THE FLEXURAL BUCKLING STRENGTH OF ALUMINIUM COLUMNS

by

ETIENNE VAN DER MERWE

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SUPERVISOR: MNR. J. PRETORIUS
CO-SUPERVISOR: PROF. P. VAN DER MERWE

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ABSTRACT

It should be the objective of every engineer to design as cost-effective as possible. This study is part of a much larger programme in which the most effective column design method is to be found by comparing the different column design criteria and methods of design which are in use for different materials. The materials under consideration are carbon and low-alloy steels (structural steels), stainless steels and an aluminium alloy that is regarded as suitable for structural design.

It has been judged that sufficient experimental data is available to the Chromium Steels Research Group (CSRG) at the Rand Afrikaans University on carbon and low-alloy steels and stainless steels and that sufficient data on the mechanical properties of the aluminium alloy 6261-T6, which is regarded suitable for structural applications, is available to the CSRG after previous research done by the author.

This short dissertation reports on a preliminary study of the different design criteria and methods for the design of columns. It also reports on the experimental study through which the results of column tests done on the abovementioned aluminium alloy are compared with different column design curves. These curves were obtained by using the previously determined mechanical properties for the aluminium alloy under consideration in the different design criteria and methods for the design of columns.
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CHAPTER 1

INTRODUCTION

A. GENERAL REMARKS

Refined materials such as steels have the natural tendency to revert to a condition of equilibrium as rust to something very much like the ore originally mined. Coating processes such as painting, galvanising, electroplating and epoxy coating, assists to stem the wasteful process of attrition, but often provide only partial success. For this reason a high proportion of the total raw steel produced in the world is used to replace corroded steel.

It is the duty of the professional engineer to find cost-effective solutions to the problems that he is requested to solve. The engineer therefore has to continuously consider and choose from various options. An example of this process is the consideration and choice of the most suitable and cost-effective materials to be used. The freedom of choice might however be restricted, due to the lack of suitable information or an unfamiliar (users unfriendly) format in which information is available.

Members of the Chromium Steel Research Group (CSRG) at the Rand Afrikaans University (RAU) have earned international recognition for their work on the development of design criteria for stainless steel structural members and connections. One of the earlier publications that has included their work, is the ASCE Specification for the Design of Cold-Formed Stainless Steel Structural Members [1]. This is an ANSI-approved design standard which has made suitable information internationally available on seven types of stainless steels for the design of structural members using a familiar (user friendly) format. Through this publication it became possible to design stainless steel structural members with confidence that are to function in different types of aggressive environments.

The RAU has developed the experience and credibility to equip the engineer with information that can serve usefully for him to make calculated choices towards aspects of cost-effective design.

Aluminium alloys also have the ability to function effectively in a variety of aggressive atmospheres. The use of aluminium alloys in structures is however not very common. This is due to factors such as (perceived) high costs of materials, unfamiliarity with methods of construction, low value of elastic modulus, but mainly unfamiliarity with the methodology (format) of design.
B. PURPOSE OF THE INVESTIGATION

This project is part of a programme that has the objective to optimise the criteria for the design of aluminium structural members by comparing the different design criteria of carbon and low-alloy steel structures, stainless steel structures and aluminium structures. This objective will be obtained through a process of comparison and exchange of methodologies and criteria where appropriate and verification by experiments.

C. SCOPE OF THE PROGRAMME

The programme referred to above is a comprehensive programme and will consist of various projects, topics and steps of investigation, all of which are finally to be brought together.

An in-depth study of the different design specifications and the origin or basis of the different design criteria and methodologies has to be done. This will identify the differences and reasons thereof and will establish if the exchange of methodologies can be done in a meaningful and justifiable way. An experimental programme on structural member behaviour will be undertaken to verify decisions where experimental data is not available. Such experimental work can only be undertaken if the mechanical properties of the materials of the structural members that are to be tested, are known. It has been judged that sufficient experimental data is available to the CSRG on carbon and low-alloy steels and stainless steels and that sufficient data on the mechanical properties of the aluminium alloy 6261-T6, which is regarded suitable for structural applications, is available to the CSRG after the research done by Van der Merwe [2].

D. SCOPE OF THIS INVESTIGATION

This investigation has two main thrusts:

• A comparative study is carried out on the criteria for the design of columns as specified by the American Institute of Steel Construction (AISC) [4] and the Canadian Standards Association (CSA) [3] for carbon and low-alloy steels, the American Society of Civil Engineers (ASCE) [1] for stainless steels and the Aluminum Association (AA) [5] of the United States of America for aluminium, by applying the mechanical properties obtained by Van der Merwe [2] into the different criteria, to do the comparison.

• A comparison of the different design criteria and methodologies with actual experiments carried out on various length columns made from the aluminium alloy 6261-T6 that is under consideration.
In the chapters that follow, a detailed discussion of the work that was done in order to achieve the stated objectives, is presented.

Chapter 2 contains a summary of the literature that was studied in order to develop an understanding of the subject at hand. A review of the literature on column behaviour as well as the derivation of column formulas are especially presented in Chapter 2.

Chapter 3 is devoted mainly to the determination of the mechanical properties, as described by Van der Merwe [2], of the aluminium alloy under consideration. The results of tensile and compression tests that were performed on specimens, that were obtained so that the longitudinal axes of the specimens coincided with the direction of extrusion of the member sections, are presented and analysed. Because aluminium yields gradually under load, its yield strength and proportional limit are determined by the offset method at 0.2 and 0.01 percent strain respectively. Values of the following properties are given: Ultimate Tensile Strength, \( f_u \), Yield Strength, \( f_y \), Proportional Limit, \( f_p \), Initial Modulus, \( E_o \), Secant and Tangent Modulus, \( E_s \) and \( E_t \), respectively, as well as the percentage elongation of a 50 mm gauge length. Analytical expressions are developed for the stress-strain curves.

There is a similarity between the gradual yielding type of behaviour of aluminium and stainless steels [6-8]. This leads to the expectation that a commonality should be found to exist in the design criteria for aluminium and stainless steel columns.

The design criteria and methods for designing structural compression members (columns) in accordance with the specifications listed below, are summarised and compared in Chapter 4.

The specifications referred to in this document are:


- Load and Resistance Factor Design Specification for Structural Steel Buildings, American Institute of Steel Construction (AISC) [4], for carbon and low-alloy steels;

- American Society of Civil Engineers (ASCE) Standard : ANSI/ASCE-8-90 : Specification for the Design of Cold-Formed Stainless Steel Structural Members [1];

- Aluminum Construction Manual, Section 1 : Specifications for Aluminum Structures, Aluminum Association (USA) [5].
In Chapter 5 a detailed report is given of the experimental work and results obtained on different column lengths made of the aluminium alloy 6261-T6, which is the aluminium alloy under consideration in this short dissertation.

This short dissertation is concluded in Chapter 6 by a summary, conclusion and the identification of the needs for future research on this topic.
CHAPTER 2

REVIEW OF LITERATURE

A. GENERAL REMARKS

This section contains a summary of the literature that was studied in order to develop an understanding of the subject at hand. The study had two objectives in mind:

- To provide a basis for conducting a credible procedure for the establishment of the required mechanical properties of a specific aluminium alloy that is suitable for structural applications and to develop a set of analytical equations for the properties;
- To develop an understanding of the various criteria and methods that are being used for the design of carbon and low-alloy steel columns, stainless steel columns and aluminium columns.

The summary of the results of the literature survey are presented below in two separate sections.

B. ALUMINIUM ALLOYS AND THE ESTABLISHMENT OF MECHANICAL PROPERTIES FOR STRUCTURAL DESIGN

1. General Remarks

This part of the literature survey, Van der Merwe [2], was done to provide a basis for conducting a credible procedure for the establishment of the required mechanical properties of a specific aluminium alloy that is suitable for structural applications and to develop a set of analytical equations for the properties. The aluminium alloy under consideration is the Aluminium Federation of South Africa (AFSA) Heat-Treatable, Al-Mg-Si, Wrought Alloy, 6261-T6. This is regarded to be a suitable alloy for structural applications due to its good corrosion resistance and level of strength. This aluminium alloy is regularly extruded in various shapes.
2. **General Classification of Aluminium [9]**

Aluminium alloys form the major group of aluminium metals being used in practice. Alloying elements are usually added in amounts ranging from 0.1 to 15 percent, where the principal alloying elements are copper, magnesium, silicon, zinc and manganese. There is a substantial variety of minor alloying elements such as iron, chromium, titanium, lead, bismuth, nickel, boron, vanadium, zirconium and beryllium.

The procedures and processes used during manufacturing divide the aluminium alloys into two different classes, namely wrought alloys and casting alloys.

Wrought alloys are those alloys which are worked by rolling, extruding, drawing, forming or forging into the desired shapes. Some examples are sheets, plates, wire, rod and extrusions.

Casting alloys are those alloys which are poured in a molten condition into a mould cavity, resulting in a shape fairly close to the shape of the finished product.

Both wrought and casting alloys may be either heat-treatable or non heat-treatable. This provides for a second major class differentiation. Alloys are described by a four digit system. Further treatment of heat-treatable alloys is denoted by the letter T followed by one or more digits; non heat-treatable alloys are denoted by the letter H. Higher heat-treatment numbers represent higher values of strength. The different heat-treatment symbols or tempergrades of both wrought and cast alloys can be seen in Table 2.1 given at the end of this chapter.

The designation system for wrought alloys, as originally compiled by the AA in the United States of America, has become the international aluminium alloy designation system. This system is also used in South Africa.

3. **Physical Properties of Aluminium Alloys**

A comprehensive comparison of the main physical properties, at room temperature, of aluminium, steel and stainless steel was done by Mazzolani [9]. A summary is shown in Table 2.2.

With regard to the most important structural parameters it can be said that:

- The density of aluminium varies for different alloys between 2600 and 2800 kg/m³. This is approximately one third of the density of steel.

- The Young's Modulus of aluminium varies between 68.5 GPa and 74.5 GPa for different alloys. This is approximately one third of that of steel.
Table 2.3 gives the characteristics of the aluminium alloy 6261 which was used in the determination of the test data for this research project. These values were obtained from a document compiled by the Aluminium Federation of South Africa (AFSA) [10].

It is noteworthy that some aluminium alloys do not lose strength at low temperatures, in fact aluminium-magnesium alloys increase in strength at temperatures below room temperature [43].

4. The Mechanical Properties of a Gradual Yielding Material

It is well known that aluminium yields gradually under load. This is also discussed in detail by Mazzolani [9]. In preparing the experimental procedures for the establishment of the required mechanical properties of an aluminium alloy, the fact that it is gradually yielding should be taken into account. The properties that are needed are illustrated in Figure 2.1 and are defined as follows:

- **The Initial Modulus**, $E_0$, is the tangent to the stress-strain curve at the origin of the curve.

- **The Tangent Modulus**, $E_t$, is the slope of the stress-strain curve at each value of stress.

- **The Secant Modulus**, $E_s$, is the ratio of the stress to the strain at each value of stress.

- **The Proportional Limit**, $f_p$, or the equivalent of the elastic limit, is the stress at an offset of 0.01% strain.

- **The Yield Strength**, $f_y$, is the stress at the offset of 0.2% strain.

Other mechanical properties also needed in design are:

- **The Ultimate or Tensile Strength**, $f_u$, is the maximum stress obtained during a tensile test.

- **Ductility** is defined as the extent to which a material can undergo plastic deformation without rupture. The ductility of a material is determined by calculating the percentage elongation of a 50 mm gauge length after fracture. This is accomplished by replacing the two broken pieces together and then measuring the elongation of the gauge length. The elongation is then expressed as a percentage of the gauge length.
a. Experimental Procedures

The CSRG has vast experience in the experimental procedures for establishing the mechanical properties of a gradual yielding material. The approximately 70 papers contained in References 11 to 14 are an indication of this fact. The information for and the knowledge needed to perform the experimental work are therefore in place. A detailed discussion on the procedures is given in Chapter 3.

b. Analytical Model for Stress-Strain Curves

Since values derived from the stress-strain curves are required for different stress levels during the design process. It is therefore necessary that an equation be developed for non-linear stress-strain curves, such as for gradual yielding materials, from which the values can easily be computed. The model that has been adopted by the CSRG as well as the ASCE [1] for stainless steels and in principle supported by Mazzolani [9] for aluminium, is known as the Ramberg-Osgood equation [Reference 1 and 15], although it is a revised form of an equation originally suggested by Osgood [16]. It has been shown that this equation represent experimental data accurately.

This equation is as follows:

$$
\varepsilon = \frac{f}{E_0} + 0.002 \left( \frac{f}{f_y} \right)^n
$$

(2.1)

where according to Hill [35] and revised by Van der Merwe [6]:

$$
\log \left( \frac{0.002}{0.0001} \right)
\log \left( \frac{f_y}{f_p} \right)
$$

(2.2)

$$
n = \frac{\log \left( \frac{0.002}{0.0001} \right)}{\log \left( \frac{f_y}{f_p} \right)}
$$
In the above equations,
\[\varepsilon = \text{normal strain}\]
\[f = \text{normal stress}\]
\[E_o = \text{initial modulus}\]
\[f_y = \text{yield strength (0.2\% offset strength)}\]
\[f_p = \text{proportional limit (0.01\% offset strength)}\]

The Secant Modulus, \(E_s\), as defined earlier, may be calculated from Eq. (2.4).

\[
E_s = \frac{f}{\varepsilon} \tag{2.3}
\]

\[
E_s = \frac{E_0}{1 + 0.002E_0 \left(\frac{f}{f_y}\right)^{n-1}} \tag{2.4}
\]

The Tangent Modulus, \(E_t\), is a function of stress and is given by Eq. (2.5). This equation is developed from the inverse of the first derivative of the strain in Eq. (2.1) with respect to stress.

\[
E_t = \frac{df}{d\varepsilon} = \frac{f_yE_0}{f_y + 0.002nE_0 \left(\frac{f}{f_y}\right)^{n-1}} \tag{2.5}
\]

C. REVIEW OF LITERATURE ON COLUMN BEHAVIOUR

1. General Remarks

An investigation of literature is needed in order to study and develop a better understanding of column behaviour. The author reports on his investigation in this section. Elastic and inelastic column failure are two column failure characteristics that are recognised.
The formulation of a general differential equation which can be used to predict the elastic behaviour of compression members will be done. This equation however results in optimistic values for compression members in the inelastic stress range. This is mainly due to the influence of residual stresses in the section and initial imperfections in the member, such as the out-of-straightness of the column.

Some of the different approaches followed by previous investigators on the behaviour of compression members in the inelastic stress range are also discussed in this section.

2. Load-Deformation Behaviour of Columns

The behaviour of columns can best be described by using two load-deformation parameters namely the axial shortening, \( z \), and the lateral deflection, \( v \), of a column under axial load \( C \). The typical load-deformation curves for columns of different lengths are shown in Figure 2.2. The load-deformation behaviour of an intermediate length column is different to that of a stub column and elastic column in the following manner.

The length of a stub column is such that it will act essentially like a compression test specimen. Under load it will only shorten and will not deflect laterally. The load on the stub column will increase until the section becomes fully plastic at the yield load, that is

\[
C_y = Af_y
\]  

(2.6)

where

- \( C_y \) = yield load of the column,
- \( A \) = cross sectional area of the column,
- \( f_y \) = yield stress of the column material.

For a gradual yielding material such as aluminium (or stainless steel) the yield load \( C_y \), can be recorded at the 0.2% offset strain, as stated before.

An intermediate length column that is initially straight will start to deflect (bend) laterally at the tangent modulus load, \( C_t \). It is important to know that at this load the deflection is instantaneous and as soon as the column buckles, unloading of the fibres may take place, but the load on the column will continue to increase until it approaches the reduced modulus load, \( C_m \), asymptotically with increase in column deflection. Only a relative small increase of load is possible beyond \( C_t \), after which unloading of the load-deformation relationship begins. The ultimate load of a column will lie between these two limits, \( C_t \) and \( C_m \).

Slender columns will start to buckle near or at the critical elastic load, \( C_e \). As long as the material remains elastic, the load will asymptotically approach the maximum critical load.
Lateral deflection occurs with a small increase of load until yielding sets in at some part of the member. The load then begins to diminish as a result.

Because of this, practical columns are categorised and designed as either inelastic columns (columns of intermediate length) or as slender columns which behave elastically.

3. Elastic Behaviour of Columns

a. The Classical Theory of Column Behaviour

The Swiss mathematician, Leonard Euler, wrote the first paper of great importance concerning the buckling of columns in 1757. The beginning of theoretical and experimental investigation into column behaviour was marked by the formulation of the most famous of all column equations, The Euler formula [17, 18].

The Euler formula was derived for a perfectly straight, concentrically loaded, homogeneous, long slender column with pinned ends. It is assumed that this perfect column is deflected laterally by a concentric load, $C$, as shown in Figure 2.3, and if the concentric load, $C$, is removed the column will straighten out completely. The assumption is made that the column behaves elastically.

From the free body diagram of Figure 2.3 it can be seen that

$$M = -Cy$$

(2.7)

By using the basic elastic theory, the differential equation is presented by

$$\frac{d^2y}{dx^2} + \frac{Cy}{E_o I} = 0$$

(2.8)

The solution to this differential equation reveals the elastic buckling load, $C_e$, and is given by Eq. (2.9).

$$C_e = \frac{\pi^2 E_o I}{L^2}$$

(2.9)

where

- $E_o$ = initial modulus
- $I$ = moment of inertia of the cross-section
- $L$ = effective length of the column
C_0, which is given by Eq. (2.9) is the smaller of the buckling loads, determined about the minor or major axis of the section to the corresponding effective length, L_y or L_x.

The classical Euler derivation is limited in the general sense that it does not include any other possibilities of failure such as failure due to torsion.

b. General Formulation of a Differential Equation for the Behaviour of Doubly Symmetric Sections

All the possibilities of column failure, including failure due to torsion, need to be considered. Galambos [19] gives the derivation of a differential equation predicting all the possibilities of the behaviour of columns.

Using an already deformed compression member, these different equations were formulated in order to ascertain the influence of each of the actions destabilising the member. Each destabilising effect cannot be considered separately but has to be combined into one formulation, because the deformation and internal forces are no longer independent of one another. In order to simplify such differential equations the following assumptions are made:

- The ends of the member are pinned and are prevented from translating with respect to one another.
- The shape of the cross-section does not change, therefore it is assumed that the section properties remain constant.
- The member is assumed to be initially straight and prismatic.
- The deflections and deformations are assumed to be small.
- The forces acting on the member are only applied at its ends.
- Finally it is assumed that the material behaves elastically.

The following three differential equations, describing the equilibrium of a doubly symmetric column subjected to an axial load, C, are given by Galambos [19].

\[ E_o I_y v'' + C v = 0 \]  \hspace{1cm} (2.10)

\[ E_o I_x u'' + C u = 0 \]  \hspace{1cm} (2.11)

\[ E_o C_0 \phi''' - \left( G_o J - C r_o^2 \right) \phi' = 0 \]  \hspace{1cm} (2.12)

where

\( v \) = centre deflection of the column in the y-direction
\( u \) = centre deflection of the column in the x-direction
\( \phi \) = centre rotation of the column
\( C_w \) = warping constant
\( G_0 \) = initial shear modulus
\( J \) = St. Venant torsion constant.

All derivatives are with respect to the direction along the z-axis of the member, as shown in Figure 2.4.

It can be shown after some algebraic procedures that the solution of the three differential equations, given in Eq. (2.10) to (2.12), is given by the following characteristic equation.

\[
ro^2 (C_{or} - C_x)(C_{or} - C_y)(C_{or} - C_z) = 0
\]

(2.13)

where

\[
ro^2 = r_x^2 + r_y^2
\]

\( C_x \) = flexural buckling load about the x-axis
\[
= \frac{\pi^2 E_o I_x}{(kL_x)^2}
\]

\( C_y \) = flexural buckling load about the y-axis
\[
= \frac{\pi^2 E_o I_y}{(kL_y)^2}
\]

\( C_z \) = torsional buckling load about the z-axis
\[
= \left( \frac{\pi^2 E_o C_w}{(kL_z)^2} + G_0 J \right)^\frac{1}{2}
\]

\( kL_{x,y,z} \) = effective length of the column in the various directions.

The critical buckling load is the lowest value of the following three solutions of the characteristic equation

\[
C_{or1} = C_x = \frac{\pi^2 E_o I_x}{(kL_x)^2}
\]

(2.14)

\[
C_{or2} = C_y = \frac{\pi^2 E_o I_y}{(kL_y)^2}
\]

(2.15)
\[ C_{cr3} = C_z = \left( \frac{\pi^2 E_z C_w}{(kL_z)^2} + G_G J \right) \frac{1}{r_o^2} \]  \hspace{1cm} (2.16)

By inspecting the above possible buckling loads, it is evident that for doubly symmetric sections, the column fails either in pure bending, Eq. (2.14) and (2.15), or in pure torsion, Eq. (2.16), depending on the column length and shape of the section.

Further information about the theory of singly symmetric sections can be obtained from Yu [20].

Eqs. (2.14) and (2.15) govern the elastic critical buckling load of a doubly symmetric column, depending on the effective length, \((kL_z)\), or \((kL_y)\) of the column. In the event that \(I_x > I_y\) then \((kL_x) \geq (kL_y)\) and Eq. (2.15) will then represent the elastic critical load, \(C_e\), of the column. This value for \(C_e\) is very similar to the value given by the classical Euler formula, Eq. (2.9).

As stated above, one of the assumptions that was made in this formulation was that the material behaves elastically. This, however, is not generally true for gradual yielding materials such as aluminium. Yet, the assumption was made knowingly that it would only apply to the members that fail at a stress below the proportional limit, \(f_p\). Columns that fail at a stress above the proportional limit, is said to behave inelastic and then other parameters influence the column strength. Inelastic column behaviour is subsequently discussed.

4. Inelastic Behaviour of Column Members

Columns with slenderness ratios varying from short to intermediate, behave inelastic. It was found according to Estuar and Tall [21] that the variables influencing column strength were numerous, however, the most important factors are as follows:

- The magnitude and distribution of residual stresses in the section.
- The static yield strength, \(f_y\), of the material and the initial modulus, \(E_0\).
- The cross-sectional dimensions of the column.
- The initial out-of-straightness of the column which includes an unsymmetrical residual stress distribution and accidental eccentricities.

Estuar and Tall [21] found that the strength of inelastic columns furthermore depend mainly on two parameters, residual stresses and the initial out-of-straightness of the column. Both variables are unfortunately not easily quantifiable by the designer. The strength of compression members are significantly reduced by these two parameters and are the main causes for the scatter of experimental results in the inelastic stress range of column behaviour.
The consideration of only the initial out-of-straightness of the column is one approach for predicting column behaviour, that can be used in design specifications. However, it is now generally accepted that both residual stresses and the above mentioned factors play important roles. In a number of cases the role of the residual stresses is predominant. The influence of residual stresses on column behaviour will be discussed in detail later in this section.

a. The Tangent and Residual Modulus Concepts

A short summary of the history and basis of inelastic column behaviour will be presented before the influence of residual stresses and the effect of initial eccentricities on column behaviour are discussed.

The tangent modulus theory and the residual modulus theory are two theories that form the basis of inelastic column behaviour. These two theories are discussed by various researchers [References 20 and 22 to 24] and can be summarised as follows.

The tangent modulus method proposed that the tangent modulus load, \( C_t \), be given by Eq. (2.17)

\[
C_t = \frac{\pi^2 E_t I}{(kL)^2}
\]  

(2.17)

where

\( E_t \) = tangent modulus at a stress corresponding to the buckling load, \( C_t \).

The reduced modulus concept or double modulus concept gives the buckling load, \( C_{rm} \), of the inelastic column as

\[
C_{rm} = \frac{\pi^2 E_{rm} I}{(kL)^2}
\]  

(2.18)

This concept was also produced because the tangent modulus concept did not include the effect of elastic unloading. The reduced modulus is a function of \( E_o \), \( E_t \) and the shape of the section, and is given by Eq. (2.19).

\[
E_{rm} = E_o \left( \frac{I_1}{I} \right) + E_t \left( \frac{I_2}{I} \right)
\]  

(2.19)

where

\( I = \) moment of inertia about the axis of bending.
\[ I_1 = \text{moment of inertia about the neutral axis of the area on the unloading side after buckling.} \]
\[ I_2 = \text{moment of inertia about the neutral axis of the area on the loading side after buckling.} \]

These two concepts, the tangent modulus and reduced modulus concepts, were derived on the assumption that the column remain straight up to the buckling load.

The tangent modulus load depends on the assumption of a constant tangent modulus across the complete cross-section. Figure 2.5 shows that as soon as the slightest deflection occurs, the material on the convex side will unload, while the material on the concave side will continue loading. Therefore, when a deflection occurs the constant tangent modulus across the cross-section is no longer constant. Because of this the theory was declared invalid and replaced by the reduced modulus concept. The reduced modulus theory appeared reasonable due to the recognition of the simultaneous loading and unloading of fibres as shown by Figure 2.5. The experimental results however always tended to be much closer to the predicted tangent modulus values than the reduced modulus values.

The correct relationship between these two theories was shown by Shanley [25], that theorised that there is nothing to prevent the column from bending simultaneously with increasing axial load upon reaching the critical tangent modulus load. This was regarded as an introduction to a completely new concept on column behaviour.

The following is a summary of the conclusions that was made by Shanley [25] on the analysis of a simplified inelastic column.

- The tangent modulus formula gives the maximum load at which an initially straight axially loaded column will remain straight.
- The column load may exceed the tangent modulus load but cannot be greater than the reduced modulus load.
- Loading beyond the tangent modulus load will cause bowing, which will produce a permanent bending deformation.
- There will be some portion of the column cross-section for which the stress will never exceed the tangent modulus stress.
- After the tangent modulus load is exceeded the compressive strain over a portion of the cross-section will increase more rapidly than the average strain.
- For most engineering materials the decrease in tangent modulus with increasing strain will limit the amount by which the column load may exceed the tangent modulus value.
- The tangent modulus equation should be used as a basis for determining the buckling strength of members in the inelastic range.

Figure 2.5 shows graphically the contribution and conclusions made by Shanley [25].

The discussion above is based on a hypothetical column that bears little resemblance to an actual column. Extensions of this theory to a more general case is largely a matter of
mathematics. Lengthy algebraic manipulations have been done on equations predicting the inelastic behaviour of columns. It is therefore necessary to know that the solution is no more than a different way of introducing curve-fitting parameters into an equation to fit a broad scatter of experimental results.

b. Influence of Residual Stresses on Column Behaviour

The Shanley [25] theory of column behaviour was used by Osgood [26] to evaluate the effect of residual stresses on column strength. The usual assumption that plane sections remain plane, stress-strain relations are the same under uniform strain and the column is initially straight, apply. An additional assumptions is made that the magnitude of the residual stresses is such that when an axial load is applied, no appreciable deflection occurs, specially no deflection large enough to invalidate the applicability of the Shanley [25] theory.

The increase in stress at any point, along the cross-section resulting from buckling is

$$E_t (\varepsilon_x - \varepsilon_o)$$ \hspace{1cm} (2.20)

where

- \(E_t\) = tangent modulus at the stress.
- \(\varepsilon_x\) = the strain at any other point along the cross-section, a distance \(x\) from the axis of constant strain.
- \(\varepsilon_o\) = constant strain during the transition from the straight to the buckled configuration.

For equilibrium the moment of the increases of stress over any cross-section must be equal to the moment of the axially applied load, \(C\), about the centroidal axis of the cross-section perpendicular to the plain of bending, that is

$$\int_A E_t (\varepsilon_x - \varepsilon_o) x dA = -Cv$$ \hspace{1cm} (2.21)

where

- \(A\) = cross-sectional area of the column.

The strains may be eliminated from Eq. (2.21) by considering that

$$\frac{d^2 v}{dx^2} = \frac{(\varepsilon_x - \varepsilon_o)}{x}$$ \hspace{1cm} (2.22)

Where \(x\) is measured parallel to the axis of the column.

Substituting Eq. (2.22) into Eq. (2.21) gives
\[
\frac{d^2v}{dx^2} \int_A E_0 x^2 dA = -Cv \tag{2.23}
\]

Eq. (2.23) is similar to the Euler formula given by Eq. (2.9) and the solution to this differential equation is given by Eq. (2.24).

\[
C = \frac{\pi^2 E_0 I}{L^2} \tag{2.24}
\]

If the applied stress, \(f\), does not exceed the elastic limit, \(E_t = E_0\), the elastic modulus, and the residual stresses have no effect.

However, if the applied stress, \(f\), exceeds the elastic limit, the column stiffness is reduced and thus the load carrying is reduced. If neglected, residual stresses will cause yielding earlier than is expected and they cause a reduction in the stiffness of the member. Since buckling is sensitive to member stiffness it is thus concluded that residual stresses play a significant role in buckling analysis.

To account for this, stub column tests are performed. The stub column test gives a stress-strain curve showing the effect of residual stresses. The proportional limit (elastic limit), \(f_p\), the static yield strength, \(f_y\), the initial modulus, \(E_0\), secant modulus, \(E_s\), as well as the tangent modulus, \(E_t\), are important data obtained from this curve. This data are necessary for the prediction of column strength. Results of experimental stub column tests performed on the specific chosen aluminium I section for this short dissertation as well as tests done by the author in collaboration with Seynaeve [27], on the same section will be discussed in Chapter 5.

The stub column tests, which are frequently used to obtain peak values of compressive residual stresses, are not influenced by the stiffness reduction effect because of the very low slenderness ratio of the column. The length of the stub column should be long enough to retain the original residual stress distribution of the section.

Lay and Ward [28] found that when columns are designed using stub column properties, no provision is needed to account for the residual stresses in inelastic columns.

By substituting the tangent modulus, \(E_t\), given by Eq. (2.5) into Eq. (2.24), where \(E_0\), \(f_y\) and \(f_p\) are the mechanical properties obtained from the stub column’s stress-strain curve, the critical buckling stress of the column can be computed by using Eq. (2.25)

\[
f_{cr} = \frac{\pi^2 f_y E_0}{\left(\frac{KL}{r}\right)^2 \left(f_y + 0.002nE_0\left(\frac{f_{cr}}{f_y}\right)^{n-1}\right)} \tag{2.25}
\]
where
\[ f_{cr} = \text{critical buckling stress of the column} \]
\[ n = \text{as stated before by Eq. (2.2)} \]

Eq. (2.25) requires an iteration procedure in order to find the critical buckling stress of the column.

Bleich [29] suggested a parabolic column curve in the inelastic stress range of slenderness in order to simplify the tangent modulus approach for practical design of column members. Non-linear column behaviour begins when the stress in the column exceeds the proportional limit, \( f_p \), invalidating the assumption of the Euler formula. Eq. (2.26) gives the parabolic column curve suggested by Bleich [29] for the inelastic stress range of column slenderness.

\[ f_{cr} = f_y \left( \frac{f_y - f_p}{f_p} \right) f_p \left( \frac{kL}{r} \right)^2 \] (2.26)

Research done by Fang [30] suggests that the proportional limit be taken as half the yield strength. The parabolic curve suggested by Bleich [29] then reduces to

\[ f_{cr} = \left( 1 - \frac{f_y}{4f_e} \right) f_y \] (2.27)

where

\[ f_e = \frac{\pi^2 E_o}{\left( \frac{kL}{r} \right)^2} \] (2.28)

The AISI Cold- Formed Steel Design Manual [31] includes Eq. (2.27) for the design of concentrically loaded compression members.

c. Initial Out-of-Straightness of Columns

Another method of predicting column behaviour is to consider the effects of initial out-of-straightness of the column. Figure 2.6 shows a pin ended strut similar to the Euler column. The only difference is that it has some initial out-of-straightness, \( v_0 \), in the unloaded state.

The differential equation governing the equilibrium of the column is
\[
\frac{d^2 y}{dx^2} + \frac{C(y + \nu_0)}{E_0 I} = 0
\]  
(2.29)

or

\[
\frac{d^2 y}{dx^2} + \frac{Cy}{E_0 I} = -\frac{C\nu_0}{E_0 I}
\]  
(2.30)

The initial deflection, \(\nu_0\), could be expressed as an infinite trigonometric series, although it can strictly only be determined by measurement. Coates et al [32] made the assumption that the initial deflection can be adequately represented by the first term of the trigonometric series, since it is not an unreasonable approximation and because it satisfies the boundary conditions.

Therefore it is assumed that

\[
\nu_0 = V \sin\left(\frac{n\pi}{L}\right)
\]  
(2.31)

where

\[V = \text{central initial deflection of the column.}\]

The solution to the nonhomogeneous differential equation, Eq. (2.30) is given by

\[
y = \left(\frac{C}{E_0 I}\right) \left(\frac{\pi^2}{L^2} - \frac{C}{E_0 I}\right) V \sin\left(\frac{n\pi}{L}\right)
\]  
(2.32)

Using the simplification of \(\rho = \frac{C}{C_e}\)

where \(C_e = \) as given by Eq. (2.9).

The Eq. (2.30) then becomes

\[
y = \frac{\rho}{1 - \rho} V \sin\left(\frac{n\pi}{L}\right)
\]  
(2.33)

The total deflection of the strut is calculated by
\[ y_t = y_o + y \]  
(2.34)

or

\[ y_t = \frac{V}{1 - \rho} \sin \left( \frac{\pi x}{L} \right) \]  
(2.35)

The maximum deflection, \( y_{\text{max}} \), will occur in the centre of the strut where \( x = \frac{L}{2} \) hence

\[ y_{\text{max}} = \frac{V}{1 - \rho} \]  
(2.36)

The initial deflection at the strut’s centre is multiplied by the ratio \( \frac{1}{1 - \rho} \) as a result of the axial compressive load and bends to infinity as \( C \) approaches \( C_e \). The deflection will therefore cease to be small compared to the length and yielding of the strut material.

Substituting \( \rho = \frac{C}{C_e} \) back into Eq. (2.36) and letting \( C = C_{\text{cr}} \) the yield load, \( C_y \), can be determined by using basic elastic theory as

\[ C_y = C_{\text{cr}} + \frac{Mc}{r^2} \]

or

\[ = C_{\text{cr}} + \frac{C_{\text{cr}}y_{\text{max}}c}{r^2} \]  
(2.37)

or

\[ \frac{cV}{r^2 \left( 1 - \frac{C_{\text{cr}}}{C_e} \right)} \]  
(2.38)

or

\[ C_{\text{cr}}^2 - C_{\text{cr}} (C_y + C_e (1 + \eta)) + C_e C_y = 0 \]  
(2.39)
where
\[ C_y = \text{the crushing load at the yield stress} = A f_y \]
\[ C_e = \text{elastic flexural buckling} = \frac{\pi^2 E_o A}{(kL/r)^2} \]
\[ \eta = \text{initial curvature parameter} = \frac{V_c}{r^2} \]
\[ r = \text{radius of gyration about the buckling axis}. \]
\[ c = \text{the distance from the neutral axis to the extreme fibres}. \]
\[ V = \text{the amplitude of the initial curvature}. \]

The critical buckling load, \( C_{cr} \), is given by the smaller root of Eq. (2.39) as

\[ C_{cr} = \left( \frac{C_y + (1 + \eta)C_e}{2} \right) - \sqrt{\left( \frac{C_y + (1 + \eta)C_e}{2} \right)^2 - C_e C_y} \quad (2.40) \]

After a series of tests Robertson [33] suggested that \( \eta \) should be taken proportional to the slenderness ratio, and taking into account that slender columns are likely to be less straight than stiff (short) ones. It was also suggested that the initial curvature parameter, \( \eta \), should be taken as \( 0.003 \left( \frac{L}{r} \right) \) rather than \( \frac{V_c}{r^2} \) for steel.

Later a better value for \( \eta \) was suggested, it should be taken as \( 0.3 \left( \frac{L}{100r} \right)^2 \). This value was used in the British design specifications BS499 [34] and known as the Perry-Robertson [33] formula.

D. CRITERIA AND METHODS FOR THE DESIGN OF COLUMNS

1. General Remarks

This part of the literature survey was done to develop an understanding of the various criteria and methods that are being used for the design of carbon and low-alloy steel, stainless steel and aluminium columns. The design criteria and methods under consideration are those in accordance with the specifications listed below.
The fundamental Euler-equation, Eq. (2.9), is normally recognised as the design equation long and slender columns where elastic behaviour is expected. The point of transition between elastic and inelastic behaviour is not consistent. The major differences appear to be in the methods or procedures for taking inelastic behaviour into account.

The different design criteria, methods and procedures are summarised in the following paragraphs.

2. **Canadian Standards Association [3]: Design of Structural Steel columns**

In accordance with the National Standard of Canada document CAN/CSA-S16.1-M89: Limit States Design of Steel Structures [3] as explained by Kulak, Adams and Gilmor [36], the maximum strength of a column is described by means of a five-part equation depending on empirically determined parameters as well as a non-dimensional slenderness factor \( \lambda \), where:

\[
\lambda = \frac{kL}{r} \sqrt{\frac{f_y}{\pi^2 E_0}} \tag{2.41}
\]

The factored compressive resistance \( C_r \) is given by the following equations:

for \( 0 \leq \lambda \leq 0.15 \)

\[
C_r = \phi A f_y \tag{2.42a}
\]
0.15 \leq \lambda \leq 1.0
\[ C_r = \phi A f_y (1.031 - 0.202 \lambda - 0.222 \lambda^2) \] (2.42b)

1.0 \leq \lambda \leq 2.0
\[ C_r = \phi A f_y (-0.111 + 0.636 \lambda^{-1} + 0.087 \lambda^{-2}) \] (2.42c)

2.0 \leq \lambda \leq 3.6
\[ C_r = \phi A f_y (0.009 + 0.877 \lambda^{-2}) \] (2.42d)

3.6 \leq \lambda
\[ C_r = \phi A f_y \lambda^{-2} \] (2.42e)

where
\begin{align*}
\lambda &= \text{non-dimensional slenderness factor} \\
r &= \text{radius of gyration} \\
kL &= \text{effective length of column} \\
C_r &= \text{factored compressive resistance} \\
\phi &= \text{resistance factor} \\
A &= \text{area of the section of the column} \\
f_y &= \text{specified minimum yield strength} \\
E &= \text{elastic modulus}
\end{align*}

The document: National Standard of Canada CAN/CSA-S16.1-M89 [3] is in the process of being superseded by an update. It is expected that Eq. (2.42a) to (2.42e) will be replaced by a single equation, Eq. (2.43).
\[ C_r = \phi Af_y \left( 1 + \lambda^{2n} \right)^{\frac{1}{n}} \]  \hspace{1cm} (2.43)

In which \( n = 1.34 \) (constant).

It can also be expected that the SABS 0162-1:1994 [42] will soon reflect the change.

3. **American Institute of Steel Construction [4]: Design of Structural Steel Columns**

In accordance with the procedures prescribed by the AISC [4] Eq. (2.44) is used for the design of carbon steel columns.

For \( \lambda < 1.5 \)

\[ C_r = \phi Af_y 0.658\lambda^2 \]  \hspace{1cm} (2.44a)

For \( \lambda \geq 1.5 \)

\[ C_r = \phi Af_y \frac{0.877}{\lambda^2} \]  \hspace{1cm} (2.44b)

4. **American Society of Civil Engineers [1]: Design of Stainless Steel Columns**

A single equation, Eq. (2.45), is used for the design of stainless steel columns:

\[ f_{cr} = \frac{\pi^2 E_t}{(kL/r)} \]  \hspace{1cm} (2.45)

Where \( E_t = \frac{f_y E_0}{f_y + 0.002nE_0 \left( \frac{f}{f_y} \right)^{n-1}} \)  \hspace{1cm} (2.5)
The gradual yielding nature of stainless steels is taken into account by replacing the initial modulus, $E_0$, by the equation for the tangent modulus, $E_t$. When Eq. (2.5) is substituted into Eq. (2.45), $f$ should be replaced by $f_{cr}$. The result of this substitution was shown in Eq. (2.25). For a given slenderness ratio $\left( \frac{kL}{r} \right)$ and a specific material ($n$, $E_0$, $f_0$ and $f_y$), the critical column buckling stress $f_{cr}$ can only be computed from Eq. (2.25) by a process of iteration.

for $f_{cr} \leq f_y$

$$f_{cr} = \frac{\pi^2 f_y E_0}{\left( \frac{kL}{r} \right)^2 \left( f_y + 0.002nE_0 \left( \frac{f_{cr}}{f_y} \right)^{n-1} \right)}$$

(2.25)

5. Aluminium Association [5]: The Design of Aluminium Columns

In the case of aluminium the design criteria and method of design is determined by the tempergrade of the aluminium alloy to be used as the column.

There are several general equations for determining the maximum permissible compression stresses ($f_{cr}$) in the columns.

Firstly there are two slenderness limits, $\lambda_1$ and $\lambda_2$, where:

$$\lambda_1 = \frac{H_e - \frac{n_u f_y}{k_e n_y}}{K_e}$$

(2.46)

and

$$\lambda_2 = N_e$$

(2.47)

For a slenderness ratio $\left( \frac{kL}{r} \right) \leq \lambda_1$:

$$f_{cr} = \frac{f_y}{k_e n_y}$$

(2.48a)

For a slenderness ratio $\left( \frac{kL}{r} \right)$ between $\lambda_1$ and $\lambda_2$, Eq. (2.48b) is used:
for \( \frac{kL}{r} < \lambda_2 \):

\[
f_{cr} = \frac{1}{n_c} \left( H_e - K_e \left( \frac{kL}{r} \right) \right)
\]  
(2.48b)

for \( \frac{kL}{r} > \lambda_2 \):

\[
f_{cr} = \frac{\pi^2 E}{n_u \left( \frac{kL}{r} \right)^2}
\]  
(2.48c)

In the above equations:

- \( k_e \) = a coefficient for compression members
- \( n_y \) = factor of safety for the yield strength
- \( n_u \) = factor of safety for the buckling strength

The buckling constants \( H_e, K_e \) and \( N_e \) are calculated from different equations, depending on the temper-designation. For temper grades 0, T1, T3 or T4 and tempered designations beginning with H, the equations for the buckling constants of compression in columns are:

\[
H_e = f_y \left[ 1 + \sqrt{\frac{f_y}{6890}} \right]
\]  
(2.49)

\[
K_e = \frac{H_e}{20} \sqrt{\frac{6H_e}{E}}
\]  
(2.50)

\[
N_e = \frac{2H_e}{3K_e}
\]  
(2.51)

The equations for the buckling constants for tempered designations T5, T6, T8, T83 or T9 are:

\[
H_e = f_y \left[ 1 + \sqrt{\frac{f_y}{15500}} \right]
\]  
(2.52)
\[ K_e = \frac{H_c}{10} \sqrt{\frac{H_c}{E}} \]  
\[ N_e = 0.41 \frac{H_c}{K_e} \]  

(2.53)  
(2.54)

Graphs to illustrate the different equations will be presented in Chapter 4 by using the actual test data developed in Chapter 3.

E. CONCLUSION

It is clear from the vast amount of literature that the behaviour of columns have extensively been researched in the past and that the different criteria and methodologies for the design of columns are soundly based on theories and/or experimentation.
<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>USA Symbol</th>
<th>UK Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>As fabricated.</td>
<td>F</td>
<td>M</td>
</tr>
<tr>
<td>Annealed soft.</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Solution heat-treated. Unstable temper. (Only applicable to natural aging alloys.)</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>Thermally treated to produce stable tempers other than F, O or H.</td>
<td>T</td>
<td></td>
</tr>
<tr>
<td>Cooled from an elevated temperature shaping process and naturally aged to a substantially stable condition.</td>
<td>T1</td>
<td></td>
</tr>
<tr>
<td>Cooled from an elevated temperature shaping process, cold worked, and naturally aged to a substantially stable condition.</td>
<td>T2</td>
<td></td>
</tr>
<tr>
<td>Solution heat-treated, cold worked, and naturally aged to a substantially stable condition.</td>
<td>T3</td>
<td>TD</td>
</tr>
<tr>
<td>Solution heat-treated and naturally aged to a substantially stable condition.</td>
<td>T4</td>
<td>TB (W)</td>
</tr>
<tr>
<td>Cooled from an elevated temperature shaping process and then artificially aged.</td>
<td>T5</td>
<td>TE (P)</td>
</tr>
<tr>
<td>Solution heat-treated and then artificially aged.</td>
<td>T6</td>
<td>TF (WP)</td>
</tr>
<tr>
<td>Solution heat-treated and then stabilized.</td>
<td>T7</td>
<td></td>
</tr>
<tr>
<td>Solution heat-treated, cold worked, and then artificially aged.</td>
<td>T8</td>
<td>TH</td>
</tr>
<tr>
<td>Solution heat-treated, artificially aged and then cold worked.</td>
<td>T9</td>
<td></td>
</tr>
<tr>
<td>Cooled from an elevated temperature shaping process cold worked and then artificially aged.</td>
<td>T10</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2.2

**MAIN PHYSICAL PROPERTIES OF ALUMINIUM, STEEL AND STAINLESS STEEL [9]**

<table>
<thead>
<tr>
<th>PROPERTIES</th>
<th>ALUMINIUM</th>
<th>STEEL</th>
<th>STAINLESS STEEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average weight density (kg/m³)</td>
<td>2700</td>
<td>7850</td>
<td>7900</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>658</td>
<td>1450-1530</td>
<td>1450</td>
</tr>
<tr>
<td>Linear thermal expansion coefficient (°C⁻¹)</td>
<td>24×10⁻⁶</td>
<td>12×10⁻⁶</td>
<td>17.3×10⁻⁶</td>
</tr>
<tr>
<td>Specific heat (cal/g°C)</td>
<td>0.225</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Electrical resistivity (μΩm)</td>
<td>2.84</td>
<td>15.5</td>
<td>70</td>
</tr>
<tr>
<td>Young’s modulus, E, (GPa)</td>
<td>68.5</td>
<td>206</td>
<td>206</td>
</tr>
</tbody>
</table>

### TABLE 2.3

**PROPERTIES OF THE ALUMINIUM ALLOY 6261 [10]**

<table>
<thead>
<tr>
<th>PROPERTIES</th>
<th>ALUMINIUM (6261)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>2710</td>
</tr>
<tr>
<td>Melting range (°C)</td>
<td>570-655</td>
</tr>
<tr>
<td>Linear thermal expansion coefficient (°C⁻¹)</td>
<td>23×10⁻⁶</td>
</tr>
<tr>
<td>Young’s modulus, E, (GPa)</td>
<td>70</td>
</tr>
<tr>
<td>0.2% Proof stress, f₀, (MPa)</td>
<td>270</td>
</tr>
<tr>
<td>Ultimate tensile stress, fₚ, (MPa)</td>
<td>310</td>
</tr>
<tr>
<td>Elongation %</td>
<td>7</td>
</tr>
</tbody>
</table>
FIGURE 2.1

ILLUSTRATION OF THE DEFINITIONS OF THE MECHANICAL PROPERTIES $E_\text{m}$, $f_r$, $f_p$, $E_\text{t}$, $E_\text{s}$
FIGURE 2.2
LOAD-DEFLECTION CURVES FOR COLUMNS

STUB COLUMN

INELASTIC COLUMN

ELASTIC COLUMN
FIGURE 2.3

DERIVATION OF THE EULER FORMULA

FREE BODY DIAGRAM

EULER COLUMN
FIGURE 2.4

FORMULATION OF A GENERAL FORMULA FOR ELASTIC COLUMN BEHAVIOUR
FIGURE 2.5
SHANLEY’S CONTRIBUTION TO COLUMN FORMULAS

STRESS DIAGRAMS

TANGENT MODULUS

Stress at tangent modulus load

Additional bending stress

REDUCED MODULUS

Stress at reduced modulus load

Additional bending stress

SHANLEY'S CONTRIBUTION

Stress above tangent modulus load additional bending stresses
FIGURE 2.6

DERIVATION OF THE EULER FORMULA WITH INITIAL CURVATURE

\[ M = -C(y + v_0) \]

FREE BODY DIAGRAM

EULER COLUMN
CHAPTER 3

THE MECHANICAL PROPERTIES OF THE ALUMINIUM ALLOY 6261-T6

* This Chapter is a revised edition of a similar Chapter previously done by the author, Van Der Merwe [2].

A. GENERAL REMARKS

The aluminium alloy under consideration is the heat-treatable, Al-Mg-Si, wrought alloy, 6261-T6. This is a suitable alloy for structural applications due to its good corrosion resistance and high level of strength. This aluminium alloy is regularly extruded in various shapes. A set of specific extrusions were produced for this project in alloy 6262-T6. A test certificate that shows the composition and certain mechanical properties of the specific material used for the production of the set of extrusions required for this project, was issued by the company. A copy of the certificate is included as Appendix A. A comparison of the information on the test certificate with the international standard of the specific alloy [5], confirms that the material meets all relevant requirements.

Four lengths of 6 m each of the I-section shown in Figure 3.1 were extruded from the same source of material (billet) to ensure that the composition of the different specimens are homogeneous and that the results for the different specimens can be compared and analysed. From these extrusions, as part of this study, a whole range of columns were prepared and tested, the results of these tests are shown in Chapter 5. The objective of this project, as formulated in Chapter 1, was to use the already established mechanical properties of the material of the extrusions (Van Der Merwe [2]) in a comparative study of the different column design criteria and methodologies, and to compare these methodologies to actual experimental data. This Chapter is a review of the determination of the mechanical properties done by Van Der Merwe [2].
B. EXPERIMENTAL PROCEDURES

1. Preparation of specimens

A 230 mm piece of extrusion was cut from each of the four I-sections. The flanges and web of each of the pieces of extrusion were cut into strips. These strips were accurately machined into tension and compression specimens. The dimensions of the tension and compression test specimens are shown in Figure 3.2.

Out of every piece of extrusion four compression and four tension test specimens were obtained from both the web and the flanges. There were also four T-section compression specimens prepared from the material where the web and flanges intersect. The longitudinal axis of the test specimens coincided with the direction of extrusion.

The tension test specimens were prepared in accordance with the dimensions outlined by the ASTM Standard A370-77 [19]. The Specification ASTM Standard E9-77 [20], together with the dimensions as described by Parks and Yu [21] were used in the preparation of the compression test specimens.

2. Test Procedure

An Instron 1195 Universal Testing Machine was used to do both tension and compression tests. Two strain gauges mounted on either side of the specimen in a full bridge configuration with temperature compensation were used to measure the average strain. In the case of the T-sections, only one strain gauge was mounted on the flange side due to practical difficulty in mounting another strain gauge on the web side. In this case a 120 Ω resistor was used to complete the bridge configuration. The influence of not using two strain gauges will be discussed later. By dividing the measured load by the initial cross-sectional area of the specimen, the stress exerted on the specimen was calculated.

An Orion Solartron datalogger controlled by a computer was used to collect data at sampling intervals of one half second. The stress and strain data were collected up to a of strain value approximately 1%. All information was stored on disk for later use.

a. Tension Tests

Due to the nature of the clamps or jaws used to secure a specimen in the Instron, an initial stress was applied to the specimen. This stress had to be removed prior to initiating the full test. The rate of separation of the cross-heads of the testing machine for the initial part
of the test, up to a strain of approximately 1%, was set at 0.5 mm/min. The rate of separation of the cross-heads was then increased to 2 mm/min for the remainder of the test. Just before failure, the ultimate load was observed for each tension specimen and recorded.

After failure, the elongation of the specimen was measured so that an indication of the ductility of the material could be calculated. This was done by fitting the two fractured ends of the specimen together and measuring the distance between the gauge marks. A gauge length of 50 mm was used.

b. Compression Tests

The compression test specimens were laterally supported by a compression test fixture to prevent overall buckling about the weak axis. The procedure of testing is very similar to that for the tension tests, except that the load was applied to the specimen via a specially manufactured sub-press and the test was stopped after a strain of 1% was reached.

C. EXPERIMENTAL RESULTS

The experimental data for each test was retrieved from the storage disk and processed by means of a spread sheet. The program enables the computation of the best fit straight line for the initial part of the stress-strain curve through a process of linear regression. The initial modulus, \( E_0 \), is the slope of the best fit straight line for the initial part of the stress-strain curve. The experimental data is then shifted along the strain axis so that the straight line that represents the initial modulus goes through the origin of the stress-strain curve. This partly compensates for zero point errors in this type of experimental work. The values of the proportional limit, \( f_p \), i.e. the stress at an offset of 0.01% strain and the yield strength, \( f_y \), i.e. the stress at an offset of 0.2% strain were obtained by simultaneously solving data strings numerically.

The above mentioned procedure was performed for each set of data of the tension and compression tests. A summary together with a statistical analysis of the mechanical properties obtained from the procedure above are given in Tables 3.1 and 3.2 at the end of this Chapter. Details of each individual test can be found in the original document, Van Der Merwe [2].
1. **Discussion on the Experimental Results**

The following observations were made after examining the test results of the experimental stress-strain curves of the tension and compression tests.

- The stress-strain curves are indeed of the gradual yielding type.
- The mechanical properties obtained in this study compare well with those given by AFSA [10] and the manufacturer of the extrusions as reflected in the test certificate in Appendix 1.
- According to the results, the initial modulus, $E_0$, is slightly less in tension than in compression. The reason for this might be the fact that in determining these values, the data of the T-sections were also used, and the T-sections had only one strain-gauge attached, which could give slightly higher values.
- The highest values for the proportional limit, $f_p$, and yield strength, $f_y$, are found in compression. That could be because of the same reason stated above.

After discarding the results of tests during which procedural problems were encountered, the remaining results of the tension and compression tests were statistically analysed. The following statistics were determined. The average of the values of $E_0$, $f_y$, $f_p$, $f_u$ and the percentage elongation for tension and the average values of $E_0$, $f_y$ and $f_p$ for compression were computed. The standard deviation and coefficient of variance (C.O.V.) of these values were calculated and were found to be low, indication that results are in a narrow band and hence acceptable.

The determination of the % elongation of the tension test specimens were performed as described in the section for tension tests. Not every test specimen fractured between the 50 mm gauge length lines. These specimens were not used for the statistical evaluation of the ductility of the aluminium alloy.

**D. ANALYTICAL STRESS-STRAIN CURVES**

The necessity for and the development of an analytical equation for the stress-strain curve was discussed in Chapter 2. Based on the results of a tension test on specimen 3TF4, the curves shown in Figure 3.3 have been drawn. The actual experimental mechanical properties of the tension test specimen 3TF4 can be seen in the original document by Van Der Merwe [2]. The properties are given below for ease of reference. The curve obtained by plotting the experimental data is compared with the curves obtained by using the Ramberg-Osgood equation, Eq. (2.1), repeated here for ease of reference.
\[ \varepsilon = \frac{f}{E_0} + 0.002 \left( \frac{f}{f_y} \right)^n \]  

(2.1)

where

\[ n = \frac{\log \left( \frac{0.002}{0.0001} \right)}{\log \left( \frac{f_y}{f_p} \right)} \]  

(2.2)

In the above equations,

- \( \varepsilon \) = normal strain
- \( f \) = normal stress

and the mechanical properties of the tension test specimen 3TF4 were

\begin{align*}
E_0 &= \text{initial modulus} = 69.22 \text{ GPa} \\
f_y &= \text{yield strength (0.2\% offset strength)} = 289.83 \text{ MPa} \\
f_p &= \text{proportional limit (0.01\% offset strength)} = 256.75 \text{ MPa} \\
n &= 24.72 \text{ (from Eq. 2.2)}
\end{align*}

Figure 3.3 shows the typical degree of accuracy achieved by using the Ramberg-Osgood equation for representing stress-strain data. At a first glance it appears that the curve labeled ‘Analytical’ however gives conservative values of the tangent modulus in the vicinity of the proportional limit. This does not come across clearly from Figure 3.3 although this has been shown to be the case in a separate analysis.

Figure 3.4 shows the analytical stress-strain curve, using the average mechanical properties given in Table 3.2, also given below for ease of reference.

\begin{align*}
E_0 &= \text{initial modulus} = 73.3 \text{ GPa} \\
f_y &= \text{yield strength (0.2\% offset strength)} = 312.1 \text{ MPa} \\
f_p &= \text{proportional limit (0.01\% offset strength)} = 282.9 \text{ MPa} \\
n &= 30.50 \text{ (from Eq. 2.2)}
\end{align*}
The graphical expression of the secant modulus, \( E_s \), and the tangent modulus, \( E_t \), as functions of stress are given in Figure 3.5. The corresponding equations developed in Chapter 2 are again repeated here for ease of reference.

\[
E_s = \frac{E_0}{1 + 0.002E_0 \left( \frac{f}{f_y} \right)^{n-1}}
\]  

and

\[
E_t = \frac{df}{d\varepsilon} = \frac{f_y E_0}{f_y + 0.002nE_0 \left( \frac{f}{f_y} \right)^{n-1}}
\]  

E. SUMMARY

The mechanical properties needed for structural design have been determined with an acceptable level of accuracy as indicated by statistical parameters. These values may now be used with confidence in the prediction of the structural behaviour of columns, which is the objective of the chapter that follows. Other mechanical properties such as the Modulus of Rigidity or Shear Modulus, \( G \), may also be determined from the information contained in Chapter 3.
### TABLE 3.1
MECHANICAL PROPERTIES BASED ON FLANGE AND WEB TENSION TESTS

<table>
<thead>
<tr>
<th></th>
<th>E₀ (MPa)</th>
<th>f₁ (MPa)</th>
<th>fₚ (MPa)</th>
<th>fᵤ (MPa)</th>
<th>% Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>70523.00</td>
<td>299.67</td>
<td>271.85</td>
<td>330.46</td>
<td>12.09</td>
</tr>
<tr>
<td>Stand. Dev.</td>
<td>475.01</td>
<td>5.95</td>
<td>7.82</td>
<td>5.31</td>
<td>1.15</td>
</tr>
<tr>
<td>C.O.V.</td>
<td>0.67</td>
<td>1.98</td>
<td>2.88</td>
<td>1.61</td>
<td>9.51</td>
</tr>
</tbody>
</table>

### TABLE 3.2
MECHANICAL PROPERTIES BASED ON FLANGE, WEB AND T-SECTION COMPRESSION TESTS

<table>
<thead>
<tr>
<th></th>
<th>E₀ (MPa)</th>
<th>f₁ (MPa)</th>
<th>fₚ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>73301.06</td>
<td>312.1</td>
<td>282.89</td>
</tr>
<tr>
<td>Stand. Dev.</td>
<td>1512.72</td>
<td>7.42</td>
<td>9.10</td>
</tr>
<tr>
<td>C.O.V.</td>
<td>2.06</td>
<td>2.38</td>
<td>3.22</td>
</tr>
</tbody>
</table>
FIGURE 3.1

DIMENSIONS OF THE CROSS-SECTION OF THE I-PROFILE

where  
\( d = 100 \text{ mm} \)
\( b = 75 \text{ mm} \)
\( t_w = 4.3 \text{ mm} \)
\( t_f = 4.3 \text{ mm} \)
\( R_1 = 1 \text{ mm} \)
\( R_2 = 3 \text{ mm} \)
\( R_3 = 1 \text{ mm} \)
FIGURE 3.2

DIMENSIONS OF THE TENSION AND COMPRESSION TEST SPECIMENS
FIGURE 3.3

COMPARISON BETWEEN THE EXPERIMENTAL AND ANALYTICAL STRESS-STRAIN CURVES OF TEST SPECIMEN 3TF4
FIGURE 3.4

ANALYTICAL STRESS-STRAIN CURVE OF THE MECHANICAL PROPERTIES (COMPRESSION) GIVEN IN TABLE 3.2
FIGURE 3.5

GRAPHICAL EXPRESSION OF $E_t$ AND $E_s$

- Secant Modulus ($E_s$)
- Tangent Modulus ($E_t$)
CHAPTER 4
A COMPARITIVE STUDY OF DIFFERENT CRITERIA AND METHODS FOR THE DESIGN OF COLUMNS

A. GENERAL REMARKS

The theories describing column behaviour and an understanding of the various criteria and methods that are being used for the design of carbon and low-alloy steel, stainless steel and aluminium columns were developed in Chapter 2. The design criteria and methods under consideration are those in accordance with the specifications listed below.


- Load and Resistance Factor Design Specification for Structural Steel Buildings, American Institute of Steel Construction (AISC) [4], for carbon and low-alloy steels;

- American Society of Civil Engineers (ASCE) Standard : ANSI/ASCE-8-90 : Specification for the Design of Cold-Formed Stainless Steel Structural Members [1];

- Aluminum Construction Manual, Section 1 : Specifications for Aluminum Structures, Aluminum Association (USA) [5].

The different design criteria are compared in this chapter.

B. THE DIFFERENT CRITERIA AND METHODS FOR THE DESIGN OF COLUMNS

Different criteria and methods are used for the design of columns. The following aspects appear to contribute towards which method or procedure is applicable :

2. Material's interest group.
4. Authority (association) in the country.
The fundamental Euler-equation, Eq. (2.9), is normally recognisable in the case of long and slender columns where elastic behaviour is expected. The point of transition between elastic and inelastic behaviour is however not consistent, not even for the same group of materials. The major differences lie in the design criteria, methods or procedures for compensating for inelastic behaviour.

The different design criteria were summarised in Chapter 2. Part of that information will be repeated in this section for ease of reference.

1. **The Design of Carbon and Low-alloy Steel (Structural Steel) Columns**

   **a. Canadian Standards Association [3]: Design of Structural Steel Columns**

   In accordance with the National Standard of Canada document CAN/CSA-S16. 1-M89: Limit States Design of Steel Structures [3] as explained by Kulak, Adams and Gilmor [36], the maximum strength of a column is described by means of a five-part equation depending on a non-dimensional slenderness factor \( \lambda \), where:

   \[
   \lambda = \frac{kL}{r} \frac{f_y}{\sqrt{\pi^2 E_0}}
   \]  

   (2.41)

   The factored compressive resistance \( C_r \), is given by the following equations:

   for \( 0 \leq \lambda \leq 0.15 \)
   
   \[
   C_r = \phi A f_y
   \]  

   (2.42a)

   for \( 0.15 \leq \lambda \leq 1.0 \)
   
   \[
   C_r = \phi A f_y \left(1.031 - 0.202 \lambda - 0.222 \lambda^2\right)
   \]  

   (2.42b)

   for \( 1.0 \leq \lambda \leq 2.0 \)
   
   \[
   C_r = \phi A f_y \left(-0.111 + 0.636 \lambda^{-1} + 0.087 \lambda^{-2}\right)
   \]  

   (2.42c)
\[ 2.0 \leq \lambda \leq 3.6 \]
\[ C_r = \phi f_y (0.009 + 0.877 \lambda^{-2}) \]  \hspace{1cm} (2.42d)

\[ 3.6 \leq \lambda \]
\[ C_r = \phi f_y \lambda^{-2} \]  \hspace{1cm} (2.42e)

where

\lambda = \text{non-dimensional slenderness factor} \\
r = \text{radius of gyration} \\
kL = \text{effective length of column} \\
C_r = \text{factored compressive resistance} \\
\phi = \text{resistance factor} \\
A = \text{area of the section of the column} \\
f_y = \text{specified minimum yield strength} \\
E = \text{elastic modulus.}

The document: National Standard of Canada CAN/CSA-S16.1-M89 [3] is in the process of being superseded by an update. It is expected that Eq. (2.42a) to (2.42e) will be replaced by a single equation, Eq. (2.43).

\[ C_r = \phi f_y (1 + \lambda^{2n})^{-\frac{1}{n}} \]  \hspace{1cm} (2.43)

in which \( n = 1, 34 \) (constant).

It can also be expected that the SABS 0162-1:1994 will soon reflect the change.

By substituting

\[ f_{cr} = \frac{C_r}{A} \]  \hspace{1cm} (4.1)
into the five-part equation of Eq. (2.42) and by omitting $\phi$ from the set of equations, hence creating a set of non-factored equations, and by dividing by $f_y$, thus making the equations non-sensitive in terms of yield strength, the following five-part equation may be written:

for $0 \leq \lambda \leq 0.15$

$$\frac{f_{cr}}{f_y} = 1 \quad (4.2a)$$

$0.15 \leq \lambda \leq 1.0$

$$\frac{f_{cr}}{f_y} = \left(1.031 - 0.202\lambda - 0.222\lambda^2\right) \quad (4.2b)$$

$1.0 \leq \lambda \leq 2.0$

$$\frac{f_{cr}}{f_y} = \left(-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2}\right) \quad (4.2c)$$

$2.0 \leq \lambda \leq 3.6$

$$\frac{f_{cr}}{f_y} = \left(0.009 + 0.877\lambda^{-2}\right) \quad (4.2d)$$

$3.6 \leq \lambda$

$$\frac{f_{cr}}{f_y} = \lambda^{-3} \quad (4.2e)$$

In the above equation, Eq. (4.2), $f_{cr}$ is described as the critical buckling stress of columns.
b. *American Institute of Steel Construction [4]: Design of Structural Steel Columns*

In accordance with the procedures prescribed by the AISC [4], and after the corresponding omission of the resistance factor $\phi$ and division by $f_y$, as described above, the two-part equation, Eq. (2.44), may be formulated by Eq. (4.3):

for

\[ \lambda < 1.5 \]

\[ \frac{f_\sigma}{f_y} = 0.658\lambda^2 \]  \hspace{1cm} \text{(4.3a)}

\[ \lambda \geq 1.5 \]

\[ \frac{f_\sigma}{f_y} = \frac{0.877}{\lambda^2} \]  \hspace{1cm} \text{(4.3b)}

The two sets of equations, that is Eq. (4.2) and Eq. (4.3), produce two different column design curves.

2. **The Design of Stainless Steel Columns**

The design criteria used for the design of stainless steel columns have been discussed in Chapter 2 and are repeated here for ease of reference.

a. *American Society of Civil Engineers [1]: Design of Stainless Steel Columns*

A single equation, Eq. (2.45), is used for the design of stainless steel columns:

\[ f_\sigma = \frac{\pi^2 E}{(kL/r)} \]  \hspace{1cm} \text{(2.45)}
where

\[
E_t = \frac{f_y E_0}{f_y + 0.002nE_0 \left( \frac{f}{f_y} \right)^{n-1}} \tag{2.5}
\]

The gradual yielding nature of stainless steels is taken into account through the equation for the tangent modulus, \( E_t \). When Eq. (2.5) is substituted into Eq. (2.45), \( f \) should be replaced by \( f_{cr} \). The result of this substitution is shown in Eq. (2.25). For a given slenderness ratio \( \frac{kL}{r} \) and a specific material property \( (n, E_0, f_p, f_y) \), the critical column buckling stress \( f_{cr} \) can only be computed from Eq. (2.25) by a process of iteration.

for \( f_{cr} \leq f_y \)

\[
f_{cr} = \frac{\pi^2 f_y E_0}{\left( \frac{kL}{r} \right)^2 \left( f_y + 0.002nE_0 \left( \frac{f_{cr}}{f_y} \right)^{n-1} \right)} \tag{2.25}
\]

Eq. (2.25) may also be written in the form given in Eq. (4.4).

\[
\frac{f_{cr}}{f_y} = \frac{\pi^2 E_0}{\left( \frac{kL}{r} \right)^2 \left( f_y + 0.002nE_0 \left( \frac{f_{cr}}{f_y} \right)^{n-1} \right)} \tag{4.4}
\]

3. **Comparison Between the Criteria for the Design of Carbon and Low-alloy Steel Columns and Stainless Steel Columns**

The identified criteria for the design of carbon and low-alloy steel columns were, for the purpose of comparison, given by Eq. (4.2) and (4.3) and graphically shown in Figure 4.1. Both of these curves make provision for elastic and inelastic behaviour, but at different
points of transition. The clearest point of transition can be identified by the boundary value of \( \lambda \) applicable to Eq. 4.2e, that is:

\[
3.6 \leq \lambda
\]

\[
\frac{f_\sigma}{f_y} = \lambda^{-2}
\]  

(4.2e)

This argument is based on the observation that all of the other Eq. (4.2) and (4.3) contain factors other than that which indicate pure elastic behaviour. Assuming that the transition of elastic to inelastic behaviour, in terms of stress, is at the proportional limit, \( f_p \), the substitution of \( f_\sigma \) in Eq. (4.2e) by \( f_p \) would be valid. With this substitution and using the value of \( \lambda = 3.6 \) in Eq. (4.2e), the ratio \( \frac{f_\sigma}{f_y} = 0.07716 \). The parameter \( n \) in the Ramberg-Osgood equation, Eq. (2.1) and (2.2) may now be computed and gives \( n = 1.1694 \). With this known, a credible comparison between the criteria for the design of carbon and low-alloy steel columns and stainless steel columns may be made. This comparison is shown graphically in Figure 4.2.

4. The Design of Aluminium Columns

The criteria for the design of aluminium columns were discussed in Chapter 2. Part of that information is repeated here for ease of reference.

The design criteria and method of design is determined by the tempergrade of the aluminium alloy to be used as the column.

There are several general equations for determining the maximum permissible stresses \( (f_\sigma) \) for columns.

Firstly there are two slenderness limits, \( \lambda_1 \) and \( \lambda_2 \), where:

\[
\lambda_1 = \frac{H_c - \frac{n_u f_y}{k c n_y}}{k c n_y}
\]  

(2.46)

and

\[
\lambda_2 = N_c
\]  

(2.47)
For a slenderness ratio \( \left( \frac{kL}{r} \right) \leq \lambda_1 \):

\[
f_{\sigma} = \frac{f_y}{k_c n_y}
\]

(2.48a)

For a slenderness ratio \( \left( \frac{kL}{r} \right) \) between \( \lambda_1 \) and \( \lambda_2 \), thus:

for \( \lambda_1 < \left( \frac{kL}{r} \right) < \lambda_2 \):

\[
f_{\sigma} = \frac{1}{n_c} \left( H_c - K_c \left( \frac{kL}{r} \right) \right)
\]

(2.48b)

for \( \left( \frac{kL}{r} \right) > \lambda_2 \):

\[
f_{\sigma} = \frac{\pi^2 E}{n_u \left( \frac{kL}{r} \right)^2}
\]

(2.48c)

By dividing Equations (2.48a) to (2.48c) by \( f_y \) and thus making it non-sensitive to the yield strength \( f_y \), we obtain:

For a slenderness ratio \( \left( \frac{kL}{r} \right) \leq \lambda_1 \):

\[
\frac{f_{\sigma}}{f_y} = \frac{1}{k_c n_y}
\]

(4.5a)

for \( \lambda_1 < \left( \frac{kL}{r} \right) < \lambda_2 \):

\[
\frac{f_{\sigma}}{f_y} = \frac{1}{n_c f_y} \left( H_c - K_c \left( \frac{kL}{r} \right) \right)
\]

(4.5b)
for \( \left( \frac{kL}{r} \right) > \lambda_2 \):

\[
\text{and } \frac{f_\sigma}{f_y} = \frac{\pi^2 E}{n_u f_y \left( \frac{kL}{r} \right)^2}
\]  

(4.5c)

In the above equations:

- \( k_c \) = a coefficient for compression members
- \( n_y \) = factor of safety on yield strength
- \( n_u \) = factor of safety of buckling strength.

The buckling constants \( H_c, K_c \) and \( N_c \) are calculated from different equations, depending on the temper-designation. For temper grades 0, T1, T3 or T4 and tempered designations beginning with \( H \), the equations for the buckling constants of compression in columns are:

\[
H_c = f_y \left[ 1 + \sqrt{\frac{f_y}{6890}} \right]
\]  

(2.49)

\[
K_c = \frac{H_c}{20} \sqrt{\frac{6H_c}{E}}
\]  

(2.50)

\[
N_c = \frac{2H_c}{3K_c}
\]  

(2.51)

The equations for the buckling constants for tempered designations T5, T6, T8, T83 or T9 are:

\[
H_c = f_y \left[ 1 + \sqrt{\frac{f_y}{15500}} \right]
\]  

(2.52)

\[
K_c = \frac{H_c}{10} \sqrt{\frac{H_c}{E}}
\]  

(2.53)

\[
N_c = 0.41 \frac{H_c}{K_c}
\]  

(2.54)

Graphs to illustrate the different temper grade equations are shown in Figure 4.3. The 'Temper T1 method' is applicable to temper grades 0, T1, T3 or T4, while the 'Temper
T6 method' is applicable to temper grades T5, T6, T8, T83 or T9. The material properties given in Table 3.2 have been used in preparing the graphs.

5. **The Design of Aluminium Columns Using Different Design Criteria**

The following criteria are compared, as if aluminium columns are to be designed:

- Aluminium Association: The Design of Aluminium Columns [5].
- American Society of Civil Engineers: Design of Stainless Steel Columns [1].

The two sets of equations, Eq. (4.4) and Eq. (4.5) were used to generate a set of two graphs which is shown in Figure 4.4. The aluminium alloy 6261-T6 has the temper grade T6, as indicated by the numbering system. For this reason the graph applicable to 'Temper T6 method' of Figure 4.3 is presented in Figure 4.4 as the AA (Aluminium) curve for purposes of comparison. The material properties given in Table 3.2 have been used in preparing the graphs.

C. **DISCUSSION OF RESULTS**

The design method for stainless steel columns as prescribed by ASCE [1] can normally not be used on carbon and low-alloy steels because there is no known value for \( f_p \) and hence value for \( n \) to be used in Eq. (4.4), except when one makes an argument that lead to the preparation of Figure 4.2. The column design criteria for aluminium as specified by the Aluminium Association can also not be used as an exercise on carbon steel column design. The reason for this is that Eqs. (2.46) to (2.54) have no apparent theoretical basis. These equations also seem to have been determined solely for aluminium on an empirical basis.

The experimental work described in Chapter 3 was specially performed so that all the mechanical properties necessary for the comparison of various column design methods could be done. Figure 4.3 shows the column design curves for the 6261-T6 aluminium alloy using the mechanical properties obtained by the experimental data and substituting them into the different column design methods.

No meaningful conclusion can be drawn from the graphs shown in Figure 4.2. The ratio \( \frac{f_p}{f_y} = 0.07716 \) is extremely low and probably not a true reflection of the real ratio of \( \frac{f_p}{f_y} \) for carbon and low-alloy steels.

Mazzaloni [9] suggested the use of the tangent modulus, \( E_t \), in the design of aluminium columns. This is very similar to the method of column design specified by the ASCE [1]
for the stainless steels. The most influential factor determining the column design curve in this method of column design is the value of $n$, which is determined by using Eq. (2.2). This equation is in turn influenced by the ratio of $\frac{f_p}{f_y}$. A high value of the proportional limit, $f_p$, in terms of the yield strength, $f_y$, gives a high value of $n$. This is the case for the 6261-T6 aluminium alloy used in this investigation.

The possible validity or usefulness of the tangent modulus equation for the design of aluminium columns can only be determined experimentally. These experiments are discussed in Chapter 5.
FIGURE 4.1

A COMPARISON BETWEEN THE CAN AND AISC COLUMN DESIGN CURVES USING CARBON STEEL PARAMETERS
FIGURE 4.2

A COMPARISON BETWEEN THE CAN, AISC AND ASCE COLUMN DESIGN CURVES USING CARBON STEEL PARAMETERS

---

Graph showing the comparison between the CAN, AISC, and ASCE column design curves using carbon steel parameters. The x-axis represents $kL/r$ and the y-axis represents $f_{cr}/f_y$. The curves represent different standards:
- ASCE (Stainless steel)
- CAN (Carbon steel)
- AISC (Carbon steel)
FIGURE 4.3

ALUMINIUM COLUMN DESIGN CURVES FOR THE DIFFERENT TEMPERGRADES (AA APPROACH)

![Diagram](image)
FIGURE 4.4

ASCE AND AA COLUMN DESIGN CURVES USING ALUMINIUM PARAMETERS

\[ \frac{f_{cr}}{f_y} \]

- ASCE (Stainless steel)
- AA (Aluminium)
CHAPTER 5
COLUMN TESTS AND RESULTS

A. GENERAL REMARKS

In this chapter the experimental results of tests done by the author on columns as well as the experimental results of tests done by the author in collaboration with Seynaeve [27] on stub-columns, are presented. The specific aluminium alloy under consideration is alloy 6261-T6. These experimental results are compared with two existing criteria and methods for the design of columns. The experimental setup is also discussed in this chapter.

B. PREPARATION OF COLUMNS

The length of the stub-columns were taken in accordance with the requirements of the 1996 edition of the AISI Cold-Formed Steel Design Specifications [31], which states: "To eliminate overall column-buckling effects, the stub-column length shall not exceed twenty times the minimum radius of gyration, \( r \) (in this case \( r = 17 \) mm), of the cross-section, \( A \). Thus in this case the length of the stub-columns may not exceed 340 mm. For unperforated columns (like those under investigation in this dissertation) the stub-column length shall not be less than three times the greatest overall width of the cross-section, \( W \). Therefore in this case the minimum length of the stub-columns may not be less than 300 mm." The length of the stub-columns were taken as 340 mm, which translates to a \( \frac{KL}{r} \) value of 17.8.

The lengths of the column test specimens were chosen for slenderness ratios, \( \frac{KL}{r} \), evenly distributed between the values of 20 and approximately 70. For values of \( \frac{KL}{r} \) less than 20, one is entering the domain for stub-columns. The results of tests on stub-columns are presented in this chapter. For values of \( \frac{KL}{r} \) greater than 70, one is entering the domain of slender columns, which should fail in accordance with the Euler equation, Eq. (2.9). The confirmation of the applicability of the Euler equation for columns that would fail elastically was considered not necessary for this investigation. The reader should note that the slenderness ratios and lengths of the aluminium columns have been adjusted to account for the pinned end fixtures, using the method explain by Osgood [40].

The columns were cut to the desired length and their ends were machined precisely flat and perpendicular to the longitudinal axis of the column. This was done to ensure that no
stress concentrations would occur at the ends of the column due to uneven ends and to minimise eccentricities because of the uneven end conditions.

Ten strain gauges in total were mounted on each column. For each column, four strain gauges were mounted at each of the bottom and top quarter lengths of the column respectively, and placed at a distance of 5 mm from the outer edges of the outside part of the flange. These strain gauges were used to align the column prior to testing. The two strain gauges mounted in the middle of the web at the centre of the column, were used to determine the buckling load of the column. Figure 5.1 shows the position of these strain gauges on the column.

C. EXPERIMENTAL SETUP

Once prepared, the columns were tested using a hydraulic Instron actuator, with a static load capacity slightly in excess of 500 kN, built into a test frame of which the height could be adjusted to suit the different column lengths. Appendix B, Figure B.1 shows the abovementioned setup.

The columns were tested with the flat ends bearing on specially manufactured loading fixtures. The loading fixtures were manufactured from gauge plates and hardened to avoid damage during the testing procedure.

The end fixtures were designed to rotate on a ball joint to allow virtually zero-restraint, thus creating a pinned-ends effect with respect to end rotation of the columns in all planes of bending, but was restrained with respect to warping. The end fixture bearing plates were also designed to allow for minor alignment via a series of set screws. Pinned-end restraints are favoured by most investigators, as described by Galambos [22]. When the column is tested in this manner, the pinned-ends create an effective length factor, \( k = 1 \). Because of this, it only uses half the column length needed for fixed end restraints, and then the critical cross-section is located near the mid length of the column, thus making the cross-section of interest remote from the boundary conditions. Figure B.2 of Appendix B, shows an example of the end fixtures.

D. EXPERIMENTAL PROCEDURE

The column tests were conducted in the following manner. Firstly the column is placed between the end fixtures as closely concentric as possible. A load not exceeding 10% of the theoretical critical load (using the ASCE [1] approach) is placed on the column. The column position is then adjusted using the set screws on the end fixtures by monitoring the strain gauges placed at the quarter lengths along the column. When these strain gauges
give measured values equal in magnitude to within 10%, it is assumed that very little eccentricities exist in the column and the pre-load is then removed.

The column is tested by applying the axial load at a displacement rate of approximately 0.5mm per minute. Strain readings of the centre two strain gauges on the web of the column and the column load are sampled at 2 second intervals. An Orion Solatron Data Logger controlled by a Pro Line computer, as can be seen in Figure B.3, is used to collect data which is saved on diskette to facilitate further data processing.

The final buckling of most of the columns were instantaneous. The columns were then violently shot away from the experimental setup, after which the tests were stopped. The load at which these events took place were recorded and used to determine the critical buckling stress, $f_{cr}$.

The testing procedure followed here is similar to that used by Van den Berg [7] and Dat [41] for axially loaded cold-formed columns.

Figures B.4 and B.5 of Appendix B, shows a column undergoing testing and the column after final buckling, respectively.

E. EXPERIMENTAL RESULTS

1. Mechanical Properties Needed for Column Design

The experimental results of tests done on coupons to establish the mechanical properties of the material from which the columns and stub-columns were prepared, were discussed in detail in Chapter 3 and in Reference [2]. The results of the tests done on stub-columns by the author in collaboration with Seynaeve [27] are given in Table 5.1.

The following mean values have been extracted from the abovementioned two sets of experiments:

<table>
<thead>
<tr>
<th></th>
<th>Stub-column tests</th>
<th>Coupon tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Modulus, $E_o$ (GPa)</td>
<td>72.9</td>
<td>73.3</td>
</tr>
<tr>
<td>Yield Strength, $f_y$ (MPa)</td>
<td>305.0</td>
<td>312.0</td>
</tr>
<tr>
<td>Proportional Limit, $f_p$ (MPa)</td>
<td>271.0</td>
<td>283.0</td>
</tr>
</tbody>
</table>

A comparison of the results given above indicates that the values of the mechanical properties as obtained by coupon tests can be used with confidence in the prediction of structural member behaviour. This finding is based on the insignificant difference between the values of the respective properties obtained through the two test procedures.
2. Results of Tests on Columns

Based on the mean values of the mechanical properties obtained through coupon tests, the column design curves, shown in Figure 5.2, have been drawn using the ASCE [1] design criteria and the AA [5] design criteria, which were discussed in detail in Chapter 3.

The results of the tests done on 9 columns are given in Table 5.2 and are also shown in Figure 5.2.

An inspection and an analysis of Figure 5.2 and Table 5.3 indicates that the experimental results generally follow the design curve and values which can be drawn and calculated in accordance with the methodology adopted by the ASCE [1], closer than the design curve and values drawn and calculated in accordance with the methodology of the AA [5]. This analysis and observation is based on a small number of tests which is not sufficient evidence for a definite conclusion.
TABLE 5.1
MECHANICAL PROPERTIES OF THE TESTED STUB-COLUMNS

<table>
<thead>
<tr>
<th>STUB-COLUMN</th>
<th>$E_o$ (GPa)</th>
<th>$f_r$ (MPa)</th>
<th>$f_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73.5</td>
<td>312</td>
<td>276</td>
</tr>
<tr>
<td>2</td>
<td>71.2</td>
<td>281</td>
<td>269</td>
</tr>
<tr>
<td>3</td>
<td>73.2</td>
<td>314</td>
<td>278</td>
</tr>
<tr>
<td>4</td>
<td>72.4</td>
<td>275</td>
<td>236</td>
</tr>
<tr>
<td>5</td>
<td>72.3</td>
<td>315</td>
<td>278</td>
</tr>
<tr>
<td>6</td>
<td>72.1</td>
<td>308</td>
<td>270</td>
</tr>
<tr>
<td>7</td>
<td>73.4</td>
<td>315</td>
<td>277</td>
</tr>
<tr>
<td>8</td>
<td>75.1</td>
<td>320</td>
<td>287</td>
</tr>
<tr>
<td>Mean</td>
<td>72.9</td>
<td>305</td>
<td>271</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.18</td>
<td>17.07</td>
<td>15.32</td>
</tr>
</tbody>
</table>

TABLE 5.2
RESULTS OF COLUMN TESTS

<table>
<thead>
<tr>
<th>COLUMN</th>
<th>$kL/r$</th>
<th>$f_{cr}$ (MPa)</th>
<th>$f_{cr}/f_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>297.5</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>300.2</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>281</td>
<td>0.90</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>274.5</td>
<td>0.88</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
<td>285.3</td>
<td>0.91</td>
</tr>
<tr>
<td>6</td>
<td>44</td>
<td>291</td>
<td>0.93</td>
</tr>
<tr>
<td>7</td>
<td>52</td>
<td>275.1</td>
<td>0.88</td>
</tr>
<tr>
<td>8</td>
<td>58</td>
<td>274</td>
<td>0.88</td>
</tr>
<tr>
<td>9</td>
<td>68</td>
<td>217.8</td>
<td>0.70</td>
</tr>
</tbody>
</table>
### TABLE 5.3

**A Comparison Between the Experimental, ASCE and the AA Column Design Values**

<table>
<thead>
<tr>
<th>COLUMN</th>
<th>Experimental $(f_{cc}/f_c)$</th>
<th>Predicted Value (ASCE) $(f_{cc}/f_c)$</th>
<th>$\frac{EXP}{ASCE}$</th>
<th>Predicted Value (AA) $(f_{cc}/f_c)$</th>
<th>$\frac{EXP}{AA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>2</td>
<td>0.96</td>
<td>0.95</td>
<td>1.01</td>
<td>0.95</td>
<td>1.01</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>0.94</td>
<td>0.96</td>
<td>0.91</td>
<td>0.99</td>
</tr>
<tr>
<td>4</td>
<td>0.88</td>
<td>0.90</td>
<td>0.98</td>
<td>0.83</td>
<td>1.06</td>
</tr>
<tr>
<td>5</td>
<td>0.91</td>
<td>0.89</td>
<td>1.02</td>
<td>0.81</td>
<td>1.12</td>
</tr>
<tr>
<td>6</td>
<td>0.93</td>
<td>0.88</td>
<td>1.06</td>
<td>0.79</td>
<td>1.18</td>
</tr>
<tr>
<td>7</td>
<td>0.88</td>
<td>0.83</td>
<td>1.06</td>
<td>0.73</td>
<td>1.21</td>
</tr>
<tr>
<td>8</td>
<td>0.88</td>
<td>0.68</td>
<td>1.29</td>
<td>0.68</td>
<td>1.29</td>
</tr>
<tr>
<td>9</td>
<td>0.70</td>
<td>0.50</td>
<td>1.40</td>
<td>0.50</td>
<td>1.40</td>
</tr>
</tbody>
</table>
FIGURE 5.1
POSITION OF THE STRAIN GAUGES ON THE COLUMNS

STRAIN GAUGES

L/4

L/4

L/4

STRAIN GAUGES
FIGURE 5.2
COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES

\[ \frac{f_{cr}}{f_y} \] vs. \( kL/r \)

- **ASCE (Stainless)**
- **AA (Aluminum)**

- Column 1
- Column 2
- Column 3
- Column 4
- Column 5
- Column 6
- Column 7
- Column 8
- Column 9
CHAPTER 6

DISCUSSION AND CONCLUSIONS

A. GENERAL REMARKS

The mechanical properties needed for structural design, of the aluminium alloy 6261-T6, which is a suitable alloy for structural applications, have previously been established by the author [2]. A detailed study of the theories describing column behaviour as well as a comparative study of the criteria and methods for the design of carbon and low-alloy steel columns, stainless steel columns and aluminium columns were also done. Tests on a small number of aluminium columns were performed and the results of the tests were plotted against appropriate column design curves. For this reason it is considered that the objectives stated in Chapter 1 have been met. The results of this project should enable the Chromium Steels Research Group to continue with the comprehensive programme on the optimisation of specifications for structural design.

This chapter primarily summarises the results of the research contained in this short dissertation and the conclusions that can be drawn from the results. At the same time the areas for future research are also identified.

B. SUMMARY OF THE RESEARCH AND CONCLUSIONS

Optimising the criteria for the design of carbon and low-alloy steel columns, stainless steel columns and aluminium columns through a process of comparison and exchange of methodologies and criteria where appropriate, together with a comparison of values that were obtained through experimental work done on columns with the values predicted by the abovementioned methodologies and criteria, constituted the primary objective of this investigation.

In order to develop an understanding of the subject matter at hand, it was necessary to undertake a literature survey. The literature survey is primarily contained in Chapter 2, but also in Chapter 3 and 4 where the topics could be served better by interaction between literature and discussion.

The mechanical properties previously determined by Van Der Merwe [2] were reported and evaluated in Chapter 3. This work enabled the establishment of a set of analytical stress-strain curves which represents the behaviour of the material accurately.
To optimise the use of the different criteria for the design of columns manufactured from various types of materials, it is necessary to compare these criteria with each other, using the mechanical properties of the various materials as basis. A comparative study of the criteria and methods for the design of carbon and low-alloy steel columns, stainless steel columns and aluminium columns was done in Chapter 4. Preliminary conclusions may be drawn from this work. These have been given at the end of Chapter 4.

The results of stub-column tests done by the author in collaboration with Seynaeve [27] as well as the results of tests that were done on columns with varying slenderness ratios are given in Chapter 5. A comparison between the experimental values and the column design values obtained through applying the mechanical properties obtained in Chapter 3 in the criteria and methods for the design of stainless steel columns and aluminium columns was also done in Chapter 5. The following conclusion that might be drawn has also been given at the end of Chapter 5.

An inspection and an analysis of experimental results and predicted values indicate that the methodology adopted by the ASCE [1] for the design of stainless steel columns might yet be proved to be useful for the design of aluminium columns.

C. FURTHER STUDY

This research is the starting point of a programme that has to cover a wide variety of different aspects of the various design criteria. More research will be required in the following areas:

- More extensive experimentation on stub columns, so that the mechanical properties of the whole section can be determined and statistically analysed.
- A more intensive experimentation on columns of different slenderness ratios to verify the applicability of the different column design curves.
- To expand on the number of different aluminium alloys for investigation purposes. The influence of the type of alloy and the tempergrade of the aluminium on the proportional limit, \( f_p \), which in turn influences the value of \( n \) which is used in the ASCE Design Specifications [1], needs to be fully investigated.
- An in-depth study of the effect of residual stresses on the behaviour of aluminium columns in the inelastic range.
- A study to find a more theoretically based set of design criteria for the gradual yielding nature of aluminium rather than the empirical method used presently.
• Experimental work have to be carried out on flexural members (beams).

• Experimental work have to be carried out on beam-columns.

• Experimental work have to be carried out on connection design.


CERTIFICATE NO. 9406PR192

DELIVERY NOTE NO. __________________________

CUSTOMER: HULET ALUMINIUM LTD
ADDRESS: 32 SAMPLE ACC EXTRUSIONS
P.O. BOX 74
PIETERMARITZBURG

ALLOY AND TEMPER 6261-T6
CAST NUMBER: ASWC

<table>
<thead>
<tr>
<th>Si%</th>
<th>Fe%</th>
<th>Cu%</th>
<th>Mn%</th>
<th>Mg%</th>
<th>Cr%</th>
<th>Zn%</th>
<th>Ti%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.630</td>
<td>0.270</td>
<td>0.322</td>
<td>0.279</td>
<td>0.750</td>
<td>0.604</td>
<td>0.015</td>
<td>0.024</td>
</tr>
</tbody>
</table>

BATCH PROOF U.T.S. ELONGATION CURVE TEST
NO (MPa) (MPa) (%) (mm.) DATE
664 299 325 13 50 15/06/94

TESTED ACCORDING TO BS EN 10002-1:1990
ON INSTRON 4505 VERIFIED ACC TO BS EN 10002-2:1992
BILLETS ULTRASONICALLY TESTED
TECHNICAL GRADE MATERIAL

Witnessed by: __________________________ (Customer Representative)
Approved: __________________________ (Head Chemist)
Approved: __________________________ (Head of Physical Testing)
APPENDIX B

FIGURE B.1

THE HYDRAULIC INSTRON ACTUATOR WITH TEST FRAME THAT WAS USED DURING COLUMN TESTING

FIGURE B.2

END FIXTURE WITH BALL JOINT THAT CAUSES A PINNED-END EFFECT
FIGURE B.3
INSTRON CONTROL PANEL WITH THE ORION SOLATRON DATA LOGGER AND PRO LINE COMPUTER

FIGURE B.4
COLUMN UNDERGOING TESTING
FIGURE B.5
COLUMN AFTER FINAL BUCKLING