Surface Integral Equation Method for Sharp Edge Structures with Junctions

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Abstract — **Complex scattering targets contain metallic structures with junctions and sharp edges that require a special procedure to be analyzed by the Method of Moments. Singular basis functions to mdel junctions with edge profile connected together are considered. At the Conference, we will show how to handle the different geometrical cases together with numerical results that validate the proposed method.**

1 INTRODUCTION

In the past, several papers have studied the case of Surface Integral Equations (SIE) modeling surface junctions [1]-[4]. Those papers show how to implement Rao-Wilton-Glisson (RWG) basis functions [5] in Method of Moments (MoM) codes in presence of junctions.

On the other hand, 3D sharp-wedge structures are studied in [6]-[11] by defining singular vector functions for higher-order MoM applications [12].

The interest in junctions come from the observation of real life objects and it is recently demonstrated in [13] where junctions among triangles and wires have been studied.

In this summary we extend the modeling capability of higher-order interpolatory vector bases [12] and singular divergence-conforming vector bases [9] to handle PEC structures with thin joined surfaces in different practical cases. Complex targets exhibit two kinds of junctions: 1) sharp edge plates joined to sharp edge plates, 2) sharp edge plates joined to smooth surfaces.

At the conference, numerical simulation results will be shown to validate the proposed method.

2 SINGULAR DIVERGENCE-CONFORMING VECTOR BASES

Singular divergence-conforming vector bases able to model 3D sharp-wedge structures are described in [9] for triangular and quadrilateral meshes. These elements model the singular charge and current densities near edges and wedges [16], [17].

Without loss of generality let us focus the attention on triangular elements.

The singular elements attached to an edge profile are shown in Fig. 1. In the case of triangular discretization of the geometry, they are of two kinds: edge singular elements or vertex singular elements.

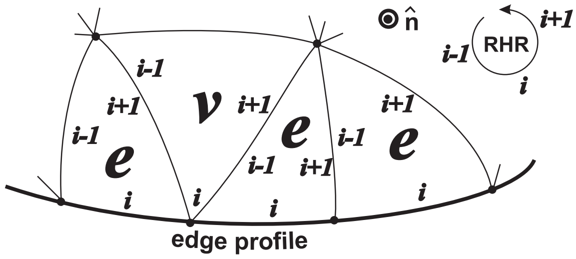

Figure 1: A face of a wedge meshed with edge singular triangles (e) and vertex singular triangles (v). From the reader's viewpoint the figure shows a right-hand-rule (RHR) numbering scheme for the edges $i-1, i, i+1$.

The vertex singular and the edge singular triangles have one vertex and one edge attached to the edge-profile, respectively. For this property we consider the vertex singular triangles *element fillers* [9]. We recall that all the bases proposed in [12] and [9] are defined in a parent domain using the triangle area coordinates ξ coordinates; the physical modeling properties depend on this definition.

The set of the singular basis functions is complete to an arbitrarily high order s and it is added to the subset of the (high order) polynomial basis functions of order p commonly used in MoM applications [12].

From the reader's viewpoint, Fig. 1 shows a right-hand-rule (RHR) numbering scheme for the edges $i-1, i, i+1$ and therefore we obtain an outward normal for the orientation of the element. The choices of RHR and thus of an outward normal are arbitrary but the reciprocal directions of them among elements is an important feature for connection of degrees of freedom in particular in surface junctions. We observe that this orientation property is trivial for simple meshed surfaces since the normals can be swapped to point in the same direction. On the contrary, in junction problems, we have more than two patches attached to a common edge, and each pair of elements might show different hand-rule according to their reciprocal orientation.

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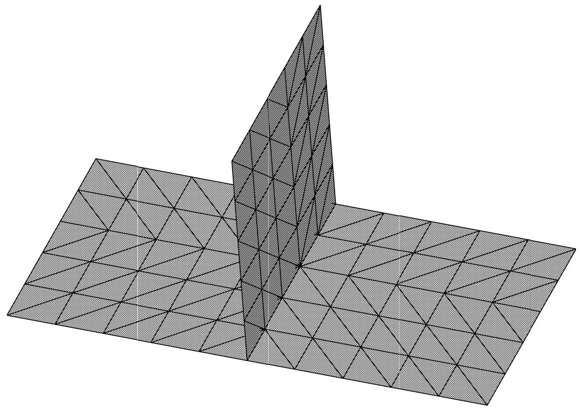

Figure 2: Tri-plate.

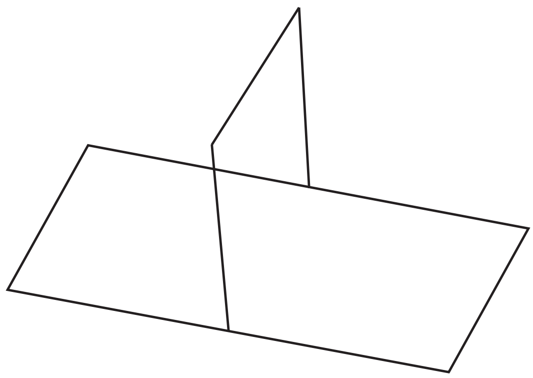

Figure 3: Tri-plate: the fields can be singular along the shown edge-border line.

3 JUNCTIONS IN JOINED ELECTRICALLY CONDUCTING PLATES

Let us consider joined electrically conducting plates. Fig. 2 shows the case of a tri-plate.

With reference to Fig. 2, we focus on the junction region that is constituted by singular elements located only in the region connected to the edge profile, see Fig. 3. Far from the edge profile the junction is modeled only by regular elements. To model the junction we use the Kirchhoff current law (KCL) formulated in terms of current density i.e. with MoM unknowns related to the basis functions. Because of the dependency relation, the KCL requires to discard one of the degrees of freedom. This procedure must be applied to the regular subset as well as to the singular subset of the bases for each interpolation point along the junction profile. Fig. 4 shows the surface junctions among three elements. We recall that the connection among the degrees of freedom (DOFs) is dependent on the relative orientation among the elements (RHR or or opposite to RHR). This feature will affect also the sign of DOFs in each el-

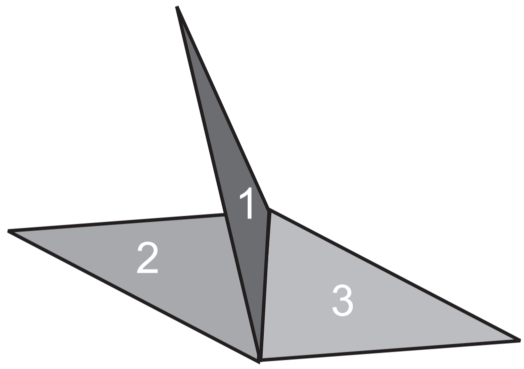

Figure 4: Three triangular patches connected together in a junction.

ement. This scheme is appropriate only for the test case referred to the first kind of junctions (see Section I). The second kind of junctions, i.e. sharp edge plates joined to (merged in) a smooth surface, requires some modeling tool that smooth the properties of singular functions of the edge structure when attached to a surface. Removing of singular dofs and adding new special bases near the attachment profile is considered. At the conference, numerical tests and tool based on singular quadrature [14],[15] will be presented.

4 CONCLUSION

The proposed method will permit to handle junctions of PEC patches with sharp edges. Extension to penetrable wedge structures [18] will be considered in future works.

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