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# Investigation of the Critical State in Soil Mechanics Using DEM

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**Abstract.** The existence and uniqueness of the so-called critical state in soil mechanics is validated in our DEM simulations of irregular polygonal particles. For different particle shape characteristics, the critical state is independent of the initial stress and density conditions. We retain low stress levels, since we do not take into account the crushing of particles. In biaxial test simulations isotropic particles evolve toward a limiting state in which the system reaches a critical void ratio and deforms with constant volume, deviatoric stress, fabric anisotropy, and mechanical coordination number. The last one has been found to be the first variable to attain a critical value making possible for the rest of micro-and-macro-mechanical variables the convergence to the critical state. In periodic shear cell tests, for large shear deformations samples with anisotropic particles reach at the macro-mechanical level the same critical value for both shear force and void ratio. At the micro-mechanical level the components of the stress tensor, the fabric tensor and the inertia tensor of the particles also reach the same stationary state. By varying the aspect ratio of the particles we stated the strong influence of particle shape anisotropy on the parameters that the granular packing attained at the critical state.

**Keywords:** Critical state, fabric anisotropy, particle shape, particle orientation

**PACS:** 01.30.Cc

## 1. INTRODUCTION

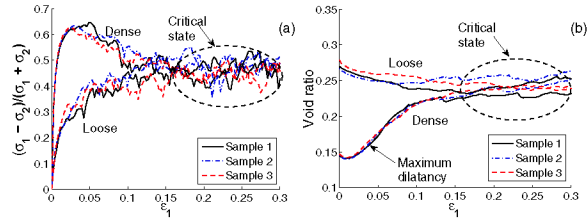
The stress-strain behavior of both dense and loose sand under shearing is first described by Casagrande in 1936 [1]. He concludes that *sand* for large shear deformations independent of the initial density state reaches a limiting state (critical void ratio) in which samples undergo unlimited deformation without further volumetric and stress increments. He also finds that the critical void ratio is only dependent on the confining pressure, and thus determines the so-called critical state line relating the critical void ratio  $e_c$  and the effective normal stress  $\sigma'$  as applied on the shear box test.

The existence and uniqueness of this critical state is a major feature in soil mechanics since it is used to define post-failure behavior of many constitutive models [2, 3]. The critical state has been experimentally proven to be independent of sample preparation and test conditions [4, 5]. Nevertheless, there are some experimental difficulties to characterize the pre-and-post peak mechanical behavior of dense samples [6]. This has led some groups of researchers to claim that the uniqueness of this state is still an open issue [7], and that the limit stage may depend on the consolidation history of sand specimens [8].

In this work, we investigate the critical state by means of DEM simulations of polygonal particles. It is assumed that the particles interact elastically with each other, i.e., the repulsive contact force between the particles is a result of their overlap. Coulomb friction is introduced in all contacts. Further details of the model are given in [9, 10]. The samples consist of isotropic and anisotropic polygonal particles in order to study the influence of particle shape anisotropy on the global mechanical response of granular media and its evolution to the critical state. We show that the DEM simulations reproduce the main features of the critical state in soil mechanics: (i) the granular media evolve toward a stationary state in which the system deforms with constant volume and deviatoric stress, and (ii) for different initial stress states the corresponding stationary values collapse onto a unique critical state line.

## 2. EXISTENCE AND UNIQUENESS

In order to assess the existence of the critical state, we first explore the macro-mechanical evolution of granular samples under biaxial compression. We characterize the density state of the samples by the void ratio  $e$ .

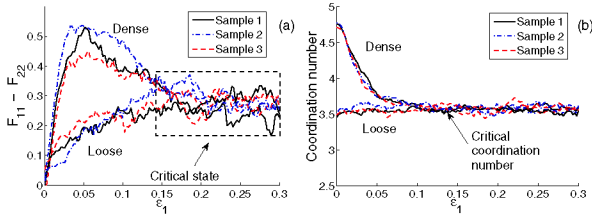


**FIGURE 1.** Evolution of (a)  $\sin \phi$  and (b) void ratio of the samples used to assess the existence of the critical state. Simulation parameters,  $p_0 = 64$  kN/m,  $N = 900$  isotropic particles and  $\mu = 0.5$ . Samples 1, 2 and 3 correspond to a different seed used in the random generation of the polygons [9, 10].

The macro-mechanical response of the granular media is presented in Figs. 1(a) and 1(b). The evolution of  $\sin \phi = (\sigma_1 - \sigma_2) / (\sigma_1 + \sigma_2)$  with axial strain  $\epsilon_1$  for the dense and the loose samples is presented in Fig. 1(a). Dense samples exhibit a peak and strain softening behavior, whereas in loose samples no peak is observed. However, both systems at large shear deformations present a tendency to stabilize around a value that one could consider as the steady state of the material ( $d \sin \phi / dt = 0$ ).

The evolution of the void ratio with axial strain is illustrated in Fig. 1(b). The dense samples initially contract and later dilate. The loose samples contract. For large axial strain values the void ratio reaches a constant value. The stationary value of  $e$  is slightly different for each sample since the parameters  $e$  and  $\phi$  at this stationary state depend on the granulometric properties of the material [11, 12]. In this stage of large deformations, the granular medium is deformed at constant volume and with the same approximate value of deviator stress. This state corresponds to the critical state of the material and it is independent of the initial sample density [2].

Another issue we address is the evolution of the anisotropy of the contact network of the granular packing [13]. This is characterized using the deviatoric component of the fabric tensor  $\mathbf{F}$  which takes into account



**FIGURE 2.** Evolution of (a) the deviatoric component  $F_{11} - F_{22}$  and (b) the trace  $F_{11} + F_{22}$  (coordination number) of the fabric tensor of the samples. Simulation parameters,  $p_0 = 64$  kN/m,  $N = 900$  isotropic particles and  $\mu = 0.5$ . Samples 1, 2 and 3 correspond to a different seed used in the random generation of the polygons [9, 10].

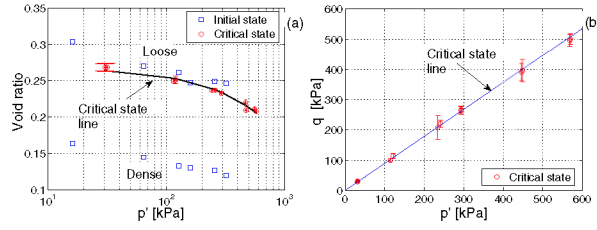
the orientational distribution of contact normal vectors  $\vec{n}$ . In Fig. 2(a) the evolution of the deviatoric component  $F_{11} - F_{22}$  of the fabric tensor with  $\epsilon_1$  is presented. One can notice that the contact network starts from a rather initial isotropic configuration  $F_{11} - F_{22} \approx 0$ , and that as soon as the shear process begins anisotropy is developed. This anisotropy is a result of the creation and reorientation of contacts and force chains along the direction of loading. For the dense samples, the anisotropy increases until the granular system develops its peak strength. The maximum anisotropy coincides with the maximum strength ( $\approx 5\%$  axial strain  $\epsilon_1$ ). The fabric anisotropy in loose samples simply increases until it saturates at a constant value between the statistical fluctuations. In the strain-softening regime the anisotropy of dense samples decreases until it converges to the same value that the loose samples have reached. Hereafter, the media deforms at a critical anisotropy. This has been previously observed in numerical simulations of biaxial tests with the DEM [14].

The creation and destruction of contacts can be studied by following the coordination number  $Z$ , which is equal to the trace of the fabric tensor  $\mathbf{F}$ . In Fig. 2(b) we present the evolution of the coordination number of the samples. At low axial strain values, the dense system contracts and as a consequence a small increment of  $Z$  is observed. This is followed by a decrease of the  $Z$  value when the system starts to dilate. This decrease is associated with the breaking of interlocking between particles. As a result, each particle begins to lose contacts. Both samples around 8% axial strain reach a similar coordination number  $Z$  close to 3.6. This critical coordination number is the first sign that the granular packing is evolving towards the critical state, and it enables the contact network to reach an steady structure.

In order to assess the uniqueness of the critical state, we also perform biaxial experiments using different initial confining pressures  $p_0$ . We retain low stress levels, since our model does not take the crushing of particles into account. In Fig. 3(a), we plot in the compression plane (void ratio  $e$  - mean stress  $p'$ ) the initial states of the samples (squares) and the values of void ratio that the loose and dense samples attain at large deformation (circles). One can see that the same limit state is reached for all the samples defining a unique steady state line. This line can be fitted by a function of the form,

$$e(e_{ref}, n) = e_{ref} \exp(-n (p'/p'_{ref})), \quad (1)$$

where  $e_{ref}$  and  $p'_{ref}$  are the void ratio and the mean stress at the critical state corresponding to the reference initial confining pressure  $p_{ref} = 16$  kN/m, and  $n$  is a constant. The collapse of the stresses (circles) onto a steady state line is also observed in Fig. 3(b). In this figure the stress plain  $q - p'$ , being  $q = (\sigma_1 - \sigma_2)$ , is



**FIGURE 3.** Critical state line in (a) the compression plane void ratio  $e$  - mean stress  $p'$ , and (b) the stress plane  $q$  -  $p'$ . System parameters,  $N = 900$  isotropic particles and  $\mu = 0.5$ . In (a) the squares indicate the initial state of the samples. The circles are the values that samples reach at the stationary state, and the error bars correspond to one standard deviation of the analyzed data.

depicted. The stress ratio  $q/p'$  at the critical state defines the strength parameter  $M$ , which for our simulations is related to the critical state friction angle  $\phi_{cs}$  as,

$$M = \frac{q_{cs}}{p'_{cs}} = 2 \sin \phi_{cs}. \quad (2)$$

The range of variation of the critical friction angle found in our simulations, i.e.,  $22^\circ \leq \phi_{cs} \leq 27^\circ$ , is smaller than the limits usually obtained for sand and gravel in three dimensional experiments in realistic soils,  $26^\circ$  to  $36^\circ$  [5, 12]. This is explained in terms of the higher coordination number of systems in 3D, namely, the higher the coordination number the higher the strength the material can mobilize.

The previous simulation results support the idea of uniqueness of the critical state [2, 6], in which a critical/steady state line links the critical states describing combinations of effective stresses and void ratio  $e : q : p'$ . We also evaluate the critical anisotropy for different stress states, and we find that a critical state line for anisotropy can also be defined.

Under biaxial compression samples with anisotropic particles do not converge to the critical state. Specifically, the anisotropic samples continue dilating without reaching a stationary value of void ratio. Similar results have been previously observed in DEM simulations [14]. This non-convergence to the critical state is explained in micro-mechanical terms as follows: whereas the isotropic samples reach a critical value of anisotropy and coordination number, the contact network of the anisotropic samples is still changing. The non-stationary state of these variables is directly related to the evolution of the particle orientation, i.e., elongated particles are continuously reoriented during the shear process without converging to a steady state. This micro-mechanical evidence does not allow the anisotropic systems to reach the critical state [10].

### 3. INFLUENCE OF ANISOTROPIC PARTICLE SHAPE

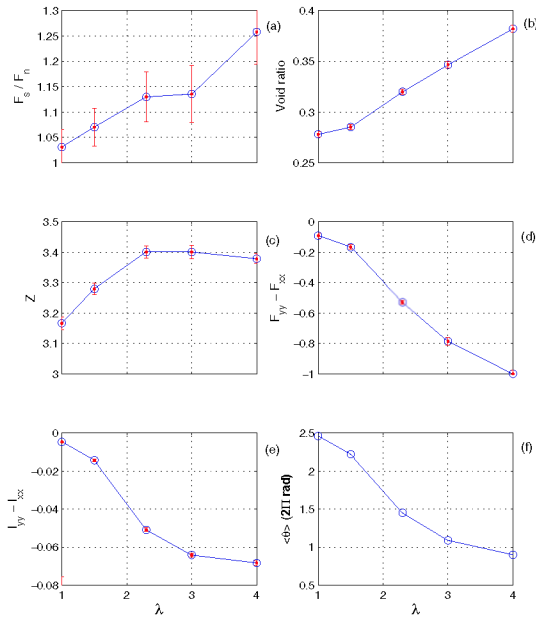
In this section, we study by means of shear cell experiments the existence of the critical state for samples consisting of anisotropic particles. For this experiment, periodic boundary conditions and fixed boundary conditions are imposed in horizontal and vertical direction, respectively. The top and bottom layers of the particles are moved in opposite directions so as to impose a constant shear rate  $\dot{\gamma}$ . Two different initial configurations are obtained for the anisotropic samples: (i) the grains are oriented parallel to the shear direction (called "horizontal" sample - H), (ii) the grains are oriented perpendicular to the shear direction (called "vertical" sample - V).

We find in our simulations that both the ratio of shear force to normal force  $F_s/F_n$  and the void ratio  $e$  evolve toward the same saturation value independently of the initial anisotropy due to contact and particle orientations. At the micro-mechanical level the deviatoric component of the fabric tensor  $\mathbf{F}$  and inertia tensor  $\mathbf{I}$  also reach a stationary value. The stationary value of the deviatoric component is directly related to the steady state at the macro-mechanical level, and seems to be a micro-mechanical requirement for the steady state at the global level.

Although the results discussed herein and presented in [10] correspond to dense samples and one value of confining pressure, they are valid for different initial density states and stress levels. These results validate in our MD simulations the existence of the so-called critical state in soil mechanics irrespective of any initial condition and particle shape characteristics.

Furthermore, in Fig. 4 we present for different aspect ratio  $\lambda$  the average value of macro-and micro-mechanical variables at the critical state. The data correspond to the average of the variables once the critical state has been reached. From Fig. 4, we can conclude that the larger the aspect ratio  $\lambda$ :

- The larger the strength of the material at the critical state, i.e., the ratio between the shear and normal force  $F_s/F_n$ , Fig. 4(a).
- The larger the void ratio at the critical state and, therefore, the larger the volumetric deformation, Fig. 4(b).
- The larger the coordination number  $Z$  of the particles for  $\lambda \leq 2.3$ , whereas for  $\lambda > 2.3$  the  $Z$  value saturates and remains constant, Fig. 4(c).
- The larger the fabric anisotropy at the critical state, i.e., the deviatoric component  $F_{yy} - F_{xx}$  of  $\mathbf{F}$ , Fig. 4(d).
- The larger the anisotropy related to particle orientation at the critical state, i.e., the deviatoric component  $I_{yy} - I_{xx}$  of  $\mathbf{I}$ , Fig. 4(e).



**FIGURE 4.** Effect of particle shape on the critical state values attained by the granular packings on shear cell tests, (a) ratio shear - normal force  $F_s/F_n$ , (b) Void ratio, (c) coordination number  $Z$ , (d) deviatoric fabric  $F_{yy} - F_{xx}$ , (e) deviatoric inertia  $I_{yy} - I_{xx}$  and, (f) mean accumulated rotation  $\langle \Theta \rangle$  of the particles till a shear strain  $\gamma = 30$ . The following aspect ratio  $\lambda$  are used: 1.0, 1.5, 2.3, 3.0 and 4.0. The shear rate  $\dot{\gamma} = 1.4 \text{ s}^{-1}$ . The error bars correspond to 1 standard deviation of the analyzed data.

- The smaller the accumulated mean particle rotation angle  $\langle \Theta \rangle$ , Fig. 4(f).
- The longer the time to reach micro-mechanical equilibrium in fabric and particle orientation [10].

The above statements concerning the influence of anisotropic particle shape on the macro-mechanical behavior of granular packings are explained in terms of the bigger interlocking among particles and the strong frustration of rotation that anisotropic particles undergo during shearing.

#### 4. CONCLUDING REMARKS

The steady state that granular materials reach under large shear deformations have been assessed for different initial conditions using isotropic and anisotropic particles. The existence and uniqueness of the critical state is verified for isotropic shaped particles using biaxial test simulations. Contrary of that, samples with anisotropic particles do not reach the critical state under biaxial loading conditions, but they do reach this state in the periodic shear test. At the critical state, the samples deform at

constant void ratio, shear stress, fabric anisotropy, particle orientation and mechanical coordination number. The last one has been found to be the first variable to attain a critical value making possible for the rest of micro-and-macro-mechanical variables the convergence to the critical state. It is also found that a steady value of fabric anisotropy and particle orientation is a micro-mechanical requirement for the existence of the critical state. Furthermore, the uniqueness of the critical state is validated in our simulations, when it is found that the critical states related to different stress states collapse onto only one critical state line. Finally, by varying the aspect ratio  $\lambda$  of the particles we can state the important effect of particle anisotropy on the macro and micro-mechanical response of granular media.

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