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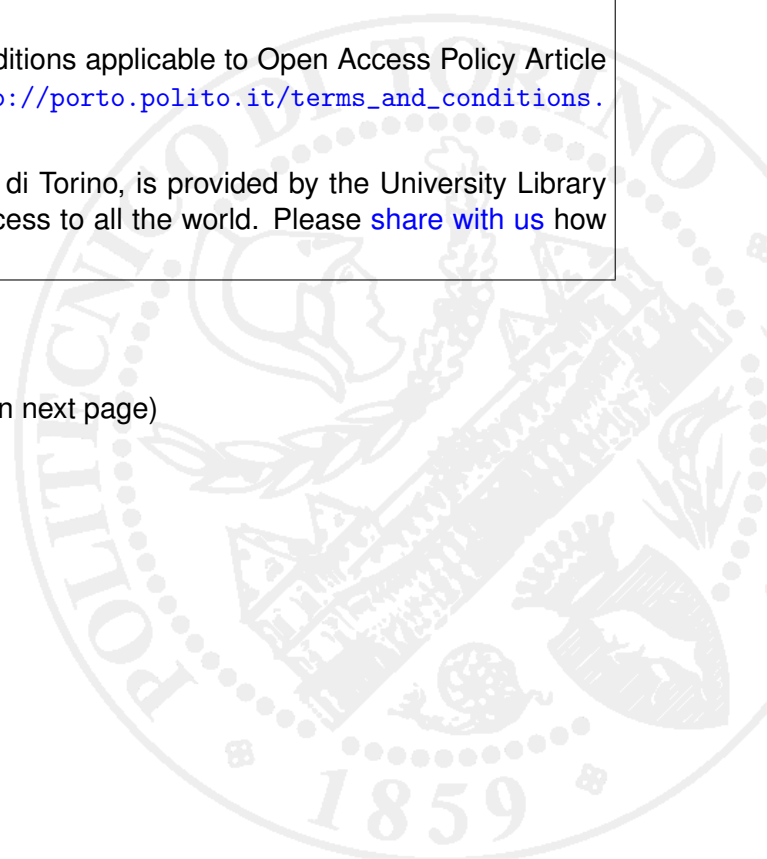
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# Cylindrical resonator filled with four alternating sectors of DPS and DNG materials

V.G. Daniele\*, R. D. Graglia\*, G. Lombardi\*, P. L. E. Uslenghi†

**Abstract** — A circular cylindrical metallic resonator with four alternating sectors of DPS and DNG materials is analyzed, in the frequency domain. The two materials are linear, lossless, homogeneous, and anti-isorefractive to each other. The electric field is assumed to be parallel to the cylinder axis. It is also shown that the resonator's dispersion relation is always satisfied for particular geometries. Numerical results for several sectors' combinations will be presented and discussed at the conference.

## 1 INTRODUCTION

A circular cylindrical resonator with metallic (PEC) walls and filled with four sectors containing alternatively double-positive (DPS) material and double-negative (DNG) metamaterial is considered.

The two DPS regions are filled with a linear, uniform and isotropic material characterized by a real positive electric permittivity  $\varepsilon$  and a real positive magnetic permeability  $\mu$ , or alternatively by a real positive wavenumber  $k = \omega\sqrt{\varepsilon\mu}$  and a real positive intrinsic impedance  $Z = \sqrt{\mu/\varepsilon}$ , where  $\omega$  is the angular frequency. The two DNG regions are filled with a linear, uniform and isotropic material characterized by a real negative permittivity  $-\varepsilon$  and a real negative permeability  $-\mu$ , or alternatively by a real negative wavenumber  $-k$  and a real positive intrinsic impedance  $Z$ . Thus, the DPS and DNG regions of the resonator are filled with materials having real refractive indexes of opposite sign and the same real intrinsic impedance.

The concept of utilizing DNG metamaterial inclusions to render a resonator size-independent was introduced by Engheta [1]-[2] for a one-dimensional structure, to show that it is possible to build resonators that function independently of dimensions at those frequencies for which the metamaterial behaves as postulated. This concept was later extended to fully three dimensional cavity resonators by Couture et al. [3]-[4] and by Uslenghi [5]-[7]. Recently, Daniele et al. [8]-[11] studied in detail a cylindrical resonator sectorally filled with metamaterial; in particular, their analysis showed that phase compensation leading to size independence is possible only if the cylinder is half-filled with metamaterial. In the present work we extend the results obtained for structures with two sectors to structures with four sectors.

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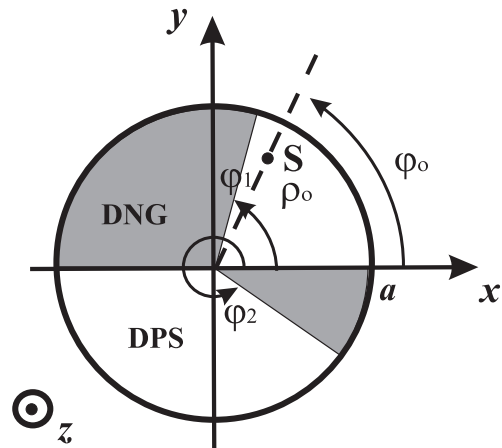


Figure 1: Cross section of the resonator. First sector  $0 < \varphi < \varphi_1$  and third sector  $\pi < \varphi < \varphi_2$  are filled with DPS material, while second sector  $\varphi_1 < \varphi < \pi$  and fourth sectors  $\varphi_2 < \varphi < 2\pi$  are filled with DNG material. The electric line source is located at  $(\rho_0, \varphi_0)$  inside the first DPS region, ( $0 < \varphi_0 < \varphi_1$ ).

The analysis is conducted in phasor domain with time-dependence factor  $exp(+j\omega t)$ . The electric field is assumed to be parallel to the axis of the cylinder, so that the boundary conditions on the bases of the resonator are satisfied and the height of the cylinder does not appear in the analysis. It is shown that the resonator's dispersion relation is always satisfied for particular geometries. Several sectors' combinations will be also considered, in particular the case of four geometrically identical sectors. The structure will be excited by a line source parallel to the axis. Numerical results for several sectors' combinations will be presented and discussed at the conference.

## 2 GEOMETRY OF THE PROBLEM

With reference to cylindrical coordinates  $(\rho, \varphi, z)$ , the metallic cylindrical resonator has the  $z$  axis as symmetry axis, the inner radius of its circumference in any plane  $z = constant$  is  $a$ , and the length of the resonator in the  $z$  direction does not come into play because the resonator is assumed to be excited by an electric line source parallel to its axis, leading to an electric field that is everywhere parallel to  $z$ , so that the boundary conditions at the two circular bases of the cylinder are

always satisfied.

With reference to Fig.1, the DPS material fills the cross-sectional area  $0 \leq \rho < a$ ,  $(0 < \varphi < \varphi_1) \cup (\pi < \varphi < \varphi_2)$ , whereas the DNG material fills the cross-sectional area  $0 \leq \rho < a$ ,  $(\varphi_1 < \varphi < \pi) \cup (\varphi_2 < \varphi < 2\pi)$ .

The electric line source  $\mathbf{J}_0$  located inside the DPS region at  $\rho_0$  ( $\rho = \rho_0$ ,  $\varphi = \varphi_0$ ) is assumed to be:

$$\mathbf{J}_0 = I\delta(\rho - \rho_0)\hat{z} = I\frac{1}{\rho_0}\delta(\rho - \rho_0)\delta(\varphi - \varphi_0)\hat{z} \quad (1)$$

where  $I$  is the intensity (in  $A$ ),  $\hat{z}$  is a unit vector parallel to the  $z$  axis, and  $\delta$  is the delta function.

### 3 SOLUTION PROCEDURE

Since we excite the structure with the line source (1), we limit our considerations to electric fields parallel to the axis  $z$  of the resonator, the boundary-value problem is two-dimensional and, the  $z$  coordinate and the length  $d$  of the cylinder do not play any role in our analysis. In this case only  $TM_{mn0}$  modes (typical of cylindrical cavities) are compatible where  $m, n, 0$  are respectively the indices related to  $\varphi, \rho, z$ . The longitudinal component  $E_z$  of the electric field must satisfy the following equation:

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + k^2\right)E_z(\rho, \varphi) = -j\omega\mu_o J_z \quad (2)$$

with boundary conditions at the DPS/DNG interface and at the cylindrical border. From the homogeneous version of (2) we obtain the resonator modes  $\Phi_{mn}(\rho, \varphi)$ , that satisfy the completeness equation

$$\frac{1}{\rho_o}\delta(\rho - \rho_o)\delta(\varphi - \varphi_o) = \sum_{m,n} w(\varphi_o)\Phi_{mn}^*(\rho_o, \varphi_o)\Phi_{mn}(\rho, \varphi) \quad (3)$$

where  $w(\varphi)$  is a suitable function. Eq. (3) allows to represent the solution of (2) in terms of the basis  $\Phi_{mn}(\rho, \varphi)$ :

$$E_z(\rho, \varphi) = -j\frac{1}{2}\omega\mu I_o \sum_{m,n} w(\varphi_o)\frac{\Phi_{mn}^*(\rho_o, \varphi_o)}{k^2 - \tau_{mn}^2}\Phi_{mn}(\rho, \varphi) \quad (4)$$

To get  $\Phi_{mn}(\rho, \varphi)$  we resort to separation of variables and we define each  $\Phi_{mn}(\rho, \varphi)$  as piece-wise functions for each sector ( $\ell = 1, 2, 3, 4$ ) as numbered in Fig. 1:

$$\Phi_{mn}^{(\ell)}(\rho, \varphi) = F_{\nu_m}^{(\ell)}(\varphi)\frac{\sqrt{2}}{a J_{\nu_m+1}(z_n(\nu_m))}J_{\nu_m}((-1)^{\ell+1}\tau_{\nu_m n}\rho) \quad (5)$$

where  $\nu_m$  is a countable parameter related to  $\varphi$  variation,  $z_n(\nu_m)$  is the  $n$ -th zero of the Bessel function of order  $\nu_m$  and,  $\tau_{\nu_m n} = z_n(\nu_m)/a$ .

While the  $\rho$  variation in (5) is obtained via the classical expression of the completeness relation in radial

direction, we focus the attention on terms related to the  $\varphi$  variation

$$\left(\frac{d^2}{d\varphi^2} + \nu^2\right)F_\nu(\varphi) = 0 \quad (6)$$

We apply the Green resolvent technique (also used in [12],[8]-[11] for Sturm-Liouville problem with classical boundary conditions) in a generalized form as reported in [13] where mixed type boundary conditions are considered. In fact our problem shows  $\varphi$  periodicity and it is not defined in a limited interval as in classical Sturm-Liouville problem.

Once the Green function is obtained, its denominator determines the dispersion relation (7) for the problem under investigation and thus a set of countable eigenvalues  $\nu_m^2$  (8)

$$\sin[\nu(2\pi - \varphi_1 - \varphi_2)] = 0 \quad (7)$$

$$\nu_m = \frac{m\pi}{2\pi - (\varphi_1 + \varphi_2)} \quad (8)$$

from which it is possible to define a set of ortho-normal basis functions to represent  $F_\nu(\varphi)$ . The procedure described in [13] is fundamental to build an organized algorithm for ortho-normal bases in mixed type boundary condition problems.

If  $\varphi_1 + \varphi_2 = 2\pi$ , the dispersion equation (7) is satisfied for any  $\nu$ . In this case the Green function does not exist [13] and a different approach to obtain the solution must be implemented.

In the case of four geometrically identical sectors, we build the solution using the image theory with a particular distribution of image sources. A similar heuristic approach has already been used for a different problem in [11]. Further details and numerical results will be presented at the conference.

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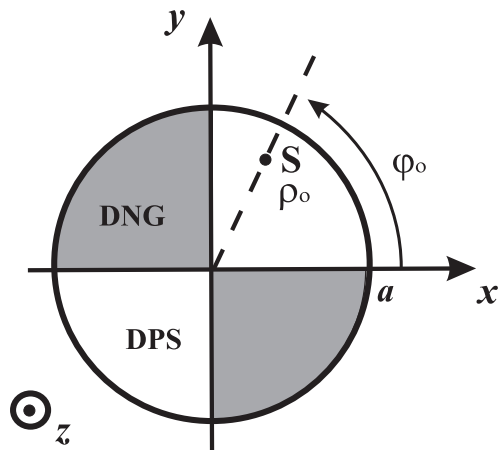


Figure 2: Cross section of the resonator filled with four geometrically identical sectors with alternating DPS and DNG materials. The electric line source is located at  $(\rho_0, \varphi_0)$  inside the DPS region.

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