$$\rho^{ze}(X^-) = R\ ln(b^{(i-n)}\ e^{-x_m/R\ b^{(j-m)}} - b^{(i-n)} + 1)$$

Derive the partial derivative of $\partial\rho(i,j)$ under concave exponential utility for j

$$\partial\rho/\partial j = \mathrm{d}/\mathrm{d}j\ (\rho(i,j))$$

$$\partial\rho/\partial j = \mathrm{d}/\mathrm{d}j\ (\rho^{ze}(n,m) - \rho^{ze}(i,j))$$

$$\partial\rho/\partial j = \partial/\partial j\ (\mathrm{R}\ ln(b^{(\mathrm{n-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{m-m})}} - b^{(\mathrm{n-n})} + 1) - (\mathrm{R}\ ln(b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)))$$

$$\partial\rho/\partial j = \partial/\partial j\ (-\mathrm{R}\ ln(b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1))$$

$$\partial\rho/\partial j = -\mathrm{R}\ \partial/\partial j\ (ln(b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1))$$

Use the chain rule with $u = b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b(\mathrm{j-m})} - b^{(\mathrm{i-n})} + 1$ and $\mathrm{d}ln(u)/\mathrm{d}u = 1/u$

$$\partial\rho/\partial j = -\mathrm{R} \times \mathrm{d}ln(u)/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}j$$

$$\partial\rho/\partial j = -\mathrm{R} \times 1/u \times \mathrm{d}/\mathrm{d}j\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = -\mathrm{R}\ b^{(\mathrm{i-n})}\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)\ \mathrm{d}/\mathrm{d}j\ (e^{-x_\mathrm{m}/\mathrm{R}\ b(\mathrm{j-m})})$$

Use the chain rule with $u = -x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}$ and $\mathrm{d}e^u/\mathrm{d}u = e^u$

$$\partial\rho/\partial j = -\mathrm{R}\ b^{(\mathrm{i-n})} \times \mathrm{d}e^u/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}j\ /\ (b^{(\mathrm{i-n})}\ e^{-x\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = -\mathrm{R}\ b^{(\mathrm{i-n})} \times e^u \times \mathrm{d}/\mathrm{d}j\ (-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})})\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = -\mathrm{R}\ b^{(\mathrm{i-n})} \times e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} \times -x_\mathrm{m}/\mathrm{R} \times \mathrm{d}/\mathrm{d}j\ (b^{(\mathrm{j-m})})\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = x_\mathrm{m} b^{(\mathrm{i-n})} e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} \times \mathrm{d}/\mathrm{d}j\ (b^{(\mathrm{j-m})})\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

Use the chain rule with $u = \mathrm{j} - \mathrm{m}$ and $\mathrm{d}b^u/\mathrm{d}u = b^u\ ln(b)$

$$\partial\rho/\partial j = x_\mathrm{m} b^{(\mathrm{i-n})} e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} \times \mathrm{d}b^u/\mathrm{d}u \times \mathrm{d}u/\mathrm{d}j\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b(\mathrm{j-m})} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = x_\mathrm{m} b^{(\mathrm{i-n})} e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} \times b^u\ ln(b) \times \mathrm{d}/\mathrm{d}j\ (\mathrm{j-m})\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = x_\mathrm{m} b^{(\mathrm{i-n})} e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} \times b^{(\mathrm{j-m})}\ ln(b) \times 1\ /\ (b^{(\mathrm{i-n})}\ e^{-x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^{(\mathrm{i-n})} + 1)$$

$$\partial\rho/\partial j = x_\mathrm{m}\ ln(b)\ b^{(\mathrm{i+j-n-m})}\ /\ (b^{(\mathrm{i-n})} - b^{(\mathrm{i-n})}\ e^{x_\mathrm{m}/\mathrm{R}\ b(\mathrm{j-m})} + e^{x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}})$$

$$\partial\rho/\partial j = x_\mathrm{m}\ ln(b)\ b^{(\mathrm{i+j-m})}\ /\ (b^\mathrm{i} - b^\mathrm{i}\ e^{x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} + b^\mathrm{n}\ e^{x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}})$$

$$\partial\rho/\partial j = x_\mathrm{m}\ ln(\mathrm{b})\ b^{(\mathrm{j+i-m})}\ /\ (b^\mathrm{n}\ e^{x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - b^\mathrm{i}\ (e^{x_\mathrm{m}/\mathrm{R}\ b^{(\mathrm{j-m})}} - 1))$$

Part 2: Find the partial derivatives of the cost function for risk reduction for j

Derive the partial derivative of $\partial\gamma(i,j)$ for j

$$\partial\rho/\partial j = \partial/\partial j\ (c(n - i + m - j))$$

$$\partial\rho/\partial j = -c$$

Part 3: Equate the partial derivatives for the zero equivalent risk measure with the partial derivatives of the cost functions for outcome reduction.

Marginal analysis for outcome reduction under linear utility

$$\partial\rho/\partial j = \partial\gamma/\partial j$$

$$x_m\ ln(b)\ (b^{(i + j - n - m)}) = -c$$

$$b^{(i + j - n - m)} = -c/(x_m\ ln(b))$$

$$ln(b^{(i + j - n - m)}) = ln(-c/(x_m\ ln(b)))$$

$$(i + j - n - m) ln(b) = ln(-c/(x_m\ ln(b)))$$

$$i\ ln(b) = ln(-c/(x_m\ ln(b))) - (j - n - m) ln(b)$$

$$i = ln(-c/(x_m\ ln(b)))\ /\ ln(b) - j + n + m$$

Marginal analysis for outcome reduction under concave exponential utility

$$\partial\rho/\partial j = \partial\gamma/\partial j$$

$$x_m\ ln(\mathrm{b})\ b^{(j + i - m)}\ /\ (b^{n}\ e^{x_m/R\, b^{(j-m)}} - b^{i}\ (e^{x_m/R\, b^{(j-m)}} - 1)) = -c$$

$$x_m\ ln(\mathrm{b})\ b^{(j - m)}\ /\ (e^{x_m/R\, b^{(j-m)}} - 1 - b^{(n - i)}\ e^{x_m/R\, b^{(j-m)}}) = c$$

$$e^{x_m/R\, b^{(j-m)}} - 1 - b^{(n - i)}\ e^{x_m/R\, b^{(j-m)}} = x_m\ ln(\mathrm{b})\ b^{(j - m)}\ /\ c$$

$$b^{(n - i)}\ e^{x_m/R\, b^{(j-m)}} = e^{x_m/R\, b(j - m)} - x_m\ ln(\mathrm{b})\ b^{(j - m)}\ /\ c - 1$$

$$b^{(n - i)} = e^{x_m/R\, b^{(j-m)}}\ /\ e^{x_m/R\, b^{(j-m)}} - x_m\ ln(\mathrm{b})\ b^{(j - m)}\ /\ (c\ e^{x_m/R\, b^{(j-m)}}) - 1\ /\ e^{x_m/R\, b^{(j-m)}}$$

$$b^{(n - i)} = c\ e^{x_m/R\, b^{(j-m)}}\ /\ (c\ e^{x_m/R\, b^{(j-m)}}) - x_m\ ln(\mathrm{b})\ b^{(j - m)}\ /\ (c\ e^{x_m/R\, b^{(j-m)}}) - c\ /\ (c\ e^{x_m/R\, b^{(j-m)}})$$

$$b^{(n - i)} = -1\ /\ (c\ e^{x_m/R\, b^{(j-m)}})\ (-c\ e^{x_m/R\, b^{(j-m)}} + x_m\ ln(\mathrm{b})\ b^{(j - m)} + c)$$

$$ln(b^{(n - i)}) = ln(-1\ /\ (c\ e^{x_m/R\, b^{(j-m)}})\ (-c\ e^{x_m/R\, b^{(j-m)}} + x_m\ ln(\mathrm{b})\ b^{(j - m)} + c))$$

$$i = (n\ ln(b) - ln(-1\ /\ (c\ e^{x_m/R\, b^{(j-m)}})\ (-c\ e^{x_m/R\, b^{(j-m)}} + x_m\ ln(\mathrm{b})\ b^{(j - m)} + c)))\ /\ ln(b).\ \square$$

3.7.3 Appendix C: Assigning cells to the acceptable or unacceptable zone

Using the procedure outlined in Goldberg et al. (1994 p.3–7 figure 3–3) the isocontours in figure 3.4 are used to determine which cells fall within and outside risk appetite. When 50% or more of the area in the cell is above the isocontour then the cell is assigned to the acceptable zone. By visual observation or integral calculus we can determine which cells are considered acceptable.

Figure 3.5 partitions the risk matrices presented in figure 3.4 into 5 × 5 cells, each identified by a coordinate with the first number designating the column and the second the row in the matrix. In figure 3.5a cells (3,3), (2,4) and (2,5) are acceptable under linear utility and unacceptable under concave exponential utility with impact reduction. In figure 3.5b this holds for cell (4,5).

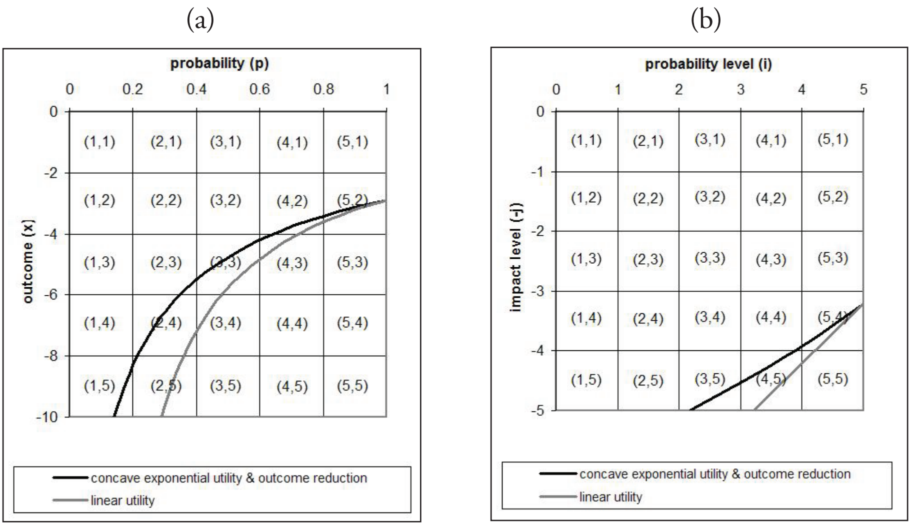

Figure 3.5 Risk matrices with 5 × 5 cells, identified by column (c) and row (r) numbers using coordinates (c,r), isocontours segregating the acceptable from the unacceptable cells, and (a) linear and (b) logarithmic probability and impact scales.

3.7.4 Appendix D: Proofs

Observation 3.1: In the risk matrix the isocontour under linear utility

$$p_k = -c / (x_j \, ln(b)) \tag{3.11a}$$

is stochastically dominated by the isocontour under concave exponential utility in combination with an outcome reduction strategy

$$p_i = c\, e^{x_j/R} / (-x_j \, ln(b) + c\,(e^{x_j/R} - 1)) \tag{3.13}$$

for all $x_j < -c / ln(b)$, which implies $p_i < p_k \; \forall \; x_j < -c / ln(b)$.

Proof:
We start by assuming this inequality holds

$$p_i < p_k$$

$$c\, e^{x_j/R} / (-x_j \, ln(b) + c\,(e^{x_j/R} - 1)) < -c / (x_j \, ln(b))$$

$c\, e^{x_j/R} / (-x_j\, ln(b) + c\, (e^{x_j/R} - 1)) < c\, e^{x_j/R} / (-x_j\, ln(b)\, e^{x_j/R})$

$-x_j\, ln(b) - c > -x_j\, ln(b)\, e^{x_j/R} - c\, e^{x_j/R}$

$x_j\, ln(b) + c < (x_j\, ln(b) + c)\, e^{x_j/R}$

The condition $x_j < -c / ln(b)$ implies $x_j\, ln(b) + c < 0$

The inequality $x_j\, ln(b) + c < (x_j\, ln(b) + c)\, e^{x_j/R}$ is conditional on the inequality $0 < e^{x_j/R} < 1$ that holds for negative outcomes ($x_j < 0$). □

Observation 3.2: Under concave exponential utility the x_j-value on the isocontour at $p = 1$ is higher for an outcome reduction strategy than a probability reduction strategy.

Proof:
Assess both equations 3.12 and 3.13 at $p = 1$

$$1 = c / ((1 - e^{-x_j/R})\, (c - R\, ln(b))) \tag{3.12}$$

$1 - e^{-x_j/R} = c / (c - R\, ln(b))$

$e^{-x_j/R} = 1 - c / (c - R\, ln(b))$

$-x_j/R = ln\, (1 - c / (c - R\, ln(b)))$

$x_j = -R\, ln\, (1 - c / (c - R\, ln(b)))$

$$1 = c\, e^{x_j/R} / (-x_j ln(b) + c\, (e^{x_j/R} - 1)) \tag{3.13}$$

$-x_j ln(b) + c\, (e^{x_j/R} - 1) = c\, e^{x_j/R}$

$-x_j ln(b) / c + e^{x_j/R} - 1 = e^{x_j/R}$

$-x_j ln(b) / c = 1$

$x_j = -c / ln(b)$

We assume that the inequality $-c / ln(b) > -R\, ln\, (1 - c / (c - R\, ln(b)))$ holds

$-c / ln(b) > -R\, ln\, (1 - c / (c - R\, ln(b)))$

$c / (R\, ln(b)) < ln\, (1 - c / (c - R\, ln(b)))$

$e^{c/(R\, ln(b))} < (c - R\, ln(b)) / (c - R\, ln(b)) - c / (c - R\, ln(b))$

$e^{c/(R\, ln(b))} < -R\, ln(b) / (c - R\, ln(b))$

$1 / e^{c/(R\, ln(b))} > -\, (c - R\, ln(b)) / R\, ln(b)$

$e^{-c/(R\, ln(b))} > 1 - c/R\, ln(b)$

Notice that $-\, c/R\, ln(b) < 0$
Let $a = -\, c/R\, ln(b) < 0$
For $a < 0$ holds that $e^a > 1 + a$. □

Observation 3.3: In the risk matrix the isocontour under concave exponential utility in combination with a probability reduction strategy

$p_k = c\, e^{x_j/R} \,/\, ((e^{x_j/R} - 1)\,(c - R\, ln(b)))$ (3.14)

is stochastically dominated by the isocontour under concave exponential utility in combination with an outcome reduction strategy

$p_i = c\, e^{x_j/R} \,/\, (-x_j\, ln(b) + c\,(e^{x_j/R} - 1))$ (3.13)

for all $x_j \in \mathbb{R}$ except for $x_j = 0$, which implies $p_i < p_k\ \forall\ x_j \in \mathbb{R} \setminus \{0\}$.

Proof:

We start by assuming this inequality holds

$p_i < p_k$

$c\, e^{x_j/R} \,/\, (-x_j\, ln(b) + c\,(e^{x_j/R} - 1)) < c\, e^{x_j/R} \,/\, ((e^{x_j/R} - 1)\,(c - R\, ln(b)))$

$c\, e^{x_j/R} \,/\, (-x_j\, ln(b) + c\,(e^{x_j/R} - 1)) < c\, e^{x_j/R} \,/\, (R\, ln(b)(1 - e^{x_j/R}) + c\,(e^{x_j/R} - 1))$

$1 \,/\, (-x_j\, ln(b) + c\,(e^{x_j/R} - 1)) < 1 \,/\, (R\, ln(b)(1 - e^{x_j/R}) + c\,(e^{x_j/R} - 1))$

$-x_j\, ln(b) + c\,(e^{x_j/R} - 1) > R\, ln(b)(1 - e^{x_j/R}) + c\,(e^{x_j/R} - 1)$

$-x_j\, ln(b) > R\, ln(b)(1 - e^{x_j/R})$

$-1 \,/\, x_j\, ln(b) < 1 \,/\, (R\, ln(b)(1 - e^{x_j/R}))$

$-x_j/R > 1 - e^{x_j/R}$

$-(1 + x_j/R) > -\, e^{x_j/R}$

$(1 + x_j/R) < e^{x_j/R}$

For $x_j = 0$ it holds that $(1 + x_j/R) = e^{x_j/R}$

Assess whether $(1 + x_j/R) < e^{x_j/R}\ \forall\ x_j \in \mathbb{R} \setminus \{0\}$

$e^{x_j/R} - 1 - x_j/R > 0\ \forall\ x_j \in \mathbb{R} \setminus \{0\}$

Let $f(x_j) = e^{x_j/R} - 1 - x_j/R$ and proof that $f(x_j) > 0\ \forall\ x_j \in \mathbb{R} \setminus \{0\}$

$f'(x_j) = (e^{x_j/R} - 1)/R$

$f'(x_j) = 0$ if $x_j = 0$ (3.16)

$f''(x_j) = e^{x_j/R}/R^2$

For all x_j holds that $f''(x_j) > 0$ (3.17)

From 3.16 and 3.17 follows that $f(x_j)$ has a minimum at $x_j = 0$ of $f(0) = 0$

If the smallest value of the function $f(x_j)$ is 0 for $x_j = 0$ then it holds that for all values unequal to 0 that $f(x_j) > 0$. □[68]

[68] The use of derivatives to formulate the final part of this proof was suggested to me by Aad van der Star.

Proposition 3.4: Under expected utility theory and concave utility a mean-preserving outcome reduction is weakly preferred to a mean-preserving probability reduction.

Proof:
Define the lottery $X^{-} = (p : x,\ 1 - p : 0)$ with $x < 0$, $0 < p \leq 1$
Let t be the cost of reducing either the outcome x to $x + t/p$ (impact reduction) or the probability p to $p + t/x$ (probability reduction) with $0 < t < -px$
Under risk neutrality the following indifferences hold:

$(p : x,\ 1 - p : 0) \sim (p : x + t/p - t,\ (1 - p) : -\,t)$

$(p : x,\ 1 - p : 0) \sim (p + t/x : x - t,\ (1 - p - t/x) : -\,t)$

We verify by calculation of expected values that the right side lotteries are mean-preserving risk reductions

$\mathrm{EV}(p : x,\ 1 - p : 0) = px$

$\mathrm{EV}(p : x + t/p - t,\ (1 - p) : -\,t) = px + t - pt - t + pt = px$

$\mathrm{EV}(p + t/x : x - t,\ (1 - p - t/x) : -\,t) = px + t - pt - t^2/x - t + pt + t^2/x = px$

Given the mean-preserving nature of the risk reductions we need to proof that expected utility theory and concavity of utility, imply a weak preference for outcome reduction over probability reduction, *i.e.*

$(p : x + t/p - t,\ (1 - p) : -\,t) \succcurlyeq (p + t/x : x - t,\ (1 - p - t/x) : -\,t)$

The aforementioned preference is under expected utility theory represented by

$p\,\mathrm{U}(x + t/p - t) + (1 - p)\,\mathrm{U}(-\,t) \geq (p + t/x)\,\mathrm{U}(x - t) + (1 - p - t/x)\,\mathrm{U}(-\,t)$

A concave utility function is defined by

$\mathrm{U}(\lambda y + (1 - \lambda)z) \geq \lambda\,\mathrm{U}(y) + (1 - \lambda)\,\mathrm{U}(z)$, (see chapter 2 equation 2.2)

with $0 < \lambda < 1$, $y, z \in \mathbb{R}$

Let $\lambda = 1 + t/(px)$, $y = (x - t)$, and $z = -t$ and insert these in the definition of concavity

$\mathrm{U}((1 + t/(px))\,(x - t) - (1 - (1 + t/(px)))t) \geq (1 + t/(px))\,\mathrm{U}(x - t) + (1 - (1 + t/(px)))\,\mathrm{U}(-t)$

$\mathrm{U}((x - t) + t(x - t)/(px) - (t - (t + t^2/(px)))) \geq (1 + t/(px))\,(\mathrm{U}(x - t) - \mathrm{U}(-t)) + \mathrm{U}(-t)$

$\mathrm{U}(x - t + t/p - t^2/(px) - t + t + t^2/(px)) - \mathrm{U}(-t) \geq (1 + t/(px))\,(\mathrm{U}(x - t) - \mathrm{U}(-t))$

$\mathrm{U}(x + t/p - t) - \mathrm{U}(-t) \geq (1 + t/(px))\,(\mathrm{U}(x - t) - \mathrm{U}(-t))$

$p\,(\mathrm{U}(x + t/p - t) - \mathrm{U}(-t)) \geq (p + t/x)\,(\mathrm{U}(x - t) - \mathrm{U}(-t))$

$p\,\mathrm{U}(x + t/p - t) - p\,\mathrm{U}(-t) + \mathrm{U}(-t) \geq (p + t/x)\,\mathrm{U}(x - t) - (p + t/x)\,\mathrm{U}(-t) + \mathrm{U}(-t)$

$p\,\mathrm{U}(x + t/p - t) + (1 - p)\,\mathrm{U}(-t) \geq (p + t/x)\,\mathrm{U}(x - t) + (1 - p - t/x)\,\mathrm{U}(-t)$

Under concave utility this reproduces the aforementioned representation under expected utility theory of weak preference for outcome reduction over probability reduction. □

4 Utility Elicitation in the Context of Organizational Risk Appetite

Summary

This chapter asserts that the conventions in decision theory to elicit utility in the domain of losses with gain-free lotteries and to elicit the loss aversion parameter with mixed lotteries with outcomes close to zero are incompatible with organizational risk appetite. It introduces a new method that is capable of eliciting utility under prospect theory over the whole domain of outcomes without requiring the assessment of the loss aversion parameter. This method is compatible with organizational risk appetite, uses the trade-off method (Wakker & Deneffe 1996) to elicit utility in the gain domain, and derives the utility function in the domain of losses from these gain utility measurements. Results of initial trial sessions of the method with entrepreneurs are reported.

4.1 Introduction

Utility functions that describe the value that senior management assigns to the degree to which organizational objectives are attained can play a crucial role in the measurement and validation of organizational risk appetite. Organizational risk appetite is defined as "the broad-based amount of risk an entity is willing to accept in pursuit of its mission/vision" (COSO 2004 p.110) and to prevent "empty and vacuous" statements on organizational risk appetite, appropriate measures for this key concept in risk management need to be identified (Anderson 2011 p.8). Risk measures, derived from utility functions, can under the assumptions of expected utility theory be used to determine the "amount of risk" that, given the risk attitude of senior management, is acceptable (see chapter 2). Experimental evidence suggests that the context in which utility functions are elicited needs to be compatible with the context in which they are applied (Hey, Morone, & Schmidt 2009 pp.213, 229). It is therefore important to verify that utility elicitation methods from decision theory are contextually compatible with organizational risk appetite.

Risk is in the context of organizational risk appetite defined as the possibility that an event will occur and adversely affect the achievement of objectives (COSO 2004 p.16). For monetary objectives organizational risk appetite can then be redefined as the willingness of the organization to accept potential losses in the pursuit of gains. Utility functions, which represent the value that a decision maker assigns to these gains and losses, are determined on the basis of a decision maker's choice between lotteries. The class of lotteries that contains both gains and losses, referred to as mixed lotteries, enables the decision maker to make trade-offs between potential gains and losses and is a natural candidate for applications in the context of organizational risk appetite. Organizations differ, furthermore, in their chances of making gains and losses and in the range of outcomes that are relevant for decision making. In the context of organizational risk appetite the realism of utility elicitation can be enhanced if organizational decision makers are allowed to express their willingness to accept risk by means of mixed lotteries that recognize both organization-specific probabilities and outcome ranges.

This chapter demonstrates that current utility elicitation methods are not fully compatible with the context of organizational risk appetite and presents an alternative utility elicitation method that is contextually appropriate for applications in risk management. It elicits utility under the assumptions of prospect theory (Tversky & Kahneman 1992, Kahneman & Tversky 1979), a descriptively accurate theory for decision making under risk, which is capable of accommodating well known empirical phenomena such as nonlinear probability weighting and loss aversion. The method proposed is nonparametric and therefore does not demand the specification of functional expressions and does not require any statistical analysis to produce the utility function. The method is an extension of the trade-off method (Wakker & Deneffe 1996). Where this method does not accommodate loss aversion (Abdellaoui, Bleichrodt, & Paraschiv 2007 p.1660), our extension captures this phenomenon without requiring an assessment of the loss aversion parameter. Our elicitation method constitutes a direct response to the call in the literature to specify a utility function on the whole of the outcome domain, in particular for large outcomes remote from zero (Wakker 2010 p.248). The aim of this chapter is to explain the utility elicitation method, demonstrate its novelty and investigate its merits and limitations.

The second section of this chapter provides a brief overview of current utility elicitation methods. The third section starts with a short account of prospect theory and subsequently discusses the four-step utility elicitation method, which is compatible with organizational risk appetite. The fourth section presents the results of initial trial sessions of this method involving eight entrepreneurs. The fifth section discusses the merits and limitations of the method. The appendices are in the final section.

4.2 Overview of utility elicitation methods

The general convention in decision analysis, the branch of science which applies decision theory to actual decision counseling, is to elicit utility functions under the assumptions of expected utility theory (Spetzler 1968, Cozzolino 1977, Stanford University Strategic Decisions Group 2010). This procedure potentially contaminates the utility function with biases that are irreconcilable with the rational axioms underlying the expected utility model (Bleichrodt, Pinto, & Wakker 2001). Elicitation of utility under the assumptions of prospect theory (Tversky & Kahneman 1992, Kahneman & Tversky 1979) prevents these deviations from the rational expected utility model to emerge in the measurement of utility.

These unbiased measurements of utility, are compatible with the rational model of expected utility and can thus be applied in counseling decision makers (see Definition 2.4, *unbiased utility function*). They can, however, still be incompatible with organizational risk appetite.

The mainstream convention in decision theory is to elicit the utility of gains using *loss-free lotteries* and to assess the utility of losses using *gain-free lotteries*. Gain-free lotteries, in which the best outcome is not higher than zero, coerce the decision maker into accepting unfavorable lotteries. An elicitation method which forces a choice between unfavorable lottery options typically measures the decision maker's willingness-to-pay for avoiding risk. The context of organizational risk appetite requires, on the contrary, the assessment of her willingness-to-accept risk on a voluntary basis.

To enable the specification of utility over the whole domain of outcomes, elicitation methods which segregate the assessment of the utility of gains and losses, necessitate a separate assessment of the exchange rate between these two curves. The assessment of this rate of exchange, called the loss aversion parameter λ, requires at least the use of one mixed lottery. While in the context of organizational risk appetite decisions typically resemble choices between mixed lotteries, their use in decision theory is often confined to the elicitation of the loss aversion parameter. It is recommended practice in decision theory to elicit the loss aversion parameter with lotteries over small monetary outcomes close to the reference point 0, a range of monetary outcomes that professional decision makers consider immaterial. Because it is plausible to assume linear utility close to zero, this procedure avoids the distortionary effects of differences between gain and loss curvature on this scalar (Wakker 2010 pp.239, 267). The fact that loss aversion is assessed for small stakes only and not for losses that really hurt, reduces the empirical meaningfulness of loss aversion (Wakker 2010 pp.247–248 problem 3).

This account of mainstream utility elicitation conventions does not do justice to the large variety in elicitation methods that have emerged in decision theory, both in terms of methods that require the specification of functional forms as well as methods that do not require such a parametric specification (nonparametric methods). Abdellaoui, Bleichrodt and Paraschiv (2007) developed a utility elicitation method that does not require a specification of the loss aversion parameter and accommodates the elicitation of utility over a specific, predefined range of gain and loss outcomes. Their elicitation method employs, however, only one mixed lottery, does not allow for organization-specific probabilities, and uses gain-free lotteries to asses utility in the loss domain. Despite being an optimally efficient elicitation method (Blavatskyy 2006) their method lacks compatibility with organizational risk appetite. A nonparametric elicitation method that is to a very high degree compatible with organizational risk appetite is the trade-off method (Wakker & Deneffe 1996). The trade-off method elicits utility under the assumptions of prospect theory and is capable of accommodating organization-specific probabilities. Fennema and van Assen (1998) used the trade-off method to assess the utility for gains and losses separately with mixed lotteries. They did not measure loss aversion and elicited utility in the loss domain over a very small range of outcomes. To elicit utility over the whole outcome domain current applications of the trade-off method require a careful specification of the method's starting values to cover a desired, predefined range of outcomes and in addition a separate assessment of the loss aversion parameter. A limited overview of utility elicitation methods is provided in appendix A.

4.3 Four-step nonparametric utility elicitation method

This section extends the trade-off method and allows it to accommodate loss aversion without imposing an additional parametric specification for the loss aversion phenomenon and the assessment of such a loss aversion parameter. The elicitation method derives utility under the assumptions of prospect theory. This descriptive theory for decision making under risk allows it to capture the decision maker's sensitivity to outcomes in the utility function and her attitude towards probability in the probability weighting function. While the analysis of the decision maker's responses employs a *descriptive* methodology, the stimuli are framed *prescriptively* as we request the respondent to indicate what lottery she *should* choose for the benefit of her organization, contrary to what she *would* choose in practice. Even under this prescriptive framing of questions it can be expected that respondents distort probabilities, which justifies the use of prospect theory.

Let $X = (p : x, (1 - p) : y)$ represent a binary lottery with $0 < p \leq 1$ and $x, y \in \mathbb{R}$. Under prospect theory the decision maker derives utility $\mathrm{U}(z)$ from outcome $z \in \mathbb{R}$, with $\mathrm{U}(.)$ being a monotonically increasing utility function over the whole domain of outcomes. Distortion of probabilities is captured by two separate probability weighting functions $\mathrm{w}^s(p)$ for gains and losses, with $s = +,-$ and $0 < \mathrm{w}^s(p) \leq 1$. For loss-free lotteries, $0 < y < x$, and gain-free lotteries, $x < y < 0$, the value that the decision maker assigns to the lottery is represented by

$$\mathrm{PT}(X) = \mathrm{w}^s(p) \times \mathrm{U}(x) + (1 - \mathrm{w}^s(p)) \times \mathrm{U}(y), \tag{4.1}$$

and for a mixed lottery with $y < 0 < x$ by

$$\mathrm{PT}(X) = \mathrm{w}^+(p) \times \mathrm{U}(x) + \mathrm{w}^-(1 - p)) \times \mathrm{U}(y). \tag{4.2}$$

For a weak preference for X_1 over X_2, denoted by $X_1 \succcurlyeq X_2$, it holds under prospect theory that $\mathrm{PT}(X_1) \geq \mathrm{PT}(X_2)$. Similarly the indifference $X_1 \sim X_2$ is represented by the equation $\mathrm{PT}(X_1) = \mathrm{PT}(X_2)$ and the strict preference $X_1 \succ X_2$ by the inequality $\mathrm{PT}(X_1) > \mathrm{PT}(X_2)$. For gain outcomes the notation x_i with $i = 1,\ldots,n$ is used and for loss outcomes the notation x_{-j} with $j = 1,\ldots,n$ while $x_0 = 0$. The whole domain of outcomes in the elicitation of utility is thus specified by the following sequence of inequalities: $x_{-n} < \ldots < x_{-(j+1)} < x_{-j} < x_0 < x_i < x_{i+1} < \ldots < x_n$.

The nonparametric procedure to elicit utility over the whole domain of outcomes consists of four steps and is summarized in table 4.1. The second column of the table describes the quantity that is assessed, the third the indifference that is sought, and the fourth the implication of this indifference under prospect theory.

Table 4.1 Four-step nonparametric elicitation procedure.

	Assessed quantity	Indifference sought	Under prospect theory
Step 1	x_i	$(p : x_i, (1-p) : g) \sim (p : x_{i-1}, (1-p) : G)$ for $i = 1,\ldots,n$ with $g < G < 0 = x_0 < x_i$	$U(x_i) = {}^{i}\!/_{n}$ $U(x_0) = 0$
Step 2	x_k	$(p : x_k, (1-p) : x_0) \sim (p : x_2, (1-p) : x_1)$ with x_1 and x_2 from step 1	$w^+(p) = 1 / (nU(x_k) - 1)$
Step 3	x_{j-n}	$(p : x_n, (1-p) : x_{j-n}) \sim (p : x_j, (1-p) : x_0)$ for $j = 0,\ldots,n-1$ with x_j and x_n from step 1	$U(x_{j-n}) = {}^{(j-n)}\!/_{n}\, w^+(p) / w^-(1-p)$
Step 4	x_f	$((1-p) : x_f, p : x_0) \sim ((1-p) : x_{-2}, p : x_{-1})$ with x_{-1} and x_{-2} from step 3	$w^-(1-p) = -1 / (nU^*(y_f) + 1)$ $U^*(x_{-i}) = -U(x_i) = -{}^{i}\!/_{n}$

The first step of the elicitation process determines utility in the domain of gains using the trade-off method. This method elicits a series of intervals on the gain dimension that are equally spaced in terms of utility. Using the outward version of the trade-off method we determine n intervals $[x_{i-1}, x_i]$ with $i = 1,..,n$ by assessing the quantity x_i using the indifference $(p : x_i, (1-p) : g) \sim (p : x_{i-1}, (1-p) : G)$ with $g < G < 0 \leq x_{i-1} < x_i$ and $x_0 = 0$. Given that under prospect theory $U(0) = 0$ and utility is unique up to a unit we start the sequence of gains with $x_0 = 0$ (Wakker 2010 p.348 OBSERVATION 12.3.5'). The trade-off method implies that

$$U(x_i) = {}^{i}\!/_{n}.\text{[69]} \tag{4.3}$$

The second step elicits the weight of probability p in the domain of gains using the indifference $(p : x_k, (1-p) : x_0) \sim (p : x_2, (1-p) : x_1)$ with x_1 and x_2 derived from step 1 and x_k being the value that is assessed. On the condition that $x_k \leq x_n$ we derive from this indifference under prospect theory

$$w^+(p) = 1 / (nU(x_k) - 1).\text{[70]} \tag{4.4}$$

[69] Let $\Delta = U(x_i) - U(x_{i-1})$ represent the equally spaced utility distance and let $U^*(.)$ represent normalized utility with $U^*(x_n) = 1$ and $U^*(x_0) = 0$.
$U^*(x_i) = (U(x_i) - U(x_0))/(U(x_n) - U(x_0)) = i\Delta/n\Delta = {}^{i}\!/_{n}$.
For ease of exposition we let $U(.)$ represent normalized utility in the main text.

[70] $w^+(p)U(x_k) + (1 - w^+(p))U(x_0) = w^+(p)U(x_2) + (1 - w^+(p))U(x_1)$
$w^+(p) = (U(x_1) - U(x_0)) / (U(x_k) - U(x_0) - U(x_2) + U(x_1))$
$w^+(p) = ({}^{1}\!/_{n} - {}^{0}\!/_{n}) / (U(x_k) - {}^{0}\!/_{n} - {}^{2}\!/_{n} + {}^{1}\!/_{n}) = {}^{1}\!/_{n} / (U(x_k) - {}^{1}\!/_{n}) = 1 / (n \times U(x_k) - 1)$.

$U(x_k)$ can be determined using interpolation or is derived from a parametric fit on the sequence of utilities in step 1.

The third step of the elicitation process determines utility in the domain of losses using the indifference $(p : x_n, (1-p) : x_{j-n}) \sim (p : x_j, (1-p) : x_0)$ for $j = 0,\dots,n-1$, the values p, x_0, x_j, and x_n derived from step 1 and x_{j-n} being the loss value that is assessed. The first lottery assesses x_{-n}, the worst loss in the entire elicitation process, by assessing the indifference $(p : x_n, (1-p) : x_{-n}) \sim (1 : 0)$. Under prospect theory we derive in total n equally spaced utility distances in the loss domain

$$U(x_{j-n}) = {}^{(j-n)}\!/_{n}\, w^{+}(p) \,/\, w^{-}(1-p).^{71} \tag{4.5}$$

The fourth step of the process aims to elicit the weight of probability $(1-p)$ in the domain of losses using the indifference $((1-p) : x_f, p : x_0) \sim ((1-p) : x_{-2}, p : x_{-1})$ with x_{-1} and x_{-2} derived from step 2 and x_f being the value that is assessed. Conditional on $x_f \geq x_{-n}$ we derive from this indifference under prospect theory

$$w^{-}(1-p) = -1 \,/\, (nU^{*}(x_f) + 1), \tag{4.6}$$

with the utility for losses rescaled to $U^{*}(x_{-i}) = -U(x_i) = -{}^{i}\!/_{n}$.[72] This rescaling is allowed as under prospect theory utility is unique up to a unit and can be rescaled by any factor in $\mathbb{R}$. $U^{*}(x_f)$ can be determined using interpolation or can be derived from a parametric fitting.

We finally consolidate the nonparametric elicitation over the domain of gains and losses by determining the values of $U(x_i)$ through equation 4.5 by imputation of the values of $w^{+}(p)$ and $w^{-}(1-p)$ derived in respectively step 2 and 4. The trade-off method is known to be susceptible to violations of procedure invariance (Fennema & van Assen 1999). For this reason the quantity that was assessed always belonged to the lottery with the largest spread in outcomes, *i.e.* the most risky lottery. In the elicitation of gain utility and the probability weight for gains (step 1 and 2) the best outcome of this risky lottery was assessed. In the domain of losses (step 3 and 4) the worst outcome of this lottery was assessed.

71 $w^{+}(p)U(x_n) + w^{-}(1-p)U(x_{j-n}) = w^{+}(p)U(x_j) + (1 - w^{+}(p))U(x_0)$
$U(x_{j-n}) = (w^{+}(p)\ (U(x_j) - U(x_n) - U(x_0)) + U(x_0)) \,/\, w^{-}(1-p)$
$U(x_{j-n}) = (w^{+}(p)\ ({}^{j}\!/_{n} - {}^{n}\!/_{n} - {}^{0}\!/_{n}) + {}^{0}\!/_{n}) \,/\, w^{-}(1-p) = {}^{(j-n)}\!/_{n} \times w^{+}(p) \,/\, w^{-}(1-p)$.

72 $w^{-}(1-p)U(x_f) + (1 - w^{-}(1-p))U(x_0) = w^{-}(1-p)U(x_{-2}) + (1 - w^{-}(1-p))U(x_{-1})$
$w^{-}(1-p) = (U(x_{-1}) - U(x_0)) \,/\, (U(x_f) - U(x_0) - U(x_{-2}) + U(x_{-1}))$.
From equation 4.5 we derive $U(x_{-i}) = -{}^{i}\!/_{n} \times w^{+}(p) \,/\, w^{-}(1-p)$. Because under prospect theory utility is unique up to a unit we rescale this utility by dividing with the constant factor $w^{+}(p) \,/\, w^{-}(1-p)$ and define $U^{*}(x_{-i}) = -{}^{i}\!/_{n}$. $w^{-}(1-p) = ({}^{-1}\!/_{n} - {}^{-0}\!/_{n}) \,/\, (U^{*}(x_f) - {}^{-0}\!/_{n} - {}^{-2}\!/_{n} + {}^{-1}\!/_{n}) = {}^{-1}\!/_{n} \,/\, (U^{*}(x_f) + {}^{1}\!/_{n}) = -1/\ (n \times U^{*}(x_f) + 1)$.

4.4 Results of trial sessions

This section presents the results of trial sessions with eight Dutch entrepreneurs. It discusses some details of the utility elicitation sessions and the methodology that was used to gear the elicitation method to the particular circumstances of the entrepreneurs. It furthermore presents for each entrepreneur the weights assigned to the firm-specific probability of being profitable and unprofitable and the utility curve over the whole domain of monetary outcomes, expressed in Euro. Finally, it presents an overview of several issues related to the elicitation procedure.

4.4.1 Details of the trial sessions

In December 2011 and January 2012 trial sessions were carried out with eight entrepreneurs from the creative industry in The Netherlands. These entrepreneurs, active in design, entertainment, architecture, media and talent development, operate from offices in the Creative Factory, a former grain silo refurbished to office space on the South bank of the Rotterdam city center port area. Our aim thereby was not to provide a representative sample of a (sub)population of decision makers but instead to test the performance of the method in a realistic, decision analysis setting. The participants were facilitated during the elicitation process by undergraduate students from the Rotterdam University of Applied Sciences who were trained for their task and followed a written protocol in administering the trials. Participants were male, aged between 26 and 36 (mean personal age 32) and their companies had been active for 4 months up to 10 years (mean active years 5). At the outset of the elicitation process an inventory of the relevant range of outcomes and the firm-specific probability of being profitable was established first. The participants were informed that the gains and losses mentioned in the elicitation task relate to earnings after interest and tax from which a compensation for the entrepreneur's personal livelihood expenses has already been deducted. During the elicitation all these "excess earnings" relate to monthly amounts. Under guidance of their facilitator participants practiced each step of the utility elicitation method in table 4.1 and subsequently completed the actual elicitation process. In the elicitation procedure a bisection method was used in which the indifference value of the quantity that was assessed was derived from a sequence of five, iterative choices between two lotteries (details are in appendix B). Finally, participants were debriefed and invited to comment on the utility curve over the whole domain of outcomes which resulted from the elicitation process.

4.4.2 Tailoring the elicitation method to entrepreneurial circumstances

To increase the likelihood that entrepreneurs consider the choices presented to them during the elicitation process as contextually meaningful, insignificantly small or excessively high gain and loss values and unrealistic probability distributions need to be avoided. To this end the elicitation procedure of each participant was tailored to the following specifications which the participants provided themselves:

1. a realistic maximum monthly earnings (x_U),
2. a negligible amount of monthly earnings (x_L),
3. a negligible amount of monthly losses (x_{-U}), and
4. a monthly loss that would terminate the enterprise (x_{-L}).

This inventory provides the decision analyst with a provisional upper limit of elicitation in the domain of gains (x_U), a lower limit for gains and an upper limit for losses (x_L and x_{-U}), which can be regarded as materiality thresholds, and a lower limit for losses (x_{-L}) at which organizational default is to be expected.

The decision maker is requested to indicate a realistic probability p used for the best outcome and $(1 - p)$ for the worst outcome in each lottery. On the assumption that entrepreneurs adopt a monthly accounting discipline the probability of being profitable (p) was specified by the participants by requesting them to indicate how many months a year they expect to be profitable in the future and derive p from this frequency distribution. In case of an advantageous business environment we expect $p > .5$ which can be conveniently applied to elicit the utility of large losses.[73]

Table 4.2 presents the specifications that the eight entrepreneurs provided and to which their individual elicitation process was tailored. Inspection of table 4.2 reveals large differences between the entrepreneurs. Maximum profits vary between 600 for a newly established company and 10,000 for two companies that both had been active for 7 years.[74] Three entrepreneurs indicated that every single dime of profit matters to them while two others consider any profit below 2,000 negligible. In general the entrepreneurs consider losses in a range between 200 and 1,500 to be immaterial. Termination of the enterprise looms already at a loss of 600 for the recent start-up while for a company active in business for 7 years this threat manifests itself only at a loss of 7,000. Noticeably four participants chose the probability of being profitable relatively close to ½ and $^2/_3$, percentages that are commonly employed in utility elicitation methods. The four remaining participants chose their probability around .9 which is an unconventional probability for elicitation studies.

4.4.3 Weights of the probabilities of being profitable and unprofitable

The probabilities of being profitable (p) chosen by the entrepreneurs are moderate to high probabilities. The complementary probability of not being profitable corresponds with low

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[73] Given the indifference $(p : x_i, (1 - p) : x_{-j}) \sim (q : x_i, (1 - q) : x_{-h})$ and $q > p$ then $x_{-h} < x_{-j}$, which implies that *ceteris paribus* the larger the probability of being profitable the more negative will be the losses.

[74] These maximum profits (x_U) correlated positively with the number of active years of the enterprise (Spearman correlation test $\rho = .832$, $p = .010$).

Table 4.2 Specifications provided by eight entrepreneurs together with figures for the personal age of the entrepreneurs and the number of active years of their companies.

ID	1	2	3	4	5	6	7	8	mean
x_U	10,000	2,000	10,000	2,000	5,000	2,500	2,000	600	4,263
x_L	2,000	0	0	0	2,000	1,900	1,000	200	888
x_{-U}	−1,000	−300	−1,500	−500	−1,000	−400	−500	−200	−675
x_{-L}	−7,000	−1,000	−2,500	−5,000	−3,000	−2,000	−2,000	−600	−2,888
p	.9	.85	.9	.6	.9	.65	.5	.5	.73
personal age	36	26	36	30	31	36	28	31	32
years active	7	3	7	2.33	5	10	3	0.33	5

Table 4.3 Probabilities, probability weights, and classifications of the participants' attitude to probability with symbols, +, − and =, respectively indicating overweighting ($w^s(p) > p$), underweighting ($w^s(p) < p$), and linear weighting at a significance level of .02 ($w^s(p) = p$).

ID	1	2	3	4	5	6	7	8
p	.90	.85	.90	.60	.90	.65	.50	.50
$w^+(p)$	.89	.44	.77	.37	.39	.37	.48	1.00
attitude to probability	=	−	−	−	−	−	=	+
$(1 - p)$	.10	.15	.10	.40	.10	.35	.50	.50
$w^-(1 - p)$	.33	.25	.22	.87	.45	.39	.16	.25
attitude to probability	+	+	+	+	+	+	−	−
$w^+(p) / w^-(1 - p)$	2.68	1.76	3.43	.43	.88	.94	2.95	4.01

to moderate probabilities. The weights that entrepreneurs assigned to these probabilities under prospect theory are presented in table 4.3.

The weight assigned to the probability of being profitable, $w^+(p)$, was for five entrepreneurs below the value of p (*underweighting*), for one entrepreneur higher than p (*overweighting*), and for two entrepreneurs almost identical to p (*linear weighting*). The majority result for this group of entrepreneurs corresponds with the common finding under prospect theory of underweighting for moderate to high probabilities. This group of entrepreneurs thus appears to have a pessimistic attitude towards their chances for profitability. Only the new start-up (ID 8) did have an extremely optimistic attitude towards its probability of being profitable. Two of the entrepreneurs weigh the probability of profitability linearly which suggests a neutral attitude towards chance.

For the two entrepreneurs with a probability of being unprofitable of .5 the probability weight $w^-(1 - p)$ indicates underweighting in the domain of losses. Underweighting for moderate probabilities is a common result under prospect theory in both the gain and loss domain and suggests for these two entrepreneurs an optimistic attitude towards their chances on making losses. Two entrepreneurs (ID 4 & 6) overweight the moderate probabilities .4 and .35. The weights of the four remaining entrepreneurs correspond with the common finding under prospect theory of overweighting for small probabilities. This result suggests a pessimistic attitude towards the probability of not being profitable.

4.4.4 Utility curves in the domain of gains and losses

Figure 4.1 presents the utility functions of the eight entrepreneurs that were elicited under the assumptions of prospect theory over the whole domain of gains and losses. The outer limits of the relevant range (x_{-L} and x_U) are indicated by the symbol □ and the symbol ◇ indicates the materiality thresholds of the range of immaterial outcomes (x_{-U} and x_L). The ID-numbers of the entrepreneurs are presented above the graphs.

The graphs of the entrepreneurs with ID 1, 2 and 3 show a striking resemblance. For all three entrepreneurs utility in the gain domain is slightly concave, shows at the origin a rather smooth transition to the loss domain, and the curves make a very sharp decline around the value at which a monthly loss would terminate the enterprise (indicated by the symbol □ on the left of the horizontal axis). The three curves do not exhibit a downward kink at the origin and thus fail to present evidence of loss aversion. It appears, however, that imminent default around the lower limit of losses does cause entrepreneurs to become strongly averse to losses. The first two entrepreneurs characterized themselves during debriefing as very risk averse and expressed their preference for a stable profit over a profit that fluctuates.

The graphs of the entrepreneurs with ID 4 and 5 show slightly convex utility curves in both the gain and loss domains. Again the transition from gains to losses is smooth at the origin. Seen from the vantage point of the origin the combined effect of a rather steep utility curve in the gain domain and a rather flat utility curve in the loss domain suggests that these entrepreneurs exhibit a gain seeking attitude. During debriefing one of these entrepreneurs (ID 4) indicated that his enterprise has always been profitable and that he will do his utmost to maintain this profitability.

The graph of the entrepreneur with ID 6 suggests several relatively haphazard alternations of convexity and concavity. Alternatively this wobbly curve could be interpreted as linear utility with errors. What favors the latter interpretation is that the curve falls almost entirely in the range of negligible, small gains and losses for which linearity is plausible (Wakker 2010).

The most striking feature of the graphs of the entrepreneurs with ID 7 and 8 is their very steep section in the utility curve in the domain of losses. For the first entrepreneur (ID 7) the steep section is located close to the materiality threshold and for the other (ID 8) close to zero within the range of negligible losses. While a kink in the utility at zero is a common finding in many studies, the slope of the curve of the second entrepreneur is extraordinarily steep. During debriefing the first entrepreneur indicated that his responses were provided under time pressure. The second entrepreneur, having just started a company, characterized himself as risk averse.

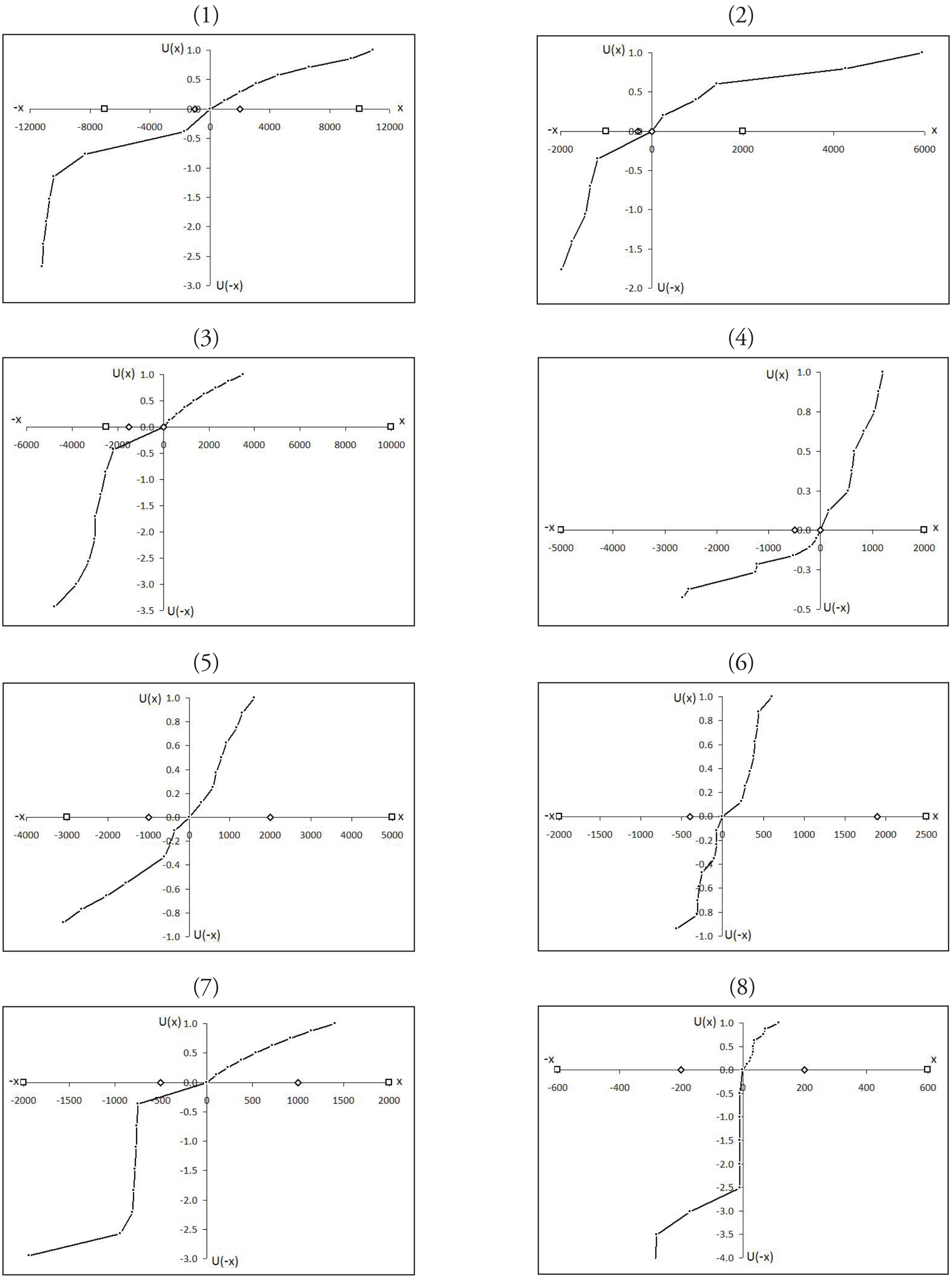

Figure 4.1 Utility curves in the domain of gains and losses with the symbol □ indicating the outer limits x_{-L} and x_U and symbol ◇ the inner limits x_{-U} and x_L.

4.4.5 Issues related to the elicitation procedure

In analyzing the results of the trial sessions several issues related to the elicitation procedure, which is discussed in appendix B, were identified. These issues are related to the range of outcomes over which utility is elicited, the iteration sequences of five choices which determine the indifference values, and the determination of probability weights.

Ideally the upper limit of utility in the gain domain should neither fall short nor exceed x_U, which is the decision maker's estimate of a realistic maximum monthly amount of earnings. To this end the upper gain limit x_U and the upper loss limit x_{-U} were in combination with predictions from cumulative prospect theory (Tversky & Kahneman 1992) used to determine the gauge values G and g in step 1 (the details are explained in appendix B). For six out of the eight entrepreneurs these gauge values were not conducive in assuring that the trade-off sequence in step 1 included x_U (the exceptions were ID 1 & 2). If x_n, the highest gain in step 1, is not sufficiently large to compensate the entrepreneur for x_{-L}, the lower limit of the loss range, then the entrepreneur will choose the largest loss x_{-n} to be less negative than this lower limit. For five of the eight entrepreneurs utility in the loss domain did cover the relevant range of losses (ID 1, 2, 3, 5, & 7).

Closer inspection of the choices of the entrepreneurs reveals that in the five iterations that lead to the quantity that is assessed, frequently either the choice option with the largest spread in outcomes or the one with the smallest spread is preferred in all five choices. From table 4.4 can be discerned that in step 3 the choice option with the largest spread of outcomes was frequently preferred all of the time. This causes the quantity that is assessed, *i.e.* x_{j-n}, to be only slightly less negative than the indifference value that resulted from the previous iteration, *i.e.* x_{j-n-1}. The steep sections in figure 4.1 for utility in the domain of losses are the result of these choice patterns.

Table 4.4 Assessed quantities for which the lottery with the largest spread or the lottery with the smallest spread in outcomes, indicated by $^{(*)}$, was preferred in all of the five iterative choices.[75]

ID	Step 1	Step 2	Step 3	Step 4
1			x_{1-n}, x_{2-n}, x_{3-n}, x_{4-n}	
2			x_{0-n}	
3			x_{4-n}	$x_f^{(*)}$
4			x_{4-n}	
5				
6			x_{2-n}, x_{7-n}	
7			x_{3-n}, x_{4-n}, x_{5-n}, x_{6-n}, x_{7-n}	
8	x_4	x_k	x_{1-n}, $x_{2-n}^{(*)}$, x_{4-n}, x_{5-n}, x_{6-n}, x_{7-n}	

[75] The choice-options with the largest spread in outcomes are the lotteries on the left of the indifference sign in table 4.1. Choice-options with the smallest spread are those on the right of the indifference sign.

In step 3 such a steep section occurs beyond the default level x_{-L} when the choice option with the largest spread in outcomes is preferred to a choice-option which contains negligible and zero gains (see ID 1). These gains are so low that the alternative choice-option, which contains the highest gain x_n is, even in combination with the loss x_{j-n}, relatively more attractive. In two other cases (ID 7 & 8) loss x_{j-n} had in previous iterations been reduced to an (almost) negligible level. As a result this loss has become so low that, in combination with the highest gain x_n, it is consistently more preferred than the outcomes offered in the alternative choice-option, even when these include substantial gains. In step 2 a consistent preference of the new start-up (ID 8) for the choice-option with the largest spread in outcomes led to the indifference $(.5 : 26, (1 - .5) : 0) \sim (.5 : 26, (1 - .5) : 14)$. Under prospect theory this indifference is resolved by the equating $1 - w^+(.5)$ with zero, which implies $w^+(.5) = 1$. From figure 4.1 can be derived that for one of the more experienced entrepreneurs (ID 3) the range between the negligible amount of monthly losses (x_{-U}) and the monthly loss which would terminate the enterprise (x_{-L}) is very small. In step 4 the value of the quantity that is assessed (x_f) is −3,140, which is located outside of this range in a very steep section of the utility curve for losses.

The probability weight for gains and losses can only be determined when respectively the conditions $x_k \leq x_n$ and $x_f \geq x_{-n}$ are fulfilled. In the trial sessions these two conditions were fulfilled for all eight entrepreneurs. For the calculation of $w^+(p)$ the condition $x_k \leq x_n$ was already satisfied within a sequence of 3 up to 4 indifference values, well within the minimum number of 5 intervals. For the calculation of $w^-(1 - p)$ the condition $x_f \geq x_{-n}$ required a sequence of 3 up to 8 indifference values.[76] It is important to note that the five and eight intervals that were elicited for the entrepreneurs with respectively ID 2 and 7 were just sufficient to fulfill these conditions and determine the $w^-(1 - p)$.[77] Probability weights are frequently entirely derived on the basis of values in a range of negligible outcomes. For gains this was the case for five entrepreneurs (ID 2, 5, 6, 7, & 8) and for losses for three entrepreneurs (ID 5, 6, & 8).

4.5 Discussion

This chapter specified the utility function on the whole domain of outcomes using an extension of the trade-off method that is compatible with organizational risk appetite. The novelty of this extension of the trade-off method is that utility in the domain of losses is not elicited independently but is instead derived from the utility of gains. The specification of the loss utility function in equation 4.5 directly derives the fraction $^{(j-n)}/_n$ from the utility

[76] The condition $x_k \leq x_n$ was satisfied by $x_k \leq x_3$ (ID 1, 3, & 8) and $x_k \leq x_4$ (ID 2, 4, 5, 6, & 7). The condition $x_f \geq x_{-n}$ was satisfied by $x_f \geq x_{0-n}$ (ID 2 & 7), $x_f \geq x_{2-n}$ (ID 3 & 8), $x_f \geq x_{3-n}$ (ID 1), $x_f \geq x_{4-n}$ (ID 5 & 6), and $x_f \geq x_{5-n}$ (ID 4).

[77] The number of intervals elicited were $n = 7$ (ID 1), $n = 5$ (ID 2), and $n = 8$ (ID 3, 4, 5, 6, 7, & 8).

of gains whereas the fraction $w^{+}(p)$ / $w^{-}(1-p)$ contributes to the expression of loss aversion by means of probability weights (Zank 2010). Our method thus does not require a separate specification and assessment of the loss aversion parameter. With the exception of the first iteration sequence in step 3 our method assesses trade-offs between two lotteries in which the same set of organization-specific probabilities p and $(1-p)$ is employed. Just as in the trade-off method these can, at the convenience of the decision maker, be exchanged for verbal probability expressions or event indicators.

Its dominant use of favorable, mixed lotteries allows this method to assess a decision maker's willingness to accept potential losses in the pursuit of gains and makes it conducive to the measurement of organizational risk appetite.[78] Most studies in decision theory measure the utility of losses by using unfavorable lotteries, usually gain-free lotteries, and therefore measure willingness-to-pay instead of willingness-to-accept. A notable exception are studies in decision analysis that use mixed lotteries in combination with a certainty equivalent of zero (Walls 1995, Walls & Dyer 1996 *zero equivalence*). To elicit utility these studies require the specification of functional forms and statistical data fitting, which is not required in our method. Additionally these studies assume expected utility theory, which makes their results prone to probability distortion. Our study assumed prospect theory, a descriptively accurate theory for analyzing decisions under risk, which isolates the distortion of probabilities from the utility of outcomes by means of probability weighting functions.

In trial sessions with eight entrepreneurs we identified the utility function over the outcome domain $[x_{-n}, x_n]$ using a maximum of 18 indifference points. The most striking result of these trial sessions is the absence of the loss aversion phenomenon in the utility curvature. The utility curves of the three entrepreneurs (ID 1, 2, & 3) for whom the utility of losses was measured beyond the bankruptcy threshold (x_{-L}) demonstrate an immediate aversion to losses beyond this threshold. Empirical findings on prospect theory suggest that the utility of money becomes concave again near ruin (Wakker 2010 p.264, hypothesized by Kahneman & Tversky 1979). For the aforementioned three entrepreneurs our preliminary results provide additional empirical support hereof. The probability weights of the entrepreneurs that participated in our study are to a very large extent in agreement with the predictions of prospect theory.

Most choice problems used in our method compare two risky lotteries and these are considered more difficult to evaluate than choice problems that compare a risky lottery with a sure outcome. For this reason each of the entrepreneurs was allowed to practice prior to the start of the elicitation process. Because the assessed quantities are chained our method is susceptible to error propagation. Participants were therefore, after each iteration sequence of five choices, required to affirm the equivalence of lotteries. Admittedly, given vulnerability to error propagation it is unclear whether our results represent the genuine preferences of the

[78] The method mainly employs favorable lotteries. Exemptions are the first sequence(s) in step 1 where unfavorable lotteries are employed and the two sequences in step 2 and 4 where respectively a loss-free and a gain-free lottery are used.

entrepreneurs. In particular the steep sections in negligible outcome ranges raise doubts in this direction. While decision analysts should be aware of the limitation of this method and intervene when this is demanded, future research should, furthermore, investigate the possibility of triangulating the results of our utility elicitation method by means of other methods. While gauge values in step 1 were chosen on the basis of cumulative prospect theory, a descriptive theory of decision making under risk, the sequence of utility values elicited in the domain of gains did not always include the upper limit of gains (x_U). Given the large heterogeneity in utility curvature in the population and the small number of entrepreneurs contributing to this study such a misfit should, in hindsight, not have come as a surprise. If we had anticipated the general lack of loss aversion in our sample and would have settled for a lower λ in determining gauge g in equation 4.8, then the value of g would have been more negative, would have induced the entrepreneurs to seek compensation for this by choosing higher values of x_i, which may have stretched the sequence to a desirable level.

A more practical limitation of this method is that entrepreneurs do not always understand the concept of gains and losses. They are more familiar with revenue as a performance indicator while earnings are frequently determined at year-end only or are assessed on a project rather than a periodic basis. They complained that occasionally the losses in step 3 were very low, leading to irrelevant choice alternatives, which suggests the introduction of a cut-off point for x_{j-n} beyond the negligible amount of monthly losses (x_{-U}). They furthermore would have appreciated more context in the choice alternatives presented to them.

The need for the elicitation method introduced in this chapter arose from a call in the risk management literature for meaningful measurements of organizational risk appetite. In addition the method addressed the call in the decision theory literature to specify a utility function on the whole of the outcome domain, in particular for large outcomes remote from zero. This study therefore emphasizes the importance of problems of a practical nature as a source of inspiration for theoretical contributions.

4.6 Appendices

4.6.1 Appendix A: Overview of utility elicitation methods

The central element in many utility elicitation methods is that they seek to elicit the indifference point between two lotteries with the general structure

$$(q : v, (1 - q) : w) \sim (p : x, (1 - p) : y), \tag{4.9}$$

with $v > w$, $x > y$ for gain and mixed lotteries and $v < w$, $x < y$ for loss lotteries, and $0 < q \leq p \leq 1$. Each method is made up of a particular combination of the elements of this general structure and by directly assessing or adjusting one of these elements each method accomplishes the elicitation of an indifference point.

Table 4.5 Elicitation methods for eliciting indifference between binary lottery pairs.

Method	**Assessed quantity**	**Indifference sought**	**Derivation from general structure**
Probability equivalent Certainty equivalent	p v	$(1 : v) \sim (p : x, (1-p) : y)$ with $x > v > y$ for gain and mixed lotteries, and $x < v < y$ for loss lotteries	$v = w$
Lottery equivalent	q	$(q : v, (1-q) : 0) \sim (p : x, (1-p) : 0)$ with $v > x$ for gains, with $v < x$ for losses, and v fixed	$w = y = 0$
Trade-off outward Trade-off inward	v x	$(p : v, (1-p) : w) \sim (p : x, (1-p) : y)$ with $v > x > y > w$ or $y > w > v > x$ for gains and mixed lotteries, with $v < x < y < w$ or $y < w < v < x$ for losses, and w and y fixed	$p = q$
Choice list	p	$(p : v, (1-p) : w) \sim (p : x, (1-p) : y)$ with $v > x > y > w$ for gains and mixed lotteries, with $v < x < y < w$ for losses, and v, w, x and y fixed in each choice list	$p = q$
Zero equivalent gain Zero equivalent loss	x y	$(1 : 0) \sim (p : x, (1-p) : y)$ with $x > 0 > y$	$v = w = 0$

Table 4.4 summarizes six elicitation formats that play a prominent role in the decision theory literature. For each of these formats the table indicates which quantity is assessed, which indifference is sought, and how this indifference is derived from the aforementioned general structure. It also indicates for each method how the outcomes in the lottery are ranked and whether they remain fixed during the process of elicitation. This overview of utility elicitation methods is restricted to methods that involve revealed choices between two lotteries (*binary choice*) and thus does not cover methods where each outcome is rated on a utility scale directly (*introspective judgment*; see Abdellaoui, Barrios, & Wakker 2007 for an experimental investigation using both approaches). The primary focus in this overview is furthermore on methods where the participant is requested to directly impute a missing value in a lottery (*direct matching*) and methods where such an indifference point is elicited by giving participants a series of two lottery choices that eventually lead to indifference between the lotteries (*sequential bisection*). A method that requires the participant to express preferences in a list of lottery choices is also discussed (*choice list* or *multiple price list*, see references in Andersen, Harrison, Lau, & Rutström 2006).[79]

[79] A fourth method demands the participant to directly choose between lotteries (*discrete choice*). It allows to combine a variety of lottery pairs that are employed in the other methods. Contrary to the other methods discrete choice does not aim to elicit an indifference point.

Traditional certainty and probability equivalent methods for eliciting utility use the standard gamble format. The probability equivalent method requires participants to respond to lottery choices in the probability-mode instead of the outcome-dimension of the lotteries, which induces a response mode bias (Hershey & Schoemaker 1985, Hershey, Kunreuther, & Schoemaker 1988). The certainty equivalent method requires participant to state a sure outcome that is equivalent to a risky lottery. Comparison of a sure and risky lottery creates probability distortion and exaggerates risk aversion for gains and risk seeking for losses (Kahneman & Tversky 1979). By comparing two risky lotteries the lottery equivalent method avoids this certainty effect and reduces boundary effects by fixing the upper-limit for gains or lower-limit for losses (McCord & de Neufville 1986). In the trade-off method the participant weighs the pros and cons of a lottery with a large variance in outcomes versus a lottery with a small variance. In each of these lotteries a gauge or reference outcome is fixed and because both lotteries employ the same probability the trade-off method is resilient to probability distortion (Wakker & Deneffe 1996). The procedures of eliciting trade-offs from the origin outward or inward to the origin create different results, violating *procedure invariance* (Fennema & van Assen 1999). All these four methods can independently be used to elicit utility without assuming a functional form of utility (*nonparametric elicitation*). When used nonparametrically the first three methods assume expected utility theory while the fourth accommodates expected utility as well as prospect theory. The next two methods require either that a functional form is specified (*parametric elicitation*) or that they are used in combination with the previous methods. The choice list method presents the participant with a list of small versus large variance lotteries with all outcomes fixed. Starting with a low probability in the lottery pair in the first row, the probability level is row by row increased to the level of certainty in the final row. For each lottery pair the respondent indicates preference and at the point where her preference switches the probability indifference point is assumed to be (Holt & Laury 2002). Although its structure resembles the trade-off method it is susceptible to probability distortion and response mode bias. In the zero equivalent method a mixed lottery is compared with a sure, zero outcome. Keeping probability fixed it either requires the respondent to specify the minimum gain that a lottery should offer to compensate for the acceptance of a potential loss or what maximum loss a lottery should be allowed to have relative to its potential gain (Walls 1995, Walls & Dyer 1996).

4.6.2 Appendix B: Elicitation procedure

During the trial sessions the indifferences in table 4.1, column 3 were elicited by means of the bisection method (Abdellaoui, Bleichrodt, & L'Haridon 2008 p.263, Abdellaoui, Bleichrodt, & Paraschiv 2007 p.1671). In this method each indifference was derived from a sequence of five, iterative choices between two lotteries (*binary choice*). When the lottery containing the quantity that is assessed is chosen by the participant, then in the next iteration this quantity is made less favorable. When the other lottery is preferred, however, then this quantity is made more favorable. After a series of five iterations the midpoint of the interval in which the indifference value of the quantity that is assessed should lie is taken as the indifference value. The participant is then requested to confirm the equivalence of the lotteries that result. In comparison with a procedure in which the participant directly states which quantity leads to indifference this choice-based elicitation procedure leads to fewer inconsistencies (Bostic, Herrnstein & Luce 1990, Luce 2000). During the trials lotteries were presented in a pie-chart format in which the size of the slices corresponds with the probabilities that are used.

In step 1 the utility for gains is elicited with the trade-off method, which uses chained questions in the assessment of equally spaced utility intervals. The standard number intervals elicited in both the gain and loss domain is at minimum 5 and at maximum 8. Because of the chained nature of these questions the largest indifference value in the elicitation process (x_n) may fall short of reaching the provisional upper limit x_U or may exceed this limit. When x_i exceeds x_U then no further indifference points in the gain domain are assessed unless the minimum number of intervals is not yet reached. If x_n falls short of x_U then the utility function for gains does not enclose the relevant range of the participant. If the indifference value exceeds x_U then utility is assessed beyond a realistic maximum amount of gains. To increase the likelihood that the indifference value x_n is at least located in the vicinity of x_U we carefully chose the gauge values g and G that are used in the elicitation process. We let $G = x_{-U}$ to honor the decision maker's materiality threshold for losses. Gauge value g is determined on the basis of predictions from cumulative prospect theory (Tversky & Kahneman 1992), using the functional specifications $u(x) = x^{\alpha}$ for $x \geq 0$, $u(x) = -\lambda((-x)^{\beta})$ for $x < 0$, and $w^s(p) = p^{\theta} / (p^{\theta} + (1-p)^{\theta})^{1/\theta}$ for $s = +,-$, $0 < p \leq 1$ with the parameter values $\alpha = .88$, $\beta = .88$, $\lambda = 2.25$, $\theta = .61$ for $s = +$ and $\theta = .69$ for $s = -$. Using these specifications and parameter values we first predict x_1 by solving equation 4.7 for $i = 1$

$$x_i = u^{-1}[u(x_U)\ {}^{i}\!/_{n}].^{80} \tag{4.7}$$

We then determine the size of g by solving equation 4.8 for i = 1

$$g = u^{-1}[(u(x_{i-1}) - u(x_i))\ w^+(p) \,/\, \lambda w^-(1-p)) + u(G)].^{81} \tag{4.8}$$

The starting, minimum and maximum values of quantity that is assessed are presented in table 4.6.

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80 $u(x_i) = u(x_n)\ {}^{i}\!/_{n}$
$x_i = u^{-1}[u(x_n)\ {}^{i}\!/_{n}] = [x_n^{\alpha}\ {}^{i}\!/_{n}]^{1/\alpha}$
Let $x_n = x_U$.

81 $w^+(p)u(x_i) + \lambda w^-(1-p)u(g) = w^+(p)u(x_{i-1}) + \lambda w^-(1-p)u(G)$
$g = u^{-1}[(u(x_{i-1}) - u(x_i))\ w^+(p) \,/\, \lambda w^-(1-p)) + u(G)]$
$g = -((-(-(((-G)^{\beta}) + ((x_{i-1})^{\alpha} - (x_i)^{\alpha})\ p^{\gamma}(p^{\delta} + (1-p)^{\delta})^{1/\delta} / \lambda(1-p)^{\delta}(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}))^{1/\beta})$.

Table 4.6 Starting, minimum and maximum values of the quantity that is assessed.

	Starting value	**Minimum value**	**Maximum value**
Step 1	x_i derived from expected value	x_{i-1}	$5 \times$ starting value of x_i
Step 2	x_k derived from expected value	x_2	$5 \times$ starting value of x_k
Step 3	$x_{j-n} = x_{-L}$ (first iteration) $x_{j-n} = .5(x_{j-n-1})$	$2(x_{-L})$ (first iteration) x_{j-n-1}	x_0
Step 4	x_f derived from expected value	$5 \times$ starting value of x_f	x_{-2}

Except for step 3 the starting value of the assessed quantity was always derived from an equivalence between the lotteries in terms of expected value. Applied to step 3 this procedure typically would result in starting values that are a multiple of the amount that would terminate the enterprise (x_{-L}). The starting value of step 3 was in the first iteration therefore fixed at x_{-L} and the minimum value at twice this amount. The minimum values in subsequent iterations in step 3 were set equal to indifference value that resulted from the previous iteration (x_{j-n-1}) and starting value was fixed at half this amount. This procedure forced the sequence of indifference values to increase monotonically.

5 Describing Organizational Risk Appetite using Decision Theory

Summary

This chapter presents the results of a field study in which the introspective judgments regarding organizational risk appetite of senior managers of a large Dutch company were compared with their professional risk attitudes, derived from predictions under prospect theory, unbiased expected utility theory, and expected value theory. In this study utility and probability weighting functions of senior managers were elicited under prospect theory using the elicitation method of Abdellaoui, Bleichrodt, and Paraschiv (2007). In the domain of losses median results are a risk seeking attitude, a utility function that is slightly concave, implying diminishing marginal utility of losses, and a probability weighting function that is considerably convex, implying an optimistic attitude towards probabilities. The median introspective judgment of organizational risk appetite expresses both risk neutrality and risk aversion and is significantly more accurately described by expected value theory than by prospect theory.

5.1 Introduction

Organizations accept different levels of risk in order to reach their objectives. Why a specific risk is acceptable to one organization and unacceptable to another remains an open empirical question. Several explanations for differences in organizational risk appetite have been suggested in the practitioner literature (for an overview see Verbaan 2009). One of these is that the professional risk attitude of senior management is driving organizational risk appetite (Hillson & Murray-Webster 2005, HM Treasury 2006a & 2006b). This explanation corresponds with the normative paradigm of management control, which states that employees are to only take on those risks that their senior management considers acceptable for the organization (Merchant 1985, Flamholtz 1983 p.154 *organizational control*). A misalignment between statements of the organization that specify its risk appetite and the intended risk attitude of its senior management is essentially a management control problem and leads it to accept more, or less, risk than intended. Because of the large impact that organizational risk-taking can have on its external environment - consider the dot-com crisis (2000) and the financial crisis (2008) - exertion of control over organizational risk appetite is not only an important topic for individual organizations but also for society as a whole.

In decision theory the attitude of senior management to events that potentially have a negative impact on organizational objectives can be analyzed from a normative and a descriptive perspective (for an overview see Starmer 2000). Prospect theory (Kahneman & Tversky 1979, Tversky & Kahneman 1992), a descriptive theory for decision making under risk, accommodates empirical evidence that shows that risk attitude is not only determined by the decision maker's attitude towards the impact of a risk but is also affected by nonlinear probability weighting, a bias that is irreconcilable with normative, rational decision models (see, however, references in Wakker 2010 p.179 *normativity of rank dependence*). It describes attitude to impact by means of a utility function, which under prospect theory is resistant to probability distortion. This utility function, elicited under the assumptions of prospect theory, can be used to describe the unbiased risk attitude (see Definition 2.6) of a decision maker under expected utility theory, a normative theory of decision making under risk (see Schoemaker 1982). Risk management has traditionally employed expected value theory, a normative theory of choice under risk in which risk neutrality is assumed. A priori it is unknown whether organizational risk appetite is most accurately described by a risk attitude in which probabilities are distorted, an unbiased risk attitude, or a neutral risk attitude.

To get an initial test of the relationship between organizational risk appetite and the risk attitude of its senior management we chose to perform a case study with a large Dutch company. In this study we compare the introspective judgments of senior managers regarding organizational risk appetite with their risk attitudes, derived from predictions under prospect theory, unbiased expected utility theory,[82] and expected value theory. We elicit the utility and probability weighting functions of 43 of the case company's senior managers under prospect theory using the elicitation method of Abdellaoui, Bleichrodt, and Paraschiv (2007). We then request the senior managers to indicate what the risk appetite of their organization should be. Finally, we establish which decision theory makes the most accurate prediction of their introspective judgment on organizational risk appetite.

The findings of our study are that in the domain of losses the utility function of the median participant is slightly concave, which implies diminishing marginal disutility of losses. The probability weighting function of the median participant is considerably convex, which implies an optimistic attitude towards probabilities in the face of losses. These results predict a risk seeking attitude under prospect theory and a risk averse attitude under unbiased expected utility theory. The median participant's introspective judgment of organizational risk appetite clearly expresses risk neutrality and risk aversion and is significantly more accurately described by expected value theory than by prospect theory.

Where studies in decision theory primarily focus on student subjects and their attitude to risk in the domain of gains, this paper provides more insight into the risk attitudes of professionals in the domain of losses. Studies on applications of decision theory are rare. Our study presents an application of decision theory to the measurement of organizational risk appetite, which is in important topic in risk management. It illustrates how senior managers

[82] For the purpose of this chapter expected utility theory is considered unbiased when the utility function is elicited under the assumptions of prospect theory.

can determine their unbiased risk attitude and how these can be expressed in statements of organizational risk appetite in order to exert control over the acceptance of risk within an organization.

This paper proceeds as follows. The second section discusses the risk matrix of the case company. The third section provides an overview of three relevant theories from decision theory. The fourth section derives isocontours in the risk matrix from these decision theories. The fifth section describes the details of the field study we performed. In the sixth section we analyze the results of this study. In the seventh section we discuss these results.

5.2 The case company risk matrix

As a formal statement to communicate its organizational risk appetite to its employees the case company uses a risk matrix. The risk matrix is an easy-to-use, popular risk management tool, which focuses on the negative consequences of risk only. Its use is recommended in the COSO-ERM (2004) guidelines and it can be applied by organizations of all sorts and sizes. Contrary to other ways to formally express risk appetite, in particular Value-at-Risk, economic capital, and risk-return diagrams, it has received relatively little attention in the academic literature. To illustrate its use we present an abbreviated version of the risk matrix of our case company in figure 5.1.[83]

A risk matrix consists of a plane in which events are plotted on the basis of their probability of occurrence and their negative impact on organizational objectives. In figure 5.1 the horizontal axis of the risk matrix contains verbal expressions for five probability/frequency levels (A to E) together with a specification of these terms. The vertical axis presents verbal expressions and specifications for five ranges (1 to 5) defining the negative impact that events may have on the case company's financial objective. The verbal probability and outcome expressions facilitate in the qualitative, subjective assessment of probability and impact. In the case company risk matrix quantitative descriptors are absent on the probability dimension and on the impact dimension they increase logarithmically by a factor 10. In combination the five probability and impact levels segregate the matrix into 25 cells. Once the risks that the company faces have been identified these are, based on an assessment of their probability and impact, allocated to cells in the matrix. Each of the events that are allocated to the cells in the risk matrix can be interpreted as a risky lottery with a negative outcome x with associated probability p, and a zero outcome with associated probability $(1 - p)$. The shades in the risk matrix indicate whether these risks are acceptable or not and specify a risk response (see the legend below figure 5.1 for an explanation of the shades). The number of cells assigned to each zone and the shape of the boundary lines between the zones specify the organizational risk appetite. For the case company it was unclear whether its current prioritization

[83] Some nonessential elements have been deleted to ensure anonymity of the case company and improve clarity.

of risks led to investments in control measures that were in line with the intentions of its leadership. Our field study therefore allowed the case company to test whether its existing formal statement of risk appetite reflected the risk attitude of its senior management.

		PROBABILITY				
IMPACT		A (Unlikely) Never heard of before in this business sector	B (Likely) Does occur in this business sector	C (Incidental) Has occured several times within organisation	D (Regularly) Has occurred many times within organisation	E (Very likely) Does occur several times per year within organisation
1. Small	Cost lower than €10.000	1A	1B	1C	1D	1E
2. Limited	Cost between €10.000 and €100.000	2A	2B	2C	2D	2E
3. Considerable	Cost between €100.000 and €1.000.000	3A	3B	3C	3D	3E
4. Large	Cost between €1.000.000 and €10.000.000	4A	4B	4C	4D	4E
5. Very large	Cost larger than €10.000.000	5A	5B	5C	5D	5E

Acceptable risk zone: no additional control effort is required.
Temporarily acceptable risk zone: additional control effort is required.
Unacceptable risk zone: additional control effort is required and risk can only be accepted by an appropriate authority within the organization.

Figure 5.1 Abbreviated version of the risk matrix of the case company.

5.3 Risk attitude and three decision theories

Decision theory studies the risk attitude of individual decision makers. In organizations it is not the private risk attitude of the decision maker that is of interest but her professional risk attitude, *i.e.* her attitude towards risks taken in the pursuit of the organization's mission. In this context it is assumed that the individual, professional risk attitudes of the organizational decision makers are absorbed in organizational policy or in behavior that can be attributed to the organization (Coumou 2003, Van den Brink 2007). In decision theory most studies elicit preferences of individuals towards their own, private objectives (for an exception see Abdellaoui, Bleichrodt, & Kammoun 2009).

The professional risk attitudes of senior managers can be elicited using methods from decision theory. In decision theory the risk attitudes of individual people are determined by revealing their preference over lotteries. A lottery is a set of outcomes with associated probabilities.[84] For risks that are allocated to the risk matrix the notation for such a lottery is

$$X = (p : x, (1 - p) : y), \tag{5.1}$$

with $x < 0$, $y = 0$, and $0 < p \leq 1$.

Commonly risk practitioners define risk as the product of probability p and negative impact x, an approach attributed to Blaise Pascal (Pfitzer, Hardwick, & Dwyer 2001). This product is referred to as the expected value which for X, given that $y = 0$, is defined by the equation

$$\mathrm{EV}(X) = p \times x \tag{5.2}$$

Expected value theory (EV) assumes a risk neutral decision maker. Traditionally boundaries between zones in the risk matrix are derived from expected value calculations.

Whether a person has a risk averse, risk neutral or a risk seeking risk attitude is determined by offering this person the choice between a risky lottery X and a sure amount which value is equal to the expected value of X. Risk aversion holds if every lottery is less preferred than its expected value, denoted by $X \preccurlyeq \mathrm{EV}(X)$. Risk seeking holds for the reverse, $X \succcurlyeq \mathrm{EV}(X)$. Being indifferent between the two options is indicative for a risk neutral attitude, denoted by $X \sim \mathrm{EV}(X)$ (Wakker 2010 p.52). Risk aversion also implies that, given identical expected values, the lottery with the smallest spread between outcome x and y is preferred (Wakker 2010 p.75 *aversion to elementary mean-preserving spreads*).[85] Naturally the opposite holds for risk seeking. A risk matrix that reflects a risk averse attitude is characterized by an unacceptable risk zone that includes relatively many cells with a large impact. A risk seeking attitude is reflected in the risk matrix by an unacceptable risk zone with relatively many cells with a large probability of occurrence. Risk aversion is in economics considered to be a rational attitude towards risk that, given equal expected values, aims to reduce variance.

A decision theory that accommodates risk aversion, risk seeking, and risk neutrality is expected utility theory (EU). The origins of this theory can be traced to Daniel Bernoulli

[84] Lotteries are in decision theory also referred to as prospects, gambles, or probability distributions.

[85] Let $X = (p : x, (1 - p) : 0)$ with $x \in \mathbb{R}$, $0 < p \leq 1$. Define X^* as lottery X with outcome x replaced by lottery $Z = (q : z, (1 - q), 0)$ with $z = x/q$, $0 < q \leq 1$. Lottery Z is an elementary mean-preserving spread. X^* can be rewritten as $X^* = (pq : z, 1 - pq) : 0)$ with $pq < p$, $z > x$ for $x > 0$, and $z < x$ for $x < 0$. The spread in outcomes is for X the distance between x and 0 which is smaller than the distance between z and 0 for X^*. Under risk aversion holds aversion to elementary mean-preserving spreads which implies $X \succcurlyeq X^*$.

(1738/1954) and the rational foundations of this theory were established by von Neumann and Morgenstern (1944). The theory replaces the negative outcome x in the product of equation 5.2 with u(x), the utility of the negative outcome x, such that

$$\mathrm{EU}(X) = p \times \mathrm{u}(x), \tag{5.3}$$

with EU(X) being referred to as the expected utility of the lottery X and u(.) being a monotonically increasing function that transforms outcomes to utility values. The utility function reflects the attitude of a decision maker towards outcome.

Under expected utility theory risk aversion holds when the shape of the utility function is concave, *i.e.* curved inward. This concave shape reflects diminishing marginal returns, which implies that a unit reduction in negative outcome at a high impact level returns more utility than that same reduction at a low impact level. The curved shape of the utility function is frequently modeled by a power function in which utility is represented by

$$\mathrm{u}(x) = -((-x)^{\beta}). \tag{5.4}$$

(Pratt 1964, *Constant Relative Risk Aversion*). Concave utility holds when the β-parameter is higher or equal than 1. A convex utility function ($\beta \leq 1$) reflects under expected utility theory a risk seeking attitude to risk. Risk neutrality is under expected utility theory reflected by linear utility ($\beta = 1$).

Descriptive studies in decision theory report empirical evidence that risk attitude is to a large degree determined by a decision maker's attitude towards probability. Prospect theory (PT, Tversky & Kahneman 1992, Kahneman & Tversky 1979) accommodates both a decision maker's attitude towards probability and attitude to outcome. Under prospect theory, probability p in the product in equation 5.3 is replaced by w(p), which stands for the weight that is assigned to probability p in making the decision, so that

$$\mathrm{PT}(X) = \mathrm{w}(p) \times \mathrm{u}(x), \tag{5.5}$$

with PT(X) referred to as the prospect theory value of X and w(.) being a monotonically increasing function that transforms probabilities to probability weights. Under prospect theory the utility function u(.) has the same properties as under expected utility theory. Only under linear probability weighting, *i.e.* w(p) = p, the shapes of these utility functions are identical. Nonlinear probability weighting describes actual choices and is in general not assigned any normative status.

Under prospect theory, risk attitude is determined by the shape of the probability weighting function as well as the utility function. For risky lotteries that have a negative outcome, a strictly concave shape of the probability weighting function indicates that a decision maker assigns more weight to probabilities than their numerical values justify. This suggests a pessimistic attitude towards probability and promotes risk averse behavior. A convex shape, on the other hand, implies underweighting of probabilities. It implies an optimistic attitude to risk and promotes risk seeking behavior.[86] Apart from this motivational aspect, being optimistic or pessimistic, decision makers differ in their sensitivity to moderate probabilities as well.

The following specification of the probability *w*eighting function

$$w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma} + (1-p)^{\gamma}} \tag{5.6}$$

(Goldstein & Einhorn 1987, Lattimore, Baker, & Witte 1992) distinguishes between motivation and sensitivity by means of the δ and γ parameters.[87] The δ-parameter mainly measures the motivation for deviating from linear probabilities, which can be pessimistic ($\delta \geq 1$) or optimistic ($\delta \leq 1$). This parameter determines the elevation of the function. The γ-parameter mainly measures how sensitive a decision maker is to moderate probabilities. This parameter thus determines the curvature of the function. If $\gamma > 1$ then choice behavior is sensitive to changes in probabilities leading to an S-shape in the probability weighting function and when $\gamma < 1$ an inverse S-shape results which promotes insensitivity to probabilities.

Tversky and Kahneman (1992) observed underweighting for high probabilities and overweighting for small probabilities, which translates into an inverse S-shape of the weighting function. Parameter values that describe this function are $\gamma = 0.64$, indicating that curvature is inversely S-shaped, and $\delta = 0.83$, suggesting an optimistic attitude towards probabilities. They observed a slightly convex shape for the utility function that is described by a power function with $\beta = 0.88$. The combined effect of these shapes of the utility and probability weighting functions results in the domain of losses in predominantly risk seeking behavior and risk aversion for lotteries that involve small probabilities.

By filtering out the phenomenon of nonlinear probability weighting from a decision maker's risk attitude, prospect theory enables us to describe the unbiased risk attitude (Definition 2.6) of senior management in terms of utility only. We assume that the true preferences of a decision maker are represented by this unbiased utility function (Assumption 2.5). Because this utility function is assumed to reflect the true or unbiased risk attitude of the decision maker, it can then be implemented in equation 5.3 to calculate EU*(*X*), with the star-symbol indicating unbiased expected utility.

5.4 Isocontours in the risk matrix under decision theory

This section specifies isocontours in the risk matrix under expected value theory, unbiased expected utility theory, and prospect theory. Isocontours in the risk matrix are curves that specify, under the three aforementioned decision theories, which lotteries are considered

[86] See Wakker (2010 p.174) for formal definitions of concavity and convexity of probability weighting functions.

[87] Gonzalez and Wu (1999 p.140) indicate that a completely independent separation of curvature and elevation is not possible for the specification in equation 5.6.

equivalent by a decision maker. These isocontours can be used to specify the boundary lines between the cells in the risk matrix (see section 3.7.3 Appendix C for an explanation of this procedure). The isocontours in the risk matrix are derived from the utility function, the probability weighting function, in combination with an isocontour that describes the equivalence condition with, respectively, EV-, EU*- and PT-values.

Under expected value theory the decision maker has a neutral attitude to risk. This implies both linear utility and linear probability weighting. Figure 5.2 contains a diagram that illustrates the relationship between EV-values, linear utility, linear probability weighting, and the risk matrix.

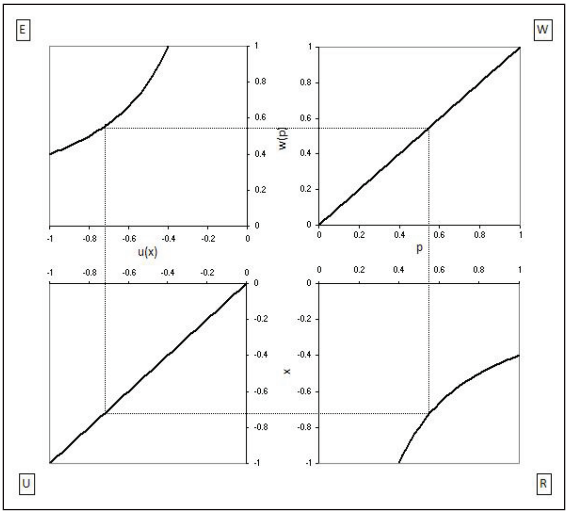

Figure 5.2 Diagram illustrating the relationship between EV-values, linear utility, linear probability weighting, and the risk matrix.

The adjacent axes of the four separate planes in this diagram are interconnected. The relationship between p and w(p) in the probability weighting function (plane W) is linear. The utility function (plane U), with horizontal and vertical axes interchanged, illustrates the linear relationship between x and u(x). Outcome x is in this utility function scaled on the range [−1,0]. The isocontour in plane E describes the equivalence condition $u(x) \times w(p) = c$, with constant $c \in [0,1]$. Under linearity of both u(.) and w(.) this equivalence condition is represented by the expected value $x \times p = c$. In the example in figure 5.2 the value of c is −.4. Under expected value theory, the isocontour in the risk matrix (plane R) is an exact reflection of the isocontour describing the equivalence condition. The dotted lines that connect the curves emphasize the alignment of the adjacent axes of the four planes and illustrate that the shape of the isocontour in the risk matrix (plane R) is derived from the equivalence condition (plane E) by mediation of both the utility function (plane U) and the probability weighting function (plane W).

Economics traditionally assumes a concave utility function and a linear probability weighting function which, under expected utility theory, implies a risk averse attitude to risk.

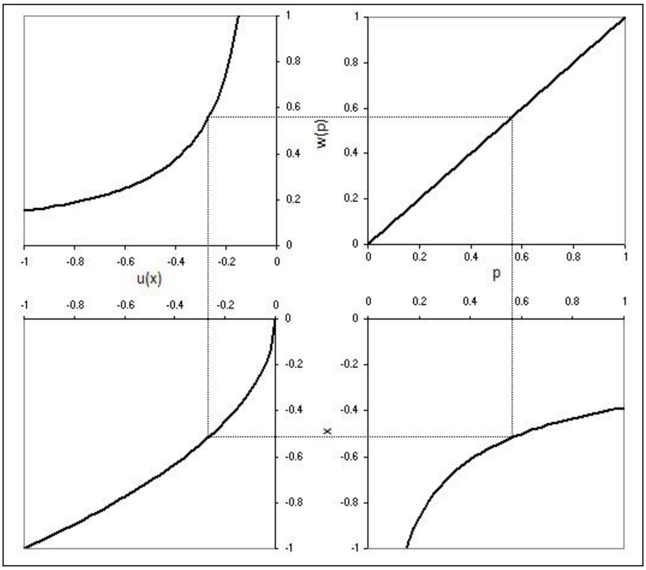

Figure 5.3 Diagram illustrating the relationship between EU*-values, a concave power utility function ($\beta = 2$), linear probability weighting, and the risk matrix.

The utility function in figure 5.3, which only appears convex because its axes are interchanged, contains a concave power function specified by $\beta = 2$. When this utility function has been elicited under prospect theory, then the isocontour that describes the equivalence condition $u(x) \times p = c$ represents the unbiased expected utility, EU*. In the example in figure 5.3 the value of c is $-.16$. At this value the isocontour in the risk matrix in figure 5.2 and 5.3 coincide with the sure "lottery" $X = (1 : -0.4)$.[88] The isocontour that under concave utility is derived in the risk matrix, designates in comparison with linear utility (see figure 5.2) a larger area in the risk matrix and relatively more small probability - large impact lotteries as unacceptable.

Tversky and Kahneman's (1992) seminal paper on cumulative prospect theory provides empirical evidence for convex utility and inverse-S probability weighting in the domain of losses. These specifications are used in figure 5.4. The isocontour that describes the equivalence condition $u(x) \times w(p) = c$ represents the prospect theory value, PT. In figure 5.4 the value of c is $-.45$. The isocontour in the risk matrix, derived

[88] At $p = 1$ there is certainty which implies the absence of risk. Therefore all three decision theories under risk that are discussed in this section should coincide at $p = 1$.

under prospect theory from functional specifications that are descriptive in nature, designates, in comparison with expected value and expected utility theory, comparatively less small probability - large impact lotteries and a smaller area in the risk matrix as unacceptable.

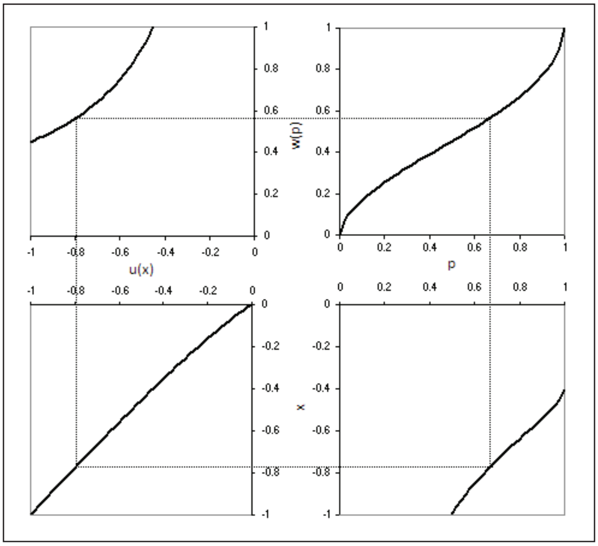

Figure 5.4 Diagram illustrating the relationship between PT-values, a convex utility function ($\beta = .88$), an inverse-S probability weighting curve ($\delta = .83$, $\gamma = .64$), and the risk matrix.

This section illustrates the relationship that in general exists between a decision maker's risk attitude and the expression of organizational risk appetite in the risk matrix. The next section describes the details of a field study in which the risk attitudes of several individual decision makers were elicited under prospect theory and compared with their individual judgments regarding organizational risk appetite.

5.5 Field study details

In a field study we elicited the professional risk attitudes to financial damages of senior managers employed by a large privatized government service body in The Netherlands. We also requested these senior managers to judge what, in their professional opinion, the risk appetite of their organization should be. The relationship between their professional risk attitude and their judgment regarding organizational risk appetite is assessed in the results section of this chapter.

The senior management, consisting of 68 managers and directors, was invited by the corporate controller of the case company to participate in the study. In total 43 members of senior management participated in the study of which 4 female participants. One third of the participants was director.[89]

Both in the invitation letter and in written instructions the participants were informed that their response would be used in the policy development of the case company. They received confirmation in advance of the confidential treatment of their individual results and were informed that their individual results would be sent to them by e-mail afterwards. The case company did not allow the use of tangible incentives, such as financial rewards, presents, or donations to charity. These types of incentives are commonly used in studies that elicit the risk attitude of individual people in relation to their own, private objectives. In this study, with its focus is on company objectives, this type of incentives creates a confound.

Participants received written and spoken instructions and were allowed to practice the decision tasks. On average participants spent 21 minutes on instructions and practice, 21 minutes on the utility and probability weight elicitation task, and 5 minutes on the risk appetite judgment task.[90] The study was conducted in July and August 2009. Monetary amounts that are mentioned are in euro.

5.5.1 Elicitation of professional risk attitudes

In the first part of the study the professional risk attitude of the senior managers was elicited in the domain of financial damages. To describe the risk attitudes of senior management a descriptive rather than a normative theory for decision under risk is required. Professional risk attitudes were therefore elicited under prospect theory. Using the nonparametric utility elicitation method of Abdellaoui, Bleichrodt, and Paraschiv (2007) we elicited each participant's utility curve for financial damages first. These individual utility curves were then used to elicit individual probability weighting functions by means of a nonparametric elicitation method using certainty-equivalents (see 4.6.1 Appendix A) in combination with interpolation. These curves can be elicited using parametric methods, that require prior specification of the functional form of the curves, and nonparametric methods, where such specification is not required and these curves can be directly observed. The method of Abdellaoui et al. was chosen because it is nonparametric and accommodates a large range of outcomes, from zero to catastrophic loss, that needs to be specified when a risk matrix is used. The nonparametric approach was selected because it

[89] Count of directors includes regional directors, program directors, and assistant-directors.

[90] In addition to what is reported in this study, participants expressed their professional risk attitude and provided their introspective judgment as well for lotteries involving verbal outcome expressions with respect to three of the case company's objectives: limiting financial damage, maintaining key performance, and avoiding reputational damage. These data are retained for future analysis and the results hereof are not reported in this dissertation.

was *ex ante* unclear whether functional forms that usually hold for risk attitudes of student subjects in laboratory experiments, also hold for the senior managers in our field study. Under nonparametric elicitation complementary parametric estimation *ex post* always remains possible.

To assess utility and probability weights indifference values were assessed using the bisection method (see 4.6.2 Appendix B), which consisted of five iterative choices between lotteries, each composed of negative outcomes. In each session, consisting of five subsequent choices, the iteration sequence was completed by a request to the participant to confirm her indifference between the lottery pair that resulted from the iterations. In each choice the participants were requested to choose the option that the case company should make according to their professional opinion. This wording was chosen to ensure that the choices made by the senior managers reflected their risk preferences as professionals. Each lottery was presented to the participants in a pie-chart format. Figure 5.5 presents a print screen of the choice menu that was used in the first iteration of the first choice session.

In figure 5.5 choice A (left) consists of a .67 probability of a financial damage of 600K and a .33 probability of a financial damage of 1M. In choice B (right) there is a .67 probability of a financial damage of 100K and a .33 probability of a financial damage of 3M. This choice menu was used in 27 choice sessions.

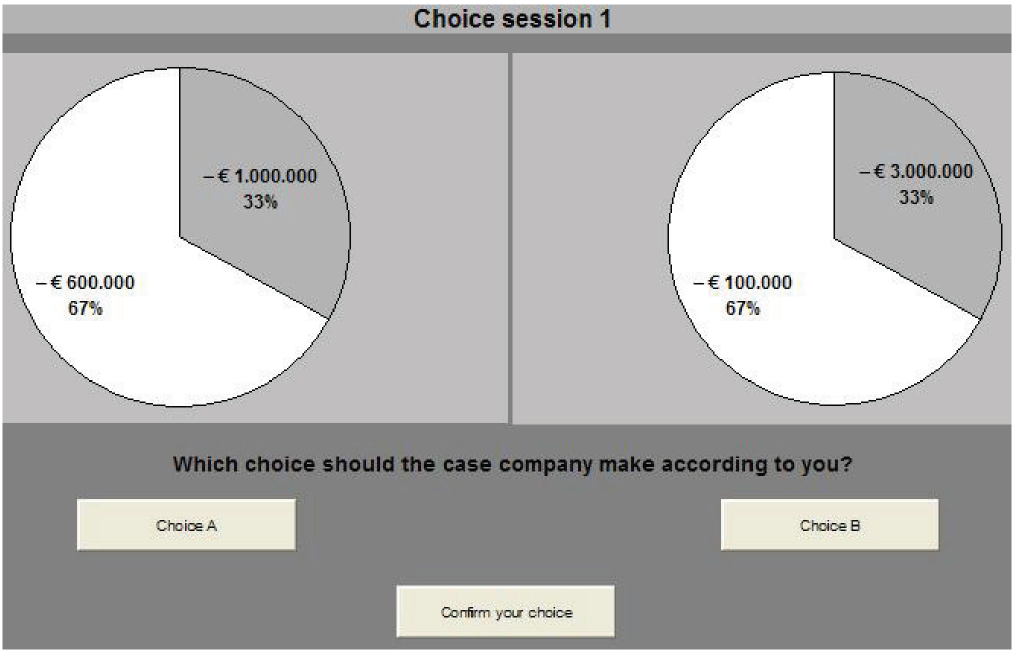

Figure 5.5 Print screen of the choice menu used in the elicitation of professional risk attitudes.

Sessions 1 & 2 were used to elicit two intervals of financial damage that are equally spaced in terms of utility using the trade-off method (Wakker & Deneffe 1996). This sequence was then used in sessions 3-to-7 to elicit the probability of occurrence of financial damage p_ℓ for which holds $w(p_\ell) = .5$ using certainty equivalents.[91] In sessions 8-to-14 the resulting probability p_ℓ was used to elicit a sequence of certainty equivalents x_i from which the utilities $u(x_i) = i$, with $i \in \{0.5, 0.75, 0.25, 0.875, 0.125, 0.938, 0.062\}$, were derived. In sessions 15-to-23 certainty equivalents were used to elicit the probability weight of the probabilities $p \in \{.01, .1, .25, .33, .5, .67, .75, .9, .99\}$. Finally in sessions 24-to-27 indifference values were elicited as a consistency check.[92]

5.5.2 Elicitation of introspective judgments of organizational risk appetite

In the second part of the study participants were requested to provide their professional, introspective judgment about the desired risk appetite of their organization regarding financial damages. Participants used a 5 point Likert-scale, a survey response scale to which most businessmen are accustomed, to indicate the degree to which, in their professional opinion, a lottery with negative outcomes should be accepted by the case company or not. The pie-chart format was maintained in this part of the study, to reduce the cognitive burden on the participants who had already familiarized themselves with this format in the first part of the study. This in addition forestalls that any difference between professional risk attitude and judgments on organizational risk appetite can be attributed to a difference in presentation format. The pie-chart format, furthermore, creates a contrast effect between financial damage and the complementary zero outcome in a lottery, which is lost when lotteries are shown in a risk matrix format.

Figure 5.6 shows a print screen of the choice menu that was used. It displays only one possible event with five choice buttons underneath. Participants were instructed to tick the most left choice button when in their professional opinion the event shown should be accepted by the case company and to tick the most right choice button when the event should not be accepted. The other three choice buttons indicate a degree of acceptability that lies between these two extremes.

[91] The probability equivalent method was not used because this method induces a response mode bias (Hershey & Schoemaker 1985, Hershey, Kunreuther, & Schoemaker 1988). During the five choice sessions, probability stayed the same in each of the five iterative choices belonging to a session. In response to the choice of the participant in the first iteration of a session, the probability that was used in the next session was adjusted. It was therefore not obvious to the participants that this change in probability resulted from a choice made by them at the start of the previous session.

[92] The consistency checks consisted of repetitions of session 1, 3, 8 and 18.

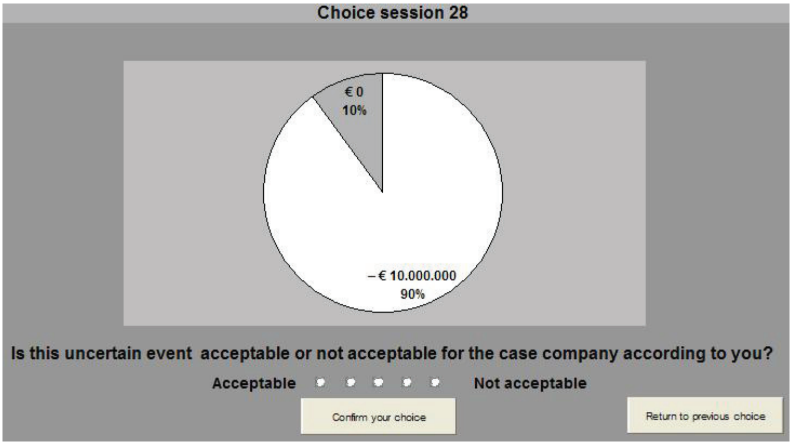

Figure 5.6 Print screen of the choice menu used in the elicitation of introspective judgments on organizational risk appetite.

The example in figure 5.6 shows the first choice presented to the participants, a possible event with a .9 probability of a financial damage of 10M and a .1 probability of no financial damage. The choice menu in figure 5.6 was used in 41 choice sessions, consisting of 30 regular choices and 11 consistency checks. For each probability in the sequence .9, .7, .5, .3, .1, and .01 the participants provided their introspective judgments for lotteries with subsequent impact levels of 10M, 3M, 1M, 300K, and 100K. When the lowest impact level in this sequence was reached, a new sequence of impact levels commenced for a lower probability level. This approach was adopted to prevent a response mode bias (Hershey & Schoemaker 1985, Hershey, Kunreuther, & Schoemaker 1988). Consistency checks consisted of six lotteries with a financial damage of 1M in combination with the aforementioned sequence of probabilities and five lotteries with a probability of .9 and with ascending impact levels.

These lotteries were almost completely derived from the case company's risk matrix shown in figure 5.1. To be able to associate the introspective judgments on organizational risk appetite in the second part of the study with the attitude towards probability in the first part of the study probability percentages were introduced. These percentages replaced the confounding mix of verbal expressions for either probability or frequency that the case company employed.[93] To reduce the cognitive burden on the participants a linear and not an

[93] The risk matrix of the case company presented in figure 5.1 employs a mix of verbal probability and frequency expressions. It was anticipated that these expressions could not be rank ordered unequivocally by the participants in the study for which reason probabilities were employed in the case study instead. The existence of this confound was confirmed by a matching task at the end of our study were participants were requested to match percentage probabilities to the verbal expressions. The median response for each expression is shown in brackets: Likely (.01), Unlikely (.125), Incidental (.1), Regularly (.5), and Very likely (.8). Notice that the relationship between the set of verbal expressions and the probabilities is not strictly increasing. In response to this finding, the case company chose a different set of verbal expressions that could be rank ordered unequivocally.

exponential probability scale was used. The scale for financial damages in the risk matrix was augmented with the amounts 3M and 300K to add precision. The amount 10K was omitted as earlier pilot tests with company representatives clearly demonstrated its irrelevance in decision making.

5.6 Results

This section presents the professional risk attitude of the senior managers, their utility functions for financial damages, and their probability weighting functions. It reports their introspective judgment regarding organizational risk appetite and reveals which decision theory makes the best prediction for this judgment of organizational risk appetite.

5.6.1 Professional risk attitudes

In the first part of the study 18 choice sessions contain a direct test of risk attitude. In these sessions participants are offered a choice between a risky lottery *X* and a sure amount equal to the expected value of *X*. For 16 out of these 18 choice problems the median response of the participants is to choose the risky lottery, which suggests a preference for risk seeking. The percentage share of risk seeking choices in all choices of the participants is 68%. A majority of participants being risk seeking in the domain of losses is a common empirical finding in decision theory.

5.6.2 Utility functions for financial damages

Contrary to most empirical studies that find convex utility for losses our nonparametric median[94] data find the utility of financial damage to be slightly concave indicating diminishing marginal returns (see figure 5.7a). This implies that the median participant in our study gains relatively more utility from a reduction of a large financial damage than from a reduction of identical size involving a small financial damage. Thus, even though our direct tests of risk attitude reveals risk seeking behavior, closer analysis reveals that participants experience a concave utility for losses which under unbiased expected utility theory suggests risk averse behavior.

[94] The medians are derived by selecting the value in the middle of the ordered set of the indifference values of all participants for each of the utilities $u(x_i) = i$ with $i \in \{0.062, 0.125, 0.25, 0.5, 0.75, 0.875, 0.938\}$.

(a) (b)

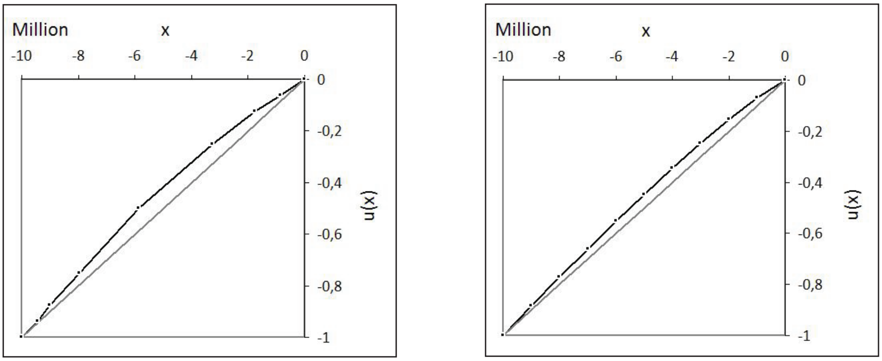

Figure 5.7 Utility function for financial damages: (a) median nonparametric results, and (b) group parametric results.

Additionally we present a parametric estimate of the utility function for financial damages for the responses of all participants together (referred to as "group parametric results"). For this we assume a model with utility and probability weighting functions respectively specified by equations 5.4 and 5.6 and use the distance measure presented in Wakker (2010) and nonlinear regression to find a parametric fit.[95] In line with the nonparametric analysis these group parametric results find utility to be slightly concave (see figure 5.7b) with a β-parameter of 1.16 ($R^2 = 0.85$). Fitting the power function in equation 5.4 to the medians in figure 5.7a we find $\beta = 1.24$. When we compare these findings with the $\beta = 0.88$ result of Tversky and Kahneman (1992) we notice that both have a slight deviation from linearity in common but differ completely in the direction that this deviation takes.

5.6.3 Probability weighting functions

The nonparametric results indicate that the probability weighting function for the medians[96] is convex (see figure 5.8a). This convexity implies that the median partici-

[95] Using the functional specifications from equations (5.4) and (5.6) we calculate certainty equivalents (CE) for each indifference in part 1. We take for all participants the sum of the squared distances between the CE of the left and right lotteries in the indifference and minimize this sum using nonlinear regression (Wakker 2010 p.361, Appendix A.2: A distance measure for parametric fitting).

[96] The median weights for the probability (p) values .1, .25, .33, .5, .67, .75, and .9 are determined using the curvature for utility and interpolation. We use a lottery where a maximum amount x_1 can be lost with probability p and where there is a $(1 - p)$ probability of a zero loss x_0 and then use the indifference $(x_1, p; x_0) \sim x_p$ to elicit the certainty equivalent x_p. Under prospect theory the indifference $(x_1, p; x_0) \sim x_p$ implies $\mathrm{w}(p) \times \mathrm{u}(x_1) + (1 - \mathrm{w}(p)) \times \mathrm{u}(x_0) = \mathrm{u}(x_p)$. Setting $\mathrm{u}(x_1) = -1$ and $\mathrm{u}(x_0) = 0$ then results $\mathrm{w}(p) = -\mathrm{u}(x_p)$. $\mathrm{u}(x_p)$ can be derived from the utility function by using interpolation. We locate the losses above (x_a) and below (x_b) the value x_p for which the utilities are known and calculate $\mathrm{u}(x_p)$ by $\mathrm{u}(x_p) = (\mathrm{u}(x_b) + (x_p - x_b) / (x_a - x_b) \times (\mathrm{u}(x_a) - \mathrm{u}(x_b))$.

pant adopts an optimistic attitude towards the probability of occurrence of financial damage. The median participant thus assigns less weight to probabilities than their numerical values justify. Optimism induces risk taking and this underweighting of probabilities explains the risk seeking attitude of most participants in this study. The group parametric results find the probability weighting curve to be convex as well (see figure 5.8b).

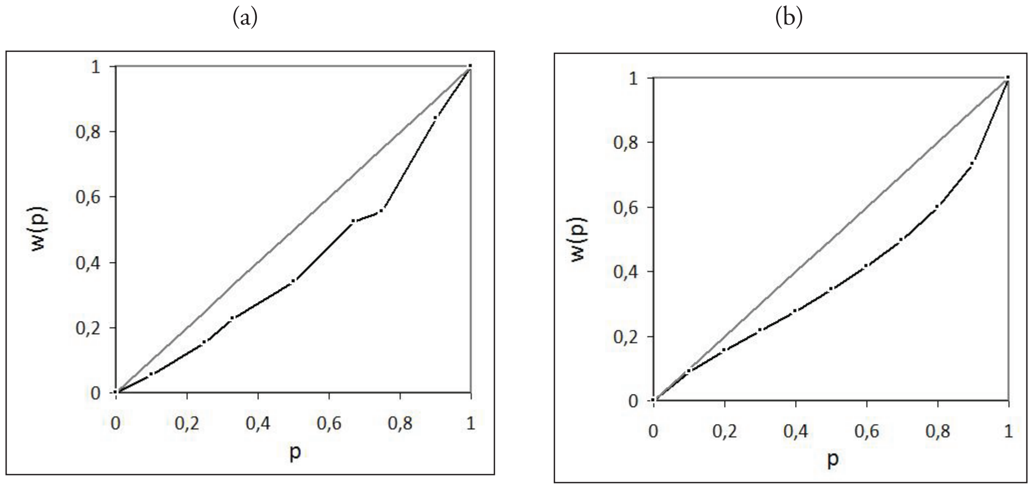

Figure 5.8 Probability weighting function: (a) median nonparametric results and (b) group parametric results.

Contrary to the findings of Tversky and Kahneman (1992) figures 5.8a and 5.8b do not display a typical inverse-S shape. Compared to a γ-parameter of 0.64 in their study, we find less insensitivity in our parametric results ($\gamma = 0.76$) and almost no insensitivity in our median nonparametric data ($\gamma = 0.96$). Nonparametric and parametric results indicate that participants in our study are more optimistic (respectively $\delta = 0.52$ and $\delta = 0.53$) than in the Tversky and Kahneman study ($\delta = 0.83$).

5.6.4 Introspective judgment regarding organizational risk appetite

The introspective judgment for the degree of acceptability of lotteries is for the median participant presented in the risk matrix in figure 5.9.

Figure 5.9 provides evidence for risk aversion as well as risk neutrality. The risk matrix identifies with a variety of shades three lottery pairs with identical expected values. On the basis of the judgment provided by the median participant the lottery with the smaller spread is in each of these pairs preferred to the lottery with the larger spread. This preference

	1%	10%	30%	50%	70%	90%
-100,000	1	1	1	1	1	1
-300,000	1	1	2	2	2	3
-1,000,000	1	2	3	4	4	4
-3,000,000	1	4	4	5	5	5
-10,000,000	3	5	5	5	5	5

Figure 5.9 Organizational risk appetite: median response for each cell in the risk matrix.[97]

implies risk aversion. From the introspective judgments in figure 5.9 we derive the preference relationships

$$\begin{aligned}(.10 : -1\text{M}, .90 : 0) &\succcurlyeq (.01 : -10\text{M}, .99 : 0)\\ (.30 : -1\text{M}, .70 : 0) &\succcurlyeq (.10 : -3\text{M}, .90 : 0)\\ (.90 : -100\text{K}, .10 : 0) &\succcurlyeq (.30 : -300\text{K}, .70 : 0)\end{aligned} \tag{5.7}$$

For five other lotteries pairs with identical expected values the median judgment of acceptability is identical for both lotteries.[98] This equality in preference implies risk neutrality.

5.6.5 Decision theoretic predictions for organizational risk appetite

In this section it is our aim to identify which decision theory makes the best prediction for the introspective judgment of organizational risk appetite provided by the participants. We do this by comparing the number of errors of expected value theory (EV), unbiased expected utility theory (EU*), and prospect theory (PT) in predicting which lotteries a participant

[97] Figure 5.9 differs from figure 5.1 in several ways. Where figure 5.1 shows the current risk matrix of the case company, figure 5.9 shows what the risk matrix should look like when we take into account the median response of the participants in our study. Where the first employs verbal probability/frequency expressions and financial impact classes, the second uses probabilities and single point monetary outcomes.

[98] These five lottery pairs are: $(.10 : -100\text{K}, .90 : 0) \sim (.01 : -1\text{M}, .99 : 0)$, $(.30 : -100\text{K}, .70 : 0) \sim (.10 : -300\text{K}, .90 : 0)$, $(.30 : -100\text{K}, .70 : 0) \sim (.01 : -3\text{M}, .90 : 0)$, $(.10 : -300\text{K}, .90 : 0) \sim (.01 : -3\text{M}, .90 : 0)$, and $(.30 : -3\text{M}, .70 : 0) \sim (.90 : -1\text{M}, .10 : 0)$.

considers unacceptable in the second part of the study. When a participant chose either one of the two points on the right of the 5 point Likert-scale (4th and 5th position), we infer from this that the participant considers the lottery unacceptable. When any of the three remaining points on the scale is selected (1st, 2nd, and 3rd position), we infer from this that the participant considers the lottery acceptable.

Each of the 30 cells in the risk matrix represents a lottery in the domain of financial damages. For each of these lotteries EV-, EU*- and PT-values are calculated. For unbiased expected utility theory and prospect theory these calculations are derived from the individual utility and probability weighting functions of the participants that were elicited in the first part of the study. We then use the EV-, EU*- and PT-values to rank order the cells in the risk matrix, with cells with higher values being preferred to cells with lower values. For each of the three decision theories this rank order predicts the shape of the unacceptable zone in the risk matrix. The size of this predicted unacceptable area is inferred by the count of cells for which participants chose the 4th and 5th position on the Likert-scale in the second part of the study. We then compare, for each of the three decision theories, two matrices: one with the lotteries that participants actually judged as unacceptable and the other with those that the decision theory predicts as unacceptable. The number of differences between the two matrices count as prediction errors. On the basis of this procedure we derive prediction errors for the median response of participants, mean prediction errors, and differences between the prediction errors of three decision theories per individual participant. The decision theory with the lowest count of prediction errors is considered to be the best predictor of the participants' introspective judgment on organizational risk appetite.

All three theories rather accurately predict the median judgment of acceptability of cells in the risk matrix. Both expected value theory and expected utility theory predict the median response without errors and prospect theory with only one error. Figure 5.10 presents the expected value theory prediction (a) and the actual median response (b).

(a)

	1%	10%	30%	50%	70%	90%
-100.000	0,000	-0,001	-0,003	-0,005	-0,007	-0,009
-300.000	0,000	-0,003	-0,009	-0,015	-0,021	-0,027
-1.000.000	-0,001	-0,010	-0,030	-0,050	-0,070	-0,090
-3.000.000	-0,003	-0,030	-0,090	-0,150	-0,210	-0,270
-10.000.000	-0,010	-0,100	-0,300	-0,500	-0,700	-0,900

(b)

	1%	10%	30%	50%	70%	90%
-100,000	1	1	1	1	1	1
-300,000	1	1	2	2	2	3
-1,000,000	1	2	3	4	4	4
-3,000,000	1	4	4	5	5	5
-10,000,000	3	5	5	5	5	5

Figure 5.10 Prediction for organizational risk appetite: (a) expected value prediction of the unacceptable area and (b) the actual median response for each cell.

For each cell in the risk matrix in figure 5.10a the expected value is calculated by multiplication of the probabilities on the horizontal axis with the amounts of financial damage on the vertical axis divided by 10M. In figure 5.10b the dark shaded, unacceptable area consists of thirteen cells with values that correspond with the 4th and 5th position on the Likert-scale. We use this amount of cells to determine the size of the unacceptable area in figure 5.10a by identifying the thirteen cells with the lowest expected value and assign dark shades to these cells as well. We then compare the dark shaded areas in the two figures to assess any difference. Whenever we identify a light shaded cell in the predicted risk matrix that is dark shaded in the risk matrix containing the actual responses, we count this as one prediction error. Comparison of figures 5.10a and 5.10b indicates that for expected value theory there is no prediction error for the median response.

Results for the median response provide a weak support for the predictive superiority of expected value and expected utility theory over prospect theory. The mean amount of prediction errors is 1.42 for expected value theory and for expected utility theory and prospect theory, both 1.74. This provides additional support for expected value theory. Nonparametric pair wise statistical tests in table 5.1 and 5.2 indicate that the number of prediction errors of expected value theory is significantly lower than those of prospect theory, that prediction errors of unbiased expected utility theory and prospect theory do not significantly differ from each other, and that the statistical comparison between expected value theory and unbiased expected utility theory remains inconclusive. Based on these data we cannot reject the hypothesis that expected value theory has the lowest number of prediction errors compared to unbiased expected utility theory and prospect theory.

Table 5.1 Wilcoxon signed ranks test.

	EV - PT	**EV - EU***	**PT - EU***
Z	−2.562[a]	−1.686[a]	−.267[a]
Asymp. Sig. (2-tailed)	.010	.092	.789

a. Based on positive ranks.
b. Based on negative ranks.

Table 5.2 Sign test.

	EV - PT	**EV - EU***	**PT - EU***
Exact Sig. (2-tailed)	.031[a]	.031[a]	1.000[a]

a. Binomial distribution used.

5.7 Discussion

The results indicate that the organizational risk appetite, as expressed by the senior management of the case company, is best described by a neutral attitude to risk. This finding corresponds with the expected value approach commonly adopted by risk practitioners in the field of system safety. The median introspective judgments for organizational risk appetite elicited in the second part of the study contain both risk neutral and, to a lesser degree, risk averse preferences. These do not correspond with the risk seeking attitude of the senior managers and their optimistic attitude to probabilities that were elicited in the first part of the study under prospect theory. The median result for the attitude of the senior managers towards outcomes is described by a concave utility function which under unbiased expected utility theory represents a risk averse attitude. This averse attitude to risk, which under unbiased expected utility theory represents the true risk preference of senior management, can be used to determine the organizational risk appetite of the case company in the risk matrix. It is concluded that the professional unbiased risk attitude of the senior managers in our study did not drive organizational risk appetite.

The high speed at which participants provided their judgment on organizational risk appetite suggests that they employed an intuitive rather than a rational approach to decision making. The fact that prospect theory, a descriptive theory of choice under risk, did not have a bearing on their judgment regarding organizational risk appetite is therefore surprising. This result resembles that of Cokely and Kelley (2009) who found that expected value choices rarely involved the actual calculation of expected values but instead resulted from trade-off like decision processes. When we assume that participants did make expected value calculations in the judgment task, then it is conceivable that an approximate amount for investments in control measures is used as a cut-off point for deciding on the acceptability of cells in the risk matrix. Such a heuristic explains the prominence of expected value in the second part of the study and explains why the behavioral traits of the participants in the first part of the study did not show up in the judgment task. An additional explanation for absence of behavioral biases in the judgment task is that the professional experience of the senior managers with organizational risk appetite in practice may have prevented them from making the same "mistakes" as students in a laboratory setting.

The approach demonstrated in this study allows an organization to make explicit what the risk attitude of its senior management is and allows it to integrate its unbiased risk attitude into organizational risk appetite. In line with the unbiased risk averse attitude of its senior management the response of the case company was to become more risk averse in its expression of organizational risk appetite in the risk matrix in order to exert more control over the acceptance of risk within its organization.

6 The Utility and Weight of Verbal Outcome and Probability Expressions: A Field Experiment

Summary

This chapter employs prospect theory to reveal the utility and probability weights that a unique population of consultants, experienced in the management of project risks, assigns to verbal probability and outcome expressions. In this study utility and probability weighting functions of consultants were elicited under prospect theory using the elicitation method of Abdellaoui, Bleichrodt, and L'Haridon (2008). The results from prior studies that find surprisingly similar response patterns in the verbal and numerical probability mode, are corroborated in this study for moderate probabilities using probability weights. In line with findings from previous studies, the utility of losses in the verbal mode is found to be much more negative than in the numerical mode, which suggests a strong aversion to vague outcomes. From these results the inference drawn is that the source of uncertainty dominates the mode of uncertainty representation whenever this source does not allow an ambiguous interpretation of the response mode.

6.1 Introduction

In organizations decisions are often made on the basis of verbal expressions of probability and outcome instead of their numerical counterparts. However, an overwhelming amount of evidence indicates that people differ considerably in their interpretation of qualitative terms like 'low probability' or 'very high impact'. Given the popular use of verbal quantifiers in daily life it is not unthinkable that this great intersubject variability in the interpretation of verbal quantifiers is causing an 'illusion of communication' in business life (Wallsten and Budescu 1995 p.53). This has led some authors in risk management (Chapman & Ward 2003 p.170) and decision analysis (Keeney 1992 p.117) to advise against the use of these vague expressions in applications altogether, favoring numerical measurements instead. Arguing from the maxim 'quantification promotes understanding' (Loosemore, Raftery, Reilly, & Higgon 2006 p.27) they are engaged in a systematic attempt to measure risk in order to build a rational risk management system.

There exists an opposing faction in risk management which emphasizes the qualitative side of risk management and whose view is summarized by the adage 'good information need not be numerical'. They state that conducting a qualitative, verbal analysis first is imperative while the need for a complementary quantitative, numerical assessment of risk is only to be contemplated for particularly important risks and is conditional on practical matters such as data availability (Loosemore et al. 2006 pp.119–120).

The large amount of intersubject variability, inherent to qualitative risk analysis, can be reduced by transforming the qualitative method to a semi-quantitative risk analysis (Loosemore et al. 2006 p.124, Cooper, Grey, Raymond, & Walker 2005 p.67) whereby the descriptive, verbal classes of probability and outcome are associated with their numerical counterparts using scaling techniques (Theil 2002). The need for a scaling of its verbal probability and outcome expressions was exactly the motivation for the Dutch subsidiary of a large international consultancy company to cooperate with the researchers in this study.

In this framed field experiment (Harrison & List 2004) introspective numerical judgments were elicited from consultants for a set of verbal probability and outcome expressions employed by the case company. Additionally the meaning that these professionals assign to these verbal probability and outcome expressions was inferred under prospect theory from their choices. Both the judged and inferred probabilities in our study appear to be prone to a base rate bias which results in a word-to-percentage scaling that seems overly optimistic. Inferred outcomes are furthermore significantly more negative than judged outcomes which suggests vagueness aversion for negative outcomes. Both judged and inferred outcomes vary exponentially with the rank order of the verbal outcome expressions and are significantly more negative for large projects compared to small projects.

This paper is a response to the call in Du & Budescu (2005 p.1801) to study the factors which affect the perception of vagueness in both the probability and outcome dimension. The contribution of this study is that it bridges the findings of choice-studies involving verbal expressions of probability and outcomes with choice-studies that aim to elicit probability weights and utility values. It furthermore demonstrates how to design a semi-quantitative risk analysis method by using probabilities and outcomes that under prospect theory can be inferred from choices involving verbal quantifiers.

The chapter proceeds as follows. The second section discusses the introspective and inferred scaling methods and introduces prospect theory as well as some necessary notations. Section 6.3 discusses vagueness studies and develops hypotheses for both the weight of verbal probability expressions and the utility of verbal outcome expressions. Section 6.4 describes the details of the field experiment including the details of the elicitation method that was employed. Section 6.5 contains an analysis of the results of this field experiment and in section 6.6 these results are discussed.

6.2 Scaling methods

Scaling techniques for verbal probability and outcome expressions can be introspective, using the respondents' direct word-to-number translations, or they infer these translations indirectly from the decision makers' choices between options in which either the probability or the outcome dimension is defined in the verbal mode. While previous studies assumed the subjective expected utility (SEU, Edwards 1962) model and focused exclusively on inferred probabilities, our study employs the descriptively more accurate prospect theory (Tversky & Kahneman 1992, Kahneman & Tversky 1979) to not only infer probabilities but also calculate the inferred outcomes from choices involving verbal expressions.

Prospect theory assumes that people, when offered a choice involving at least one risky option (prospect), assign a particular weight to its probabilities and derive utility from its outcomes. To designate a two-outcome prospect in the domain of monetary losses, with one outcome zero, the notation $x_{i}p_{i}0$ is used, with $x_{i} < 0$, $i = 1,\ldots,n$, representing the loss outcome with probability p_{i}, with $0 < p_{i} \leq 1$, and a zero outcome for which its probability of occurrence $(1 - p_{i})$ is understood.[99] The symbol ~ is used to designate that the decision maker is indifferent between two prospects. The attitude of a decision maker towards losses is modeled by a monotonically increasing utility function $u(x_{i})$, normalized with $u(0) = 0$ and $u(x_{n}) = -1$. The decision maker's attitude towards probabilities is modeled by the monotonic probability weighting function $w(p_{i})$, with $w(0) = 0$ and $w(1) = 1$. Under prospect theory the decision maker's preferences for prospects of the type $x_{i}p_{i}0$ are represented by the multiplication of the weight of the probability, $w(p_{i})$, and the utility of the outcome, $u(x_{i})$, *i.e.* $w(p_{i}) \times u(x_{i})$. For the utility of verbal outcomes expressions we use the notation $u_{v}(.)$, with subscript 'v' designating "verbal", in order to clarify that the inverse utility of the utility of a verbal outcome expression is not the identity of this expression, *i.e.* $u^{-1}(u_{v}(r_{i})) \neq r_{i}$. For the weight of verbal probability expressions we use $w_{v}(.)$ to clarify that $w^{-1}(w_{v}(q_{i})) \neq q_{i}$.

Incorporation of verbal outcome expressions r_{i} and verbal probability expressions q_{i} into prospects allows us to determine the utility and probability weights associated with these verbal quantifiers, *i.e.* $u_{v}(r_{i})$ and $w_{v}(q_{i})$. The notation $r_{i}p_{i}r_{0}$ then represents a prospect for which one outcome is verbally expressed by r_{i} and the alternative outcome r_{0} represents a non-perceptible loss. A prospect for which only the probability is verbally expressed is described by $x_{i}q_{i}0$. From the values $u_{v}(r_{i})$ and $w_{v}(q_{i})$ we can, using the inverse utility and probability weighting functions $u^{-1}(.)$ and $w^{-1}(.)$, infer a precise, numerical outcome $u^{-1}(u_{v}(r_{i}))$ and probability $w^{-1}(w_{v}(q_{i}))$. This inferred outcome (probability) is a precise outcome (probability) that leads to the same choice as a vague outcome (probability) expression. Inferring subjective probabilities from preferences is standard practice in decision theory (see Tversky & Kahneman 1974 p.1130 for a reality check on this convention). Inferred probabilities have been calculated for verbal probability expressions under subjective expected utility (SEU) assuming linear utility (Budescu, Weinberg, & Wallsten 1988, Wallsten 1971, *adjusted bids*) and for probability intervals under prospect theory (Baillon, Cabantous, & Wakker 2011, *matching probabilities*). Inferred probabilities and outcomes are choice-based and are thus aligned with the revealed preference paradigm.

For word-to-number scalings which are based on introspective judgments we employ the notation $s(r_{i})$ for a precise, judged outcome that matches a verbal outcome expression

[99] Alternatively prospects are in decision theory referred to as lotteries or gambles.

and $s(q_i)$ for a judged probability associated with a verbal probability expression. Using the utility and probability weighting functions we can on the basis of these values calculate the utility associated with judged outcomes, $u(s(r_i))$ and the probability weight that is associated with the judged probability, $w(s(q_i))$.

6.3 Vagueness studies

This section provides an overview of studies on verbal probability and outcome expressions. The main concepts in this overview are defined in the first section. Hypotheses on the weight of verbal probability expressions and the utility of verbal outcome expressions, formulated on the basis of this literature review, are presented at the end of the second and third section.

6.3.1 Vagueness, source of uncertainty, and mode of representation

Vagueness is defined as an inexactness where there are no precise boundaries to the meaning of an expression (Zwick & Wallsten 1989 p.70, Black 1937 p.430). The Ellsberg paradox (1961), a demonstration of the human preference for precision over vagueness, inspired an abundance of studies on *vagueness aversion*.[100] Vagueness aversion is a typical finding in the gain domain. For losses the majority of studies finds vagueness seeking behavior, albeit that in this domain there is much mixed and opposing evidence (Wakker 2010 p.354, Camerer and Weber 1992 table 3 *stylized fact 9*). Empirical studies have demonstrated that risk attitude and vagueness attitude are independent constructs (*stylized fact 10*) and have found vagueness seeking for low probabilities in the domain of gains and for high probabilities in the domain of losses (*stylized fact 7*). Fox and Tversky (1995) found that vagueness aversion was prevalent when decision makers were offered choices which facilitate a direct comparison between vagueness and imprecision, whereas in the absence of such a clear comparison this preference for precision did not hold. In the first, comparative choice condition vagueness and precision become salient characteristics of the decision context whereas in the second condition decision makers, acting in ignorance of this comparative condition, assign less value to these characteristics (*comparative ignorance hypothesis*).

The need to explain noticeable exceptions to vagueness aversion, such as vagueness seeking in areas in which one is competent or knowledgeable (Tversky & Fox 1995 p.280

[100] In decision theory, *vagueness aversion* is usually referred to as *ambiguity aversion*. Arguments in favor of using the term 'vagueness' instead of 'ambiguity' can be found in Budescu, Weinberg and Wallsten (1988 footnote 1 p.282) and an explanation of the difference between these two terms in Zwick and Wallsten (1989 p.70). In this chapter the term ambiguity is reserved for describing a kind of inexactness where there is a finite number of alternative, precise meanings to the same phrase. When referring to articles that use the term *ambiguity*, we will nevertheless use the term *vagueness*.

competence hypothesis), led to the generalization of vagueness aversion to *source preference*. In studies on source preference a source of uncertainty is defined as a group of events that is generated by the same mechanism of uncertainty (Tversky & Wakker 1995, Abdellaoui, Baillon, Placido, & Wakker 2011). A source can be a group of events with known objective probabilities (*risk*), unknown probabilities (*uncertainty*), probability intervals (*imprecision*), or conflicting probability estimates. Examples of sources of uncertainty are the value of a stock index, the temperature in a particular city, and the number of colored marbles in an opaque urn. Source preference is logically independent of *source sensitivity*, which measures the degree of nonadditivity of decision weights of a single source (Fox & Tversky 1995 p.601), and which for decision making under risk is expressed in the nonlinearity of probability weighting functions.

In this study our focus is on events generated by a single mechanism of uncertainty. This single source of uncertainty is, however, expressed by different representation modes of uncertainty. Examples of uncertainty representations are numeric probability intervals, verbally qualified point estimates, second order probability distributions, unreliable probabilities, imprecise probability estimates, and linguistic probabilities (see also the review by Budescu & Wallsten 1987). In terms of the three-way taxonomy offered by Budescu and Wallsten (1995 pp.277–279) our study holds the stimuli 'nature of the event' and 'the nature of the underlying uncertainty' constant while it varies the stimulus 'representation of uncertainty'. The next section discusses empirical findings on vagueness in linguistic probabilities, also referred to as verbal probability expressions.

6.3.2 Verbal probability expressions

In studies that compare the verbal and numerical mode of expressing probability three main themes can be distinguished: (1) mode preference, (2) mode translation, and (3) mode effectiveness. The findings of these three lines of research are discussed in this section.

6.3.2.1 Mode preference studies

Various studies demonstrate that people prefer conveying information about uncertainty in the verbal mode while preferring to receive this information in the numerical mode (Erev & Cohen 1990 *communication mode preference paradox*, Wallsten, Budescu, Zwick, & Kemp 1993). This communication mode preference reversal holds in particular when the decision making context itself lacks precise boundaries (Budescu & Wallsten 1995 principle 3, Olson & Budescu 1997). One explanation for the preference to provide likelihood information in the verbal mode is that the vagueness surrounding the verbal assessments makes these messages more justifiable to the conveyor than would be the case if the precise numerical mode would have been used (Piercey 2009). Another explanation is that verbal phrases more clearly suggest the inferences that should be drawn from the estimates (Teigen & Brun 1999) and are more consistent with human reasoning which is similar to putting forward linguistic arguments as opposed to computing numerical parameters (Moxey & Sanford

2000). Whether the prediction of the probability of occurrence of an event is better served by the deliberate and rule-based reasoning that is induced in the numeric mode or the associative and intuitive thinking that is invoked by the verbal mode (see Windschitl & Wells 1996), is an open empirical question. An explanation for the preference to receive probabilistic information in the numerical format is that numbers suggest an accuracy which makes these estimates appear superior to vague, verbal estimates (Erev & Cohen 1990).

6.3.2.2 Mode translation studies

Empirical studies on the meaning of verbal probability expressions unequivocally find a great intersubject variability in the numerical values assigned to probability terms (Theil 2002, Sanford, Moxey, & Paterson 1994, Wallsten, Budescu, & Zwick 1993, Mosteller & Youtz 1990 and comments by Wallsten and Budescu, Wallsten & Budescu 1985, Beyth-Marom 1982). This intersubject variability is contrasted by the within-subject variability which is not minor, but considerably less (Wallsten, Budescu, Rapoport, Zwick, & Forsyth 1986 p.349). The numerical outputs that can be used to match with the probability phrases can be single point estimates (Mosteller & Youtz 1990), probability ranges (Tversky & Koehler 1994 p.563 *staircase method*), fuzzy membership functions (Verkuilen 2005, Wallsten, & Budescu 1995 p.46), and probability signatures (Wallsten & Jang 2008). The interpretation of verbal probability expressions in terms of these outputs is structurally influenced by context (Pepper & Prytulak 1974, Brun & Teigen 1988, Visschers, Meertens, Passchier, & de Vries 2009 p.275). Contextual factors that have been found to influence the interpretation of verbal probability expressions are the base rate of the event (Wallsten, Fillenbaum, & Cox 1986), the emotional desirability or valence of potential outcomes (Cohen 1986, Weber, & Hilton 1990), the severity of the consequences (Weber & Hilton 1990, Cohen & Wallsten 1992), whether a set of verbal expressions is self-selected or not (Fillenbaum, Wallsten, Cohen, & Cox 1991), various contextual and framing manipulations (Windschitl & Wells 1996), arbitrary anchors (McGlone & Reed 1998), and preferences of superiors (Piercey 2009). In a reversed interpretation task, where a verbal interpretation of a numerical probability is requested, the translation is in the presence of hazardous activities influenced by one's attitude towards these activities (Verplanken 1997). Receivers of probabilistic phrases typically interpret these terms to be closer to 0.50 and with a broader interval than intended by the sender (Fillenbaum, Wallsten, Cohen, & Cox 1991, Budescu & Wallsten 1990, Wallsten, Budescu, & Erev 1988, see also Wallsten & Budescu 1990 p.24, Budescu, Por, & Broomell 2011), in particular when negative wording is used (Smithson, Budescu, Broomell, & Por 2011).

6.3.2.3 Mode effectiveness studies

Given the aforementioned mode preferences, which suggest that people regard either the verbal or the numerical mode as superior, and the large variability in verbal-to-numerical translations, it is surprising that studies on the use of verbal probability expressions in various tasks show relatively small differences in the decision makers' response between the verbal and numerical mode (Wallsten, Budescu, & Erev 1988 p.291). Studies employing bids, rankings and choices involving verbal and numerical expressions find similar response patterns between the two modes (Budescu, Weinberg, & Wallsten 1988 p.291, González-Vallejo & Wallsten 1992), no significant differences in profits earned (Erev & Cohen 1990), and no significant response differences when the role of forecaster and decision maker are separated (Budescu & Wallsten 1990 *dyadic decisions*). However, the verbal mode does induce fewer preference reversals (González-Vallejo & Wallsten 1992), in the numerical mode rank-orderings correlate with probabilities and in the verbal mode with outcomes (González-Vallejo, Erev, & Wallsten 1994), and better decisions are obtained when the precision/vagueness of the decision context matches that of the mode of uncertainty expression (Burkell 2004 p.206, Olson & Budescu 1997).

On the expectation that forcing people to predict in the numerical mode requires them to exert more mental effort and therefore induces more bias (Zimmer 1983, 1984 p.123), several studies have investigated the mode-effect on biases. In a Bayesian updating (*revision*) task verbal probability judgments were indeed less biased towards conservatism than numerical estimates (Rapoport, Wallsten, Erev, & Cohen 1990). However, the conjunction bias and wishful thinking (*motivated reasoning*) bias appear equally present in either mode (Erev & Cohen 1990).

Finally, in a selection of other studies the verbal mode was found to induce more overconfidence (Wallsten, Budescu, & Zwick 1993), reduce the speed of comparison (Jaffe-Katz, Budescu & Wallsten 1989), improve judgment consistency and consensus among experienced auditors (Stone & Dilla 1994), and improve probability comprehension of females on the topic of breast cancer (Vahabi 2010).

While vagueness aversion is able to account for the preference for receiving precise probabilities in the preference mode reversal paradox, its apparent absence in the bidding, ranking and choice studies involving verbal probability expressions was unanticipated. The main results of studies on verbal and numerical mode effectiveness suggest that participants will be equally sensitive to verbal expressions as to numerical expressions. On the basis of this null-hypothesis of no difference between the verbal and numerical mode we expect that in the domain of losses the probability weights of a set of verbal probability expressions q_i with $i = 1,\ldots,n$, will be equal to the probability weights of $s(q_i)$, the numerical interpretations of these expressions by the respondents.

Hypothesis 1: In the domain of losses the probability weight of a verbal probability expression $[w_v(q_i)]$ is not significantly different from the probability weight of its single point numerical translation $[w(s(q_i))]$.

6.3.3 Verbal outcome expressions

It is a convention in decision theory to restrict the modeling of vagueness to the probability dimension on the argument that the outcomes are probability-contingent because of which outcome vagueness can be reinterpreted as probability vagueness.[101] The potential confound in modeling vagueness in the outcome dimension is then to confuse vagueness attitude for risk attitude (Camerer & Weber 1992 p.331; see, however, arguments by Du & Budescu 2005 pp.1793–1794 and Budescu & Wallsten 1995 p.279 challenging this view). Confounding as it may be, given the wide use of verbal outcome expressions in practical applications of risk management, further investigation into the utility assigned by practitioners to vague expressions of outcome seems warranted. Studies that measure vagueness attitude in the outcome dimension typically model imprecision through outcome ranges and, as yet, no studies on vagueness attitude have been performed with choices involving verbal outcome expressions. Generally studies employing outcome intervals find support for a preference for vagueness in the gain domain, a preference for precision in the loss domain and overall stronger preferences in the outcome dimension in comparison with the probability dimension (Du & Budescu 2005, Budescu, Kuhn, Kramer, & Johnson 2002, Kuhn, Budescu, Hershey, Kramer, & Rantilla 1999, Kuhn & Budescu 1996, González-Vallejo, Bonazzi, & Shapiro 1996). The opposite directions of vagueness preference in both domains suggests that outcome valence, *i.e.* whether the outcome is positively or negatively valued (Wallsten & Budescu 1995 p.51), is important. The general pattern of vagueness seeking for gains and vagueness aversion for losses has met several psychological motivational explanations (*goal framing* Levin, Schneider, & Gaeth 1998, *desirability-bias* Krizan & Windschitl 2007, *asymmetrical loss functions* Weber 1994) and perceptual explanations (*attentional salience* attributed to Daniel Kahneman in Weber 1994 footnote 8). The common idea behind the motivational explanations is that humans have a need for security which, driven by fear, leads to a focus on negative outcomes, and a need for potential which, driven by hope, focuses on positive outcomes. On the basis of these findings we expect that in the domain of losses the utility of a set of verbal outcome expressions r_i with $i = 1,...,n$, will be significantly higher than the utility of $s(r_i)$, the numerical interpretation of these expressions by the respondents.

Hypothesis 2: In the domain of losses the utility value of a verbal outcome expression $[u_v(r_i)]$ is significantly more negative than the utility value of its single point numerical translation $[u(s(r_i))]$.

The field experiment described in the next section enables the test of these two hypotheses in the domain of losses.

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[101] The following example illustrates how outcome vagueness can be reinterpreted as probability vagueness. A 10% probability of gaining outcomes in the range (0,5) suggest that outcomes are vague and that probability is precise. However, given that the probabilities of the outcomes in the range (0,5) are unknown, it is the probabilities that are vague and not the outcomes. Applied to verbal outcome expressions this line of argumentation suggests that the probabilities of the outcomes in the range $(-\infty, +\infty)$ are unknown.

6.4 Field experiment

The request of the Dutch subsidiary of a large worldwide consultancy firm (referred to as the case company) to calibrate its set of verbal probability and outcome expressions allowed us to test the two hypotheses in section 6.3. Standard procedure within the case company is to assess the probability of occurrence and the impact of events that threaten the company's projects using a standard set of verbal expressions for probability (q_i) and outcome (r_i). The sets of verbal probability and outcome expressions are with their respective rank-orders $i = 0,\ldots,10$ shown in table 6.1.

Table 6.1 Case company's verbal probability and outcome expressions and their rank order.

Probability (q_i)	**Rank (i)**	**Impact (r_i)**
Absolute certain	10	Catastrophic
Very high	9	Very high
High	8	High
Fairly high	7	Fairly high
Above average	6	Above average
Average	5	Average
Below average	4	Below average
Fairly low	3	Fairly low
Low	2	Low
Very low	1	Very low
Impossible	0	Not perceptible

Apart from the fact that participants did not preselect the set of verbal probability expressions, the set employed by the company fulfills the requirements of the *modified equal spacing method* (Wallsten, Budescu, & Zwick 1993). Such a set is composed of no more than 11 to 15 phrases, contains anchor phrases for the two end and the middle points of the probability continuum to which the probabilities 0, 1 and 0.5 are assigned, and assumes that the remaining expressions are equally spaced between these three anchor phrases.

The request to participate in the study was issued to 212 employees and a total of 143 staff members participated in the study (5% female).[102] The average age of the participants was 45 years of which on average 13 years with the case company. Participants were invited by their colleagues to motivate them to participate in the study. They were informed in advance and at the start of their response tasks, that their participation would contribute to the development of the risk policy of their organization and that their participation

[102] One additional employee did not finish the questionnaire and was discarded from the analysis.

would be anonymous. Participants were already acquainted with the set of verbal probability and outcome expressions which had been in use with the case company for several years. A software application developed with VBA in MS Excel which contained all choice options and questions was sent to the participants and returned by e-mail. The study was conducted during December 2010 – January 2011.

In two tasks, utility and probability weights (task 1) and introspective judgments of the meaning of verbal outcome and probability expressions (task 2) were elicited. Participants were randomly assigned to either a small or large project manipulation (financial revenue 500K *versus* 2M)[103], different task orders (1–2 *versus* 2–1), and eight randomized question orderings.[104] When relevant, the full set of verbal outcome or probability expressions (table 6.1) was shown to the participants with the expression v_i or q_i that was under consideration highlighted. The participants were requested to respond to the questions descriptively in their role as employee of the case company.[105] Despite the absence of financial incentives in this study, which is a general characteristic of survey studies, we believe that participants provided informative responses because of the personal, professional significance of the work-related events presented in our study (Manski 2004 footnote 11, p.1370).

6.4.1 Elicitation of utility and probability weights

The participants' attitudes to risk and vagueness are assessed by eliciting utility values and probability weights in both the numeric and verbal mode. Discussions with case company personnel suggested that relevance, realism and less (cognitive) burden could be attained by using certainty equivalents (x_{Si}) in the elicitation process. In all four steps of the elicitation procedure certainty equivalents are employed, which results in a homogeneous task, that does not require participants to switch cognitively, and prevents results from being impaired by changes in elicitation procedure (Delquié 1993). A financial penalty (claim) from a client was put forward by the case company as an understandable and unambiguous operationalization of the negative impact of events in a project setting. Utility functions for financial claims were elicited using the semi-parametric method by Abdellaoui, Bleichrodt and L'Haridon (2008) and the exponential family of utility (CARA, Pratt 1964, *Constant*

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103 All monetary amounts are in Euro. The case company distinguishes between small projects (< € 1,000,000) and large projects (≥ € 1,000,000). We choose € 500,000 as being representative for small projects and € 2,000,000 for large projects.

104 Sales and Delivery personnel were randomly assigned to these conditions as well. Randomization was applied using the procedure outlined in Shadish, Cook and Campbell (2002 pp.297–298, p.313).

105 After finishing the two main tasks participants were asked questions on the following topics: manipulation check for the small (500K) *versus* large (2M) project conditions, perceived representativeness of answers for sales *versus* delivery personnel, age, gender, function, number of years of working experience in sector and case company, involvement in public versus the private segments, experience in small or large projects, desire to receive feedback on study results, and 10 questions added by request of the case company that were not used in this study.

Absolute Risk Aversion), which specification depends on the parameter θ, an index for concavity (Wakker 2010 p.80).

In a four-step procedure we elicited the utility function $u(x_i)$, the utility values $u_v(r_i)$ for five verbal outcome expressions, the probability weighting function $w(p_i)$, and the probability weights $w_v(q_i)$ for five verbal probability expressions.

Table 6.2 Four-step elicitation procedure for utility and probability weights.[106]

	Assessed quantity	Indifference sought	Under prospect theory
Step 1	x_{Si}	$x_i p 0 \sim x_{Si}$, with $x_i \in \{-200K, -600K, -1100K, -1500K, -2M\}$, $p = .5$	$w(p)u(x_i) = u(x_{Si})$, with $u(0) = 0$ and $u(-2M) = -1$
Step 2	x_{Si}	$r_i p r_0 \sim x_{Si}$, with $r_i \in$ {very low, low, average, high, very high}, $p = .5$, and r_0 = {not perceptible}	$u_v(r_i) = u(x_{Si}) / w(p)$, with $w(p)$ and $u(.)$ from step 1
Step 3	x_{Si}	$x_w p_i 0 \sim x_{Si}$, with $p_i \in \{.05, .25, .50, .75, .95\}$, $x_w = -2M$	$w(p_i) = u(x_{Si}) / u(x_w)$, with $u(.)$ from step 1
Step 4	x_{Si}	$x_w q_i 0 \sim x_{Si}$, with $q_i \in$ {very low, low, average, high, very high}, $x_w = -2M$	$w_v(q_i) = u(x_{Si}) / u(x_w)$, with $u(.)$ from step 1

In steps 1 to 4 participants choose between a risky prospect with a particular chance of having to pay a financial penalty or a prospect which offers to settle this claim by a sure penalty of amount x_{Si}. In step 1 and 2 utility is elicited by varying the outcomes in respectively the numerical and verbal mode and holding probability constant at $p = .5$. Step 1 derives the parameter θ which specifies u(.), and the value of w(.5), the probability weight of $p = .5$. Using these specifications of u(.) and w(.5) we derive in step 2 the utility of five verbal outcome expressions, $u_v(r_i)$. In step 3 and 4 probability weights are consecutively elicited in the percentage and linguistic format while holding financial penalty constant at its *worst* outcome, $x_w = -2M$. For five numeric probabilities (p_i) and five verbal probability expressions (q_i) step 3 and 4 respectively derive probability weights using the u(.) specification from step 1. Relatively more measurements are taken at the low and high end extremities because there typical behavioral patterns have been observed in previous studies. The comparative igno-

[106] In addition the experiment involved the elicitation of 3 indifferences between a sure revenue and a risky revenue prior to step one, 4 indifferences to assess multriattitribute scaling constants after completing step four, and 25 introspective judgments of risk appetite at the very end of the experiment. Publication of these results in a multiattribute study is in preparation.

rance effect (Fox & Tversky 1995) was neutralized by avoiding a direct comparison between verbal expressions and their numeric equivalent. One group of participants determined their choices for a small project (500K) and the other for a large project (2M).

A novelty in this study is that it fully takes into account the information on the elevation of the probability weighting function which is elicited in step 1 using the Abdellaoui et al. (2008) method. From the value of w(.5) estimated in step 1 we can derive the parameter δ, which mainly measures the elevation of the Goldstein and Einhorn (1987) probability weighting function

$$w(p) = \delta p^{\gamma} / (\delta p^{\gamma} + (1 - p)^{\gamma}), \tag{6.1}$$

using the equality

$$\delta = w(.5) / (1 - w(.5)).^{107} \tag{6.2}$$

The value of w(.5) was estimated in step 1 on the basis of five data points. Insertion of equation 6.2 in equation 6.1 and fitting this equation to the five probability weights $w(p_i)$ derived from step 3, then only requires the estimation of parameter γ, which mainly measures the curvature of the probability weighting function, using nonlinear regression. This approach enables the integration of the semi-parametric utility elicitation method of Abdellaoui et al. (2008) and the full parametric elicitation of probability weights. This integrated procedure is an alternative to a full parametric fitting.

The utility specification u(.) from step 1 allows us to infer the outcome value x_i and probability value p_i that are implied in the utility values of the verbal expressions elicited in step 2 and 4.[108] These inferred estimates can be contrasted with introspective judgments of outcomes and probabilities.

6.4.2 Introspective translation of verbal outcome and probability expressions

In line with the mode translation studies in §6.3.2.2 we assess the meaning that participants assign to verbal expressions of probability and outcome by letting the participants match each expression with a single point estimate that, according to them, best represents the meaning of that expression. Consecutively participants match the verbal out-

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[107] Because the equality $.5^{\gamma} = (1 - .5)^{\gamma}$ holds then given that $p = .5$ we find that $p^{\gamma} = (1 - p)^{\gamma}$. For $p = .5$ equation 6.1 can be rewritten as $w(p) = \delta p^{\gamma} / (\delta p^{\gamma} + p^{\gamma}) \Rightarrow \delta p^{\gamma} = w(p)\ (\delta p^{\gamma} + p^{\gamma}) \Rightarrow p^{\gamma} = w(p)\ p^{\gamma} + w(p)\ p^{\gamma}/\delta \Rightarrow 1 = w(p) + w(p)/\delta \Rightarrow \delta = w(p) / (1 - w(p))$.

[108] Using the CARA exponential utility specification (Pratt 1964) the inferred outcome $x_i = u^{-1}(u_v(r_i))$ can be calculated using the formulae $x_i = -ln\ (1 - u_v(r_i)) / \theta$, for $\theta > 0$; $x_i = u_v(r_i)$, for $\theta = 0$; $x_i = -ln\ (1 + u_v(r_i)) / \theta$, for $\theta < 0$. Using the Goldstein and Einhorn (1987) specification of the probability weighting function the inferred probability $p_i = w^{-1}(w_v(q_i))$ can be calculated using the formula $p_i = ([\delta / w_v(q_i) - \delta]^{1/\gamma} + 1)^{-1}$.

come and probability expressions r_i, q_i, ∈ {very low, low, average, high, very high} with their respective monetary best estimate $s(r_i)$ or best percentual estimate $s(q_i)$. Because the interpretation of verbal expressions is structurally influenced by context, these introspective judgments were not elicited in the 'unknown context condition' (Clark 1990 p.14) but instead were made in a contextual setting which is identical to the one used to elicit utility and probability weights. For verbal outcome expressions this contextual setting is described as 'a deal with a size of 500K (2M) with a .5 likelihood on a claim with r_i impact' and for verbal probability expressions the context is 'a deal with a size of 500K (2M) with a q_i likelihood on a claim of 2M'.

6.5 Results

6.5.1 Manipulation and reliability checks

The small *versus* large project manipulation was successful. Respondents interpreted their exposure to either the 500K or the 2M financial revenue manipulation as representative for respectively small *versus* large projects (n = 143, Mann-Whitney test, p = .003).

The certainty equivalent for the prospect $-2M_{.5}0$ was elicited both in step 1 and 3 which enabled a test of reliability of the responses of the participants. Correlation between the two measurements was $\rho = .72$ (n = 143, Spearman, significant at the p = 0.01 level, 2-tailed) and the median value of the measurements was significantly more negative in step 1 than in step 3 (n = 143, Wilcoxon Signed Ranks test, p = .000). These results indicate that the participants in this study were not always consistent in their choices and suggest an order effect. On the assumption that there exists a general tendency in our data to stimulate the elicitation of less negative certainty equivalents x_{Si} as the experiment proceeds, the ordering in table 6.2 is expected to produce lower values for $u_v(r_i)$, $w(p_i)$, and $w_v(q_i)$ than would have been the case under a randomized ordering of steps 1-to-4.

6.5.2 Median utility and probability weights

The median response of participants (n = 143) for utility in the domain of financial claims (step 1) illustrates that the median response assigns relatively more utility to a decrease in a small claim *vis-a-vis* a large claim. The curvature of this utility function, represented by the index of concavity $\theta = -5.09 \times 10^{-7}$ significantly differs from a straight line (Wilcoxon Signed Ranks test, p = .000, IQR: $1 \times 96\ 10^{-6}$).[109] This result indicates that the median utility function for financial claims is convex, a typical finding in the domain of losses, which expresses increasing marginal utility of losses.

[109] IQR = interquartile range = the difference between the upper and lower quartiles.

For probability weighting (step 3) the results indicate that the median responses do not distort probability but weigh probability linearly. The median probability weight for $p = .5$ was $w(.5) = 0.48$, a value which is not significantly different from .5 (Wilcoxon Signed Ranks test, p = .377, IQR: 0.34). Both the median value for the elevation indicator of the probability weighting function $\delta = 0.93$ and the curvature indicator $\gamma = 0.94$ were not significantly different from one (Wilcoxon Signed Ranks tests, p = .270 and p = .150, IQR: 1.45 and 0.60). All three tests suggest the absence of probability distortion for the median respondent.

The combination of convexity of utility and linearity in probability weighting suggests that the median response is in line with expected utility theory and is risk seeking in the domain of losses. The choices of individual respondents may, however, differ considerably from this median response.

6.5.3 Verbal probability expressions

When the weight of a verbal probability expression is higher than the probability weight transformation of the numerical translation of this expression, *i.e.* $w_v(q_i) > w(s(q_i))$, then the participant assigns relatively more weight to vagueness than to precision, which in the domain of losses suggests vagueness aversion. The reverse inequality $w_v(q_i) < w(s(q_i))$ in this domain suggests vagueness seeking behavior. Equality implies vagueness neutrality.

The participants' individual utility and probability weighting functions were used to calculate the probability weights and inferred probabilities of the verbal expressions in table 6.3.[110] Each participant's introspective numerical translation of a verbal probability expression was, using the participant's individual probability weighting function, transformed to the probability weight $w(s(q_i))$. These transformed numerical estimates of the verbal probability expressions did not significantly differ from the choice-based probability weights of the verbal probability expressions that were derived from the choices of the participants (n = 630, Wilcoxon Signed Ranks test p = .428).

When a verbal expression's inferred probability differs from its judged probability then this implies that choices involving the linguistic probability descriptor will be different

[110] Excluded from the analysis in §6.5.3 were the responses of 17 participants: three participants whose introspective judgment of probability monotonically decreased with the rank of the verbal probability expressions (S025, S064, S101) and two participants who responded using extreme percentages only (S042, S084) and apparently expressed their strength of belief in the statement (see also Wallsten, Fillenbaum & Cox 1986 p.578), three participants with erratic or invariant responses (S030, S081, S098), one participant whose probability weighting is monotonically decreasing (S115), and nine participants for which R^2 could not be computed for the nonlinear estimation of γ and thus no reliable model specification could be found (S016, S068, S077, S098, S110, S119, S177, S184, S202). Participant S098 is mentioned twice but not double counted.

from those involving its judged numerical identity. An inferred probability is calculated by transforming the probability weight of a verbal expression $w_v(q_i)$ to a numerical probability using the inverse probability weighting function $w^{-1}(.)$. Vagueness aversion is implied when in the domain of losses the inferred probability is higher than the judged probability, *i.e.* $w^{-1}(w_v(q_i)) > s(q_i)$. The reverse inequality implies vagueness seeking. The introspective numerical estimates of the verbal probability expressions did not significantly differ from the probabilities that were, using the participants' individual inverse probability weighting functions, inferred from the choices of the participants (n = 630, Wilcoxon Signed Ranks test p = .199). The inferred probabilities did not include much more variability than the judged probabilities (standard deviation of $w^{-1}(w_v(q_i)) = 0.33$ and of $s(q_i) = 0.30$) which suggests that the degree of vagueness of the two measures is similar (Budescu & Wallsten 1990 p.258).

In these tests we use five observations from the same respondents, *i.e.* one for each verbal probability expression. The results of the Wilcoxon Signed Ranks test, which assumes statistical independence, may therefore have been inaccurate. For this reason we apply the same test to each verbal probability expression separately. Table 6.3 presents median results for probability weights, inferred and judged probabilities for five verbal probability expressions together with a test of significance.

Table 6.3 Median choice-based probability weights [$w_v(q_i)$], and probability weight transformations of numerical estimates [$w(s(q_i))$], inferred probabilities [$w^{-1}(w_v(q_i))$], and judged probabilities [$s(q_i)$] for five verbal probability expressions (q_i) and Wilcoxon Signed Ranks tests (p), n = 126.[111]

term (q_i)	**rank (i)**	**$w_v(q_i)$**	**$w(s(q_i))$**	**p**	**$w^{-1}(w_v(q_i))$**	**$s(q_i)$**	**p**
very high	9	0.79	0.75	**.046**	0.84	0.75	**.043**
high	8	0.59	0.57	.521	0.62	0.60	.296
average	5	0.25	0.31	.143	0.30	0.33	.270
low	2	0.10	0.11	.190	0.10	0.10	.769
very low	1	0.04	0.06	**.027**	0.04	0.05	.822

The significant differences in table 6.3 suggest vagueness seeking for the 'very low' probability expression q_1 and vagueness aversion for the 'very high' expression q_9. For q_9 this

[111] Because of nonlinearities in $w^{-1}(.)$ the p-values of the Wilcoxon Signed Ranks tests for each of the verbal probability expressions in table 6.3 differ when the test is executed over probability weights in comparison with probabilities. Let $\varepsilon = w_v(q_i) - w(s(q_i))$ and $\varepsilon' = w^{-1}(w_v(q_i)) - s(q_i)$ then under a linear transformation the equality $\varepsilon' = w^{-1}(\varepsilon)$ holds and test results would be identical.

contradicts the common finding of vagueness seeking for high probabilities in the domain of losses (Camerer and Weber 1992 table 3 *stylized fact 7*).

The correlation between the probability weights of the verbal probability expressions and the probability weights calculated on the basis of judged probabilities is $\rho = .76$ and inferred and judged probabilities correlate at $\rho = .70$ (n = 630, Spearman, significant at the p = 0.01 level, 2-tailed). This corresponds with results in previous studies (*e.g.* in Budescu, Weinberg, & Wallsten 1988 p.288 correlation between inferred and displayed probability is $\rho = .70$).

Both the inferred and the judged probabilities were significantly lower than the values of the modified equal spacing method (n = 630, Wilcoxon Signed Ranks tests p = .000). If the modified equal spacing method is chosen as a benchmark, then the inferred and judged probabilities in table 6.3 reflect an optimistic attitude towards probability. Even though for a single project any of the verbal probability expressions in table 6.1 may be applicable, the percentages and probability expressions employed in step 3 and 4 are not representative for the average frequency of the occurrence of a claim in the case-company. Because the base rate for the occurrence of a claim within the case company is much lower than the percentages shown in table 6.3, the optimistic attitude of the respondents can be explained by a base rate effect (Wallsten, Fillenbaum, & Cox 1986).

The sequencing of the choice task versus the introspective judgment task (1–2 *versus* 2–1) had a significant influence on judged probabilities (n = 630, Mann-Whitney test, p = .000). When introspective judgments were elicited after subjects had expressed their preferences for prospects, the introspective translations of the verbal probability expressions resulted in significantly higher single point estimates for the expressions q_5, q_8, and q_9 (n = 126, Mann-Whitney tests, p = .000). This significant order effect suggests that exposure of the participants to the choice tasks made claim occurrence more available to them and therefore increased the subjective estimate associated with the verbal probability expression (Tversky & Kahneman 1973). When judgment precedes choice then judged probabilities are significantly lower than inferred probabilities, which suggests a preference for precise probabilities in the domain of losses (n = 315, Wilcoxon Signed Ranks test, p = .000).[112] But when instead choice precedes judgment then this relationship is reversed (n = 315, p = .041) and reveals itself as well in probability weights (n = 315, p = .008), in both cases suggesting a preference for vague probabilities in the loss domain.

Table 6.4 presents median results for probability weights, inferred and judged probabilities for five verbal probability expressions and demonstrates that taking the reversing of task order into account reveals more significant differences between choice-based and judged estimates.

[112] The probability weights of verbal probability expressions $w_v(q_i)$ and the transformed numerical estimates $w(s(q_i))$ did not differ significantly (p = .119).

Table 6.4 Median choice-based probability weights [$w_v(q_i)$], probability weight transformations of numerical estimates elicited 1st or 2nd [$w(s(q_i))$], inferred probabilities [$w^{-1}(w_v(q_i))$], and judged probabilities elicited 1st or 2nd [$s(q_i)$] for five verbal probability expressions (q_i) and Wilcoxon Signed Ranks tests (p), n = 126.

term (q_i)	**rank (i)**	**$w_v(q_i)$**	**$w(s(q_i))$ 1st**	**p**	**$w(s(q_i))$ 2nd**	**p**	**$w^{-1}(w_v(q_i))$**	**$s(q_i)$ 1st**	**p**	**$s(q_i)$ 2nd**	**p**
very high	9	0.79	0.67	**.006**	0.80	.978	0.84	0.70	**.001**	0.85	.691
high	8	0.59	0.50	**.010**	0.63	.163	0.62	0.50	**.005**	0.75	.244
average	5	0.25	0.24	.612	0.34	**.010**	0.30	0.25	.273	0.50	**.013**
low	2	0.10	0.11	.396	0.10	.295	0.10	0.10	.579	0.10	.827
very low	1	0.04	0.07	**.013**	0.06	.603	0.04	0.05	.816	0.05	.603

Table 6.4 repeats for q_9 and in addition demonstrates for q_8 significant differences which contradict the common finding in the loss domain of vagueness seeking for high probabilities. The significant differences for q_5 and q_1 suggest vagueness seeking for the 'average' and 'very low' probability expression.

In the experimental studies Tversky and Fox (1995 p.278), Fox and Tversky (1998 p.883) and Baillon, Cabantous and Wakker (2011) choice-tasks precede judgment-tasks. In Budescu, Weinberg and Wallsten (1988) judgment precedes choice. There are no definitive arguments why either of the two task orderings better represents the preferences of the participants. For studies that let judgments precede choice, the fact that introspective estimates are then not in any way biased by the experimental choice task, argues in favor of the judge-choose sequence. On the other hand, participants in the choice-task involving verbal probability expressions (step 4) had already been involved in several earlier choice tasks (step 1 – 3). The prior exposure to choices in both step 4 and the successive judgment task argues in favor of the choose-judge sequence. Without a clear preference for either of the two, their combined effect as expressed in table 6.3 is presumed to be best representing the participants' preferences. The results in this section demonstrate that judgments and choices lead to approximately the same probability weights for moderate probabilities but not for extreme probabilities. We conclude that the null-hypothesis of 'no-difference', emanating from previous studies, cannot be rejected for moderate probabilities but is rejected for extreme probabilities.

6.5.4 Verbal outcome expressions

Each participant's numerical interpretation of a verbal outcome expression was, on the basis of the utility function of the individual participant, transformed to the utility value

$u(s(r_i))$. These utility transformations of the single point estimates of the verbal outcome expressions were significantly less negative than the choice-based utility values of the verbal outcome expressions, *i.e.* $u_v(r_i) < u(s(r_i))$ (n = 674, Wilcoxon Signed Ranks test, p = .000). The value of the outcomes implied by choices with verbal outcome expressions is significantly more negative than the value of the outcomes that were matched with the verbal outcome expressions in the introspective judgment task, *i.e.* $u^{-1}(u_v(r_i)) < s(r_i)$ (n = 617, Wilcoxon Signed Ranks test, p = .000).[113] Both results suggest strong aversion to vagueness in the domain of losses.

The inferred outcomes included much more variability than the judged outcomes (standard deviation of $u^{-1}(u_v(r_i))$ = 968K and of $s(r_i)$ = 583K) which reflects the greater vagueness associated with the former in comparison with the latter (Budescu & Wallsten 1990 p.258).

Table 6.5 presents median results for utility values, inferred and judged outcomes for five verbal outcome expressions.

For all verbal outcome expressions the tests in table 6.5 indicate that utility values and outcomes inferred from choices are significantly more negative than those resulting from introspective judgments. This means that *ceteris paribus* the median respondent prefers a prospect with a numerical expression to its counterpart in the verbal mode. This suggests vagueness aversion for outcomes in the domain of losses, a finding which can be explained by the salience of the outcome dimension due to its compatibility with the monetary response scale and the participants' need for security motivating them to focus on the negative outcomes (Budescu, Kuhn, Kramer, & Johnson 2002, Du & Budescu 2005, Fischer & Hawkins 1993 p.376 *scale compatibility effect*).

The correlation between the utility of the verbal outcome expressions and the utility calculated from the judged outcomes is ρ = .73 (n = 674) whereas inferred and judged out-

[113] Excluded from the analysis §6.5.4 were six participants: four participants whose introspective judgment of outcome monotonically decreased with the rank of the verbal outcome expressions (S064, S085, S103, S112), two participants with erratic responses (S081, S154). For 41 participants the inferred outcome of one or more verbal outcome expressions could not always be calculated because the $u_v(r_i)$ is smaller than the limit to minus infinity of the CARA-utility function with $\theta < 0$ (amounting to 8% of all responses, which is an aggregrate of 62 from the responses of the following participants: S003, S004, S005, S007, S008, S015, S020, S024, S026, S041, S046, S058, S061, S062, S067, S078, S080, S082, S084, S095, S098, S103, S112, S114, S133, S138, S142, S145, S147, S158, S169, S178, S179, S181, S189, S190, S191, S192, S196, S197, S207). For 12 participants some outcomes were not judged and others judged twice due to a programming error (S039, S047, S058, S098, S103, S104, S127, S162, S166, S181, S191, S207). These double judgments were averaged and missing observations were left blank. For judged and choice-based outcomes n = 617 results from (143 – 6) × 5 – 62 – 12 + 6 multiple counts (S103 (2×), S112, S058, S098, S181). For weight of the verbal outcome expression and probability weight transformations of judged probabilities n = 674 results from (143 – 6) × 5 – 12 + 1 multiple count (S103). In determining the utility of verbal outcome expressions an indifference point was not always reached. When these responses were exempted from the analysis the aforementioned test results remain the same (n = 512, Wilcoxon Signed Ranks test, p = .000).

Table 6.5 Median choice-based utilities [$u_v(r_i)$], utility transformations of numerical estimates [$u(s(r_i))$], inferred outcomes [$u^{-1}(u_v(r_i))$], and judged outcomes [$s(r_i)$], for five verbal outcomes expressions (r_i) and Wilcoxon Signed Ranks tests (p).[114]

term (r_i)	**rank (i)**	**$u_v(r_i)$**	**$u(s(r_i))$**	**p**	**$u^{-1}(u_v(r_i))$**	**$s(r_i)$**	**p**
very high	9	−0.97 (n = 137)	−0.40 (n = 137)	**.000**	−1368K (n = 103)	−500K (n = 137)	**.000**
high	8	−0.77 (n = 132)	−0.27 (n = 132)	**.000**	−1029K (n = 110)	−275K (n = 132)	**.000**
average	5	−0.26 (n = 137)	−0.10 (n = 137)	**.000**	−325K (n = 136)	−100K (n = 137)	**.000**
low	2	−0.06 (n = 136)	−0.03 (n = 136)	**.000**	−75K (n = 136)	−45K (n = 136)	**.000**
very low	1	−0.02 (n = 132)	−0.01 (n = 132)	**.001**	−25K (n = 132)	−10K (n = 132)	**.000**

comes correlate at $\rho = .67$ (n = 617, Spearman, significant at the p = 0.01 level, 2-tailed). These results are comparable with the aforementioned findings for verbal probability expressions.

Just as was the case with verbal probability expressions the sequencing of the choice task versus the introspective judgment task (1–2 *versus* 2–1) had a significant influence on judged outcome (n = 674, Mann-Whitney test, p = .000). When introspective judgments were elicited after subjects had expressed their preferences for prospects, the introspective translations of the verbal probability expressions were approximately two times more negative than in

114 For inferred and judged outcomes the number of responses for which the Wilcoxon Signed Rank tests were calculated were the total number of participants *minus* six monotonically decreasing and erratic responses *minus* the missing responses due to programming error *minus* the incalculable values for high verbal expressions *plus* double counts.
r_1: 132 = 143 – 6 – 5 – 0;
r_2: 136 = 143 – 6 – 1 – 0;
r_5: 136 = 143 – 6 – 0 – 1;
r_8: 110 = 143 – 6 – 6 – 26 + 5 (S058, S098, S103, S112, S181);
r_9: 103 = 143 – 6 – 0 – 35 + 1 (S103).
For the utility of verbal outcome expressions and the utility transformation of judged outcomes the number of responses for which the Wilcoxon Signed Rank tests were calculated were the total number of participants *minus* six monotonically decreasing and erratic responses *minus* the missing responses due to programming error *plus* double counts.
r_1: 132 = 143 – 6 – 5;
r_2: 136 = 143 – 6 – 1;
r_5: 136 = 143 – 6 – 0;
r_8: 110 = 143 – 6 – 6 + 1 (S103)
r_9: 103 = 143 – 6 – 0.

the reverse condition and significantly different for the expressions r_1 (Mann-Whitney test, p = .041), r_5 (p = .001), r_8 (p = .003), and r_9 (p = .015). This suggests that the exposure to an abundance of losses under outcomes creates an availability effect similar to that under verbal expressions. Despite this difference inferred outcomes were in both task orders more negative than judged outcomes (Wilcoxon Signed Ranks tests, 1–2 sequence: n = 319, p = .000; 2–1 sequence, n = 298, p = .000).

Under the large project condition the responses of the participants were significantly more negative than those under the small project condition for the utility of judged outcomes, judged outcomes (n = 674, Mann-Whitney tests, both p = .000) and inferred outcomes (n = 617, p = .013).[115] The descriptors for 'average' and 'very high' impact were in terms of inferred outcomes significantly different between small and large projects (Mann-Whitney-tests, r_5: p = .026 and r_9: p = .035) whereas other expressions were not significantly different at $\alpha = .05$. For judged outcomes all verbal outcome expressions were significantly different between small and large projects (Mann-Whitney-tests, r_1: p = .038, r_2: p = .023, r_5: p = .003, r_8: p = .001, r_9: p = .001).

For small and large projects and five verbal outcome expressions median results are presented in table 6.6 for inferred and judged outcomes. Figure 6.1 presents choice-based utility values of verbal outcome expressions, utility transformed judged outcomes, and linear utility defined over the ranks of the verbal probability expressions.

Table 6.6 Median inferred outcomes [$u^{-1}(u_v(r_i))$] and judged outcomes [$s(r_i)$] for small and large projects for five verbal outcomes expressions (r_i) and Wilcoxon Signed Ranks tests (p).[116]

term (r_i)	**rank (i)**	**small project (500K)**			**large project (2M)**		
		$\mathbf{u^{-1}(u_v(r_i))}$	$\mathbf{s(r_i)}$	**p**	$\mathbf{u^{-1}(u_v(r_i))}$	$\mathbf{s(r_i)}$	**p**
very high	9	−1236K (n = 47)	−400K (n = 67)	**.000**	−1735K (n = 56)	−1000K (n = 70)	**.000**
high	8	−922K (n = 52)	−250K (n = 65)	**.000**	−1259K (n = 58)	−500K (n = 67)	**.000**
average	5	−256K (n = 66)	−100K (n = 67)	**.000**	−387K (n = 70)	−150 (n = 70)	**.000**
low	2	−70K (n = 67)	−25K (n = 67)	**.000**	−88K (n = 69)	−50K (n = 69)	**.003**
very low	1	−26K (n = 65)	−10K (n = 65)	**.000**	−24K (n = 67)	−15K (n = 67)	.090

[115] The utility of the verbal outcome expressions was not significantly different (n = 674, Mann-Whitney test, p = .085).

[116] The number of respondents in the Wilcoxon Signed Ranks tests correspond with the numbers in the inferred outcome columns.

(a) (b)

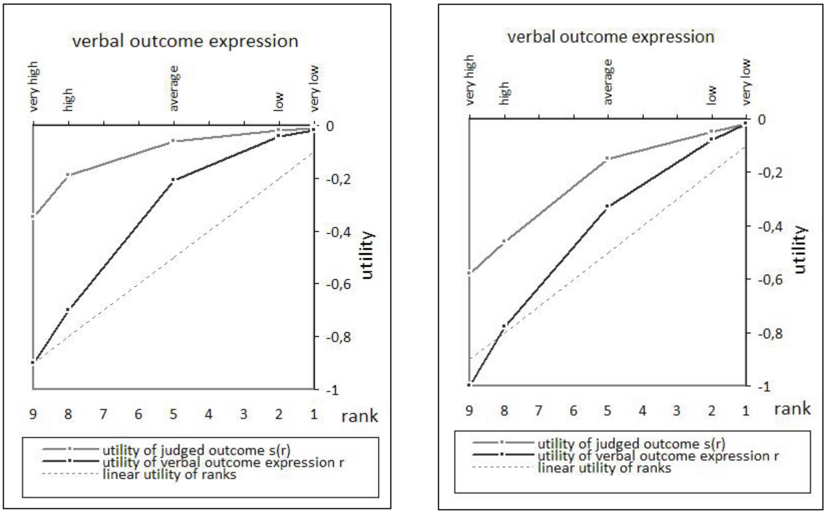

Figure 6.1 Median choice-based utility values of verbal outcome expressions [$u_v(r_i)$] and utility transformed judged outcomes [$u(s(r_i))$] for (a) small projects [500K] and (b) large projects [2M] for five verbal outcomes expressions (r_i) with rank $i = 1, 2, 5, 8, 9$.

Both table 6.6 and figure 6.1 demonstrate that the step size between verbal outcome expressions measured in outcomes and utility is increasing with the rank of the expressions. This corresponds with a tradition in risk management to express the severity of negative events exponentially (*e.g.* Clemens et al. 2005). Figure 6.1 together with the test results in table 6.6 and at the start of this section all indicate that in the domain of losses the utility value of a verbal outcome expression is significantly more negative than the utility value of its single point numerical translation. From this we conclude that the null-hypothesis can be rejected.

6.6 Discussion

This study was undertaken among a population of consultants with a unique, professional experience in applying a common set of verbal probability and outcome expressions to assess project risks. For a relevant selection of these expressions a probability weight or utility value was determined under prospect theory, thereby extending the application of this descriptive theory for decision making under risk to vague expressions of both probability and outcome. An interesting finding is that, while prospect theory was assumed in the elicitation process, the median response of the participants corresponds with expected utility theory. This result was derived using a novel estimation procedure for the probability weighting function which is compatible with the Abdellaoui et al. (2008) semi-parametric utility elicitation method. For moderate probabilities the study corroborates in the field earlier, counterintuitive results discovered in laboratory studies, revealing surprisingly similar response patterns in the verbal and numerical probability mode using probability weights. As in many studies results for large and small probabilities were different, with the note that the usual vagueness seeking for high probabilities is in this field study replaced by vagueness aversion. The analysis of choices involving verbal outcome expressions, not studied previously, upholds the previous finding of vagueness aversion in the domain of losses, established in prior studies under alternative vague representations of uncertainty.

The fact that the null-hypothesis of 'no-difference' between the verbal and numerical probability representations of uncertainty could not be rejected for moderate probabilities in this and previous studies suggests that the nature of the event and its uncertainty, but not its representation mode, are the drivers of a decision maker's choices under the two modes. When the nature of uncertainty is precise, such as in laboratory experiments with spinners, then decision makers apparently are successful in finding clues for the real mechanisms of objective chance in both the verbal and numerical mode. When the nature of uncertainty is vague, such as in our field experiment, then decision makers are obviously equally susceptible to contextual influences in either the verbal or the numerical mode in the construction of their preferences. The finding that groups of events and their uncertainty mechanisms easily drown any effects of the verbal and numerical representation mode suggests that for verbal probability expressions *source dominance over representation* holds. The consultants in our study, not used to receiving precise numerical estimates for the vague uncertainties in their professional life, assign an equal degree of vagueness to the numerical as to the verbal representations of those uncertainties. A precise representation of a vague source of uncertainty in the form of a probability percentage will then not be interpreted as an objective measure of risk but as a best estimate from a range of possible values. Numerical probability estimates can thus be ambiguous, portraying alternative meanings dependent on whether the nature of their source of uncertainty is precise or vague. On the other hand, being accustomed to processing monetary figures in their daily profession they will, when provided with a precise monetary estimate, reasonably assume that the rationale for it being mentioned in precise terms is its precise nature. People's degree of familiarity with working with numerical estimates may thus help to explain the *mode indifference* phenomenon for probabilities and the *mode preference* phenomenon for outcomes. This suggests that source dominance over representation does not hold when it is unknown to the decision maker if the nature of uncertainty of a source is vague of precise. Future studies should assess under what circumstances source dominance over representation holds by manipulating the ambiguity that is associated with both the numerical and verbal modes of representation.

The large amount of verbal probability expressions employed by the case company and their symmetry around the verb 'average', suggests that a natural and reasonable word-to-number translation consists of a linear scaling using an equal probability space of .1 between the probability descriptors (*modified equal spacing* method). Both the introspective judgments and inferred probabilities of the participants fell, however, significantly below this line, suggesting that participants anchored their interpretations and choices on the low base rate of claim occurrence. The application of a formalized scaling of its verbal probability expressions would enable the case company to employ an 'anchoring-and-adjustment strategy' (Einhorn & Hogarth 1985 p.436) but then in reverse. Instead of leaving the editing operation of 'vagueness resolution' (Budescu, Kuhn, Kramer, & Johnson 2002 p.765) to the professional, the case company itself can resolve the vagueness by prescribing the best numerical anchors for its set of expressions and reduce the large intersubject variability that exists in the interpretation of verbal expressions. To further reduce cognitive biases under uncertainty these anchors could additionally be expressed in frequencies instead of probabilities (Ayton & Pascoe 1995 pp.37–38, Gigerenzer 1991, Gigerenzer, Hoffrage, & Kleinbölting 1991).

Both the inferred and judged outcomes in our study become exponentially more negative with the rank of verbal outcome expressions, both for small and large projects. Furthermore, the absolute size of the outcomes assigned to these expressions is for large projects significantly larger than for small projects. Tradition in risk management endorses both the exponential scaling of impact, which creates a natural, large divide between the 'minor' and the 'major' impact, and the aforementioned relationship between impact and entity size, suggesting that large entities can have a larger risk appetite than small entities. A formalized scaling of these qualitative terms to outcome ranges can take account of this exponentiality and project size thereby reducing the large intersubject variability that exists in the interpretation of verbal outcome expressions.

It should be noted that the case company's tradition to employ imprecise natural language only, is conducive to the social interaction between employees with multiple, sometimes conflicting goals (comparable with social undesirability of precision in Erev, Wallsten, & Neal 1991). A transformation of the case company's current qualitative risk assessment method to a semi-quantitative method with its verbal expressions scaled to numerical probability and outcome ranges, enables its staff to engage both in rule-based reasoning in the numerical mode and associative and intuitive thinking in the verbal mode.

In this field study we demonstrate how to design a semi-quantitative risk analysis with a solid decision theoretic foundation. The focus of this study is on one source of uncertainty (a group of events resulting in financial claims) and contrasted two modes of uncertainty representation (numerical *versus* verbal expressions). While rating scales in risk management appear to be entirely based on introspective judgments, our application of prospect theory to verbal expressions allows us to derive rating scales entirely from a decision maker's revealed choices. The field accommodates an abundance of sources of uncertainty, a variety of modes of uncertainty representation, and numerous companies that are currently engaged in risk management. This offers a clear and present opportunity for decision theorists to study risk in daily-life while at the same time provide a relevant contribution to the management of risk.

7 Curbing Risk Appetite with Performance-based Incentives

Summary

This chapter reports the results of a laboratory experiment among lecturers of the Rotterdam University of Applied Sciences that was designed to test the effect of performance-based financial incentives on the participants' efficient use of risk management resources and on their effectiveness in assuring that financial buffers exceed the losses that they sustain. The experiment is designed around the risk matrix, a tool commonly used in risk management to visualize the portfolio of risks that an entity faces. A fixed incentive scheme induced an averse appetite to risk in the risk matrix and a variable incentive scheme promoted a neutral risk appetite. In comparison with the fixed scheme, participants were under a variable scheme equally effective in assuring that losses are absorbed by financial buffers but were significantly more efficient in their use of risk management resources. Participants could have attained more optimum levels of efficiency and effectiveness by choosing to reduce risk by impact reduction instead of probability reduction.

7.1 Introduction

In the wake of high-profile company failures and preventable large losses, the mid-1990s saw the dawn of a systematic and integrated approach to the management of all risks that a company faces, called enterprise risk management (Dickinson 2001). The premise underlying enterprise risk management (ERM) is that every entity exists to provide value for its stakeholders and, faced by risk and opportunity that have the potential to erode or enhance value, needs to determine how much uncertainty it accepts in order to grow stakeholder value. The relationship between enterprise risk management and value is formulated by the COSO-ERM (2004) framework as follows:

> "Enterprise risk management enables management to effectively deal with uncertainty and associated risk and opportunity, enhancing the capacity to build value. Value is maximized when management sets strategy and objectives to strike an optimal balance between growth and return goals and related risks, and efficiently and effectively deploys resources in pursuit of the entity's objectives." (p.3)

Empirical evidence on the relation between enterprise risk management and firm value is mixed, finding either a positive relationship (Hoyt & Liebenberg 2011, Gordon, Loeb, & Tseng 2009) or the absence of any value added compared to traditional risk management practices (McShane, Nair, & Rustambekov 2011). While enterprise risk management aims to provide a "reasonable assurance regarding the achievement of entity objectives" (COSO 2004 p.4), its criticasters argue that this assurance is at best limited and illusory at worst, resulting in "the risk management of nothing" (Power 2009).

According to COSO-ERM, maximization of stakeholder value is conditional on the efficient and effective deployment of an entity's resources (see the aforementioned quote from COSO). This implies that resources available for risk management should be used efficiently and effectively as well. The entity's *risk appetite*, *i.e.* its willingness to accept potential losses in the pursuit of value (COSO 2004 p.6), is managed by an entity by striking a balance between the efficiency and effectiveness of risk management. The management of this organizational risk appetite is considered to be one of the least mature competencies of organizations (RIMS 2009, 2008, Spinard, Faris, Culp & Nunes 2010) and a failure to manage risk appetite properly is seen as one of the causes of the 2007–2009 financial crisis (McDonald 2009). Financial incentives, directed at aligning the actions of agents with the objectives of the principals of the organization, have since the crisis been blamed for stimulating an excessively large appetite for risk by neglecting low-probability, high-impact events (Edwards 2010 pp.255, 297). The resulting call for a compensation reform in risk management led to various legislative and regulatory initiatives that aimed at "balancing risk, performance and pay" (Reda 2009). To assess the value added of different incentives for the risk management of an enterprise this chapter explores experimentally the effect of two very different financial incentive schemes on risk appetite, the efficient use of risk management resources, and the effectiveness of maintaining sufficiently large financial buffers to absorb losses.

The study reports the choices of 34 lecturers from the Rotterdam University of Applied Sciences who participated in an experiment designed to test these effects, as a prelude to future artifactual field experiments (Harrison & List 2004). The study employed a fixed incentive scheme, which awarded a fixed fee contingent on the participant being able to effectively avoid default, and a variable incentive scheme that awarded a percentage of the participant's risk management resources that remained at the end of the experiment. The results indicate that the fixed incentive scheme induced an averse appetite to risk and that a variable incentive scheme promoted a neutral risk appetite. Under the variable incentive condition participants were able to accomplish the same level of effectiveness in a more efficient way. However, in neither compensation scheme participants attained a choice that would have maximized their earnings potential.

This study contributes to the design of institutions which provide proper incentives for economic agents, which is a central theme in economics (Laffont & Martimort 2002 p.1) and as old as economics itself (Laffont 2003 p.xi). The principal-agent theory of incentives has up to now motivated empirical studies in risk management that employ general indicators of risk, such as investment in R & D, leverage, stock volatility, and derivatives usage (*e.g.* Coles, Daniel, & Naveen 2006, Daníelsson, Jorgensen, & De Vries 2002, Rogers 2002, Tufano 1996). Our experimental design allows us to study in detail how participants choose to manage specific risks by reducing their impact and probability and is arranged around the risk matrix, a popular risk management tool. This novel experimental set-up allows us to study when participants prefer impact reduction, as in studies on insurance, and when they choose to reduce a risk's probability of occurrence, which is a more preventive control measure. Our study contributes to the ongoing debate about the role of performance-based incentive schemes in risk management by assessing the effectiveness of two very dissimilar incentive schemes in curbing excessive risk taking as well as excessive expenditure on risk management.

This chapter is organized as follows. The next section discusses the disputed role of performance-based incentives and their hypothesized influence on organizational risk appetite. The third section discusses the empirical finding of risk seeking for losses in relation organizational risk appetite. The fourth section describes the experimental design, the optimum choices that maximize the earnings potential of participants in the experiment and develops the hypotheses for this experimental study. In the fifth section the results of the experiment are analyzed. These results are discussed in the final section. Appendices contain additional figures, proofs and computer codes for data-fitting.

7.2 Performance-based incentives and organizational risk appetite

This section discusses how the public and the professional community's attitude towards the use of performance-based incentives in risk management has changed because of the 2007–2009 financial crisis. Next the influence of incentives on risk taking are discussed from the viewpoint of principal-agent theory.

7.2.1 The influence of the 2007–2009 financial crisis on performance-based incentives

Performance-based compensation packages have been blamed for excessive risk-taking practices in the financial sector during the 2007–2009 financial crisis. In the USA this has led regulatory agencies to jointly issue a guidance on incentive compensation for banks (Cai, Cherny, & Milbourn 2010) and in the UK this had led to the introduction of a new Financial Services Bill which aims to promote effective risk management by regulating financial sector remuneration (Stilitz 2011, Smith 2010, Wighton 2009). The reason why performance-based compensation schemes allegedly gave an incentive to gamble, is their asymmetric pay-off structure which allows management to share in the organizational gains but does not let them share equally in its losses (Edwards 2010 p.286, Panning 2008). When based on general performance measures, such as current year income, these incentives align the motivation of those responsible for risk management with the rest of management (Lindorff 2009). Instead of inducing a more conservative stance towards risk such an incentive scheme improperly motivates those responsible for risk management to take on a more aggressive and entrepreneurial role and “chances are that they will cut corners in order to maximize profits” (Taleb, Goldstein, & Spitznagel 2009 p.81).

Because of their presumed detrimental effects on risk management, there is a call for a reform of performance-based incentives. At the executive level this reform is poised to encourage executives to focus on and commit to the long-term value of the firm, by means of restricted stock, restricted stock options (Bhagat & Romano 2009), indexing, and claw-backs (Faulkender, Kadyrzhanova, Prabhala, & Senbet 2010). At lower organizational levels reform initiatives aim to tie incentives to specific performance measures of secondary managers (Schwarcz 2009) and key professionals who are able to directly influence the risk profile of the organization (Taylor 2007). Some authors emphasize the continuing importance of performance-based incentives for risk management because they believe that the “systemic-risk-generating incentives”, which have caused the financial crisis, are still intact and cannot be curbed by more rules and regulation (Edwards 2010 p.255). Contrary to a move “from incentives to controls”, as suggested by the subtitle of Lam’s (2003) textbook on enterprise risk management, this suggests a move in the opposite direction. A reformation in performance-based incentives is believed to be able to induce desirable behavior in economic agents and contribute to the avoidance of catastrophic risk management failures.

Best practice guidelines in enterprise risk management advise companies to create and promote incentives and disincentives for risk management throughout the firm, with features that influence individual behavior and assure that staff members are focused on optimizing risk on a day-to-day basis (Hoffman 2002 p.167, *best practice #9*). However, it is argued that the more such an incentive scheme is based on quantitative performance measures, the more it distorts and corrupts the action patterns and thoughts of the individuals it is intended to monitor, and causes them to commit actions that are accountable but not responsible (Darley 1994, *Darley’s Law*). Based on this argument an incentive scheme should be composed of an appropriate blend of rules and subjectivity, in which incentives are reinvested in the company, and can be forfeited afterwards (Koenig 2006). Designing incentive schemes in risk management that reward desirable behavior with compensation, requires more understanding of how humans respond to incentives (Koenig 2008 p.12).

7.2.2 The influence of compensation on risk taking

Many economic studies have investigated the “behavior-altering abilities of incentives” (Skog 2009 p.6) albeit with a completely opposite concern in mind: the problem that the degree of risk taking by the agent is less than that desired by the principal. In this principal-agent incentive conflict the agent, unable to diversify risks specific to her managerial wealth (salary, bonus, shares) and human capital tied up in the firm (firm-specific competencies), acts risk averse, in fear of losing this capital, and does not maximize the value of the organization (Smith & Stulz 1985, *managerial self-interest hypothesis*). Shareholders, being able diversify their personal wealth, are assumed to be risk-neutral (Milgrom & Roberts 2000) whereas executives, precluded from effectively diversifying employment and personal wealth risk, are assumed to be risk-averse (Jensen & Meckling 1976). Executives who, for self interested reasons, avoid taking risk do this, assuming large returns accrue to large risks (Sharpe 1964), at the expense of organizational returns. Principal-agent theory suggests that incentive pay has the potential to align the risk preference of executives with those of shareholders by curbing executive opportunism and discouraging risk aversion (Devers, Cannella, & Yoder 2007 pp.1025, 1028).

Theoretical models in principal-agent theory suggests several ways in which incentive pay can aid in solving the problem of self-interested unobservable behavior by the agent, also known as moral hazard (see Prendergast 1999 for a literature overview and Laffont & Martimort 2002 for a textbook treatment). In an incentive scheme in which the compensation of the agent is a linear function of performance, the sensitivity (slope) and fixed fee component (intercept) of this line influence risk-taking by the agent. A high sensitivity to performance promotes risk-taking and a high fixed fee promotes risk-aversion (Holmstrom & Milgrom 1987, 1991). The result of this linear function of performance can be approximated by a two-wage scheme which pays a fixed fee (*bonus*) unless performance is below a threshold in which case a very low fee is paid (Holmstrom & Hilgrom 1987 pp.305–306). Less risk aversion can be induced in the agent by making her incentives a more *convex* function of the measure of performance (Ross 2004 p.224). This is comparable with owning a call option, *i.e.* a right to buy a security, where performance below the threshold results in a flat reward (*out-of-the-money*) while rewards increase sharply when above this hurdle (*in-the-money*). Call options on the stock of the entity only offer upside potential and limit downside risk, resulting in the aforementioned asymmetric pay-off structure. The introduction of a maximum limit on rewards (*cap*) can prevent unwanted excessive risk taking while excessive risk aversion, which results in low performance, can be penalized by introducing a minimum reward (*floor*) and a bonus which is triggered only when performance exceeds a particular hurdle rate (Grinblat & Titman 1989 p.819).

Several empirical, archival-based studies on the efficacy of incentive pay in aligning the risk preferences of executives and shareholders find that stock option pay encourages CEOs to undertake riskier investments (Datta, Iskander-Datta, & Raman 2001, Rajgopal & Shevlin 2002, Sanders 2001) while stock ownership negatively influences risk taking (Sanders 2001). However, in other studies high levels of stock option awards were found to have a negative effect on risk-taking (Chen & Ma 2011, Knopf, Nam, & Thorton 2002), and, unless firms were well-governed, stock option pay did not result in risk preference alignment (Desai & Dharmapala 2006). The behavioral agency model (Wiseman & Gomez-Mejia 1998, BAM) challenges principal-agent theory's simplistic depictions of risk by suggesting that certain decision situations (*e.g.*, monitoring, problem framing, and performance) differentially influence executive risk taking. BAM assumes executives to be loss averse, such that their desire to minimize losses exceeds their desire to maximize gains, which is descriptively more accurate than the more normatively oriented assumption of risk aversion adopted in principal-agent theory. Most theoretical models in the executive compensation research literature assume a normative rather than a descriptive executive risk attitude and empirical studies use ambiguous proxies for executive risk preferences. The influence of compensation on risk taking therefore largely remains an open question (Devers, Cannella, Reilly, & Yoder 2007 pp.1040–1041).

7.3 Risk preference in the domain of losses and organizational risk appetite

Experimental studies on decision making under risk typically report risk aversion in the domain of gains, risk seeking in the domain of losses, and loss aversion (Tversky & Kahneman 1992, Kahneman & Tversky 1979, *prospect theory*). The empirical finding of risk seeking in the domain of losses suggests that humans, given a choice between a risky and a sure loss with the same mathematical expectation, generally tend to behave in a risk seeking manner by choosing the risky alternative. This descriptive finding of risk seeking for losses is in stark contrast with the content of textbooks on risk management that typically employ expected loss calculations, which assume a risk neutral decision maker (*e.g.* Coleman 2012, Crouhy, Galai, & Mark 2006, Lam 2003). Recent risk management failures have led some authors in risk management to promote the adoption of a more averse attitude to risk whereby more attention is assigned to low probability – high impact events (Edwards 2010 footnote 35, Ai & Brockett 2008, Taleb 2007). Risk averse behavior is also promoted by the aforementioned new remuneration rules that emerged from the financial crisis (Wighton 2009). These rules specify that the ratio of variable to fixed fees in the compensation of the executive should be decreased while the absolute level of the executive's fixed fee should be increased (Smith 2010 p.38). This lower sensitivity of compensation to the entity's performance will make the executive incentive scheme under the new remuneration rules less convex and, according to principal-agent theory, stimulate risk averse behavior. The inherent stimulus in these new remuneration rules to promote risk aversion can then be interpreted as a method to curb the general tendency of humans to be risk seeking in the domain of losses.

The empirical finding from decision theory that decision makers are risk seeking in the domain of losses derives from stand-alone risk and need not necessarily apply to the context of organizational risk appetite, which is concerned with portfolio risk.[117] In decision theory the decision maker evaluates pairs of one-shot events whereas in risk management a portfolio of multiple risky events is evaluated. In choice experiments in the laboratory typically only one randomly selected choice is played for real whereas this luxury is not available in the reality of risk management. Survival of the organization during a particular period depends on the final outcome of all risky events in a portfolio taken together. Portfolio risk

[117] In this paper a portfolio choice is defined as a choice between two different sets of lotteries. This definition differs from that employed by Charness and Gneezy (2009) who define portfolio choice as the decision on "how much to invest in a risky asset". In their experimental framework this emanates to investing in individual lotteries contrary to sets of lotteries.

thus seems to be more concerned with final wealth, while stand-alone risk is concerned with the gains and losses associated with individual lotteries.

The steady pace of making a large number of consecutive choices in experimental studies and the typical absence of calculative aids in the laboratory promotes the use of an intuitive, reactive, quick and holistic system of thinking. This system, referred to as System 1, relies on cognitive heuristics to arrive at judgments and is particularly helpful when immediate action is required (Evans 2003). Actual decisions in risk management can be based on System 1 thinking but can alternatively be based on System 2 thinking, which is the deliberative, reflective, computational and rule governed system of thinking. System 2 is helpful when one encounters unfamiliar situations and when there is no time pressure. While the experimental finding of risk seeking for losses is induced under conditions that promote System 1 thinking, these conditions can be very different from the circumstances under which decisions about organizational risk appetite are made.

Decisions on organizational risk appetite are made by professional decision makers whose age profile and expertise differs considerably from that of the average participant in a laboratory experiment. It is an unresolved empirical question whether age has a significant influence on risk preferences in the domain of losses. Evidence on age-differences in the domain of monetary losses is mixed, reporting for similar decision tasks either no significant effect of age (Rönnlund, Karlsson, Laggnäs, Larsson, & Lindström 2005, Mayhorn, Fisk, & Whittle 2002) or a significant age difference with only young adults exhibiting risk seeking behavior (Mikels & Reed 2009). Age-related differences are attributed to an age-related decline in some aspects of attention, memory, learning, and cognitive control as well as motivational and affective changes (see for a meta-analysis Mata, Josef, Samanez-Larkin, & Hertwig 2011). Older adults show less responsiveness to losses relative to younger adults (Samanez-Larkin, Gibbs, Khanna, Nielsen, Carstensen, & Knutson 2007) and while younger adults strive for gains, older adults appear more likely to work toward preventing losses (Ebner, Freund, & Baltes 2006). Experimental evidence suggests that professional experience has a significant effect on performance in complex decision problems (Abdolmohammadi & Wright 1987) and when the contextual setting of the experiment is familiar to the "expert" decision maker (Dyer, Kagel, & Levin 1989, Cooper, Kagel, Lo, & Gu 1999). In addition it is argued that differences may occur when professional and social conventions and norms play a significant role in reality (Potters & Van Winden 2000, Carpenter, Burks, & Verhoogen 2005). From these empirical findings it is concluded that both age and expertise potentially have a profound influence on decisions with respect to risk appetite.

Decisions on organizational risk appetite are made in the context of portfolio risk, in an environment in which both intuitive and reflective thinking systems can be employed, and are made by experienced and older adult decision makers. On these three characteristics decisions on organizational risk appetite differ considerably from usual laboratory experiments that typically assess risk preferences for stand-alone risk only, involving inexperienced, young adult decision makers under conditions that promote the use of intuitive thinking.

It is therefore unclear whether the finding of risk seeking in the domain of losses applies in the decision context of organizational risk appetite.

7.4 Experimental design

This section contains an experimental design that enables us to directly observe the influence of compensation on the risk preference of the participant and assess whether the finding of risk seeking in the domain of losses applies in the context of organizational risk appetite. The design allows us to assess the effect of two very different financial incentive schemes on the risk appetite of the participants, their effectiveness in maintaining sufficiently large financial buffers to absorb losses, and their efficient use of risk management resources. First definitions for the effective and efficient use of risk management resources are provided. Then the experimental task is introduced which requires participants to simultaneously manage the risk of a portfolio of 25 mutually independent random loss events. Drawing on Monte Carlo simulation techniques we then identify a set of optimal choices along an efficient frontier. Next a fixed and variable incentive scheme is introduced and hypotheses for the influence of these conditions on the choices of the participants are formulated. At the end of the section details on the execution of the experiment are presented.

7.4.1 The experimental task

We design an experimental task in which participants face a portfolio of 25 mutually independent lotteries with negative financial consequences only. The experimental task requires participants to decide between reducing and accepting the inherent risk of the lotteries in this portfolio and thus demands them to express their risk appetite, *i.e.* their willingness to accept potential losses in the pursuit of value. Contrary to conventions in decision theory participants do not face the 25 lotteries consecutively but instead evaluate them simultaneously. The task accommodates two different responses to risk.

Enterprise risk management identifies four risk responses – avoiding, reducing, sharing, or accepting risk – that can be used in the management of risk (COSO 2004 p.3). Avoiding risk implies that the activity from which the risk originates is completely terminated. Risk is reduced when the entity takes action to reduce either the probability of occurrence or the impact of a risky event. Risk sharing, by means of insurance, outsourcing, and partnerships, allows the entity to share risk with other parties. Finally, accepting risk means that an entity does not take any further action to control risk and uses its financial buffers to absorb the consequences of risk. For the purpose of this chapter we refer to the entity's expenditure on risk reduction efforts as the *ex ante* use of risk management resources. The expenses made from the entity's financial buffers to absorb the losses that occur when the anticipated risky events become manifest, even after risk reduction efforts have been taken, are referred to as the *ex post* use of risk management resources.

For each lottery participants choose between two risk responses, to reduce or to accept risk, and they are therefore not allowed to share or avoid any lottery. Each participant receives a fixed budget that they divide between expenditures on risk reduction (*ex ante* use of risk management resources) and the maintenance of a financial buffer that can be used for the absorption of losses (*ex post* use of risk management resources). By making expenditures on risk reduction participants reduce the initial probability and negative impact levels of the lotteries in the portfolio to a level that they consider acceptable. These *ex ante* expenditures do, however, reduce the financial buffer that remains for the *ex post* absorption of losses. Participants are thus required to trade-off the *ex ante* use of risk management resources, which is a sure expenditure, with the *ex post* use of these resources, which is uncertain. Once the participant has determined the final probability and impact level for each lottery in the portfolio, which can also be its initial level, the participant accepts the final lottery composition of the portfolio. Once the portfolio is accepted a single draw is taken from each of its 25 lotteries. The total negative financial impact that results from this simultaneous draw from all 25 lotteries determines the *ex post* use of risk management resources. When the amount of required *ex post* risk management resources exceeds the financial buffer of the participant, the participants loses. If the financial buffer is sufficient to cover the total impact of the lottery draws, the participant wins. From the group of winning participants only one participant is randomly selected and paid out for real. The size of the financial reward depends on the incentive scheme that the participant has been randomly assigned to.

The experimental task is programmed in VBA for MS Excel and is designed around the risk matrix, which is a tool commonly used in risk management to visualize the portfolio of risky events that an entity faces. The screenshot in figure 7.1 shows the two risk matrices that are used in the experiment to elicit choices from the participants.

Both matrices show five probability levels (s) on the vertical axis and five levels of negative financial impact (i) on the horizontal axis. In the left matrix the 25 lotteries, signified by the symbol "x", are uniformly distributed over the 25 cells of the risk matrix. Each cell in this five-by-five matrix contains a binary lottery $x = (p_s : x_i, 0)$ with $.05 \leq p_s \leq .8$ and $-80 \leq x_i \leq -5$.[118] The complementary probability that there is no negative financial impact, $(1 - p_s)$, is understood and is not explicitly mentioned in the risk matrix. By clicking on a cell in the left risk matrix participants select a lottery which initial probability or impact level they want to reduce and by consecutively clicking on a cell in the right matrix they select a final probability and/or impact level for this lottery. When next the button "move the cross" is clicked on, the lottery moves in the left matrix from its original position to its new destination. Moving a lottery one cell leftward or downward halves the expected loss of this lottery. The cost of such a move is 5K. For each destination cell the right matrix displays the cost of moving the selected lottery to its new destination. The budget available for risk reduction (*i.e.* moving lotteries) and risk absorption in the experiment is in total 300K. Moving a selected lottery

[118] All monetary amounts are expressed in euro.

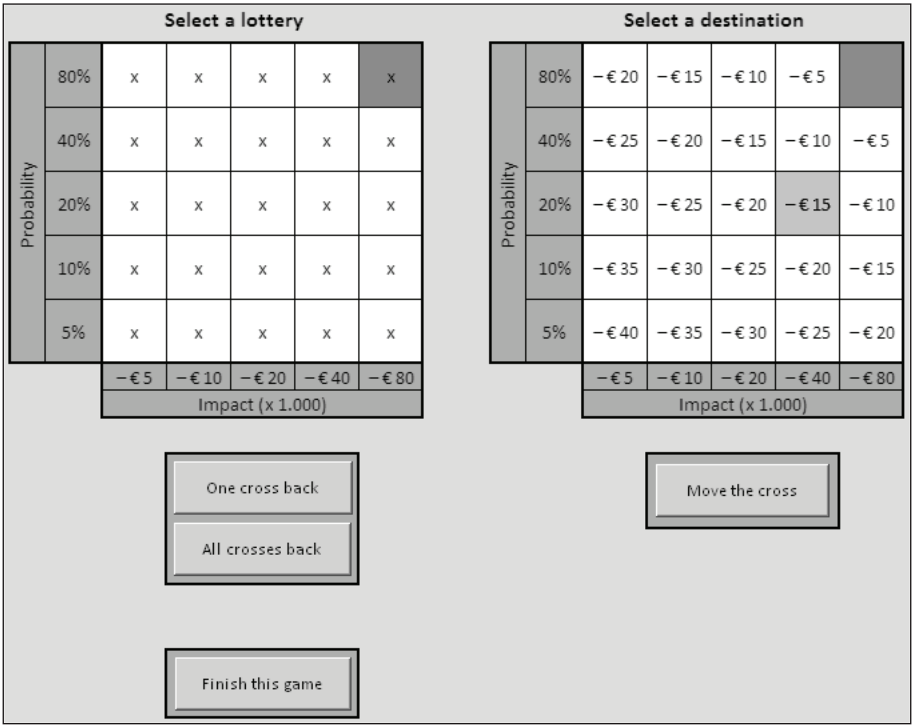

Figure 7.1 Left side screenshot of the computer program used to elicit choices.

upward or to the right is not possible. Depending on her risk appetite the participant can move any number of lotteries to any destination available, provided that her budget is still sufficient to accommodate these moves. While the cost of each move remains constant at 5K, the benefits of risk reduction decrease incrementally. This implies that the risk matrix in figure 7.1 exhibits marginal diminishing returns with risk reduction effort, an assumption used by several other authors in risk management (Cooper, Grey, Raymond, & Walker 2005 fig. 6.4 p.81, Li, Pollard, Kendall, Soane, & Davies 2009 fig. 2 p.1731, Daníelsson, Jorgensen, & De Vries 2002 p.1411).

Figure 7.1 displays an example in which the cell of the lottery that is to be moved, at the intersection of $p_5 = .8$ and $x_5 = -80$K, is highlighted in both risk matrices. In addition the right risk matrix highlights the destination cell for this lottery, at the intersection of $p_3 = .2$ and $x_4 = -40$K, together with the *ex ante* expenditure of 15K that effectuates the three-step lottery move from its original position (.8 : −80, 0) to its new destination (.2 : −40, 0). Participants were randomly assigned to either the matrix format presented in figure 7.1 or a format in which the axes for probability and impact were swapped. This alternation was introduced to exclude the possibility that a particular presentation of the axes would bias the experimental results.

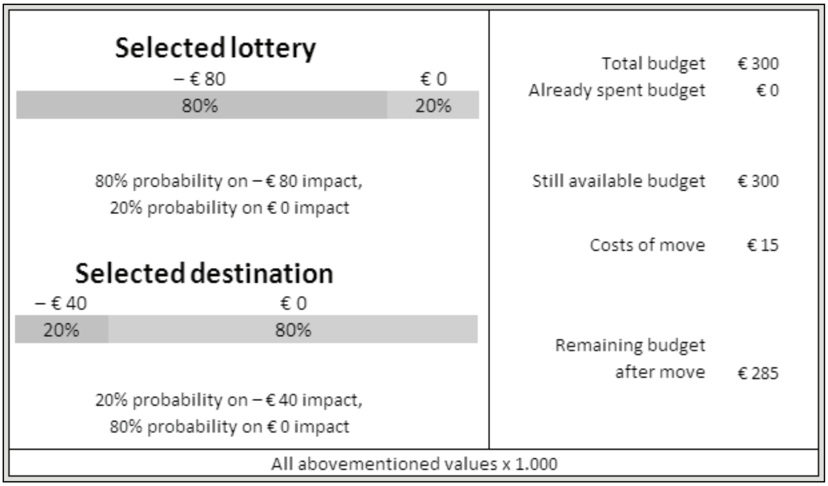

Figure 7.2 Right side screenshot of the computer program used to elicit choices.

The screenshot in figure 7.2 shows two bar-charts and a box with summary information, which are located on the right side of the risk matrices shown in figure 7.1. The box on the far right contains summary information on the budget that is still available, the cost of risk reduction, and the budget that remains after risk reduction. Both visually and in writing the bar-charts show the distribution of probability and impact of the selected lottery and its destination cell. In line with practices in risk management the cost of risk reduction is not integrated in the amounts of impact in the lower bar chart. Instead these costs are presented in the risk matrix on the right as well as the box with the summary information.

7.4.2 Effective and efficient use of risk management resources

Prior to executing the experiment we explored which final lottery compositions of the portfolio in the experimental task result in an effective and efficient use of risk management resources. For the purpose of this chapter the effective use of risk management resources is defined as attaining a desired level of assurance that losses do not exceed financial buffers. This desired level of assurance is specified by $P(d > b) \leq a$ in which $P(d > b)$ represents the cumulative probability P that losses d exceed financial buffers b, with b, $d > 0$, and a represents the desired level of assurance, with $0 \leq a \leq 1$. We define efficiency as the minimum use of risk management resources, consisting of both *ex ante* and *ex post* expenditures, to obtain a specific level of assurance. We identify an efficient final lottery composition by the minimum sum of *ex ante* expenditure c and *ex post* expenditure d (*i.e.* losses that manifest themselves) that attains a desired level of assurance, *i.e.* $\inf \{c + d \mid P(d > b) \leq a\}$. In our experiment the size of the financial buffer is $b = 300\text{K} - c$.

We constructed 251 final lottery compositions and determined for each the *ex ante* expenditure c and three cumulative distribution functions for the *ex post* expenditure d

based on Monte Carlo simulations of 10,000 independent lottery draws each. For each final lottery composition the three cumulative distribution functions were used to calculate average values of the mean use of risk management resources $\mu(c + d)$, the standard deviation of the use of risk management resources $\sigma(c + d)$, and the probability that losses exceed the financial buffer $P(d > b)$. Figure 7.3 displays results of these 251 final lottery compositions with the probability of exceeding budget $P(d > b)$ on the horizontal axis and the mean use of risk management resources $\mu(c + d)$, presented in negative values, on the vertical axis.[119] The starting position of participants in the experimental task, *i.e.* the status quo where no risk reduction expenditures have yet been made, is displayed by the horizontal dashed line expressing a level of *ex post* losses of 240.25K and a vertical dashed line expressing the corresponding probability of exceeding financial buffers of 20.8%. An efficient frontier displays for each level of assurance the most efficient mean use of risk management resources. The most efficient and effective final lottery compositions on the efficient frontier, which we refer to as *efficiency 1st* and *effectiveness 1st*, are respectively indicated by the symbols Δ and □. Table 7.1 presents the $\mu(c + d)$, $\sigma(c + d)$, and $P(d > b)$ values for the final lottery compositions of the status quo, "efficiency 1st" and "effectiveness 1st".

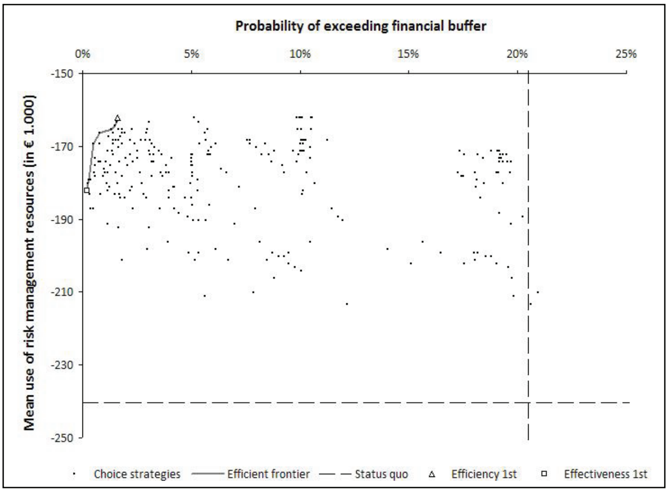

Figure 7.3 Comparison of the mean use of risk management resources, $\mu(c + d)$, and the probability that losses exceed the financial buffer, $P(d > b)$, for 251 final lottery compositions (displayed are average values resulting from three Monte Carlo simulations with n = 10,000 performed in @Risk).

[119] In figure 7.8 in appendix A an additional scatter diagram is presented for comparison of $\mu(c + d)$ with $\sigma(c + d)$.

Table 7.1 Mean use of risk management resources, $\mu(c + d)$, the probability that losses exceed the financial buffer, $P(d > b)$, and the standard deviation of the use of risk management resources, $\sigma(c + d)$, of the status quo, "efficiency 1st" and "effectiveness 1st" final portfolio compositions (displayed are average values resulting from three Monte Carlo simulations with n = 10,000 performed in @Risk).

	$\mu(c + d)$	$P(d > b)$	$\sigma(c + d)$
status quo	–240K	.208	77K
efficiency 1st	–162K	.016	59K
effectiveness 1st	–182K	.002	37K

On the criteria mentioned in table 7.1 the status quo is dominated by the "efficiency 1st" and "effectiveness 1st" final lottery compositions. A comparison of the marginal cost of risk reduction with its marginal benefit, expressed as changes in the risk's expected value (EV), allows one to identify efficient levels of risk reduction (Vaughan 1997 p.67). "Efficiency 1st" consists of moving six lotteries in the top-right corner of figure 7.1, each characterized by an expected loss of 16K or higher, to the top-to-right diagonal in the risk matrix defined by an expected loss of 8K. In this composition the lowest mean use of risk management resources, *i.e.* 162K, is attained. Given this minimum expected loss, the smallest attainable probability that losses exceed the financial buffer is 1.6%, which is effected by moving all six lotteries to the left, which reduces their impact. While the selection of lotteries to transfer is under "efficiency 1st" consistent with risk neutrality, the choice to reduce the impact of these lotteries instead of their probability is consistent with risk aversion (see Appendix B for proof). In "effectiveness 1st" the aforementioned six lotteries are all transferred to the –10K column. In addition the two lowest-probability lotteries in the –80K column are transferred to the –20K column. This lottery move is both in the selection of lotteries as well as the direction of their transfer consistent with a risk averse attitude. We expect that many participants will not be able to attain the optima on the efficient frontier in figure 7.2 because $\mu(c + d)$ and $P(d > b)$ are not directly observable in the experiment which makes it hard for them to realize optimum choices.

The cumulative distribution functions of the final lottery compositions in table 7.1 are presented in figure 7.4. The slopes of the curves indicate the degree of risk aversion of the three lottery compositions: the steeper the slope, the more risk averse. Inspection of the intersection of the curves with the vertical line at –300K reveals the large difference between the probability of exceeding budget in the status quo and "effectiveness 1st".

7.4.3 Incentive conditions

To assess the influence of performance-based incentives on the performance of participants in the experimental task we elicit their risk appetite under two very different incentive conditions, a fixed and a variable incentive scheme. Participants are randomly assigned to either

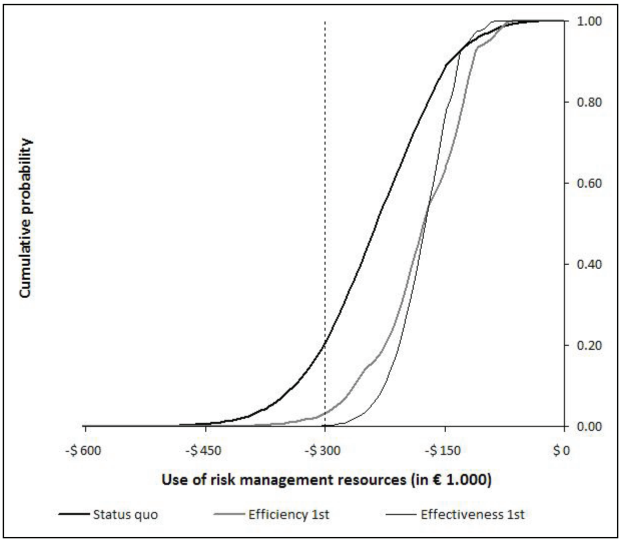

Figure 7.4 Cumulative distribution functions of the status quo, "efficiency 1st" and "effectiveness 1st" lottery compositions.

of these two conditions. When the amount of funds needed to absorb losses exceeds the financial buffer that remains after risk reduction, participants in both conditions lose their eligibility for payment of a personal financial reward. When the financial buffer is not exceeded then participants remain eligible for receiving a personal fixed fee of 2,067 in the fixed incentive condition. When in the variable incentive condition the financial buffer is not exceeded, the participants remain eligible for receiving a personal fee of .015 of the financial buffer that remains after absorption of losses, which implies a maximum fee of 4,500.[120] Eligible participants enter a random lottery that selects one participant who is paid out for real.

The fixed fee is performance-contingent in the sense that the variability of this compensation depends on the performance of the participant in assuring that losses do not exceed the financial buffer. Under this fixed incentive scheme the optimum risk management effort for the participant is one which would almost eliminate the probability of losses exceeding the financial buffer, regardless of its cost (Han 1996 p.382, Kim, Nam, & Thornton 2008 p.230). Under the fixed incentive scheme a principal, deliberately or not, allows the agent to aim for an optimum in effectiveness in which efficiency is completely ignored (Van den Brink 2007 p.2, Daníelsson, Jorgensen, & De Vries 2002 p.1410). A fixed incentive scheme

[120] When the participant decides not to move any lottery and incurs no losses then the participant is eligible for receiving a financial reward of .015 300K = 4,500. Monte Carlo simulations in @Risk indicate that the lowest expected loss in the experimental task is −162.25K. Given our aim to provide a high incentive of around 2K, we decided for the fixed fee on the amount 2,067 derived from the calculation .015 × (300K – 162.25K).

provides a disincentive for under spending this budget and creates an incentive for demanding budget raises. Similar to up front high compensation, it on the other hand prevents the agent from seeking overly ambitious, difficult-to-implement strategies (Dow and Raposo's 2005 pp.2704, 2718, *ex ante contracting*). In figure 7.3 the optimum choice under the fixed incentive scheme is the "effectiveness 1st" lottery composition indicated by the symbol □ on the efficient frontier.

In both the fixed and variable incentive conditions participants, assuming self-interest, strive for effectiveness. Participants in the variable incentive condition are in addition interested to use risk management resources efficiently because more efficient use implies that a higher personal financial reward can be earned. Because risk reduction is costly, a variable incentive scheme curbs excessive risk aversion while excessive risk taking is curbed because the size of the allocated budget acts as a natural cap on rewards. In comparison with the fixed incentive scheme the risk manager in the variable scheme bears more risk and her incentives are to a much higher degree tied to her performance (Han 1996 p.382). Under the variable incentive scheme optimum lottery compositions are located somewhere on the efficient frontier depending on the trade-off that the participant makes between an efficient and an effective use of risk management resources.

Section 7.2 concluded that the influence of compensation on risk taking largely remains an open empirical question. The influence of performance-based incentives on the participants' risk appetite and efficient and effective use of risk management resources is in this experiment tested on the basis of the following three hypotheses. The first hypothesis is derived from the prediction from the theory of incentives that a fixed fee relative to a variable fee induces more risk aversion in the agent. Both the recent thrust towards risk averse regulation and the call for behavioral instruments to curb excessive risk seeking behavior warrant further empirical investigation of this prediction in the context of organizational risk appetite.

Hypothesis 1: In the fixed incentive condition the agent's risk appetite exhibits more risk aversion than in the variable incentive condition.

The other two hypotheses are related to the efficient and effective use of risk management resources and state that comparatively a fixed fee promotes effectiveness and a variable fee promotes efficiency.

Hypothesis 2a: In the variable incentive condition efficiency in the use of risk management resources is attained to a higher degree than in the fixed incentive condition.

Hypothesis 2b: In the fixed incentive condition effectiveness in the use of risk management resources is attained to a higher degree than in the variable incentive condition.

Section 7.3 concluded that risk seeking behavior in the context of organizational risk appetite remains open empirical question as well. We therefore, in addition, address in this experiment the question to what degree individual choices that are made in a portfolio context correspond with the empirical finding of risk seeking in the domain of losses.

7.4.4 Details of the experiment

The teaching staff of the Rotterdam University of Applied Sciences was invited by e-mail to participate in a study in which a game was to be played which offered the prospect of earning a substantial sum of money. This message informed participants that only one participant would be randomly selected for actual payment. Participants were randomly assigned to one of the two incentive conditions and the two axis-presentations. Upon arrival participants watched individually a 20 minute video instruction on the game which was tailored to their incentive condition and axis-presentation. In an individual meeting with the experimenter they subsequently were allowed to ask questions and were interviewed in order to verify whether the instruction by video had been successful. Participants then played the game on an individual basis without any time-pressure and with the possibility to use calculative aids. The experiment took place on December 16, 2011. A pilot test with student participants was carried out two months before the experiment.

A few days after the experiment took place a single lottery draw was taken for each participant from all of the 25 lotteries in their final destination to determine how much financial impact was sustained by each of the participants. Lottery draws were made using Monte Carlo simulation software @Risk and were recorded on video. Out of the participants whose remaining budget was sufficient to absorb the financial impact of the draws one participant was randomly selected for receiving payment. Participants were subsequently informed by e-mail about their results and were provided with a web link to the video recordings of the drawing.

In total 36 staff members from a variety of departments participated in the study.[121] Nine participants who intended to participate failed to show up. Out of the 36 who did participate one quit the study during participation while the results of a participant who skipped parts of the video instruction and ignored instructions were excluded from the study. This study reports the results of the remaining 34 participants, 24 male and 10 female. With an average age of 42 years, the age-profile of the participants is clearly different from that of regular laboratory experiments. Participants spent

[121] At the moment that the study ran teaching staff at the university amounted to approximately 2,000 members. Potential reasons why the number of participants in the experiment was low relative to the population size (1.8%) are the unfamiliarity of the teaching staff with acting as participants in research projects, the heavy teaching load at the university of applied sciences, a fair amount of part-time employees that do not work on Fridays, large travelling distances between university buildings, and heavy rain at the time the experiment took place. The background of the 36 participants was Finance (20), International business (3), Technical (3), Management (2), Marketing (2), Health (1), Social (1), Arts (1), and Media (1).

on average 26 minutes on playing the game and between their arrival and departure on average 63 minutes lapsed.

After finishing the experimental task participants were requested to state to what degree they concur with the following ten statements on a scale between 1 (fully agree) and 7 (fully disagree).

Table 7.2 Questionnaire with ten statements.

1	I consider the size of the financial rewards that can be won in the game attractive.
2	I found it difficult to make choices in this game.
3	The choices I made during the game have reduced my chances on losing.
4	I find that the study has been clearly explained to me.
5	The prospect of winning the price stimulated me to think and act carefully.
6	My objective in playing the game was to minimize my chances on losing.
7	I find that I made daring choices during the game.
8	In playing the game it was my objective to maximize the difference between the buffer and the total financial impact.
9	In playing the game I made prudent choices.
10	In general I like to calculate.

Additionally participants were requested to state their age, sex, fulltime equivalent and department.

7.5 Results

We first discuss some general findings and then analyze several qualitative judgments of the participants in relation to the game. The effect of the incentives on risk appetite (*hypothesis 1*) is assessed by independent and paired samples tests. On the basis of the performance of participants in the game and simulation statistics derived from this the effect of the incentive conditions on efficiency and effectiveness is assessed (*hypothesis 2a* and *2b*).

7.5.1 General findings

Participants spent on average 95K on moving lotteries in the game, $\mu(c)$, the mean financial impact they sustained was 98K, $\mu(d)$, and the amount of funds remaining after sustaining this impact was on average 106K, $\mu(b - d)$. For 3 participants funds were not sufficient to absorb impact. The potential earnings made by the participants were on average 1,783. The two axis-presentations that were used in the experiment did not have a significant impact on the results of this study.[122]

7.5.2 *Analysis of qualitative judgments in relation to the game*

After reaffirming their choices in the game participants stated to what degree they concur with ten qualitative statements.[123] From their responses the following inferences are made.

More than half of the participants fully agree that the financial rewards offered in the study are attractive (question 1: mean 2.38, SD 1.907). Even though participants on average neither agree nor disagree whether the prospect of winning a prize had stimulated them to think and act carefully (question 5: mean 3.47, SD 1.862), there is a positive relationship between judgments of the reward's attractiveness and its perceived positive effect on careful participation in the game (Spearman correlation test, $\rho = .591$, $p = .000$).

Participants did in general agree with the game's objective being "the minimization of one's chances on losing" (question 6: mean 2.53, SD 1.796) but also with its objective being "the maximization of the difference between buffer and total financial impact" (question 8: mean 2.59, SD 1.794). The median responses to both objectives were not significantly different (Wilcoxon signed rank test, $p = .849$) and were positively related (Spearman correlation test, $\rho = .355$, $p = .039$). In the fixed incentive condition participants judge their choices as significantly more prudent than daring (question 9 *vs* 7). In the variable reward condition the same relationship did not prove to be significant (Wilcoxon signed rank tests, respectively $p = .028$ and $p = .068$).

A large majority of the participants indicated that the study had been clearly explained to them (question 4: mean 1.56, SD .960). Participants in the variable reward condition

[122] Given $n = 25$ cells and a significance level $\alpha = .05$ for testing one hypothesis then a significance level of $\alpha = .05 / 25 = .002$ is required for testing the significance for each cell (Bonferroni correction). The p-value for the distribution of cell (.2 : −10K, 0) was higher than this significance level (Mann-Whitney U test, $p = .038$). It was therefore concluded that there is no significant difference between the two axis-presentation formats.

[123] Mean responses and standard deviations are mentioned between brackets.

Q1) I consider the size of the financial rewards that can be won in the game attractive. (2.38, 1.907).

Q2) I found it difficult to make choices in this game. (4.06, 2.117).

Q3) The choices I made during the game have reduced my chances on losing. (3.03, 1.660).

Q4) I find that the study has been clearly explained to me. (1.56, .960).

Q5) The prospect of winning the price stimulated me to think and act carefully. (3.47, 1.862).

Q6) My objective in playing the game was to minimize my chances on losing. (2.53, 1.796).

Q7) I find that I made daring choices during the game. (4.47, 1.846).

Q8) In playing the game it was my objective to maximize the difference between the buffer and the total financial impact. (2.59, 1.794).

Q9) In playing the game I made prudent choices. (3.09, 1.545).

Q10) In general I like to calculate. (2.82, 1.817).

were even more positive about the clarity of explanations in the study than in the fixed reward condition (Mann-Whitney U test, $p = .001$). Participants' opinions regarding the difficulty of making decisions in the game were greatly dispersed and on average neutral (question 2: mean 4.06, SD 2.117). The assessment of difficulty was negatively associated with one's preference for making calculations ($\rho = -.565$, $p = .000$).

7.5.3 *The effect of incentive conditions on the expression of risk appetite in the risk matrix*

In this section we describe the choices made by the participants under the variable and fixed reward conditions and use independent and paired sample tests to test hypothesis 1.

7.5.3.1 Independent samples tests

The median number of lotteries that remain in the matrix after participants have chosen which lotteries to move are presented in figure 7.5 under variable and fixed incentive conditions.

(a)

Probability						
	80%	1	1	0	0	0
	40%	1	1	1.5	0	0
	20%	1	1	1.5	2	0
	10%	1	1	1	1	0.5
	5%	1	1	1	1	0.5
		-€ 5	-€ 10	-€ 20	-€ 40	-€ 80
		Impact (x 1,000)				

(b)

Probability						
	80%	1	1	0	0	0
	40%	1	1	1	0	0
	20%	1	1	1.5	0	0
	10%	1	1	1	1	0
	5%	1	1	1	1	1
		-€ 5	-€ 10	-€ 20	-€ 40	-€ 80
		Impact (x 1,000)				

Figure 7.5 Median number of lotteries that remain in the matrix after participants have moved lotteries under (a) the variable incentive and (b) the fixed incentive condition.

Under the variable incentive condition the area in figure 7.5a that does not contain lotteries is bounded by a top-to-right diagonal which contains cells with identical expected values. The triangular shape of this area, symmetrically spaced along the probability and impact axes, is in conformity with expected value theory. In figure 7.5b, under the fixed incentive condition, the number of cells that do not contain lotteries is larger and in addition suggests an aversion to low probability – high impact lotteries. Where in figure 7.5a the cells ($.2 : -40K, 0$) and ($.1 : -80K, 0$) contain respectively 2 and 0.5 lotteries, these low probability – high impact cells do not contain any lotteries in figure 7.5b. The medians and the distribution of lotteries in cell (.2 : −40K, 0) in the two conditions are significantly different (Median test, $p = .012$; Mann-Whitney U test, $p = .019$).

Interestingly the distributions of lotteries in cell (.05 : −5K, 0), the cell in the bottom-left corner of the matrix, are in the fixed incentive condition significantly different from those in the variable incentive condition (Mann Whitney test, $p = .008$).[124] Apparently the fixed reward stimulus conditioned some participants to take risk aversion to the extreme by choosing the destination which enabled maximum risk reduction.

These observations on figure 7.5 are consistent with the hypothesis that the fixed incentive condition induces more risk aversion than the variable incentive condition. This impression is corroborated by the median number of lottery moves which with 11.5 moves in the variable incentive condition is significantly lower than the 21.5 moves in the fixed incentive condition (Median test, $p = .039$).[125] Median expenditure in the fixed incentive condition exceeds that in the variable reward condition by 50K, which entails 1/6th of the available budget. In the variable incentive condition the median number of lottery moves is equally distributed between downward moves along the probability axis and leftward moves along the impact axis, with respectively 5.5 and 5 and in the fixed incentive condition these numbers were respectively 8 and 12. The medians of both the number of moves along the probability axis as well as the number of moves along the impact axis are in the variable and fixed incentive conditions not significantly different from each other (Median tests, respectively $p = .311$ and $p = .169$). The preliminary result of our analysis is that our first hypothesis cannot be rejected on the basis of independent samples tests.

7.5.3.2 Paired samples tests

Our next step is to use paired samples tests to assess whether the number of lotteries in cells with identical mathematical expectation, which are located on the top-to-right diagonals of the matrix, is significantly different between pairs of cells. Even though lotteries on any of the diagonals from the top to the right side of the matrix have the same mathematical expectation, the cells on these diagonals differ in the maximum number of lotteries that they can accommodate. The maximum number of lotteries that each cell potentially can receive is determined by their relative position to the other cells in the matrix in combination with the restriction that lotteries in the experiment can only be moved to the left and downward. These maxima are the smallest at the end points of each diagonal

[124] Even though the medians of cell (.05 : −5K, 0) are identical in both conditions, observation of the two distributions shows that for 7 out of the 16 participants in the fixed incentive condition this cell contains more than one lottery while in the variable incentive condition this is only true for 1 out of 18 participants.

[125] The distributions from which these medians are derived are significantly different between these two conditions as well (Mann Whitney test, $p = .010$).

and largest at its center. Crosswise along the diagonal from the top-right downwards to the bottom-left they furthermore increase exponentially from zero to a maximum of 24 (see figure 7.6).

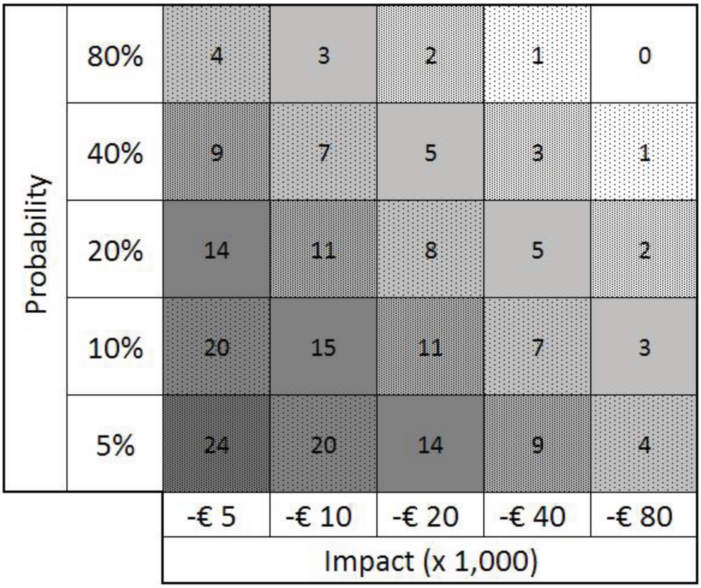

Figure 7.6 Distribution of the maximum number of lotteries that each cell of the matrix potentially can receive (cells on the top-to-right diagonals are identical in expected value and share the same shading).

Under expected value theory cells in figure 7.6 with identical maxima are equally qualified as recipients of lotteries originating from other cells. Given that under expected value theory a move leftward or downward is equally preferred we can expect that the observed distribution of lotteries within the risk matrix will under risk neutrality follow the distribution of the maxima in figure 7.6. Any significant difference which runs opposite to this direction is then attributed to the risk preference of the participants.

For each pair of cells along the top-to-right diagonals we assessed whether the number of lotteries contained in these cells was significantly different. The cells (.8 : −5K, 0) and (.05 : −80K, 0), both having the same expected value and identical maxima, differed significantly from each other in terms of the number of lotteries they contain (Sign test, $n = 34$, $p = .021$). Given that both the expected values and the maxima of these cells are the same, the preference for cell (.8 : −5K, 0) as a lottery destination over (.05 : −80K, 0) is interpreted as a risk averse preference. To indicate that the median difference in the observed number of lotteries between cells is significantly positive, we use the notation (.8 : −5K, 0) > (.05 : −80K, 0). For 7 pairs significant differences were found that directionally correspond with differences in the maxima of these cells and thus conform with predictions under expected value theory.[126] For the remaining 22 pairs nonparametric paired sample tests did not reveal significant differences.

The variable incentive condition revealed (.05 : −20, 0) ≻ (.1 : −10, 0), a preference that runs opposite to the difference in the maxima of these cells (Wilcoxon signed rank test, $n = 18$, $p = .038$). In combination with the fact that the expected values of these cells are the same, this preference is interpreted as an indication of risk seeking behavior in the domain of losses. Significant differences that directionally correspond with differences in cell maxima were found for 7 pairs and none for the remaining 22 pairs.[127]

Our prediction that the fixed incentive condition induces risk aversion was confirmed by two significant differences, in particular (.8 : −10K, 0) ≻ (.2 : −40K, 0) (Wilcoxon signed rank test, $n = 16$, $p = .029$, one-sided) and (.8 : −10K, 0) ≻ (.1 : −80K, 0) (Sign test, $p = .035$, one-sided). Both in the full sample and in the variable incentive condition significant differences were found that directionally correspond with differences in cell maxima. Such differences were, however, completely absent in the fixed incentive condition. This is another indication that the distributions of lotteries under the two incentive conditions are clearly different.

The results of both the paired and independent samples test suggest that in the fixed incentive condition the risk appetite as expressed in the risk matrix exhibits more risk aversion than in the variable incentive condition. We therefore conclude that hypothesis 1 cannot be rejected.

7.5.4 The effect of incentive conditions on risk management efficiency and effectiveness

In order to assess the effect of the participants' choices on the risk management objectives of efficiency (*hypothesis 2a*) and effectiveness (*hypothesis 2b*) we analyze their performance within the experiment itself and use their choices to derive Monte Carlo simulation statistics for $\mu(c + d)$, $\sigma(c + d)$, and $P(d > b)$.

[126] After each pair comparison significance levels are mentioned between brackets. Unless otherwise stated use of Wilcoxon signed rank tests is assumed. The following seven significant results correspond directionally with what is to be expected on the basis of the maximum number of lotteries that each of the cells can contain under expected value theory.
(.4 : −20K, 0) ≻ (.1 : −80K, 0) [p = .011], (.2 : −40K, 0) ≻ (.1 : −80K, 0) [p = .018],
(.2 : −20K, 0) ≻ (.8 : −5K, 0) [Sign test, p = .035], (.2 : −20K, 0) ≻ (.4 : −10K, 0) [p = .030],
(.4 : −10K, 0) ≻ (.05 : −80K, 0) [Sign test, p = .008], (.2 : −20K, 0) ≻ (.1 : −40K, 0) [p = .020],
(.2 : −20K, 0) ≻ (.05 : −80K, 0) [p = .010].

[127] The following seven significant results correspond directionally with what is to be expected on the basis of the maximum number of lotteries that each of the cells can contain under expected value theory.
(.4 : −20K, 0) ≻ (.8 : −10K, 0) [p = .039], (.2 : −40K, 0) ≻ (.8 : −10K, 0) [p = .023],
(.4 : −20K, 0) ≻ (.1 : −80K, 0) [p = .036], (.2 : −40K, 0) ≻ (.1 : −80K, 0) [p = .004],
(.2 : −20K, 0) ≻ (.8 : −5K, 0) [p = .031], (.4 : −10K, 0) ≻ (.05 : −80K, 0) [p = .025],
(.2 : −20K, 0) ≻ (.05 : −80K, 0) [p = .008].

7.5.4.1 Performance of participants in the experiment

Between the two incentive conditions no differences were observed in the amount of losses sustained, the financial buffer that remains *ex post*, the number of participants for which losses exceeded their financial buffer, and their earnings (Mann-Whitney tests, respectively p = .717, p = .104, p = .058, p = .337).[128]

As already reported in the previous section expenditures on risk reduction are significantly higher in the fixed incentive condition than in the variable incentive condition. For the variable incentive group this result is corroborated by the distribution of the empty median cells in figure 7.5a which corresponds with the marginal benefit-marginal cost rule (Vaughan 1997), while the distribution of empty cells for the fixed incentive group in figure 7.5b does not. This suggests that the fixed incentive is not promoting efficient use of resources and thus seems to confirm our hypothesis 2a.

This line of argumentation is, however, far from satisfactory for two reasons. For one, risk aversion implies that a sure loss is preferred to a risky loss with the same mathematical expectation and thus by definition implies inefficient expenditure of resources in risk reduction. Second, efficient use of risk management resources implies that the combination of *ex-ante* risk reduction expenses and the *ex-post* absorption of losses is minimized. This suggests that the size of post-loss resources should be used as a measure of efficiency. However, this can just as well be construed as a measure for effectiveness which then blurs the distinction between the two objectives. In line with the definition of efficiency and effectiveness in section 7.4.2 hypotheses 2a and 2b are therefore in the next paragraph tested using simulation statistics.

7.5.4.2 Performance of participants on the basis of simulation statistics

Table 7.3 presents descriptive statistics of the Monte Carlo simulations for the status quo and the final lottery compositions of participants in the variable and fixed incentive conditions.

Table 7.3 Comparison of simulation statistics in the status quo with the median (mean) simulation statistics in the variable and fixed incentive conditions of mean use of risk management resources, $\mu(c + d)$, the probability of losses exceeding the financial buffer, $P(d > b)$, and the standard deviation for use of risk management resources, $\sigma(c + d)$.

	$\mu(c + d)$	$P(d > b)$	$\sigma(c + d)$
status quo	–240K	.21	77K
variable incentive condition	–173K (–180K)	.06 (.09)	78K (77K)
fixed incentive condition	–194K (–210K)	.06 (.11)	63K (65K)

[128] The null-hypothesis that the distribution of earnings, *i.e.* the monetary amount which a participant would obtain in case random selection indicated her as winner, is the same in both conditions could not be rejected on the basis of a Mann-Whitney U test (p = .337) but was rejected in a Kolmogorov-Smirnov test (p = .029). A test for identical medians revealed significant differences as well (Median test, p = .001). In the variable incentive condition median earnings were 2,025 and for the alternative condition these had been fixed in advance at 2,067. Mean earnings were 1,875 in the variable incentive condition and 1,679 in the fixed incentive condition.

Compared to the status quo, participants in both conditions managed to significantly reduce the mean use of risk management resources, $\mu(c + d)$, and thus improve efficiency. Compared with the status quo the simulated mean use of risk management resources of the variable incentive condition is less negative at a significance level of $\alpha = .001$ (Wilcoxon signed rank test, p = .000) while in the fixed incentive condition this decrease is only significant at the $\alpha = .05$ level (p = .049). These different levels of significance suggest that in the variable incentive condition the risk management objective of efficiency is attained to a higher degree than in the fixed incentive condition. A Kolmogorov-Smirnov test (p = .023) and a one-sided Mann-Whitney U test (p = .033) indeed confirm that average use of risk management resources in the fixed incentive condition is significantly higher than in the variable incentive condition. We therefore cannot reject hypothesis 2a and conclude, as predicted by the principal-agent theory of incentives, that variable incentives serve efficiency better than fixed incentives. However, in both conditions participants failed to minimize mean use of risk management resources to the optimum levels (see table 7.1) which were significantly lower than theirs (Wilcoxon signed rank test, in both conditions p = .000). Thus, even though participants were able to make significant improvements on efficiency they were not able to attain optimum levels.

By reducing the probability of losses exceeding the financial buffer, $P(d > b)$, participants were in comparison with the status quo in both the variable and fixed incentive conditions able to provide more assurance that post-loss resources are sufficient (Wilcoxon signed rank tests, respectively p = .000 and p = .030). Inspection of table 7.3 reveals that the median probability of exceeding is in both conditions approximately reduced by two-thirds. These medians do not differ significantly between conditions (Mann Whitney U test, p = .730). Contrary to our prediction we therefore reject hypothesis 2b and conclude that in both incentive conditions participants attained comparable levels of effectiveness. Again these levels fall significantly short of the optimum levels of effectiveness (see table 7.1) (Wilcoxon signed rank tests at significance level $\alpha = .01$).

Because standard deviation is often used as a measure of risk in finance, it is relevant to analyze its low value in the fixed incentive condition compared with its higher value in the status quo and variable incentive condition (see table 7.3). While a Mann Whitney U test on the simulated standard deviations of the use of risk management resources, $\sigma(c + d)$, did not reveal a significant difference between the two conditions (p = .168), a one-sided Wilcoxon signed rank test comparison with the status quo did reveal a significant difference with the fixed incentive condition (p = .040).[129] It is revealing that such a reduction in spread did not result, however, in a lower probability of exceeding budget in the fixed incentive condition.

The scatter diagram in figure 7.7 presents the simulation results of the participants together with the efficient frontier and the status quo. The triangles, which designate results under the variable incentive condition, represent with one exception an improvement on

[129] A Wilcoxon signed rank test comparing the simulated standard deviation of the status quo with the variable incentive condition did not reveal significant different (p = .879).

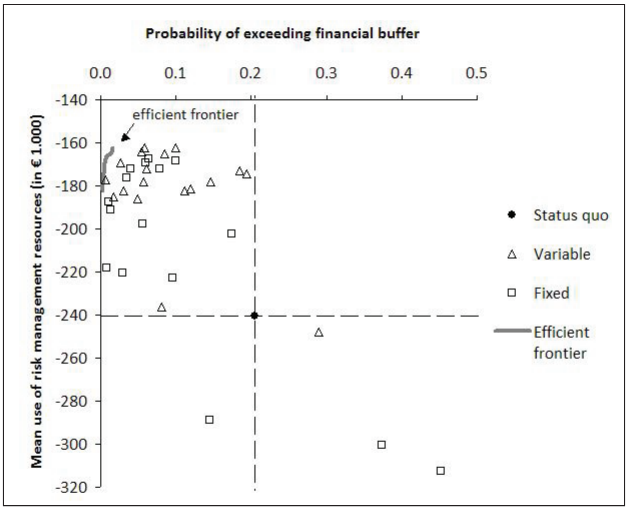

Figure 7.7 Scatter diagram of the simulated mean use of risk management resources (in € 1.000) and the probability of losses exceeding the financial buffer in the variable (n = 18) and fixed incentive condition (n = 16).

the status quo. While these triangles mostly populate the upper half of the top-left quadrant, the squares, which represent results under the fixed incentive condition, are in comparison much more dispersed. Triangles and squares do not appear to differ in dispersion in terms of the probability that losses exceed the financial buffer.[130] These observations corroborate the conclusions derived from the statistical analysis that both conditions do not significantly differ in terms of effectiveness but do significantly differ in terms of efficiency.

7.5.5 Assessment of utility under expected utility using Maximum Likelihood Estimation

Under the assumptions of expected utility (EU) theory the choices made by the participants are used to estimate a utility function using Maximum Likelihood Estimation (MLE). Each of these choices implies that a destination cell together with its associated cost is preferred to all other alternative destinations and associated costs in the matrix. For a lottery that originates from a cell from the i^{th} column and the s^{th} row its new destination is preferred to $i \times s - 1$ alternative destinations in the risk matrix, including the cell of origin. When for example the lottery (.8 : −80K, 0) is moved to another destination, then this destination is preferred to all other 24 destinations, including the cell of departure itself. For this specific lottery one move in the matrix thus implies 24 discrete preferences. For lotteries which originate from other cells in the risk matrix the number of discrete preferences is lower because the game only

[130] A scatter diagram comparing the simulated mean and standard deviation of the use of risk management resources is presented in appendix A.

accommodates leftward and downward moves. Because expected utility theory and portfolio choice share a focus on final wealth, we performed a MLE assuming linearity in probability and nonlinearity in utility. As MLE requires large samples this analysis is undertaken at the group level.[131]

The discrete preferences resulting from the experiment were fitted to an expected utility (EU) model using MLE assuming a power utility function (CRRA, Pratt 1964).[132] The best fit for the utility function $u(x) = -((-x)^{\beta})$, with x representing monetary outcomes, is $\beta = 0.86$ which under expected utility theory suggests a convex utility function in the domain of losses (n = 3,956, log pseudo likelihood = −2376). Under expected utility theory a convex utility function in the domain of losses implies risk seeking behavior. A test of conformity of these results with expected value theory indicates that the hypothesis of risk neutrality, implying $\beta = 1$, could not be rejected (Chi-square, p = .0826). In a second model specification the incentive conditions, the axis-formats and their interaction did not prove to be a significant influence on the β-parameter (p-values were respectively p = .550, p = .852, p = 843, log pseudo likelihood = −2306).[133]

These results are in conformity with the tendency of participants in the variable reward condition to behave according to expected value theory with a slight inclination to risk seeking behavior. They do, however, contradict the results of the nonparametric statistical tests in section 7.5.3 for the fixed incentive condition which suggest risk averse behavior. Furthermore, in contrast with these statistical tests the two incentive conditions do not appear as significant variables in the EU choice model.

7.6 Discussion

In a laboratory experiment we assess the effect of a fixed and variable incentive scheme on choices regarding risk appetite. The results suggest that fixed incentives induce risk aversion, by making participants spend more on risk reduction, in particular for the low probability – high impact risks. In comparison with variable incentives these efforts result in a lower standard deviation of the use of risk management resources but do not result in a significantly smaller probability of exceeding one's risk management budget. One reason for this result is that it is possible to reduce this probability of default by increasing the skewness of the distribution of outcomes, which has a negative relationship

131 It should be noted that our experimental design does not accommodate the incentivization of individual discrete choices such as in the random-lottery incentive scheme (Bardsley, Cubitt, Loomes, Moffatt, Starmer, & Sugden 2010 p.266).

132 See appendix C for the specification of these MLE-models and data fittings. The model employs a Fechner error specification and its code was derived from Harrison (2008).

133 In addition the β-parameter in this specification was not significant as well (p = .297).

with standard deviation.[134] Contrary to our predictions fixed incentives, in comparison with variable incentives, fail to deliver more assurance regarding the effective use of risk management resources. In line with our predictions the fixed incentive scheme proves to be comparatively less efficient, as measured by the significantly higher mean use of risk management resources. Indicative for the efficiency induced by variable incentives is the correspondence of its median responses with expected value theory and the marginal benefit – marginal cost rule (Vaughan 1997).

In neither the fixed nor the variable incentive conditions participants are able to attain optimum levels of efficiency and effectiveness. Under both the fixed and variable incentive conditions participants could have reduced the probability of exceeding their budget much further by reducing the risk of lotteries in the risk matrix along the impact-axis instead of the probability axis. Participants do not seem to notice that by being indifferent between the direction of risk reduction they were "stuffing risk into the tails" (Nocera 2009). In the management of risk the choice which event to *select* for risk reduction can be based one kind of risk preference while the decision which *direction* this risk reduction should take can be based on another preference. A preference for impact reduction over probability reduction, given equal lottery means, is evidence of a risk averse preference to risk (see appendix B). By reducing the impact of lotteries participants under the variable and fixed incentive schemes would, compared to the status quo, have been able to reduce the probability of exceeding budget by a factor 10 and 100 respectively.

The risk matrix offers several advantages to conventional elicitation techniques that are used in decision theory. The large number of individual lotteries that can be evaluated simultaneously, makes it a useful instrument for analyzing portfolio risk. The risk matrix allows the participant a wider spectrum of choices, requiring the participant to choose which lottery needs to be moved and choose its destination in addition. Moving a lottery ($p_s : x_i$, 0) to another destination in the risk matrix yields $si - 1$ discrete preferences, which makes it quite an efficient elicitation method. These discrete choices can then be used to estimate model parameters, such as in our case the utility of the participants at the group level using Maximum Likelihood Estimation. However, our attempt to estimate the utility function parameter under expected utility theory did not substantiate findings that resulted from the independent and paired samples tests in our study, which revealed risk neutrality and risk aversion under respectively the variable and fixed incentive condition. Under MLE the null-hypotheses of a neutral risk attitude of the participants and of the absence of any effect of incentives on their choices could not be rejected. Given that the dominant choice of participants was a leftward, risk averse move of lotteries in the risk matrix, this is a surprising result that demands further investigation.

[134] The Pearson mode skewness coefficient is defined by the formula (mean – mode) / standard deviation (Kenney & Keeping 1962 p.101).

While our experimental design serves the purpose of assessing the influence of incentives on risk taking behavior, it does so using a rather abstract experimental task. Even though the task centers around a very popular tool in risk management, the risk matrix, it is not fully representative of a company-context. All lotteries in the experiment have known probabilities and outcomes and the cost and effects of risk reduction are precisely known, which are luxuries that are available in the laboratory but usually absent in practice. While participants in the experiment lose their eligibility for financial rewards when losses exceed the financial buffer, golden handshakes and other contractual obligations prevent organizations from withholding financial rewards in practice. An important limitation of our study is furthermore that participants in the experiment are neither active as executives and lack experience in risk management. Finally, it cannot be ruled out that events at the time at which the experiment took place, a period of renewed financial turmoil, have influenced the responses of the participants.

In future studies we aim to replicate the findings of this study with representatives from the business community. When these studies replicate the finding that efficient use of risk management resources is identical in both the variable and fixed incentive condition then this calls for a theoretical explanation, which is currently lacking. In addition we intend to vary the size of the financial buffer to assess whether extremely difficult and easy circumstances stimulate excessive risk taking or exacerbate risk aversion (Carpenter 2000, Devers, McNamara, Wiseman, & Arrfelt 2008, Larraza-Kintana, Wiseman, Gomez-Mejia, & Welbourne 2007).

The implication of this experimental study is that it suggests that performance-based incentive schemes, despite their bad publicity since the 2007–2009 financial crisis, need not result in excessive risk taking but instead may promote the efficient and effective use of risk management resources. These findings result from an experimental design in which risk management resources can be construed as *revenues*, expenditures on risk reduction and sustained losses can be considered *costs*, and *ex post* risk management resources can be seen as *profits*. This experimental set-up corresponds with organizing the risk management process as a profit-generating business (Culp 2001 p.209), which contrasts conventional corporate risk cultures in which risk management is usually organized as a cost center (Daníelsson, Jorgensen, & De Vries 2002 p.1410, Culp 2001 p.ix). The results of our experiment suggest that when the financial compensation of the executive who is responsible for this risk management profit center is based on a variable performance-based incentive both the efficiency and effectiveness of risk management are promoted. A difficulty of organizing risk management as a profit center in practice is, however, that *ex ante* expenditures on risk reduction are known, while the *ex post* benefits of risk reduction and the time at which losses manifest themselves are unknown. In such a context, organizing risk management as a business requires safeguards that prevent opportunistic behavior and gambling (Culp 2001 p.209). In this light it is noteworthy that in our experiment both incentive schemes did not seem to stimulate risk seeking behavior, which is predicted under prospect theory, but instead promoted either risk neutrality or risk aversion. In this exploratory study we find that, at least in the laboratory, it is possible to curb risk appetite using performance-based incentives and that the default risk attitude of decision makers under portfolio risk need not be risk seeking in the domain of losses. However, careful experimentation with incentives in the practice of risk management and collection of more evidence on the aforementioned relationships are still required to prevent that incentives become a poison in risk management rather than a cure.

7.7 Appendices

7.7.1 Appendix A: Additional scatter diagrams of simulation results

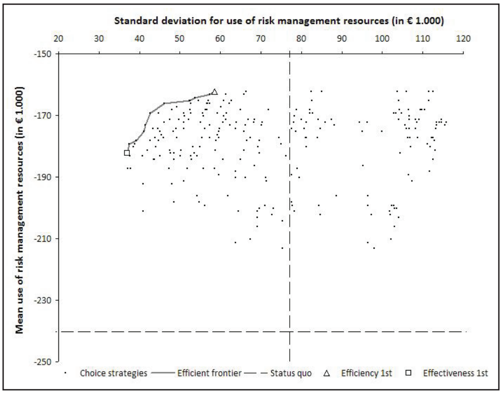

Figure 7.8 Comparison of mean use of risk management resources, $\mu(c+d)$, with the standard deviation for use of risk management resources, $\sigma(c+d)$, for 251 final lottery compositions.

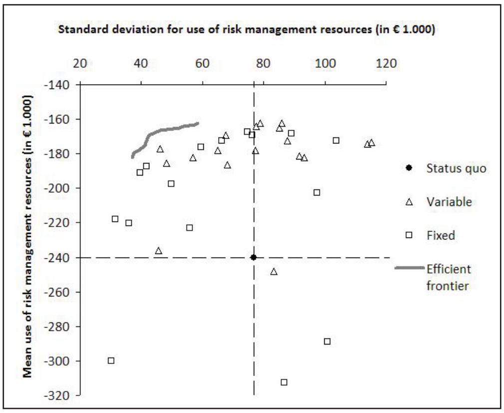

Figure 7.9 Scatter diagram of the simulated mean and standard deviation for the use of risk management resources (in € 1.000) in the variable (n = 18) and fixed incentive condition (n = 16).

7.7.2 Appendix B: Proof

Proof that under risk aversion reducing the risk of a lottery by impact reduction is preferred to probability reduction, when it is given that the means of the lotteries after risk reduction are identical.

Proof:
Define the lottery $X = (p : x, 1 - p : 0)$ with $x < 0$, $0 < p \leq 1$. Let c be the cost of reducing either the outcome x to $x + b/p$ (impact reduction) or the probability p to $p + b/x$ (probability reduction) with $b, c > 0$. Define lottery $Y = (p : x + b/p - c, 1 - p : -c)$ as the lottery that results from impact reduction and lottery $Z = (p + b/x : x - c, 1 - p - b/x : -c)$ as the lottery resulting from probability reduction. It can be verified that the expected values of lotteries Y and Z are both $px + b - c$.

We are now ready to prove that under risk aversion $Y \succcurlyeq Z$. Given that aversion to mean-preserving spreads implies risk aversion (Wakker 2010 p.75) this requires us to proof that Z is a reduction of a compound lottery in which one of the outcomes of single lottery Y has been exchanged for a mean preserving spread (Wakker 2010 p.60 footnote 4, *reduction of compound prospects assumption*). Define lottery $S = (1 + b/px : x - c, -b/px : -c)$ with expected value $x + b/p - c$. Lottery S is a mean-preserving spread of outcome $x + b/p - c$. Replace outcome $x + b/p - c$ in lottery Y by lottery S, its mean-preserving spread, defining the compound lottery $Y^* = (p : S, 1 - p : -c)$. We next reduce this compound lottery to a single lottery by imputation of lottery S in lottery Y^*.

$$Y^* = (p : (1 + b/px : x - c, -b/px : -c), 1 - p : -c)$$

$$Y^* = (p + b/x : x - c, -b/x : -c, 1 - p : -c)$$

$$Y^* = (p + b/x : x - c, 1 - p - b/x : -c) = Z. \square$$

7.7.3 Appendix C: Maximum Likelihood Estimation model and data fittings in STATA

STATA-code for fitting an EU-model to all discrete preferences.

```
* create a log file
  log using ml_mainv1_eu.log, text replace
```

```
* load a text-file and assign names to the columns
  insheet id prob0l prob1l prob0r prob1r m0l m1l m0r m1r choice reward axes interact using
  mainv1.txt, clear
* describe content of data in the file
  describe
```

```
* list the values of the variables in the file
  list
* save the file to stata-format (.dta-file)
  save mainv1, replace
* use the .dta file
  use mainv1, clear
```

```
* eliminate program from memory
  program drop ML_eut0
```

```
* define EUT with CRRA and Fechner error, and replace earlier versions of this program
  program define ML_eut0
* specify the arguments of this program
  args lnf r LNmu
* declare the temporary variables to be used
  tempvar prob0l prob1l prob0r prob1r m0l m1l m0r m1r y0l y1l y0r y1r euL euR euDiff mu
* please do not display all these steps on the screen, for every ML iteration!
  quietly {
* define the noise parameter
  generate double ‘mu’ = exp(‘LNmu’)
* initialize the data
  generate double ‘prob0l’ = $ML_y1
  generate double ‘prob1l’ = $ML_y2
  generate double ‘prob0r’ = $ML_y3
  generate double ‘prob1r’ = $ML_y4
  generate double ‘m0l’ = $ML_y5
  generate double ‘m1l’ = $ML_y6
  generate double ‘m0r’ = $ML_y7
  generate double ‘m1r’ = $ML_y8
* evaluate the utility function
  generate double ‘y0l’ = –((–‘m0l’)^ ‘r’)
  generate double ‘y1l’ = –((–‘m1l’)^ ‘r’)
  generate double ‘y0r’ = –((–‘m0r’)^ ‘r’)
  generate double ‘y1r’ = –((–‘m1r’)^ ‘r’)
* calculate EU of each lottery
  generate double ‘euL’ = (‘prob0l’*‘y0l’) + (‘prob1l’*‘y1l’)
  generate double ‘euR’ = (‘prob0r’*‘y0r’) + (‘prob1r’*‘y1r’)
* get the Fechner index
  generate double ‘euDiff’ = (‘euR’ – ‘euL’)/‘mu’
* evaluate the likelihood
  replace ‘lnf’ = ln($cdf(‘euDiff’))
  }
  end
```

```
* to check for syntax errors
  ml model lf ML_eut0 (r: prob0l prob1l prob0r prob1r m0l m1l m0r m1r = $version) (LNmu:)
  ml check
  ml search
```

```
* the actual command to run the MLE model
  ml model lf ML_eut0 (r: prob0l prob1l prob0r prob1r m0l m1l m0r m1r = $version) (LNmu:),
  cluster (id)
  technique (nr) maximize

* create output
  ml display

* recover the core parameter mu
  nlcom (mu: exp ([LNmu]_b[_cons]))

* for testing whether a variable is significantly different from a particular value (such as 0 or 1)
  test [r]_cons = 1

* global programme for adding controls
  global version "reward axes interact"

* global programme for choosing error theory, change for example to "invlogit"
  global cdf "normal"

* close the log
  log close
```

STATA-output for the EU-model.

```
* create output
  ml display                                        Number of obs      =       3956
                                                    Wald chi2(0)       =          .
Log pseudolikelihood = −2376.3459                   Prob > chi2        =          .
                                                      (Std. Err. adjusted for 34 clusters in id)
```

	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
r						
_cons	.8551918	.08342	10.25	0.000	.6916915	1.018692
LNmu						
_cons	−1.89187	.3663621	−5.16	0.000	−2.609927	−1.173814

```
* recover the core parameter mu
  nlcom (mu: exp([LNmu]_b[_cons]))
  mu: exp([LNmu]_b[_cons])
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
mu	.1507895	.0552436	2.73	0.006	.0425141	.2590649

```
* for testing whether a variable is significantly different from a particular value (such as 0 or 1)
  test [r]_cons = 1
  (1) [r]_cons = 1
  chi2(1) = 3.01
  Prob > chi2 = 0.0826
```

STATA-output for the EU-model with the binary control variables “reward” (the two incentive conditions), “axes” (the two axis-formats), and “interact” (“reward” × “axes”).

```
* create output
  ml display                                              Number of obs    =      3956
                                                          Wald chi2(3)     =      1.31
Log pseudolikelihood = −2306.3213                         Prob > chi2      =    0.7277

                                               (Std. Err. adjusted for 34 clusters in id)
```

	Coef.	Robust Std. Err.	z	P>| z |	[95% Conf. Interval]	
r						
reward	−.9814279	1.640714	−0.60	0.550	−4.197168	2.234312
axes	−.328002	1.758144	−0.19	0.852	−3.7739	3.117896
interact	.3484799	1.761938	0.20	0.843	−3.104854	3.801814
_cons	1.71738	1.645471	1.04	0.297	−1.507684	4.942443
LNmu						
_cons	−2.165276	.2645722	−8.18	0.000	−2.683828	−1.646724

```
* recover the core parameter mu
  nlcom (mu: exp([LNmu]_b[_cons]))
  mu: exp([LNmu]_b[_cons])
```

	Coef.	Std. Err.	z	P>| z |	[95% Conf. Interval]	
mu	.1147182	.0303513	3.78	0.000	.0552309	.1742056

```
* for testing whether a variable is significantly different from a particular value (such as 0 or 1)
  test [r]_cons =1
  (1) [r]_cons = 1
  chi2(1) = 0.19
  Prob > chi2 = 0.6629
```

8 Conclusion

This final chapter provides a discussion of the main conclusions drawn in this thesis along the lines of the five questions formulated in the introduction and identifies areas for future research.

8.1 Decision theory, measure theory and organizational risk appetite

Chapter 2 discussed models in decision theory and methods from measure theory that ensure that organizational risk appetite both complies with the risk attitude of senior management and a rational model of decision-making. It provided definitions for organizational risk appetite (Definition 2.2), an unbiased utility function (Definition 2.4), unbiased risk attitudes (Definition 2.6) and the zero equivalent risk measure (Definition 2.7). It furthermore derived two decision rules for enterprise risk management from this risk measure.

While the concept of unbiased expected utility theory, as defined in this thesis, is not new in decision theory (see Bleichrodt, Pinto, & Wakker 2001), it is not generally applied in decision analysis. In applications, utility is usually elicited under the assumptions of expected utility theory (the *classical elicitation assumption*), despite empirical evidence that refutes the descriptive validity of this theory. At the other extreme, some applications that do assume a descriptively accurate theory of choice under risk, such as prospect theory or rank-dependent utility theory, consider probability weighting to be normative. In both these approaches, descriptive findings are by default assumed to have a normative status. A theory, in which descriptive reality and normative prescription by definition coincide and in which, as a consequence, natural behavior equals desired behavior, offers no added value for decision making (Howard 1988). Unbiased expected utility theory offers the best of both the descriptive and normative spectrum of decision theory. It adopts a descriptively valid theory of choice under risk, elicits a utility function that is free from well known empirical biases and applies this unbiased utility function in analyses within the classical expected utility framework. Unfamiliarity with this unbiased utility concept may be one cause for the lack of its application. Another cause may be that it requires the decision analyst to assume that the true risk preferences of a decision maker are represented by an unbiased utility function (Assumption 2.5). Future research should investigate the reasonableness of this assumption, which, in fact, was accepted *a priori* by the case company in chapter 5. The variety of models, conventions, tools and techniques in decision theory and decision analysis results in an ambiguity that broadens rather than tightens the gap between the practice and promise of these disciplines. Decision theorists and analysts are therefore urged to define a set of generally agreed and accepted principles and conventions for the application of decision theory and analysis to practice.

The relationship between decision theory and measure theory has been investigated before by researchers from both these disciplines (Goovaerts, Kaas, & Laeven 2010). This thesis contributes to this literature by introducing the zero equivalent risk measure, a measure for risk that is derived from the willingness-to-accept valuation method from decision theory and the concept of unbiased expected utility. The zero equivalent risk measure belongs to the family of utility based shortfall risk measures (UBSR). While utility, a concept from economics, has found adoption in measure theory through UBSR, its literature treats the specification of this utility function as an exogenous variable. While the literature on measure theory leaves the source of the utility function unspecified, we propose eliciting this function under prospect theory and applying this function to utility based risk measures under unbiased expected utility theory. Based upon the zero equivalent risk measure, we derived a limited number of decision rules for enterprise risk management. Future research is needed to extend this set with additional decision rules for organizational risk appetite and to increase their applicability by reducing the number of restrictive assumptions that were presented in chapter 2.

The definition of organizational risk appetite in terms of risk measures (Definition 2.2) is a response to the call in the risk management practitioner literature to identify appropriate measures for organizational risk appetite (Anderson 2011 p.8). Risk measures offer a unified framework for addressing the measurement of organizational risk appetite in both financial risk management, where risk measures are already applied, and operational risk management, where their application is still lacking. A refinement of Definition 2.2, in which the current, restrictive assumption of the independence of lotteries is avoided, is left for future research. Under expected utility theory, the utility function expresses both a decision maker's sensitivity to outcomes and her risk attitude. In the same vein, both the need for the measurement of organizational risk appetite and the incorporation of the risk attitude of senior management are, under unbiased expected utility theory, facilitated by the application of the unbiased utility function to risk measures.

Chapter 3 discussed what effect risk attitudes have on the expression of organizational risk appetite in the risk matrix, one of the most popular tools for risk evaluation in enterprise risk management. While traditionally formal expressions of organizational risk appetite have been based on a neutral attitude to risk, several authors in risk management agree that small probability – large impact risks should be ranked high in the risk prioritization, which implies risk aversion. While this already provides some guidance for decisions on the acceptability of risks, practitioners find it hard to decide on the exact location and shape of the boundary for organizational risk appetite in the risk matrix. To elucidate this we applied a marginal cost-benefit analysis to the risk matrix, based on unbiased expected utility theory (chapter 3), and provided diagrams that illustrate how probability weighting and utility functions translate to the zoning in the risk matrix (chapter 5). We illustrate that a risk averse attitude, in comparison with a neutral attitude to risk, designates a larger area in the risk matrix as unacceptable, in particular the area with a small probability of occurrence and a large impact.

Apart from specifying the risk appetite boundary in the risk matrix, decision theory is also able to specify which risk reduction strategy is compatible with the risk attitude of a decision maker. A decrease in the expected loss of a risk can be attained either by reducing its probability of occurrence or its negative impact. *Ceteris paribus* this decrease in expected loss is preferably attained under expected utility theory and risk aversion by means of an impact reduction strategy (PROPOSITION 3.4, *preference for mean-preserving outcome reduction*). Both by categorizing small probability – large impact as unacceptable and reverting risks to lower impact levels, decision makers are prevented from concentrating risk in the tail of the loss distribution. An illustration of this was the optimum default probability that could be attained in the laboratory experiment in chapter 7. Many participants in this experiment were, however, indifferent between a probability and an impact reduction strategy and, as a result, did not attain the optimum levels along the efficient frontier in this study. Future research is required to test whether practitioners in risk management and management show the same indifference to these two risk reduction strategies. By broadening the scope of decisions from preferences over lotteries to preference relationships over risk reduction strategies, the value added of decision theory to risk management is enhanced. The study of a decision maker's consistency in relation to these two types of preferences is also of interest to decision theory itself. A decision maker may, for instance, be risk neutral in her preference over lotteries while, on the other hand, favoring an averse risk reduction strategy. Future research should investigate to what degree decision makers exert these inconsistencies.

8.2 Utility elicitation and organizational risk appetite

Chapter 4 investigated which utility elicitation method is compatible with the elicitation of utility in the context of organizational risk appetite. This resulted in the design of a new nonparametric utility elicitation method, which is an extension of the trade-off method (Wakker & Deneffe 1996). The novelty of our elicitation method is that utility in the domain of losses is not elicited independently from, but is instead derived from the utility of gains. The method is capable of eliciting utility over the whole domain of outcomes without requiring the elicitation of a loss aversion parameter.

Based on reviews of the risk management literature in chapters 1 and 7, it is concluded that in the context of organizational risk appetite there is an increased call for prudence, which precludes risk seeking for losses and the use of unbiased convex utility functions in risk management. In decision theory, convex utility in the domain of losses is, however, reported in a small majority of studies (Wakker 2010 p.264). Our median results for the case company in chapter 5 indicate a concave utility function for financial damages, while those for the case company in chapter 6 showed a convex utility function for claims. Both these studies elicited utility under prospect theory and predominantly compared risky losses with certainty equivalents. This reflects the mainstream convention in decision theory to elicit the utility of losses using gain-free lotteries, which measures the decision maker's willingness-to-pay to avoid risk and which usually induces a risk seeking response from a decision maker. While choices between two gain-free alternatives occur in practice, they are not the focal point of decisions in organizational risk appetite and should therefore be avoided in this particular context.[135] The elicitation method in chapter 4 allowed decision makers to express their willingness to accept risk mainly by means of mixed lotteries. This enhances the realism of utility elicitation in the context of organizational risk appetite where probability distributions that resemble mixed lotteries are common.

Our utility elicitation method should be able to assess the loss aversion phenomenon for large stakes where losses really hurt. As such it has the potential to enhance the empirical meaningfulness of loss aversion studies. The number of trial sessions carried out in chapter 4, however, was too small to draw any preliminary conclusions. Nevertheless it is quite remarkable that the distinctive kink around zero is mostly absent in our method and that for several entrepreneurs concavity of the utility of losses was found near ruin. With the aid of our elicitation method, future research should investigate the nature of both loss aversion and concavity near ruin, phenomena that were both hypothesized by Kahneman and Tversky (1979).

It is not uncommon to find academics and practitioners in risk management who have serious concerns about elicitations of utility and the application of these to organizational risk appetite. At the same time, it is common practice in risk management to employ introspective ratings of probability and impact which are grounded in pragmatism and lack any theoretical foundation. An example of this is the rating scales of Fine (1971), presented in the appendix to chapter 2. In the light of the aforementioned controversy surrounding utility elicitation, future research should, however, investigate the possibility of testing the validity of our elicitation method and those of others by triangulating its results with other measurement techniques.

8.3 Describing organizational risk appetite using decision theory

Chapter 5 assessed whether expected value theory, unbiased expected utility theory or prospect theory performed best in describing the organizational risk appetite expressed by the senior management of a large Dutch privatized government service. Being a non-profit organization, this case company lacked a focus on financial gains. For this reason, the elicitation

[135] A vivid example of a choice between two gain-free "lotteries" was presented in the BBC One television series Sherlock, where its lead character in the episode *A Study in Pink* is faced with a choice between certain death and a lottery with an even chance between life and certain death.

method that was introduced in chapter 4 could not be used in measuring utility. Instead the nonparametric utility elicitation method of Abdellaoui, Bleichrodt, and Paraschiv (2007) was employed.

In this study, the introspective judgment of the participants, which measured their professional opinion with respect to organizational risk appetite, neither corresponded with their elicited nor their unbiased risk attitudes. To the decision theorist it may come as a surprise that the introspective judgments in this study were best described by expected value theory, which is not considered to be a very accurate decision theory for descriptive purposes. Given that introspective judgments of the acceptability of lotteries imply that more acceptable lotteries are preferred to less acceptable lotteries, the preference relationships in the first part of the study should in theory have corresponded with the acceptability judgments in its second part. A possible explanation for the inability of prospect theory to describe organizational risk appetite relates to the difference in cognitive load of both tasks. The high cognitive load of the choice task, consisting of many iterative sessions between pairs of lotteries, may have acted as a stimulus for cognitive biases, such as nonlinear probability weighting. Under prospect theory this bias does not interfere with utility, but is completely captured by the probability weighting function. The low cognitive load of the judgment task, requiring one judgment to be made for single lotteries, may have stimulated trade-off like decision processes with results that correspond with expected value theory (Cokely & Kelley 2009). In our study, unbiased expected utility theory did not drive organizational risk appetite, but was instead accepted by the case company as a prescriptive foundation for setting organizational risk appetite. If the prescriptive, professional unbiased risk attitude and introspective judgment of organizational risk appetite of the participants had been the same, then the elicitation of an unbiased utility function under prospect theory would have lost its added value.

8.4 The utility and probability weight of verbal expressions

Chapter 6 investigated whether the value of numerical translations of verbal probability and outcome expressions differ from those that can be inferred from choices under prospect theory. The main academic contribution of this study is that it bridges findings of choice-studies involving verbal probability and outcome expressions with those that elicit probability weights and utility values. In a field study involving consultants of a large, global advisory firm the utility for verbal outcome expressions and the weight of verbal probability expressions were elicited. This study employed the semi-parametric method of Abdellaoui, Bleichrodt, and L'Haridon (2008) to elicit the utility of financial claims and introduced an estimation procedure for the probability weighting function that is compatible with this method.

This field study replicated results from laboratory studies that demonstrated a large degree of similarity between the verbal and numerical probability response mode. In our study, the weight of single point percentage translations of verbal probability expressions did not differ significantly from the weight of verbal probability expressions inferred from choices under prospect theory (*Hypothesis 1*). This suggests that the consultants were indifferent between the two representation modes. As in previous studies, the context of the decision appeared to have a profound influence on word-to-percentage scaling. Both the judged and inferred percentages were lower than was expected, suggesting that participants anchored their interpretations and choices on the low base rate of financial claim occurrence.

The utility of single point monetary translations of verbal outcome expressions was significantly less negative than the utility of verbal outcome expressions inferred from the consultants' choices under prospect theory (*Hypothesis 2*). This result suggests a dispreference for the verbal mode, which corresponds with findings from previous studies that report vagueness aversion in the domain of losses. Inferred outcomes were significantly more negative than judged outcomes. Both judged and inferred outcomes varied exponentially with the rank order of the verbal outcome expressions.

On the basis of these results, the study hypothesized that in the probability dimension *source dominance over representation* holds. In a context in which objective probabilities are employed, such as in many laboratory experiments, all decision makers are able to understand that the source of uncertainty is precise and that vague or precise modes of representation are merely expressions of precise probability. In a business context, objective probabilities rarely exist, and decision makers understand that the source of uncertainty must be vague, regardless of the mode of representation that is used. In these examples, the nature of uncertainty in the probability dimension is unambiguous, while the numerical and verbal probability modes are ambiguous, portraying alternative meanings depending on the nature of uncertainty. In the outcome dimension, on the other hand, such a source dominance over representation may not hold because the nature of uncertainty itself is ambiguous. In a business context, the nature of outcome estimates can be either precise, based on meticulous calculation of the effects of an event, or vague, based on a best guess. When it is unknown *a priori* to the decision maker whether what the nature of outcome estimates is vague or precise, then the mode of representation provides a signal to the decision maker of the nature of its underlying uncertainty. Given that the source itself is ambiguous, it is unable to dominate the representation mode. By manipulating the ambiguity of sources of uncertainty, future research should assess whether this hypothesized source dominance over representation holds.

It has already been mentioned that the utility elicitation method presented in chapter 4 is capable of accommodating verbal probability expressions. It is, however, important to assess whether the pair of verbal probability expressions that are used in this method complement each other such that their numerical translations by the decision maker sum up to 100%.

The scales of risk matrices presented in chapters 3 and 5 are often composed of a set of verbal probability and outcome expressions. The techniques presented in chapter 6 can be used to infer for each verbal probability expression its numerical counterpart and for each verbal outcome expression its utility value. By calculation of the product of this numerical probability and utility value we derive for each cell its unbiased expected utility value, which can be used to articulate organizational risk appetite.

The rating scales of Fine (1971), presented in chapter 2, can be considered as prototypes of the weights and utility values of verbal probability and outcome expressions introduced in chapter 6. In risk management, rating scales are derived from introspective judgments. Our application of prospect theory to verbal expressions allows us to derive rating scales entirely from a decision-maker's revealed choices. It allows us to distinguish between a verbal probability (outcome) expression's weight (utility) and its inferred probability (outcome). This provides a link between qualitative and quantitative risk analysis that has a solid decision theoretic foundation.

8.5 Performance-based incentives and organizational risk appetite

Chapter 7 assessed experimentally what the influence of performance-based incentives is on expressions of risk appetite in the risk matrix. This laboratory study contributes to the ongoing debate about the role of performance-based incentives in risk management by highlighting their capacity to curb excessive risk taking as well as excessive expenditure on risk management.

Participants who were exposed to the variable incentive scheme in this study were significantly more efficient in their use of risk management resources than those under the fixed incentive condition. In addition, participants under the variable incentive scheme minimized the probability of default equally well as those exposed to the fixed incentive scheme. The variable incentive condition thus performed equally well in curbing excessive risk taking and outperformed the fixed incentive scheme in curbing excessive expenditure of risk management resources.

The fixed incentive scheme induced a risk averse response in the participants, making them spend more on risk reduction, in particular for the low probability – high impact risks. Such a risk averse articulation of risk appetite in the risk matrix was given a normative status in chapter 2 and 3. However, in chapter 7, in comparison with a more risk neutral risk reduction strategy, risk aversion did not result in a significantly lower probability of default. Such a lower probability of default would have been attained if the aforementioned risk averse response had been accompanied by a preference for outcome reduction over probability reduction (see PROPOSITION 3.4). This finding emphasizes the need to educate decision makers in choosing appropriate risk reduction strategies.

Principal-agent theory assumes that agents are risk averse, while decision theory provides empirical evidence of risk seeking in the domain of losses. The introspective judgments of the acceptability of lotteries in chapter 5 and movements of lotteries in the risk matrix in chapter 7 provide evidence for risk aversion and risk neutrality. The results of the choice-based elicitation tasks in chapters 5 and 6 correspond with risk seeking behavior. These findings suggest that judgments of risk appetite need not correspond with the elicited risk attitudes and are partially in agreement with the assumptions of agency theory. Future research should investigate these relationships with a larger sample of organizations in field studies and participants in laboratory studies.

An unsolved puzzle is that the expression of risk appetite in the risk matrices in chapter 7 suggested risk neutral and risk averse behavior while using Maximum Likelihood Estimation under expected utility theory suggested risk seeking behavior. Future research is necessary to assess whether the risk matrix format that was used in chapter 7 is a useful tool for decision theorists to elicit risk preferences.

8.6 Epilogue

The inspiration for this thesis arose from a question posed by a bachelor's student of the Rotterdam University of Applied Sciences in one of my risk management courses. The question was whether there is a normatively preferred way to articulate risk appetite in the risk matrix. The aim of this thesis has been to formulate provisional answers to this question. While there are many unresolved issues remaining, decision theory has enabled me to unravel some of the questions related to organizational risk appetite. I hope that this thesis will inspire risk practitioners to consciously apply the concepts of decision theory in their measurement of organizational risk appetite and that it will attract researchers from decision theory to explore this new applied field of research.

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Samenvatting

Het ontrafelen van risicoacceptatiegraad

Toepassingen van besliskunde in de evaluatie van de risicoacceptatiegraad van organisaties

1 Inleiding

In deze tijd van mondiale uitdagingen en wereldwijde crises zijn organisaties continu op zoek naar oplossingen voor het fundamentele economische probleem van het vaststellen welke doelen moeten worden nagestreefd en de manier waarop schaarse middelen moeten worden verdeeld. In hun streven om de nadelige effecten van risico te minimaliseren tegen zo gering mogelijke kosten, balanceren organisaties tussen de *ex ante* allocatie van middelen om risico's te reduceren en de *ex post* beschikbaarheid van middelen om de gevolgen van risico's te absorberen. De keuzes die een organisatie hierin maakt, weerspiegelen de bereidheid van de organisatie om risico's te accepteren, ofwel de risicoacceptatiegraad van de organisatie. Schaarsheid van middelen impliceert dat niet alle risico's kunnen worden weggenomen of verkleind, met als gevolg dat er een rangorde in de risico's dient te worden vastgesteld. Het besliskundige begrip risicohouding bepaalt de rangorde van de geïdentificeerde risico's. De risicoacceptatiegraad bepaalt vervolgens voor welk deel van deze geordende verzameling risico's beheersingsmaatregelen worden genomen en voor welk deel niet.

De risicoacceptatiegraad van een organisatie is niet onbeperkt. In dit proefschrift wordt daarom onderzocht hoe een gemeten risicohouding een zinvolle bijdrage kan leveren aan de oplossing van het economische probleem van de schaarsheid van middelen binnen risicomanagement. De uitkomst van dit onderzoek is een belangrijke schakel, tussen enerzijds de op de risicohouding gebaseerde risicoacceptatiegraad van de leiding van een organisatie, en anderzijds het eenduidig vertalen van deze risicoacceptatiegraad naar methoden en technieken die door medewerkers van de organisatie dienen te worden gehanteerd bij het bewaken van die risicoacceptatiegraad.

In het COSO (2004a) raamwerk voor ondernemingsrisicomanagement is risicoacceptatiegraad gedefinieerd als 'de hoeveelheid risico in brede zin die een onderneming of entiteit bereid is te accepteren in het nastreven van haar missie' (p.110). De algemene opinie binnen risicomanagement is dat de risicoacceptatiegraad formeel dient te worden vastgesteld en vastgelegd en dat zij de risicohouding van de leiding van de organisatie moet weerspiegelen. Recente publicaties van beoefenaars van risicomanagement benoemen expliciet de behoefte aan het inbedden van de risicohouding van de leiding in de risicoacceptatiegraad van de organisatie (Semple 2007, Van den Brink 2007, PwC 2006, Barfield 2005, Zurich 2004). Voor de validatie van de risicoacceptatiegraad van organisaties kunnen gedragseconomische modellen en methoden worden gebruikt, waarmee de risicohouding van een beslisser gemeten en geanalyseerd wordt.

Besliskunde is de discipline waarin gedragseconomen en cognitieve psychologen samenwerken om individueel beslissingsgedrag onder risico en onzekerheid te onderzoeken. In de besliskunde bestaan beschrijvende modellen om de professionele risicohouding van belangrijke beslissers in een organisatie te meten, en normatieve modellen om de strategische besluitvorming over de risicoacceptatiegraad te verbeteren. Eén model uit de besliskunde dat in ondernemingsrisicomanagement gebruikt wordt, is de verwachte-waardetheorie (Huygens 1657). Dit beslismodel veronderstelt een risiconeutrale beslisser, die de prioriteit van de risico's bepaalt aan de hand van de verwachtingswaarde. Een ander model voor beslissen onder risico is de verwachte-nuttheorie (Cramer 1728/1954, Bernoulli 1738/1954). De verwachte-nuttheorie is bekend in de economie, maar wordt nog bijna niet toegepast in ondernemingsrisicomanagement. Deze theorie veronderstelt dat een beslisser risicoavers is. De verwachte-nuttheorie geeft deze risicoaverse houding van een beslisser weer met een concave nutscurve, waarbij het nut de subjectieve waardering van een beslisser is voor de uitkomsten van een risico. Deze theorie kent prioriteit toe aan risico's op basis van de wiskundige verwachting over de uitkomsten van een risico, die door middel van de nutscurve naar nut zijn getransformeerd. Zowel de verwachte-waardetheorie als de verwachte-nuttheorie heeft een normatieve status als rationeel model voor beslissen onder risico. De rationaliteit van de verwachte-waardetheorie is verbonden aan de wet van de grote aantallen. De rationaliteit van de verwachte-nuttheorie is vastgesteld door Von Neumann en Morgenstern (1944), die vier axiomata voor rationele besluitvorming hebben geformuleerd en hebben aangetoond dat deze equivalent zijn met de verwachte-nuttheorie. De verwachte-waardetheorie en de verwachte-nuttheorie zijn vanuit een normatief oogpunt aantrekkelijk, maar empirisch onderzoek toont aan dat beide theorieën beslissingsbedrag onder risico niet accuraat beschrijven.

Een accurate, beschrijvende theorie voor beslissen onder risico die in staat is om de rationele componenten en de irrationele componenten van beslissingsgedrag te onderscheiden, is prospecttheorie (Kahneman & Tversky 1979, Tversky & Kahneman 1992). Prospecttheorie beschrijft door middel van nuts- en kanswegingsfuncties voor winsten en verliezen diverse afwijkingen van de verwachte-nuttheorie die al eerder in de literatuur zijn beschreven. Eén van deze afwijkingen is het fenomeen van risicozoekend gedrag in het domein van verliezen. Empirisch onderzoek suggereert vervolgens dat beslissers niet alleen de uitkomsten van een risico transformeren maar ook de kans op het optreden van een risico. Een andere afwijking van de verwachte-nuttheorie betreft verliesaversie, het fenomeen dat verliezen zwaarder wegen dan winsten (Tversky & Kahneman 1991).

Bewijs voor deze afwijkingen biedt meer inzicht in beslissingsgedrag onder risico. Echter, dit vergrote inzicht beperkt de vrijheid van de besliskundig analist om nutscurves aan de hand van de verwachte-nuttheorie te meten. De gangbare toepassing van besliskunde op besluitvorming door ondernemingen (*e.g.* Spetzler 1968, Walls & Dyer 1996, Stanford University Strategic Decisions Group 2010) houdt echter vast aan nutsmeting onder de veronderstellingen van de verwachte-nuttheorie. Een alternatief is de nutscurve te meten onder de veronderstellingen van prospecttheorie en deze normatief te gebruiken in analyses met de verwachte-nuttheorie (Bleichrodt, Pinto, & Wakker 2001). Diverse nutsmeetmethoden zijn in staat om een dergelijke 'zuivere' nutscurve onder de veronderstellingen van prospecttheorie te meten (Wakker & Deneffe 1996, Abdellaoui, Bleichrodt, & Paraschiv 2007). Deze nutscurves worden verondersteld de werkelijke preferenties van beslissers in organisaties te vertegenwoordigen en kunnen onder de rationele verwachte-nuttheorie worden toegepast om de risicoacceptatiegraad van een organisatie te formuleren.

2 Onderzoeksvragen

Eén van de grootste struikelblokken bij het toepassen van besliskunde binnen ondernemingsrisicomanagement, is de empirische bevinding van risicozoekend gedrag ten aanzien van verliezen. Als de risicoacceptatiegraad van een organisatie gebaseerd is op een convexe, 'zuivere' nutsfunctie voor verliezen, die de werkelijke preferenties van haar leiding weerspiegelt, dan stimuleert het toepassen van deze functie binnen de verwachte-nuttheorie de organisatie om te gokken. Traditioneel wordt in risicomanagement een risiconeutrale en niet een risicozoekende houding verondersteld. Daarnaast stellen diverse auteurs binnen het risicomanagement, in reactie op recente financiële schandalen en verscheidene crises, een risicoaverse houding voor. Impliciet toegeven aan gokken binnen risicomanagement is ook vanuit de economische wet van afnemend marginaal nut onacceptabel.

In operationeel risicomanagement is het gangbaar risico's te prioriteren aan de hand van een risicomatrix, een assenstelsel met een schaal voor de kans van optreden en een schaal voor de negatieve gevolgen van een risico. De risicomatrix geeft uitdrukking aan de risicoacceptatiegraad door aan te geven welke risico's acceptabel zijn, en welke risico's onacceptabel zijn en dienen te worden beheerst. Beoefenaars van risicomanagement vinden het moeilijk om te bepalen waar de grens tussen acceptabele en onacceptabele risico's ligt. Een 'zuivere' nutsfunctie van de leiding van een organisatie zou gebruikt kunnen worden om deze grens te bepalen, maar het is onduidelijk hoe dit moet worden gedaan.

In organisaties zijn er veel situaties die lijken op gemengde loterijen, dat wil zeggen loterijen die zowel winsten als verliezen bevatten. In de context van de risicoacceptatiegraad zijn vooral gemengde loterijen met grote financiële gevolgen relevant. In de besliskunde worden gemengde loterijen voornamelijk gebruikt voor het meten van de verliesaversieparameter. De meettechniek voor deze parameter noodzaakt de besliskundig analist om juist kleine bedragen te gebruiken. Gemengde loterijen met uitkomsten rond de nul zijn binnen de besliskunde volledig geaccepteerd, maar passen niet bij de werkelijke omstandigheden waaraan de leiding van een organisatie in de praktijk is blootgesteld.

Organisaties maken hun formeel vastgestelde risicoacceptatiegraad over het algemeen niet publiek. Het is daarom onbekend welke risicohouding wordt weerspiegeld in beleidsdocumenten die de risicoacceptatiegraad van de organisatie vastleggen. Het is onduidelijk of een risicozoekende tendens verwacht kan worden, zoals voor verliezen wordt voorspeld door prospecttheorie, een risicoaverse houding, zoals wordt verondersteld in economische modellen, of een risiconeutrale redenering conform de verwachte-waardetheorie.

Terwijl besliskunde erg kwantitatief is, is ondernemingsrisicomanagement vaak erg kwalitatief van karakter. Kwantificering van risico is prominent aanwezig in financieel risicomanagement. In operationeel risicomanagement is het echter gebruikelijk om in risicoanalyses verbale uitdrukkingen van kansen en uitkomsten te gebruiken, zoals 'onwaarschijnlijk' en 'bijna zeker' voor kansen, en 'marginaal' en 'catastrofaal' voor uitkomsten. Het gevolg hiervan is dat het begrip risicoacceptatiegraad vaak wordt gezien als iets vaags en kwalitatiefs. De vraag is dan wat de relevantie is voor operationeel risicomanagement van een kwantitatieve aanpak zoals wordt gehanteerd binnen de besliskunde. De toegevoegde waarde van besliskunde kan liggen in het met elkaar verbinden van kwalitatieve en kwantitatieve risicoanalyses door het bepalen van een nutswaarde voor verbale uitdrukkingen van uitkomsten en een kansweging voor verbale uitdrukkingen van kansen.

Het uitgangspunt van de principaal-agenttheorie (Laffont & Martimort 2002) is dat de economische agent risicoavers is. Voor het op één lijn brengen van de doelstellingen van de agent en de doelstellingen van de principaal kunnen financiële prikkels worden gebruikt. De empirische bevinding van risicozoekend gedrag in het domein van verliezen, is in tegenspraak met de veronderstelling van een risicoaverse agent. Terwijl binnen de principaal-agenttheorie het doel is om de agent meer risico te laten nemen, is er in reactie op de financiële crisis van 2008 een toenemende behoefte aan resultaatgerelateerde prikkels die roekeloos gedrag van economische agenten tegengaan. Het is onduidelijk wat voor prikkels ontworpen kunnen worden om binnen organisaties een risicoaverse risicohouding en risicoacceptatiegraad te bevorderen.

Aan de hand van het bovenstaande kunnen vijf centrale vragen worden geformuleerd:

1. Welke modellen uit de besliskunde en welke methoden uit meettheorie zorgen ervoor dat de risicoacceptatiegraad van een organisatie overeenkomt met zowel de risicohouding van de leiding van de organisatie als een rationeel beslismodel? Welk effect heeft een risicohouding op de uitdrukking van de risicoacceptatiegraad van een organisatie in de risicomatrix?
2. Welke nutsmeetmethode sluit aan op de context van de risicoacceptatiegraad van organisaties?
3. Als het doel het beschrijven van de risicoacceptatiegraad van beoefenaars van risicomanagement is, welke theorie uit de besliskunde (verwachte-waardetheorie, 'zuivere' verwachte-nuttheorie of prospecttheorie) kan dan het beste gebruikt worden?

4. Als verbale uitdrukkingen van kansen en uitkomsten worden gebruikt in de risico-evaluatie, verschillen de introspectieve schattingen van hun waarde dan van de waarden die vanuit het keuzegedrag kunnen worden afgeleid onder de veronderstellingen van prospecttheorie?
5. Wat is de invloed van resultaatgerelateerde financiële prikkels op uitdrukkingen van de risicoacceptatiegraad in de risicomatrix?

Om deze vijf vragen te beantwoorden is allereerst een studie gemaakt van de relevante literatuur uit de vakgebieden risicomanagement, besliskunde, meettheorie en linguïstiek, en van de principaal-agenttheorie. Bij gebrek aan databases op het gebied van de risicoacceptatiegraad van organisaties, leende de verkennende aard van de onderzoeksvragen zich bij uitstek voor een analyse met surveys en experimenten. Voor het testen van de meetmethode die voortkwam uit de tweede onderzoeksvraag, is met de surveymethode de risicohouding van verschillende jonge ondernemers gemeten. De derde onderzoeksvraag is beantwoord door de surveymethode in te zetten bij het meten van zowel de risicohouding als de risicoacceptatiegraad van de beslissers van één grote onderneming. De vierde en vijfde onderzoeksvraag zijn beantwoord aan de hand van de experimentele dataverzamelingsmethode. In het eerste experiment is bij één grote onderneming de houding van managers ten aanzien van verbale uitdrukkingen van kansen en uitkomsten met betrekking tot projectrisico's gemeten bij kleine en grote projecten. Het tweede experiment is uitgevoerd onder docenten van de Hogeschool Rotterdam om het effect van verschillende financiële prikkels op de risicoacceptatiegraad van de deelnemers te meten.

3 Resultaten

3.1 Besliskunde, meettheorie en risicoacceptatiegraad

Het doel van hoofdstuk 2 is om duidelijk te krijgen welke modellen uit de besliskunde en welke methoden uit de meettheorie ervoor zorgen dat de risicoacceptatiegraad van een organisatie overeenkomt met zowel de risicohouding van haar leiding als een rationeel beslismodel. De belangrijkste bijdrage van dit hoofdstuk is dat het tegemoetkomt aan de behoefte van beoefenaars van risicomanagement aan een heldere maatstaf voor de risicoacceptatiegraad van organisaties en het inbedden van de risicohouding van de leiding daarin. Aan beide behoeften, het vaststellen van risicoacceptatiegraad en het inbedden van de risicohouding, kan worden voldaan met de verwachte-nuttheorie. In deze theorie wordt de risicohouding van een beslisser uitgedrukt met een nutsfunctie. Deze functie kan vervolgens worden gebruikt om een risicomaatstaf te bepalen, door het berekenen van het minimumbedrag aan kapitaal dat een organisatie verlangt om risico te kunnen accepteren (Föllmer & Schied 2004).

Dit proefschrift introduceert de nul-equivalentrisicomaatstaf, een maatstaf voor risico die is afgeleid van de 'willingness-to-accept' waarderingsmethode uit de besliskunde in combinatie met het concept van 'zuiver' verwacht nut. De nul-equivalentrisicomaatstaf behoort tot de categorie 'utility based shortfall risk measures' (UBSR). Terwijl nut, een concept uit de economie, binnen meettheorie met UBSR al wordt toegepast, behandelt UBSR de nutsfunctie zelf als een exogene variabele. Waar meettheorie de bron van de nutsfunctie niet definieert, stelt dit hoofdstuk voor om de nutsfunctie te meten onder de veronderstellingen van prospecttheorie en deze functie toe te passen op de op nut gebaseerde risicomaatstaven onder de veronderstellingen van de 'zuivere' verwachte-nuttheorie. Dit hoofdstuk definieert aan de hand van risicomaatstaven het begrip risicoacceptatiegraad en leidt een beperkt aantal beslissingsregels af voor ondernemingsrisicomanagement. Toekomstig onderzoek is noodzakelijk om deze beslissingsregels uit te breiden met aanvullende regels en de toepasbaarheid hiervan te verhogen door het verminderen van het aantal in het hoofdstuk genoemde restrictieve veronderstellingen.

Het doel van hoofdstuk 3 is te onderzoeken welk effect verschillende risicohoudingen hebben op de uitdrukking van de risicoacceptatiegraad van een organisatie in de risicomatrix, één van de meest populaire hulpmiddelen binnen risicomanagement. Op basis van methoden uit de besliskunde wordt in dit hoofdstuk onder de veronderstellingen van de verwachte-nuttheorie de grootte van iedere zone in de risicomatrix bepaald alsook de vorm van de grenslijn. Traditioneel adviseren risicomanagementtekstboeken om een risiconeutrale houding aan te nemen bij het bepalen van de risicoacceptatiegraad. Diverse auteurs binnen het risicomanagement propageren echter dat aan risico's met een kleine kans van optreden en een grote impact, een grote prioriteit dient te worden toegekend, wat een risicoaverse houding impliceert. De belangrijkste bijdrage van dit hoofdstuk is dat het verklaart op welke manier methoden uit de besliskunde bijdragen aan het vastleggen van deze risicoaverse houding in de risicomatrix. Er wordt onderzocht hoe de 'zuivere' risicohouding van een beslisser kan worden uitgedrukt in de risicomatrix aan de hand van de nul-equivalentrisicomaatstaf. Met isocontouren worden de vorm en de locatie van de onacceptabele risicozone in de risicomatrix afgeleid op basis van een analyse van de marginale opbrengsten en marginale kosten. Deze toont aan dat met een risicoaverse houding, in vergelijking met een risiconeutrale houding, een groter gebied binnen de risicomatrix als onacceptabel wordt bestempeld, in het bijzonder het gebied met een kleine kans van optreden en grote gevolgen. Deze analyse impliceert dat een concave nutsfunctie, die onder de verwachte-nuttheorie een risicoaverse houding weerspiegelt, consistent is met een voorkeur voor het verminderen van de negatieve uitkomsten van een risico ten opzichte van een strategie van het verminderen van de kans van optreden van een risico. Toekomstig onderzoek is nodig om vast te stellen wat de voorkeur van beoefenaars van risicomanagement en managers is ten aanzien van deze twee risicoreducerende strategieën. Voor besliskunde is het interessant om de consistentie te bestuderen tussen de risicohouding die blijkt uit de selectie van onacceptabele risico's in de risicomatrix en de keuze voor gevolg- dan wel kansreductie binnen de risicomatrix.

3.2 Nutmeting en risicoacceptatiegraad

Hoofdstuk 4 stelt vast dat bepaalde conventies binnen de besliskunde niet aansluiten op de context waarin de risicoacceptatiegraad van organisaties wordt bepaald. De conventie om het nut van verliezen te meten aan de hand van winstloze loterijen, meet de bereidheid van de beslisser om te betalen voor het vermijden van risico's, wat gewoonlijk resulteert in een risicozoekende reactie van de beslisser. Hoewel keuzes tussen twee winstloze alternatieven in de praktijk voorkomen, bepalen deze niet de kern van beslissingen die betrekking hebben op de risicoacceptatiegraad van een organisatie en moeten deze in deze specifieke context worden vermeden. Het meten van de bereidheid om risico's te accepteren door middel van gemengde loterijen, verhoogt het realisme van een nutmeting binnen de context van de risicoacceptatiegraad, waar kansverdelingen die lijken op gemengde loterijen, gemeengoed zijn. De conventie binnen de besliskunde is echter om deze gemengde loterijen alleen te gebruiken voor het meten van de verliesaversieparameter voor uitkomsten dicht bij nul.

Hoofdstuk 4 introduceert een nieuwe meetmethode, die in staat is nut te meten onder de veronderstellingen van prospecttheorie over het volledige domein van uitkomsten, zonder dat daarbij een verliesaversieparameter dient te worden bepaald. Deze methode sluit aan op de context waarin organisaties hun risicoacceptatiegraad bepalen en maakt gebruik van de 'trade-off' methode (Wakker & Deneffe 1996) om nut in het domein van winst te meten. Deze methode is non-parametrisch en behoeft daarom geen specificatie van wiskundige functies of gebruik van statistische analyses om een nutsfunctie te bepalen. Het vernieuwende van deze meetmethode is dat nut in het domein van de verliezen niet onafhankelijk wordt gemeten, maar dat dit wordt afgeleid van het nut van winsten. Deze meetmethode beantwoordt aan de oproep in de besliskundeliteratuur om een nutsfunctie te specificeren voor het gehele domein van uitkomsten, in het bijzonder voor grote uitkomsten ver van het nulpunt (Wakker 2010). Met behulp van deze meetmethode dient toekomstig onderzoek het karakter vast te stellen van de fenomenen verliesaversie en concaviteit rond catastrofale uitkomsten. In het hoofdstuk worden de resultaten van proefsessies met ondernemers getoond, samen met een aantal daaruit volgende beperkingen van de meetmethode. Toekomstige studies dienen de validiteit van deze meetmethode te bepalen door middel van het vergelijken van de resultaten met andere meetmethoden.

3.3 Het beschrijven van de risicoacceptatiegraad met besliskunde

Hoofdstuk 5 onderzoekt de relatie tussen de risicoacceptatiegraad en de risicohouding van de leiding van een grote Nederlandse onderneming. In dit onderzoek wordt het introspectieve oordeel van leidinggevenden over de risicoacceptatiegraad van de onderneming vergeleken met hun professionele risicohouding, die is gebaseerd op voorspellingen vanuit prospecttheorie, de 'zuivere' verwachte-nuttheorie en de verwachte-waardetheorie. De nutsfuncties en kanswegingsfuncties van 43 leidinggevenden van de onderneming worden gemeten aan de hand van de meetmethode van Abdellaoui, Bleichrodt en Paraschiv (2007). Vervolgens worden de leidinggevenden verzocht om aan te geven wat de risicoacceptatiegraad van de onderneming zou moeten zijn. Ten slotte wordt vastgesteld welke theorie uit de besliskunde de meest accurate voorspelling geeft voor het introspectieve oordeel van de leiding over de risicoacceptatiegraad.

De bevindingen van de studie zijn dat in het domein van de verliezen de nutsfunctie van de mediaan deelnemer licht concaaf is, wat een afnemend grensnut impliceert. De kanswegingsfunctie is zeer convex, wat een optimistische houding ten aanzien van de kans op het optreden van verliezen weerspiegelt. Gezamenlijk voorspellen deze resultaten een risicozoekende houding onder prospecttheorie en een risicoaverse houding onder de 'zuivere' verwachte-nuttheorie. Het introspectieve oordeel van de mediaan deelnemer over de risicoacceptatiegraad geeft duidelijk uitdrukking aan risiconeutraliteit en risicoaversie en wordt significant meer accuraat beschreven door de verwachte-waardetheorie dan door prospecttheorie. Dit laatste is een verassing, aangezien de verwachte-waardetheorie binnen de besliskunde niet gezien wordt als een erg accurate, beschrijvende beslissingstheorie.

3.4 Het nut en de kansweging van verbale uitdrukkingen

Hoofdstuk 6 maakt gebruik van prospecttheorie om het nut en de kansweging te meten die 143 consultants van een groot, wereldwijd opererend adviesbureau toekennen aan een verzameling verbale kans- en gevolguitdrukkingen die binnen deze organisatie worden gehanteerd om projectrisico's te managen. Naast de waarde van deze verbale uitdrukkingen die uit de keuzes van de consultants kan worden afgeleid, meet deze studie tevens het introspectieve oordeel van de consultants over de waarde van deze verbale uitdrukkingen. Om het nut van financiële claims te meten, gebruikt deze studie de semi-parametrische methode van Abdellaoui, Bleichrodt en l'Haridon (2008). Tevens wordt een schattingsmethode voor de kanswegingsfunctie geïntroduceerd die aansluit op deze nutsmeetmethode.

In deze veldstudie worden resultaten uit laboratoriumstudies bevestigd, die een grote overeenkomst aantonen tussen keuzes aan de hand van verbale kansuitdrukkingen en keuzes aan de hand van numerieke kansuitdrukkingen. Het kansgewicht van een direct naar een kanspercentage vertaalde verbale kansuitdrukking verschilt niet significant van het kansgewicht van deze verbale kansuitdrukking dat onder de veronderstellingen van prospecttheorie kan worden afgeleid uit de keuzes van de consultants (*hypothese 1*). Dit suggereert dat de consultants onverschillig staan tegenover de twee manieren waarop de kans van optreden wordt uitgedrukt. Net als in eerdere studies blijkt de context waarin de beslissing plaatsvindt, aantoonbare invloed te hebben op de relatie tussen de verbale kansuitdrukking en het kanspercentage. Zowel de kanspercentages gebaseerd op het introspectieve oordeel van de consultants als de kanspercentages die uit de keuzes van de consultants kunnen worden afgeleid, zijn lager dan verwacht. Dit suggereert dat de consultants hun interpretaties en keuzes verankeren op het lage onderliggende kanspercentage voor het optreden van een financiële claim.

Het nut van een direct naar een bedrag vertaalde verbale gevolguitdrukking is significant meer negatief dan het nut van deze verbale gevolguitdrukking dat kan worden afgeleid uit de keuzes van de consultants onder de veronderstellingen van prospecttheorie (*hypothese 2*). Dit resultaat suggereert een afkeer van de vage uitdrukkingsvorm voor gevolgen, wat overeenkomt met bevindingen uit eerdere studies die een aversie voor vaagheid aantonen in het domein van verliezen. Bedragen die uit de keuzes van de consultants kunnen worden afgeleid, zijn significant meer negatief dan de bedragen die gebaseerd zijn op het introspectieve oordeel van de consultants. Beide bedragen worden exponentieel positiever met de rangorde van de verbale gevolguitdrukkingen.

Deze resultaten suggereren dat wanneer de aard van de onzekerheid ondubbelzinnig duidelijk is, zij de uitdrukkingsvorm domineert. Wanneer de beslisser *a priori* echter niet weet of de aard van informatie over kansen of gevolgen vaag is of precies, dan geeft de uitdrukkingsvorm hiervan een signaal over de aard van de onderliggende onzekerheid aan de beslisser. Toekomstig onderzoek moet door de dubbelzinnigheid van de aard van onzekerheid te manipuleren, vaststellen of deze veronderstelde dominantie van de aard van de onzekerheid over de uitdrukkingsvorm standhoudt.

De academische bijdrage van deze studie is dat zij de bevindingen van keuzestudies met betrekking tot verbale kans- en gevolguitdrukkingen en de bevindingen van keuzestudies die nut en kansweging meten, met elkaar verbindt. De relevantie van de studie voor beoefenaars van risicomanagement is dat de studie aangeeft hoe een semi-kwantitatieve risicoanalyse kan worden vormgegeven met kanspercentages en gevolgbedragen die onder de veronderstellingen van prospecttheorie kunnen worden afgeleid uit de keuzes met verbale kans- en gevolguitdrukkingen.

3.5 Resultaatgerelateerde prikkels en risicoacceptatiegraad

Hoofdstuk 7 toont de keuzes van 34 docenten van de Hogeschool Rotterdam die deelnemen aan een laboratoriumexperiment dat is ontworpen om het effect van resultaatgerelateerde financiële prikkels op de risicoacceptatiegraad te bepalen. De resultaten laten zien dat een vaste vergoeding een risicoaverse risicoacceptatiegraad tot gevolg heeft, wat resulteert in meer bestedingen aan risicoreductie, in het bijzonder voor de risico's met een lage kans en groot gevolg. Een variabele vergoeding resulteert in een risiconeutrale risicoacceptatiegraad. Deelnemers met de variabele vergoeding zijn significant meer efficiënt in hun gebruik van risicomanagementmiddelen dan diegenen met een vaste vergoeding. Deelnemers met de variabele vergoeding zijn net zo goed in staat om de kans op verlies te verkleinen als de deelnemers met een vaste vergoeding. Deelnemers met een variabele vergoeding zijn dus net zo effectief in het verkleinen van de kans op verlies als de deelnemers met een vaste vergoeding, maar kunnen dit resultaat op een efficiëntere manier bereiken. De deelnemers zouden een kleinere kans op verlies hebben gehad wanneer zij gekozen hadden voor het reduceren van de gevolgen van een risico in plaats van het reduceren van de kans op het optreden van een risico. Deze bevinding onderstreept het belang van het onderwijzen van beslissers in het hanteren van de juiste strategieën voor risicoreductie.

Deze studie draagt bij aan het ontwerpen van effectieve en efficiënte prikkels voor economische agenten, een centraal thema in de economie. Het draagt tevens bij aan het voortgaande debat over de rol van resultaatgerelateerde prikkels bij het aan banden leggen van het nemen van excessieve risico's en het beperken van excessieve uitgaven aan risicomanagement.

4 Tot slot

De aanleiding voor dit proefschrift was een vraag van een bachelor-student van de Hogeschool Rotterdam in één van mijn lessen risicomanagement. De vraag was wat normatief de beste manier was om de risicoacceptatiegraad in de risicomatrix weer te geven. De doelstelling van dit proefschrift was om een voorlopig antwoord op deze vraag te formuleren. De verschillende studies in dit proefschrift beantwoorden vanuit verschillende invalshoeken de vraag hoe besliskunde een bijdrage kan leveren aan de koppeling tussen enerzijds de op de risicohouding gebaseerde risicoacceptatiegraad van de leiding van een organisatie, en anderzijds de vertaling van deze risicoacceptatiegraad naar methoden en technieken die door medewerkers van de organisatie worden gebruikt bij het implementeren van de risicoacceptatiegraad binnen de organisatie. Hoewel er nog vele onbeantwoorde vragen overblijven, heeft de besliskunde mij in staat gesteld om een deel van de problematiek rondom de risicoacceptatiegraad te ontrafelen. Ik hoop dat dit proefschrift beoefenaars van risicomanagement zal inspireren om de concepten van de besliskunde toe te passen bij het meten van de risicoacceptatiegraad in hun organisaties en dat het onderzoekers vanuit de besliskunde zal aantrekken om dit nieuwe gebied van toegepast besliskundig onderzoek verder te ontginnen.

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Unraveling Risk Appetite

Applications of Decision Theory in the Evaluation of Organizational Risk Appetite

One of the most difficult choices that organizations face is the choice to spend resources today to reduce the probability or negative impact of events that may happen tomorrow. In hindsight, it seems to be a waste to spend organizational resources on reducing the risk of low probability events that up to now never did materialize. Intuitively it appears much more prudent for an organization to spend resources on events that have a higher frequency of occurrence and for which it is easier to assess that resources have been well spent. But what if the consequences of the low probability events are catastrophic and threaten business continuity? Should the leadership of an organization be gambling on a catastrophic low probability event not to occur?

The central theme of this PhD-dissertation is measuring an organization's willingness to accept risk in the pursuit of its objectives. It attempts to unravel this *organizational risk appetite* using decision theory. The study proposes to measure organizational risk appetite using unbiased measurements of the pleasure and pain (utility) that a leadership associates with the consequences of risky events. This unbiased utility is measured under prospect theory and used normatively under expected utility theory to validate tools that communicate organizational risk appetite. In one of these tools, the risk matrix, the economic law of diminishing marginal utility identifies the low probability and large impact events as unacceptable. The dissertation introduces a new design for the measurement of utility under businesslike circumstances, evaluates risk appetite at a large organization, assesses values that decision makers associate with verbal expressions of probability and outcome, and experimentally tests the effect of performance-based incentives on risk appetite.

Arie de Wild is Professor in Behavioral Economics at the R&D Centre for Entrepreneurship & Business Innovation of Rotterdam University of Applied Sciences. He develops and teaches Risk management courses and provides training sessions for professionals on risk simulation, risk assessment and organizational risk appetite.

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