Impacts of Revenue Management on Estimates of Spilled Passenger Demand

by

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Abstract

In the airline industry, spill refers to passenger demand turned away from a flight because demand has exceeded capacity. The accurate estimation of spill and the lost revenue it implies is an important parameter in airline fleet assignment models, where improved estimates lead to more profitable assignments.

Previous models for spill estimation did not take into account the effects of passenger choice and airline revenue management. Since revenue management systems protect seats for later-arriving higher fare passengers, revenue management controls will influence the number of spilled passengers and their value because they will restrict availability to lower fare passengers even if seats on the aircraft are available.

This thesis examines the effect of various revenue management systems and fare structures on spill, and, in turn, the marginal value of incremental capacity. The Passenger Origin Destination Simulator is used to simulate realistic passenger booking scenarios and to measure the value of spilled demand. A major finding of the research is that in less restricted fare structures and with traditional revenue management systems, increasing capacity on a flight leads to buy-down which can result in negative marginal revenues and therefore revenue losses. This behavior is contrary to conventional wisdom and is not considered in existing spill models. On the other hand, marginal revenues at low capacities are greater than would be predicted by first-choice-only spill models because some passengers will sell-up to higher fares to avoid spilling out. Additionally, because of passenger recapture between flights, adding capacity to one flight can lead to revenue losses on another. Therefore, the marginal value of incremental capacity is not always positive. Negative marginal revenues and associated revenue losses with increasing capacity can at least be partially mitigated by using more advanced revenue management forecasting and optimization algorithms which take into account passenger willingness to pay.

The thesis also develops a heuristic analytical method for estimating spill costs which takes into account the effects of passenger sell-up, where previous models tend to underestimate the spill cost by only modeling passengers' first choices. The heuristic demonstrates improved estimates of passenger spill: in particular, in restricted fare structures and for moderate amounts of spill, the model exhibits approximate relative errors on the order of 5%, a factor of two improvement over previous models.

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Chapter 1 – Introduction

1.1 Airline Spill

The airline industry in the United States generates $200 billion in annual revenues and is responsible for the movement of over 600 million passengers within the US and around the globe [1]. Since deregulation in 1978, the industry has become extremely competitive with airlines frequently posting yearly operating losses. Between 2001 and 2010, US airlines had a cumulative profit margin of –4% [2]. In order to increase these profit margins, airlines must be extremely careful to maximize revenue and minimize costs. The proper estimation of airline spill leads to both improved revenue and cost structures for airlines.

Spill, within the airline industry, refers to passengers who are unable to purchase tickets for a flight because the demand for that flight has exceeded the number of seats made available for sale. In other words, spill is passenger demand turned away. For example, an aircraft with 200 seats for sale and a demand of 230 passengers will result in 30 passengers being spilled from the flight. The loss of these passengers' revenue contributions is an opportunity cost for the airline and is termed the spill cost. If these 30 spilled passengers would have purchased $100 tickets, the spill cost is $3 000 and the airline has given up $3 000 in potential revenues.

The spill cost will depend on the value of the passengers spilled, that is, on what fares they would have purchased had they not been turned away. The fares available for sale at any one time depend on an airline's choice of fare structure and revenue management system, brief explanations of which follow in Sections 1.2 and 1.3.

The spill cost is also a measure of the marginal revenue per seat as a function of aircraft capacity. For example, if the spill cost for a 100 seat plane is $1 000, the airline has given up $1 000 in potential revenue. If the spill cost for a 101 seat plane is $900, the airline has gained $100 by increasing the capacity of the aircraft by one seat. Thus, the marginal revenue obtained from increasing the capacity is $100. That is, the reduction in spill cost between two capacities is the marginal revenue per seat.

Spill modeling is the practice of estimating the spilled passenger count and spill cost given demands for a flight and a selected capacity. The contribution of this thesis is to explain how an airline's revenue management system affects passenger spill and to improve and extend existing spill models so that airlines may better estimate the value of incremental capacity.

1.2 Fare Structures

Typically, airlines offer many different fares in a market in order to segment demand and to extract fares at passengers' maximum willingness to pay, serving to maximize revenues. An example of passengers' willingness to pay is shown in Figure 1.1. There is a relationship between the price of a ticket and the demand for it. In the example, the relationship is linear, although it need not be in general. Some passengers are price insensitive and are willing to pay very high prices to travel, although there are fewer of these passengers (low demand willing to pay a high price). There are also many passengers who are only willing to pay low fares in order to travel (high demand at this price).

If an airline were to sell tickets at a single price, the revenue maximizing strategy would be to price tickets in the middle of the price-demand curve. This maximizes the area captured under the price-demand (quantity) curve, which is the revenue. This is illustrated in the Single Price graph in Figure 1.1. However, an airline can increase its revenues by practicing differential pricing. If an airline offers five different fares on a flight and is able to sell tickets at these different fares, the airline stands to increase its revenue, as illustrated in the Differential Pricing graph in the figure. An airline is able to sell some very high priced tickets because some passengers are price insensitive and are willing to pay more for a ticket. The captured (shaded) area under the price-demand curve has increased, and the airline has increased its revenue. Thus, an airline practicing differential pricing will increase its revenues above one using a single price structure, as long as passengers who would be willing to pay a very high price do not purchase a lower priced ticket instead. A situation in which a passenger with a high willingness to pay purchases a low fare ticket is termed "buy-down." Given that passengers will, all else equal, always wish to buy the least expensive tickets, an airline must work to ensure less expensive tickets are made unattractive to passengers who are willing to pay more.

In order to ensure high willingness to pay passengers purchase only more expensive tickets, airlines typically define different "fare products" in a fare structure and attempt to segment overall demand into demands for these products. Less expensive products are made unattractive to high willingness to pay customers through the application of various restrictions, such as restricting the refundability, upgradeability, or frequent flier mileage accrual of these tickets. By implementing these restrictions on lower fares, airlines can mitigate buy-down and steer passengers with a high willingness to pay to purchase higher fares.

A sample fare structure is shown in Figure 1.2. In such a structure, five fares, all at different prices, are available for sale to the consumer on a flight. While all fares are for a standard seat on the plane, the fares are divided into booking classes (or fare classes) and fares sold within each class are subject to certain restrictions. Some fares have advance purchase requirements (the class 5 fare must be booked at least two weeks before travel), some require a Saturday night stay, some have change fees, and so on. The differentiation between the fares leads to the term fare products, where passengers thus have a demand for a certain product (and its associated conditions) and not simply for a seat on the aircraft.

For example, a passenger traveling on business typically has a high willingness to pay (as his employer is likely paying for the ticket), but values flexibility in his travels and is therefore less likely to prefer a non-refundable fare, likely does not wish to remain at his destination over a Saturday night, and might only be able to purchase a ticket just before his travel date. Business travelers are thus unlikely to purchase lower class tickets (such as classes 3, 4, or 5) as they find the restrictions inconvenient, despite the lower price points. Conversely, a leisure traveler booking far in advance is less likely bothered by a non-refundable fare, may desire to visit a city for a weekend (thereby staying over a Saturday night), and likely has a much lower maximum willingness to pay. Thus, leisure passengers will usually book lower class fares and would often consider the higher classes (here, 1 and 2) prohibitively expensive.

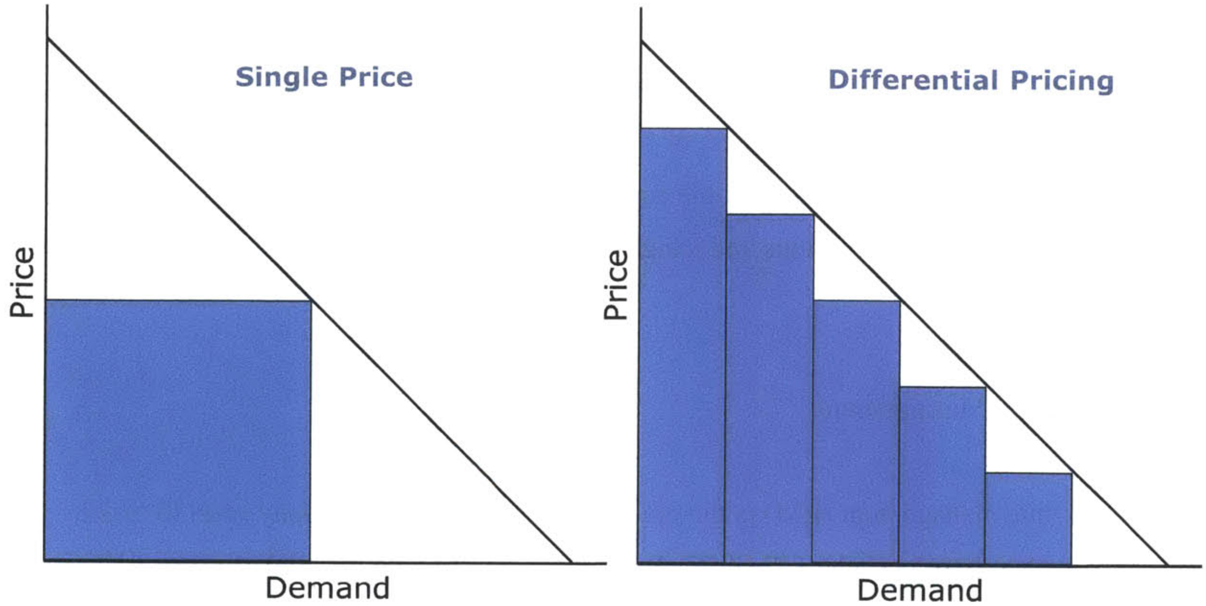


Figure 1.1: Price and demand relationship for an airline offering (*left*) a single ticket price and (*right*) differential pricing

Class	One-Way Fare	Advance Purchase	Minimum Stay	Change Fee	Refunds	Round Trip Required
1	$600	None	None	None	Yes	No
2	$500	3 days	None	None	Yes	No
3	$400	7 days	Sat night	$150	No	Yes
4	$300	10 days	Sat night	$150	No	Yes
5	$200	14 days	Sat night	$150	No	Yes

Figure 1.2: Sample fare structure with five different fare products for sale in a market

In practice, the actual differentiation strategies airlines use vary significantly. As in the example, airlines may offer non-refundable fares with round-trip travel requirements. They may also prohibit lower fare classes from earning frequent flier miles, upgrading to premium cabins, or selecting seats in advance. Any such restrictions effectively segment demand and prevent passengers with a high willingness to pay from buying down to lower fares.

It should be noted that fare products are typically offered for a single class of service. In other words, all the fares presented in Figure 1.2 are for economy class seats. A similar fare product structure can be constructed for business or first class. Demands for premium cabins are typically considered independent from the economy cabin. This thesis considers spill only in the economy cabin, but the discussions and insights into spill may be similarly applied to premium cabins.

1.3 Revenue Management

Airline revenue management (RM) is the practice of determining how many seats to make available for purchase in each of the different booking classes with the goal of maximizing revenues (where booking classes contain the various fare products) . The implementation of RM was credited with increasing the revenue of American Airlines by $500 million annually as early as 1992 [3]. The demands for each of the different booking classes must be estimated by the revenue management system and are typically forecast from an airline database of historical bookings.

Typically, passengers buying higher fares are business passengers who purchase tickets closer to departure, while passengers buying lower fares are leisure travelers who book further in advance. Thus, a revenue management system must protect seats for the later arriving business passengers. An overview of revenue management optimization and demand forecasting is given in Chapter 2.

Given that the revenue management system controls the number of seats in each class that are made available to passengers, it affects the number of passengers who are unable to get seats at their desired fare. Passengers can thus be spilled even if the total demand for a flight does not exceed capacity: as long as the demand for any given fare class exceeds the number of seats the RM system has allocated for it, spill will occur. Using a revenue management system, an airline is more likely to spill lower fare passengers, as high willingness to pay passengers should have higher fare seats protected for them and therefore should usually be accepted for travel. It is thus apparent that there is significant interaction between the spill cost and the choice of revenue management system and fare structure.

1.4 Spill Terminology and Passenger Choice

As discussed in Section 1.2, passengers buy tickets for a certain itinerary in a certain fare class. For example, an airline (Airline 1) may serve a Boston (BOS) – Los Angeles (LAX) nonstop market with 3 flights per day, as in Figure 1.3. The flights are scheduled at different times during the day, for example, morning, afternoon, and evening flights are operated. Airline 1 competes on the route with Airline 2.

The number of ticketed passengers on each of the flights is different as demands for each of the flights vary. For example, the evening flight from BOS may have a lower total demand as the arrival time into LAX is late at night and inconvenient. Each flight thus has a different load factor, where load factor is the fraction of the total capacity that is booked.

Consider a (leisure) passenger who wants to fly from BOS to LAX for the lowest possible fare, such as the $200 fare in the fare structure in Figure 1.2. He may prefer to fly with Airline 1 and would like to travel on the first flight of the day. The passenger may get his first choice of route and fare, for example, if he books a ticket far in advance of the travel date. This passenger has received his first choice (shown in red in Figure 1.3). If the lowest fare is unavailable on this flight, the passenger may purchase a ticket on another one of Airline 1's flights, at either an equal or higher fare. This passenger is thus recaptured (purple) and the airline does not lose his revenue contribution.

If the passenger's first choice fare is unavailable, the passenger may also be willing to buy a higher fare in order to get a ticket for the first flight of the day. In this case, the passenger has sold up (green) to a higher fare class and a more expensive ticket.

However, if a passenger does not receive his first choice and other alternatives on Airline 1 are not appealing (being recaptured or selling-up), the passenger is spilled and his revenue is lost to the airline. In this case, the passenger may spill-out (pink) to a competing airline (here, Airline 2), which also serves the BOS – LAX market. The passenger may also elect not to travel (at least by air), termed "no go" (orange). Similarly, a passenger who wanted to travel with Airline

2 and spilled-out may be spilled in (blue) to Airline 1. Thus, a passenger may fall into one of many different choice categories in the analysis of spill. Further, each of Airline 1's flights will have a different mix of passengers within the choice categories: first choice, recaptured, spilled-in, and sold-up passengers.

The mix of passenger choice categories will change if Airline 1 decides to reduce the capacity on one of these flights, as in Figure 1.4. The capacity of Flight 1 has decreased, as indicated by the blocked out seats in the figure. On Flight 1, fewer empty seats are available and the load factor increases. Thus, fewer passengers are likely to receive their first choices. Recapture likely increases on other flights, since with fewer total seats, fewer passengers are getting their first choices. Similarly, sell-up is likely to increase as some passengers will be willing to pay more to remain on the flight. Spill-out and no go would also increase given the capacity reduction.

Changes to the passenger choice mix would also be evident on other flights (2 and 3) given this capacity reduction on Flight 1. Recaptured passengers would likely increase as they are not able to obtain seats on Flight 1. The actual choice composition on these flights is not immediately intuitive. It is possible, for example, that sell-up on Flight 2 will increase: if the demands for the flight were high before the capacity reduction on Flight 1, more passengers will now be competing for the same seats, given the increased recapture from Flight 1. The exact passenger choice mix in such scenarios is of experimental interest in order to create accurate analytical spill models. This thesis examines the natural of the passenger choice mix and its changes as a function of capacity in several different fare structures and revenue management systems.

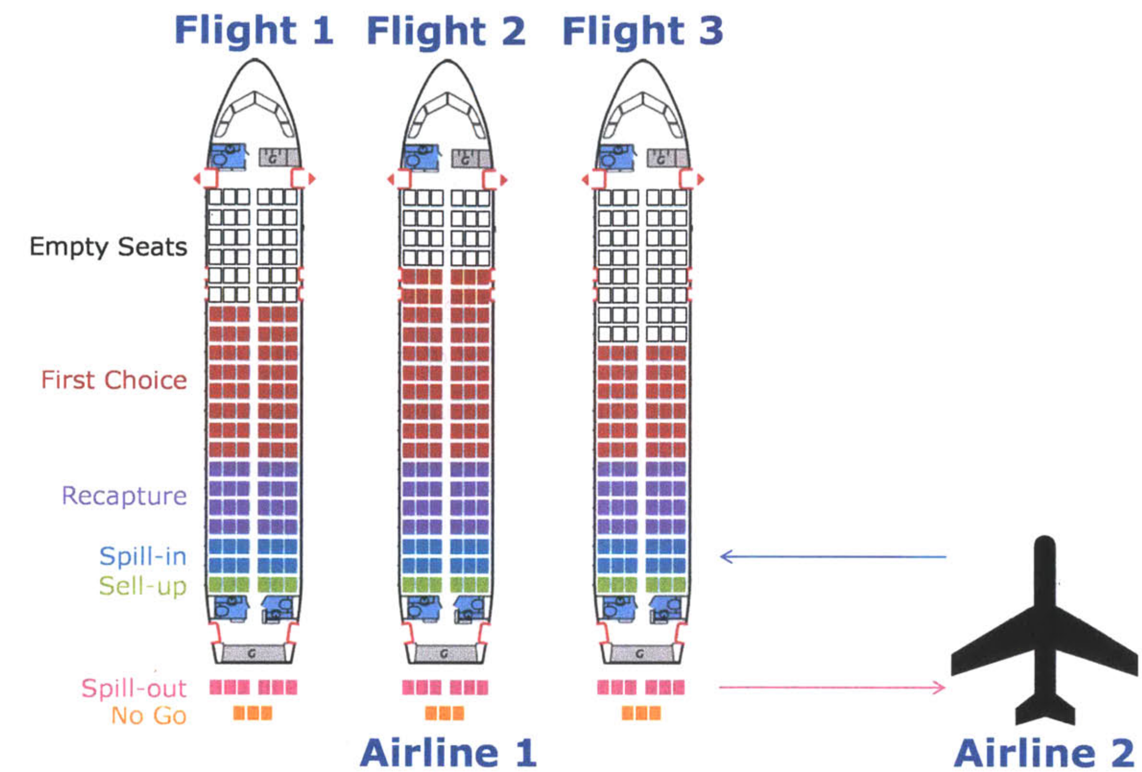


Figure 1.3: Options for passenger behavior when purchasing tickets for a flight

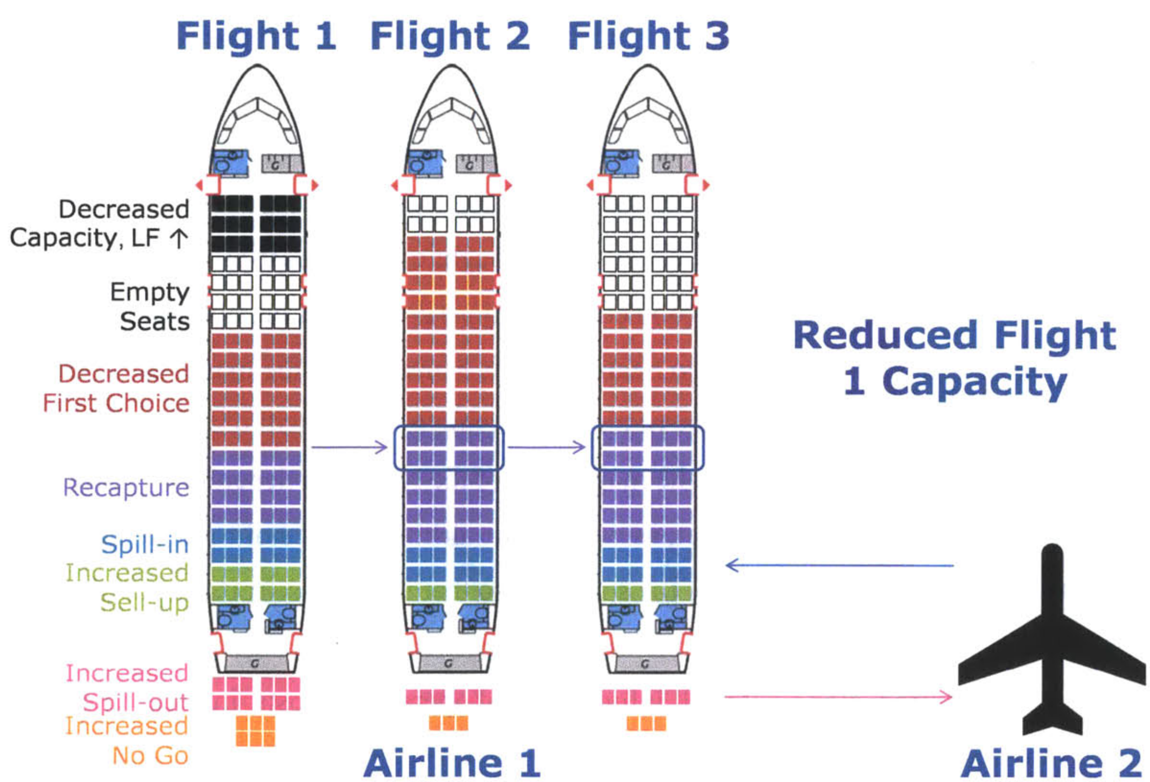


Figure 1.4: Change in passenger choice with reduced capacity on flight 1

1.5 Motivation: Importance of Accurate Spill Estimation

Accurate estimates of spilled demand and spill cost derived from spill models are important in both airline fleet assignment and in manufacturers' aircraft sizing. Accurate estimates of the spill cost given demands for a flight are important in airline fleet assignment. Fleet assignment is the matching of an airline's available aircraft to cover its schedule and is an important problem in an airline's operations, further discussed by Barnhart *et al.* in [4]. An airline desires an assignment which maximizes its profit contribution. This contribution consists of ticketed passenger revenues less operating costs of the aircraft operating the schedule. The operating costs depend on the airline's selected fleet assignment. For example, operating costs will change depending on the size of aircraft selected to operate certain flight legs – assigning larger aircraft to shorter haul flights will lead to different operating costs than operating the larger aircraft on longer haul flights. Subramanian *et al.* describe some operational costs and issues Delta Air Lines considered in developing its fleet assignment methods in [5].

Ticketed passenger revenue on a market depends on the size of the aircraft operating the flight legs making up the market. Assigning a larger plane to a flight leg is likely to result in increased revenues as additional seats are now available for previously-spilled passengers. However, this increased revenue comes with increased costs in operating the larger aircraft. Thus, the profit-maximizing fleet assignment best trades off revenues against costs in covering the schedule.

The profit-maximizing fleet assignment can also be thought of as the cost-minimizing assignment, where the assignment cost (objective function) consists of the operating cost and the spill cost. The spill cost, as mentioned in Section 1.1, is a measure of the marginal revenue of incremental capacity. Increasing the size of aircraft serving a market will result in a reduction of the spill cost (i.e. a revenue gain), with the magnitude of the reduction depending on the demands and fares offered in that market, as well on the airline's choice of revenue management system. An airline requires estimates of the spill cost so that it may assign its larger aircraft to markets that stand to gain the most revenue from the larger capacity. Thus, accurate estimates of the spill cost are required, along with full information of operational costs, in order to obtain profit-maximizing fleet assignments.

Accurate spill estimates are also important from aircraft manufacturers' perspectives. Aircraft manufacturers wish to be able to size future aircraft to fulfill demand on a set of airline routes. With proper understanding of the spill cost, an airline manufacturer can trade off the marginal revenue as a function of capacity against the cost of operating a larger air frame and may approach an airline with a strong case for purchasing an aircraft of a specific size that the manufacturer offers.

1.6 Shortcomings in Current Spill Modeling

Spill modeling was studied extensively at Boeing Commercial Airplanes in the 1970s. Boeing's approach for spill modeling used a record of historical bookings to obtain average load factors for a flight in order to predict spill on future flights [6]. This approach became known as the Boeing Spill Model and does not explicitly consider the revenue management system or the mix of fare products offered for sale. The approach was effective in the era of regulated aviation when load factors on flights were low (62% US average in 1978 [7]) and airlines were only permitted to offer a single fare for each cabin on a flight. However, as the airline industry deregulated, airlines came to offer tens of different fares for sale. With the large number of fares offered, a revenue management system controls availability of seats at each of these fares, and ignoring the effect of the revenue management system leads to inaccurate estimates of the spill cost. The inaccuracy becomes especially pronounced as the airline industry cuts capacity, load factors continue to increase (83% US average in 2011 [8]), and more passengers are spilled. Some of the weaknesses of the Boeing Spill Model are illustrated in the experimental portion of the thesis in Chapter 5.

Swan [9] extended the Boeing Spill Model to consider demands for fare products and approximate the effects of revenue management in 1992. Swan's model obtained the estimated number of spilled passengers by creating a joint demand distribution of the demands for the different fare classes offered for sale and assumed the spill cost to be equal to a weighted average of the fares for sale on a flight multiplied by the number of passengers spilled. In particular, Swan indicates that Boeing used a weighted average consisting of 80% of the lowest available fare for sale in a market and 20% of the average fare, finding this heuristic reasonably effective in their applications. The weights in the average may be adjusted based on an airline's knowledge of its demands. With the weighting, Swan estimates that the value of a spilled passenger is only slightly higher than the value of a lowest-fare passenger. This approximates the effect of revenue management – a good revenue management system should protect seats for higher fare passengers, such that most higher fare passengers may always purchase seats. Increasing the capacity by one seat should only reduce the spill cost by approximately the lowest fare.

More recently, a model developed by Farkas [10] explicitly considers the revenue management system in its estimates of spill. However, the model considers only first choice demand and does not account for recapture or sell-up in its spill estimates. Farkas's model is likely to overestimate spill costs, given some portion of passengers are likely to sell-up to higher fares or be recaptured on other flights in the airline's network rather than spilling out.

Finally, both the Farkas and Boeing spill models only consider spill on a single flight leg. However, most larger airlines operate hub and spoke networks with many flights per day between city pairs and connecting itineraries on each aircraft flight leg. Estimating spill on such networks is complicated as passengers have many options when traveling: a passenger wishing to travel from Boston to Los Angeles can be recaptured on another nonstop flight on the same day or may be recaptured on a connecting itinerary through, for example, Chicago. More recent methods for fleet assignment attempt to consider the probability of a passenger being recaptured in an airline's network, such as assigning recapture rates based on the attractiveness of alternative paths to spilled passengers, such as a model by Wang *et al.* [11], but these methods again do not consider the effect of revenue management: the passengers more likely to be spilled and recaptured are lower class passengers. The experimental work in this thesis illustrates some of the shortcomings of current spill models and explores the interactions of revenue management in single flight and multiple flight scenarios.

1.7 Thesis Contribution

This thesis examines the effect of various revenue management systems and fare structures on the composition and value of spilled passenger demand. It explores the interactions between revenue management and the marginal value of incremental capacity through a series of experimental simulations using the Boeing-developed Passenger Origin Destination Simulator (PODS). In particular, it explains how passenger choices, such as choosing to sell-up to a higher fare, to travel at a particular time of the day, and to be recaptured on another flight, affect the marginal revenues based on the revenue management system and fare structure an airline has chosen to use. The effect of revenue management is explored in both monopolistic and competitive scenarios. It is hoped that the thesis will provide a more complete picture on the interactions between revenue management and spill in a realistic airline market: a market in which multiple airlines operate several flights per day and compete for passengers who are looking for low fares and will purchase tickets on any airline depending on the available capacity.

The thesis also tests the Farkas and Boeing spill models against "true" simulated spill results, highlighting the differences in the spill model estimates from the simulated spill results and therefore illustrating the importance of considering passenger choices and revenue management in estimations of the spill cost. The thesis also presents a heuristic method which extends the Farkas spill model to take into account passenger sell-up on a single flight and shows the improved performance of this method as compared to the Farkas and Boeing spill models. It is hoped that this extension serves as a starting point to improving spill models further down the road.

1.8 Outline of the Thesis

The remainder of the thesis is organized into several chapters. Chapter 2 gives an overview of basic demand forecasting and revenue management methods and explains existing spill models in more detail. Chapter 3 outlines the experimental methods used in the thesis and introduces the PODS revenue management simulator tool. Chapter 4 examines spill in a single market and serves as the primary results chapter in the thesis. It provides a complete discussion on the effects of revenue management, fare structure, and passenger choice on spill. Chapter 5 briefly tests existing spill methods against simulated spill results and presents an extension to the Farkas model which incorporates sell-up in estimates of the spill cost. Finally, Chapter 6 summarizes the results of the thesis and outlines some possible areas of future research.

Chapter 2 – Literature Review

2.1 Introduction

Revenue management in the airline industry has been developed over the past 30 years and a large quantity of literature on revenue management topics exists. This chapter attempts to summarize some of the major developments in revenue management and discusses the components of a modern revenue management system in more detail.

This chapter also outlines the major developments in airline spill modeling, while illustrating some of the shortcomings in existing models. This discussion presents the motivation for the work in the thesis and also serves to provide some background on the formulations of the models, given that an extension to them will be presented in subsequent chapters.

2.2 Revenue Management Overview

The use of revenue management (RM) has been credited with an increase in revenues for the airlines of 4-6% [12] above simply using differentiated pricing strategies. It is important to note that the revenue management problem is not one of pricing. While an airline has the freedom to set fares on its routes as desired, due to the extremely competitive nature of the industry, an airline's strategy is to typically match its competitors' fares. In this case, the path to maximizing revenue lies in selling the correct amount of inventory (seats) at each of the different fares (where passengers purchase fares that best match their willingness to pay, as discussed in Section 1.2). The function of an RM system is to protect the correct number of seats for the different booking classes in order to maximize revenues. Recall that higher fare passengers (such as business travelers) typically book closer to departure while lower fare passengers (such as leisure travelers) typically book further in advance. The RM system must thus ensure that the correct number of seats is protected for these later-booking higher class passengers.

The design of modern RM systems is described by Barnhart *et al.* in [13]. In essence, an RM system has three major components – a forecaster, an optimizer, and an overbooking model.

The forecaster uses actual (current) bookings and data from historical bookings as its inputs. For a given flight leg or itinerary, an airline keeps track of both historical and current bookings in order to estimate the demands for each of the booking classes on the flight. Two forecasting models, pick-up and hybrid forecasting, are further discussed in this thesis. The estimated demands obtained from the forecaster are fed into the optimizer and overbooking model.

The optimizer takes estimated demands for each booking class as inputs and computes seat protections for each of the different booking classes. Most modern RM systems do not allocate seats to the different classes, but protect seats in a nested structure, as shown in Figure 2.1. In this example, the airline has 4 booking classes in order of decreasing fare and the optimizer has computed nested booking limits BL1 through BL4. For the top class, BL1 = C seats are available, where C is the capacity of the aircraft. It is clear that an airline would want to accept all of its highest fare passengers. To ensure the airline provides space for its class 1 passengers, it protects (C – BL2) seats exclusively for them. It then protects (C – BL3) seats for class 1 and 2 passengers. Thus, the inventory offered for a certain class comes out of a shared pool of all lower (or less valuable) classes. In such a fashion, an airline never rejects higher class bookings for lower class ones if demands for them exist. If more passengers purchase higher class tickets, this simply results in fewer seats available for lower class passengers.

Thus, the job of the RM optimizer is to compute the booking limits BL1 through BL4 in order to maximize revenues. Many different RM optimizers have been studied in the literature. Two of the most popular optimizers, the Expected Marginal Seat Revenue heuristic and Displacement Adjusted Virtual Nesting, are further discussed in this thesis.

Overbooking models are not considered in this thesis, although they are in practice an important component of the airline revenue maximization problem. A more complete literature review of the development of overbooking methods is found in [13].

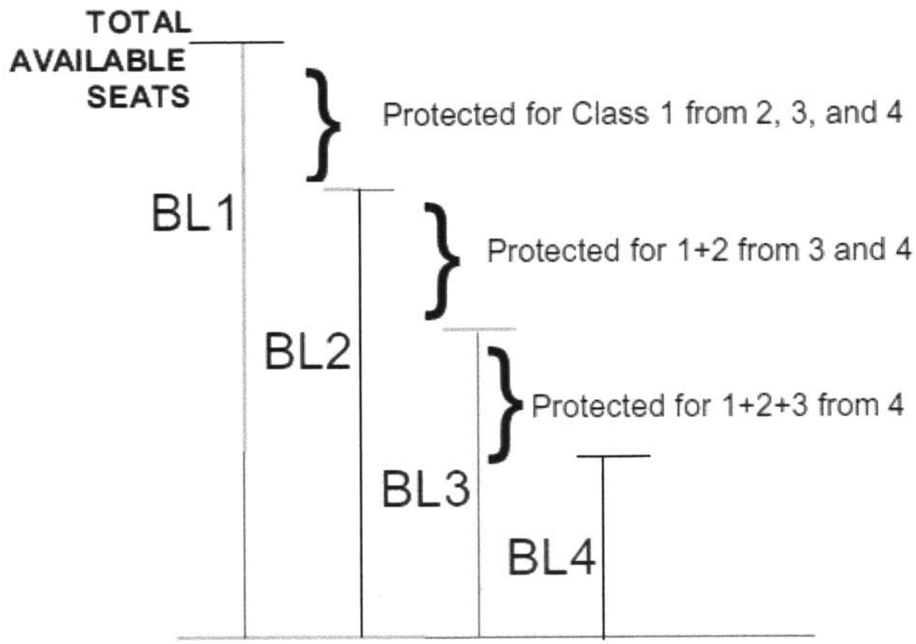


Figure 2.1: Nested booking limit structure often employed by airlines, as discussed by Barnhart *et al.* in [13].

2.3 The Forecaster in Revenue Management Systems

As mentioned in Section 2.2, the job of the forecaster in an RM system is to estimate the demand for each of the different booking classes offered for sale on any given itinerary so that these demands can be fed into the optimizer to generate booking limits. Recall that the goal of offering different fare products is to segment demand according to willingness to pay. Historically, demands for each of the different fare products were taken to be independent and the forecaster needed only to estimate demand for each product (that is, for each booking class). To this end, Belobaba [14] writes that pick-up forecasting, described in the following section, is most commonly used in airline RM systems when demands can, at least approximately, be assumed to be independent. This occurs in restricted (differentiated) fare structures, such as the example structure given in Figure 1.2. In a restricted fare structure, a passenger is assumed to be interested in a single fare product only. For example, a passenger may require a refundable fare and may wish to purchase his ticket three days in advance of travel. In this case, the passenger wishes to purchase a class 2 fare. For pick-up forecasting to be effective, the passenger must not consider purchasing the expensive and less restrictive class 1 fare if the class 2 fare is sold out (he must not consider selling up). In this fashion, the demand for class 2 fares is independent of the demand for class 1 fares. Of course, some passengers may be willing to pay this more expensive fare – if many passengers are allowed to purchase fares outside of their first choices, as in some other (less restricted) fare structures, more advanced forecasting techniques are required, such as hybrid forecasting, which is discussed in Section 2.3.3.

2.3.1 Pick-up Forecasting

In pick-up forecasting, an airline attempts to forecast bookings-to-come for a flight in each of its booking classes at prescribed time frames before departure, beginning from when bookings open for a flight through to a frame immediately before the flight's departure. In any given time frame, the pick-up forecaster estimates bookings-to-come by averaging historically-recorded bookings of previous flight departures. This pick-up is added to the current bookings received in order to form a total estimate of demand for a flight. For example, three weeks before the departure of some Friday afternoon flight, the pick-up forecaster may estimate bookings-to-come by averaging the number of historical bookings recorded within three weeks of departure on previous Friday afternoon flights. This estimate is added to bookings currently received for the flight in order to forecast the total number of bookings for the flight. The forecasting is done for each booking class and demands for each class are fed to the optimizer. Given pick-up forecasting's frequent use in the industry, it is used as a baseline forecaster in this thesis and is referred to standard forecasting in the experimental section of the thesis. Gorin discusses pick-up forecasting in more detail and provides a worked example in [15].

2.3.2 Forecasting in Less Restricted Fare Structures

Recent trends in the airline industry, especially the emergence of low cast carriers (LCCs) described by Belobaba in [16], have led to the simplification of fare structures into so-called less restricted or completely unrestricted (undifferentiated) ones. These fare structures differ from the differentiated structure in Figure 1.2 in that many fare products are identical but for their prices. An example of a less restricted fare structure is shown in Figure 2.2. Here, the minimum stay and round trip requirements for the lower fares are dropped and many of the fare products are undifferentiated (for example, classes 1 and 2 and classes 3 and 4 offer the same fare products, differing only in their prices).

In such a fare structure, traditional forecasting techniques (such as pick-up forecasting), in which demands for each fare class are assumed to be independent, are entirely ineffective. Given a choice of classes 1 and 2 (or 3 and 4), a passenger will always buy the lower priced ticket if it is available, given the fare products differ only in price. Therefore, no bookings of the higher classes 1 and 3 would be recorded in the historical booking database, causing the pick-up forecaster to interpret no demand for such classes. Without any demand for these classes, the optimizer would not protect any seats for them, and instead more seats would be made available to the lower classes. With these increased lower class bookings now being recorded in the database, the forecaster would interpret these as increased demand for lower classes, and so the optimizer would make still more lower class seats available. The cycle would continue until most of the bookings would only be found in the lowest classes. The airline has lost much of its revenue as minimal passengers have booked valuable upper class fares, even though some passengers are likely willing to pay higher fares to travel. This effect is called "spiral down" and must be avoided by an airline's RM system. Spiral down is described in more detail by Cooper *et al.* in [17].

To mitigate the effect of spiral down and to better estimate demands for different classes in less restricted fare structures, more advanced forecasting methods were developed, such as hybrid forecasting, discussed in the following section.

Class	One-Way Fare	Advance Purchase	Minimum Stay	Change Fee	Refunds	Round Trip Required
1	$600	None	None	None	Yes	No
2	$500	None	None	None	Yes	No
3	$400	None	None	$150	No	No
4	$300	None	None	$150	No	No
5	$200	14 days	None	$150	No	No

Figure 2.2: Sample less restricted fare structure

2.3.3 Q- and Hybrid Forecasting

Given the limitations of pick-up forecasting in less restricted fare structures, Belobaba and Hopperstad developed a method known as Q-forecasting in 2004 [18]. Q-forecasting is appropriate for an entirely undifferentiated fare structure in which the only distinction between fare products is their price. An undifferentiated fare structure may have the same fares as the structure in Figure 2.2, but all characteristics of the fares would be the same (such as no advance purchase requirements, no minimum stay, a change fee, no refunds, and no round trip required).

Q-forecasting first estimates demand only for the lowest class (where the lowest class frequently has the letter code *Q* in an airline's RM system) and then considers the probability of a passenger selling-up to a higher fare by incorporating estimates of passengers' willingness to pay. For example, if 10% of passengers would be willing to pay a top class fare, the RM system would close down lower booking classes throughout the booking process so that these 10% of passengers would be forced to buy the highest class fares (if they desired to travel). Thus, these passengers have been made to pay an amount closer to their maximum willingness to pay. The Q-forecaster is thus different from the pick-up forecaster in that it does not directly use historical recorded bookings by class to predict how many passengers are willing to pay for upper class fares.

While Q-forecasting is appropriate for use in a completely undifferentiated fare structure, Boyd and Kallesen [19] introduce hybrid forecasting for use in semi-restricted fare structures (or simply "less restricted" fare structures), such as the one exhibited in Figure 2.2. In hybrid forecasting, pick-up forecasting is used for classes in which demands may be assumed to be independent (that is, classes where the fare products differ, such as for classes 2 and 4) and Q-forecasting is used to estimate willingness to pay and elicit sell-up in classes for which the fare products are identical (such as between classes 3 and 4). Reyes outlined the revenue benefits of using hybrid forecasting in [20] and found hybrid forecasting greatly outperformed pick-up forecasting in semi-restricted fare structures. Hybrid forecasting is often used with fare adjustment (explained in Section 2.5) to best deal with less restricted fare structures. Hybrid forecasting is used as a more advanced revenue management forecaster within this thesis.

2.4 The Optimizer in Revenue Management Systems

As discussed in Section 2.2, the optimizer in an RM system is responsible for determining booking limits for each of the different booking classes in the fare structure. The Expected Marginal Seat Revenue heuristic and Displacement Adjusted Virtual Nesting are two popular revenue management optimizers in use throughout the airline industry and are discussed here in more detail.

2.4.1 Expected Marginal Seat Revenue Heuristic

Early work on optimal seat protections for nested booking limits was done by Littlewood [21]. Littlewood's model considered two types of fares – low-yield and high-yield – with fares averaged into one of these two classifications. Littlewood's model determined how many low-yield passengers to accept on a flight, assuming low-yield passengers booked before high-yield ones. The number of low-yield passengers to be accepted on a flight was determined by considering the estimated probability that remaining seats would be filled by high-yield passengers. If such a probability was high, more seats were protected for these high-yield passengers, and vice versa.

Littlewood's model was extended by Belobaba in his PhD thesis [22] and a subsequent paper [23]. The extended model was termed the Expected Marginal Seat Revenue (EMSR) heuristic and could handle an arbitrary number of nested booking classes. With some further modifications for improved robustness, Belobaba developed the EMSRb heuristic in 1992 [24].

The EMSRb model assumes normally distributed and independent demands for each booking class. It also assumes that bookings occur bottom-up, that is, that all lower class bookings arrive before the first booking in the next higher class is recorded. While, in general, lower class passengers book before upper class ones, it is unlikely that classes will always fill exactly bottom-to-top. However, upper class passengers booking early simply results in higher revenues – lower classes tend to fill anyway and an early arriving upper class passenger is now accepted instead of a lower class one. In any case, airline RM systems recompute booking limits (reoptimize) frequently during the booking process based on bookings already received.

Booking limits for any given class using EMSRb are determined by considering the marginal revenue gain if the booking limit for that class were increased by one seat. Given the normally distributed demands, there is an expected value from protecting one more seat for the class given by the probability of receiving one more booking multiplied by the fare for that class. As the number of protected seats increases, the expected marginal revenue decreases as the probability of receiving additional bookings decreases. The EMSRb heuristic protects seats (i.e. generates a booking limit) for a given class until the marginal revenue of protecting an additional seat is less than the fare of the next lowest class. A graphical interpretation of EMSRb for a four class flight leg is given in Figure 2.3. A complete treatment of EMSRb is given in [24]. Given EMSRb's widespread use in the industry, it is used as a base-case optimizer in this thesis.

EMSRb is a leg-based optimizer in that it tries to maximize revenues for any given leg in a network and controls availability for connecting itineraries by examining booking classes on each leg: a passenger will be able to purchase a class 5 BOS – YYZ – FRA fare if booking class 5 is available on both the BOS – YYZ local leg and the YYZ – FRA local leg. If class 5 is closed on one of these legs, the class 5 connecting itinerary will be unavailable and the airline may be losing on substantial revenue since the long haul passenger is likely valuable to the

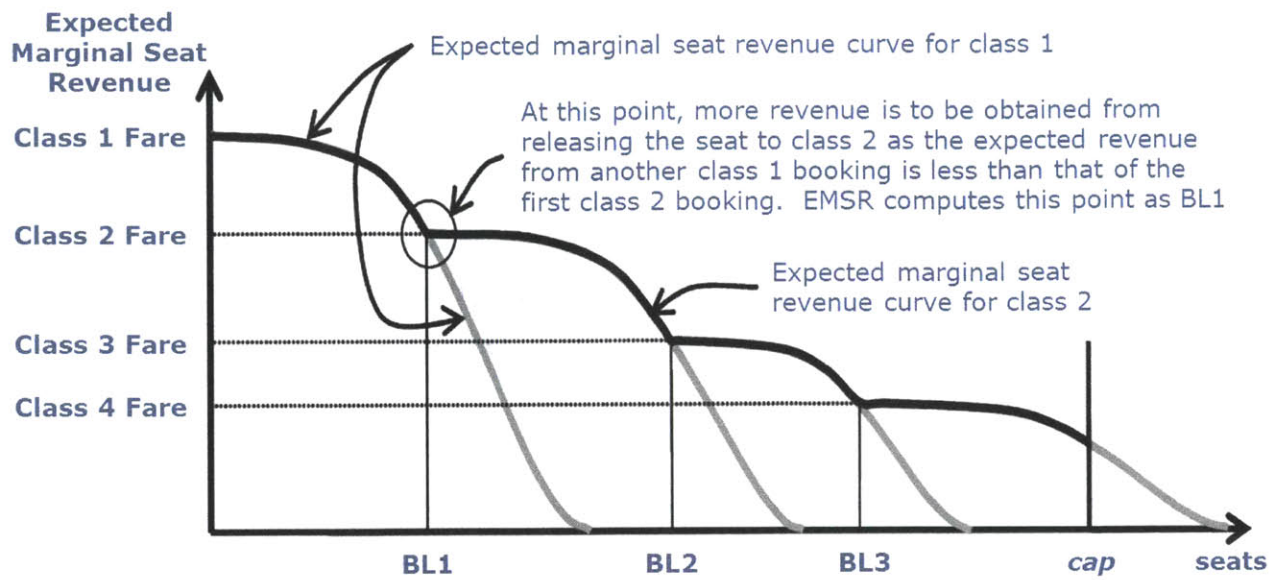


Figure 2.3: Expected Marginal Seat Revenue curves for a four class flight, showing computation of booking limits BL1 through BL4. Adapted from Belobaba [14].

Airline, even if the fare is of a lower booking class. The goal is to maximize revenue on the network and not simply on any given leg – airlines have thus begun to develop network revenue management systems, introduced in the following section.

2.4.2 Displacement Adjusted Virtual Nesting

Given the limitations of leg-based methods such as the EMSRb heuristic, airlines have begun to move to more advanced network revenue management systems which serve to maximize revenues over the airline's network and not simply a single flight leg. Belobaba has found that network revenue management systems lead to a further revenue gain of 1-2% over their leg-based counterparts [12].

Belobaba outlines some of the advantages in network revenue management, or "origin-destination (OD)" control, over leg-based control in [25]. In essence, a lower class long haul connecting itinerary, such as a $750 BOS – YYZ – FRA itinerary, is a higher revenue contribution to the airline than a higher class short haul $400 BOS – YYZ fare. A leg-based system will protect seats for higher classes on each leg and may not accept the lower class connecting itinerary if the BOS – YYZ leg is in demand and has few empty seats. Thus, a leg-based RM system may be led to accept the $400 fare over the $750 fare and forgo $350 in revenue.

If available seats are few and demands exist for high fare local traffic on both of these legs, such as the $400 BOS – YYZ fare and a $600 YYZ – FRA fare, then the airline should not accept the lower class connecting itinerary as it can earn $1000 instead of $750 for the same one seat on both legs.

It becomes clear that revenue must be maximized over the network so that the most valuable itineraries to the airline are always accepted. Good OD control, Belobaba explains [25], must thus follow two strategies: (1) when seats are available on flights which make up connecting itineraries, availability should always be made to high-revenue long haul passengers, even if these itineraries are in lower classes; and (2) if flights are more likely to be full, the airline must protect seats for upper class local traffic.

An OD control method that effectively implements these strategies is known as Displacement Adjusted Virtual Nesting (DAVN). DAVN estimates the "displacement cost" of accepting a connecting passenger at the expense of local passengers and the revenue contribution to the airline of any itinerary is taken to be the *fare – displacement cost* (since by accepting a connecting passenger, the airline may be unable to accept a local one on some or all of the legs of the connection). If flights tend to have many available seats and do not fill to capacity, the displacement costs will be low and the RM system will tend to accept high-revenue connecting traffic. If flights tend to have few available seats, displacement costs will be higher, and the system will prioritize higher fare local traffic.

The mechanics of a DAVN optimizer are as follows. The forecaster provides demand estimates of traffic for each itinerary on the network. The DAVN optimizer then solves a revenue-maximizing deterministic linear program, where the objective function is to maximize revenue on the network given the different itineraries which could be accepted and subject to the constraints that the number of accepted passengers must be less than the (assumed deterministic) demands for each itinerary and the capacity of each aircraft. The shadow prices of the solution become the displacement costs of accepting connecting traffic on each leg. The displacement-adjusted fares are then ranked into "virtual buckets," much like the ranking of classes in an EMSRb leg-based optimizer. The availability of itineraries is then obtained out of these buckets by EMSRb control. In this way, the DAVN optimizer successfully implements the two strategies described by Belobaba above. The DAVN method is described in further detail by Williamson [26] and Bratu [27].

Other origin-destination methods exist, such as Probabilistic Bid Price control, also described by Bratu. Research into further improving network revenue optimizers continues. This thesis deals exclusively with a single flight leg and no network methods are thus required. Any extensions to the network level based on the work in this thesis would likely use DAVN as a starting point.

2.5 Fare Adjustment

Fare adjustment is an advanced optimization technique developed by Fiig and Isler [28] in order to deal with buy-down in less restricted fare structures.

Consider a two class fare structure in which only the top class has seats available for sale (is open). In such a structure, there will be some number of passengers willing to purchase tickets in the top fare class. If the bottom class is then opened, another group of passengers will be willing to purchase these tickets (who, for example, may have otherwise elected not to travel as the top fare class was too expensive). In a restricted fare structure, the revenue gain would simply be the fare contribution of all these new passengers because the demands for each of the classes are (assumed) independent. However, in a less or unrestricted structure, some passengers from the top class group are likely to buy down into the low class. Therefore, the revenue gain from opening the low class is not simply the number of new bookings multiplied by the fare of the low class, but is some lesser value due to the buy-down from the top class passengers.

Fare adjustment estimates the revenue loss due to passengers buying down and computes adjusted fares for the low class bookings, accounting for the buy-down. For example, if the lower class fare is $200, the value to the airline of any seat sold in the low class may be only $180 because of the buy-down from the top fare class.

Fare adjustment then allows the transformation of a less or unrestricted fare structure with dependent demands into an equivalent structure with independent demands: hybrid forecasting gives the demands for each class and the adjusted fare gives the marginal value of a booking in any given fare class.

By using fare adjustment, an airline can continue to use DAVN and EMSRb optimizers as if the fare structure were a restricted one, simply by utilizing the adjusted fares in the optimizers' computation of booking limits. Without using fare adjustment, optimizers would release too many lower class seats as they would overestimate the revenue contribution of these lower classes. Fare adjustment corrects for this by valuing the lower classes at a reduced dollar amount.

Therefore, in less or unrestricted fare structures, both hybrid forecasting and fare adjustment should be used to incorporate the impact of passengers selling up and buying down to upper and lower classes, respectively. A full treatment of fare adjustment is given by Fiig and Isler in [28]. Hybrid forecasting with fare adjustment is used as an advanced revenue management system in the experimental portion of this thesis.

2.6 Spill Models

Spill modeling is the estimation of rejected passenger demand and its value from any given flight. Spill models assume known (or forecast) demand profiles. The importance of accurate spill modeling was detailed in Section 1.5 and centered on a requirement for good fleet assignments.

Two of the major spill models in the literature are discussed in this thesis. The first is an early spill model developed by Boeing, the Boeing Spill Model, with an extension to incorporate revenue management given later by Swan. The second is a more advanced spill model developed by Farkas designed to better capture the effects of revenue management.

2.6.1 The Boeing Spill Model

Much of the early work on airline spill estimation was performed at Boeing Commercial Airplanes in the 1970s [6] and came to be known as the Boeing Spill Model (BSM). The BSM assumes a single normally distributed unconstrained passenger demand distribution $f(x)$ for a given flight leg, defined by mean demand μ and standard deviation σ. Spill will occur when the realized demand on any given departure is larger than the capacity C of the operating aircraft. Over many departures of the flight, an average number of spilled passengers can be estimated. The number of expected spilled passengers is given by the tail end of the demand distribution, where demand is greater than the aircraft's capacity, as seen in Figure 2.4. Mathematically, the number of spilled passengers is

$$\mathrm{SP} = \int_C^\infty f(x)(x-C)\,dx, \tag{2.1}$$

where

SP is the total spill,
C is the capacity of the aircraft, and
$f(x)$ is the normal demand distribution.

Thus, for every drawn demand x above the capacity of the aircraft, the number of spilled passengers is equal to the number of booking requests larger than the capacity, or $x - C$. These spilled passengers are weighted by the probabilities of obtaining such a drawn demand.

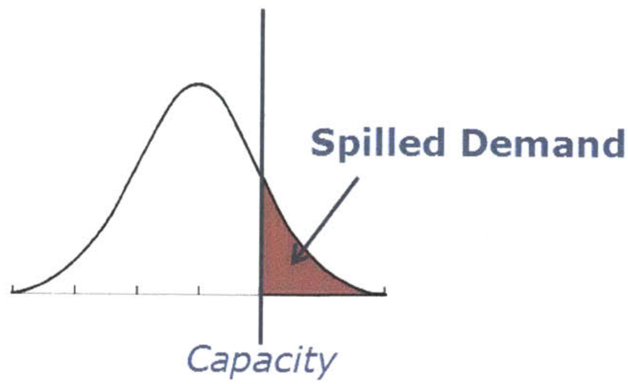


Figure 2.4: Graphical interpretation of spilled demand

However, an airline does not typically know the unconstrained demand distribution for any of its flights. In describing the Boeing Spill Model, Boeing presents a method to estimate the unconstrained demand for a flight based on observations of historical load factors, given that an airline is easily able to record historic loads on its aircraft. Boeing defines an unconstrained demand factor $D = \frac{\mu}{C}$ which is to be estimated from an airline's average load factor L for the flight. With a calculated L and an estimated D, the number of spilled passengers SP and the associated spill factor $S = \mathrm{SP}/C$ may be estimated.

Averaging the load factors of previous flight departures results in a sample mean demand $\bar{x} = L \times C$. The sample mean, based on observations of load factors, underestimates the true mean μ because the right tail of the normal distribution is truncated by the capacity of the aircraft, as seen in Figure 2.5. Boeing assumes a K factor in order to unconstrain the truncated distribution, where $K = \frac{\sigma}{\mu}$. The K factor is a measure of the variability of the mean demand and directly influences the spill, since a larger K factor indicates that variability is large and observations of load factors therefore vary wildly. In practice, an airline will typically assume a K factor based on its knowledge of the market in which the flight operates. For example, the K factor for a largely business market, such as JFK – ORD, will be smaller than the K factor for a leisure destination in which departures around holidays and in the summer will have much more demand than departures on other dates.

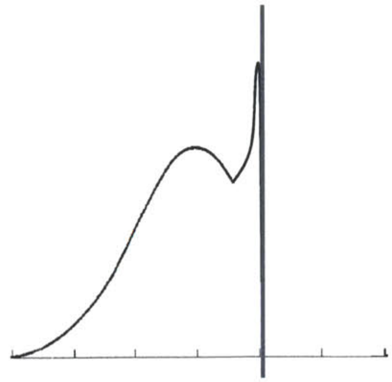


Figure 2.5: Observed distribution of historical bookings

The use of K to obtain D is an attempt to detruncate the observed demand distribution. Mathematically, Boeing presents a formula which relates L, K, and D, given as

$$L = (D-1)F_0\left(\tfrac{1}{KD}-\tfrac{1}{K}\right) - KDf_0\left(\tfrac{1}{KD}-\tfrac{1}{K}\right)+1, \tag{2.2}$$

where

$$K = \tfrac{\sigma}{\mu}$$
$$f_0(x) = \tfrac{1}{\sqrt{2\pi}}\exp\left(\tfrac{-x^2}{2}\right), \text{ the standard normal}$$
$$F_0(x) = \int_{-\infty}^{x} f_0(t)dt, \text{ the standard normal cdf}$$
$$L = \tfrac{\text{average load}}{\text{capacity}}, \text{ the load factor}$$
$$D = \tfrac{\text{average demand}}{\text{capacity}}, \text{ the demand factor.}$$

Therefore, with an assumed K factor, the formula allows for the estimation of any unconstrained demand distribution from a constrained one. The spill factor S and the expected spill SP are then given by the difference between the demand factor and the load factor as

$$\begin{aligned} S &= D-L \\ \text{SP} &= C(D-L) = C\times S, \end{aligned} \tag{2.3}$$

that is, spill is given by how much the average unconstrained demand exceeds the average load factor. In [6], Boeing tabulates expected spill for various values of K, D, and L. In order to use the BSM when the demand distribution for a flight is unknown, an airline observes L and estimates K. It may then consult the tables in order to immediately obtain D and S, multiplying by the capacity C to obtain SP.

Another approach to obtaining the K factor, instead of simply assuming one, is to use the load distribution for a set of departures to estimate the mean demand and standard deviation. As shown in Figure 2.5, the recorded load distribution is a truncated normal distribution. Boeing suggests using the median of a set of departure data to approximate the sample mean. It also suggests using the difference in sampled loads between the 16th and 50th percentile as the sample standard deviation, since one standard deviation covers 34% of the demand distribution. This is shown graphically in Figure 2.6. With the sample mean $\bar{x}$ and sample deviation s, $K = \frac{s}{\bar{x}}$ can be computed.

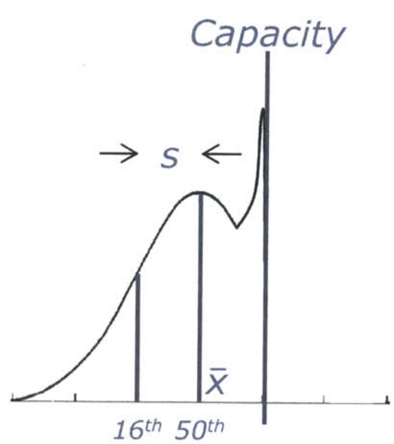


Figure 2.6: Estimating sample mean and sample standard deviation from observed load distribution

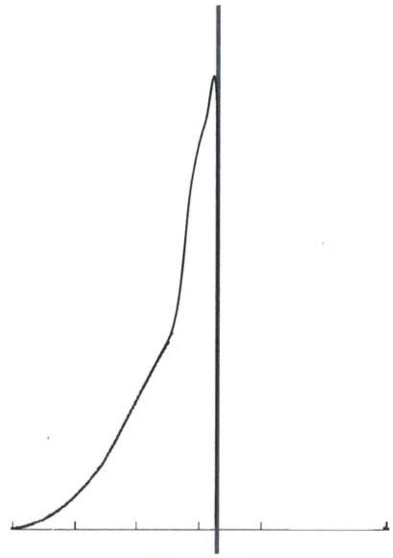


Figure 2.7: Observed distribution of historical loads with mean demand approaching capacity of aircraft

The use of the median to approximate the mean in order to obtain the K and demand factors for a flight is effective for moderate load factors. However, as average load factors continue to increase, the mean demand approaches the capacity of the aircraft. Using the median demand to approximate the mean demand will then result in a poor estimate as many demand observations will be simply the capacity of the aircraft, as shown in Figure 2.7. The method gives more accurate estimates of passenger spill at lower average load factors when the left half of the demand distribution is shaped as a normal one.

To compute the value of spilled passengers, the BSM defines a "spill fare" as the fare that each spilled passenger would have paid had he been able to purchase a seat. This is the revenue loss to the airline for each passenger who was spilled. The spill fare will vary for each passenger as passengers book into different booking classes and desire different fares. Therefore, Boeing suggests using the average fare on the flight to approximate the true spill fare. The average fare is defined to be a weighted average of all fares passengers paid to travel on the flight. For example, if two fares, \$100 and \$200, were offered on the flight, and 50 passengers purchased the lower \$100 fare and \$25 purchased the higher \$200 fare, the average fare is \$133.33. The spill fare multiplied by the expected spill SP gives the spill cost – the lost revenue potential or the opportunity cost of the spilled demand. In practice, using the average fare as a spill fare was not a large limitation in the era of regulation or shortly thereafter, since only a small number of differently priced fares would be made available for sale on any given flight and these perhaps could be simply averaged to obtain an estimate of the value of the spilled demand. However, as modern airlines offer tens of different fares for sale on a market, using the average fare will introduce large errors in the spill cost. Further, since the only input to the model is a single forecast demand factor for the flight, it is clear that different booking classes, and thus the revenue management system, are not considered in estimating either the spilled demand or its value. As mentioned in Section 1.6, the revenue management has a large influence on the demand turned away and neglecting to consider the RM system leads to poor estimates of the value of spilled demand.

To deal with some of these shortcomings, Swan extended the Boeing Spill Model to account for revenue management and the presence of different booking classes. In [9], Swan writes that simulations of leg-based RM methods at Boeing have shown that as long as spill is not very small, the spill fare can be estimated to be a weighted average of 80% of the lowest fare available for sale in a market and 20% of the overall average fare. In other words, Swan estimates that a spilled passenger is worth slightly more than a lowest class fare, since lower fare passengers are more likely to be the spilled given that a good RM system protects seats for higher fare passengers.

Swan's approach for estimating spill with multiple booking classes is described in [29] and was further developed in an unpublished 1992 paper at Boeing referenced by Farkas [10]. In this model, estimated normally distributed demands for each booking class, perhaps specified by the forecaster, are taken as inputs. The demands are aggregated into a joint demand distribution $f'(x)$, giving a joint mean and a joint standard deviation. The expected spill is then given as

$$\mathrm{SP} = \int_C^\infty f'(x)(x-C)\,dx, \tag{2.4}$$

where

SP is the total spill,

C is the capacity of the aircraft, and

$f'(x)$ is the aggregated normal demand distribution.

This extended model thus approximates the effect of the revenue management system by using a weighted average of the lowest fare and the average fare and takes into account the demands for the different booking classes offered for sale on the flight. However, the extended model still does not consider the booking limits computed by the RM system.

The BSM and Swan's extended model are used as baseline spill models in this thesis, given their use in the industry.

2.6.2 Farkas Spill Model

Farkas introduced a new approach for estimating airline spill in his PhD thesis [30] and in a 1999 paper [10]. The method explicitly considers the impact of the revenue management system through the use of booking limits taken as inputs in the calculations of expected spill.

Farkas's spill model does not aggregate demand from all booking classes into a single demand distribution, but instead considers demand distributions and corresponding booking limits for each fare class (in a nested fare structure). The model is leg-based and requires an RM optimizer to generate protection levels and hence booking limits for each fare class. Thus, the revenue management system affects the spill calculation through the input booking limits.

Farkas's spill model makes several assumptions. Demands for each fare class are assumed to be independent, so that each class may be considered separately. It should be noted that some form of fare adjustment and marginal revenue transformation, as discussed in Section 2.5, would allow Farkas's spill formulation to be applied in cases of dependent demands and would require little reformulation. The assumption of independent demand will overestimate spill, given that some lower class passengers will sell-up to higher fares instead of being turned away, especially if the fare difference between classes is not large. Farkas also assumes no overbookings or no-show passengers.

Lower fare passengers are assumed to book in their entirety before higher fare passengers. Such an assumption is required in order to compute spilled passengers class-by-class. For example, on a certain flight with 100 seat capacity, the booking limit for the lowest class may be 50 seats and 60 lowest class passengers may have attempted to book seats. 10 have been spilled. If low class demand books in its entirety before higher class demand, no more lower class booking requests are expected and the remaining spill may now be computed only from examining bookings arriving from higher classes, where 50 seats now remain on the aircraft. This parallels the assumptions in the EMSRb heuristic.

The assumption that low fare demand books before higher fare demand can be relaxed by breaking up the booking procedure into time frames and only assuming that low fare demand books before high frame demand within a time frame. As long as there are a sufficient number of time frames (e.g. weekly frames instead of a single frame for the booking process), this simplification will be acceptable as it captures the typical booking pattern of low fare demand booking before high fare demand, while still allowing some, for example, early booking business passengers. Spill may still be computed class-by-class. Farkas discusses the spill calculations for multiple time frames further in [10] and notes that the computational complexity of computing the spill within each time frame is challenging as many additional integrations must be performed.

Farkas gives the total passenger spill SP as

$$\begin{aligned} \mathrm{SP}_k[S] &= \int_0^{\mathrm{BL}_k - S} f_k(x)\,\mathrm{SP}_{k-1}[x+S]\,dx \\ &\quad + \int_{\mathrm{BL}_k - S}^{\infty} f_k(x)\left\{x - (\mathrm{BL}_k - S) + \mathrm{SP}_{k-1}[\mathrm{BL}_k]\right\}dx \\ \mathrm{SP}_0 &= 0, \end{aligned} \tag{2.5}$$

where

SP_k is the spill from classes 1 through k,
S is the number of allocated seats from previous recursions,
BL_k is the booking limit of class k, and
$f_k(x)$ is the normal demand distribution for class k.

SP_k is computed as the total number of spilled passengers from classes 1 through k, organized in order of decreasing fare. In a fare structure with 5 classes, the total number of spilled passengers is SP_5. The demand for each fare class k is given by the normally distributed demand $f_k(x)$. The formulation in (2.5) is recursive and works through all booking classes in order to compute the total spill, while the number of seats S already accounted for in the recursion is tracked. The recursion begins with $S = 0$. Note that the revenue management system is accounted for in the spill equation through the booking limits for each fare class and the capacity is accounted for in the booking limit of the highest fare class. The booking limits are commonly obtained through EMSRb protections. As the booking limits are changed (either by changing the offered fares or by changing the demands), the number of spilled passengers will also change, thereby illustrating the relationship between the RM system and spill.

Equation (2.5) considers all possible numbers of passenger requests for each class drawn from the demand distribution $f_k(x)$. The integral is broken into two terms. The first term considers the case in which the demand is less than the booking limit of class k, weighted by the probability of this happening for any number of drawn passengers x, given the integration of the pdf. In this case, all of the spill would come from higher classes (i.e. classes $k-1$, $k-2$, and so on), and the recursion continues to SP_{k-1}. However, x seats are then accounted for on the aircraft, and so the higher class spill begins with x seats already accounted for, plus S, the number of seats already accounted for from the previous recursion for still lower classes. $x + S$ is thus passed to the spill function for the next recursion. The second term considers the case in which the demand x exceeds the available seats for that fare class, namely, the booking limit minus the number of seats accounted for (due to bookings in lower classes). Thus, the bounds on the integral are $\mathrm{BL}_k - S$ through to ∞. The number of spilled passengers then becomes the excess drawn demand above the booking limit and allocated seats, $x-(\mathrm{BL}_k - S)$, weighted by the probability of the draw when integrated. As the recursion moves to higher classes, booking limits of the current class are passed as the number of allocated seats S, given spill only occurs if the current class is full to its booking limit. The recursion terminates when all booking classes have been traversed. Finally, the value of the spilled passengers is simply given by the spill from each class multiplied by the fare each of these passengers would have paid had they been able to purchase tickets.

It is important to note that recapture and sell-up are not considered in Farkas's spill model. In practice, passengers turned away from a flight may purchase tickets either in a higher class on the same flight or on another flight with the same airline and Farkas's model will thus tend to overestimate spilled demand. Additionally, Farkas's spill model considers spill only on a single leg and not throughout a network, where passengers may accept rerouting through other hubs. Some of the experimental work in this thesis evaluates the accuracy of Farkas's spill estimates in fare structures with significant sell-up while presenting an extension to Farkas's model to improve the spill estimates.

2.7 Additional Spill Models in the Literature

While the Boeing and Farkas spill models are two major spill models on which much of the literature has centered, other attempts at quantifying spill exist.

Li and Oum question the validity of using a normal demand distribution in spill analysis in a 2000 paper [31], citing empirical evidence of Swan [9] which explains that the normal distribution does not fit well in many cases, especially in premium cabins. They present a spill model which assumes demands are either logistic, log-normal, or gamma distributions. The spill model is similar to the Boeing Spill Model in that Li and Oum generate spill tables relating an observed load factor to a demand factor and spill factor. While the model writes that demand distributions may be prescribed for a fare class with an effective capacity specified for each class, the revenue management system's nested booking limits are not considered in the paper and Li and Oum acknowledge the challenge of integrating their model with RM. The paper finds that as demand becomes more volatile (in the context of the Boeing Spill Model, has a larger K

factor), the use of a normal distribution becomes problematic as a normal demand distribution may now have a negative tail. In these cases, Oum and Li explain that the normal distribution overestimates spill while using other distributions in the modeling does not.

Other research has attempted to extend spill analysis to the network level, again largely in the context of fleet assignment. Barnhart *et al.* [4] describe an itinerary-based fleet assignment model which takes input passenger demands by itinerary in a 2002 paper. However, the model values spilled passengers at an average fare and so does not explicitly consider the revenue management system. Additionally, the model assigns a probability of recapturing a spilled passenger based on a quality of service index of alternative paths, where, for example, a spilled passenger is more likely to be recaptured if a non-stop alternative exists. With revenue management, an alternative non-stop is less likely to have lower class fares available as it is a more in-demand flight – the recapture probability should therefore depend on the fare structure and the booking limits prescribed by the RM system, not addressed in Barnhart's model. Subsequent fleet assignment models are similar in their approaches to spill modeling. For example, a 2012 paper by Wang *et al.* [11] describes a model which defines an "attractiveness" of alternatives to determine spill and recapture rates. Many recent fleet assignment models still consider single demands by path and average fares to value spilled demand. In short, literature on the interaction between revenue management and spill is limited.

2.8 Summary

This chapter presented a review of literature on topics in airline revenue management, focusing on methods that will be used in the experimental section of the thesis. It also outlined some developments in airline spill modeling, serving to illustrate the formulations of these models as well as some of the shortcomings in these current models, thereby building the motivation for the spill studies conducted in the remainder of the thesis.

Chapter 3 – The Passenger Origin Destination Simulator

3.1 Introduction

This chapter serves to give some background on the Passenger Origin Destination Simulator (PODS), the simulation tool used to test spill in this thesis. The overall architecture of the PODS tool and its components is discussed and some of the passenger choice and revenue management models in the simulator are described in more detail. Further information about PODS is available in [14].

3.2 The Passenger Origin Destination Simulator

PODS was first developed by Boeing Commercial Airplanes in the 1990s and has been updated and extended over the years as part of research performed by the MIT PODS Revenue Management Consortium. PODS simulates a fully competitive airline environment in which passengers purchase tickets for flights in a network and allows for the testing of various revenue management systems and strategies. Essentially, PODS attempts to capture the behavior of real-world passengers booking travel on scheduled airlines. In particular, PODS features a full passenger choice model in which passengers with individual characteristics are generated and therefore can have preferences for a given airline, fare product, itinerary, and time of day departure. Within PODS, if a passenger does not get her first choice of itinerary or fare class, she will replan and consider other options, such as flying another airline or in a higher booking class. This passenger choice model is a feature that is not generally present in other revenue management simulators and PODS therefore captures reality better than other revenue management simulators do. For example, as a passenger will travel with another airline if her first choice airline only has high fares available, the PODS simulator better captures the competitive aspects of the airlines: passengers will spill and be recaptured as in reality.

PODS takes as input the description of an airline network. Each airline simulated within PODS specifies the flight schedules for markets in which it operates service. The fare structure for each market is also specified. PODS also takes as input information the number of passengers to generate for every origin-destination market. This number represents the total number of people who desire to travel – this is the unconstrained demand by market. For each of these passengers, passenger characteristics are specified, such as passenger type (business or leisure), willingness to pay, and airline preference.

Output from a PODS simulation run gives revenues and common airline metrics, such as load factors and yields, for each simulated airline. Results are available at the aggregated network level, as well as for any individual flight or market. Therefore, a large amount of information is available to the user to understand the performance and behavior of the particular revenue management system, fare structure, or aircraft capacity being tested. For example, the average fare class mix on every flight, the number of spilled passengers by market, and the marginal revenue change with capacity may all be extracted from PODS.

PODS also gives results on generated passenger choices. PODS makes available a generated passenger's first choice of booking class or path, as well as the actual choice the passenger made (such as selling up to a higher fare or traveling on a different path, depending on what the airline's revenue management system made available). Thus, PODS can provide information on sell-up and recapture rates. The PODS-reported sell-up rate will be used as an input to the extended Farkas spill model developed in the experimental sections of this thesis.

Results are given as averages for a specified number of samples. Each sample represents a single day of departure, simulated repeatedly. For each such day, a new set of passengers is generated, and the airlines within the scenario use a historical database of bookings recorded from previous samples to forecast demands and generate booking limits. Thus, demands for an itinerary and passenger willingness to pay vary by sample. The booking process for a sample consists of 63 simulated pre-departure days. The RM systems of the airlines continually reoptimize booking limits as passengers book within this 63 day period.

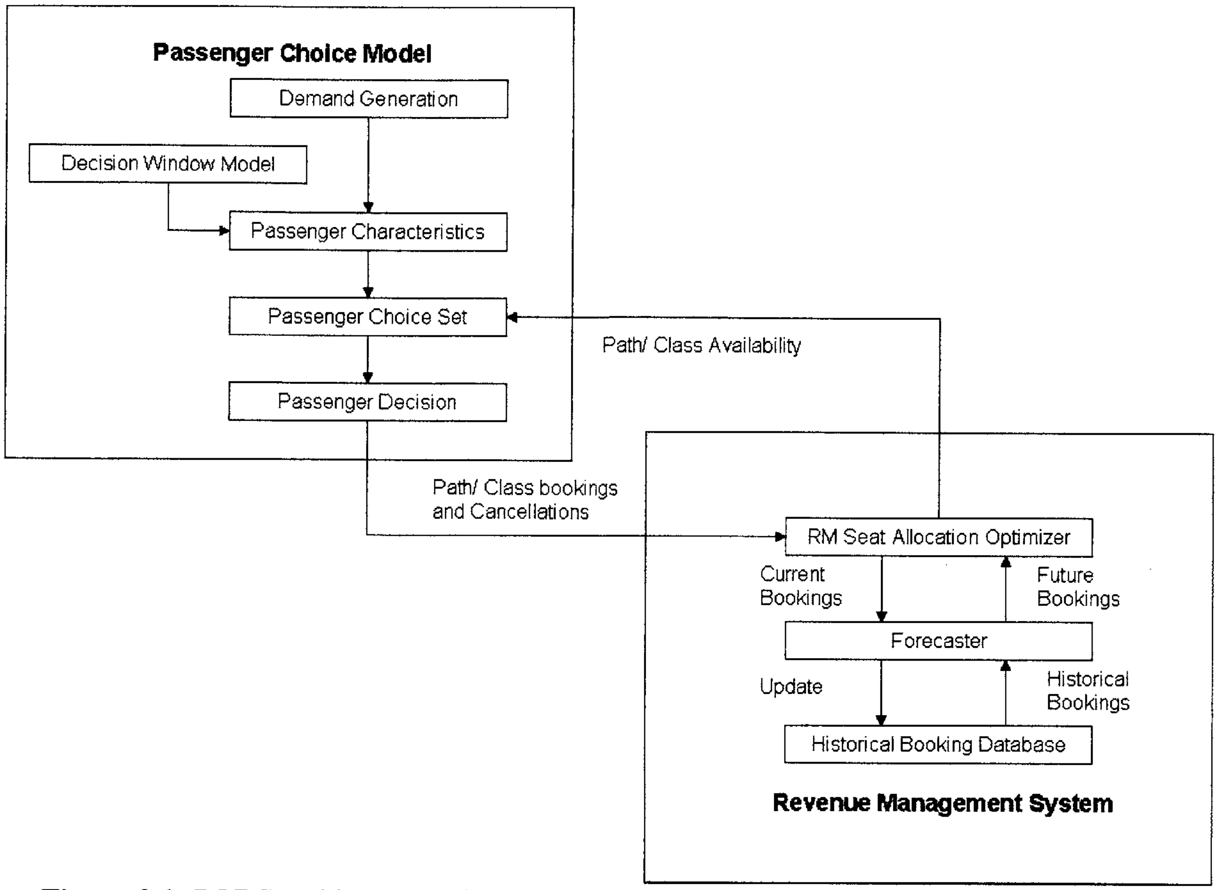


Figure 3.1: PODS architecture schematic, illustrating passenger choice model and revenue management system [14]

3.3 Overview of PODS Structure

The general structure of the PODS tool is shown in Figure 3.1. As mentioned, PODS features a passenger choice model and a selected revenue management system. The RM system options utilized within PODS have been discussed in the Literature Review. Within this thesis, EMSRb is used as an RM optimizer, while pick-up (standard) forecasting and hybrid forecasting are used as forecasters. Marginal revenue optimization, using fare adjustment, is used as an advanced RM optimization technique. Subsequent background on the PODS simulator focuses on the Passenger Choice Model.

3.4 Passenger Choice Model

As shown in Figure 3.1, the Passenger Choice Model consists of several components. First, *Demand Generation* generates some number of passengers who are interested in traveling in any given market. The Boeing *Decision Window Model* is used to generate some *Passenger Characteristics*, which model passenger schedule and fare preferences. In particular, PODS generates two passenger types, business passengers and leisure passengers. These types have different characteristics in terms of price sensitivity and schedule flexibility, as would be expected in the real world. From these generated characteristics, a *Passenger Choice Set* of acceptable travel options is created, based on options made available by an airline's revenue management system. Finally, a *Passenger Decision* is made based on this choice set. The passenger's decision is reported to the airline's RM system (i.e. the passenger purchased a ticket), indicating the interaction between the passenger choice model and the airline's RM system. The mechanics of these processes are discussed in subsequent sections.

It is important to note that the airline's revenue management system is kept separate from the passenger choice model, which is reflected in the graphic. That is, airlines within PODS are not provided passenger demands and instead must forecast passenger behavior in order to maximize revenues. Such a set-up attempts to model real-world behavior, where airlines are only able to estimate passenger characteristics and demands.

3.4.1 Demand Generation

Within PODS, mean demands for business and leisure passenger types are specified as inputs for each origin-destination market. The actual number of passengers wishing to travel on any given sample (simulated departure day) is obtained from a normal distribution around the specified means. In typical PODS experiments, business passengers make up 30-40% of total passengers. This business and leisure mix has been recommended by several airlines participating in the MIT PODS Revenue Management Consortium.

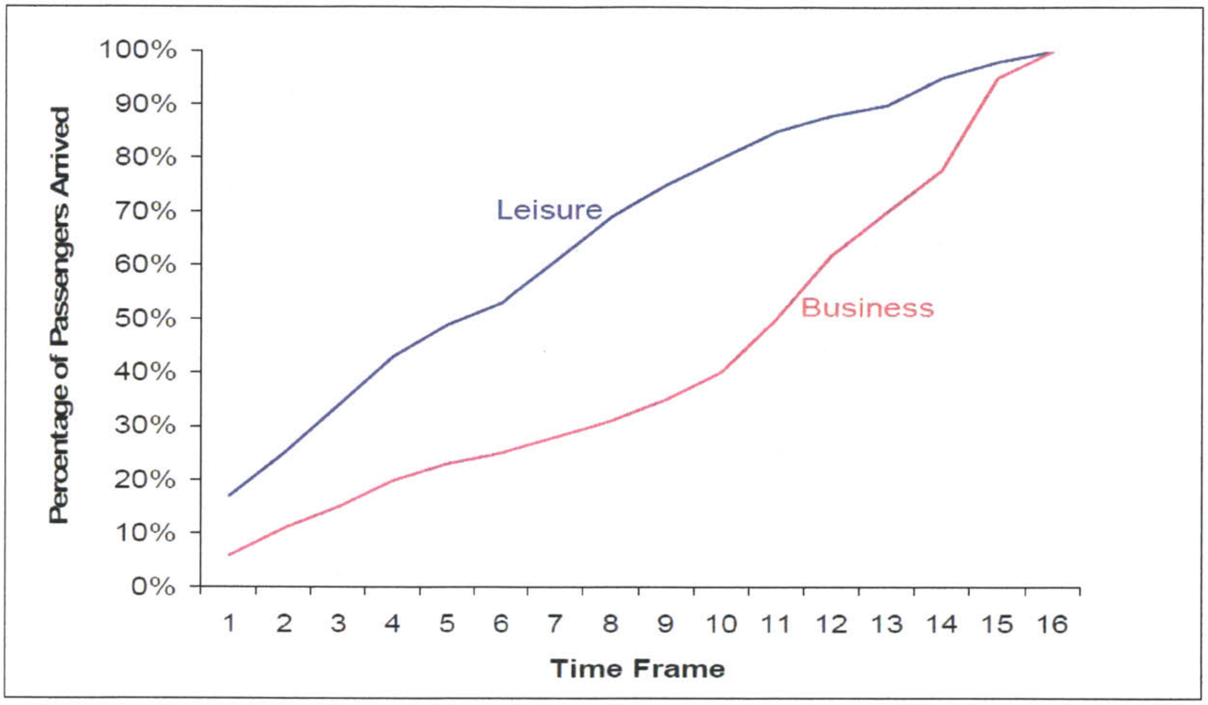


Figure 3.2: Booking arrival curves by time frame for business and leisure passengers [14]

Within the 63 day pre-departure period, passengers book according to the arrival curves shown in Figure 3.2. The 63 day period is divided into 16 time frames, where time frame 1 starts on the 63rd day out from departure and time frame 1 includes the day of departure. On average, passenger arrivals within a time frame follow the curves in the figure. Leisure passengers typically book earlier (further from departure) than business passengers do, as is captured in the booking arrival curves.

3.4.2 Decision Window Model

As in reality, passengers within PODS prefer different departure times throughout the day. These time preferences are determined by the underlying Boeing Decision Window Model (DWM), described in more detail in [32].

DWM defines, for each passenger, a window of the day in which the passenger will prefer to travel. This is defined as the window between the earliest possible departure time and the latest possible arrival time that the passenger will consider. The width of this window for any given generated passenger depends on the sum of two factors, the market ΔT and the schedule tolerance for any given passenger, while the position of the window within a day has been determined by "time of day demand" surveys conducted by Boeing.

The market ΔT is the difference between the departure time and arrival time of the best paths serving the market. For a market in which an airline provides non-stop service, the best path is the non-stop itinerary. If a flight departs its origin city at 9 am local time, and arrives at its destination city at 1 pm local time, the market ΔT is 4 hours, even if the flight time is longer. This 4 hour ΔT defines part of a passenger's decision window. Boeing's surveys have found that a passenger's decision window does not depend as strongly on the flight time as it does on the elapsed time between origin and destination city, adjusted for changing time zones, that is, the market ΔT.

The schedule tolerance is a measure of a passenger's flexibility. This tolerance depends on the stage length of a market (where long haul passengers are usually more flexible in which flight times are acceptable) and on passenger type (where business passengers typically have less flexibility than leisure travelers do). Together, the schedule tolerance and the market ΔT determine the width of the decision window for any given passenger, as shown in Figure 3.3.

The actual time of day demand, that is, the position of passengers' decision windows within the day, has been determined by surveys carried out by Boeing. Boeing surveys provide departure time preferences for passengers which are aggregated into "time of day demand" curves. These curves are generated for every origin-destination market in PODS, based on its ΔT. A sample time of day demand curve for a 9 hour ΔT flight is shown in Figure 3.4. A 9 hour ΔT is approximately a 6 hour flight from Los Angeles to New York with a 3 hour time difference. In such a market, many passengers prefer to depart Los Angeles in the morning, arriving in New York towards the evening. Afternoon departures are not desirable, given very early morning arrival times in New York. Some passengers also prefer evening departures from Los Angeles. These are red-eye flights which arrive in New York in the mid-morning. The preferences for either morning or evening departures from Los Angeles generate a bimodal time of day demand curve, as observed in the figure. It is the positions of passengers' decision windows within the day which lead to some flights being more in demand than others. The time of day demands and

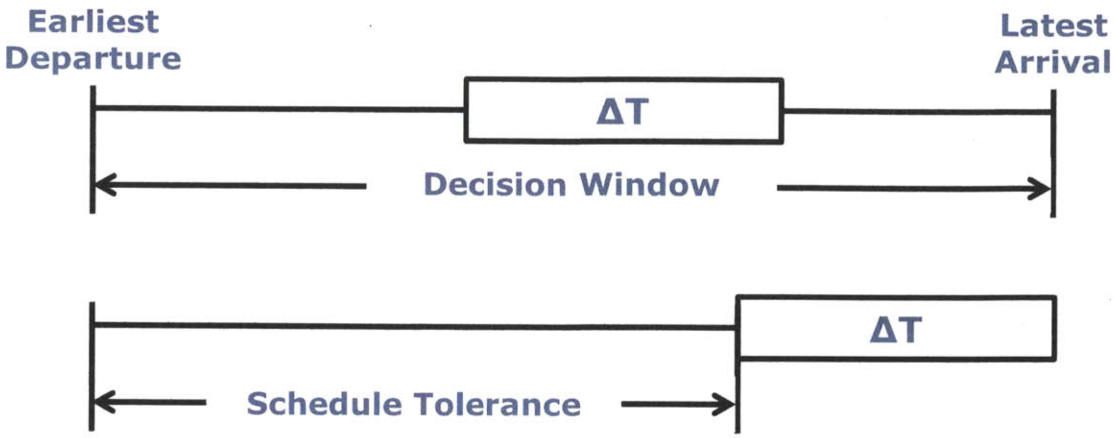


Figure 3.3: Generated decision window for a given passenger, determined as the sum of the market ΔT and the schedule tolerance. Adapted from [32]

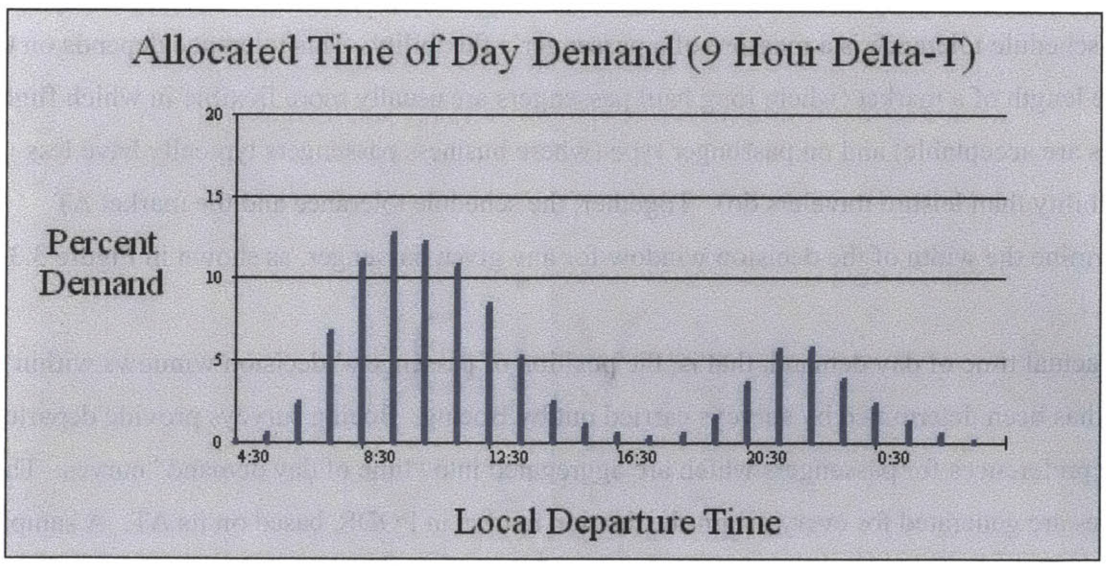


Figure 3.4: Boeing Decision Window Model Time of Day Demand curve for 9 hour ΔT

a passenger's corresponding schedule preferences are important as they will impact recapture and spill-in, as discussed in the experimental sections of this thesis.

3.4.3 Passenger Characteristics

As mentioned, a passenger generated within PODS has several characteristics. One of these characteristics is the width of a passenger's decision window, as discussed above. Other characteristics for a passenger are maximum willingness to pay and a set of disutility costs capturing a passenger's preference for different fare products, such as a preference for refundable fares, for fares without Saturday night stay requirements, for certain airlines, and for non-stop flights.

A passenger's maximum willingness to pay is randomly determined by a negative exponential probability distribution around a *base fare* for a market. On average, all passengers wishing to travel in the market will pay this base fare. The base fare is specified for both business and leisure passengers in a given market, where the business base is fare is typically 2.5 times higher than the leisure base fare, capturing the willingness of business passengers to pay more to travel.

The probability P that a given passenger pays a fare f larger than the base fare f_b is given as

$$P(\text{pay at least } f) = \min\left[1, \exp\left(\frac{\ln\frac{1}{2}(f-f_b)}{(e-1)f_b}\right)\right],$$

where e is the elasticity multiplier, determining how sensitive passengers are to changing prices. Namely, 50% of passengers are willing to pay a fare $f_b e$. Business passengers have a larger elasticity multiplier than leisure passengers do, indicating their reduced sensitivities to increasing prices. With the use of the willingness to pay (WTP) probability distribution, every generated passenger will have a different WTP, again attempting to capture the large variance in WTPs which exists in reality. A passenger will never pay for a ticket which has a price higher than his maximum WTP.

The WTP distributions for business and leisure passengers are shown in Figure 3.5. For business passengers, the probability to pay a fare up to 2.5 times the base fare remains 1. Additionally, the negative slope of leisure passengers is steeper than those of business passengers, again indicating their higher elasticity multipliers.

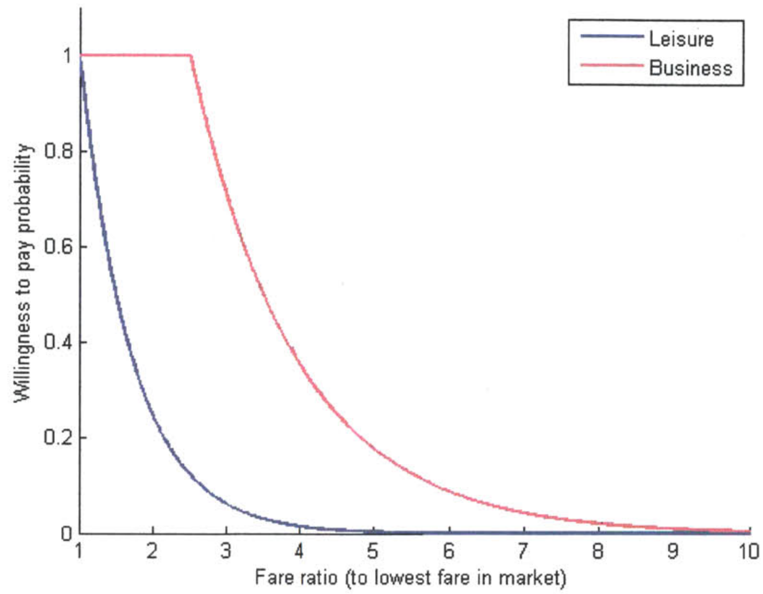


Figure 3.5: Willingness to pay curves for business and leisure passengers

Finally, passengers also have associated with them various disutility costs, modeling their preference for various fare products, flight times, non-stop itineraries, and certain airlines. All such disutilities are randomly drawn for any passenger from a normal distribution of disutility costs, the average values of which have been determined through surveys of airlines. The methodology of disutility costs is further discussed by Lee in [33].

Within PODS, three disutility costs are associated with restrictions on fare products, denoted as R1, R2, and R3. The restrictions are binary: a fare in a certain booking class either has a given restriction or does not. The strongest restriction, R1, has the largest disutility costs, and typically represents a Saturday night stay restriction on lower fares. Any passenger with a large R1 disutility will find fares with an R1 restriction unattractive. Similarly, R2 and R3 typically model lesser restrictions, such as non-upgradeable and non-refundable fares. Business passengers have higher average disutility costs for all restrictions as they value flexibility in their travels, while leisure passengers are generally more interested in traveling at the lowest price. The use of restrictions segments demands into the various booking classes. For example, the use of a disutility cost for a Saturday night stay requirement on lower fares effectively forces most business passengers to only purchase upper class fares.

Passengers also have disutilities representing replanning flexibility, as well as preferences for certain airlines and high path quality. Replanning flexibility refers to how willing a passenger is to accept a flight outside of his generated decision window, while path quality indicates a preference for non-stop flights. As with restrictions on fare products, disutilities for replanning flexibility, airline preference, and path quality are higher on average for business passengers than for leisure passengers, as leisure passengers typically exhibit a preference for lower ticket prices and are more willing to accept less convenient flights times or connections.

The use of decision windows, willingness to pay probabilities, and disutilities allows for the modeling of individual passengers within the PODS tool, thereby creating a simulation environment that closely captures passenger behavior in the real world. Passenger characteristics within the PODS simulator tool are discussed in more detail in [14] and [33]. Carrier also provides a comprehensive description of the passenger choice model within PODS in [34].

3.4.4 Passenger Choice Set

Given a passenger wishes to travel in a certain origin-destination market, she is likely to have many options to make the trip. Travel in an origin-destination market may be offered by several airlines, through various connecting hubs, and in multiple booking classes. All of these itineraries are initially within a passenger's choice set – a set of travel alternatives a passenger will consider.

Some of the itineraries may be removed from the choice set: those for which the passenger does not meet the advance purchase restrictions of a fare, those for which the airline's RM system has closed down the desired booking class, and those which the passenger cannot afford (larger than the passenger's maximum WTP).

Additionally, a passenger's choice set always includes the no-go option, for which a passenger chooses not to travel. This option will be selected if no travel alternatives are found suitable to the passenger. A passenger makes a decision on one of the options within her choice set (including the no-go option), as discussed in Section 3.4.5.

3.4.5 Passenger Decision

Within PODS, a passenger makes a decision for an itinerary within his choice set by considering the generalized cost of the travel options. The generalized cost of a fare for a passenger is the sum of the ticket price and the disutility costs associated with the fare. For example, if a passenger has a disutility cost of $100 for a non-refundable fare and $200 for a connecting itinerary, and all of his other disutility costs are $0, the total generalized cost of a non-refundable, connecting itinerary with a price of $250 is $550.

A passenger makes a decision for an itinerary by ranking all travel options by generalized costs and selecting the option with the lowest generalized cost. A passenger will also only consider a travel option with a ticket price less than his maximum willingness to pay. Again, if no available fares are lower than his maximum WTP, the passenger will elect not to travel (no-go).

Once the passenger selects an available fare, the airline's revenue management system records the booking in its historical database and decreases available inventory in the booking class the passenger booked. Thus, the PODS passenger choice model has created a passenger with some characteristics, allowed the passenger to consider his travel alternatives, and has had the passenger make a selection on a certain fare.

3.5 PODS Revenue Management Systems

As shown in Figure 3.1, PODS has several revenue management systems which may be selected for use in an experiment, where the most common revenue management optimizers and forecasters used in PODS have been discussed in the Literature Review.

This thesis will utilize EMSRb as a baseline leg-based revenue management optimizer. Pick-up forecasting (also referred to as standard forecasting within the PODS literature) is used, while hybrid forecasting is utilized as a more advanced forecaster. A brief note on the implementation of hybrid forecasting within PODS follows.

3.5.1 Hybrid Forecasting within PODS

As discussed in the Literature Review, hybrid forecasting takes into account passenger willingness to pay and uses estimated sell-up rates to forecast demand for different booking classes in less restricted fare structures. Within PODS, sell-up rates are provided as input "FRAT5" curves to the simulator. A FRAT5 value is the fare ratio of a higher fare to the lowest fare for sale in a market at which 50% of passengers would sell up to the higher fare (if the lowest fare was no longer offered). For example, if demand for the lowest fare available for sale in a market is 100, the fare is $250, and the FRAT5 value is 3.2, 50 passengers are willing to pay a fare of $800.

A typical FRAT5 curve is shown in Figure 3.6. The FRAT5 value varies by time frame: it is expected that passengers booking nearer to departure (in the later time frames) have higher maximum WTPs, as compared to those booking in the earlier time frames. In this curve, 50% of passengers booking within time frame 16 will sell-up to a fare 4 times higher than the lowest fare available for sale. The *s*-shape of the FRAT5 curve has been developed based on discussions with member airlines of the MIT PODS Revenue Management Consortium. Further, the input FRAT5 curve is itself an estimate of passenger willingness to pay. A PODS user may specify different FRAT5 curves to examine revenue changes based on changing estimates of passengers' willingness to pay (and therefore to sell up).

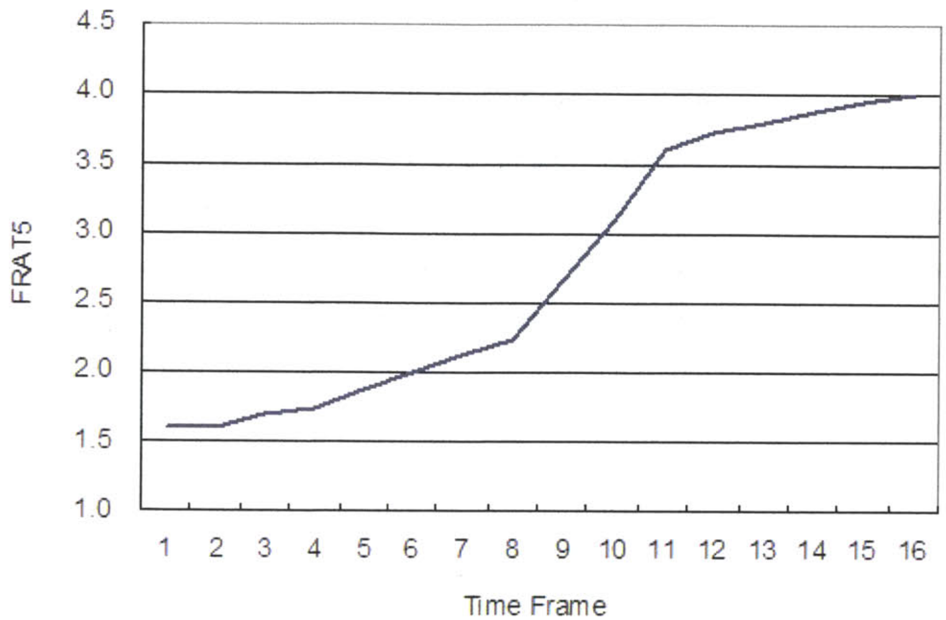


Figure 3.6: Typical FRAT5 curve used to determine sell-up rates [14]

3.6 Summary

This chapter provided a basic overview of the architecture of the Passenger Origin Destination Simulator, with a focus on the passenger choice model. The components of the choice model were explained and an explanation of how a generated passenger within PODS selects a travel alternative was provided. Further explanation of the PODS simulator is available in [14].

The use of a passenger choice model allows PODS to reasonably capture realistic passenger behavior and using PODS to validate and test spill models will allow for the capturing and understanding of real-world spill behavior. The remainder of the thesis uses PODS as an experimental tool to understand spill and to test an extended spill model which better takes into account passenger sell-up.

Chapter 4 – Spill in a Single Market

4.1 Introduction

This chapter explores and details the interactions between revenue management and spill in an isolated market. The discussion begins with the most basic scenario in which an airline operates a single flight leg with no competition and builds through to examine spill in a scenario in which an airline operates several flights per day and competes against another carrier: in the latter scenario, the revenue management system influences sell-up, spill-in, spill-out, and recapture. The analysis highlights the value of additional capacity on a flight and illustrates how the revenue management system and fare structure influence this value.

4.2 Experimental Overview

The Passenger Origin Destination Simulator is used to observe and understand "real-world" spill. The mechanisms of PODS were explained in Chapter 3, where it was discussed that given PODS's sophisticated passenger choice model and the separation between the passenger choice model and an airline's RM system, it is an acceptable simulation tool to use in order to capture real-world passenger behavior.

Simulations are performed in PODS to experimentally test spill behavior. Within PODS, results are averaged over 10 000 samples – thus, the booking process for every flight is repeated 10 000 times. One may think of these samples as simulating the same Friday 5 pm flight over 10 000 departures, with the previous (historic) departures being used to forecast the upcoming ones.

The single market tested features two cities, AAA and BBB, and will use two revenue management systems, a standard system and a more advanced system. EMSRb optimization with pick-up forecasting is used as a standard leg-based RM system which represents a common revenue management system found in the industry, while the more advanced revenue management system uses an EMSRb optimizer with hybrid forecasting and fare adjustment, perhaps representing an airline that has recently invested strongly in RM.

Throughout the study, two fare structures are used. The first is a more restricted fare structure which attempts to emulate traditional legacy airline fare structures in which passenger demand is segmented across the booking classes. The restricted fare structure is shown in Figure 4.1. In line with general industry practices, fare ratios widen towards the top of the structure given business passengers targeted for the higher classes have a higher willingness to pay. Advance purchase requirements are used in order to further segment demands across classes. The restrictions R1, R2, and R3 may represent any restrictions of the fares, such as fares requiring a Saturday night stay, requiring change fees, and being non-refundable. R1 is the strongest restriction and typically represents a Saturday night stay: again, the Saturday night stay requirement is an effective restriction to force business travelers to pay for fares in the higher classes.

In addition to the restricted fare structure, a less restricted structure is also tested, as shown in Figure 4.2. This structure attempts to emulate more recent fare structures in which an airline must compete against the less restricted fare structure of an LCC. In this structure, the R1 restriction is lifted and the bottom four classes offer the same fare product which differs only in price and advance purchase requirements. Passenger sell-up becomes much more important in such a structure.

Class	One-Way Fare	Advance Purchase	R1	R2	R3
1	$500	None	0	0	0
2	$400	3 days	0	1	0
3	$315	7 days	0	1	1
4	$175	10 days	1	1	1
5	$145	14 days	1	1	1
6	$125	21 days	1	1	1

Figure 4.1: Restricted fare structure

Class	One-Way Fare	Advance Purchase	R1	R2	R3
1	$500	None	0	0	0
2	$400	3 days	0	1	0
3	$315	7 days	0	1	1
4	$175	10 days	0	1	1
5	$145	14 days	0	1	1
6	$125	21 days	0	1	1

Figure 4.2: Less restricted fare structure – lack of R1 restrictions highlighted

4.3 Spill on a Single Flight

As mentioned, the most basic environment in which to study the interaction between revenue management and spill is on a single flight leg in the absence of competition. In such a scenario, an airline does not spill out any traffic to its competitor: spilled passengers simply do not travel (no-go).

The baseline experiment in the single flight network thus has an airline, Airline 1, operate one flight daily between the two cities AAA and BBB. The flight schedule is shown in Figure 4.3. The distance between the cities is 1400 miles. The baseline capacity of the aircraft is taken to be 100 seats, and this capacity is varied in order to study spill and the marginal revenue of additional seats. There is no recapture given the airline operates only one flight per day.

Spill on a single flight leg may be further simplified by limiting passenger choice in the simulation – this is the study described in Section 4.3.1. Limiting passenger choice removes sell-up so that any passenger who does not receive his first choice of fare does not travel (is spilled). The idea is to start from the simplest spill environment and then add in sell-up, recapture, and competition to build an understanding of the interactions between spill and passenger choice.

Airline	Flight	Depart AAA	Arrive BBB	Flight Time
1	1	12:37	16:00	3:23

Figure 4.3: Schedule for single flight tests

4.3.1 Baseline First Choice (BFC)

Test Parameters: First choice only, restricted fare structure, standard RM (EMSRb optimizer with pick-up forecasting)

In this test, a passenger does not consider alternatives beyond his first choice – for example, a passenger whose generalized cost is lowest for a class 5 ticket will not consider his second choice of a class 4 ticket in this model. If the class 5 fare is unavailable, the passenger will spill. Hence, this test is referred to as the *Baseline First Choice* (BFC) test.

The BFC test uses the standard RM system and must use the restricted fare structure as the less restricted structure would simply have all bookings in lower classes given passengers do not have the option of selling up to a higher fare. The baseline capacity of the aircraft is 100 seats and varies between 70 and 250 seats.

The BFC test is important as removing a passenger's ability to sell up represents the state of spill modeling in other simulators and in existing spill models: neither the Boeing Spill Model nor the Farkas spill model take into account passengers making a decision to sell up or travel on a different flight (recapture). Therefore, comparisons between the BFC test and later full-choice tests will serve to highlight the importance of the interaction between passenger choice and revenue management in solving the spill problem.

The baseline average revenue for the BFC single flight test is \$19 268 at an average load factor of 85.04%. The average fare class mix for both business and leisure passengers for the 100 seat baseline capacity is shown in Figure 4.4. On average, there are 30 business passenger bookings and 55 leisure passenger bookings, such that business passengers make up 35% of total travelers.

It should be noted that most business passengers book in the top classes while nearly all leisure passengers book in the bottom classes. Given leisure passengers are usually most interested in price, it is to be expected that most are found in class 6, especially as sell-up is not simulated in this first-choice-only model. It is of course the restricted fare structure which forces business class passengers to purchase higher fares through the use of the restrictions.

In order to investigate the relationship between revenue management and spill, the BFC study is performed with capacities ranging from 70 to 250 seats in intervals of 10 seats. Note that throughout this chapter, results are plotted across all capacities to highlight trends, although simulated values exist only at capacity multiples of 10 (70 seats, 80 seats, and so on). Further, any referenced values, such as revenues or load factors, are averages of values obtained in each of the 10 000 simulated samples (days of departure).

Revenues and load factors as a function of capacity are shown in Figure 4.5. As would be expected, revenue rises with increasing capacity as previously spilled passengers are now able to purchase tickets. However, revenues stagnate at the higher capacities, revealing an important observation: increasing capacity does not in itself stimulate additional demand and will not lead to further increasing revenues.

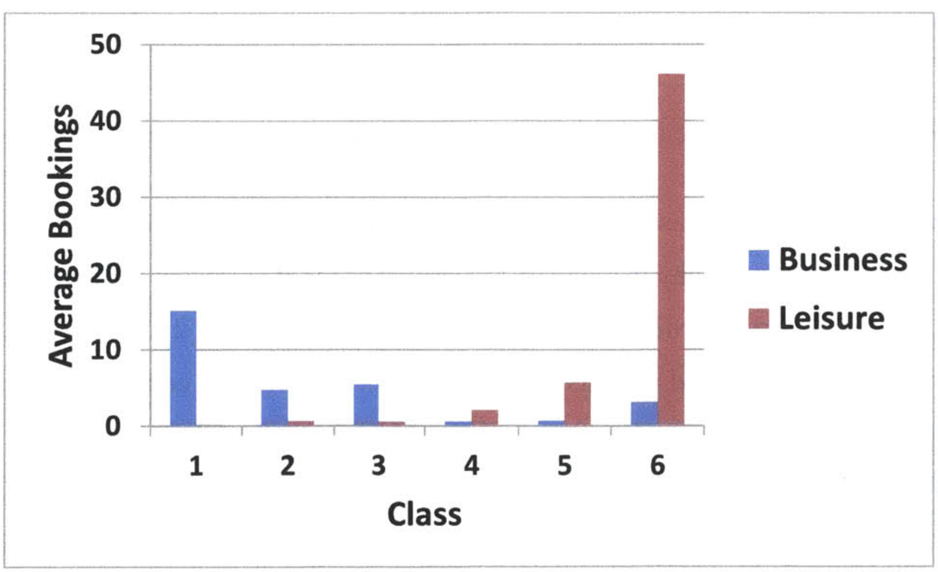


Figure 4.4: Average fare class mix for business and leisure passengers in BFC 100 seat capacity test

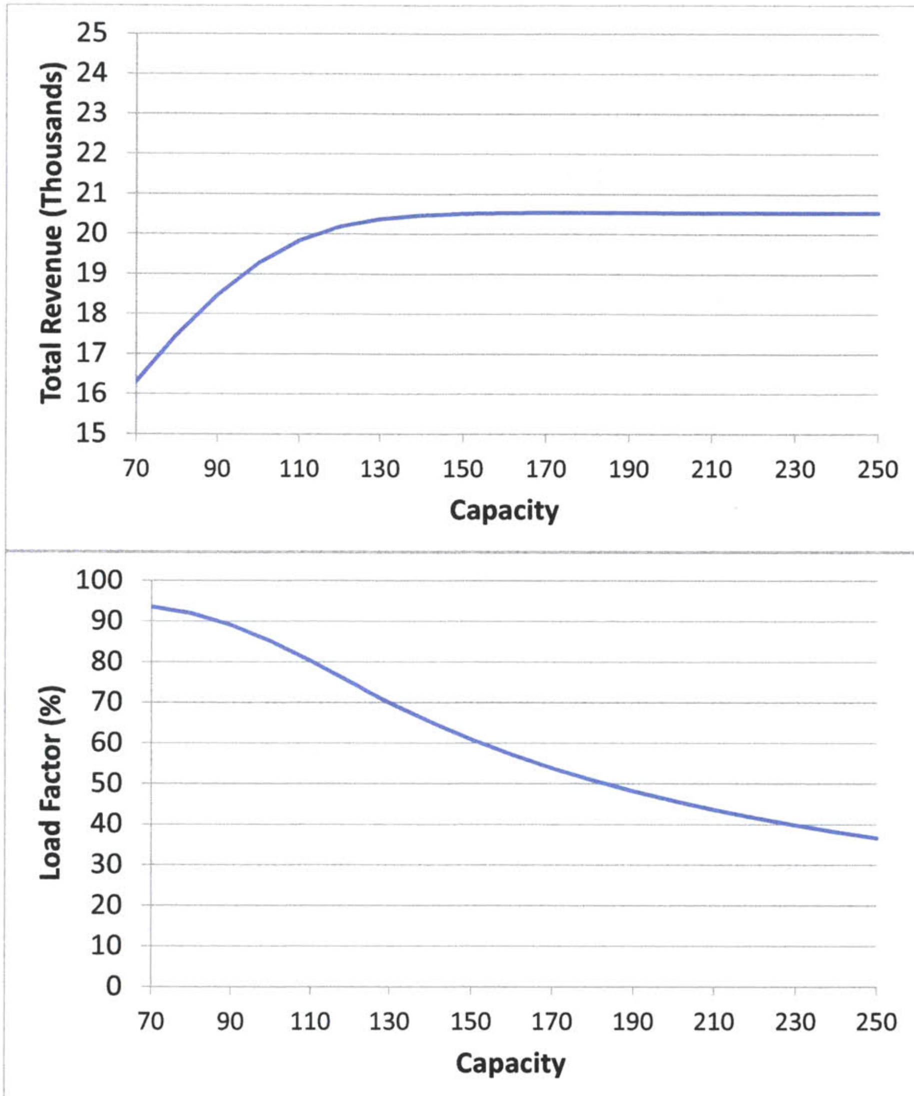


Figure 4.5: Revenue and load factor as a function of capacity for BFC tests

As expected, load factors decrease with increasing capacity, although the relationship is not linear. At lower capacities, many passengers are spilled as the RM system restricts availability to all but the upper classes in order to maximize revenues. As capacity increases, many additional passengers are carried as seats become available for them. Load factors thus do not decrease strongly with the increasing capacity around 70 seats – there is only a 4 point drop in load factor between 70 and 90 seat capacity. As seats continue to be added, fewer and fewer additional passengers book tickets as most of the demand is being carried. Load factors then

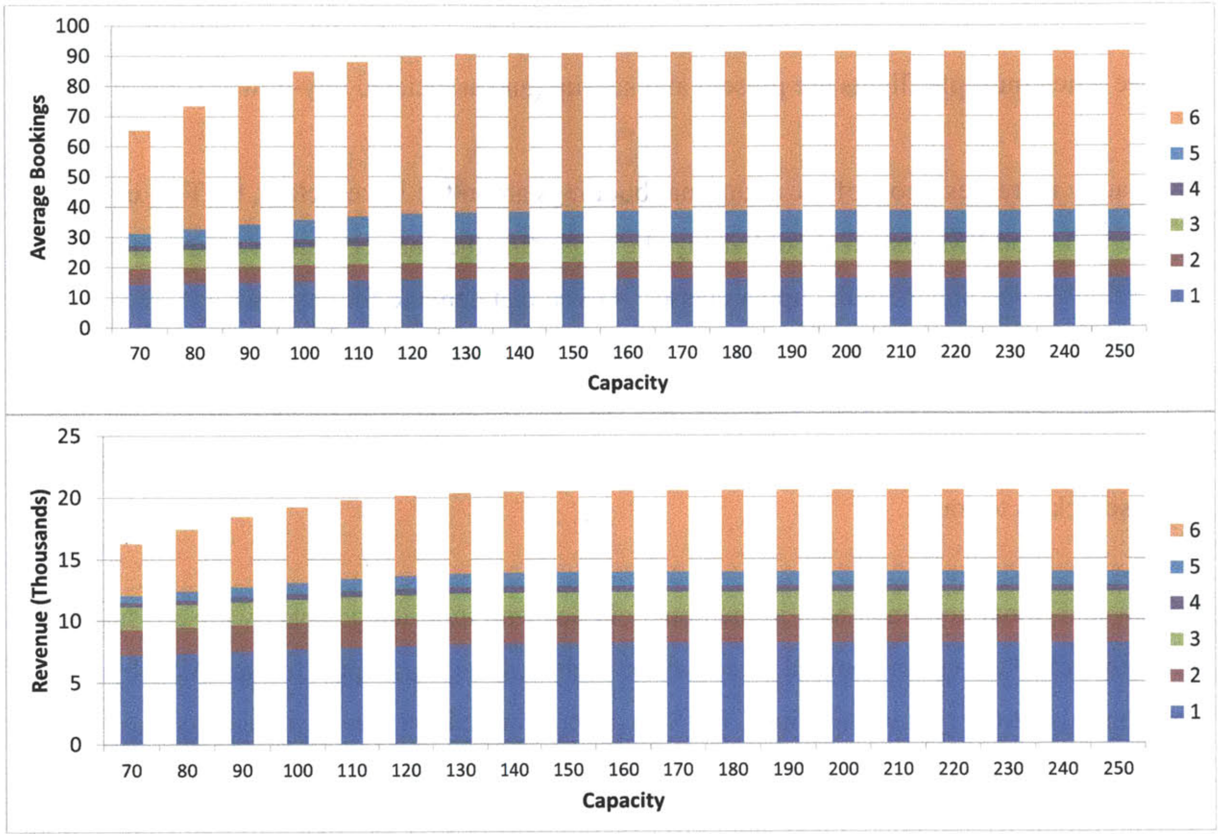


Figure 4.6: (*top*) Average bookings by fare class and (*bottom*) average revenue by fare class as a function of capacity for BFC tests

drop substantially with the still-increasing capacity, leading to load factors of under 40% at 250 seats.

In order to gain further insight into how the revenue management system controls seat availability with changing capacity, the fare class mix for all capacities is examined. The bookings and revenues by fare class as a function of capacity are shown in Figure 4.6. The bars represent the contributions of each fare class, with class 1 (blue) making up the base of the bar and class 6 (orange) making up the top.

Many insights into the interaction between RM and spill can be observed from the revenue fare class mix. Firstly, in this first-choice-only model, no buy-down (or sell-up) is simulated. In this case, adding capacity will never result in decreasing bookings or revenue in any given class, so that adding capacity cannot result in decreasing revenues for the flight (i.e. never resulting in a negative marginal revenue per additional seat capacity), although it will be seen that this is not the case when multiple flights are present and sell-up (and associated buy-down) is permitted.

As capacity increases, most of the additional bookings are in the lowest class, 6. This is to be expected: a good RM system should always be protecting the correct number of seats for the more valuable passengers in the upper classes. Given that demand for any class is fixed, no additional upper class passengers should be arriving with increasing capacity, and the booking limits for the upper classes should not change. Instead, the increasing capacity allows for more bookings in the lowest class and class 6 passengers who had previously spilled are now able to purchase tickets to travel.

The few additional bookings seen in the upper classes with increasing capacity are a result of the probabilistically uncertain demand forecasts for each class used as inputs to the EMSRb booking limit controls. At very low capacities, EMSRb accepts more lower class passengers and spills upper class ones as flights are more likely to depart full. Lower class, earlier arriving passengers can purchase seats, but upper class, later arriving passengers are more likely to find flights more fully booked. These passengers are thus spilled as booking limits for their class have already been reached, especially as passengers may not sell-up to a still-higher class in this first-choice-only model. Some class 1 passengers may be spilled as the flight is more likely to be sold out. For these reasons, some additional upper class bookings are obtained when increasing capacities within the very high load factor regions of 70, 80, and 90 seat capacities. However, most additional bookings with increasing capacity are from class 6 passengers.

Recall that the Boeing Spill Model predicts how many additional passengers will be carried at an increased capacity based on the observed load factor at a given baseline capacity (when the underlying demand distribution is unknown). It also estimates the value of each of these spilled passengers to be equal to the average fare in the market. However, as seen in Figure 4.6, most additional bookings with the increased capacity are class 6 fares. Therefore, the value of additional capacity (or, similarly, the spill cost when decreasing capacity) is much closer to the class 6 fare, given that the revenue management system always ensures (most) upper class passengers are still able to purchase seats. In this fashion, the Boeing Spill Model may

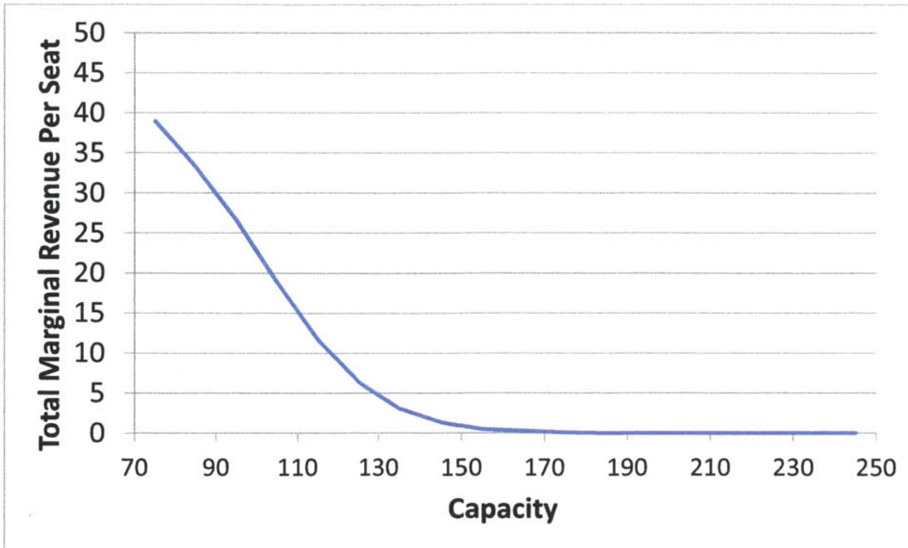


Figure 4.7: Marginal revenue per seat for BFC test

overestimate the value of additional capacity, even without more sophisticated passenger decisions such as choosing to sell-up or to be recaptured. Indeed, Swan proposed an update to the Boeing Spill Model which attempted to rectify this average fare discrepancy by using a weighted average for the spill fare consisting of 80% of the lowest fare and 20% of the average fare [9]. The findings presented here indicate that, in this test, the value of incremental capacity is only slightly higher than the lowest fare for sale on a flight, exactly what Swan's fix addresses.

The value of incremental capacity on the flight can be summarized by computing the marginal revenue (MR) per additional seat added to the flight, as illustrated in Figure 4.7. Given the simulation capacities are spaced 10 seats apart, the MR per seat is approximated by examining the change in revenue between two simulation points divided by the 10 seats added. MR values are plotted at the midpoints between simulation capacities.

As would be expected, the MR per seat decreases with additional capacity, given the fewer additional passengers who book with continually increasing capacity. It should be noted that on this single flight leg and restricted fare structure, the MR is always non-negative. As buy-down is not simulated, there is no mechanism for revenue loss with increasing capacity. Finally, increasing capacity beyond approximately 170 seats results in all demand being carried and marginal revenue going to zero.

The Baseline First Choice test introduced the interaction between the revenue management system and passenger spill and illustrated that the value of incremental capacity was almost entirely equal to the lowest fare offered for sale in a market: increasing capacity does not increase demand and the revenue management system always ensures that the correct number of seats are protected for upper class passengers. Additional bookings with incremental capacity must therefore only be found in lower classes.

The BFC test is intentionally designed as a simplistic case in which to examine revenue management and spill with limited passenger choice. Its importance is characterized by comparisons with the Boeing Spill Model, which does not fully consider revenue management, and the Farkas spill model, which does not consider passenger choice. Subsequent tests which allow sell-up and consider less restrictive fare structures will highlight the differences in the value of incremental capacity relative to this baseline first choice test.

4.3.2 Single Flight Spill with Full Passenger Choice (FPC)

Test Parameters: Full passenger choice, restricted fare structure, standard RM

Extending the BFC simulation allows for *full passenger choice* (FPC), such that passengers may sell-up to higher fares if their desired booking classes are unavailable. The FPC test is of course more reflective of reality – many passengers will pay more money to secure seats on a flight if their first choice lower fare is sold out. Results of the FPC test are compared to the BFC test in order to illustrate how passenger choice influences marginal revenue per incremental seat capacity.

All parameters used in the FPC test are identical to those used in the BFC test – both tests use the restricted fare structure shown in Figure 4.1 and vary capacity from 70 through 250 seats. Demands are not adjusted from the levels used in the BFC test: the same number of business and leisure passengers intends to travel on the flight from AAA – BBB. In other words, the baseline load factor at 100 seat capacity is no longer 85.03%, given that some passengers may now sell

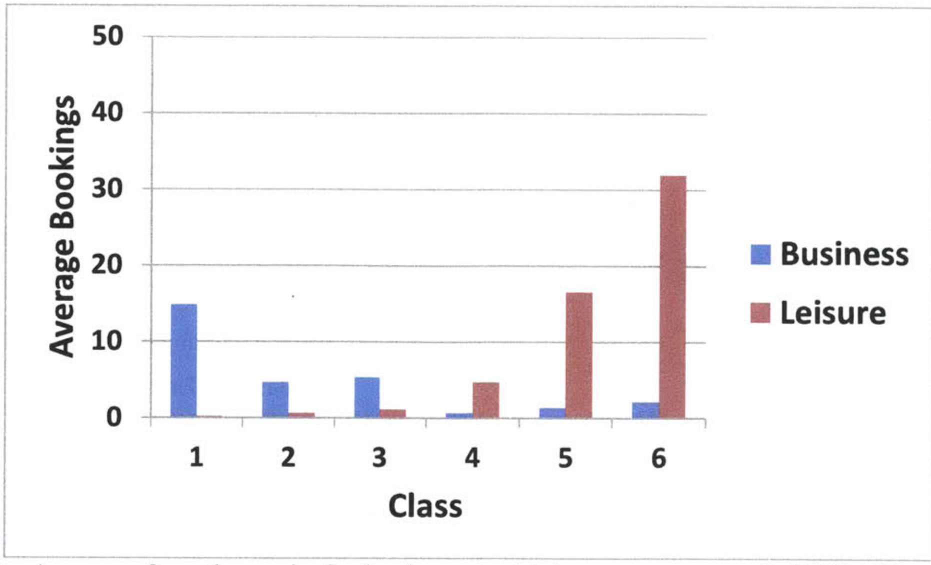


Figure 4.8: Average fare class mix for business and leisure passengers in FPC 100 seat capacity test

up instead of being spilled and the RM system will change seat availabilities based on this behavior.

The revenue at 100 seat capacity for the FPC test is \$19 622 at an average load factor of 84.59%. The average fare class mix for both business and leisure passengers is shown in Figure 4.8.

Note many additional bookings are seen in class 4 and especially in class 5 as compared to the BFC test. With sell-up, passengers will consider purchasing a class 5 fare if class 6 is closed. Note also that with sell-up, the airline is able to slightly increase its revenues (as compared to a \$19 268 revenue for the BFC test), given that some passengers will pay more to avoid being spilled. Sell-up effects become readily apparent when considering the changes in revenues with capacity, as illustrated in Figure 4.9. As capacity continues to decrease, few lower class fares are available and most class 6 passengers will spill unless they sell up to higher fares. Given that the fare difference between class 6 and class 5 is only \$20, the class 5 fare falls within the range of many passengers' maximum willingness to pay and many passengers choose to sell up. This increased sell-up in the FPC test results in an additional \$1 504 of revenue over the BFC case for the 70 seat capacity.

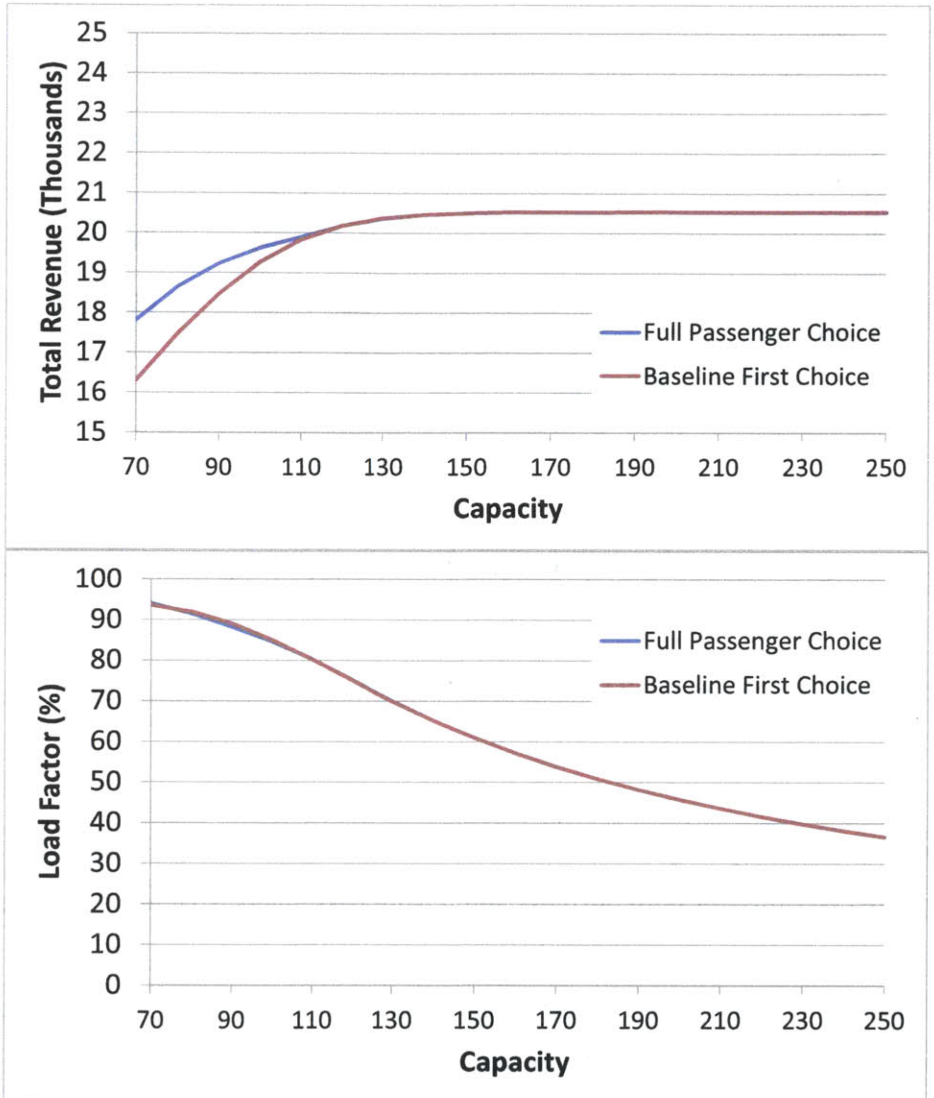


Figure 4.9: Revenue and load factor as a function of capacity for FPC and BFC tests

Load factors remain very similar in the BFC and FPC tests due to the behavior of the revenue management system, as seen in Figure 4.9. In the case without sell-up (the BFC test), at low capacities, the revenue management system will record virtually no demand for class 5, since bookings into this class normally result only from passengers selling up, and will adjust the booking limits to increase availability for the lowest class 6 accordingly. Passengers then book into class 6 until it is full. In the sell-up case, passengers sell up into class 5 to avoid being spilled and the RM system records demand for this booking class, thereby reducing the booking limit for class 6 in order to protect more seats for the passengers selling up. Given demands are high relative to the capacities, there is enough demand to fill seats to the booking limit in either case – the average load factors therefore remain very similar. At the higher capacities, passengers are able to obtain their first choices in both the BFC and FPC scenarios and the load factors are therefore identical – note that the RM system's standard (pick-up) forecaster does not estimate passenger willingness to pay and therefore does not reduce class 6 booking limits in order to elicit sell-up at the higher capacities.

Further insights into the differences between the BFC and FPC tests and the effect of sell-up are evident in considering the FPC fare class mix shown in Figure 4.10 as compared to the BFC fare class mix (seen in Figure 4.6). At lower capacities, many more upper class bookings are seen in the FPC test: passengers are selling up to higher fares to avoid spilling out given the limited capacity. This sell-up leads to the much higher revenues observed with the FPC test over the BFC test at lower capacities (Figure 4.9). However, as capacity increases, more lower class seats are made available by the RM system and passengers are no longer forced to sell up to higher classes, instead purchasing the lower class seats. At higher capacities, nearly all passengers receive their first choices, no sell-up is observed, and the fare class mix and revenues for both the BFC and FPC tests are nearly identical. Again, note that the RM system is not capturing passengers' maximum willingness to pay in these tests and releases more lower class seats as capacity increases, thereby leading to reduced sell-up – moving to a more advanced RM system with hybrid forecasting, discussed in subsequent sections, preserves some of the upper class bookings at higher capacities.

Again, the value of incremental capacity in the BFC and FPC test can be summarized by examining the marginal revenue per seat, as shown in Figure 4.11. Clearly, marginal revenues at lower capacities for the FPC test are higher than those for the BFC tests given the sell-up in the FPC test. However, as FPC sell-up decreases with increasing capacity, the marginal revenue per seat curve has a much larger negative slope than for the BFC test as passenger bookings move from higher classes to lower ones. As capacity continues to increase, all demand is carried and both marginal revenue curves approach zero. Note that the flat portion of the FPC marginal revenue curve observed around 110 seats represents the transition region from the lower capacities, where revenues are sell-up dominated, to the higher capacities, where revenues are made up largely of passengers receiving their first choices. This is further seen by examining the

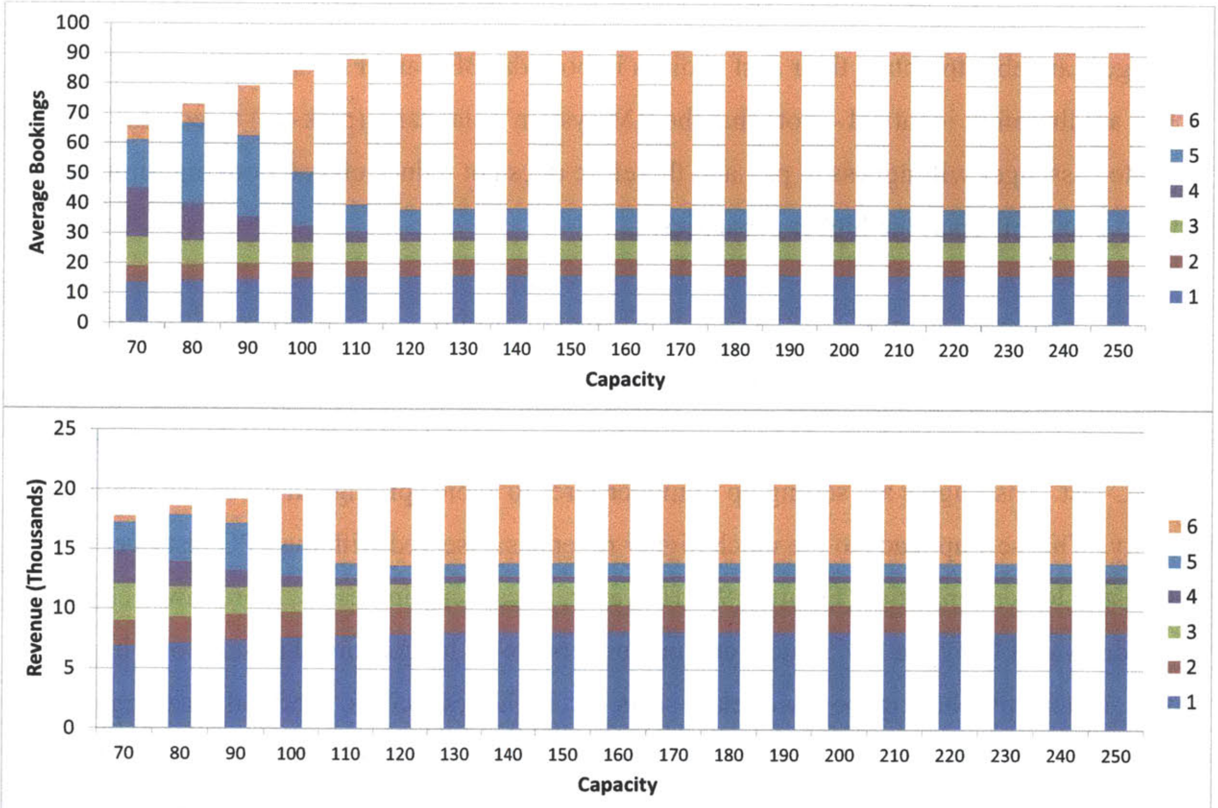


Figure 4.10: (*top*) Average bookings by fare class and (*bottom*) average revenue by fare class as a function of capacity for FPC tests

fare class mix for the FPC test in Figure 4.10, where beyond 110 seats, little buy-down is seen. Thus, the flat portion of the MR curve essentially joins two MR curves with distinct slopes: the sell-up dominated curve at lower capacities, where marginal revenues are very high due to sell-up, and the mostly first choice curve at higher capacities with a smaller negative slope.

While approximately the same number of passengers are spilled (do not travel) in both the FPC and BFC tests (as illustrated by the load factor profiles in Figure 4.9), reducing capacity in the presence of sell-up leads to lesser revenue losses. This is not captured by the standard Boeing Spill Model or the Farkas spill model. The Boeing Spill Model assumes the value of a spilled passenger is equal to the average fare in the market, while the Farkas spill model takes into account revenue management in computing spill, and therefore will estimate that most of the spilled demand is from class 6 passengers and will estimate that the value of a spilled passenger is much closer to the class 6 fare offered for sale in the market, as it was in the BFC test.

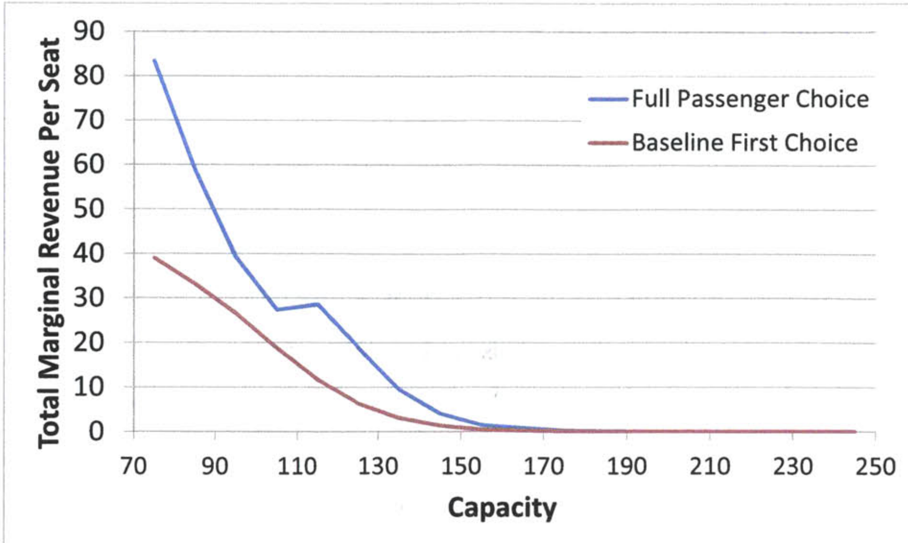


Figure 4.11: Marginal revenue per seat for FPC and BFC tests

In reality, the "true" value of a spilled passenger differs from the values predicted by the models: the Boeing Spill Model overestimates the value of spilled demand as the RM system is much more likely to spill lower class passengers whose value is less than the average fare, while the Farkas spill model underestimates the value of spilled demand as it assumes the spill cost is simply equal to the fare of the spilled passenger, which neglects the possibility of passengers selling up to avoid spilling out, increasing demand in other classes. The Farkas spill cost underestimation is described further in Chapter 5. To an airline, these marginal revenue results are important, as swapping in a smaller plane on a route results in different revenue losses due to spill than as predicted by both the Boeing and Farkas spill models.

Finally, it should be noted that in this restricted fare structure, increasing capacity always leads to increasing revenue for both the FPC and BFC tests. While passengers no longer sell up with increasing capacity (i.e. yields decrease), enough additional bookings in the lower classes are obtained such that revenues do not decrease – it will be seen that as the complexity of the single market network grows with the introduction of more flights and competitive effects, incremental capacity can lead to negative marginal revenues and revenue losses.

4.3.3 Full Passenger Choice in a Less Restricted Fare Structure (FPC-LR)

Test Parameters: Full passenger choice, less restricted fare structure, standard RM

The restricted (differentiated) fare structure utilized in the BFC and FPC tests in Sections 4.3.1 and 4.3.2 limits passenger buy-down as the restrictions across most of the fare products serve to segment demands into particular booking classes. In a less restricted fare structure, like the one described in Figure 4.2, many of the fare products differ only in price and advance purchase restrictions. Sell-up and buy-down become much more important as business travelers are much more likely to purchase lower class fares given the strong Saturday night stay restriction is lifted on all fares.

As less restricted (and less differentiated) fare structures have become more common with the growth of LCCs, it is important to examine how revenue management systems behave and influence spill in such structures. The full passenger choice with less restricted fare structure (FPC-LR) test examines spill in the fare structure shown in Figure 4.2.

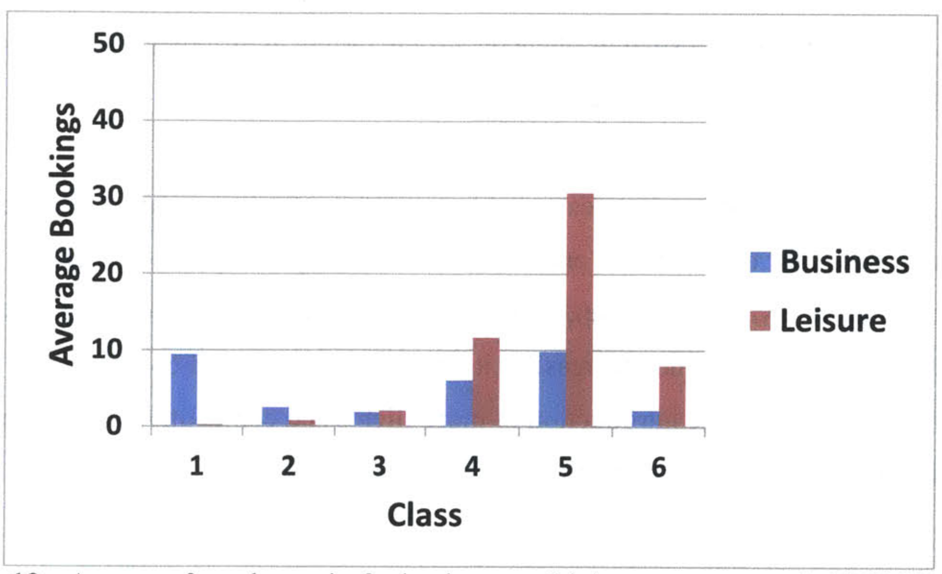


Figure 4.12: Average fare class mix for business and leisure passengers in FPC-LR 100 seat capacity test

Baseline demands in the FPC-LR test were increased from the BFC and FPC tests such that the load factor at 100 seat capacity remains approximately 85%. As such, revenues may not be directly compared between FPC and FPC-LR tests – clearly, revenues in a less restricted fare structure will be less than those in a restricted fare structure at equivalent levels of demand. Instead, the 85% baseline load factor should lead to comparable amounts of spill. Attention should be paid to the differences in the fare class mixes and the shape of the marginal revenue curves between the FPC and FPC-LR tests.

The revenue at 100 seat capacity for the passenger full choice test is $17 555 at an average load factor of 85.01%. The average fare class mix for both business and leisure passengers is shown in Figure 4.12.

Differences in the fare class mix are immediately observed in the FPC-LR test as compared to the FPC test (although the input demands are higher in the FPC-LR test). Fewer business travelers book into the top classes, having instead bought down to lower ones: as the Saturday night stay requirement is lifted, a business traveler will purchase low class fares as long as he can meet the AP requirements. Therefore, the number of passengers buying down below their maximum willingness to pay is much increased in the less restricted fare structure. This increased buy-down decreases the value of incremental capacity, as many more passengers will purchase lower fares when the airline increases booking limits for the lower classes with increasing capacity.

Note that the increase in leisure bookings in class 5 over the FPC test is reflective of the increased demand levels and associated sell-up to class 5 from class 6. The only difference between the class 5 and 6 fares in both the restricted and less restricted fare structures is price: for the average leisure traveler, class 6 will have the lowest generalized cost, followed by class 5 (where many leisure travelers arrive early enough not to be affected by the AP restrictions). Thus, without the difference in the underlying demand levels, similar fare class mixes in classes 5 and 6 for leisure travelers would be observed in both fare structures.

Revenues and load factors as a function of capacity for the FPC-LR test are shown in Figure 4.13. Clearly, the revenue as a function of capacity for the FPC-LR test differs significantly from that of the previous FPC test (Figure 4.9). In particular, while revenue increases with capacity between 70 and 100 seats, there is a drop in revenue between 100 and 120 seats, before revenues increase again and stabilize at higher capacities. Insight into the behavior of the revenue management system and the value of the incremental capacity for the FPC-LR test can be gained by considering the fare class mix shown in Figure 4.14 and in particular by comparing the FPC and FPC-LR fare class mixes at lower capacities, shown in Figure 4.15.

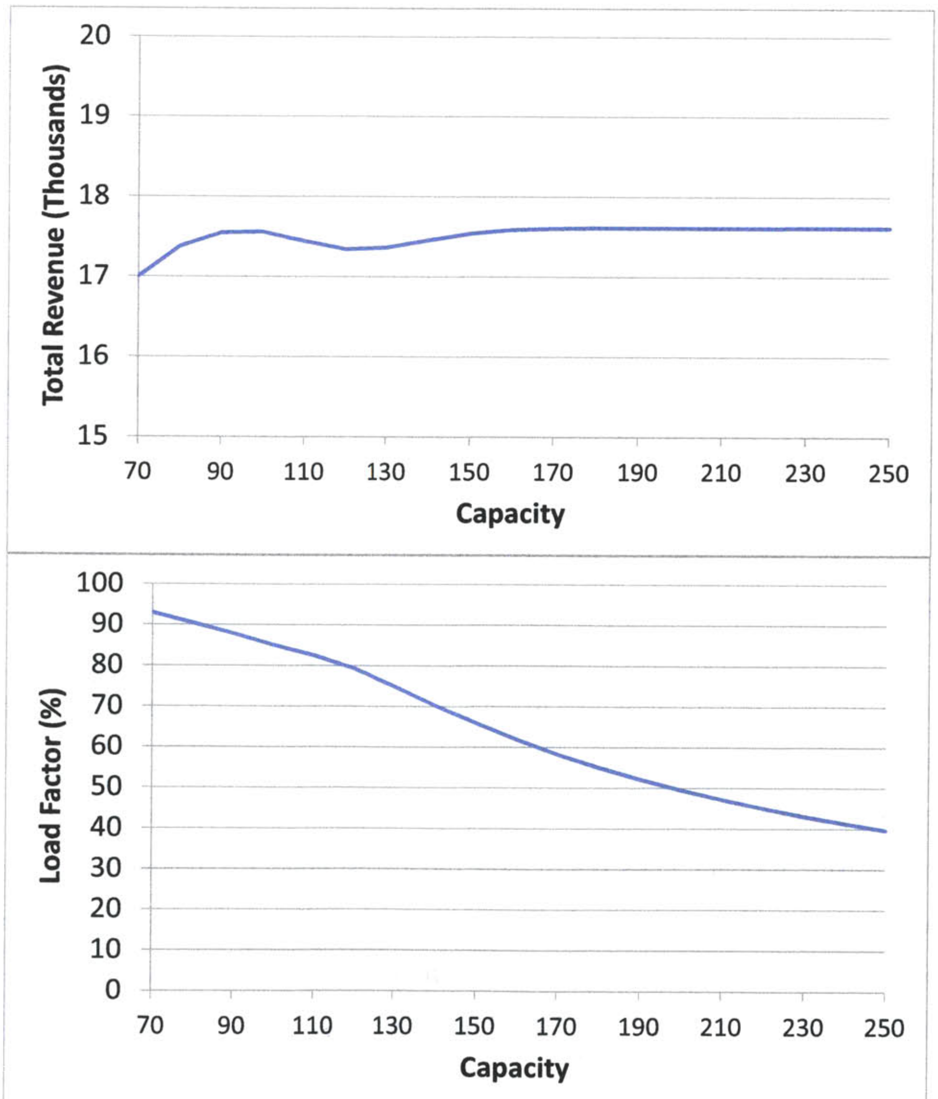
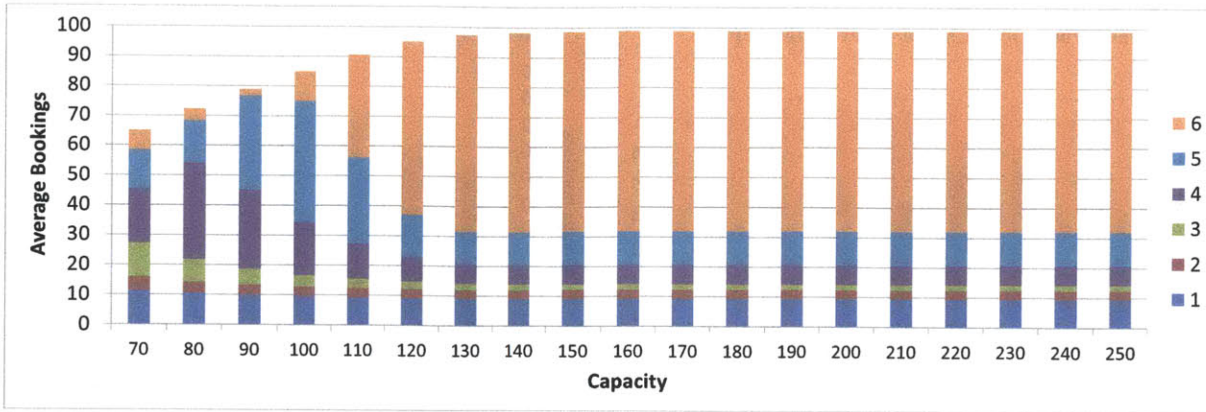


Figure 4.13: Revenue and load factor as a function of capacity for FPC-LR tests



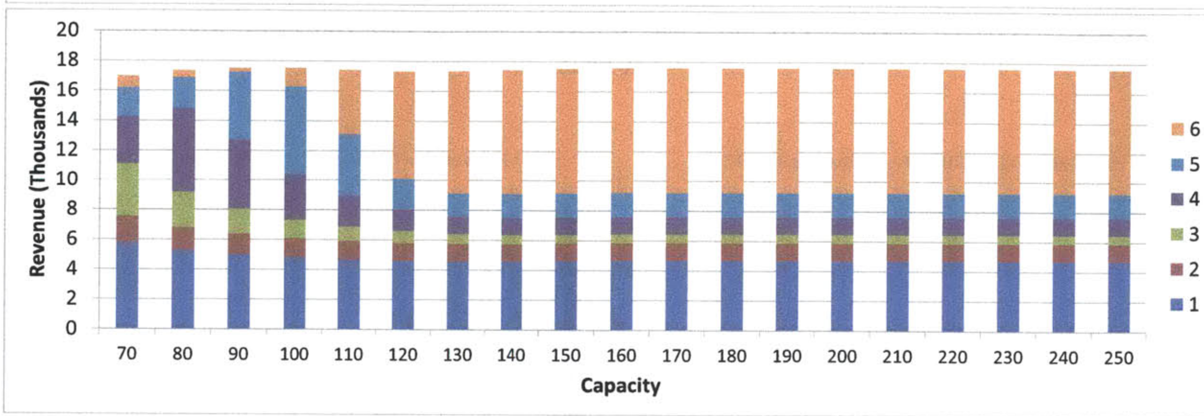


Figure 4.14: (*top*) Average bookings by fare class and (*bottom*) average revenue by fare class as a function of capacity for FPC-LR tests

In the FPC-LR structure, the amount of buy-down with increasing capacity is much greater than in the FPC case, highlighted by the loss of higher class passengers when increasing capacity from 90 to 130 seats, as seen in Figure 4.15. In particular, note that the buy-down from all classes is much larger in the FPC-LR test. In the restricted fare structure, some buy-down was observed from class 5 to class 6, for example, when increasing capacity from 70 to 100 seats. However, the upper class bookings did not vary largely with increasing capacity as the restricted fare structure better segmented demands. In the less restricted fare structure, buy-down occurs in most classes – note the decreases in class 2 and 3 passengers as capacity increases from 70 to 80 seats and the large decline in class 4 passengers between 80 to 90 seats. As capacity continues to increase, most bookings are in class 6 and the fare class mix no longer changes: all passengers are receiving their first choice preferences.

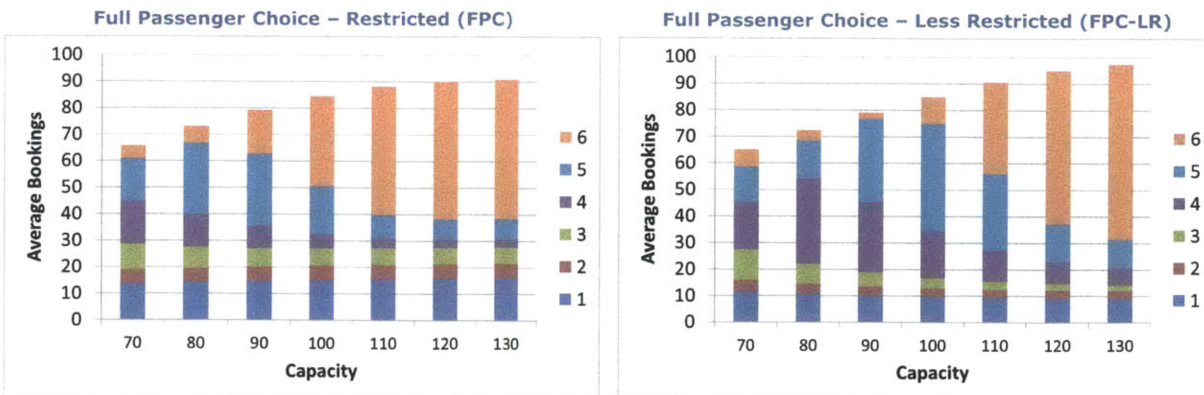


Figure 4.15: Average bookings by fare class at lesser capacities for (*left*) FPC and (*right*) FPC-LR tests

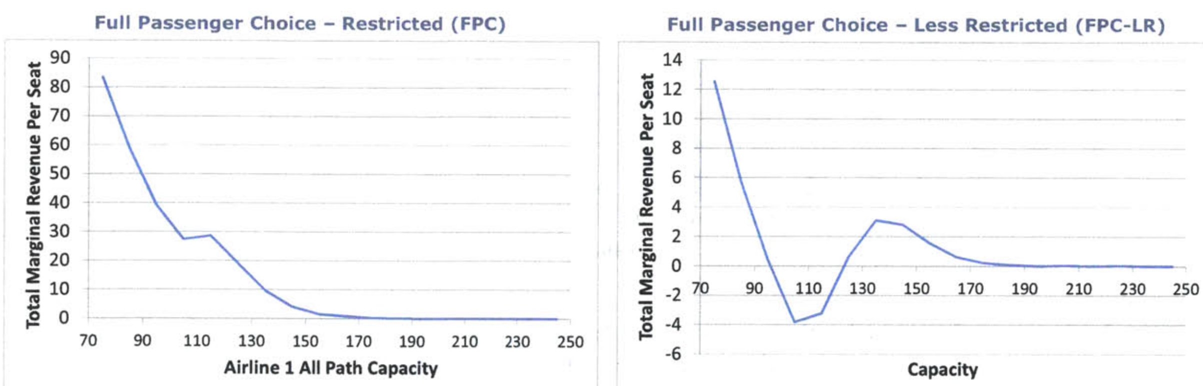


Figure 4.16: Marginal revenue per seat for (*left*) FPC and (*right*) FPC-LR tests. Note different scales

The value of incremental capacity is summarized by considering the marginal revenue per seat curves, shown in Figure 4.16 for both the FPC and FPC-LR tests. Clearly, the marginal value per seat for the FPC-LR test is very different from the MR for the FPC test (note, however, the different scales and the differing baseline demands between the tests). First, the marginal revenue per additional seat is negative between approximately 100 and 120 seat capacity. This is due to the previously discussed buy-down of all classes at these capacities, especially evident in the decrease in class 5 bookings as seen in Figure 4.14. Because bookings have eroded from higher classes to lower ones, marginal revenue becomes negative as the incremental lower class bookings do not overcome the revenue loss of previous higher class passengers purchasing lower class fares. Total revenue thus decreases with increased capacity between 100 and 120 seats, as was seen in the revenue curve in Figure 4.13. Above 120 seats, most passengers are receiving their first choice preferences and additional capacity results in additional class 6 bookings, much like in the FPC and BFC tests, with steadily decreasing positive marginal revenues through to zero at which point all demand is carried.

The FPC-LR test is important as it illustrates the interaction between the fare structure used in a market and the value of incremental capacity. The marginal revenue curve for the FPC-LR test is very different from the FPC test (with the restricted fare structure) – due to increased buy-down in the less restricted fare structure, additional capacity can lead to revenue losses. In particular, the analysis performed here further illustrates the need for spill models which consider sell-up (and therefore buy-down) in order to correctly capture the value of incremental capacity, especially as less restricted fare structures become more common.

4.3.4 Full Passenger Choice with Advanced Revenue Management System (LR-HFFA)

Test Parameters: Full passenger choice, less restricted fare structure, hybrid forecasting and fare adjustment

The less restricted (less differentiated) fare structure utilized in the FPC-LR test in Section 4.3.3 results in increasing passenger buy-down because the revenue management system does not account for passenger willingness to pay. As more seats are made available on the aircraft, the revenue management system will increase lower class availability. Except for demand for class 1, first choice demand will only be for the lowest class as the fare products are the same and differ only in price (as long as passengers arrive while lower classes have not yet been closed due to AP). Therefore, the RM system will continually record increased demand for the lower classes at the expense of higher ones and will make more seats available to these classes – partial spiral down occurs (with complete spiral down avoided due to the AP). To mitigate this spiral down, hybrid forecasting and fare adjustment can be used in more advanced RM systems. Given hybrid forecasting accounts for passenger willingness to pay, the RM system should restrict availability to the lowest classes, recognizing that many passengers who are booking into class 6 (because it is cheapest) would book into higher classes if forced to do so by the RM system's closure of the class. Thus, testing a less restricted fare structure with hybrid forecasting and fare adjustment (LR-HFFA) is a test of a recent fare structure with advanced revenue management – essentially at the forefront of leg-based revenue management methods.

Baseline demands and all other parameters in the LR-HFFA test are set to be equal to those of the FPC-LR test. In particular, revenues between the two structures may be compared and the advantages of using the more advanced revenue management system are made apparent.

The revenue at 100 seat capacity for the LR-HFFA test is \$18 182 at an average load factor of 81.47%, while the revenue for the FPC-LR test was \$17 555 at a load factor of 85.01%. The LR-HFFA test therefore exhibits a revenue gain of 3.57% compared to the FPC-LR test, where the greater revenue is a result of the more advanced revenue management system used in the LR-HFFA test. The average fare class mix for both business and leisure passengers in the LR-HFFA test is shown in Figure 4.17, presented along with the fare class mix of the FPC-LR test for comparison.

First, note the higher number of business passengers in class 1 in the LR-HFFA test, which uses a more advanced RM system. The hybrid forecaster accounts for passenger willingness to pay and reduces the booking limit of some of the lower classes so that more business passengers are forced to book into class 1. Similarly, fewer business passengers are able to book class 6 tickets as the RM system forces them into classes 4 and 5. The advanced RM system reduces passenger buy-down and leads to higher revenues.

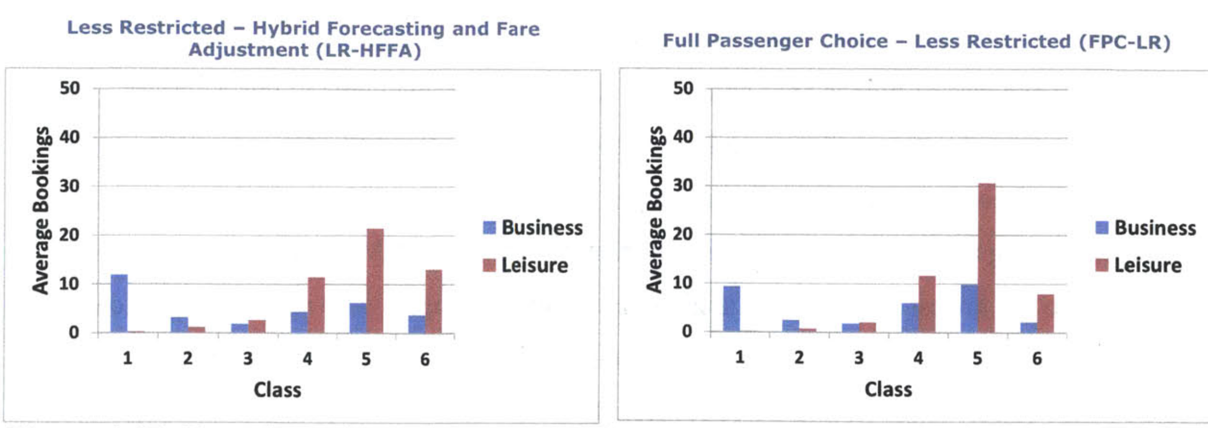


Figure 4.17: Fare class mixes at 100 seat capacity for (*left*) LR-HFFA and (*right*) FPC-LR tests

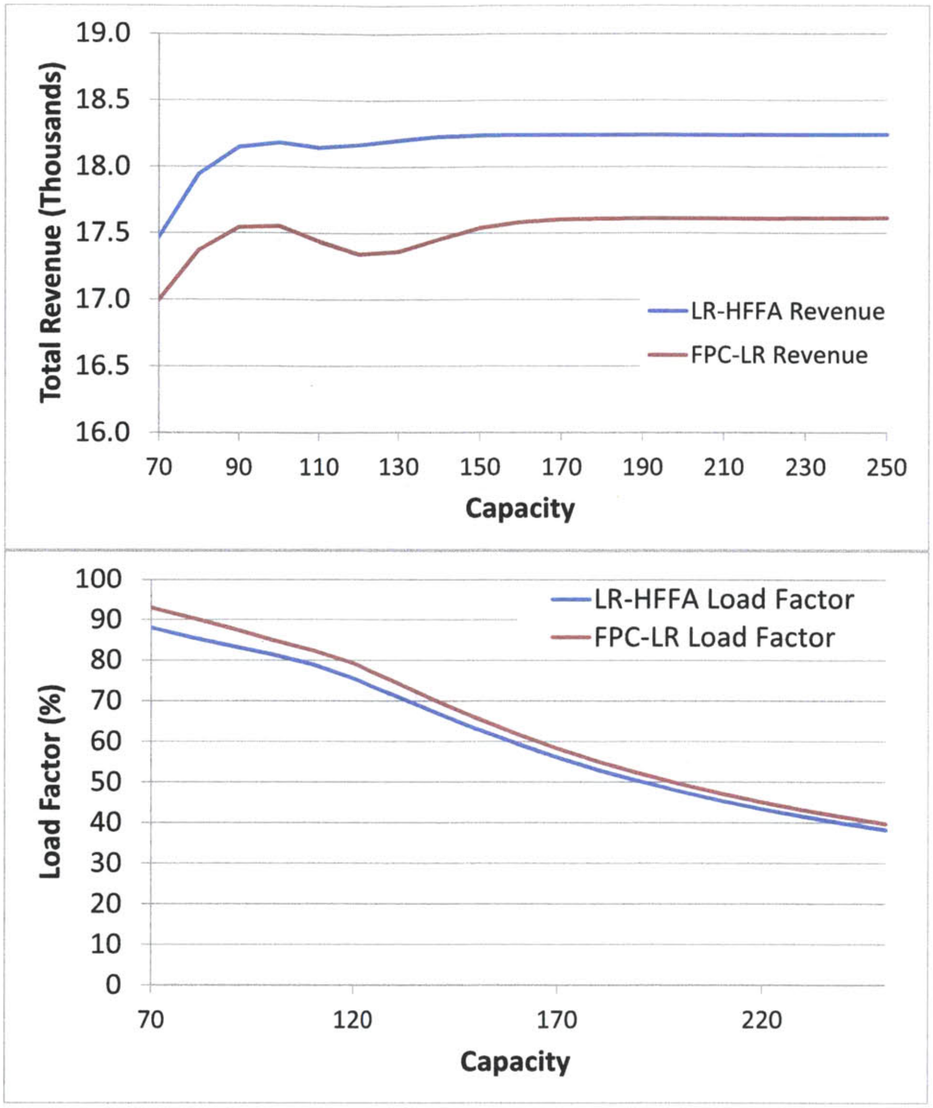


Figure 4.18: Revenue and load factor as a function of capacity for LR-HFFA and FPC-LR tests

Revenues and load factors for both the LR-HFFA and FPC-LR tests are shown in Figure 4.18. While the shapes of the revenue and load factor curves are similar, the LR-HFFA test has higher revenues and lower load factors across all capacities – where the lower load factor is a result of the RM system reducing booking limits on lower class seats such that fewer passengers buy down: more passengers spill out, but those who do not, pay higher fares.

While the FPC-LR revenue shows a decline from approximately 90 to 120 seats (as discussed in Section 4.3.3), the LR-HFFA results do not show as large a decline – the more advanced RM

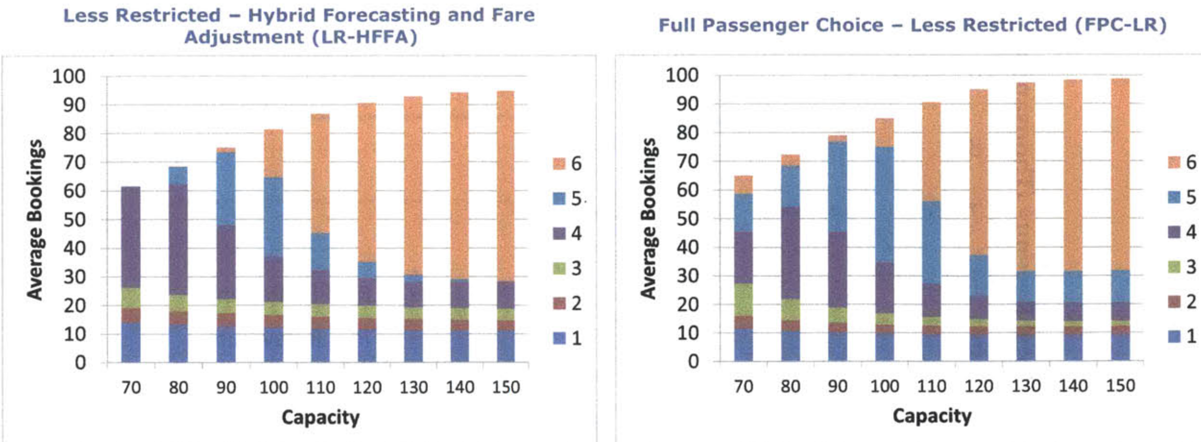


Figure 4.19: Fare class mix for (*left*) LR-HFFA and (*right*) LR-FPC tests

system prevents some of the revenue loss with increasing capacity. The revenue curves can be fully understood by examining the fare class mix for the two tests, as shown in Figure 4.19. The fare class mixes between the two tests are radically different. Again, the more advanced RM system reduces the booking limits on lower classes, forcing passengers to sell up to avoid spilling. The changes to the fare class mix as a result of the HFFA RM system are readily apparent at the lower capacities. At 70 seats, for example, the LR-HFFA test only has passengers book in classes 1 through 4, while the FPC-LR test still allows bookings in classes 5 and 6.

As capacity increases, the HFFA RM system is able to retain bookings in the higher classes. In increasing capacity from 70 to 80 seats, the HFFA system largely retains all upper class bookings and only adds class 5 bookings with the extra capacity, while the standard leg-based RM system used in the FPC-LR test is, for example, not able to retain many of its class 3 passengers, instead trading them largely for class 4s. This trade-off of higher class passengers for lower class ones continues as capacity increases. At 150 seat capacity, for example, the fare class mixes for the two tests are similar, except the HFFA system is able to book more passengers into class 4 and keep class 5 empty, while the standard leg-based system books fewer passengers into 4 and instead accepts more bookings into 5. The HFFA system avoids revenue losses from the lesser revenue contribution of booking additional lower class passengers (as in the FPC-LR test), instead receiving fewer bookings but keeping them in more valuable classes.

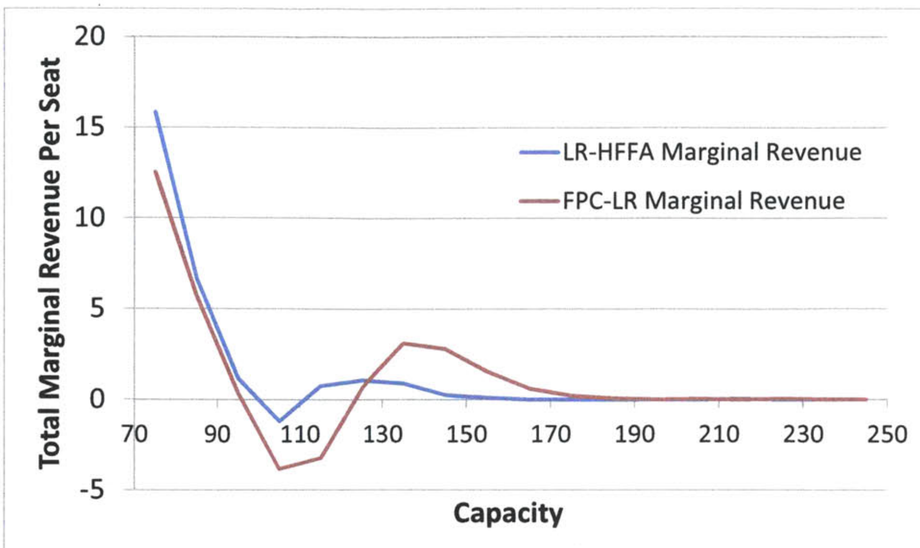


Figure 4.20: Marginal revenue per seat for LR-HFFA and FPC-LR tests

The value of additional capacity for the LR-HFFA case is summarized by considering the marginal revenue per seat curve, shown in Figure 4.20. The use of the more advanced RM system nearly eliminates the negative marginal revenue between 90 and 120 seats as compared to the FPC-LR test. As mentioned, the HFFA system is able to avoid revenue loss by protecting for more higher class seats and preventing passengers from buying down, while the standard leg-based RM system loses revenue with additional capacity until there is enough capacity such that all passengers receive their first choices. With a less restricted fare structure, additional capacity is not nearly as detrimental to revenue when using the more advanced HFFA RM system as compared to the standard leg-based system.

4.3.5 Single Flight Spill Summary

The single flight spill tests have explored spill from the simplest case of limited passenger choice in a traditional restricted airline fare structure through to full passenger choice under a more recent less restricted fare structure with a more advanced RM system. The impact of revenue management on spill and the value of additional capacity was introduced.

In particular, because of passenger buy-down, adding capacity can lead to negative marginal revenues in less-restricted fare structures. This finding is contrary to conventional wisdom and previous studies, which have assumed non-negative marginal revenues with increasing capacity, stemming from an assumption of independent demands between booking classes. The buy-down can be mitigated by using a more advanced revenue management system which takes into account passenger willingness to pay. It is therefore important that spill models take into account passenger choice whenever they are used for fleet assignment, fleet planning, or other purposes.

4.4 Multiple Flight Spill

In the single flight tests, all passengers either purchased tickets on the single flight and traveled or were spilled out and did not travel. If additional flights are added to the schedule, passengers who do not secure seats on one flight may travel on others (they are recaptured) or may be spilled and not travel (no-go).

With additional flights, time of day preferences become important: different flights will have different load factors and revenues as passengers prefer to travel at certain times and may sell up to higher fares to secure tickets on flights departing at preferred times. It is therefore important to study spill in a multiple flight scenario in order to understand how marginal value is affected by time of day preferences and recapture between flights.

In this section, the interaction between revenue management and the value of incremental capacity is tested in a market in which with an airline operates multiple flights throughout the day.

Airline	Flight	Depart AAA	Arrive BBB	Flight Time
1	1	12:37	16:00	3:23
1	2	17:07	20:30	3:23

Figure 4.21: Schedule for multiple flight tests

4.4.1 Multiple Flight Baseline Spill (MFB)

Test Parameters: Full passenger choice, restricted fare structure, standard RM system, varying capacity on flight 1 (noon)

The baseline multiple flight experiment has one airline, Airline 1, operate two flights daily between AAA and BBB. The flight schedule is shown in Figure 4.21. The distance between the cities remains 1400 miles. The baseline capacity of both aircraft is 100 seats. Full passenger choice is permitted and recapture is present given the two flights. The traditional, restricted fare structure is used in this *multiple flight baseline* (MFB) test. Finally, most parameters are unchanged from the single flight BFC test, although demands are increased in order to obtain an average market load factor of approximately 85%.

For a 3 hour and 23 minute flight time, as in the schedule, the Boeing Decision Window model [32] indicates the most preferred departure time is at midday, while flight times early or late in the day are less preferred. Given the schedule of flights, it is expected that incremental capacity on flight 1 will be worth more as demands are higher for this flight. Therefore, the capacity of flight 1 is varied between 70 and 250 seats in order to study spill and the marginal revenue of additional seats, while the capacity of flight 2 is unchanged from the baseline 100 seats.

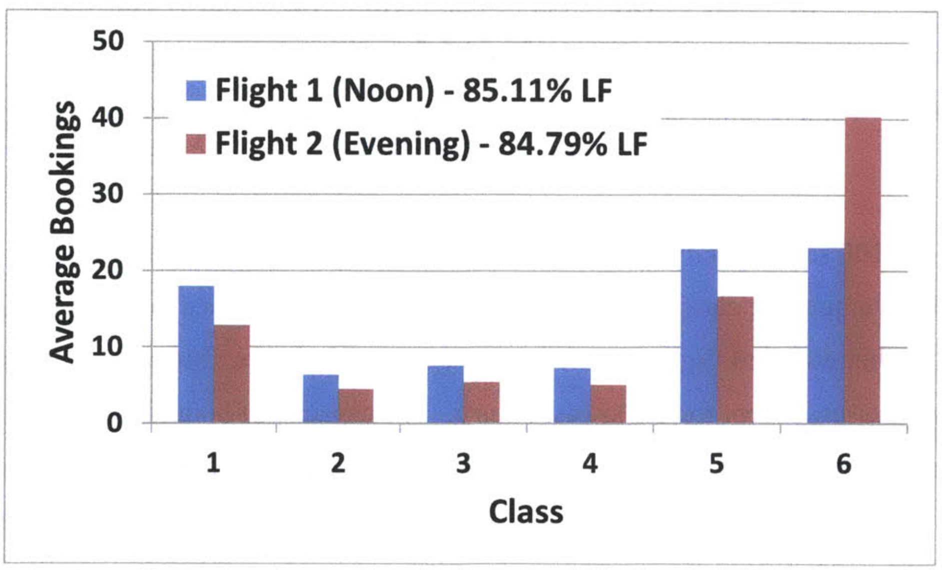


Figure 4.22: Fare class mix for each of the flights in the MFB test

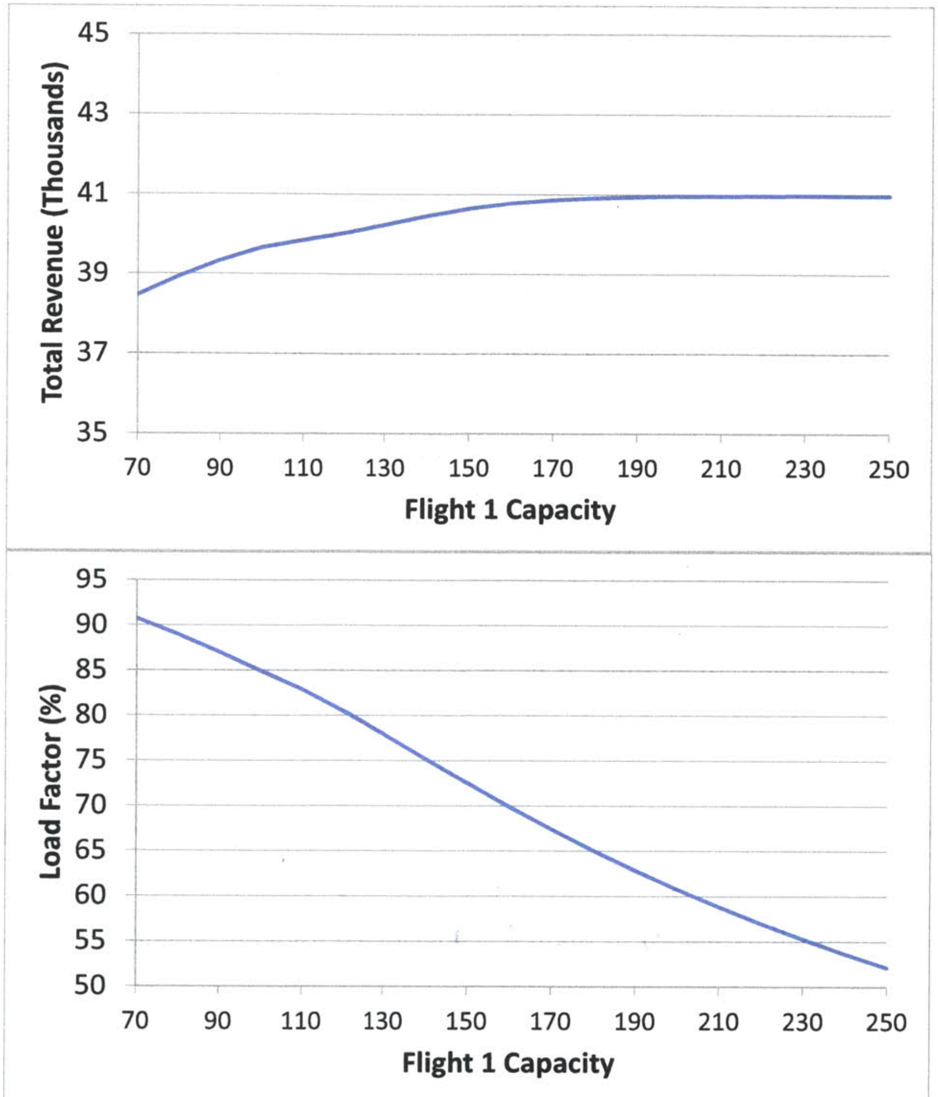


Figure 4.23: Total revenue and average load factor (across the two flights) as a function of capacity for MFB test

The revenue at 100 seat capacity for the MFB test is \$39 659 at an average load factor of 84.94%. The average fare class mix for each of the two flights is shown in Figure 4.22. As mentioned, the better-timed flight 1 carries a stronger fare class mix. More passengers are booked in all classes, except for the lowest class 6. The capacity of both flights is 100 seats and the difference in the fare class mix is only a result of time of day preferences and higher demand for flight 1: with more passengers desiring to travel on flight 1, the RM system protects more upper class seats and obtains the strong fare class mix.

Total revenue and average load factor as a function of capacity are shown in Figure 4.23. Recall only flight 1's capacity is changing, as indicated by the x-axis label. At the market level, the revenue and load factor curves are very similar to the FPC tests: revenues rise with increasing capacity as previously spilled out passengers are now able to purchase tickets, while load factors decrease with increasing capacity as more and more of the total demand is carried.

Adding capacity will impact the revenue and load factors on each of the flights differently due to the time of day preferences of the passengers and the corresponding differing demands for each of the flights. Revenues and load factors for each flight are shown in Figure 4.24. At the 70 seat flight 1 capacity, flight 2's revenues are slightly higher than flight 1's as many passengers are being spilled off flight 1 (due to the low capacity) and are being recaptured onto flight 2. Load factors for both flights are high because of this recapture: flight 1 is very highly loaded due to its limited capacity, while flight 2 attracts many of the passengers recaptured from flight 1 and has a high load factor even though its capacity is 100 seats. As capacity increases above 70 seats, revenues for the more in-demand flight 1 increase sharply. The flight 1 load factor remains approximately constant as passengers fill the additional seats on flight 1, given demands for flight 1 are higher because of the preferred departure time. Flight 2's load factors and revenues thus decrease as passengers book on flight 1 given the newly available capacity. As capacity further increases, load factors on both flights decrease as there is not enough additional demand to fill up the incremental capacity. Above approximately 150 seat capacity, flight 2's load factors no longer decrease as all passengers traveling on the flight receive their first choice and additional capacity on flight 1 will not result in more passenger bookings on flight 2.

Examining the revenue curve immediately shows the revenue coupling, through recapture, between flight 1 and flight 2. Flight 2's revenues decrease even without it changing capacity: at lower flight 1 capacities, flight 2 is full of recaptured passengers. As flight 1 capacity increases, passengers no longer are forced to be recaptured and can purchase tickets on flight 1. Thus, even in a restricted fare structure (with limited buy-down), increasing capacity on one flight can lead to revenue losses on other flights. It is therefore apparent that recapture is an important component of spill and must be considered in spill models.

Insight into the behavior of the revenue management system and the value of the incremental capacity for each flight in the MFB test can be gained by considering the fare class mix shown in Figure 4.25. The fare class mix for flight 1 is similar to the fare class mix for the FPC test (Figure 4.10). Increasing capacity leads to increasing bookings, while some buy-down occurs. However, given the fare structure is restricted, upper class bookings are largely preserved even at the higher capacities.

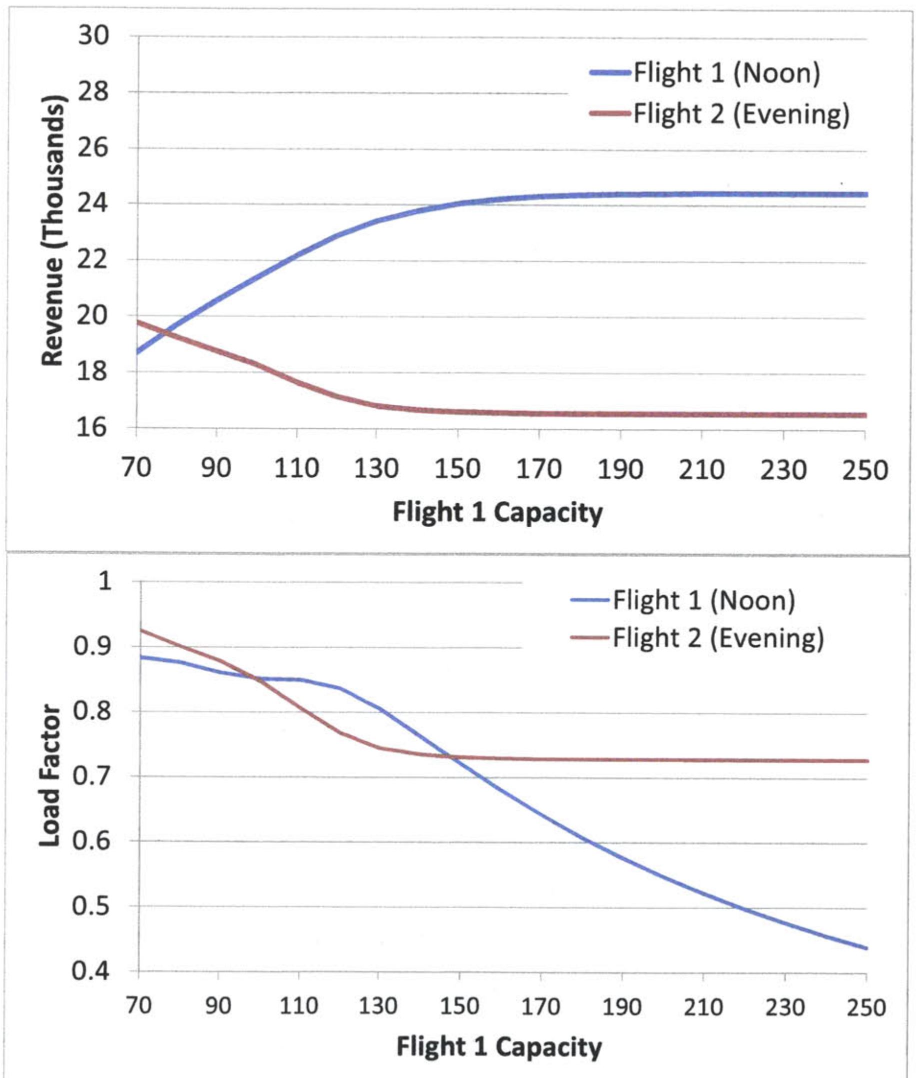


Figure 4.24: Revenue and load factor as a function of flight 1 capacity for MFB test and for each of the two flights on the AAA – BBB market

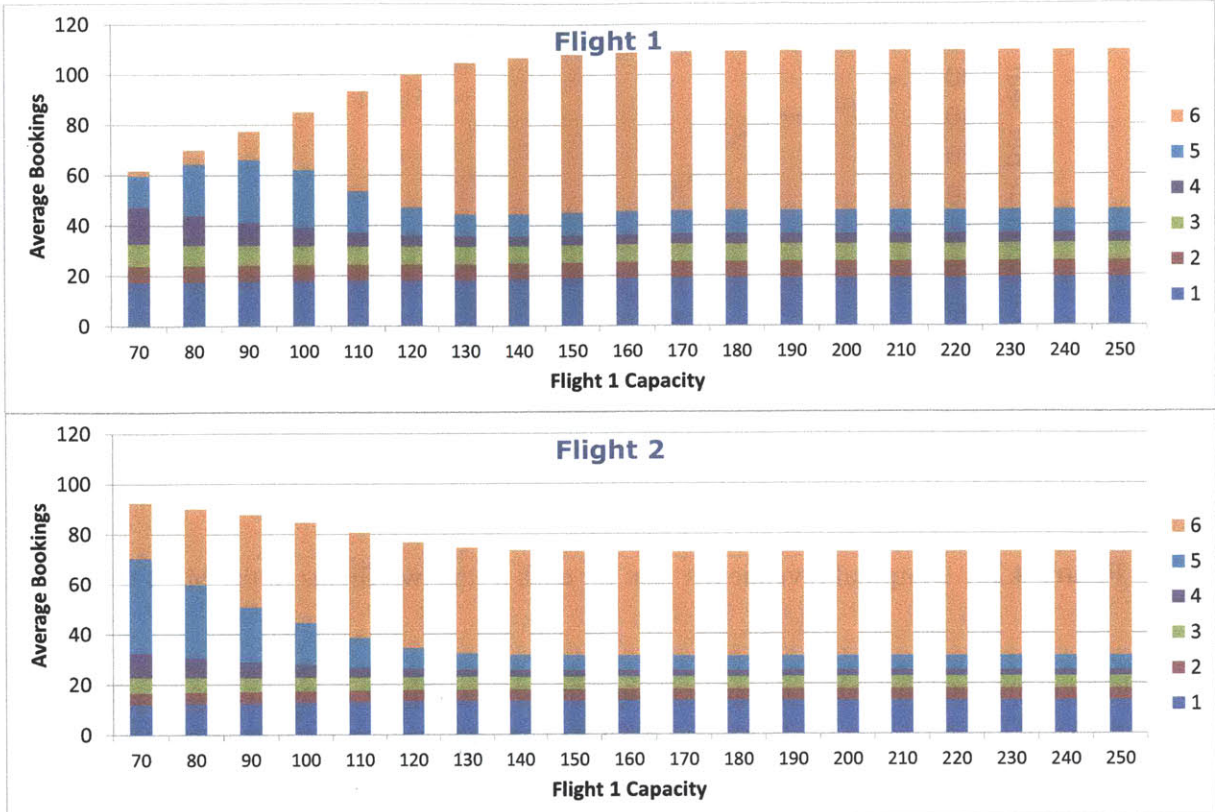


Figure 4.25: Average fare class mix for (*top*) flight 1 and (*bottom*) flight 2 in MFB test

The fare class mix for flight 2 differs substantially from that of flight 1. When flight 1 capacity is low, passengers are recaptured onto flight 2 to avoid spilling out completely (not traveling). There is therefore an increased demand for flight 2: more bookings are recorded and some passengers sell up to secure seats on the flight, as seen by the increased bookings in class 5. As flight 1 capacity increases, passengers move back to their first choice flight, demands for flight 2 are reduced, and passengers no longer sell up. The value of incremental capacity can again be summarized by considering the marginal revenue per seat curve for each of flights 1and 2, shown in Figure 4.26. Again, while flight 1's marginal revenue is always non-negative, adding capacity on flight 1 results in a strongly negative marginal revenue on flight 2 because of passengers' lower preferences to travel on that flight. The changes in the shapes of the marginal revenue curves at approximately 110 seats again define a transition region between sell-up dominated low capacities and mostly-first-choice higher capacities, as in the FPC test (Figure 4.11).

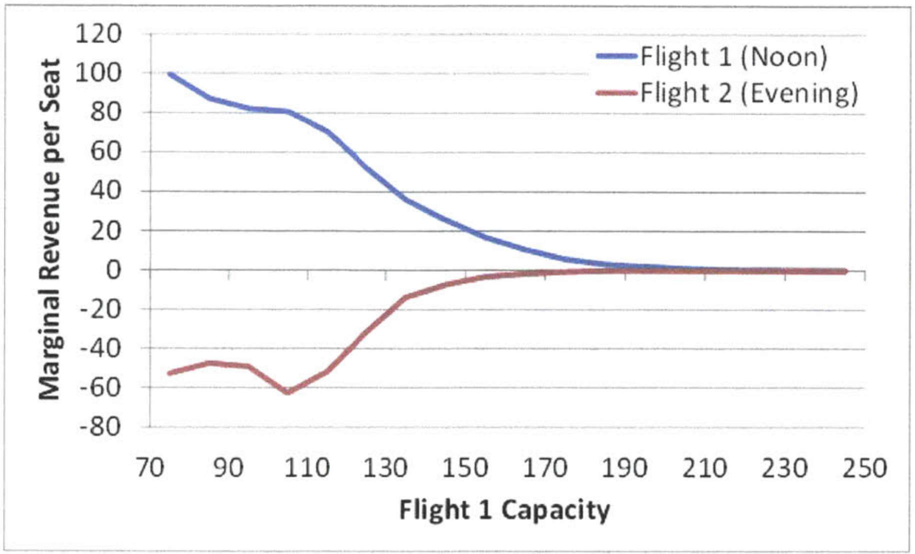


Figure 4.26: Marginal revenue per seat for each of the two flights in the MFB test

It should be mentioned that while capacity is changing only on flight 1, most revenue and fare class mix characteristics would remain very similar in a scenario in which both flights change capacity. Load factors on flight 2 would of course continue to decrease if capacity were added on that flight, but the negative marginal revenue observed on flight 2 would remain since simply adding capacity would not result in more bookings. More sell-up would be observed at low capacities as more seats are removed from the market (since both flights change capacity), but the buy-down with increasing capacity would remain (and would likely be more pronounced). Regardless of whether one or more flights change capacity, it is readily apparent that changing capacity on one flight affects revenues and load factors on other flights due to passenger recapture, especially if some flights are more in-demand than others, due to time of day preferences or any other reasons.

4.4.2 Multiple Flight Spill with Advanced Revenue Management System (MF-HFFA)

Test Parameters: Full passenger choice, less restricted fare structure, hybrid forecasting and fare adjustment, varying capacity on flight 1 (noon)

The MFB test in the preceding section introduced recapture in a traditional, restricted fare structure with a standard leg-based revenue management system. It is also important to discuss

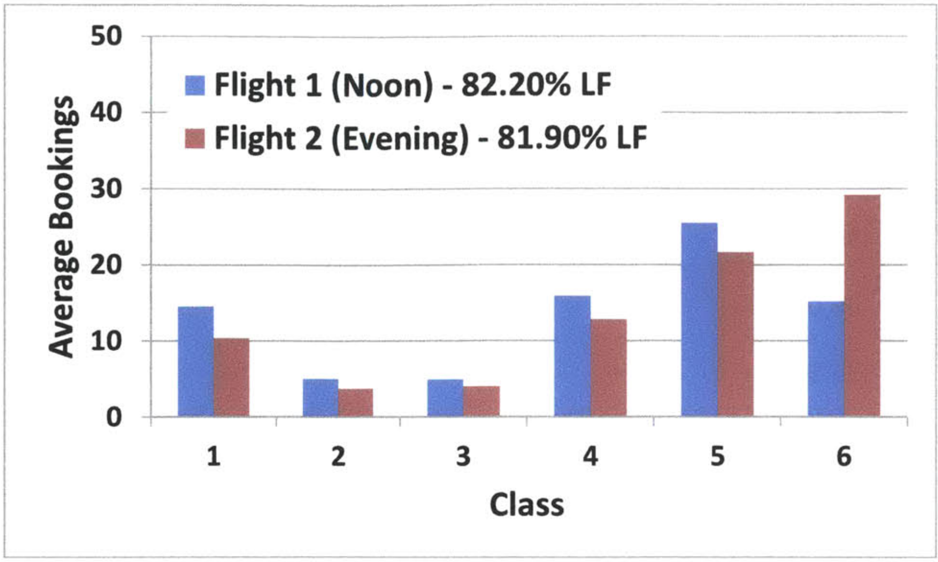


Figure 4.27: Fare class mix for each of the flights in the MF-HFFA test

recapture in the context of more recent, less restricted fare structures. As discussed in Section 4.3.3 under the FPC-LR test, increasing buy-down occurs in less restricted fare structures: flight 2 in the MFB test would thus likely exhibit even more revenue loss when capacity on flight 1 is increased if the fare structure were less restricted. It therefore makes sense to test multiple flight spill in a less restricted fare structure with the more advanced hybrid forecasting and fare adjustment revenue management system (MF-HFFA) in order to mitigate buy-down. Such a test reveals the impact of the more advanced revenue management system on recapture and therefore on the value of incremental capacity.

For consistency, baseline demands for the MF-HFFA test are set equal to the demand that generates an approximate 85% load factor in a multiple flight test in a less restricted fare structure with standard leg-based revenue management, exactly as the LR-HFFA test utilized the same demands as the FPC-LR test. Otherwise, parameters remain identical to the MFB test with only flight 1 varying in capacity.

The revenue at 100 seat capacity for the MF-HFFA test is \$36 259 at an average load factor of 81.55%. The average fare class mix for each of the two flights is shown in Figure 4.27. The fare class mix is similar to that of the MFB test (Figure 4.22), although the advanced RM system

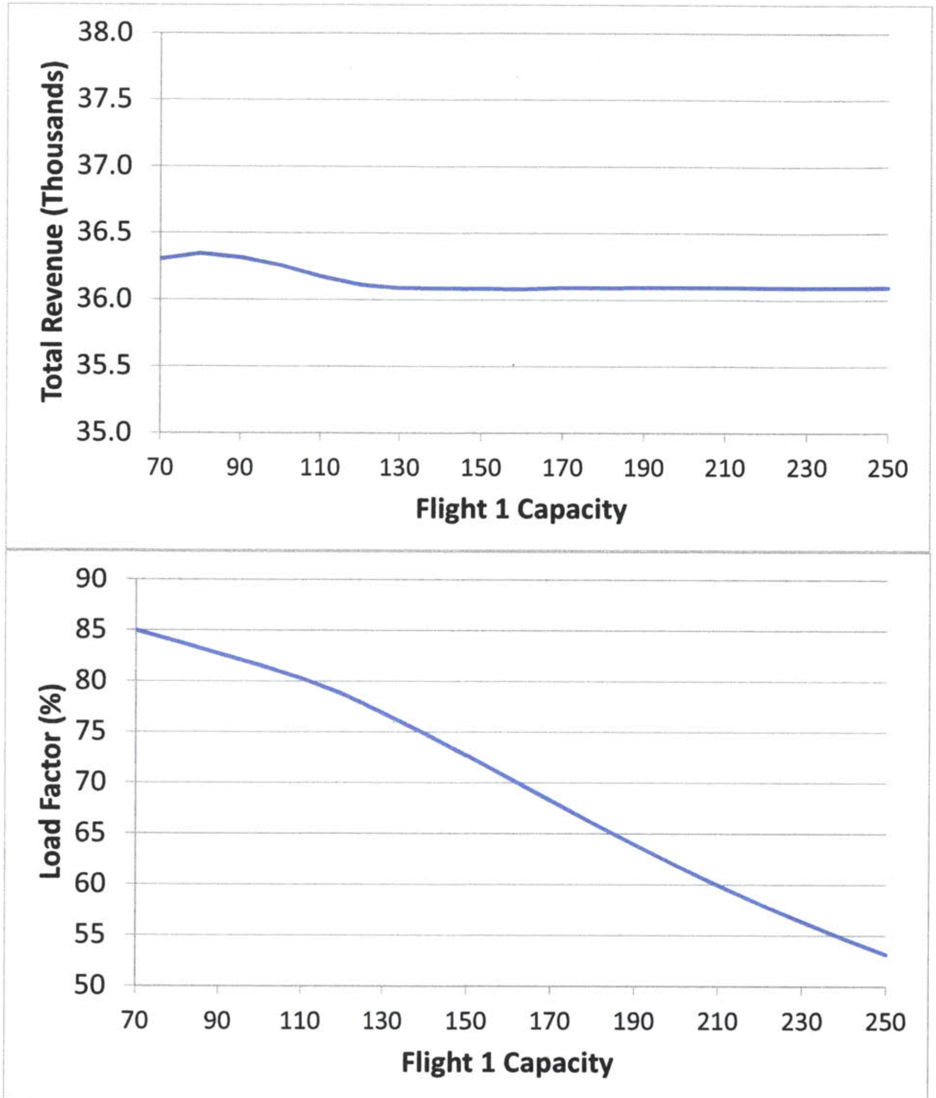


Figure 4.28: Total revenue and average load factor (across the two flights) as a function of capacity for MF-HFFA test

takes into account passenger willingness to pay and is able to secure more class 5 bookings than class 6 bookings on the in-demand flight 1 in the MF-HFFA case (note, however, that baseline demands are higher in the MF-HFFA case than in the MFB case).

Total revenue and average load factor as a function of capacity at the market level are shown in Figure 4.28. Recall only flight 1's capacity is changing, as indicated by the *x*-axis label. At the market level, the revenue and load factor curves are very similar to those of the LR-HFFA tests: adding capacity first results in a small revenue gain (between 70 and 80 seats), then a revenue

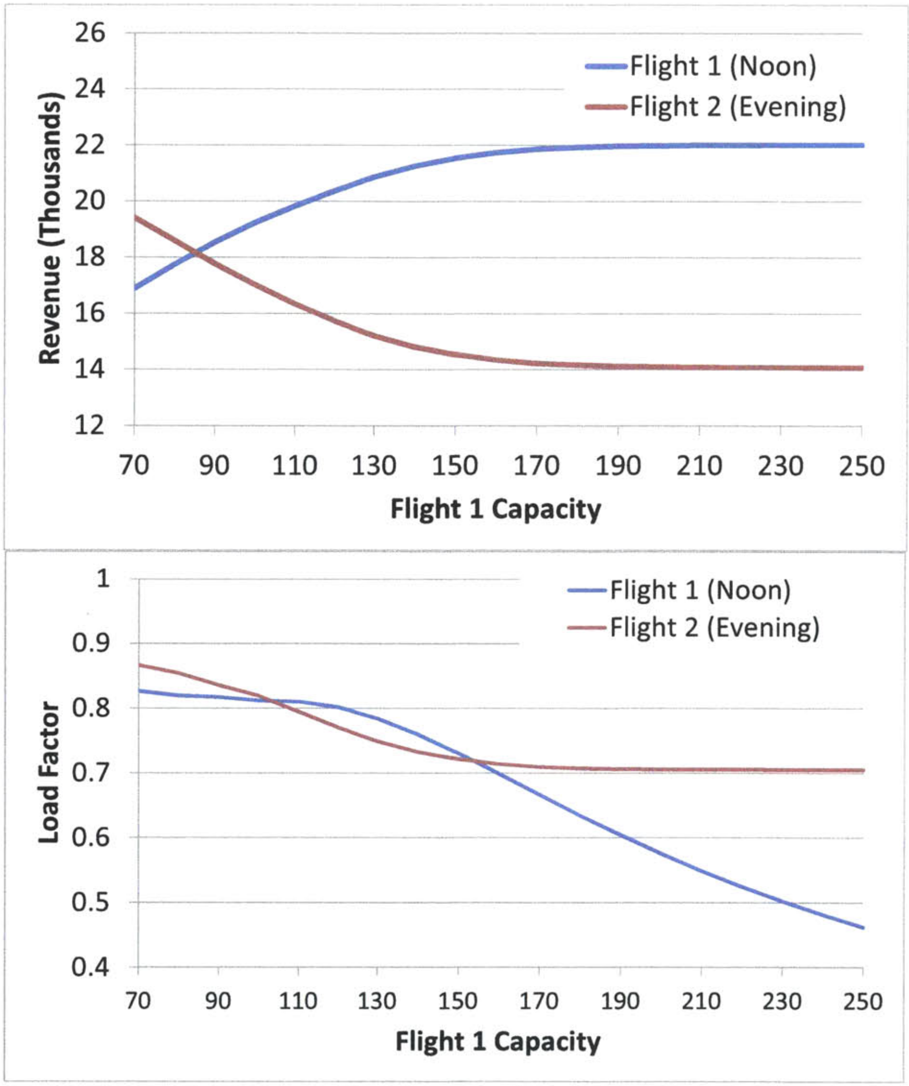


Figure 4.29: Revenue and load factor for as a function of flight 1 capacity for MF-HFFA test and for each of the two flights in the AAA – BBB market

loss as passengers buy-down (through to 120 seats), and then a revenue stabilization as all demand is carried and passengers receive their first choices. As with the FPC-LR and LR-HFFA tests, the revenue change across capacities is not as large as in the restricted fare structure as the revenue gains from additional bookings at increased capacities are approximately balanced by the greater number of passengers buying down.

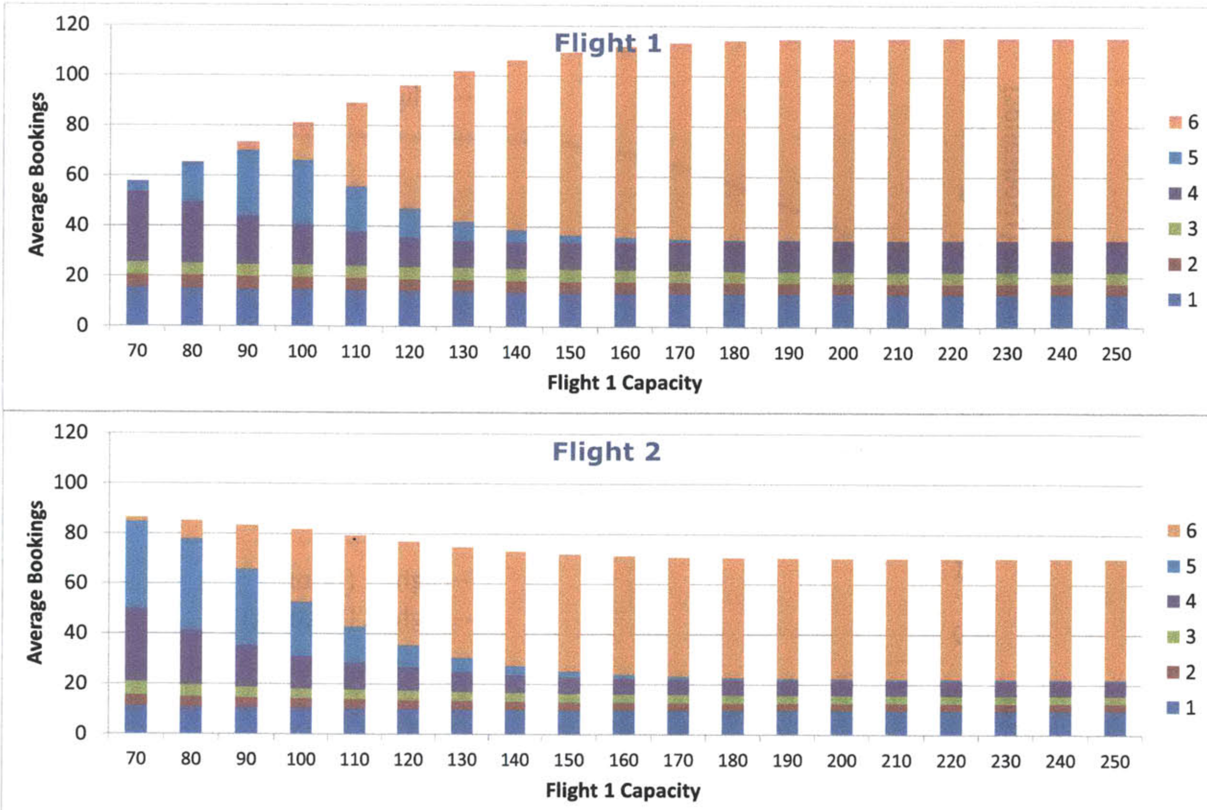


Figure 4.30: Average fare class mix for (*top*) flight 1 and (*bottom*) flight 2 in MF-HFFA test

The revenue and load factor for each of the two flights is shown in Figure 4.29. The revenue and load factor profiles, in general, appear to be similar to those of the MFB test (Figure 4.24). As the HFFA RM system is able to elicit more sell-up, flight 2 revenues in the MF-HFFA test appear to be slightly higher, relative to flight 1, than those of flight 2 in the MFB test. The decrease in load factor on flight 2 is more gradual than it is in the MFB test. Further insight into the revenue and load factor profiles may be gained by considering the fare class mix for each of the two flights, shown in Figure 4.30.

The fare class mix changes with varying capacity in a similar fashion to previously conducted tests. On flight 1, increasing capacity leads to additional bookings but increasing buy-down because of the less restricted fare structure, similar to the LR-HFFA single flight test. The flight 2 fare class mix is similar to that of the MFB test: increasing capacity on flight 1 again leads to buy-down and revenue losses onto flight 2, though these losses are exacerbated by the less

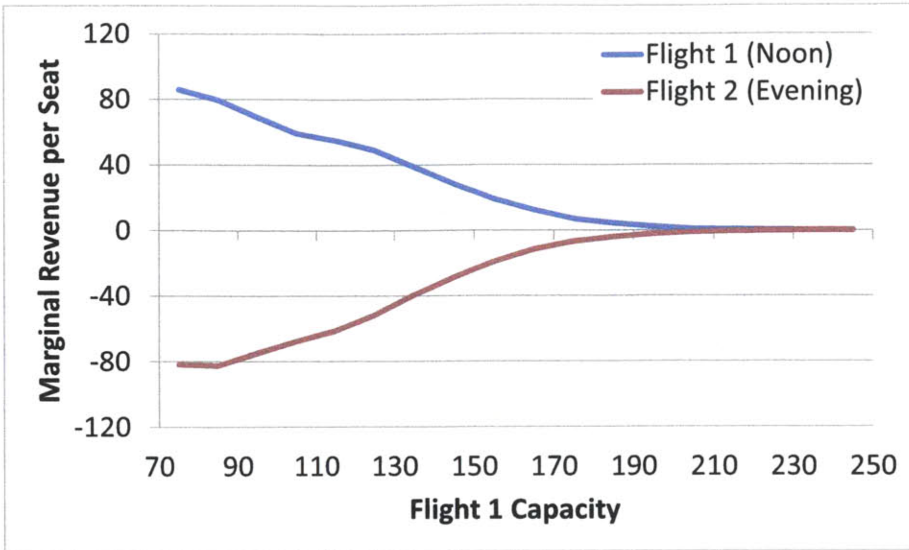


Figure 4.31: Marginal revenue per seat for each of the two flights in the MF-HFFA test

restricted fare structure here. Load factors on flight 2 are higher than in the MFB test: because the revenue management system is accounting for passenger willingness to pay and protecting some higher fare seats on flight 1, more passengers are unable to receive their first choice preferences for flight 1 and are instead recaptured onto flight 2. The MF-HFFA test reveals how several factors come into play when considering multiple flight spill in a less restricted fare structure. Recapture again couples revenues between the flights, while sell-up and buy-down affect marginal revenues, as seen in the single flight less restricted tests.

The value of incremental capacity for the MF-HFFA test can be summarized by considering the marginal revenue per seat curves, as seen in Figure 4.31. While the curves are similar to those of the MFB test (Figure 4.26), the use of the more advanced revenue management system smooths the marginal revenue curves for each of the two flights. Again, adding capacity on one flight can lead to revenue losses on other lesser in-demand flights, as seen in the MFB case.

4.4.3 Multiple Flight Spill Summary

The multiple flight spill tests built on the spill picture developed through single flight tests and introduced the effects of recapture by giving passengers the choice to travel on one of two flights. In particular, the multiple flight spill tests revealed that recapture couples revenues between flights such that adding capacity on one flight can lead to negative marginal revenues on others in both restricted and less restricted fare structures.

4.5 Competitive Spill

The preceding sections have explored the interactions between revenue management and spill with regards to passenger choice – time of day preferences, sell-up, and recapture – but have not considered competitive effects. The airline industry is extremely competitive and an airline's competitor's actions can affect the airline's revenues, load factors, and fare class mixes. It is important to study spill within a competitive environment in order to build a complete picture of the interactions between revenue management and the value of incremental capacity.

4.5.1 Competitive Baseline Spill (CFB)

Test Parameters: Two airlines, full passenger choice, restricted fare structure, standard RM system, varying capacity on all flights

The baseline competitive flight experiment has two airlines, Airline 1 and Airline 2, operate three flights daily between AAA and BBB. The flight schedule is shown in Figure 4.32. Note both airlines operate flights at the same time in order to eliminate schedule effects. The distance between the cities remains 1400 miles, while the baseline capacity of both aircraft is 100 seats. Passengers have a choice between three flights on each of the two airlines. The traditional, restricted fare structure is used in this *competitive flights baseline* (CFB) test. As before, baseline demands are set in order to obtain an average load factor of approximately 85% for both airlines.

Marginal revenue is studied from the perspective of Airline 1. In order to test marginal revenues, the capacity of *all* of Airline 1's flights is varied between 70 and 250 seats. Because of the time of day preferences, where flight 2 is most preferred, marginal revenues will be different for the different flights. Recall, as discussed in Section 4.4.1 for the MFB test, the composition of fare class mixes when changing capacity on one or all flights does not change the results greatly if one flight is much more in-demand than others: the largest difference will be in the amount of

Airline	Flight	Depart AAA	Arrive BBB	Flight Time
1 and 2	1	8:07	11:30	3:23
1 and 2	2	12:37	16:00	3:23
1 and 2	3	17:07	20:30	3:23

Figure 4.32: Schedule for competitive flight tests

sell-up, as reducing capacity on all flights simply results in fewer total seats in the market, such that more passengers are forced to sell up if they desire to travel.

The revenue at 100 seat capacity for the competitive baseline spill test is \$69 255 at an average load factor of 85.62% and \$69 104 at an average load factor of 85.21% for Airlines 1 and 2, respectively. The revenues and load factors for both airlines are very similar, as expected, given the symmetry of the fare structure and schedule.

Total revenues and average load factors for each of the two airlines in the CFB test are shown in Figure 4.33. Note the capacity labels on the x-axis refer to the capacity on each of Airline 1's flights (while Airline 2 does not change its capacity). From the baseline capacity of 100 seats, as Airline 1 increases capacity, it gains revenue, while Airline 2 loses revenue. This is due to spill-out to Airline 2 – essentially, this is recapture, but to a different airline (so that the revenue is lost to Airline 2). Passengers move to Airline 1 when more lower class seats are made available given the additional capacity. On the other hand, when Airline 1 removes capacity, its revenues decrease, while Airline 2's increase. As Airline 1 removes seats, there is less capacity in the market and Airline 1 passengers are spilled out to Airline 2. As there is now higher demand for fewer seats in the market, passengers are forced to sell up, thereby leading to both higher revenues and load factors for Airline 2. In this fashion, it becomes apparent that the actions of one airline (Airline 1) will influence revenues for other airlines (Airline 2). As such, revenues are not only coupled across flights through recapture, but are also coupled between airlines through spill-out and spill-in.

Revenues and load factors for Airline 1 can be broken down by flight, as shown in Figure 4.34. As discussed in the MFB tests, flight 2, departing at midday, is most in-demand. Flight 2's revenues thus increase through to above 210 seats. Flight 1 and flight 3, lesser in-demand, do not add revenues above approximately 130 seat capacity. In this restricted fare structure, buy-down is limited, and Airline 1 does not lose revenue on any one flight with increasing capacity (neglecting the very slight decrease in revenues at higher capacities). Reducing capacity leads to revenue loss on all flights. Similarly, flights 2's load factor is always higher than flight 1's and 3's, given the higher demands for flight 2.

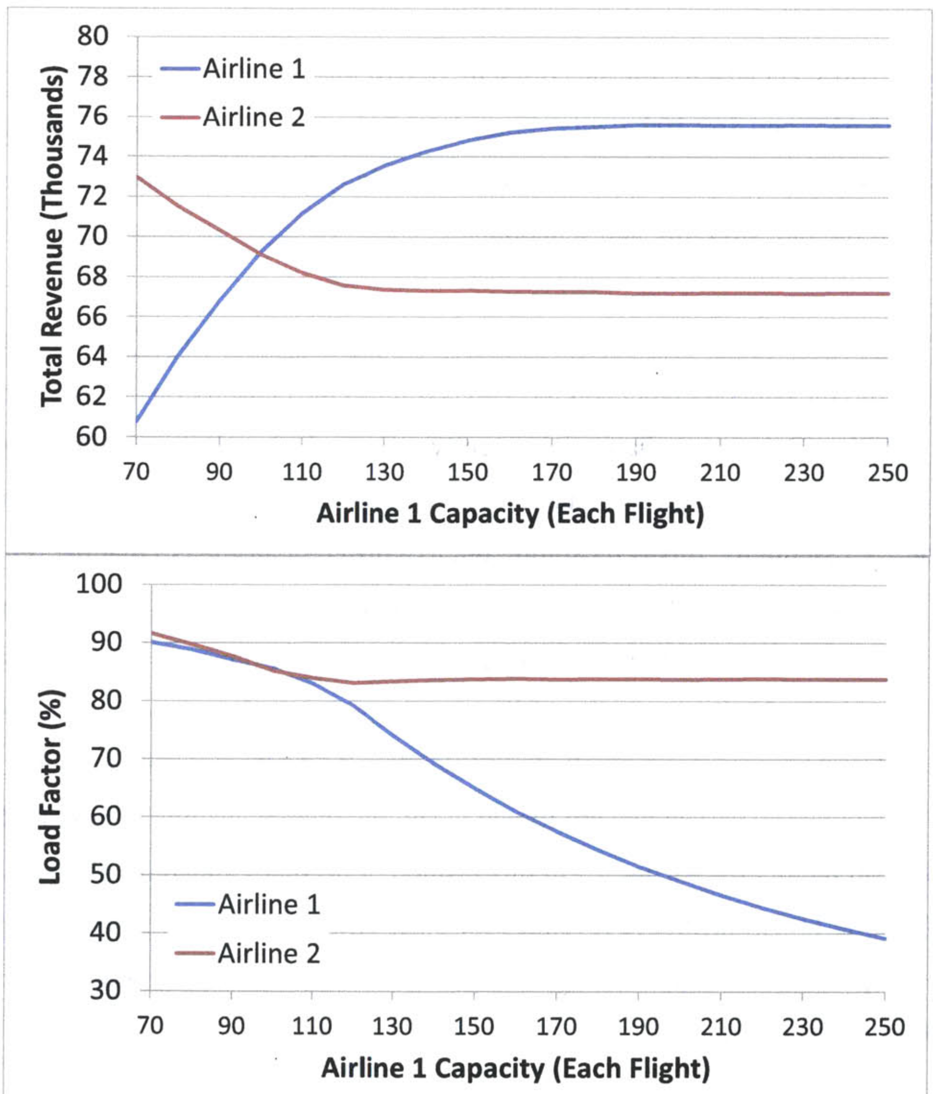


Figure 4.33: Total revenues and average market load factors for each airline in the CFB test

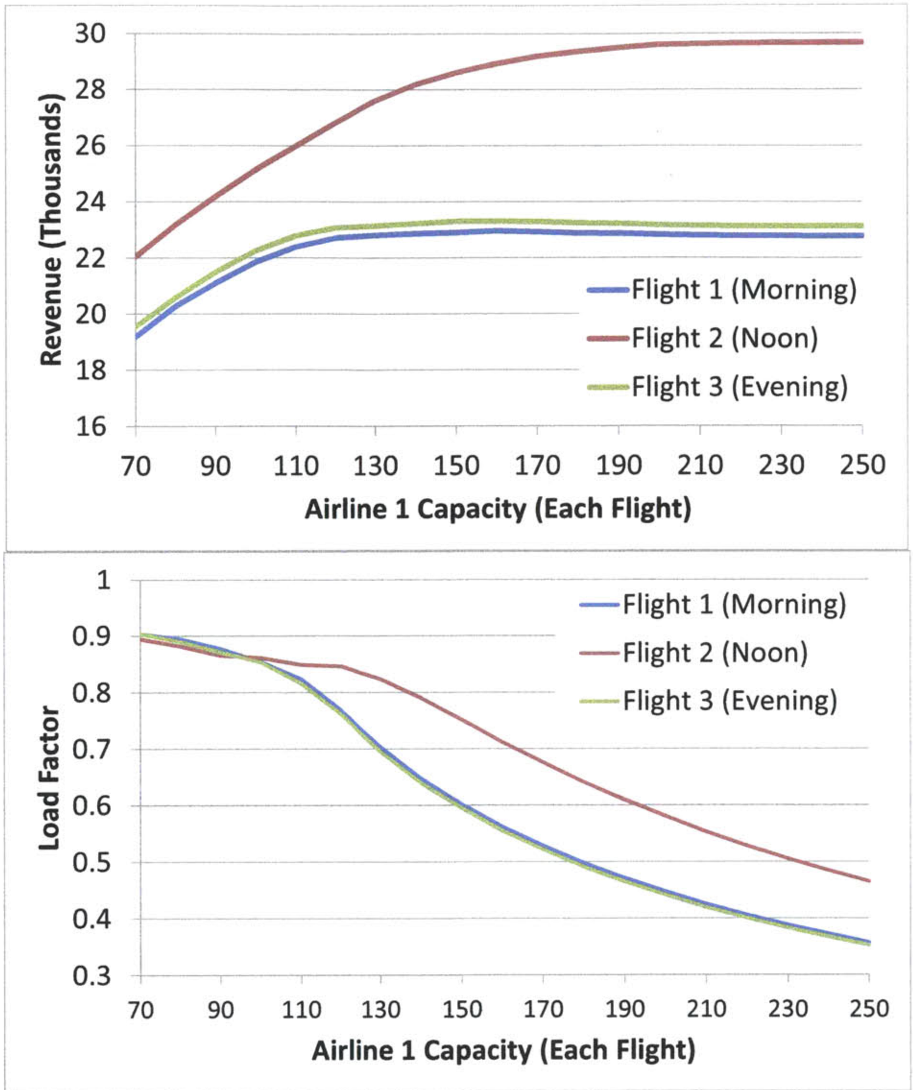


Figure 4.34: Revenue and load factor for CFB test as a function of capacity on each of Airline 1's flights

It is also interesting to consider the "revenue mix" for Airline 1. As PODS provides complete information on passenger choices, the revenue for Airline 1 can be sorted into categories by passenger choice in order to examine the revenue contribution from passengers who received their first choices, who sold-up, spilled-in, or were recaptured. The revenue mix for Airline 1 is shown in Figure 4.35. The revenue at each flight capacity is sorted into first choice, recapture, spill-in, and sell-up revenue. Note the scale on the *y*-axis begins from $50 000 as most of the revenue at any capacity remains first choice revenue.

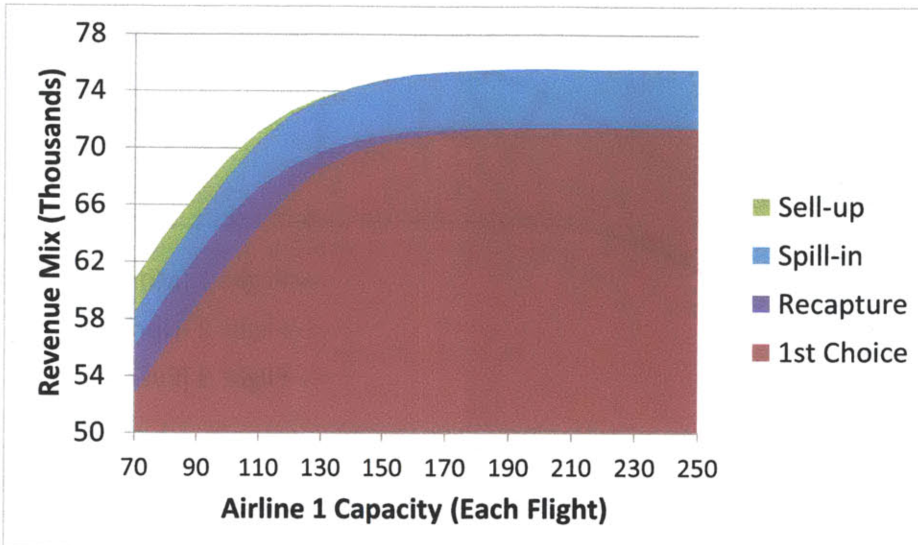


Figure 4.35: Revenue mix for Airline 1 in CFB test

The revenue mix illustrates how each of the revenue categories are important in computing the total revenue contribution at any given capacity: as all categories make up a sizeable portion of total revenue, none can be ignored in spill models if accurate revenue estimates are required.

At low capacities, sell-up is important and some passengers will pay a premium to secure seats for travel (as explained in the FPC and FPC-LR tests). Recapture also makes up an important component of total revenue: even though all flights are reduced in capacity, flight 2 is more likely to fill up first, given it is most in-demand. Thus, some passengers will be recaptured onto flights 1 and 3. Of course, other passengers will spill out to Airline 2 or choose not to travel.

The revenue contribution of spill-in makes up a large component of Airline 1's total revenue. At lower capacities, fewer passengers are spilling in from the competing airline as available seats are limited. However, as capacity increases, many passengers will move from Airline 2 to Airline 1. At large enough capacities, all passengers traveling on Airline 1 are either receiving their first choices or are spilling in from Airline 2.

The impacts of recapture and spill-in can be observed by considering the fare class mix for each of Airline 1's three flights, seen in Figure 4.36. Many characteristics of the fare class mix have been previously explored throughout the preceding sections of this chapter. First, the fare structure is more restricted, so that limited buy-down occurs, seen mostly in the decreasing class 5 bookings as capacities on all three flights increase from 80 through to 110 seats, where this buy-down is shown in the decreasing sell-up portion of the revenue mix. Both flight 1 and flight 3 show decreasing total bookings above approximately 120 seats, indicating the reduction of recapture as passengers move onto their preferred flight 2. Most of the additional bookings are in class 6, as seen in the BFC test, where many of the additional bookings are spilling in from the competing airline.

The value of the incremental capacity may again be summarizing by considering the marginal revenue per seat for each of the three flights, as shown in Figure 4.37. As previously discussed, given the restricted fare structure, the marginal revenue per seat is never (strongly) negative. The steep negative slopes of flight 1 and 3 marginal revenues, and the only gradual decline of flight 2 marginal revenues between 90 and 130 seats, are a result of recaptured passengers now moving back to their first choice of flight 2. With these passengers moving to flight 2, there are reduced demands on flights 1 and 3 and therefore increasing buy-down given the additional capacity and corresponding increased lower class booking limits. This follows the mechanism of the MFB test. Clearly, adding capacity on the most in-demand flight results in the largest revenue gains and is the most valuable place to add seats.

The CFB test built upon previously discussed tests and introduced competitive effects of spill by giving passengers the additional choice of selecting which airline to travel with, illustrating the coupling of revenues across airlines through spill-in. An airline's revenues can be affected by the actions of a competitor and the revenue at any given capacity is dependent on the total capacity available within the market. Therefore, depending on the desired accuracy of a spill model, it may be necessary to include competitors' possible actions or spill-in/spill-out forecasts.

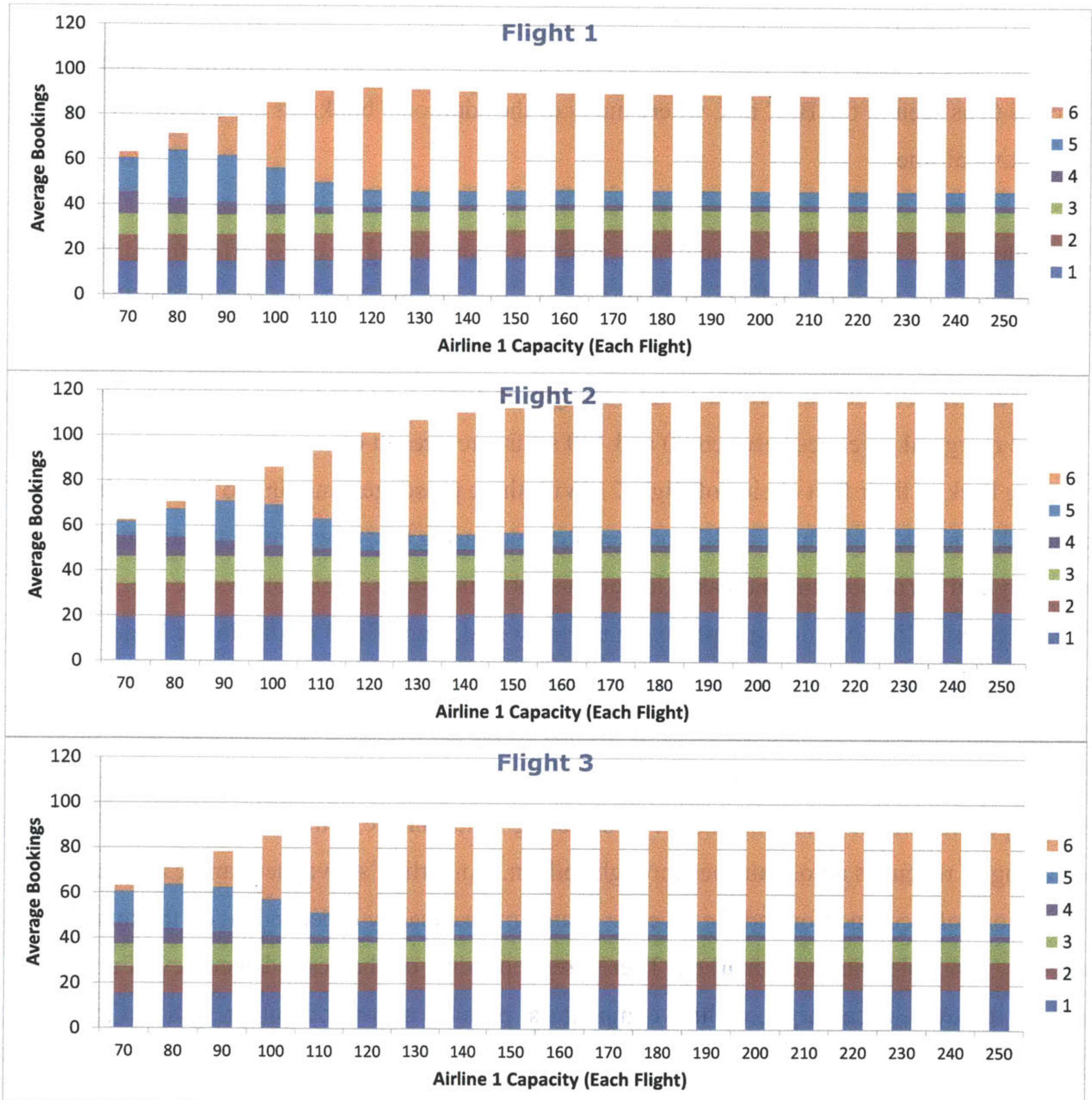


Figure 4.36: Average fare class mix for each of Airline 1's three flights in the CFB test

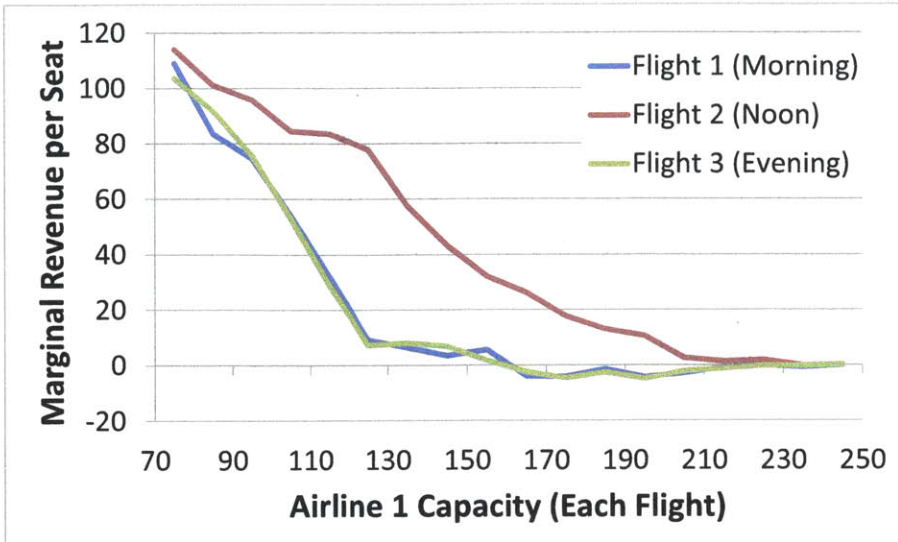


Figure 4.37: Marginal revenue per seat curve for each of Airline 1's three flights in the CFB test

4.5.2 Competitive Spill with Advanced Revenue Management System (CF-HFFA)

Test Parameters: Two airlines, full passenger choice, less restricted fare structure, hybrid forecasting and fare adjustment, varying capacity on all flights

The CFB test in the preceding section introduced competitive effects and spill-in in a traditional, restricted fare structure with a standard leg-based revenue management system. As with the single flight and multiple flight test cases, it is important to discuss competitive spill effects in the context of more recent, less restricted fare structures, where sell-up and buy-down play more prominent roles. Given the increased buy-down, the more advanced hybrid forecasting and fare adjustment revenue management system (CF-HFFA) is tested. Such a test reveals the impact of differing revenue management systems on spill-in and therefore on the value of incremental capacity.

Baseline demands for the CF-HFFA test are set equal to the demands that generate an approximate 85% load factor in a competitive flight test with a less restricted fare structure and under standard leg-based revenue management, exactly as in the case of the MF-HFFA test. All other parameters, including schedule, remain identical to the CFB test. All of Airline 1's flights change capacity, as before.

The revenue at 100 seat capacity for the CF-HFFA test is \$61 558 at an average load factor of 80.87% and \$61 564 at an average load factor of 80.79% for each of Airline 1 and 2, respectively. Of course, given the fare structure is less restricted, buy-down occurs and revenues are less than those in the restricted fare structure, even with the increased baseline demands and more advanced revenue management system.

Total revenues and average load factors for each of the two airlines in the CF-HFFA test are shown in Figure 4.38. The revenue and load factor curves are similar to those of the CFB test (Figure 4.33). However, note the revenue changes to both airlines when capacity is cut are larger than in the CFB case. As the HFFA RM system is more aggressive and protects more higher class seats, more sell-up occurs when capacity is reduced. Thus, the revenue loss to Airline 1 is less than in the CFB case, while the revenue gain to Airline 2 is larger. In a less restricted fare structure where sell-up is important, removing capacity results in less of a revenue loss than would be expected if passengers only made one choice (did not sell-up), given a good RM system.

Similarly, the change of fare structure and revenue management system affects load factors on both airlines. As expected, given the HFFA RM system, Airline 1's load factors are lower at all capacities. However, at low capacities, Airline 2's load factors are higher than Airline 1's, not observed in the CFB test. As the HFFA RM system protects more upper class seats, fewer seats are made available to lower class Airline 1 passengers. Thus, more of these passengers spill out to Airline 2 (where capacity has not been reduced), leading to Airline 2's higher revenues and load factors relative to the CFB test (with a restricted fare structure).

Revenues for Airline 1 can be broken down by flight, as shown in Figure 4.39. Here, several differences are seen as compared to the restricted fare structure CFB test. On flight 2, revenues continue to increase beyond 250 seat capacity, while flights 1 and 3 both exhibit revenue losses with increasing capacity, due to the buy-down. The revenue mix for Airline 1 fully explains the revenues observed on each flight.

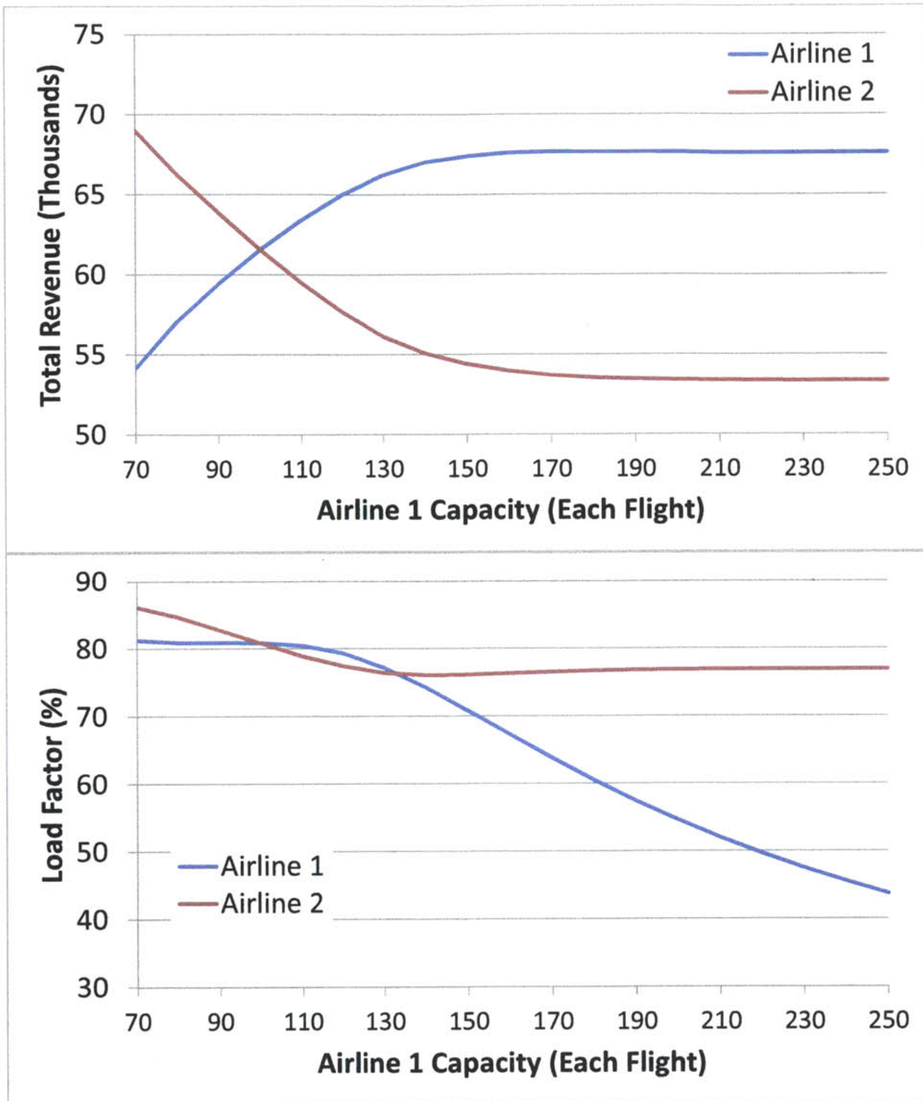


Figure 4.38: Total revenues and average market load factors for each airline in the CF-HFFA test

The revenue mix for the CF-HFFA test is shown in Figure 4.40, with the CFB mix presented for comparison. Clearly, sell-up makes up a much larger portion of the revenues than with the CFB test, given the less restricted fare structure. This larger sell-up results in a lesser revenue loss to Airline 1 when cutting capacity, but the increasing buy-down at larger capacities leads to flights 1 and 3 losing revenue. Additionally, sell-up and spill-in revenue components are much larger in the CF-HFFA test than in the CFB test at the higher capacities. This is due to the more aggressive RM system: since both airlines are using hybrid forecasting and fare adjustment, they account for passenger willingness to pay and protect more higher fare seats. However, as Airline

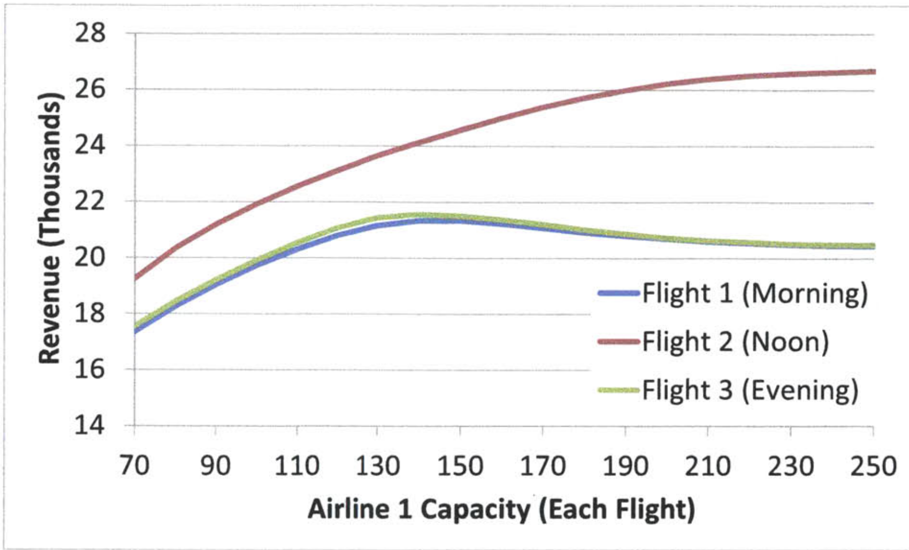


Figure 4.39: Revenue for CF-HFFA test as a function of capacity on each of Airline 1's flights

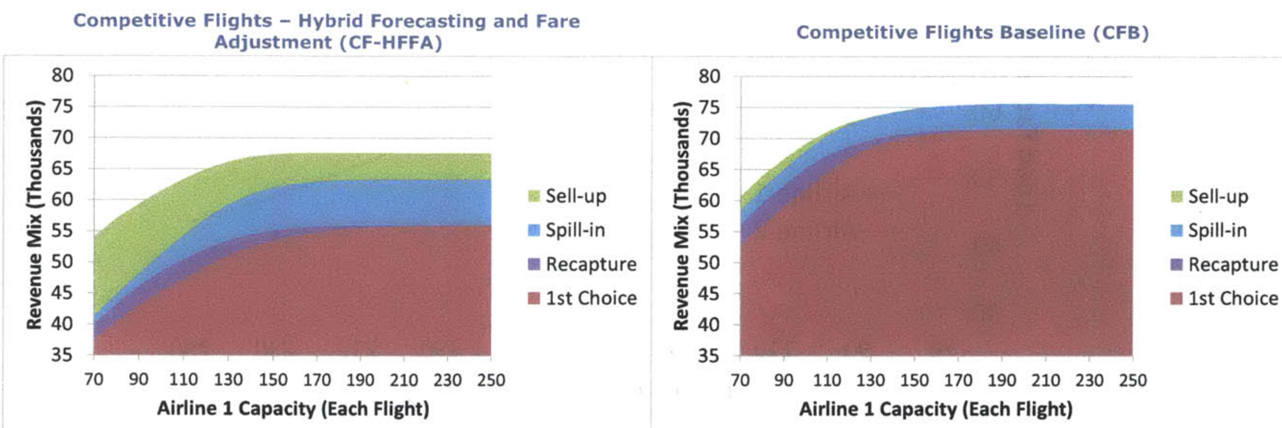


Figure 4.40: Revenue mixes for (*left*) CF-HFFA and (*right*) CFB tests

1 has spare capacity, more passengers spill-in to avoid paying a premium on Airline 2. There is therefore both more sell-up and more spill-in, and Airline 1's flight 2 revenues continue to increase beyond 250 seat capacity. It should also be noted that the recapture revenue component does not change substantially between the two fare structures and revenue management systems. Note that the average fare class mix for each of the three flights is presented for completeness in Figure 4.41.

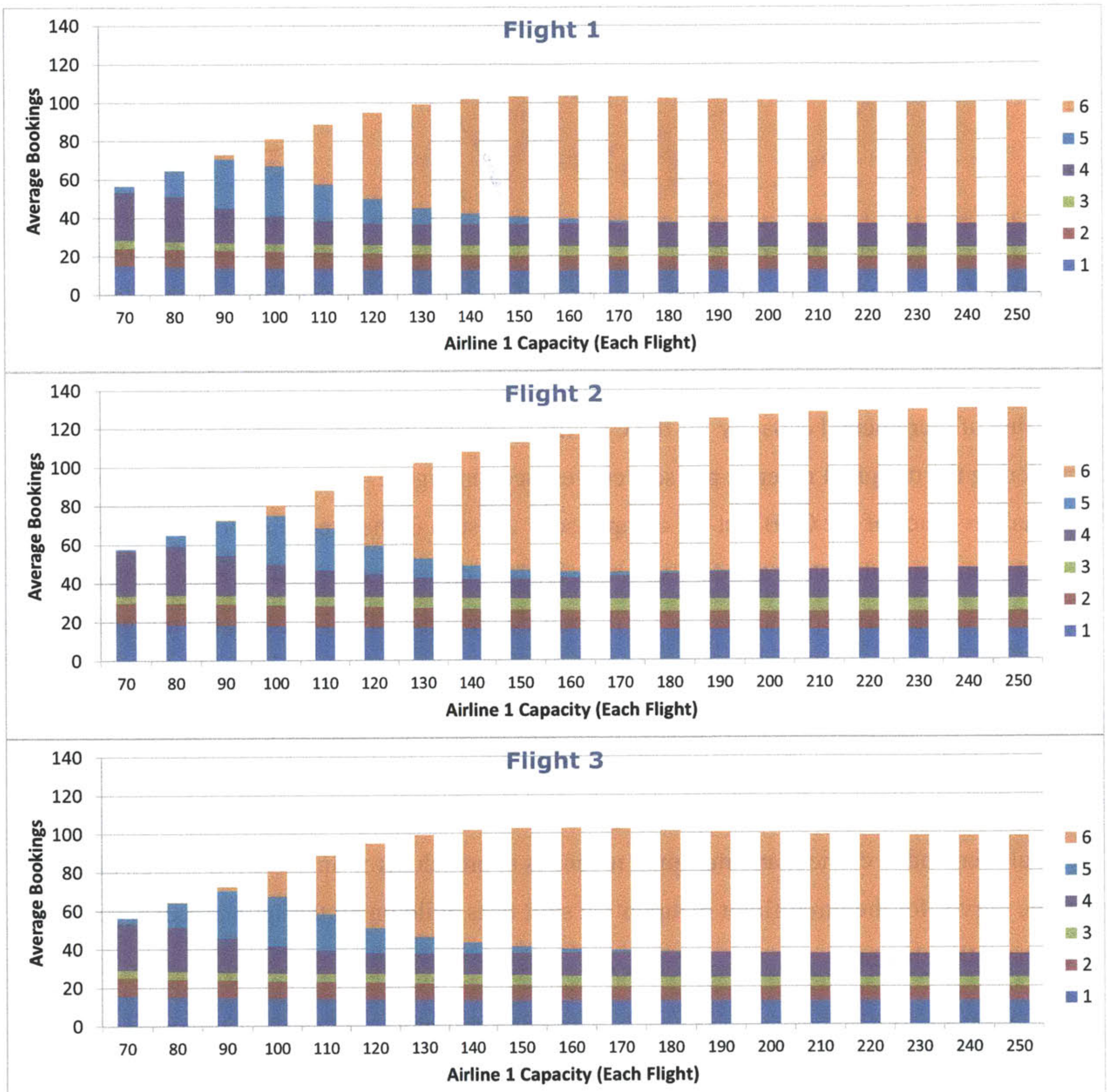


Figure 4.41: Average fare class mix for each of Airline 1's three flights in the CFB test

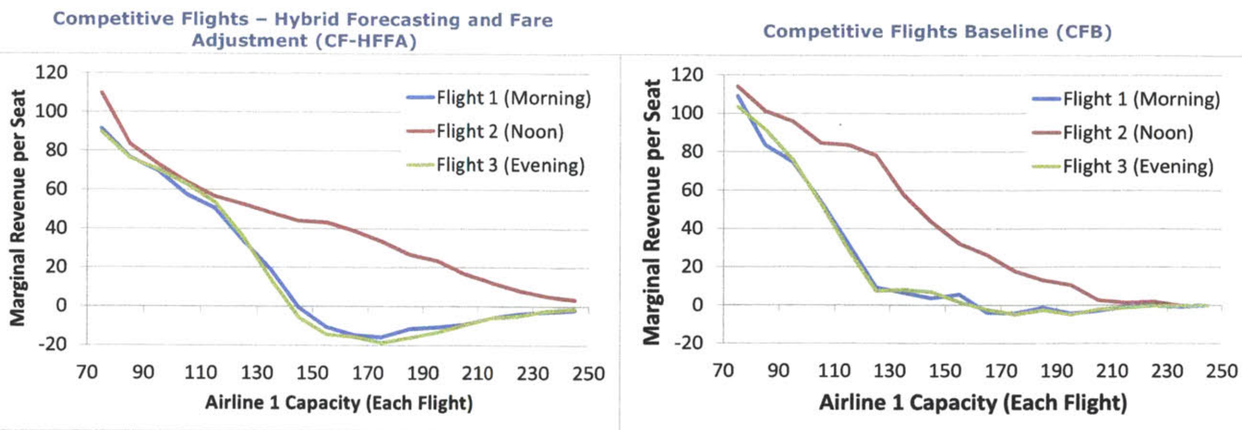


Figure 4.42: Marginal revenue per seat curves for each of the three flights in the (*left*) CF-HFFA less restricted fare structure and (*right*) CFB restricted fare structure tests

The value of incremental capacity for the CF-HFFA test can again be summarized by considering the marginal revenue per seat curves, seen in Figure 4.42, again shown with the marginal revenues of the CFB test for comparison. As seen in the preceding tests, the change of revenue management system and fare structure leads to different marginal revenue curves for each of the competitive spill test cases. With buy-down in the CF-HFFA test, the marginal revenue on flights 1 and 3 is negative at higher capacities. However, the marginal revenue curve for flight 2 in the CF-HFFA test has a less negative slope than in the CFB test due to increased spill-in as Airline 2 makes fewer lower class seats available, even as Airline 1 adds capacity. When enough capacity is added, all passengers receive their first choice of fare, airline, and flight, all demand is carried, and marginal revenues approach zero. In short, the marginal revenue curves for the three-flight competitive spill tests build on all the spill concepts described throughout the chapter.

4.5.3 Competitive Spill Summary

The competitive spill tests described in this section extended previous single airline spill tests and investigated the effect of spill-in on marginal revenues. It was shown that competitors' actions may influence the value of incremental capacity in a market and that the amount of spill-in revenue is dependent on the fare structure and revenue management system, where more spill-in revenue to Airline 1 was observed when Airline 2 attempted to elicit more sell-up by protecting more upper class seats under the more advanced HFFA RM system.

4.6 Conclusion

This chapter served to test the interactions between spill and revenue management in a realistic, competitive simulation environment. It explored and explained the effects of sell-up, recapture, and spill-in through the study of single flight, multiple flight, and competitive spill scenarios on a single city pair (single market).

The chapter began by testing spill in a first-choice-only model, representing the state of existing spill models, such as the Farkas and Boeing spill models. It was shown that modeling sell-up resulted in much higher estimates of marginal revenue at lower capacities, given some passengers would sell-up to avoid spilling. This improved the fare class mix and led to higher revenues.

An important finding was the possibility of an airline losing revenue by increasing capacity in less restricted fare structures, contrary to conventional wisdom. In such structures, increasing capacity has the revenue management system release more lower class seats, such that passengers with higher willingness to pay are able to buy down to lower fares. This may result in negative marginal revenues at higher capacities. As such, passenger sell-up and buy-down, influenced by the choice of fare structure, strongly drive the value of incremental capacity. Some of the revenue loss with increasing capacity can be mitigated by using a more advanced revenue management system which takes into account passenger willingness to pay.

The effect of recapture was studied by testing a scenario in which an airline operated two flights. The tests showed that due to passengers' time of day preferences, increasing capacity was much more valuable on higher in-demand flights. However, adding capacity to these flights resulted in revenue losses on other flights as previously recaptured passengers now received their first choice preferences to travel on the better-timed flight. These revenue losses were more prominent in less restricted fare structures where passengers were also able to buy down when capacities were increased.

In the competitive tests, it was shown that spill-in revenue depended strongly on the revenue management system used. More spill-in was observed in less restricted fare structures as passengers attempted to avoid selling up to higher fares by moving to the airline with increasing capacity. In essence, competitive effects were similar to recapture effects, except that revenues between flights were now coupled across airlines instead of simply across flights. The studies have shown that measuring the value of incremental capacity is difficult and all of the fare structure, revenue management system, and competitive environment must be considered to obtain accurate spill estimates.

The chapter also served to illustrate the importance of considering passenger choice in spill modeling, therefore justifying the extension to the Farkas spill model developed in the subsequent chapter.

While not within the scope of this thesis, further work on spill and revenue management could explore interactions between the value of incremental capacity and networks effects, including the study of recapture on other itineraries between cities and the effects of more advanced network revenue management systems.

Chapter 5 – Analytical Estimations of Spill

5.1 Introduction

This chapter introduces a heuristic spill estimator which extends the Farkas spill model in order to take into account passenger sell-up in estimating spill costs. As discussed in Chapter 4, taking sell-up into account will, in general, provide lower estimates of the number of spilled passengers compared to a first-choice-only model, given that some passengers will purchase higher fare tickets to ensure they are able to travel.

The extended sell-up spill heuristic is introduced and explained, while its limitations are presented. The sell-up heuristic is tested against the Farkas and Swan-extended Boeing spill models. PODS is again used to simulate the booking process in various fare structures, where the number of spilled passengers and the spill cost from the PODS runs is compared to the estimated spills from the analytical spill models.

5.2 Heuristic Sell-up Spill Estimator

In this section, the Farkas Spill Model is extended to account for sell-up. Recall that the single period Farkas spill model, as discussed in Section 2.6.2, is

$$\begin{aligned}\mathrm{SP}_k[S] = &\int_0^{\mathrm{BL}_k - S} f_k(x)\mathrm{SP}_{k-1}[x+S]dx \\ &+\int_{\mathrm{BL}_k - S}^{\infty} f_k(x)\{x-(\mathrm{BL}_k - S)+\mathrm{SP}_{k-1}[\mathrm{BL}_k]\}dx \qquad (2.5)\\ \mathrm{SP}_0 = &\ 0.\end{aligned}$$

In the model, all $x-(\mathrm{BL}_k - S)$ passengers who arrive after the booking limit for class k has been reached will spill (weighted by the probability of x passengers arriving). As mentioned, this approach overestimates spill as it does not allow for some class k passengers to purchase tickets in class $k - 1$ or higher if class k is closed. The proposed heuristic to take into account this sell-up is

$$\begin{aligned} \mathrm{SP}_k[S] = & \underbrace{\int_0^{\mathrm{BL}_k - S} f_k(x)\mathrm{SP}_{k-1}[x+S]dx}_{\text{Term 1}} \\ & + \underbrace{\int_{\mathrm{BL}_k - S}^{\infty} f_k(x)(1-p_k)\left[x-(\mathrm{BL}_k - S)\right]dx}_{\text{Term 2}} \\ & + \underbrace{\int_{\mathrm{BL}_k - S}^{\infty} f_k(x)\mathrm{SP}_{k-1}\left[\mathrm{BL}_k + p_k\left(x-(\mathrm{BL}_k - S)\right)\right]dx}_{\text{Term 3}} \\ \mathrm{SP}_0 = & \ 0, \end{aligned} \tag{5.1}$$

where

SP_k is the spill from classes 1 through k,
S is the number of allocated seats from previous recursions,
BL_k is the booking limit of class k,
$f_k(x)$ is the normal demand distribution for class k, and
p_k is the sell-up rate from class k to class $k-1$.

Here, the first term remains identical to the Farkas model and accounts for the cases in which the number of arriving passengers x is less than the number of remaining seats in the booking class k. In this case, there is no spill from class k, and the recursion continues to the next higher class $k - 1$.

The second term contains the sell-up rate p_k from class k to $k - 1$, where the sell-up rate p_k is defined as the fraction of passengers who sold up to a higher class $k - 1$ given they were not able to purchase their first choice fare in class k. Given p_k is the sell-up rate, $(1 - p_k)$ is the fraction of passengers who do not sell up. The second term of the extended spill model is therefore modified from the Farkas equation. Of the $x-(\mathrm{BL}_k - S)$ arriving passengers who exceed the number of seats available for sale in class k, $(1-p_k)\left[x-(\mathrm{BL}_k - S)\right]$ of them are spilled, instead of the full $x-(\mathrm{BL}_k - S)$ as in the Farkas spill model.

The third term accounts for the p_k fraction of passengers who sell up to the next highest class to avoid spilling out. Of course, the sell-up rate p_1 is 0. In this heuristic, the passengers who sell-up are assumed to purchase seats in class $k-1$, with the logic that they have arrived after class k is closed but before any class $k-1$ first choice passengers have begun to arrive. Thus, of the $x-(\mathrm{BL}_k - S)$ passengers who were not able to obtain seats in class k, $p_k\left[x-(\mathrm{BL}_k - S)\right]$ of them sell up and do not spill. Therefore, the number of occupied seats counting against class $k-1$'s booking limit, even before any class $k-1$ passengers begin to arrive, is $\mathrm{BL}_k + p_k\left(x-(\mathrm{BL}_k - S)\right)$, where the first addend is the number of seats occupied by class k passengers (with the class full to the booking limit and as per the Farkas model) and the second addend is the number of passengers selling up to the higher class. Thus, the modified equation (5.1) takes into account sell-up and should result in better spill estimates in fare structures where passengers are likely to sell up.

Equation (5.1) requires the sell-up rates p_k from any class k to $k-1$. These can be obtained from PODS as PODS reports the decisions of every individual simulated passenger. The number of passengers who preferred class k as a first choice and were not able to purchase it are noted. The number of these passengers who purchased a seat in a higher class sold up, and the sell-up rate is calculated as the fraction of passengers who purchased higher class tickets given their first choice seats were unavailable.

The sell-up heuristic was implemented in MATLAB. The numerical implementation is straightforward, with the upper bound at infinity in terms 2 and 3 approximated by a bound at four standard deviations above the mean demand, such that the probability of a drawn demand being that high is extremely unlikely and introduced errors from the finite integration are minimal.

5.3 Limitations of the Sell-up Heuristic

The sell-up extension to the Farkas model developed here is effective in a mostly-restricted fare structure in which most of the sell-up occurs only to the next highest class (given there is only a single probability p_k for selling up from one class to the next highest). To account for sell-up to all other, higher, classes, a sell-up probability p_{kj} would be required from any class k to any higher class j. This adds substantial complexity to the heuristic and affects computational performance as additional recursions would be required.

The implementation of the sell-up heuristic is based on the single period Farkas model. PODS, simulating a realistic airline revenue management system, reoptimizes its booking limits for each class at certain intervals prior to departure, while the heuristic assumes a single booking period. In the single-period approach, it is assumed that all bookings occur bottom-to-top, such that all bookings for a lower class are made before those in an upper class. Given that PODS reoptimizes, the fare class mix of rejected passengers from the heuristic will likely have some error from the mix simulated in PODS. For example, in samples with more arriving upper class demand, PODS can decrease the booking limits for the lower classes to accept more upper class traffic. This will result in more spill than a static, single-period model (and more revenue as well).

It should also be noted that, within PODS, a passenger arriving after his preferred class has been closed by an advance purchase deadline is recorded as making his first choice for a higher class (and is not considered to have sold up). For example, a passenger who wants to purchase a class 5 fare and arrives after class 5 has been closed due to AP is recorded as having made a first choice for a higher class, such as class 4. This passenger is not considered to have sold up to 4. Therefore, any passengers arriving after the AP closure of their desired class who choose not to sell up are not considered to have spilled.

5.4 Experimental Overview and Methodology

For each of the Swan-extended Boeing Spill Model, Farkas spill model, and sell-up heuristic, the number of spilled passengers and the spill cost is computed at a variety of capacities and compared to simulation results in PODS in order to evaluate the performance of the methods.

The tests are performed on a single flight leg with a baseline capacity of 100 seats. Demands are selected such that an approximate 85% average load factor is obtained on the flight leg in the baseline case. To test spill, the capacity is varied between 50 and 150 seats in intervals of 10 seats. The idea of testing around a baseline capacity is to mimic an airline performing its fleet assignment. An airline may currently operate a 100 seat aircraft and wishes to know the change in the assignment cost by assigning either a smaller or larger aircraft to the leg. It may know the operating costs of operating a smaller or larger aircraft but requires estimates of the spill cost for aircraft of other sizes.

Both the Farkas and extended sell-up models require the booking limits for each class as inputs. As EMSRb is used as the standard leg-based optimizer, estimates of the demand and standard deviations by class are required. While it is possible to use the RM system's forecaster to obtain these estimates, the academic study here uses the "true" assumed normal demand distributions for each class, obtained by averaging first choice demands by class for every generated passenger across all the departure samples. In other words, the first choice preference for every individual arriving passenger is recorded at the baseline 100 seat capacity. The first sample may record 30 passengers whose first choices are class 5 fares, while the second may record 35. Recording all such preferences across samples builds demand distributions by class $f_k(x)$ for which (sample) means and standard deviations can be calculated (assuming a normal demand distribution). The unconstrained demand distribution is not a function of the capacity. These means and standard deviations are used to calculate booking limits BL_k at every tested capacity.

The sell-up heuristic also requires input sell-up rates p_k. In reality, an airline would have to estimate sell-up rates. In the tests here, the "actual" sell-up rates are used in order to remove any willingness to pay estimation errors, so that the performance of the heuristic itself may be evaluated (and not the performance of a sell-up estimator). This is in line with all methods being provided first choice demands. The sell-up rates are obtained from the baseline PODS simulation, as were the demands. The idea is that an airline was able to estimate, with sufficient accuracy, sell-up rates at some reference capacity (100 seats). It now wishes to apply its spill model to estimate spill costs at other capacities. As sell-up rates are generated from simulated observations in which passengers did not receive their first choices, these rates will differ for each of the different capacities (because at different capacities, different numbers of passengers will or will not receive their first choice). However, the sell-up rates are only used from the baseline capacity in order to better approximate reality (as an airline is unlikely to know how the sell-up rate changes with capacity). The sell-up rates are obtained in a similar fashion to the first choice demands. At the baseline capacity, every arriving passenger who did not get his first choice is recorded, along with the passenger's actual choice of selling up or choosing not to travel. The sell-up rate is then the total number of passengers who sold up divided by the number of passengers who did not get their first choices. In summary, each of $f_k(x)$, BL_k, and p_k are obtained from a PODS simulation at the baseline capacity of 100 seats.

The Boeing Spill Model is also tested in this chapter. Recall that the Boeing Spill Model assumes a single normal demand distribution for the flight, as explained in Section 2.6.1. However, the Boeing Spill Model is most appropriately used when the number of booking classes is limited. The Swan-extended Boeing Spill Model allows for the consideration of multiple booking classes as

$$\mathrm{SP} = \int_C^\infty f'(x)(x-C)\,dx, \tag{2.4}$$

where

SP is the total spill,
C is the capacity of the aircraft, and
$f'(x)$ is the aggregated normal demand distribution.

While the demand distribution can be estimated by assuming a K factor, the "true" demands are again available from the PODS simulation, as discussed above. Therefore, the first choice demand distributions by class $f_k(x)$ may be aggregated into a single demand distribution $f'(x)$. Thus, all three models are provided with "true" demand distributions – the idea of the experiments here is to test the accuracy of the models and not the methods needed to obtain the models' inputs. In particular, there is no need to use the Boeing spill tables to estimate demand factors, given the demands are provided to the Swan-extended Boeing Spill Model in this

experiment. Recall that the Swan-extended Boeing Spill Model does not take into account revenue management given its use of a single demand distribution for the flight. The Swan-extended Boeing Spill Model is hereafter referred to as the Swan model for brevity.
Two approaches for defining a spill fare to obtain the spill cost in the Swan model were discussed in Section 2.6.1. A traditional use of the Boeing Spill Model would use the average fare class mix to obtain the spill fare. At the 100 seat baseline capacity, the total number of bookings received may be N, with n_k bookings recording in each booking class, such that $\sum n_k = N$. If an airline has fares F_1 through F_n, the traditional spill fare is $F_s = \frac{1}{N}(n_1F_1 + \ldots + n_nF_n)$. This spill fare is then used to compute the spill cost at all other capacities. This spill fare assumes a spilled passenger is worth the average fare, which will lead to high estimates of the spill cost as most incremental passengers book into the lowest class because an RM system will protect seats for upper class bookings. Therefore, an alternative method is to use a spill fare which weights the lowest fare more strongly, also described in Section 2.6.1. This spill fare is $F_s' = 0.2(F_s) + 0.8F_n$. Both spill fares and their associated spill costs are tested in the experiments in this chapter.

Two fare structures are tested: a fully differentiated fare structure (most restricted) and a less restricted fare structure (similar to the one tested in Chapter 4). EMSRb and standard forecasting are used as the revenue management optimizer and forecaster, respectively.

It should be noted that all tests use 5 booking classes in the fare structure. 5 classes are used (instead of the more traditional 6 for PODS runs) as the additional recursion necessary in the sell-up heuristic greatly increases the computation time necessary for each spill estimate. Run time for capacities of 100 seats is on the order of one hour.

5.5 Fully Differentiated Fare Structure Test

Given that the Swan and Farkas spill models assume independent demands, the most appropriate fare structure in which to test their performance is a fully differentiated fare structure in which sell-up is limited. In such a structure, the advantage of the sell-up heuristic should be minimal.

Class	One-Way Fare	Advance Purchase	R1	R2	R3
1	$450	None	0	0	0
2	$350	3 days	0	1	0
3	$240	7 days	1	0	1
4	$190	14 days	1	1	0
5	$150	21 days	1	1	1

Figure 5.1: Fully differentiated fare structure

Such a test can serve to validate the Farkas spill model. The fully differentiated fare structure tested is shown in Figure 5.1. Every booking class offers a different fare product as the combination of restrictions vary for every class. In such a fare structure, demands are expected to be approximately independent.

At the baseline capacity, the PODS-reported first choice demands and standard deviations by class k are

$$\mu_k = [20.72 \quad 8.56 \quad 8.05 \quad 12.68 \quad 40.48]$$
$$\sigma_k = [8.15 \quad 4.45 \quad 4.20 \quad 5.61 \quad 13.13].$$

For the Swan model, these demands by class are aggregated into a total demand of $f' \sim N(\mu,\sigma) = N(90.50,17.54)$. The sell-up rates for the sell-up heuristic are computed to be $p_k = [0 \quad 0.20 \quad 0.26 \quad 0.60 \quad 0.67]$.

These first choice demands result in an average load factor of 85.07%. The average fare class mix and average number of spilled passengers for the baseline capacity resulting from these first choice demands is shown in Figure 5.2. As with the tests in Chapter 4, there are both business and leisure passengers. Business passengers tend to book the upper classes, while leisure passengers tend to book the lower ones. On average, 5.41 passengers are spilled in the fully

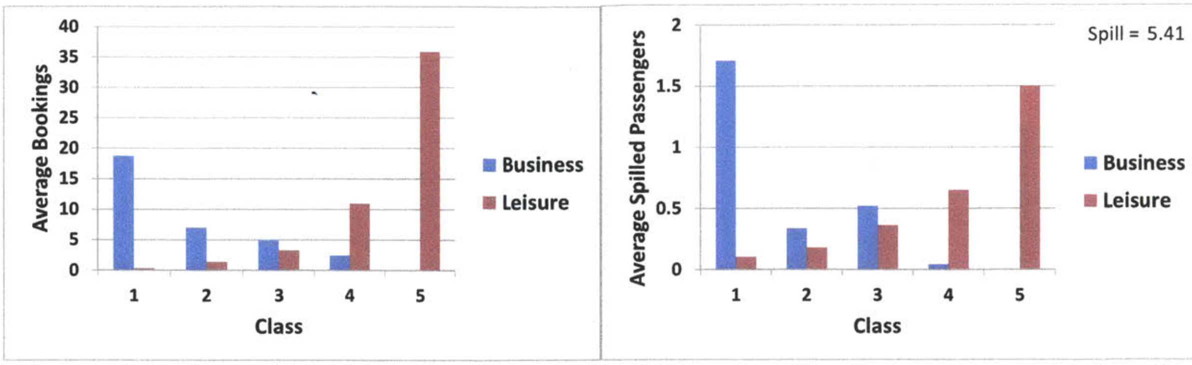


Figure 5.2: Average (*left*) fare class mix and (*right*) number of spilled passengers for baseline capacity in fully differentiated fare structure test

differentiated fare structure test at the baseline capacity. Note that the spill cost of these passengers is computed simply by multiplying spilled passengers from every class by the fare they would have paid, had they been able to purchase a seat. In the baseline capacity case, the actual spill cost is $1 561.

The average number of spilled passengers as a function of capacity for the fully differentiated fare structure is shown in Figure 5.3. There are four curves plotted. The blue curve is the "true" spilled demand, that is, the simulated spill from PODS. The analytical methods' estimates of spill are compared against the PODS spill.

The Swan model predicted spill is shown in purple. In this test of a fully differentiated fare structure, the Swan model underestimates spill at all capacities. Recall that the Swan model does not take into account revenue management. It is possible that it may overestimate spill because, like the Farkas model, it does not take sell-up into account. However, it is also possible that it underestimates spill, as observed here. For example, as seen by the baseline fare class mix in Figure 5.2, approximately 19 passengers have booked into class 1; they likely arrive later in the booking process and the RM system has protected seats for them. To protect more upper class seats, the RM system will decrease the booking limits for the lower classes and spill more lower class passengers than the Swan model would otherwise predict. For example, the mean demand for class 5 passengers is 40.48, while PODS accepts only 36 of these passengers, while the mean

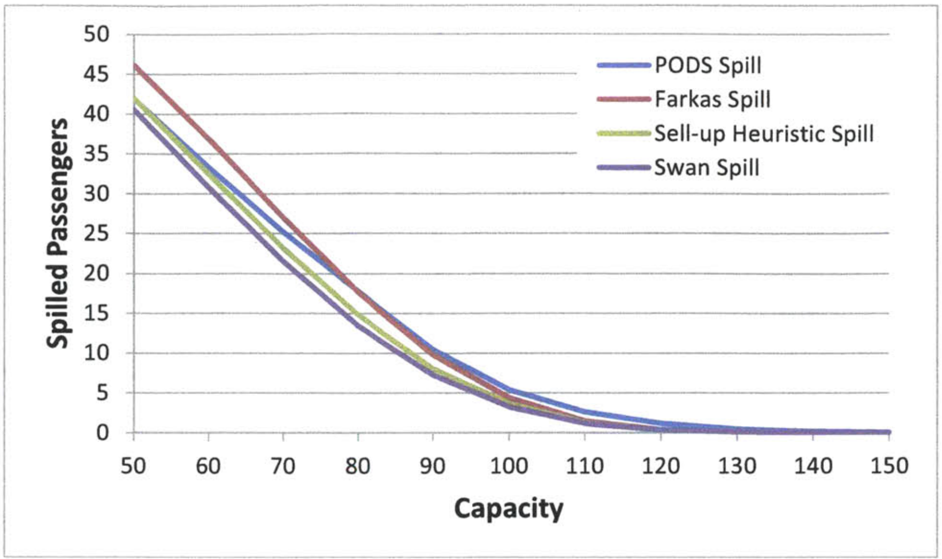


Figure 5.3: Average number of spilled passengers in fully differentiated fare structure test

demand for class 1 is 20.72 passengers and PODS accepts 19. In this fashion, the Swan model will underestimate spill across all capacities.

The Farkas Spill Model predicted spill is shown in red. At most capacities, the Farkas spill model overestimates the actual number of spilled passengers as it does not take into account passenger sell-up, as explained in Chapter 4. As capacity increases, more passengers receive their first choices, passenger sell-up decreases, and the Farkas-predicted spill approaches the simulated spill. It should be noted that sell-up is present even in this fully differentiated fare structure.

The sell-up heuristic predicted spill is shown in green. For lower capacities, the sell-up heuristic well-approximates the simulated spill count. As the fare structure is fully differentiated, most sell-up occurs from one class to the next higher class, exactly as captured by the sell-up heuristic. In this fashion, at the lower capacities, the heuristic predicts spill more accurately than the Farkas spill model does. At larger capacities, the sell-up heuristic underestimates the actual number of spilled passengers. As mentioned in Section 5.3, this is likely due to the reoptimization of booking limits within a PODS simulation. On samples with higher than average demands, PODS may reduce lower class availability to protect more seats for the increased number of

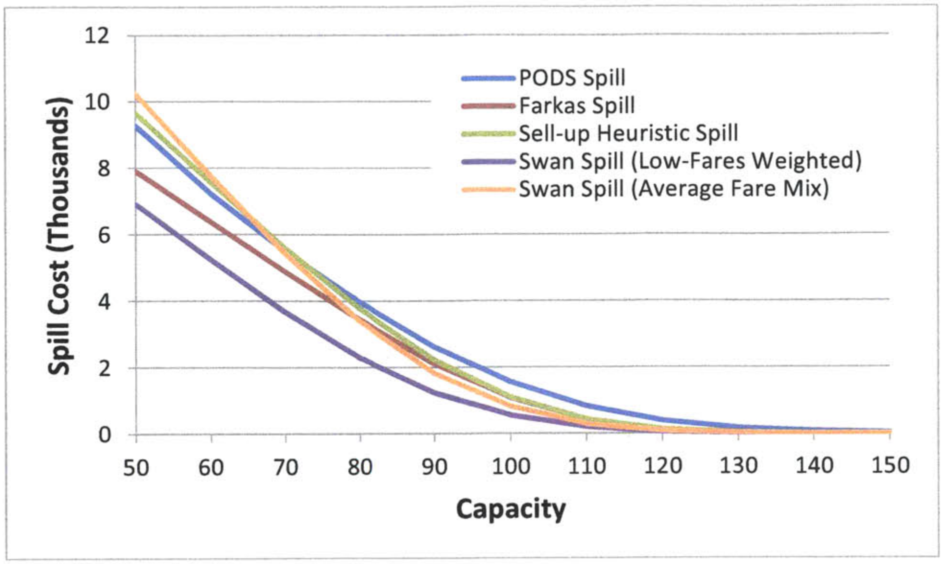


Figure 5.4: Average spill costs in fully differentiated fare structure test

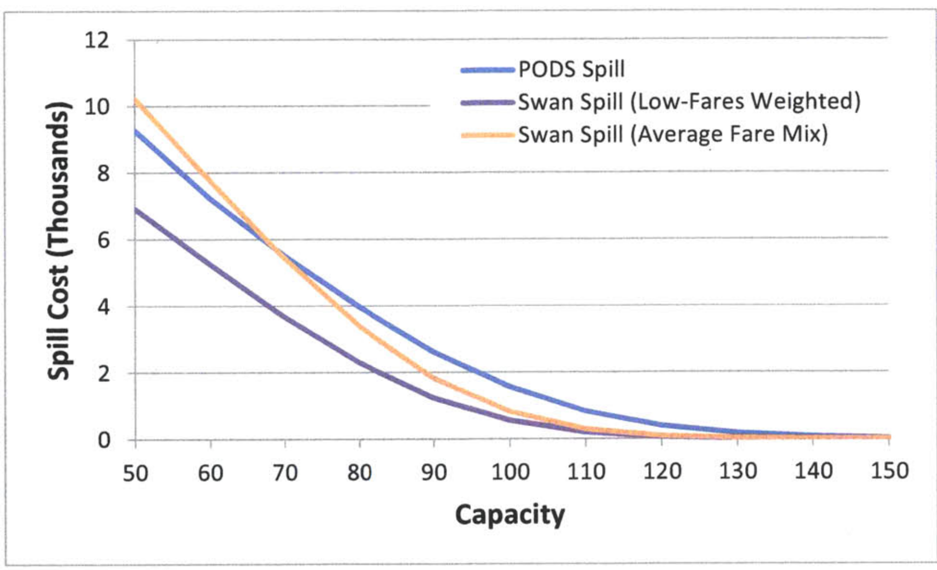


Figure 5.5: Average spill costs for Swan model in fully differentiated fare structure test

upper class passengers. This results in more spilled passengers than predicted by the Farkas and sell-up extended models which do not reoptimize. Such an effect is more pronounced at larger capacities where classes are less likely to close due to bookings reaching the average booking limits and more likely to close only in samples with higher than average demands.

The spill cost as a function of capacity for each method is shown in Figure 5.4. Recall that the spill cost is a measure of the value of passenger demand turned away, where the reduction in spill cost by adding one seat on a plane is the marginal revenue per seat of the incremental capacity.

As mentioned in Section 5.4, the Swan spill cost may be computed in two ways, using either the baseline average fare class mix to obtain the spill fare or else to use a spill fare which weights more heavily the lowest fare available for sale in the market. At the baseline capacity, the PODS-reported fare class mix, as shown in Figure 5.2, was (19.13, 8.43, 8.15, 13.46, and 35.9) for classes 1 through 5 and for a total of 85.07 passengers. The average of the fare mix then becomes $\frac{1}{85.07}\left[19.13(\$450)+8.43(\$350)+8.15(\$240)+13.46(\$190)+35.90(\$150)\right]=\$252$. This is the "average fare mix" spill cost and is shown in orange in the spill cost plot. Weighting the lowest fare more heavily gives the "low-fares weighted" spill fare as $0.8(\$150)+0.2(\$252)=\$170$. This spill cost is shown in purple in the spill cost plot. Using the average fare class mix $252 spill fare will result in a much higher estimate of the spill cost which, in general, would overvalue spill, since increasing or reducing capacity should largely only change the number of lower class passengers carried.

For clarity, the average spill cost for both Swan methods is reproduced in Figure 5.5. In this experiment, the average fare class mix spill fare better approximated the simulated spill cost at all capacities as compared to the low-fares weighted spill cost. At low capacities, passengers are spilling from all classes and weighting the low class too highly will result in error. In fact, in this test, weighting the low fare so highly is not appropriate – in the baseline spill by class shown in Figure 5.2, passengers are spilling from all classes even at the 100 seat capacity. For these reasons, the average fare class mix spill cost better approximates the simulated spill. However, the good performance of the average far mix spill cost could simply be the result of good fortune in this experiment (as the method does not take revenue management into account).

The sell-up heuristic estimates of the spill cost are in good agreement with the simulated spill costs at all capacities. The improved estimate of the spill cost is particularly evident when compared to the lower spill costs given by the Farkas model. In the fully differentiated fare structure, the difference in fare between class 5 and class 4 is only $40, such that many class 5 passengers are willing to pay the class 4 fare if class 5 is closed. This sell-up propensity is reflected in the high sell-up rate $p_5 = 67\%$ from class 5 to 4, such that two-thirds of passengers who do not get their first choices will sell up. Therefore, in addition to the first choice demand for class 4, there is demand from class 5 passengers who sold up and are assumed in the heuristic to occupy seats against the class 4 booking limit. With this additional demand, more class 4 passengers are spilled than would be in a first-choice-only model. This same process follows for all classes. Farkas therefore underestimates the spill cost by not taking into account passenger sell-up. The higher spill costs due to sell-up are also reflected in the higher marginal revenues per seat at low capacities for the full passenger choice test (with sell-up) as compared to the baseline first choice test (no sell-up modeled), as seen in Chapter 4.

To summarize the performance of the different spill models, the absolute error of the spill costs estimated by the models compared to the simulated PODS spill cost is shown in Figure 5.6. The absolute error is the difference between the spill cost estimate of each method and the simulated PODS spill cost. The error is positive for an overestimate of the spill cost and negative for an underestimate. Note as capacity becomes large, few passengers are spilled and all methods' absolute errors approach zero. Error comparisons are therefore valid at lower capacities and it is in any case more important to evaluate the performance of the spill models at lower capacities in which there is a moderate amount of spill.

At lower capacities where spill is moderate, the sell-up heuristic exhibits the lowest absolute error of the methods, on the order of approximately ±$500 for the lower capacities between 50 and 100 seats. This represents a relative error on the order of ±5%. The Farkas spill model, by comparison, underestimates the simulated spill cost by an amount on the order of –$1 000, an approximate –10% relative error. As capacity increases and more passengers get their first choice preferences, the Farkas and sell-up extended models report the same estimates of the spill cost and therefore exhibit the same error. Thus, in this test, when spill is moderate, it appears the sell-up heuristic reduces the error in the Farkas spill model by approximately a factor of 2.

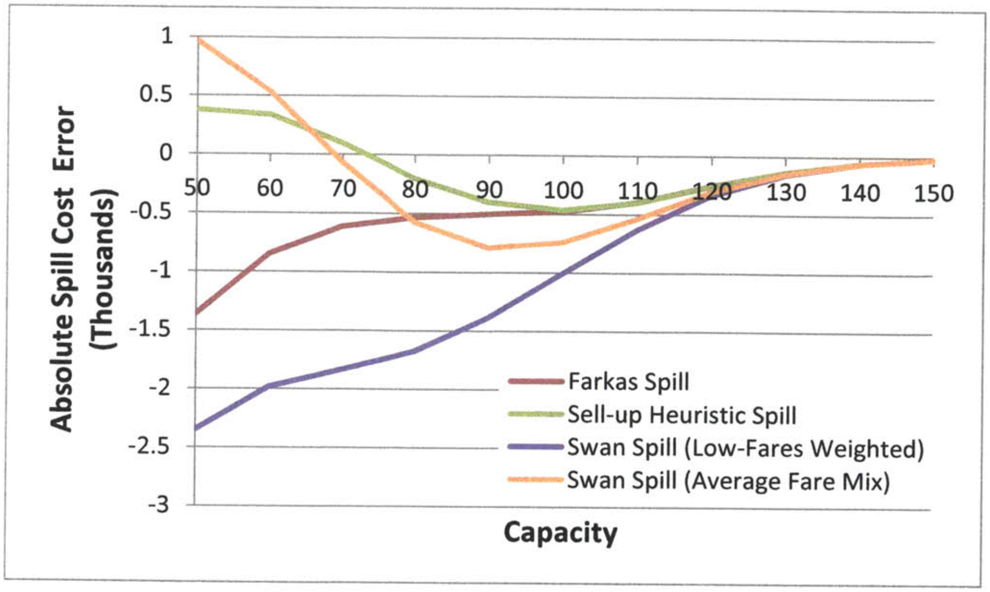


Figure 5.6: Absolute error in the spill cost of spill methods as compared to simulated PODS results for fully differentiated fare structure test

It should be noted that even at the baseline capacity, the models' spill cost estimates differ widely from one another, reflected in very different relative errors. The average fare class mix Swan approach (in orange) has an error of approximately –\$750 from the simulated results, a relative error of approximately –50%. The Farkas and sell-up extended models exhibit errors of approximately –\$500, approximate –30% relative errors (although the error of the sell-up heuristic is reduced at lower capacities). The large spill errors across models in the most basic fully differentiated fare structure speak to the challenge of the spill problem and the difficulty of accurately estimating the value of spilled demand.

5.6 Less Restricted Fare Structure Test

The sell-up heuristic was shown to be effective in the fully differentiated fare structure. Given the increasing prominence of less restricted fare structures, it is important to test the heuristic in a more recent less restricted fare structure, given sell-up makes up a large component of total revenue and buy-down reduces the marginal value of incremental capacity, as shown in Chapter 4.

Class	One-Way Fare	Advance Purchase	R1	R2	R3
1	$450	None	0	0	0
2	$350	3 days	0	1	0
3	$240	7 days	0	1	1
4	$190	14 days	0	1	1
5	$150	21 days	0	1	1

Figure 5.7: Less restricted fare structure

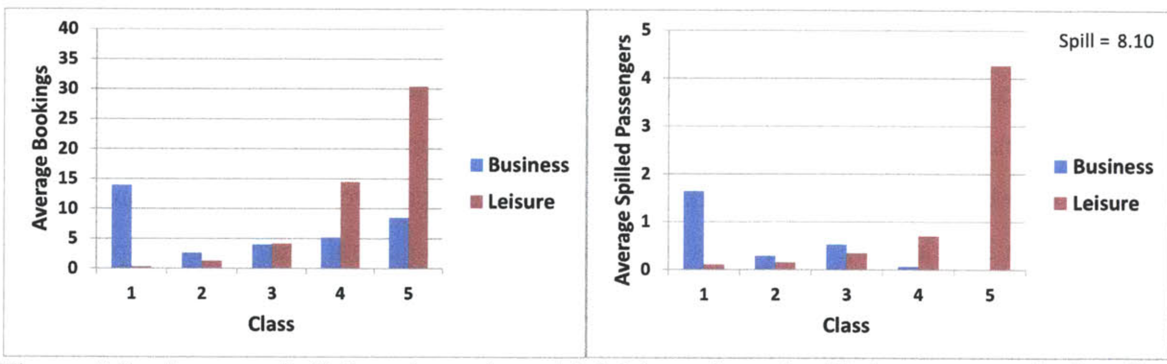


Figure 5.8: Average (*left*) fare class mix and (*right*) number of spilled passengers for baseline capacity in less restricted fare structure

The less restricted fare structure tested is shown in Figure 5.7 and parallels the structure used for the FPC-LR and LR-HFFA tests in Chapter 4. There is no R1 restriction on the fares. As in previous tests, demands are selected such that the baseline load factor is approximately 85%.

At the baseline capacity, the PODS-reported first choice demands, standard deviations, and sell-up rates by class k are

$$\mu_k = \begin{bmatrix} 14.79 & 3.64 & 7.16 & 8.55 & 59.06 \end{bmatrix}$$
$$\sigma_k = \begin{bmatrix} 6.59 & 2.67 & 3.92 & 4.21 & 16.20 \end{bmatrix}$$
$$p_k = \begin{bmatrix} 0 & 0.18 & 0.41 & 0.73 & 0.79 \end{bmatrix}.$$

The average fare class mix and average number of spilled passengers for the baseline capacity is shown in Figure 5.8. Given the lack of an R1 restriction, many business passengers buy down to the lower classes. On average, 8.10 passengers are spilled in the less restricted fare structure test at the baseline capacity. The baseline spill cost is $2 190, where the higher cost relative to the restricted fare structure is reflective of the increased demand level necessary to obtain an 85% load factor. For the Swan spill cost, the average fare class mix spill fare is $228, while the low-fare weighted spill fare is $166.

The average number of spilled passengers as a function of capacity for the less restricted fare structure is shown in Figure 5.9. Note the Swan spill (purple) and sell-up heuristic spill (green) have similar curves. In the less restricted fare structure, the sell-up heuristic underestimates spill at all capacities, although its estimates are improved at lower capacities when the number of spilled passengers increases (in fact, the heuristic agrees well with the PODS-reported spill at capacities of less than 50 seats, such as 30 and 40 seat capacities, although these are not shown here, with the idea that an airline is unlikely to reduce a 100 seat aircraft operating at an 85% load factor to a 30 seat one). The Farkas spill model, which in general overestimates spill (and does so here at lower capacities) in fact better approximates the number of spilled passengers than the sell-up heuristic in this experiment.

The average spill costs are shown in Figure 5.10. Here, both the low-fares weighted and average fare mix Swan models underestimate the spill cost, as does the Farkas spill model. In the less restricted fare structure, first choice demands for classes 2, 3, and 4 are very low. For classes 3 and 4, a passenger only has a first choice demand for these classes if arriving after the AP-closure of a lower class. Therefore, most passengers spilled in the Swan models and Farkas models are valued as class 5 passengers. Classes 2, 3, and 4 are populated largely by passengers selling up, so there are more bookings into these classes than the first choice demands would indicate. As such, more class 3 and 4 passengers are spilled than would be estimated by the Farkas model and the Swan models. In the Swan models, given that the average fare class mix is used to obtain the spill fare, the spill fare is too low as there are many more class 5 bookings at the baseline capacity than at other capacities. For clarity, the Swan model spill costs are

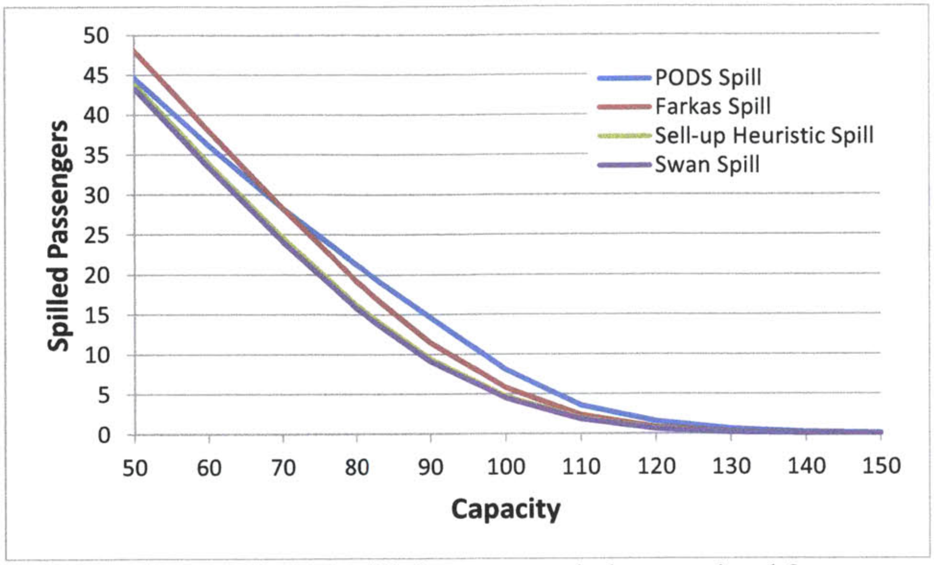


Figure 5.9: Average number of spilled passengers in less restricted fare structure test

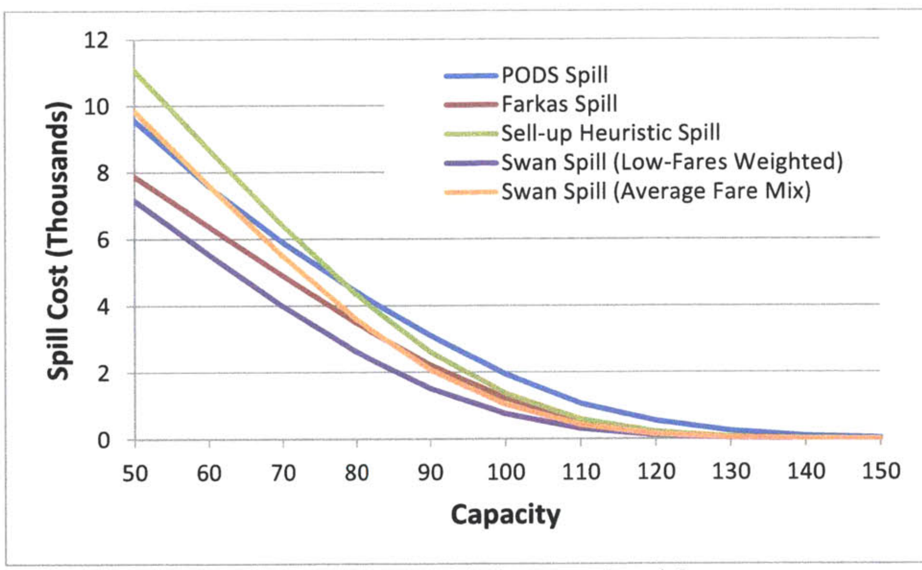


Figure 5.10: Average spill costs in less restricted fare structure test

reproduced in Figure 5.11. Here, given the spill fare estimate of the average fare class mix is already too low, discounting the spill fare even further by the increased weighting of the lowest fare results in a still worse estimate of the spill cost.

The sell-up heuristic, taking into account sell-up, performs much better than both the Swan and Farkas models. Of course, the heuristic sell-up spill cost estimate still has some error. Recall the heuristic is most effective when sell-up occurs only to the next highest class. In a less restricted fare structure, sell-up may occur across multiple classes, which is not modeled by the heuristic. Additionally, as in the fully differentiated fare structure, reoptimization at larger capacities introduces some error in the spill cost estimates. In any case, of the four spill cost estimates, the sell-up heuristic appears to be most accurate.

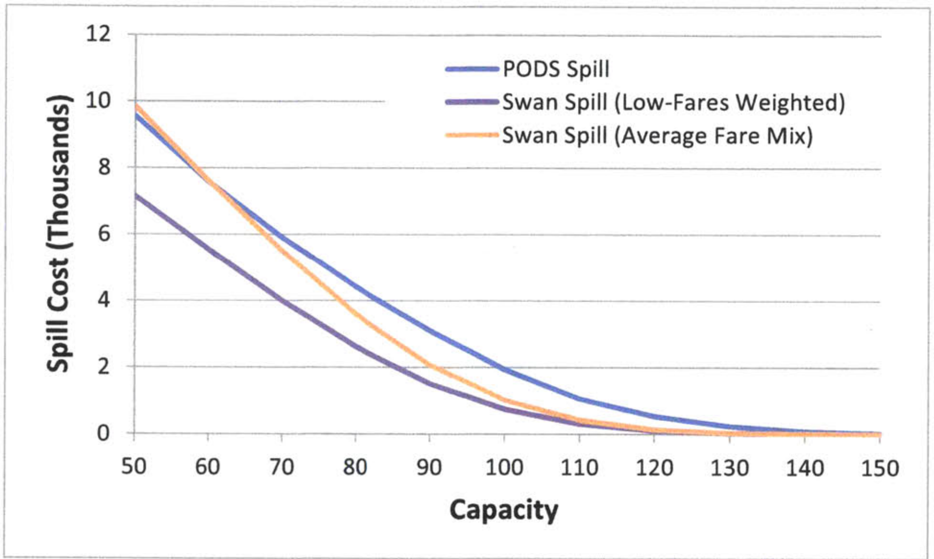


Figure 5.11: Average spill costs for Swan model for less restricted fare structure test

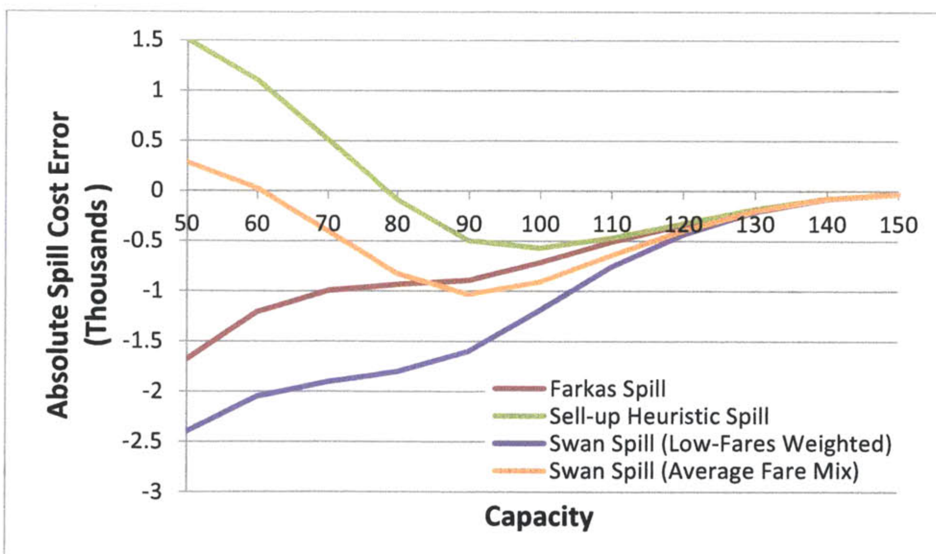


Figure 5.12: Absolute error in the spill cost of spill methods as compared to simulated PODS results for less restricted fare structure test

To summarize the performance of the different spill models, the absolute error of the spill costs estimated by the models compared to the simulated PODS spill cost for the less restricted fare structure is shown in Figure 5.12. The absolute and relative errors in the spill cost between the sell-up heuristic and the simulated spill are slightly larger in the less restricted fare structure test than in the fully differentiated fare structure test, given sell-up occurs between multiple classes, which the heuristic does not model. As in the fully differentiated fare structure, at lower capacities where spill is moderate, the sell-up heuristic has the lowest error (performing better than the average fare mix Swan spill over the range of 50 to 100 seats). The Farkas spill model exhibits a larger error and underestimates the spill cost at all capacities. As capacity increases and more passengers get their first choice preferences, the Farkas and sell-up extended models report the same estimates of the spill cost and therefore exhibit the same error. In the Swan model, weighting the lowest fare more heavily results in too-low estimates of the spill cost, while using the average fare class mix as the spill fare results in slightly better estimates.

It should be noted that the sell-up heuristic, as implemented, will not work under hybrid forecasting and fare adjustment (HFFA). In the tests here, the PODS-reported first choice demands are a function only of the fare structure, where changing the RM methods (to HFFA) will not change this structure. As such, tests with both standard forecasting with EMSRb and HFFA with EMSRb will report the same first choice demands, while the booking limits will be very different (the HFFA system, presumably, will reduce the booking limits of the lower classes to elicit more sell-up). In order to use the heuristic, the forecasts, instead of the first choice demands, must be used to generate the correct booking limits and sell-up rates. In such a case, the quality of the forecaster is likely to play as important a role as the quality of the sell-up heuristic in estimating spill costs.

5.7 Summary

This chapter proposed a heuristic spill estimator which extended the Farkas spill model to take into account passenger spill, applicable for use in a standard leg-based RM system. The heuristic limited sell-up from one class to the next higher class, but demonstrated improved spill cost estimates over the Farkas and Swan-extended Boeing spill models in several fare structures when compared to simulated spill costs from PODS. The heuristic was most effective in the fully differentiated fare structures when a moderate amount of passengers spilled, exhibiting a relative error of the spill cost on the order of 5%, a factor of 2 improvement over the Farkas model.

The chapter served to analytically capture some of the experimental results with regards to sell-up observed in Chapter 4. It is hoped that the sell-up heuristic presented could be the starting point for the further development of spill models which more completely take into account other passenger choices, such as recapture.

The chapter completes the discussion and analysis of the interactions between revenue management and spill. Chapter 6 serves as a brief conclusion to the thesis and outlines some possible directions for future research.

Chapter 6 – Conclusions

6.1 Summary of Thesis

In the airline industry, spill refers to passenger demand turned away because demand for a flight has exceeded capacity. However, given the presence of revenue management, passengers may be spilled because their desired fare classes have been closed, even if seats are available on the aircraft. Passenger spill is an opportunity cost to the airline because the revenue contribution of spilled passengers is not realized. This opportunity cost is termed the spill cost and is a measure of the marginal value of incremental capacity.

The number of spilled passengers, and their associated spill cost, has important applications in airline fleet assignment and manufacturers' aircraft sizing. In particular, the increased revenue potential from flying a larger plane in a market, that is, the reduction in the spill cost, must be traded off against increased operating costs of the larger aircraft in order to select an assignment that maximizes an airline's profits (or minimizes an airline's assignment cost). Chapter 1 explained the importance of accurate spill modeling, illustrated some shortcomings in current spill modeling, and presented the motivation for the importance of a complete understanding of airline spill.

The Boeing Spill Model was an early model for estimating the spill costs on a flight. The Boeing Spill Model assumes a single normal demand distribution for a flight and calculates the expected spill to be the portion of the distribution which exceeds the capacity of the aircraft. The model does not take into account the revenue management system protecting more seats for upper class passengers and restricting availability to lower class passengers, even though capacity for them may be available. It also does not take into account the revenue management system's effect on the value of spilled passengers, instead estimating that every spilled passenger pays some weighted average fare and that the spill cost is therefore this fare multiplied by the number of spilled passengers.

The Farkas spill model is a more recent model which incorporates an airline's revenue management system in estimates of the spill cost by spilling passengers of a given class whenever the booking limit of that class has been reached. However, the model assumes independent demands between fare classes and only models passengers' first choice preferences. Passenger sell-up to higher fares when unable to purchase a first choice fare is not modeled. Additionally, the Farkas spill model estimates spill on a single flight and does not take into account passenger recapture between several flights operating in a market. In general, the Farkas spill model will overestimate the number of spilled passengers because some passengers will choose to sell up to higher fares rather than be spilled. Chapter 2 explained existing spill models in further detail.

6.2 Summary of Contributions

The main contribution of this thesis is to present a complete picture of the interactions between revenue management, passenger choice, and the value of spilled demand for a sample flight leg. Additionally, the Farkas spill model was extended with a basic heuristic which takes into account passenger sell-up. This extended heuristic, together with the more complete spill picture constructed through the experimental tests, could become a starting point for future spill models which will result in more accurate estimates of passenger spill counts and costs.

The experiments conducted in this thesis explored the value of incremental capacity in several scenarios in order to provide a comprehensive overview of the effects of revenue management on passenger spill. The Passenger Origin-Destination Simulator was used as a simulation tool with which to conduct the tests and served to approximate the passenger ticket-buying process leading up to a flight departure. The Passenger Origin-Destination Simulator was explained in more detail in Chapter 3.

The research has led to several important findings which center on the fact that sell-up and recapture strongly affect the fare class mix and therefore the value of additional capacity as compared to more traditional first-choice-only spill models. In particular, when capacity on an aircraft is reduced, some passengers will sell up instead of spilling out and the spill cost is therefore less than would be predicted by a first-choice-only model.

One of the most important findings among the studies conducted in Chapter 4 is the possibility of revenue losses on a flight when capacity is increased. This loss can be present in less restricted fare structures where passengers may buy down to lower fares as the revenue management system releases more lower fare seats with the incremental capacity. Therefore, the marginal revenue per incremental seat can be negative as the revenue loss of passengers buying down to lower fares is larger than the revenue gain of carrying additional previously-spilled passengers. This revenue loss was found to be partially mitigated by using a more advanced revenue management system (with hybrid forecasting and fare adjustment).

Further, it was found that recapture played an important role in determining the marginal value of an additional seat. In particular, it was found that adding capacity to one flight could lead to negative marginal revenue on other flights, as passengers prefer to fly on flights at convenient times and bookings on flights departing at lesser convenient times will be reduced if capacity is added to more popular flights. In short, the inclusion of recapture couples an airline's revenues between its flights. Neglecting the effects of recapture therefore results in poor estimates of the marginal revenue of incremental capacity, given that spilled passengers from one flight may be recaptured on another.

The thesis also investigated the effects of additional capacity and revenue management in a competitive airline environment, where it was found that revenue was additionally coupled between airlines by spill-in and spill-out as well as between flights by recapture. An airline could lose or gain revenue depending on whether a competing airline increased or decreased capacity. This was due to spill-in and spill-out where many lower fare passengers chose to travel on the airline with higher capacity because it made more lower fare seats available for purchase. The complete spill picture has showed that the marginal value of a seat depends on many factors, including the schedule of other flights, the competition between market carriers, and the revenue management system and fare structure employed in the market.

The thesis also introduced a heuristic extension to the Farkas spill model which modeled passenger sell-up in Chapter 5. The method used input sell-up rates to identify a portion of demand for a given class that would sell up to seats in the next higher class. This heuristic sell-up extension led to improved estimates of the spill cost over the Farkas and Boeing spill models when compared to simulated spill costs from PODS simulation runs. The heuristic best performed in a fully differentiated fare structure in which most sell-up indeed occurred to the next highest class. In this case, for moderate levels of spill, the heuristic exhibited a relative error on the order of 5% when compared to simulated spill results. This was about a factor of 2 improvement over the Farkas spill model. Further work would be required in order to extend the heuristic so that it could be effective in a more advanced revenue management system, such as one using hybrid forecasting and fare adjustment.

6.3 Possible Directions for Future Research

While the research conducted within this thesis provided a comprehensive analysis of the effects of revenue management on spill, it did so only for a set of flights legs operating on a single route. Further research could take the direction of expanding the study to cover an airline network. In a larger network, the spill picture is further complicated as passengers travel on connecting itineraries between cities. As such, passengers can be recaptured by traveling on different paths and connecting across different intermediate cities.

Further, studying spill in the context of a network revenue management system could be important. Airlines are increasingly moving to network revenue management systems such that spill costs must be computed by considering all origin-destination itineraries a passenger may take. For example, if one additional seat is added to a flight from Boston to New York, the additional passenger carried may be terminating in New York, or else may be connecting to Miami, Houston, or anywhere else. The marginal revenue gain is not immediately obvious, and further research is required to generate spill estimates for connecting passengers.

Additionally, the sell-up extension for estimating spill costs presented in Chapter 5 could be further developed in order to work with more advanced revenue management systems. The model could also be extended to take into account recapture so that it could be applied to realistic scenarios in which airlines operate more than one flight between city-pairs. By building on the research findings in this thesis, airlines can continue to improve spill estimates and therefore increase profitability.

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