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Recent developments and research needs in modeling lane changing

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Abstract: This paper comprehensively reviews recent developments in modeling lane-changing behavior. The major lane changing models in the literature are categorized into two groups: models that aim to capture the lane changing decision-making process, and models that aim to quantify the impact of lane changing behavior on surrounding vehicles. The methodologies and important features (including their limitations) of representative models in each category are outlined and discussed. Future research needs are determined.

Keywords: Lane changing; Car following; Lane changing decision; Lane changing's impact; Driver behavior

1. Introduction

Car following (CF) and lane changing (LC) are two primary driving tasks observed in traffic flow, and are thus vital components of traffic flow theories. CF and LC rules describe vehicular longitudinal and lateral interactions on the road, respectively. Although CF has been widely studied for many years, LC did not receive much attention until recently. This is because of the increasing evidence of (1) LC's negative impact on traffic safety; and (2) its linkage to macroscopic traffic flow characteristics, as elaborated below.

LC's adverse impact on traffic safety has been frequently investigated and confirmed (Winsum et al., 1999; Mattes, 2003; Pande and Abdel-Aty, 2006; Zheng et al., 2010). Obviously, driver workload and stress are likely to significantly increase during the LC; this makes driving more error-prone, and thus, more dangerous. For instance, approximately 539000 two-vehicle lane change crashes occurred in the U.S. in 1999 (Sen et al., 2003).

LC's negative impact on traffic breakdowns and bottleneck discharge rate reduction at the onset of congestion (i.e., capacity drop) is reported in Cassidy and Rudjanakanoknad (2005). The significant roles played by LC in formation and propagation of stop-and-go oscillations have also been revealed (Kerner and Rehborn, 1996; Mauch and Cassidy, 2002; Ahn and Cassidy, 2007). More recently, Zheng et al. (2011), using high-resolution vehicle trajectories, further showed that LC is a primary trigger of oscillations and is responsible for transforming subtle, localized oscillations into substantial disturbances.

LC modeling

With the realization of LC's significant impacts on traffic safety and traffic congestion, efforts to model it have rapidly increased over the last decade. Briefly, LC is often distinguished as either 'discretionary' or 'mandatory'; this is because each of these LC types generally involves different decision making processes and has different impacts on surrounding traffic. The primary purpose of a discretionary LC is to gain a speed advantage or a better driving environment, whereas the primary motivation of a mandatory LC is to reach the planned destination. Meanwhile, the modeling efforts in the literature roughly fall into two themes: modeling the LC decision-making process (i.e., how a driver reaches the LC decision when facing conflicting goals), and modeling the LC's impact on surrounding vehicles (In this latter case, LC decision making is not specifically considered). Note that, for the convenience of discussion from this point on, the LC decision-making process is denoted as LCD, while the LC's impact on surrounding traffic is denoted as LCI. Meanwhile, LC is used for denoting the entire LC behavior consisting of both LCD and LCI.

Although there has been significant progress in modeling LCD and LCI, a complete understanding of LC, for the most part, remains elusive. Existing simulation packages that represent the state-of-the-practice are widely criticized as inadequate (Prevedouros and Wang, 1999; Hidas, 2005; Laval and Daganzo, 2006); e.g., the waiting time before a LC vehicle finds an acceptable gap can be unrealistically long so that upstream traffic in the same lane are totally blocked. A common strategy used in many simulation packages to suppress the impact of this inadequacy is to simply remove vehicles that have waited too long to execute the attempted LCs (TSS, 2002). Clearly, a traffic modeling tool that fully describes LC is still lacking. However, several serious issues in recent developments of LC modeling need to be resolved before any breakthrough can be achieved. These issues are discussed later in this paper.

The research gap

A comprehensive review of LC modeling which scrutinizes and summarizes notable endeavors and achievements in LC modeling (i.e., LCD and LCI), pinpoints the issues in need of further research, and inspires more efficient and effective methodologies for this research, is long overdue. Unlike CF modeling, which has been frequently reviewed (Chandler et al., 1958; Brackstone and McDonald, 1999; Hoogendoorn and Bovy, 2001; Brackstone et al., 2002; Nagel et al., 2003; Olstam and Tapani, 2004; Helbing, 2005), LC has not been comprehensively reviewed. Although there are two reviews of previous studies of modeling LCD (Toledo, 2007; Moridpour et al., 2010), the current review (that is, the review offered in this paper) of LCD models is more comprehensive and detailed.

More importantly, neither Toledo (2007) nor Moridpour et al. (2010) discussed models of the impact of lane changing. To the best of our knowledge, nobody has taken a holistic approach by reviewing studies in both LCD and LCI; this is a significant shortcoming, as both of these factors are critical to a complete understanding of LC. Even worse, LCD and LCI have not been clearly distinguished in the literature; this causes confusions, as evidenced by the fact that models of LCI are sometimes compared with those of LCD, and vice versa.

The purpose of this paper, therefore, is to address this gap. For the sake of clarity and focus, the paper concentrates on representative LC models in the literature, rather than attempting to exhaustively cover all existing models. Similarly, studies related to LC but not specifically to the modeling of LCD or LCI are excluded. In this author's view, understanding how previous studies have attempted to capture critical issues arising in real traffic is very important for moving forward. More specifically, for modeling LCD, critical issues are: the logic that governs the LC decision-making process, driver characteristics, and inevitable human uncertainties. For modeling LCI, critical issues are: how to (accurately) measure such impact and, consequently, how to integrate such a measure into a conventional CF model. Some general modeling issues include data requirements, model calibration and justification (statistically and empirically). In selecting and reviewing previous studies, these important issues have been paid special attention. Furthermore, to facilitate future research, limitations and research needs in modeling LCD, LCI and LC in general are respectively discussed in detail in the final two sections of the paper.

The remainder of this paper is organized as follows: Section 2 reviews major LCD modeling endeavors in the literature; Section 3 reviews major LCI modeling attempts; Section 4 discusses major issues arising from these previous attempts, and determines what future research is needed in the area; and Section 5 summaries main conclusions of this study.

2. Modeling LCD

This section reviews major studies of modeling LCD, focusing on logic of these models, factors that have significant roles in a driver's decision on whether or not to execute a LC, and rules that govern LCD. Note that models totally focusing on gap acceptance are not included in this review. For readers interested in these models, see Ahmed (1999) for a review. Meanwhile, incidents occurred at downstream can induce or force upstream vehicles to execute lane changes. Most models reviewed in this section can be (directly or indirectly) used to incorporate incident's impact (e.g., lane blockage, impact on speed in the current and/or adjacent lanes) into a driver's lane changing decision-making process by treating the incident as an obstruction.

For the convenience of discussion, a typical LC schematic is presented in Figure 1, where v_2 is the LC vehicle (i.e., the lane changer), the lane in which v_2 is currently travelling is noted as the initial lane, the lane to which v_2 will insert is noted as the target lane, v_1 and v_3 are the immediate preceding vehicle (i.e., the leader) and the immediate following vehicle (i.e., the follower) in the initial lane respectively, and v_4 and v_5 in the target lane are similarly defined. Note that different notations may be used in the literature. However, for the sake of consistency and clarity, the same notations depicted in Figure 1 are used throughout this paper.

Figure 1

The Gipps-type LCD models

Gipps (1986) was among the first to introduce a structure of LCD for drivers who face conflicting goals. The model covers various driving situations in an urban street context, in which a driver's behavior is governed by two basic considerations: maintaining a desired speed and being in the correct lane for an intended turning maneuver. A driver decides whether or not to change lanes by considering the possibility, necessity, and desirability of executing the change. More specifically, factors that impact LCD in Gipps' model include possibility of changing lanes without an unacceptable risk of collision, locations of permanent obstructions, the presence of heavy vehicles, the presence of special purpose lanes such as transit lanes, the driver's intended turning movement, the possibility of gaining a speed advantage, and etc. The relative importances of these factors are determined by a set of deterministic rules, which imply that each of the rules is evaluated sequentially according to its importance.

Thus, driver behavior in Gipps' model is considered as deterministic. Trade-offs among considerations, and variation among different drivers and inconsistency in a driver's behavior over time are ignored. In this deterministic model, depending on the distance to the intended turn, a driver decides to maintain the desired speed or prepare for the turning maneuver. More specifically, as the driver approaches to the intended turn, the priority shifts from maintaining the desired speed to staying in the correct lane for the turning maneuver. When more than one lane is acceptable as the target lane, the conflict is resolved deterministically according to a set of priority rules considering locations of obstructions, presence of heavy vehicles, potential speed gains, and etc. To test the soundness of the LCD structure, Gipps' model was configured to simulate drivers' LC decision-making processes under a scenario involving several features typical in the real world. And it was reported that Gipps model generated a consistent and realistic description of driver behavior under the Australian road condition.

Although Gipps' LC model was designed for use with Gipps' CF model (Gipps, 1981), the LCD structure proposed by Gipps is generic and can be used with other CF models if the follower's speed is bounded under an appropriate condition similar to as in Equation (1).

$$v_n(t+T) = b_nT + \{b_n^2T^2 - b_n[2\big(x_{n-1}(t) - s_{n-1} - x_n(t)\big) - v_n(t)T - \frac{v_{n-1}(t)^2}{\hat{b}}]\}^{1/2} \quad (1)$$

where $v_n(t+T)$ is the maximum safe speed for vehicle *n* with respect to the leader *n-1* at time t+T; b_n is the most severe braking the driver of vehicle *n* is prepared to undertake; $\hat{b}$ is an estimate of b_{n-1}employed by the driver of vehicle *n*; T is the time step of updating speed and position; $x_n(t)$ is the location of the front of vehicle *n* at time *t*; and s_{n-1} is the effective length of vehicle *n-1*.

After Gipps' pioneering work, many efforts ensued to either extend or improve his LCD modeling framework, as discussed in the remaining of this section. By extending Gipps' model to freeways, Yang and Koutsopoulos (1996) developed and implemented a LC model in the microscopic traffic simulator, MITSIM. They classified LC as either mandatory or discretionary, and modeled LCD as a sequential four steps: decision to consider a LC, choice of the target lane, and search for an acceptable gap, and executing the change. The gap acceptance algorithm examines the lead and follow gaps in the target lane for the execution of the desired lane change. As illustrated in Figure 1, the lead gap is the clear spacing between the front of the lane changer and the rear of the leader in the target lane; the follow gap is the clear spacing between the rear of the lane changer and the front of the follower in the target lane. The target (desired) lane has to meet several criteria including lane use regulation, lane connection, prevailing traffic condition, driver's desired speed, and etc.

Although the rule-based modeling framework in Yang and Koutsopoulos (1996) is similar to Gipps' model (1986), a distinct feature of their model is that, instead of treating LCD as a deterministic process, LC probability was introduced to make the model more realistic. More specifically, the probability for a driver to start a mandatory LC at a distance x_n from the downstream node (or incident, lane drop, and etc.) is defined in Equation (2):

$$p_n = \begin{cases} \exp\left[{(x_n - x_0)^2}/{\sigma_n^2} \right] & x_n > x_0 \\ 1 & x_n \leq x_0 \end{cases} \quad (2)$$

where p_n is the probability that vehicle *n* starts a mandatory LC; x_n is the distance from the downstream node or lane blockage point; x_0 is a critical distance; $\sigma_n = \alpha_0 + \alpha_1 m_n + \alpha_2 K$, where m_n is the number of lanes between the initial lane and the target lane; K is the traffic density of the segment; and α_0, α_1, and α_2 are parameters.

The gap acceptance model for a mandatory LC is defined in Equation (3):

$$\bar{g}_n^i = \bar{\varepsilon}_n + \begin{cases} \bar{g}_{max}^i & x_n \geq x_{max} \\ \bar{g}_{min}^i + \left(\bar{g}_{max}^i - \bar{g}_{min}^i\right) \frac{(x_n - x_{min})}{(x_{max} - x_{min})} & x_{min} < x_n < x_{max} \\ \bar{g}_{min}^i & x_n \leq x_{min} \end{cases} \quad (3)$$

where *i* = lead, or lag; $\bar{g}_n^i$ is the minimum acceptable gap for a mandatory LC; $\bar{g}_{min}^i$ and $\bar{g}_{max}^i$ are respectively lower and upper bounds; x_n is the vehicle's current position; x_{min} and $x_{max}$are distances corresponding to $\bar{g}_{min}^i$ and $\bar{g}_{max}^i$, respectively; and $\bar{\varepsilon}_n$ is an error term.

Meanwhile, for a discretionary LC, a driver first checks traffic conditions of both the initial and target lanes, and then uses several factors, such as an impatient factor and a speed indifference factor, to determine whether the speed difference between these lanes is big enough for considering LC. In their widely cited paper, Yang and Koutsopoulos (1996) did not define these parameters.

Like a mandatory LC, before a discretionary LC is executed, the gaps need to be checked to ensure that an acceptable gap as defined in Equation (4) is available.

$$\bar{g}_n^i = \bar{g}^i + \bar{\varepsilon}_n^i \quad (4)$$

where *i*=lead, or lag; $\bar{g}_n^i$ is the minimum acceptable gap for driver *n*; $\bar{g}^i$ is the average acceptable gap; and $\bar{\varepsilon}_n^i$ is an error term, whose distribution was not specified in Yang and Koutsopoulos (1996) but mentioned that the distribution is provided in the parameter file.

Neither formal estimation of the parameters nor validation of the model was conducted in Yang and Koutsopoulos (1996).

Another serious limitation of Gipps (1986) is the assumption that LC occurs only when it is safe, i.e., when a gap of sufficient size is available in the target lane. Therefore, the interaction between the lane changer and the follower in the target lane is minimal, which is unrealistic when traffic is heavy, congested, or impacted by an incident (Zheng et al., 2013). To overcome this limitation, Hidas (2002, 2005) proposed an improved modeling framework for both urban streets and freeways to capture the vehicular interaction induced by LC, which was explicitly classified into three categories based on the observations from video-recording footages: free, cooperative, and forced LCs. In a cooperative or forced LC, close interactions between the lane changer and the follower in the target lane occur, in which the follower slows down either reluctantly (i.e., in a forced LC) or willingly (i.e., in a cooperative LC) to create a sufficient space for the lane changer to insert. Factors that may have impact on the follower's decision of slowing down or not include the follower and the lane changer's aggressiveness, the follower's driving experience, the necessity and urgency, the mental state, the traffic conditions, and etc. Hidas (2002, 2005) assumed the follower to be willing to accept a certain maximum speed decrease and adopted the "time-to-end-of-lane" to reflect the urgency of LC. Other factors were ignored. The LC model was tested on two simple hypothetical road networks by using simulations: a freeway segment with an on-ramp and a weaving section. Average maximum speed decrease, minimum safe constant gap (which can be approximated as the jam gap) and acceptable gap parameter were estimated from the video footages. No detail was provided on how these parameters were personalized for individual drivers according to their aggressiveness or how other parameters were estimated and calibrated. The simulation results showed the consistence between the speed-flow curves generated by the LC model and the expected one (or the curve obtained by using the Highway Capacity Manual (Transportation Research Board, 1994)).

Kesting et al. (2007) proposed a novel logic for simplifying and modelling LCD, that is, the anticipated advantages and disadvantages of a potential lane change can be measured using single-lane accelerations, based upon which a LCD model, MOBIL ("minimizing overall braking induced by lane changes"), was developed. This review paper regards MOBIL as a variant of the Gipps-type models mainly because MOBIL is essentially governed by two LC rules, i.e., safety rule and desirability rule (incentive criterion in the original paper). The difference is that the rules in MOBIL are acceleration-based.

The safety rule in MOBIL is defined as:
$\tilde{a}_n \geq -b_{safe}$, where $\tilde{a}_n$ is the deceleration of the immediate follower in the target lane; b_{safe} is the safe limit.

For countries (e.g., Germany) where right lane is the default lane and the left lane should only be used for the purpose of overtaking, passing in the right lane is forbidden unless traffic is congested, the desirability rule in MOBIL neglect the disadvantage of the immediate follower in the right lane because the left lane has priority as explicitly defined below:

Left→Right: $\tilde{a}_c^{eur} - a_c + p(\tilde{a}_o - a_o) > \Delta a_{th} - \Delta a_{bias}$
Right→Left: $\tilde{a}_c - a_c^{eur} + p(\tilde{a}_n - a_n) > \Delta a_{th} + \Delta a_{bias}$
Where $a_c^{eur} = \begin{cases} min(a_c, \tilde{a}_c) \ if \ v_c > \tilde{v}_{lead} > v_{crit} \\ a_c \quad otherwise \end{cases}$, where a_c^{eur} is the acceleration in the right lane, $\tilde{a}_c$ is the acceleration in the left lane, $\tilde{v}_{lead}$ is the speed of the preceding vehicle in the left lane, v_c is the speed of the lane changer in the right lane, and v_{crit} is a critical speed below which traffic is congested; a_n and $\tilde{a}_n$ denote the acceleration of the immediate follower in the target lane before and after the lane change, respectively; a_o and $\tilde{a}_o$ denote the acceleration of the immediate follower in the initial lane before and after the lane change, respectively; p is a politeness factor and Δa_{th} is a threshold; Δa_{bias} is a constant bias.

For countries without such asymmetric lane usage rules (e.g., US), the desirability rule is:
$\tilde{a}_c - a_c + p(\tilde{a}_n - a_n + \tilde{a}_o - a_o) > \Delta a_{th}$

Where a_c and $\tilde{a}_c$ denote the lane changer's acceleration before and after the lane change, respectively; other variables are the same as previously defined.

As indicated in the lane changing rules above, MOBIL considers the advantage or disadvantage of the followers via a politeness factor. By adjusting this parameter, the motivations for lane changing can be varied from purely egoistic to more cooperative behaviour, e.g., increasing the combined accelerations of the lane changer and affected neighbours.

Using accelerations in MOBIL has two main advantages: 1) the lane change decision-making process is dramatically simplified, which leads to the parsimoniousness of MOBIL; and 2) accelerations can be readily calculated with an underlying microscopic longitudinal traffic model, which enables MOBIL to be easily integrated with a typical CF model. However, the logic of MOBIL has yet to be empirically justified, and MOBIL itself has yet to be calibrated and validated.

Utility theory based LCD models

Ahmed et al. (1996) adopted utility theory to model the decision process of LC. The proposed LCD structure consists of four latent (i.e., unobservable) levels of decision hierarchy, which is similar to the four sequential steps in Yang and Koutsopoulos (1996). Their models also considered driver heterogeneity and state dependence (The current choice's dependence on previous driving experiences and LCD). The utility of LC at time *t* for driver *n* is defined as in Equation (5):

$$U_{tn} = \gamma^T X_{tn} + v_n + \varepsilon_{tn} \quad (5)$$

where U_{tn} is the utility for driver *n* at time *t*; X_{tn} is a vector of explanatory variables; γ is a vector of unknown parameters; v_n is an individual-specific random term; ε_{tn} is a random term that varies across different time period for a given individual, as well as across individuals. The conditional probability of observing a LC pattern for driver *n* can be expressed as in Equation (6):

$$\Pr(\{J_{1n}, J_{2n}, \dots, J_{tn}, \dots, J_{T_n n}\} | v_n) = \prod_{t=1}^{T_n} \Pr(J_{tn} | v_n)$$

$$= \prod_{t=1}^{T_n} Pr_t(L|v_n)^{\delta_{tn}^L} Pr_t(R|v_n)^{\delta_{tn}^R} Pr_t(C|v_n)^{\delta_{tn}^C} \qquad (6)$$

where $J_{tn} \in \{L, R, C\}$; *L*: change to the left lane; *R*: changing to the right lane; *C*: continuing in the current lane;

$$\delta_{tn}^L = \begin{cases} 1 \; if \; driver \; n \; changes \; to \; the \; left \; lane \; at \; time \; t \\ 0 \qquad\qquad otherwise \end{cases}$$

δ_{tn}^R and δ_{tn}^C are similarly defined.
The likelihood function is given by

$$L^* = \prod_{n=1}^{N} \int_{-\infty}^{\infty} \left[\prod_{t=1}^{T_n} Pr_t(L|v_n)^{\delta_{tn}^L} Pr_t(R|v_n)^{\delta_{tn}^R} Pr_t(C|v_n)^{\delta_{tn}^C} \right] f(v_n) dv_n \qquad (7)$$

where $f(v_n)$ is the distribution of v_n and *N* is the sample size.

Ahmed et al. (1996) estimated the parameters of the model for a special case using vehicle trajectories: merging to the left lane from a freeway on-ramp, which only requires two decision levels, i.e., gap acceptance and LC execution. They found that for the data they used, unlike the lead gap, the critical follow gap was sensitive to traffic conditions. Possible impact of the previous LCD on the current decision was not considered because the number of possible state sequences increases exponentially with the number of observations, which is computationally prohibitive.

Ahmed (1999) extended the mandatory LC model to specifically accommodate heavily congested traffic, where forced merging behaviors frequently occur because of lacking of normally acceptable gaps. A simplified version of the LCD structure proposed in Ahmed et al. (1996) was used, in which the decision process for a forced merging LC involves two levels: intention of merging into the target lane, and perception on the establishment of a mutual understanding on right of way, i.e., an understanding between v_2 in the initial lane and v_5 in the target lane has been reached such that v_5 would allow v_2 to insert (See Figure 1). This two-level decision process is evaluated at every discrete time point and the forced merging begins (this state is denoted as M) if (1) the intention of merging is valid and (2) v_2 believes that the right of way is given by v_5. Otherwise, v_2 remains in the initial lane (this state is denoted as $\bar{M}$) and the evaluation/communication process continues.

Mathematically, the forced merging model is defined as in Equation (8):

$$P\{S_n(t) = M | S_n(t-1) = \bar{M}, v_n\} = 1 / \left(1 + e^{-X_n(t)\beta - \alpha v_n}\right) \quad (8)$$

where $P\{S_n(t) = M | S_n(t-1) = \bar{M}, v_n\}$ is the probability of switching to state M from state $\bar{M}$, conditional on v_n, which is the random term for each driver and assumed to capture the correlation between the utilities of different states at different times; X_n is a vector of important explanatory variables, including the leader's relative speed (only when the leader is slower); the follower's relative speed; remaining distance to the point by which LC must be completed; delay; total clear gap; indicator for heavy vehicles, and etc.

The unconditional likelihood function of observing a LC at time T_n is given by Equation (9):

$$\mathcal{L}_n(\beta, \alpha) = \int_{-\infty}^{\infty} \left(\sum_{t=1}^{T_n} P_n\{state\ sequence_t | v\} \times \delta_n(t)\right) f(v) dv \quad (9)$$

And if the observations for different drivers in the sample are independent, the log-likelihood function for all observations is given by equations (10-11):

$$\mathcal{L}_n(\beta, \alpha) = \sum_{n=1}^{N} \ln \left\{\int_{-\infty}^{\infty} \left(\sum_{t=1}^{T_n} P_n\{state\ sequence_t | v\} \times \delta_n(t)\right) f(v) dv\right\} \quad (10)$$

$$\delta_n(t) = \begin{cases} 1\ if\ the\ \ gap\ at\ time\ t\ is\ the\ same\ gap\ driver\ n\ ultimately\ selected \\ 0 \qquad otherwise \end{cases} \quad (11)$$

Although the meaning of $\delta_n(t)$ was graphically explained (Figure 4-7 in Ahmed (1999)), it is unclear how to determine its value in the implementation.

This model was calibrated using vehicle trajectories collected from I-93, Boston, US. However, the discretionary LC model and the mandatory LC model were estimated separately due to the lack of data.

Although theoretically the discretionary LC and the mandatory LC can be distinguished from each other (i.e., as previously discussed, the discretionary LC is to gain a speed advantage or a better driving environment, whereas the mandatory LC is to reach the planned destination), boundaries between them are sometimes vague. For example, when considering a mandatory LC, a driver may decide to overtake a heavy vehicle in front first (i.e., executing a discretionary LC first). However, a rigid separation between the mandatory LC and the discretionary LC remains in the models mentioned above, which implies that these models fail to capture the trade-offs between mandatory and discretionary conditions (Toledo et al., 2003). Moreover, the mandatory LC situation is not always perceived by the driver (except for special cases like on-ramp merging traffic). Hence, the conditions that trigger a mandatory LC have not been estimated in the models above. To overcome this problem, Toledo et al.

(2003) proposed an integrated LC model where mandatory and discretionary conditions were joined together in a single utility model. The model captured the trade-offs between the utility of being in the correct lane (i.e., the mandatory LC consideration) and that of the speed advantage offered by a faster lane (i.e., the discretionary LC consideration). The model also considered a driver specific random term that represents unobservable characteristics of the driver and correlations between observations of the same driver over time. Parameters of the model were jointly estimated using vehicle trajectories collected from I-395 Southbound, Arlington, Virginia, US. And results showed the importance of incorporating trade-offs between the mandatory and the discretionary LC into the model.

Although driver characteristics (e.g., level of aggressiveness, alertness) naturally have significant impact on various aspects of lane change decision making process, they are missing from most of the existing LCD models. To explicitly incorporate the effect of driver characteristics, Sun and Elefteriadou (2011) conducted a focus group study to identify and understand drivers' concerns and responses under various lane-changing scenarios. From the focus group study, driver types, and reasons and main factors for each driver type in lane changing decision-making processes were revealed and linked. To observe drivers' actions under various lane-changing scenarios, and to obtain field-measured values for the important factors identified in the focus group study, field data were collected using instrumented vehicles (Sun and Elefteriadou, 2012).

Data from the focus group study and from the field were utilised to develop more realistic LCD models. More specifically, the lane-changing probability for each discretionary LC scenario was modelled as a function of corresponding important factors and driver types. For example, for the "stopped bus" scenario, five important factors were identified from the focus group study: traffic congestion in the target lane (Cgst); Queue ahead (Que); Location of the next downstream stop (LocStop, mile); Distance to the bus (Dist, feet); and number of persons at the bus-stop (NPson). The corresponding utility function of changing lanes is

$$V(LC) = \beta_0 + \beta_1 * Cgst + \beta_2 * Que + \beta_3 * LocStop + \beta_4 * Dist + \beta_5 * NPson + \alpha_1 * DriverTypeA + \alpha_2 * DriverTypeB + \alpha_3 * DriverTypeC$$

Where driver types (Type A, B, C) are dummy variables.

The proposed lane-changing model was implemented in a microscopic traffic simulator, CORSIM. Traffic data were collected along a congested arterial in the City of Gainesville, FL, and used for model calibration and validation purposes. Performance of the proposed model was compared against the original lane-changing model in CORSIM, using three measures (i.e., average lane-based travel time, lane distribution, and cumulative lane changes). It was reported that the driver characteristics based model better replicated the observed traffic under different levels of congestion (Sun, 2009).

Cellular automata based LCD models

(1) Introduction

Cellular automata (CA) were historically proposed in the 1940s (Neumann, 1948) and popularized in the 1980s (Wolfram, 1983) to accurately reproduce macroscopic behavior of a complex system using minimal microscopic descriptions. A typical CA model constitutes four key components: the physical environment, the cells' states, the cells' neighborhoods, and local transition rules, as denoted in (12).

$\mathrm{CA} = (\zeta, \Sigma, \mathcal{N}, \delta)$ (12)

where ζ is for the physical environment represented by the discrete lattice; Σ is for the set of possible states; $\mathcal{N}$ is for the neighboring cells; and δ is for the local transition rules, which are commonly given by a rule table.

CA models have been frequently applied in various fields, including traffic flow modeling. Several notable traffic CA (TCA) models were developed for reproducing CF & LC behaviors, such as single-cell models, multi-cell models, deterministic models (e.g., Wolfram's rule 184 (Wolfram, 1983), deterministic Fukui-Ishibashi TCA (Fukui and Ishibashi, 1996)); Stochastic models (e.g., Nagel-Schreckenberg TCA (Nagel and Schreckenberg, 1992; Nagel, 1995), STCA with cruise control (Nagel and Paczuski, 1995), Stochastic Fukui-Ishibashi TCA (Fukui and Ishibashi, 1996)); Slow-to-start models (e.g., Takayasu-Takayasu TCA (Takayasu and Takayasu, 1993)); Velocity-dependent randomization TCA (e.g., Barlovi´c et al., 1998; Barlovi´c, 2003)). To demonstrate the setup of a typical TCA, a single-cell CA model using Wolfram's rule 184 (which is defined later) for a single lane road is presented here. For other models, see excellent reviews by Chowdhury et al. (2000), Knospe et al. (2004), Nagel (1996), Schadschneider (2000; 2002), Schreckenberg et al. (2001), and Maerivoet and Moor (2005).

The physical environment of applying CA for modeling traffic flow is obviously the road segment of interest, which consists of a one-dimensional lattice for a single lane road. The lattice and the time are discretized into equal-length cells typically equal to the vehicle length and the driver's average reaction time, respectively. The corresponding speed increment is computed as $\Delta x/\Delta t$. The state of each cell can be 0 (empty) or 1 (occupied) with two implicit assumptions: typically each cell is exactly occupied by one vehicle, and drivers cannot react to any events between consecutive time steps.

Similar to traditional traffic flow theories, the longitudinal movements of individual vehicles in TCA are also governed by CF. In fact, TCA is closely connected to traditional traffic flow theories. For example, a TCA can be derived from Gipps' CF model (Gipps, 1981); and Daganzo (2004) proved two TCA models' equivalence to the kinematic wave model with a triangular fundamental diagram.

CF in TCA is represented by a set of local transition rules. By applying the local transition rules to all vehicles either in parallel or in sequence, their states are updated with new speeds & positions. Wofram's rule 184 essentially consists of two rules: rule for acceleration and braking as shown in Equation (13), and rule for vehicle movement as shown in Equation (14).

$$v_i(t) \leftarrow min\{g_{s_i}(t-1), 1\} \quad (13)$$

$$x_i(t) \leftarrow x_i(t-1) + v_i(t) \quad (14)$$

where v_i and x_i are speed and position of vehicle *i*, respectively; $\mathrm{g}_{\mathrm{s_i}}$ is the net space gap (measured by number of cells) between the vehicle *i* and the leader. Obviously, if $\mathrm{g}_{\mathrm{s_i}} = 0$, vehicle *i* has to stop to avoid collision, i.e., $v_i(t) = 0$.

In Equation (13), the maximum speed is 1 cell per time step, which can be relaxed to enable a vehicle's maximum speed up to several cells per time step. The rule defined in Equation (14) is self-explanatory.

(2) TCA for modeling LCD

TCA were extended to accommodate multi-lane traffic streams by adding LC rules to CA-based CF models discussed above. Chowdhury et al. (2000) and Maerivoet and Moor (2005) provided excellent reviews on CA-based LC models. Like the traditional LC models, CA-based LC models also consist of two basic steps: desirability and/or necessity of LC and gap acceptance. For a road segment with L_ς lanes and K_ς cells per lane, a successful LC must satisfy the following constraints:

$$g_{s_i}^l \geq 0 \wedge g_{s_i}^f \geq 0$$

where $g_{s_i}^l$ and $g_{s_i}^f$ are the lane changer (i)'s net space gaps with the leader and the follower in the target lane, respectively. Note that all these space gaps are measured as number of cells. The implementation of a TCA model with a LC module follows two essential steps: moving vehicle laterally by executing the LC module and then moving vehicle longitudinally by executing the CF module.

One of the first CA-based LC models was proposed by Nagatani (1993; 1994) based on the deterministic Wofram's rule 184. Rickert et al. (1996) improved Nagatani's model by incorporating a stochastic term. Further improvements were obtained by Wagner et al. (1997) and by Nagel et al. (1998) to capture the lane usage inversion phenomenon* observed in Germany roads. To avoid scheduling conflicts, which may occur for a road with three lanes or more when more than one vehicle intends to insert to the same cell, two schemes were suggested by Maerivoet and Moor (2005): randomly choosing a winner from competing vehicles, or considering a left-to-right LC and a right-to-left LC alternately at consecutive time steps. Maerivoet and Moor (2005) neither evaluated these schemes nor cautioned any potential consequence of these schemes. Meanwhile, to reproduce the phenomenon that a small size vehicle (e.g., motorcycle) can pass the leader in the same lane, the longitudinal multi-cell concept (i.e., a vehicle may occupy more than one cell longitudinally) was extended to a lateral multi-cell structure by allowing a cell width smaller than the lane width (Gundaliya et al., 2004; Mallikarjuna and Rao, 2005). Consequently, more than one (small size) vehicle may occasionally travel side by side on the same lane. However, this approach was criticized by Maerivoet and Moor (2005) for 'unnecessary complexity'.

Like other approaches used to model LC, CA-based LC models face several challenges. First of all, CA-based LC models are artifact-prone. One obvious artifact induced by these models is that the LC duration is implicitly fixed as the length of one time step (typically not longer than 1 s), which is unrealistically short and inconsistent with observations (Zheng et al., 2013). Another reported artifact is the infamous ping-pong traffic that vehicles laterally bounce back and forth between lanes without forward moving (Nagatani, 1993; Nagatani, 1994). Although most of these reported artifacts can be avoided, e.g., the ping pong phenomenon can be eliminated by randomizing decisions of LC (Nagatani, 1994), vehicle movements need to be scrutinized before any conclusions should be drawn whenever CA-based LC models are used.

Meanwhile, despite the macroscopic nature of the CA-based models, the temptation to develop a CA-based LC model with the capability of reproducing microscopic characteristics of LC is strong, which leads to unnecessary complexities. As recommended by Maerivoet and Moor (2005), the CA-based models may be extended to accommodate the following heterogeneities of traffic: vehicle lengths, maximum speeds, acceleration characteristics, anticipation levels, and stochastic noise of distinct classes of vehicle/drivers, while other elements should not be considered. Along the same lines, the CA-based LC models should be evaluated at a macroscopic level, e.g., evaluating the LC frequency, the lane-wise capacity and the combined capacity, the critical density, and etc.2.4 Scenario-based LCD models

* The passing lane becomes more crowded than the one for slower vehicles when traffic flow is high. This phenomenon is frequently observed on freeways in countries like Germany where passing is only allowed on a dedicated lane (Wagner et al., 1997).

Markov process based LCD models

LC has also been modeled as a Markov process. The first Markov-based LC model was perhaps proposed in Worrall et al. (1970), where a stochastic LC model was developed as a homogeneous Markov chain and calibrated using data collected on a section of 6-lane freeway in Chicago.

In a broader context of treating human as a device with a large number of unobservable internal mental states, Pentland and Liu (1999) modeled the driving behavior using a Markov dynamic model. LC experiments using driving simulator were used to demonstrate the soundness of the proposed modeling framework. LC was broken down into a chain of states: (1) a preparatory centering the car in the initial lane; (2) looking around to make sure the target lane is clear; (3) steering to initiate LC; (4) the change itself; (5) steering to terminate the change; and (6) a final re-centering of the car in the target lane. Results supported the view that human actions are best described as a sequence of control steps rather than as a sequence of raw positions and velocities. In the case of driving, this means that the action is defined by the pattern of acceleration and heading.

Sheu and Ritchie (2001) modelled the mandatory LC induced by incidents using the Markov process. All state variables in the stochastic system follow homogeneous Gaussian-Markov processes. Unlike the previous studies (e.g., Worral et al, 1970) in which stable traffic conditions were often assumed to justify the use of a time-invariant transitional LC probability, a noise term which follows a Gaussian process was introduced to accommodate time-varying traffic conditions that are caused by incidents.

To estimate traffic densities using loop detector data for roads where LCs are frequent, Singh and Li (2012) incorporated a Markov chain into the state space model to describe the LC behaviour. In the Markov chain process, each lane was characterized as a state of the process. Specifically, each vehicle in the roadway segment was assumed to stay in the current state (lane) or to change from one state (lane) to another with a certain probability. The transition probabilities for LC were further assumed to remain approximately constant over time in the stable traffic flow. The LC procedure was not broken down to any sub-states because for their research purpose, obtaining accurate vehicle counts from each loop detector is more important than understanding the decision process of LC.

The models described above in this section aimed to reproduce LC frequency but cannot explain the decision process: why or why not LC occurs. Thus, they are not suitable for microscopic simulations. This limitation is overcome in Toledo et al. (2009) by integrating a hidden Markov model (HMM) with the utility theory based modelling framework discussed in Section 2.2, as elaborated below.

Similar to other LC models developed by Toledo and his collaborators, an individual-specific error term in all components of the model was included to capture correlations among the decisions made by the same driver across choices and over time. However, the target lane choice, which is unobservable, was modelled as a HMM by assuming the state dependence. The gap acceptance model connects the HMM and the observable outcome (i.e., the lane in which the vehicle is moving at any time step). Meanwhile, other factors related to driving goals, personal characteristics and surrounding traffic were also considered. More specifically, the state dependence was captured in the lane choice model by introducing into the utility function a parameter that represents the strength of the state dependence in the lane choice. Using vehicle trajectories collected at I-395 southbound in Arlington, Virginia, all parameters of the model were estimated jointly using the maximum likelihood method, and a positive and significant state-dependency coefficient was reported, which indicates that the lane that was chosen as the target lane at the previous time step has a larger utility.

Furthermore, the model with the HMM was compared with the model without HMM and result of the likelihood ratio test revealed a significant gain by considering the state dependency (Toledo et al., 2009).

Hazard-based (survival) LCD models

Hamdar (2009) criticized previous LC models for neither sufficiently nor explicitly considering stochasticity and possibly unsafe character of the cognitive processes (e.g., perception, judgment and execution) followed by drivers. Thus, a hazard-based duration model was proposed. Unlike rule-based LC models, the hazard-based duration model treats driver behaviors as a multiple duration process: Free flow, CF, or LC. Three parametric hazard functions were adopted in Hamdar (2009): the increasing monotonic dependence; non-monotonic dependence; and the third one was based on an increasing positive correlation between duration and hazard before a given time, followed by a constant hazard value. The proportional hazard form was employed to accommodate effects of exogenous factors (e.g., headways, speed, speed difference, and etc.). Driver heterogeneity was considered using a Gamma distribution as shown in Equation (15).

$$\begin{aligned} Prob(t_i = k|w_i) &= G(\delta_k - \beta' x_i + w_{hi}) - G(\delta_{k-1} - \beta' x_i + w_{hi}) \\ &= \exp\left[-\left\{I_{i,k-1}\exp(w_{hi})\right\}\right] - \exp\left[-\left\{I_{i,k}\exp(w_{hi})\right\}\right] \end{aligned} \quad (15)$$

where $I_{i,k} = \wedge_0 (u^k)\exp(-\beta' x_i)$; w_{hi} represents the unobserved heterogeneity and $\exp(w_{hi})$ is assumed to have a Gamma distribution with a mean of 1 and a variance σ^2. $\wedge_0$ is the integrated baseline hazard. $Prob(t_i = k)$ is the probability that driver *i* ends the CF process in the discrete time period k.

Equation (15) was expanded to accommodate the fact that multiple types of events may end a CF, LC, or free-flow process. Two strategies were discussed: (1) the utility-based strategy: each potential event that ends a particular state (termed as an exit strategy in Hamdar (2009)) is considered as an alternative with a given utility. Then the appropriate exit strategy is determined according to these utilities; and (2) the hazard-based strategy: instead of using utilities to determine the appropriate exit strategy, the exit strategy with the associated highest hazard is selected. It is not clear which strategy was employed in their study. The NGSIM vehicular trajectories (Alexiadis et al., 2004) were used to calibrate and validate their model.

Fuzzy logic based LCD models

Several fuzzy logic based LCD models were developed (McDonald et al., 1997; Brackstone et al., 1998; Das et al., 1999; Wu et al., 2000; Moridpour et al., 2009). The overall structure of fuzzy logic based LCD models is similar to the LCD models discussed above (e.g., LC often categorized as either mandatory or discretionary; Two essential decisions are often considered: desirability (or necessity) of LC and gap acceptance), except that the LCD rules are fuzzified as *IF-THEN* rules and presented in a natural language. The following is a typical *IF-THEN* LC rule:

IF: (vehicle i is eligible for using the left lane) and (the gap between vehicle i and the leader in the left lane is large) and (the gap between vehicle i and the follower in the left lane is large) and (the speed in the current lane is low) and (the speed in the left lane is high)

THEN: (vehicle i changes to the left lane).

Technically speaking, all the models discussed above can be fuzzified. For example, Yeldan et al. (2012) proposed a TCA model based on fuzzy logic.

Using fuzzy logic is often reported to be capable of better mimicking a driver's actual decision process because fuzzy logic is well equipped to handle human's cognitive and perceptional uncertainties frequently encountered in real-world LC processes (Brackstone et al., 1998). However, among many issues, defining fuzzy sets and their associated membership functions are challenging (Ross, 2010), which consequently makes calibrating and validating fuzzy logic based LCD models extremely difficult.

LCD models using other intelligent algorithms, e.g., Neutral Network (Hunt and Lyons, 1994), game theory (Kita, 1999; Kita *et al.*, 2002) are not reviewed here because of the high complexity in training, implementing, and interpreting these models.

3. Modeling LCI

The models discussed so far primarily tackle LCD, and by and large ignore LC's impact on surrounding vehicles (LCI). Such impact is significant as reported by several empirical studies that examined microscopic features of a LC, particularly the transition process during a LC maneuver. This transition process arises as the (equilibrium) CF states of the lane changer and the follower are disrupted due to LC and then recover gradually, during which the involved vehicles are willing to accept spacings much shorter than the equilibrium ones and then relax to a normal spacing (the so-called relaxation phenomenon). Smith (1985) first reported that the transition typically persists for 25 seconds. Wang and Coifman (2008) and Ma and Ahn (2008) reported similar findings. In a more recent study, Zheng et al. (2013) detected three distinct effects of LC on the follower in the target lane: anticipation (i.e., the transition of the follower in the target lane after the lane changer's intention is noticed and before the lane changer inserts), relaxation (as previously explained), and the regressive effect on driver behavior (i.e., LC "neutralizes" the follower's behavior by encouraging a timid (aggressive) driver to become less timid (aggressive)). Not surprisingly, researchers (Cassidy and Rudjanakanoknad, 2005; Laval and Daganzo, 2006) have postulated the linkage between LC's complex impact on the surrounding traffic and some long-puzzling traffic phenomena, such as breakdown, capacity drop, traffic oscillations, and etc.

CF models are not capable of describing LCI because these vehicles involved in LC are in non-equilibrium states. Using CF models, a sharp reduction of the spacing between the leader (the lane changer) and the lane changer (the follower) will lead to an emergency braking, which is often not the case in real traffic (Hidas, 2005; Laval and Leclercq, 2008).

Evidently, understanding LCI is critical for modeling the full spectrum of LC. Thus, the important models that mainly attempted to capture LCI are reviewed in this section.

Models by Laval and Daganzo (2006), Laval and Leclercq (2008) and others

Extending the celebrated kinematic wave (KW) theory (Lighthill and Whitham, 1955; Richards, 1956) to incorporate LC is a natural idea and has been attempted since 1970s (Munjal and Pipes, 1971; Munjal et al., 1971; Michalopoulos et al., 1984; Daganzo, 1997; Daganzo et al., 1997; Daganzo, 2002a,b). The lane-specific conservation law is shown in (16).

$$\frac{\partial k_i}{\partial t} + \frac{\partial q_i}{\partial x} = \emptyset_i \quad i = 1,2, \dots, n \quad (16)$$

where $k_i(t, x)$ and $q_i(t, x)$ are the density and the flow on lane i at the time-space point (t, x); $\emptyset_i$ is the net LC rate from other lanes to lane i.

These models are widely criticized for treating the LC vehicles as fluid that can accelerate instantaneously, and hence have no slowing down effect on the following vehicles (Laval and Daganzo, 2006). To overcome this issue, Laval and Daganzo (2006) developed a hybrid model in which traffic in each lane was modeled as a separate KW stream linked to neighboring lane traffic by LC vehicles. More specifically, the LC vehicles are approximated as dimensionless moving bottlenecks that completely block traffic behind them.

Assuming a triangular fundamental diagram, Laval and Daganzo (2006) found

$$\emptyset_{ji} = \frac{\mu_i}{T_i + \sum_{i \neq j} \Delta x L_{ji}} L_{ji} \quad (17)$$

where μ_i is the available capacity of lane i; T_i is the desired through flow of lane i; L_{ji} is the desired LC rate from lane j to i. Both T_i and L_{ji} are determined by k, t & x.

They further demonstrated that in the discretized time-space dimensions, the limit of the net LC rate (i.e., $\Delta x \rightarrow 0$ & $\Delta t \rightarrow 0$) as expressed in Equation (17) is finite, which indicates a stable LC rate as defined in Equation (18).

$$\lim_{\Delta x \rightarrow 0} \emptyset_{ji} = \frac{\mu_i \, \pi_{ji} S_j}{u S_i} \quad (18)$$

where π_{ji} is the fraction of choice-makers per unit time with intension of changing lanes from j to i and assumed to be proportional to the speed difference between lanes j and i, which can be approximated as $\frac{\Delta v_{ji}}{u\tau}$; τ is the time a driver takes to decide and execute LC; S is the desired amount of advancing vehicles in Δt and can be approximated as $\Delta t \times min\{uk, Q\}$; u is the free-flow speed.

To accommodate the lane changer's impact on the follower in the target lane, the LC rate was quantized to generate discrete particles that move with bounded accelerations and temporarily block the traffic behind. Trajectories of these particles were constructed using the constrained motion model proposed by Laval and Daganzo(2003).

Besides the three commonly used KW parameters (i.e., free-flow speed, jam density, and capacity), this hybrid model only requires one additional parameter, τ. Numerical experiments were used to demonstrate the model's capability of reproducing two LC related phenomena: capacity drop caused by lane-drops (Cassidy and Bertini, 1999; Bertini and Leal, 2003) and the relationship between the speed of moving bottlenecks and their capacities (Munoz and Daganzo, 2002).

Laval and Leclercq (2008) incorporated the macroscopic LC model discussed above into a microscopic modeling framework to capture the relaxation phenomenon. To achieve this, a microscopic version of the moving bottleneck model was first developed. Assuming a triangular fundamental diagram and using a discretized version of Newell's simplified CF model (Newell, 2002; Daganzo, 2006), Equation (19) was derived to describe the relaxation process. In this equation, macroscopic variables are used in a microscopic framework and individual lane changers (or their new leaders in the target lane) are treated as moving bottlenecks.

$$\Delta N^{i+1}(t) = \Delta N^{i+1}(0) + \frac{\varepsilon w k}{\beta} ln\left[1 + \frac{\beta t}{v^i(0) + \omega}\right] \quad (19)$$

where $\Delta N^{i+1}(t)$ is the difference in cumulative number of vehicles between the new leader (the lane changer) i and the lane changer (the follower) $i+1$ at time t. Note that $\Delta N^{i+1}(t) < 1$ if vehicle $i+1$ is in non-equilibrium (i.e., in the relaxation process) and $\Delta N^{i+1}(t) = 1$ if vehicle $i+1$ is in equilibrium; k is the jam density; β is a constant acceleration rate of the leader; ε is the speed difference vehicle $i+1$ is willing to accept during the relaxation process; $v^i(0)$ is the initial speed of vehicle i, and w is the wave speed.

Only one parameter in this model, ε, needs to be calibrated while the others can be measured directly from observations. This model was verified in Leclercq et al. (2007) at macroscopic and microscopic levels using vehicle trajectories collected by NGSIM (Alexiadis et al., 2004). It was found that the model agreed well with macroscopic observations, such as the reduction in bottleneck discharge rate (i.e., capacity drop), and microscopic observations, such as individual vehicle trajectories.

To facilitate more straightforward calibrations, Duret et al. (2011) reformulated the model by Laval and Leclercq (2008) using microscopic variables, such as the maximum passing rate, which can be readily measured as the reciprocal of τ, a parameter in Newell's simplified CF theory (Newell, 2002). This reformulation as mathematically defined in Equation (20) used the same logic as proposed in Laval and Leclercq (2008), which is characterized by the initially non-equilibrium passing rate that gradually converges to the equilibrium one.

$$r^{i+1}(t) = \left[\frac{1}{r^{i+1}(0)} + \frac{\varepsilon}{\beta} ln(1 + \frac{\beta t}{w+v^i(0)})\right]^{-1} \qquad (20)$$

where $r^{i+1}(t)$ is the passing rate of vehicle *i+1* at time *t*; $r^{i+1}(0)$ is the initial passing rate of vehicle *i+1*; other variables and parameters are the same as in Equation (19).

Note that the parsimony of the model by Laval and Leclercq (2008) remains in Equation (20). They calibrated the model macroscopically using NGSIM data (Alexiadis et al., 2004), and to keep the model efficient, obtained the parameters using the average values across vehicles. Their analysis showed that the relaxation process produced from the model reasonably matched the observed one.

Zheng et al. (2013) further extended the above model to describe the entire transition period caused by the LC maneuver that typically consists of anticipation and relaxation processes. More specifically, for the anticipation process, parameters were measured between the lane changer and the follower because their analysis showed that the follower's trajectory in the anticipation process was more correlated with the lane changer's trajectory in the initial lane rather than the leader's trajectory in the target lane. They also evaluated the model's performance macroscopically using NGSIM data (Alexiadis et al., 2004) and found that the extended model can simultaneously describe anticipation and relaxation processes with reasonable accuracy. Note that Zheng et al. (2013) also found strong evidence that LC can at least temporarily change driver characteristics (i.e., the so-called regressive effect previously discussed) and proposed an extended framework of Newell's simplified CF model to capture this change.

Models by Jin (2010; 2013)

To remedy the issue that the traditional fundamental diagram captures only longitudinal interactions between vehicles when they follow each other, but not lateral interactions when they change lanes, a novel approach was proposed by Jin (2010; 2013) to extend KW theory for modeling the LC traffic flow. In this approach, Jin introduced a new concept, the LC intensity $\epsilon(x; t)$ that was defined as the ratio of the total LC time over the total travel time in an Edie's spatial-temporal domain (Edie, 1963). Note that Edie's generalized definition of density was also adopted in Leclercq et al. (2007). $\epsilon(x; t)$ is generally time- and location-dependent, which is determined by drivers' LCD and their characteristics at the microscopic level, and also determined by locations, geometric features, on- and off-ramp flows, and etc. at the macroscopic level. For uniform traffic (i.e., traffic density is the same across the region, and all vehicles travel at the same speed), the LC intensity is defined as

$$\epsilon = \alpha \frac{\rho_{LC} t_{LC}}{\rho T}$$

where ρ_{LC} is the density of LC traffic; t_{LC} is the LC duration; $\alpha = \frac{N_{LC}}{\rho_{LC} L}$ and $T = \frac{L}{v}$; L is the length of the LC region; N_{LC} is the total number of LC maneuvers in the LC region during T.

In Jin (2010), a lane changer's contribution to total density was doubled because it essentially uses two lanes during the LC execution. They further defined the fundamental diagram with the LC effect as

$$q = \rho V((1+\epsilon)\rho)$$

where q & ρ are flow and density, respectively; $(1+\epsilon) \times \rho$ is the effective total traffic density with the additional density caused by LC; and speed $v = V((1+\epsilon)\rho)$.

The fundamental diagrams with the LC effect is

$$q = \rho V((1+\epsilon)\rho)$$

For the traffic flow free of LC with the same density, the flow rate is $(1+\epsilon)$ times larger as shown in Equation (21):

$$q = (1+\epsilon)\rho V((1+\epsilon)\rho) \quad (21)$$

Thus, traffic conservation incorporating the LC effect in the framework of KW theories follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V((1+\epsilon)\rho))}{\partial x} = 0$$

Jin (2010) calibrated LC intensities and corresponding fundamental diagrams using vehicle trajectories on I-80 collected by NGSIM (Alexiadis et al., 2004) and demonstrated that this modeling framework was capable of capturing several widely-observed traffic phenomena closely related to LC, such as capacity drop, different jam densities, reverse-λ shape fundamental diagram. From this model, LC traffic can also affect the formation and dissipation of traffic oscillations, which is consistent with findings from the empirical analysis by Zheng et al. (2011).

Built on the same concept of LC intensity and the corresponding speed-density relation, Jin (2013) developed a multi-commodity KW model of LC traffic flow by treating LC vehicles and non-LC vehicles as two commodities. For total traffic, its density, speed, and flow-rate are denoted as k, v, and q, respectively; for weaving traffic, its density, speed, and flow-rate are denoted as ρ, ψ, and ϕ, respectively. Then, for non-weaving traffic, its density, speed, and flow-rate are k-ρ, ${(kv-\rho\psi)}/{(k-\rho)}$, and q- ϕ, respectively. The LC fundamental diagram constitutes the following equations:

$$\xi = \frac{\rho}{k};\ v = F\left(\frac{(1+\alpha\xi v)k}{n}\right);$$

$$q = kF\left(\frac{k+\alpha\phi}{n}\right);\ \rho = \frac{\phi}{F\left(\frac{k+\alpha\phi}{n}\right)}$$

where $\alpha = \frac{n-1}{2L}\pi$, ϕ is the lane-changing flow rate, n is the number of lanes, π is the average LC duration (t_{LC} in Jin (2010)), L is the length of the LC area.

Jin calibrated and validated a LC fundamental diagram based on a triangular CF fundamental diagram, also using the vehicle trajectories collected on I-80 collected by NGSIM (Alexiadis et al., 2004). Jin found that the derived fundamental diagram of LC traffic approximately matched the observed data, although significant discrepancies existed due to some of the assumptions discussed later. Furthermore, Jin derived a multi-commodity KW model for understanding traffic dynamics around a lane-drop location, investigating capacities of lane-drop areas and smoothing effects of HOV lanes (i.e., discharging rates on regular lanes increase because of implementation of HOV or other lane restrictions) observed by several studies (Chen et al., 2005; Menendez and Daganzo, 2007; Daganzo and Cassidy, 2008; Cassidy et al., 2010).

Using this model, Jin analytically showed that systematic LCs cause capacity drop, and that the drop percentage depends on the proportion of weaving vehicles. The analysis also confirmed that LC is the primary reason of HOV lane's smoothing effects (Cassidy et al., 2010). The consistency demonstrates that the multi-commodity KW theory of LC traffic flow can be used to quantify such effects analytically.

Note that the KW model in Jin (2010) is phenomenological because the LC intensity was calibrated as a function of total density; in contrast, the multi-commodity KW model in Jin (2013) is behavioral because the LC intensity was derived as a function of traffic composition, road geometry, and CF & LC characteristics. Although the phenomenological fundamental diagram fitted NGSIM data better (e.g., the reported larger R-squares), the behavioral model is easier to calibrate and more importantly provides us more insights into the impacts of traffic composition on capacity and traffic dynamics (Jin, 2012). The LC models in Jin (2010; 2013) were further validated by Gan and Jin (2013) and a satisfactory performance was reported.

As acknowledged by the author, several assumptions were used in developing the models, such as inside a LC area, speed-density relations are location- and time-independent, and lane-identical; weaving and non-weaving vehicles have the same speed at the same location and time; the number of LC is proportional to the weaving flow-rate and LC flow rate is constant over time; merging vehicles from the on-ramp are evenly distributed across lanes; and LC duration is constant. Meanwhile, the proposed LC intensity can only capture the impact of the LC execution and the anticipation and relaxation phenomena may not be covered. Nevertheless, these models can be powerful tools of analytically analyzing and quantifying observed traffic flow phenomena at the macroscopic level.

Finally, Jin's models (Jin, 2010; Jin, 2013) were developed specifically for dealing with lane changing maneuvers in weaving sections with on/off ramps, where mandatory lane changes are dominant. More specifically, Jin's models assume that the number of lane changes is proportional to the weaving flow-rate (e.g., merging vehicles from the on-ramp). Thus, Jin's models are inherently more suitable for modeling the mandatory LC. In contrast, Laval and Daganzo (2006) assumes that (intended) lane change maneuvers per unit time are proportional to the speed difference between the current and adjacent lanes as indicated in Equation (18), which implicitly indicates that this model and its extensions are inherently more suitable for modeling the discretionary LC because speed difference between two lanes is consistently adopted as a key factor in modeling discretionary lane changing decisions by most LCD models in the literature.

4. Discussion

Over the last decade, researchers have slowly but surely realized the critical role that LC plays in traffic operations and traffic safety; this realization has motivated significant attempts to model LC decision-making and its impact on traffic. Meanwhile, thanks to advances in data collection and communication technologies, there is now mainstream access to high-resolution vehicular data; this access provides an unprecedented opportunity for researchers to fully understand the highly complex LC procedure. Thus, notable progress has now been made in modeling various aspects of LC; nevertheless, our knowledge of LC remains incomplete.

This paper roughly categorizes the major LC models in the literature into two groups: models that aim to capture the LC decision-making process, and models that aim to quantify LC's impact on traffic flow. Representative models in each of these categories are comprehensively reviewed to assist future researchers in this important field to efficiently grasp the historical development of LC modeling, the current state-of-the-art, and its future research needs. More specifically, their methodologies are outlined, and important features (including their limitations) are summarized. However, before any further research breakthroughs can be made, the major issues arising from the existing modeling need to be reviewed and subsequently addressed. Thus, the major issues shared by the modeling efforts discussed in this paper are identified and discussed below.

LCD modeling issues

The first issue in current modeling is that the models are largely based on how the modelers themselves would make lane changing decisions, rather than on the general driving experience. This is the result of the modelers' implicit assumption that others would share their particular perspective. Unfortunately, this is often not true, and leads to numerous LCD factors that are often not empirically justified being reported in the literature. For the existing LCD models, only a few have identified factors and developed lane changing rules based on video evidence (e.g., Hidas, 2002; Hidas, 2005), or by interviewing drivers (e.g., Sun and Elefteriadou, 2011; Sun and Elefteriadou, 2012).

A second issue is that the driver's role in LC may be over-simplified. LC is a typical choice making process in which the choice maker (the driver in our case) inevitably plays a vital role; however, this role is more active than the one that is presumed in the existing models. For example, when drivers in a real traffic situation are deciding whether or not to change lanes, they can simultaneously evaluate two or more spacings in the target lane, and their next decision is where and how to execute the change. Among many possibilities, the driver may accelerate to move into a more comfortable spacing ahead, or may deliberately decelerate to wait for a desired spacing behind. In the existing LC decision models reviewed in Section 2, however, such complexity is not considered and only one gap – the one nearest to the lane changer – is evaluated.

A third issue is that a LC decision is often modeled as a one-player (the lane changer) decision-making process. However, our observations and experience tell a different story: in heavy traffic, a typical LC decision-making process closely involves at least two players – the lane changer and the follower in the target lane. This is because the follower is often also required to make decisions as a result of someone else's LC decision. Thus, at least two decision-making players and processes are involved in the LC process in heavy traffic.

Another issue with the existing models is that failed lane changing attempts are often ignored in calibrating and validating LCD models due to a lack of data; thus, current LCD models do not have the capability of reproducing failed attempts. However, failed lane changing attempts are likely to have significant impacts on surrounding traffic and have important safety-related implications. Meanwhile, LCD models are also criticized because the LC frequency depends on the number of times that the decision-making process has been evaluated; this indicates that the duration of the time step becomes a parameter of the model (Laval and Leclercq, 2008).

The scheduling conflict issue (i.e., the situation where more than one vehicle is attempting to move into the same location) in implementing a LCD model is discussed in several CA-based models (Maerivoet and Moor, 2005). However, this issue is not mentioned in other types of LCD models.

A final note on LCD modeling is that many models have been developed either for freeways or for urban streets. Although lane changes on freeways and those on urban streets have different complexities (lane changes on urban streets are generally more complex), there are unlikely to be fundamental differences in modeling the lane changing decision-making processes.

LCI modeling issues

The (few) models developed for measuring LCI, are macroscopic (or hybrid), which is parsimonious and consistent with KW theories; however, their compatibility with microscopic LCD and CF models may be a significant issue. Thus, in the author's view, for the purpose of micro-simulation, there is a need to develop microscopic LC models, which are capable of providing detailed information on LC's impact at an individual vehicle level, and capable of being easily integrated into existing CF models.

Unlike LCD models where LC is often categorized into different types (e.g., mandatory LC and discretionary LC; or free, cooperative, and forced LC), LC types (scenarios) are ignored in the LCI models. This may have serious consequence for the models' performance. For example, if a LC is freely executed, its impact should be negligible; if a LC is cooperatively executed, its impact should be very different from that of a forced LC.

LC's impact can be highly complex. Spatially, a typical LC can have an impact on three individuals: 1) the lane changer; 2) the immediate follower in the initial lane; 3) the immediate follower in the target lane. Temporally, a typical LC's impact consists of: 1) impact before the insertion (i.e., anticipation); 2) impact of the insertion; 3) impact after the insertion (i.e., relaxation). In total, a LC can have 9 different impacts (i.e., 9 combinations of the spatial and the temporal impacts), depending on the time and location. The number is even bigger if impacts on non-immediate followers and on driver characteristics are considered. The existing models only attempt to describe a few of these impacts. For example, Laval and Leclercq (2008) investigated the impact after insertion on the lane changer and on the immediate follower in the target lane, while Jin (2010) studied (implicitly because of the macroscopic nature of his model) the impact of the insertion on the lane changer, and on the followers in the initial and target lanes. Evidently, a comprehensive model is lacking. More importantly, empirical studies aiming to more accurately and more reliably measure different components of LC's impact at a microscopic level are greatly needed. These studies would provide the foundation for the development of a comprehensive LCI model.

Finally, the modeling of LCI has received much less attention than the modeling of LCD. The current literature is dominated by LCD modeling. Although notable progress has recently been made in modeling LCI, research on this front is still at the early stage. Compared with the long history and vast family of LCD models, only two types of LCI models have been proposed (Laval and Leclercq, 2008; Jin, 2010); this limited attention is clearly not proportional to its importance.

General discussion

The first general issue with the existing LC modeling is that, although LC behavior is affected by both personal attributes and interpersonal interactions, psychological, perceptional, and cognitive factors are, by and large, ignored. There can be large discrepancies in LC decision making and execution among drivers, and these differences can significantly influence how, and to what extent, their LC impacts surrounding vehicles. Indeed, in the few models where driver characteristics are considered, this important dimension is over-simplified, with only one or two parameters being relied on to indirectly capture the total impact of drivers' individual characteristics and interpersonal interactions. Examples of these parameters are: the impatience factor and the speed indifference factor in Yang and Koutsopoulos (1996); a driver-specific random term that represents unobservable characteristics of the driver and correlations between observations of the same driver over time in Toledo et al. (2003); and ε, the speed difference vehicle $i+1$ is willing to accept during the relaxation process, in Laval and Leclercq (2008). In even further simplification of this important dimension, such parameters are often assumed to be constant across individuals in calibration and validation. The data source used for developing these models – loop detector data or trajectories, at best – is the root of this problem. This type of data can only provide basic vehicular information with no information on driver characteristics; this makes it impossible to decipher drivers' thinking processes during the LC procedure.

The related issue is that the use of high-resolution vehicular data alone is insufficient, and innovative data collection methods aiming to capture drivers' psychological disposition, perceptional performance, and cognitive function during LC are clearly needed. Sun and Elefteriadou (2011) conducted a focus group study with 17 participants to identify important factors that they frequently considered during their LC experiences. Sun and Elefteriadou (2012) also used instrumented vehicles to obtain field-measured values for these factors. While this study had its limitations (i.e., participants in focus groups may over-think their actions, and the sample size was small), it is, nevertheless, one of the very few studies in the LC literature that used non-vehicular data. In our view, there is a need to adapt the data collection methods and devices that are commonly used in the Social Science disciplines to gather psychological, perceptional, and cognitive information to complement the vehicular data during LC.

Another important issue is the balance between improving the model's performance and controlling the model's complexity, which leads to an important question: How good is good enough? Or: How complex is too complex? Of course, there is no easy answer to such a challenging question. On the one hand, to better mimic drivers' lane changing decision-making process, various features (e.g., human uncertainty, driver heterogeneity) have been considered in LCD modelling, among which many features are not convincingly justified[†]. On the other hand, it seems that there is a need to incorporate additional variables related to psychological, perceptional, and cognitive factor. In the author's view, since any model is only a simplification of reality, to avoid the model being ever-increasingly complex and over-fitting, assumptions about how a driver might think alone is not sufficient reason for including new features. The bottom line is that the performance gains from adding new variables should outweigh the disadvantage associated with the model's extra complexity. A two-step approach can be used to test this: the first step is to rigorously implement a series of model comparison analyses, analogous to the comparison tests of statistical models (e.g. the likelihood ratio test (Casella and Berger, 2001)); and the second step is to obtain empirical evidences (e.g., field observations and surveys) for the target driver population because the same factor can vary substantially across different driver populations. For example, same-lane passing is considered in some CA-based models, which have been criticized for being unnecessarily complex (Maerivoet and Moor, 2005). However, same-lane passing is a common phenomenon in some developing countries (e.g., China, where drivers are generally more aggressive and often treat the whole road as one lane when traffic is congested (Sun and Zheng, 2013)). Thus, it seems reasonable to include same-lane passing when modeling lane changings of Chinese drivers.

[†] For example, most of the models distinguish lane changes as either 'mandatory LC' or 'discretionary LC' because of their distinctive mechanisms. Some models even break LC into three categories: free, cooperative and forced LC to more realistically and more completely cover lane changing scenarios in the real world. However, in the author's view, more research is needed on whether it is necessary to consider the potential trade-offs between mandatory LC and discretionary LC (Toledo et al., 2003), and whether it is necessary to consider the state dependency (Toledo et al., 2009).

Another serious issue relates to the calibration and validation of LC models. The majority of models were either numerically tested or validated by demonstrating their potential to produce outcomes consistent with certain macroscopic traffic flow features. While this approach may be acceptable for macroscopic LC models, it can only invalidate microscopic LC models. More specifically, performance measures directly linked to LC are not used in the literature. Possible measures include: detection rates (i.e., percentage of LCs that have been successfully detected/generated by the LCD model) and false alarm rates (i.e., percentage of LCs that are falsely detected/generated by the model); time error (i.e., the difference between the LC time predicted by the model and the actual LC time); and location error (i.e., the difference between where the LC occurs as predicted by the model and where it actually occurs). There is little discussion on the systematic and rigorous calibration and validation of LC models. Furthermore, the majority of the existing LC models were calibrated and validated using data collected in developed countries where drivers are generally less aggressive, compared with their counterparts in developing countries. More specifically, NGSIM data are frequently used. Although NGSIM data are a valuable resource for developing LC models, there may be a danger of over-utilizing it. Data containing more diverse driving behaviors, particularly more aggressive driving behavior, is clearly needed.

Finally, LCD models often ignore the impact of LC. In heavy traffic, LC directly impacts at least three vehicles (the lane changer, and the follower in both the target lane and the lane of origin); furthermore, such direct impact is likely to have a domino effect on other following vehicles in both lanes, as reported in Zheng et al. (2011; 2013). Models that ignore LC's impact on surrounding vehicles are incapable of reproducing LC-related traffic phenomena (e.g., anticipation, relaxation, and capacity drop). Hidas' (2005) model is an exception in that it considers the anticipation and relaxation reactions[‡] of the follower in the target lane, as reported in Zheng et al. (2013). However, Hidas (2005) considered such impact by imposing simple assumptions on the CF rules, and without attempting to validate these assumptions. In addition, LC's impact on surrounding vehicles may induce secondary LCs, which cannot be predicted by models that ignore such impact.

Meanwhile, the LCD component in LCI models is also inadequate. In these models, LC decision-making processes and driver characteristics are either ignored, or unrealistically aggregated and generalized.

5. Conclusion

In summary, there is a clear need to develop a comprehensive model that captures the (mandatory or discretionary) LC decision-making process and its consequent impact on surrounding traffic. In developing a new LC model, the multi-level evaluation strategy should be generally preferred: at the macroscopic level, outputs of the model should be consistent with typical traffic flow characteristics; at the microscopic level, lane changing decisions need to be matched with observations with a reasonably low prediction error rate, and trajectories of the vehicles involved in LC should be close to actual trajectories.

It is also important to maintain the balance between maximizing the model's predictive and explanatory power and minimizing the model's complexity. Factors considered in the model need to be empirically and statistically justified for the target driver population. Furthermore, the LC model should be able to be easily integrated into mainstream car following modeling frameworks.

[‡] Hidas (2005) did not explicitly use these terms.

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