The flow of water through gravels

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The linear flow law of Darcy, relating the flux to the hydraulic gradient by a constant of proportionality, the hydraulic conductivity (K), is almost universally employed to analyse and predict the flow of fluids through soils and aquifers. Laminar flow is a prerequisite for its application and while not a problem in soils, it may be problematic in high velocity flows through gravel aquifers and drains. In the present study, the velocity of water flow through 30 screened gravel aggregates, comprising pit and broken stone gravels and ranging in particle diameter from 38.1 to 1.6 mm was measured. Some of the aggregates were reconstituted, derived by combining in various proportions fractions from laboratory screenings. The experimental arrangement comprised a 1.53-m long, 60-mm bore PVC smooth-walled pipe with retainer screens at both ends. This pipe was carefully packed with the particular gravel aggregate in 0.5 kg aliquots, placed horizontally on adjustable supports and connected to a 3-m³ reservoir held at a constant level. Hydraulic gradients applied were generally in the range 0.05–0.56. Each test was repeated at least twice. The flow of water through the gravels did not obey Darcy’s Law. The relationship between velocity, v (m/s), and the hydraulic gradient (i) was of the form, v = aib, with a and b (–) constants for a particular gravel. Darcy’s Law can be applied by considering a gradient-dependent hydraulic conductivity, v = K(i), from which K(i) = aib−1. Under unit gradient K(i) = a. In the coarser gravel aggregates, the value of b approached 0.5, similar to that for turbulent flow in rough-walled pipes. In fine-particle gravels, b tended toward unity indicating that Darcy’s Law could be applied without too much error. Results of tests in this series were in good agreement with those from another laboratory. Log-log curves and equations relating K(i) of individual aggregates to i are provided. K(i) values ranged over two orders of magnitude from 120,000 m/day for coarse gravel to about 700 m/day for a 4.76 to 1.6 mm gravel. There were very large declines in K(i) with increase in i in coarse gravels and small declines in the finer gravels. Results are discussed in relation to drainage and filter gravels and gravel aquifers.

Keywords: Darcy’s law; hydraulic conductivity; hydraulic gradient; non-proportional flow in gravels
Introduction
Gravel aggregates are commonly found among glacial and floodplain soil deposits and are also manufactured by crushing solid rocks. Apart from their ubiquitous use in construction, gravel aggregates are widely used in drainage and as filters. In drainage, they can act as a transport medium to carry a flow of water, as a connector (e.g. to connect flows from one elevation to another in soil), as a filter to restrain a base soil such as a quick sand from entering drains or blankets of other aggregates, and as a drain enlarger to reduce entry resistance into drains. In gravel mole drains, apart from a transport function, the gravel has also a structural function. In wastewater treatment, gravel layers are employed in stratified sand filters (Nichols et al., 1997). In groundwater hydrology, gravel layers are commonly exploited as aquifers and may also give rise to hillside seepage and springs (Selim and Kirkham, 1972). Gravel layers may also serve to transport contaminants with significant dispersion at high pore velocities (Pfannkuch, 1963 quoted in Leij and van Genuchten, 1999, their Figure 9-3).

The classic law of Darcy is almost universally employed to analyse the flow of fluids through soils and sands. This linear flow law states that the flux density is proportional to the hydraulic gradient and the constant of proportionality is the hydraulic conductivity, K. Darcy’s law is widely used in the analysis and modelling of groundwater flow. However, Darcy’s law cannot be uncritically applied; there are some basic principles that must be observed. In particular, the flow must be laminar. This is not usually a problem in normal soils where flow velocities are slow but may be a problem in groundwater flows through gravel aquifers and in engineering gravels.

In the above occurrences and applications, a knowledge of the flow properties of a range of gravel aggregates is required and this paper is a contribution in this regard. The applicability of Darcy’s linear flow law is reviewed and the results of laboratory flow tests on a selected range of gravels commonly found or used in Ireland are presented. Equations that can be used for design purposes in a variety of applications are also given.

Theory
Under isothermal conditions, the 1-dimensional flow of water through homogeneous isotropic saturated porous media, such as soils, is laminar and described by the classic equation of Darcy,

\[ q = -K \frac{\partial H}{\partial z} \]  

[1]

where \( q \) is the flux density (m/s), \( K \) the hydraulic conductivity (m/s), \( H \) the hydraulic head (m), the sum of pore-water pressure head (h) and elevation head (z) and \( \frac{\partial H}{\partial z} \) the hydraulic gradient, i (m/m).

Darcy’s Law can be extended to 2- and 3-dimensional flows by breaking down the total flow into a set of components in the two horizontal and one vertical directions. Equivalent equations for cylindrical and spherical co-ordinates can also be written. In saturated soils, \( K \) values ranging from \( 10^{-4} \) m/s for sandy soils to \( 10^{-10} \) m/s for fine-grained compacted landfill clays are commonly found (Rawls, Brakensiek and Saxton, 1982; Olson and Daniel, 1981; Youngs, Leeds-Harrison and Elrick, 1995). Hydraulic gradients in saturated soils generally vary by only half an order of magnitude, although in the surface layers of unsaturated soils at low water contents hydraulic gradients may be very large, e.g. 10 to 1000 (Hosty and Mulqueen, 1996).

Soils of high porosity, many large pores and good interconnectivity between them,
have high K values. The dependence of K on particle size and hence on pore size is reflected in the Hazen (1893) formula (Kezdi, 1974) often used as a first approximation for K (m/s) in sands and gravels (Equation 2)

$$K = 10^{-2} d_{10}^{-2}$$  \[2\]

where $d_{10}$ is the diameter (mm) of the smallest 10% fraction of a sand or gravel from a grading curve.

A further elaboration of the Hazen formula is the Kozeny (1927) equation (Kezdi, 1974), developed for a particular model of a bundle of uniform capillary tubes (Equation 3)

$$K = C d_{e} \left[ \frac{\frac{e^3}{1+e}}{\frac{\gamma_{w}}{\eta}} \right]$$  \[3\]

where C is an empirical co-efficient depending on the pore shape among other factors, $d_{e}$ the effective grain size usually taken as the $d_{10}$ (mm), e the void ratio, $\gamma_{w}$ the unit weight of water (10 kN/m³) and $\eta$ the dynamic viscosity of water (1.3 × 10⁻³ Ns/m² at 10 °C).

Equation 3 reflects the effects of pore size and shape, the void ratio and the physical properties of water which depend on temperature. However, this equation still only provides an approximation of K values.

Darcy’s law can only be applied if the water velocity through a porous medium is slow enough for flow to be laminar. While initially an empirical law, Darcy’s law is theoretically grounded on the Stokes-Navier equation (Childs, 1969). It is a consequence of the latter equation that the rate of flow of water through a saturated column of porous medium is proportional to the potential difference between the inflow and outflow faces of the column. This holds only if the flow is slow enough that the acceleration terms in the Stokes-Navier equation are small. It has been shown (Fancher, Lewis and Barnes, 1933 quoted in Childs, 1969) that Darcy’s law cannot safely be applied to flow in porous media if the Reynolds number (Re, Equation 4) exceeds one, a fact known for some time (Scheidegger, 1963)

$$Re = \frac{vr\rho}{\eta}$$  \[4\]

where $v$ is the mean flow velocity (m/s), $r$ the pore radius (m) and $\rho$ the density of water (10³ Ns²/m⁴).

In the case of water infiltrating under a unit gradient through a saturated coarse sand with a pore diameter of 0.5 mm and having a K value by the Hazen formula of $2.5 \times 10^{-3}$ m/s, Re is 0.48. This is close to 1 and to being as unfavourable as natural soils can get for laminar flow. Reynolds number for gravels can exceed 1 because of large pore size and fast flow rates indicating that Darcy’s law may not apply (Scheidegger, 1963). Exceptions to Darcy’s law are found in flow through soil cracks and gravels, where the flow rate increases less rapidly than the hydraulic gradient (Swartzendruber, 1962, 1968).

The finite-difference model MODFLOW (McDonald and Harbaugh, 1988), developed in the US Geological Survey, is one of the most widely used, tested and verified groundwater models in use today. There are a number of packages associated with the model, e.g. river, recharge, well, drain and evapotranspiration packages. In the model, Darcy’s law ($v = K(i)$), is assumed to hold, which may not be the case for gravel aquifers. In such cases, modifications may be required to the equations in the model.
Review of experimental results
Experimental results from flow tests through gravels indicate that non-linear relationships exist between flow velocity and hydraulic gradient (Scheidegger, 1963). Dudgeon (1966) found that the relationship between the rate of flow of water through gravels and the hydraulic gradient was a discontinuous exponential one of the form

\[ v = \frac{i^{1/n}}{a^{1/n}} \]  

[5]

where \( a \) is a constant (s/m) and \( n \) is also a constant.

Dudgeon (1966) characterised the domains as: prelinear at very low gradients with the exponent \( n \) less than 1; linear at intermediate gradients with the exponent \( n \) equal to 1; postlinear where the exponent \( n \) lies between 1 and 2. For the coarse gravel aggregates Dudgeon (1966) employed, the value of \( n \) lay between 1.8 and 1.9. In the prelinear domain, the non-Darcian behaviour maybe due to surface tension effects and the polar nature of water molecules. Mulqueen and Harrington (1976) in a brief paper reported that the flow of water through gravels obeyed the relationship

\[ v = a_i b \]  

[6]

where \( v \) is the velocity (m/s), \( a \) is a constant (m/s) and \( b \) a constant.

The co-efficient of determination was in excess of 99% for this relationship. However, no flow data were provided. Other authors, quoted in Dudgeon (1966) and Leps (1973), found similar relationships. Deviations from Darcy’s law can be considered in terms of a hydraulic gradient-dependent hydraulic conductivity, \( K(i) \), as already employed by Swartzendruber (1968). Then

\[ v = K(i) i \]  

[7]

Substituting equation 6 into equation 7 yields the expression

\[ K(i) = a_i b^{1-1} \]  

[8]

For the case where there is a unit gradient, \( K(i) \) equals \( a \) and is a constant. Equation 8 is employed in this paper to derive values for \( K(i) \) from flow tests. A discussion of some experimental data on hydraulic conductivity can be found in Lambe and Whitman (1969).

Materials and Methods
The gravels were packed in a smooth-walled PVC pipe that was 1.53-m long and 60-mm bore. This was placed horizontally on adjustable supports and connected to a 3-m³ water reservoir (Figure 1). The water
The level in the reservoir tank was held constant by a supply and overflow arrangement. The temperature of the water was between 10 and 11.5 °C. A range of hydraulic gradients was achieved by adjusting the elevation of the test pipe. The outflow from the test pipe was passed through a 90° bend that was swivelled on the pipe to an angle of about 45° to direct the outflow into a graduated bucket.

Tests were carried out on 30 aggregates which were either commercial aggregates or reconstituted aggregates obtained by thoroughly mixing together fractions pre-screened in the laboratory in varying proportions. Aggregates were sourced at gravel pits (pit gravel) and at quarries as crushed limestone (broken stone).

In packing the test pipe with aggregate, care was taken to minimise segregation of particles. Each test aggregate was divided into aliquots of about 0.5 kg and each aliquot was poured individually into the test pipe to minimise segregation until the pipe was full. The aggregates were retained by mesh screens at both ends of the pipe. The filled pipe was then connected to the reservoir tank. Tests were carried out over a range of hydraulic gradients generally varying from 0.04 to 0.6 with duplicate tests at each gradient. When the tests were completed on each aggregate, it was removed from the pipe, dried, screened and weighed. Porosity and void ratio were calculated for each aggregate.

Two aggregates, gravels 4 and 27, contained large particles with 8% and 21%, respectively retained on a 25.4-mm screen and 22% and 16%, respectively, passing a 19.1-mm screen. While the test pipe was 60-mm diameter, it is considered that, with the amount of smaller fractions present and the filling precautions taken, wall effects were minimised.

Results
The grading analyses of the commercial aggregates are shown in Figures 2 and 3 and those of the reconstituted aggregates in Table 1. Except for aggregate numbers 1,
2, 4 and 27, 100% of all the other aggregates passed the 19.1-mm screen. Aggregates 1 to 7 show a wide variation in grading analyses with d10 values (mm) of: 1.2, 1.9, 1.8, 15, 4.76, 5.8 and 7.9, respectively. Reconstituted aggregates of pit gravel and broken stone were compared, i.e. 12 and 15, 14 and 17, 16 and 19, 18 and 20, and 22 and 21, respectively. A number of aggregates had similar grading, e.g. 20, 28 and 26, 30 (Figure 3).

For aggregates with similar grading the broken stone was generally about 200 kg/m³ greater in dry bulk density than the

<table>
<thead>
<tr>
<th>Aggregate no.</th>
<th>Type of aggregate</th>
<th>Bulk density (t/m³)</th>
<th>Sieve size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>Pit gravel</td>
<td>1.15</td>
<td>100 0 0 0 0</td>
</tr>
<tr>
<td>9</td>
<td>Broken stone</td>
<td>1.39</td>
<td>100 0 0 0 0</td>
</tr>
<tr>
<td>10</td>
<td>Broken stone</td>
<td>1.31</td>
<td>100 10 10 10 0</td>
</tr>
<tr>
<td>11</td>
<td>Pit gravel</td>
<td>1.20</td>
<td>100 10 10 10 0</td>
</tr>
<tr>
<td>12</td>
<td>Pit gravel</td>
<td>1.20</td>
<td>100 100 100 100 0</td>
</tr>
<tr>
<td>13</td>
<td>Broken stone</td>
<td>1.41</td>
<td>100 10 10 0</td>
</tr>
<tr>
<td>14</td>
<td>Pit gravel</td>
<td>1.31</td>
<td>100 50 50 50 0</td>
</tr>
<tr>
<td>15</td>
<td>Broken stone</td>
<td>1.43</td>
<td>100 100 100 100 0</td>
</tr>
<tr>
<td>16</td>
<td>Pit gravel</td>
<td>1.28</td>
<td>100 25 25 25 0</td>
</tr>
<tr>
<td>17</td>
<td>Broken stone</td>
<td>1.54</td>
<td>100 50 50 50 0</td>
</tr>
<tr>
<td>18</td>
<td>Pit gravel</td>
<td>1.31</td>
<td>100 75 75 75 0</td>
</tr>
<tr>
<td>19</td>
<td>Broken stone</td>
<td>1.50</td>
<td>100 25 25 25 0</td>
</tr>
<tr>
<td>20</td>
<td>Broken stone</td>
<td>1.50</td>
<td>100 75 75 75 0</td>
</tr>
<tr>
<td>21</td>
<td>Broken stone</td>
<td>1.35</td>
<td>100 0 0 0 0</td>
</tr>
<tr>
<td>22</td>
<td>Pit gravel</td>
<td>1.16</td>
<td>100 0 0 0 0</td>
</tr>
</tbody>
</table>

Figure 3: Grading curves for aggregate numbers 23 to 30.
pit gravel (Table 1). This difference was due to the chert gravel used, the particles of which exhibited pitting and as a result had less mass per unit bulk volume. This indicates that 9 to 21% (commonly 14 to 17%) more bulk volume can be obtained using pit gravel for the same weight.

The K(i) values of aggregates 1 to 30, inclusive, are shown in log-log scale in Figures 4 to 7. (In Figure 4 aggregate 13 is coincident with aggregate 11 and aggregate 12 (not shown) was off-scale with K(i) values as low as 650 m/day at a hydraulic gradient of 0.56 and in Figure 6 K(i) of aggregate 22 is coincident with aggregate 21). K(i) values through the aggregates ranged over two orders of magnitude from 120,000 m/day to 700 m/day. The highest K(i), 120,000 m/day, was obtained for aggregate 27, a coarse gravel with 100% passing 38.1-mm screen and 16% through a 19.1-mm screen. There was a six-fold decline in K(i) of this gravel as the hydraulic gradient increased from 0.02 to 0.56. The next highest declined from a K(i) of 70,000 to 25,000 m/day with increasing hydraulic gradient.

Four predominantly 4.76 to 1.6 mm sized gravel aggregates, commonly used as an infill in slit trenches for the drainage of sports grounds, had the lowest K(i), varying from about 700 to 1900 m/day under unit gradient, being lowest for the pit gravels. All aggregates exhibited a marked decline in K(i) with increase in hydraulic gradient. On the arithmetic scale, all curves show a steep decline in K(i) at low gradients followed by a flattening thereafter as illustrated in Figure 8. In Figure 4 aggregate 4 (9.53- to 25.4-mm broken stone) had the highest values of K(i), falling from 70,000 m/day to 25,000 m/day as hydraulic gradient increased from 0.06 to 0.55, respectively. Aggregate 7 (9.53-mm to 19.1-mm broken stone) had K(i) values of 29,000 and 13,000 m/day at similar hydraulic gradients.

**Discussion**

Aggregates 16 and 18 (reconstituted pit gravel) had similar bulk densities and, therefore, similar porosities (52%).

![Figure 4: Hydraulic conductivity [K(i)] of aggregates 1 to 9 in relation to hydraulic gradient.](image)
However, gravel 16, with only 25% passing the 4.76-mm screen, was much coarser than 18 (75% passing the same screen). K(i) values varied from 5300 to 3600 m/day for aggregate 16 and from 1700 to 1200 m/day for aggregate 18, with K(i) arranged in accordance with an increase in hydraulic gradient, indicating that K(i) is strongly influenced by particle size and hence pore size as indicated in Equation 3. A comparison of K(i) curves in Figure 4 for aggregates 4 (9.53 to 25.4 mm) and 7 (9.53 to 19.1) also illustrates the influence of particle size, and hence of pore size, on K(i).

Aggregate 1, over 50% in the 1.6-mm to 4.76-mm range and 13% in the 0.8- to 1.6-mm range, had the lowest K(i), range 4000 to 2000 m/day, in Figure 4. All aggregates with about 75% in the 1.6-mm to 4.76-mm range (No. 1 in Figure 4, No. 18 in Figure 5 and No. 20 in Figure 6) had low overall K(i) while aggregate 12 (not shown) had the lowest of all at 650 m/day under a hydraulic gradient of 0.56. It is clear that significant amounts of fine-grained particles in aggregates control the flow rate as recognised in the Hazen empirical formula (Equation 2), in which the effective particle size of an aggregate is taken as the diameter of its smallest 10% fraction in a grading curve. There was a small decline in K(i) through fine-grained gravels in repeated tests at high hydraulic gradients, e.g. at a hydraulic gradient of 0.56, the K(i) of aggregate 12 declined from 770 m/day to 615 m/day over 10 consecutive tests, suggesting some slight redistribution of particles within the pipe.

Four comparisons were made between broken stone and pit gravel. Three of these, 14 v. 17 (Figure 5), 16 (Figure 5) v. 19 (Figure 6) and 18 (Figure 5) v. 20 (Figure 6) indicate a substantially higher K(i) in the broken stone aggregates, while aggregates 21 and 22 (Figure 6) were coincident. The ratio of K(i) for aggregate 17 to that for aggregate 14 was 1.74 at i = 0.07 and 1.32 at i = 0.56. Corresponding ratios for aggregates 19 and 16 were 1.89 and 1.51, respectively. While the pit gravels had more

Figure 5: Hydraulic conductivity [K(i)] of aggregates 10 to 18 in relation to hydraulic gradient.
volume per tonne, this advantage would be more than offset by the greater $K(i)$ of the broken stone aggregates as a result of their smoother pore walls. This shape effect is recognised in the $C$ co-efficient of the Kozeny equation (Equation 3).

Measurements of water flow through gravels were also carried out by Courtney (1978) for 10-mm and 20-mm aggregates, using a 2.9-m long 75-mm diameter sloping pipe arrangement with the outlet at a higher elevation than the inlet, different from that employed in the tests presented here. A comparison with his results is presented in Figure 8. There was good agreement between the

*Figure 6: Hydraulic conductivity $[K(i)]$ of aggregates 19 to 24 in relation to hydraulic gradient.*

*Figure 7: Hydraulic conductivity $[K(i)]$ of aggregates 25 to 30 in relation to hydraulic gradient.*
Courtney data and those from the present study. Mulqueen and Harrington (1976) obtained very good fits for the relationship between flow rate and hydraulic gradient using Equation 6. These relationships were transformed using Equation 8 to obtain $K(i)$ and a selection of the equations for coarse and fine aggregates used in the present study is given in Table 2. Applying Darcy’s law to the equations for gravels 4 and 18 (Table 2), for example, yields $v = 18377i^{0.52}$ and $1115i^{0.88}$, respectively. It is apparent that coarser aggregates exhibit flow characteristics similar to those for rough walled pipes while flow in the finer aggregates tends toward laminar flow in porous media with $v$ moving toward a direct proportionality with the hydraulic gradient. This indicates that Darcy’s law can be applied to flow through the latter without much error.

![Figure 8: A comparison of Courtney (1978) data with some data from the present series of tests.](image)

Table 2. Equations of best fit between hydraulic conductivity [$K(i)$] and hydraulic gradient ($i$) for coarse and fine aggregates

<table>
<thead>
<tr>
<th>Aggregate no.</th>
<th>Type of aggregate</th>
<th>Size range (mm)</th>
<th>Hydraulic conductivity (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Broken stone</td>
<td>9.5 to 25.4</td>
<td>$K = 18377i^{0.48}$</td>
</tr>
<tr>
<td>8</td>
<td>Gravel</td>
<td>9.5 to 19.1</td>
<td>$K = 9003i^{0.40}$</td>
</tr>
<tr>
<td>9</td>
<td>Broken stone</td>
<td>9.5 to 19.1</td>
<td>$K = 7776i^{0.38}$</td>
</tr>
<tr>
<td>21</td>
<td>Broken stone</td>
<td>9.5 to 19.1</td>
<td>$K = 8796i^{0.46}$</td>
</tr>
<tr>
<td>22</td>
<td>Gravel</td>
<td>9.5 to 19.1</td>
<td>$K = 9478i^{0.42}$</td>
</tr>
<tr>
<td>14</td>
<td>Gravel (50%)</td>
<td>1.6 to 4.76</td>
<td>$K = 1607i^{0.12}$</td>
</tr>
<tr>
<td>15</td>
<td>Broken stone</td>
<td>1.6 to 4.76</td>
<td>$K = 1607i^{0.12}$</td>
</tr>
<tr>
<td>18</td>
<td>Gravel (75%)</td>
<td>1.6 to 4.76</td>
<td>$K = 1115i^{0.12}$</td>
</tr>
<tr>
<td>1</td>
<td>Gravel</td>
<td>1.6 to 19.1</td>
<td>$K = 2143i^{0.17}$</td>
</tr>
<tr>
<td>3</td>
<td>Gravel</td>
<td>1.6 to 19.1</td>
<td>$K = 4285i^{0.24}$</td>
</tr>
</tbody>
</table>

†( ) indicates the percentage of the aggregate composed of the listed fraction.
Conclusions
The non-linear relationship \( v = a_i^b \) reported here between flow velocity \( v \) of water through aggregates, over a wide range of diameters (about 0.8 mm to 38 mm) and hydraulic gradient \( i \) shows that Darcy's Law does not hold for aggregates. In the coarser aggregates, the value of \( b \) approached 0.5, similar to that for turbulent flow through rough-walled pipes. In the finer aggregates, the value of \( b \) approached 1 similar to that in the classic relationship of Darcy for laminar flow through sands and soils, indicating that Darcy's Law could be applied without too much error in such cases. Flow velocity was primarily dependent on particle size, increasing rapidly as particle size increased, and secondarily on particle surface rugosity, declining substantially with increased surface roughness. The inclusion of significant amounts of finer fractions in otherwise coarse gravels can drastically reduce their \( K(i) \) and this should be recognised in drain and filter design.

The results from the present study show that the flow equations used in models, such as MODFLOW, for groundwater flow in gravel aquifers need to be modified to account for the exponential relationship between hydraulic conductivity and hydraulic gradient when modelling coarse gravel aquifers.

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