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Correlated equilibria, good and bad: an experimental study

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Abstract

We report results from an experiment that explores the empirical validity of *correlated equilibrium*, an important generalization of the Nash equilibrium concept. Specifically, we seek to understand the conditions under which subjects playing the game of Chicken will condition their behavior on private, third–party recommendations drawn from known distributions. In a "good–recommendations" treatment, the distribution we use is a correlated equilibrium with payoffs better than any symmetric payoff in the convex hull of Nash equilibrium payoff vectors. In a "bad–recommendations" treatment, the distribution is a correlated equilibrium with payoffs worse than any Nash equilibrium payoff vector. In a "Nash–recommendations" treatment, the distribution is a convex combination of Nash equilibrium outcomes (which is also a correlated equilibrium), and in a fourth "very–good–recommendations" treatment, the distribution yields high payoffs, but is not a correlated equilibrium. We compare behavior in all of these treatments to the case where subjects do not receive recommendations. We find that when recommendations are not given to subjects, behavior is very close to mixed–strategy Nash equilibrium play. When recommendations are given, behavior does differ from mixed–strategy Nash equilibrium, with the nature of the differences varying according to the treatment. Our main finding is that subjects will follow third–party recommendations only if those recommendations derive from a correlated equilibrium, and further, if that correlated equilibrium is payoff–enhancing relative to the available Nash equilibria.

Journal of Economic Literature classifications: D83, C72, C73.

Keywords: correlated equilibrium, recommendation, coordination problem, chicken game, hawk–dove game, experiment, experimenter demand effects.

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1 Introduction

A standard assumption in noncooperative game theory is that players' strategies—whether pure or mixed—are probabilistically independent. However, researchers at least as long ago as Aumann (1974, 1987) recognized that relaxing this assumption by allowing *correlation* in players' strategies could greatly enlarge the set of equilibrium possibilities beyond the set of Nash equilibria. The equilibria that result are known as *correlated equilibria*.[1] As an illustration, consider the two–player game of Chicken, shown in Figure 1; strategies are defect (D) and cooperate (C). This game has two asymmetric pure–strategy Nash equilibria—(D,C) and (C,D)—as well as a mixed–strategy Nash equilibrium in which each player chooses D with probability two–fifths.

		Player 2	
		D	C
Player	D	0,0	9,3
1	C	3,9	7,7

Figure 1: The basic Chicken game

The mixed–strategy equilibrium of this game has the attractive feature of symmetry—thus avoiding the "symmetry–breaking" question implicit in asymmetric equilibria (see, for example, Crawford (1998)). Evolutionary dynamics often favor such symmetry and indeed, the Nash equilibrium mixed strategy is the unique evolutionarily stable strategy of this game (see, for example, Hofbauer and Sigmund (1998)). However, as Skyrms (1996) and others have observed, this mixed–strategy equilibrium is inefficient: in the Chicken game of Figure 1, it yields expected payoffs of just 5.4 for each player. By contrast, if the players somehow agreed to condition their behavior on a fair coin toss, playing (for example) the strategy profile (D,C) after Heads and (C,D) after Tails, each could improve her ex ante expected payoff to 6. Moreover, since both recommended outcomes are pure–strategy Nash equilibria, both would strictly prefer to honor the agreement as long as they believed that the other would, even after knowing which recommendation was received.[2]

Furthermore, as Aumann pointed out, the players could actually do even better in this game by enlisting the services of a monitor—a non–strategic third party—who has access to a randomizing device. If this device, for example, chooses the three outcomes (D,C), (C,D), and (C,C) with equal probability, and the monitor tells each player *privately* that player's own action according to the chosen outcome (e.g., if the outcome randomly drawn is (C,D), the monitor recommends C privately to Player 1 and D privately to Player 2), and players follow these recommendations, then expected payoffs increase to $6\frac{1}{3}$ for each player. Furthermore, under the assumption that the players know the distribution obeyed by the randomizing device, as well as their own recommendation, *but not the recommendation given to the other player*, both players will strictly prefer to follow their own

[1] An early example of a correlated equilibrium is also found in Luce and Raiffa (1957, pp. 115-120).

[2] By contrast, see Young (2005, Chapter 3) for a discussion of a variant on this setup (due to Moulin and Vial (1978)) in which players choose—*before* receiving recommendations—whether to commit to following them or not. Young calls a distribution of recommendations under this setup a *coarse* correlated equilibrium if no player prefers not to commit to following recommendations, given that the others have chosen to commit.

recommendation if they believe that the other will also do so.

A correlated equilibrium is a probability distribution over outcomes—that is, a joint distribution over players' strategies—such that under the assumptions mentioned above, all players prefer to follow their own recommendations.[3] Then, a Nash equilibrium is just a special case of correlated equilibrium, in which the joint distribution of strategies is the product of the corresponding marginals (that is, the resulting players' strategies are probabilistically independent of one another). Indeed, as Hart and Mas-Colell (2000, p. 1128) observe:

> "...from a practical point of view, it could be argued that correlated equilibrium may be the most relevant noncooperative solution concept. Indeed, with the possible exception of well–controlled environments, it is hard to exclude a priori the possibility that correlating signals are amply available to players, and thus find their way into the equilibrium."

The purpose of this paper is to examine the empirical validity of this "most relevant noncooperative solution concept." Perhaps ironically, we study correlated equilibria in the well–controlled environment of the laboratory, as this enables us to clearly assess the role of well–defined, correlated signals as coordinating devices. Specifically, we design and conduct an experiment in which human subjects play the game shown in Figure 1. Prior to making their choices, subjects receive private signals ("recommendations") generated according to a known distribution of outcomes that serves as our main treatment variable. Three of the distributions we use are symmetric correlated equilibria. In one of our treatments, which we call our "Nash–recommendations" treatment, the correlated equilibrium we attempt to implement is simply a convex combination of Nash equilibria. In a second treatment—our "good–recommendations" treatment—the correlated equilibrium is the one described above, which yields payoffs that are Pareto superior to all symmetric payoff vectors in the convex hull of Nash equilibrium payoff vectors.

It is often forgotten, though, that there also exist correlated equilibria in which payoffs are *Pareto inferior* to all symmetric payoff vectors in the convex hull of Nash equilibrium payoff vectors. If correlated equilibrium is to be taken seriously as a descriptive device, and not just a theoretical curiosity, then it should be possible to induce these bad correlated equilibria as well as the good ones. To our knowledge, however, there has never been an experimental test of a bad correlated equilibrium. We remedy this, with what we call our "bad–recommendations" treatment. Despite the "bad" moniker, the distribution over outcomes we use in this treatment is every bit as much a correlated equilibrium as that in our good– and Nash–recommendations treatments. In particular, it is still optimal for a player to follow her recommendations, as long as she believes her opponent will follow the recommendations given him.

Finally, we attempt to distinguish between subjects' following recommendations as part of a correlated equilibrium and their following of recommendations for other reasons—for example, out of a

[3] We do not claim that recommendations from a third party are the only way of achieving correlated equilibrium. Hart and Mas-Colell (2000), for example, show that under a simple adaptive learning algorithm, the empirical distribution of outcomes converges to the set of correlated equilibria; in that case, players are able to correlate their strategies according to the history of play rather than third–party recommendations. See also Foster and Vohra (1997), Fudenberg and Levine (1998 Chapter 8, 1999), Vanderschraaf (2001), Vanderschraaf and Skyrms (2003), and Brandenburger and Friedenberg (2008).

desire to please the experimenter (also known as "experimenter demand effects")—with our "very–good–recommendations" treatment. In this treatment, the distribution of recommended outcomes is *not* a correlated equilibrium, but the temptation to follow recommendations may be great, because if both players follow recommendations, payoffs are Pareto superior to all symmetric correlated–equilibrium payoff vectors.

In the experiment, subjects play the game shown in Figure 1 repeatedly against changing opponents. In half of the rounds, they receive recommendations (always according to the same correlated equilibrium distribution), while in the remaining rounds, they do not receive any recommendations. The main results are as follows. When players do not receive recommendations, their behavior is well–described by the mixed–strategy Nash equilibrium. Giving subjects recommendations has an effect that depends on which underlying distribution of outcomes is used. The likelihood of following a recommendation is higher in the good– and Nash–recommendations treatments and lower in the bad– and very–good–recommendations treatments, and also varies somewhat with which of the available actions is recommended. In nearly all cases, subjects follow recommendations more than chance would predict, but there is no treatment where subjects follow recommendations all the time.

2 Correlated equilibrium—theory and tests

The game we use is the Chicken game shown in Figure 1 above. We chose Chicken as it is perhaps the simplest game with the property that there exist correlated equilibrium payoff pairs that lie outside the convex hull of Nash equilibrium payoff pairs. The game has three Nash equilibria: (D,C), (C,D), and a mixed–strategy Nash equilibrium in which each player chooses D with probability $\frac{2}{5}$. Payoffs in these three equilibria are, respectively, (9,3), (3,9), and (5.4,5.4).

As mentioned in the introduction, one way to think about correlated equilibria is as involving a "monitor"—a non–strategic third party—in the game. The monitor chooses one of the four pure–strategy profiles according to a commonly–known probability distribution, and to each player "recommends" that player's component in the profile. (The monitor never recommends a mixed strategy.) The probability distribution is a correlated equilibrium of the original game if each player at least weakly prefers following her recommended action to choosing any other action. (Thus, a correlated equilibrium of the original game corresponds to a Nash equilibrium of this new game, in which players' strategies are mappings from recommended actions to chosen actions.[4])

Define p_{DD}, p_{DC}, p_{CD}, and p_{CC} to be the probabilities of the outcomes (D,D), (D,C), (C,D), and (C,C), according to the commonly–known distribution characterizing the monitor's behavior. Suppose Player 1 is given a recommendation of D. Then, the conditional probability that the chosen outcome was (D,D) is $\frac{p_{DD}}{p_{DD}+p_{DC}}$, and the probability that the chosen outcome was (D,C) is $\frac{p_{DC}}{p_{DD}+p_{DC}}$. If Player 1 believes that Player 2 will follow the recommendation given to him, then Player 1's conditional expected payoff from following her recommendation of D is

$$\frac{p_{DD}}{p_{DD}+p_{DC}} \cdot 0 + \frac{p_{DC}}{p_{DD}+p_{DC}} \cdot 9 = \frac{9p_{DC}}{p_{DD}+p_{DC}},$$

[4]This Nash equilibrium is not unique. There always exist three "babbling" equilibria corresponding to the three Nash equilibria of the original game, in which both players completely ignore the recommendations given them, and play Nash equilibrium strategies instead. There exist additional equilibria as well.

and her conditional expected payoff from choosing C instead is

$$\frac{p_{DD}}{p_{DD}+p_{DC}}\cdot 3+\frac{p_{DC}}{p_{DD}+p_{DC}}\cdot 7=\frac{3p_{DD}+7p_{DC}}{p_{DD}+p_{DC}},$$

so she prefers to follow the D recommendation if $\frac{9p_{DC}}{p_{DD}+p_{DC}}\geq\frac{3p_{DD}+7p_{DC}}{p_{DD}+p_{DC}}$—that is, if $2p_{DC}\geq 3p_{DD}$. Using similar reasoning for Player 1 following a C recommendation, Player 2 following an D recommendation, and Player 2 following a C recommendation gives us a total of four inequalities:

$$\begin{aligned}2p_{DC} &\geq 3p_{DD}\\ 3p_{CD} &\geq 3p_{CC}\\ 2p_{CD} &\geq 3p_{DD}\\ 3p_{DC} &\geq 3p_{CC}.\end{aligned}$$

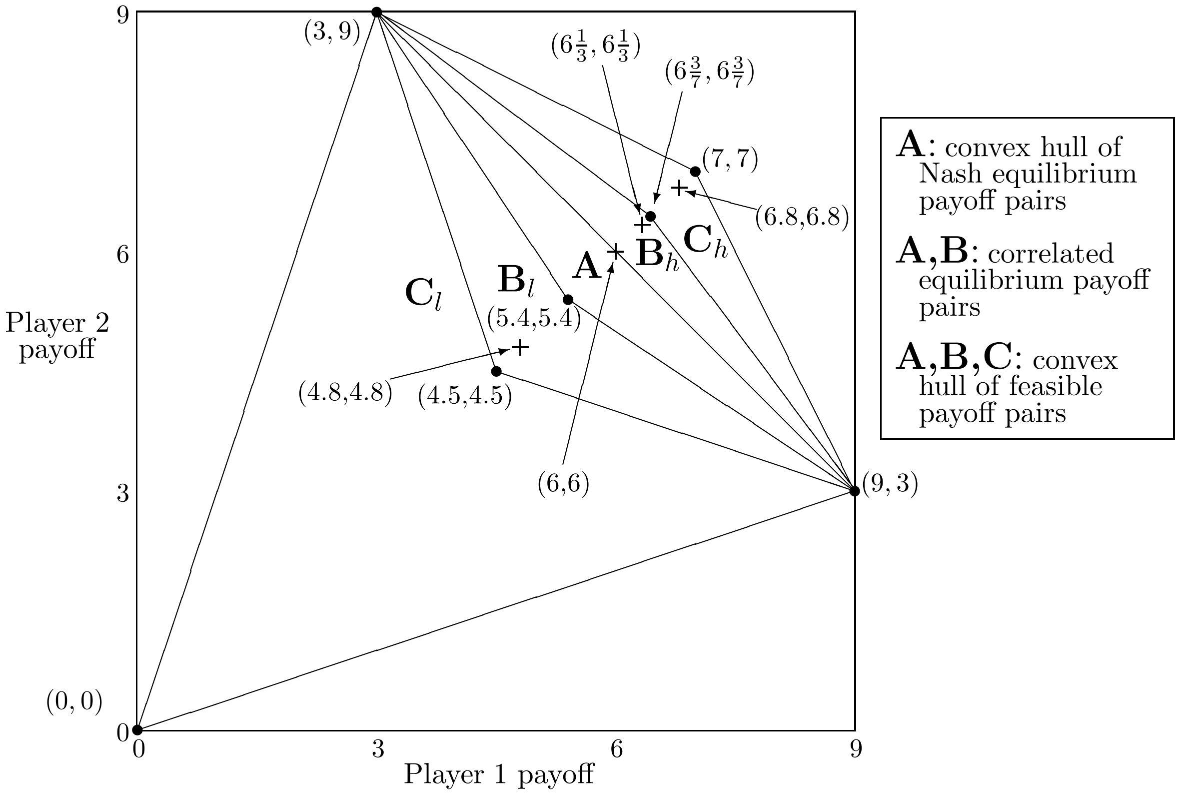
A correlated equilibrium is a quadruple $(p_{DD},p_{DC},p_{CD},p_{CC})$ that satisfies these four inequalities, along with $p_{DD}+p_{DC}+p_{CD}+p_{CC}=1$.

Since the set of correlated equilibria can be characterized as an intersection of sets defined by linear equations and inequalities, it is a convex set, and because it contains the set of Nash equilibria, it must also contain the convex hull of Nash equilibria. The same is true in payoff space; that is, the set of correlated–equilibrium payoffs of a game always contains the convex hull of the set of Nash equilibrium payoff pairs. However, in most games—including ours—there also exist correlated equilibria that are *not* in the convex hull of Nash equilibrium payoff pairs. Figure 2 shows the regions corresponding to the sets of Nash equilibrium payoff pairs and correlated equilibrium payoff pairs. The Nash equilibrium payoff pairs of this game are (3,9) (corresponding to the equilibrium (C,D)), (9,3) (corresponding to (D,C)), and (5.4,5.4) (corresponding to the mixed–strategy equilibrium). Therefore, the convex hull of Nash equilibrium payoff pairs is the triangle with these three points as vertices (region A in the figure); in particular, 6 is the highest symmetric payoff in this convex hull, and 5.4 the lowest. The set of correlated equilibrium payoff pairs is the quadrilateral with vertices (3,9), (4.5,4.5), (9,3), and $(6\frac{3}{7},6\frac{3}{7})$ (the union of regions A, B_l, and B_h in the figure), so that $6\frac{3}{7}$ is the highest symmetric correlated equilibrium payoff and 4.5 the lowest.

Relatively little experimental research has looked at correlated equilibria that are not convex combinations of Nash equilibria.[5] The earliest such study that we know of is that by Moreno and Wooders (1998), who examine the ability of several game–theoretic solution concepts (including Nash equilibrium and correlated equilibrium) to characterize subject behavior in a three–player version of a one–shot matching pennies game, in which two of the players have perfectly aligned interests; their game is shown on the left of Figure 3. Instead of giving players recommendations as we do, they allowed subjects to participate in a round of cheap talk prior to play of the game; subjects could send messages to either other player individually, or to both at once. Moreno and Wooders found that the choices of the players with aligned interests were highly correlated, so that mixed–strategy Nash equilibrium poorly described the distribution of outcomes. Rather, they concluded

[5]Experimental studies of correlated equilibria that are convex combinations of Nash equilibrium include Van Huyck, Gilette, and Battalio (1992), Brandts and McLeod (1995), and Seely, Van Huyck, and Battalio (2005). In a market setting, Duffy and Fisher (2005) examine whether subjects will coordinate on the closely related concept of a sunspot equilibrium involving a randomization over two certainty equilibria.

Figure 2: Characteristics of the Game



that the best–performing solution concept was coalition–proof correlated equilibrium (Einy and Peleg (1995), Moreno and Wooders (1996)), a generalization of Bernheim et al.'s (1987) coalition–proof Nash equilibrium.

More recently, Cason and Sharma (2007) attempted to induce a correlated equilibrium through the use of private recommendations to subjects, as we do. The game they use is a version of Chicken, shown on the right of Figure 3. The correlated equilibrium they attempt to induce has (Up, Right)

Moreno and Wooders (1998)

Player 3: H

		Player 2: H	Player 2: T
Player 1	H	1,1,–2	–1,–1,2
	T	–1,–1,2	–1,–1,2

Player 3: T

		Player 2: H	Player 2: T
Player 1	H	–1,–1,2	–1,–1,2
	T	–1,–1,2	1,1,–2

Cason and Sharma (2007)

		Player 2: Left	Player 2: Right
Player 1	Up	3,3	48,9
	Down	9,48	39,39

Figure 3: Games used in previous correlated–equilibrium experiments

and (Down, Left) occurring with probability 0.375 each, and (Down, Right) with probability 0.25, with (Up, Left) never occurring. This correlated equilibrium yields expected payoffs of 31.125 for each player: higher than the mixed–strategy Nash equilibrium expected payoffs of 20.4, and indeed, higher than any symmetric payoff pair in the convex hull of Nash equilibrium expected payoffs. In the experiment, subjects often did follow recommendations, doing so roughly 80% of the time in their baseline treatment, and earning payoffs well above the mixed–strategy Nash equilibrium prediction (though below the prediction of the correlated equilibrium) as a result.[6]

However, by only considering a correlated equilibrium that was *payoff–enhancing* relative to Nash equilibrium, Cason and Sharma's study risks confounding the coordinating role of third-party recommendations with a general interest by subjects in earning higher payoffs. Further, in Cason and Sharma's experimental instructions, they explicitly tell subjects that they ought to follow recommendations, as doing so will result in higher payoffs.[7]

By contrast with Cason and Sharma's (2007) experiment, which considered a single type of correlated equilibrium, our experimental design considers three different correlated equilibria, each associated with a different probability distribution for recommended play. In our "Nash–recommendations" treatment, the recommendations (D, C) and (C, D) are each selected with probability one–half, and (C, C) and (D, D) are selected with probability zero. This distribution of recommended outcomes is a correlated equilibrium, and moreover, is a convex combination of Nash equilibria, with payoffs of 6

[6] Cason and Sharma—somewhat pessimistically, in our opinion—conclude from these results that "players frequently reject recommendations," and their experiment includes additional treatments designed to increase the likelihood that recommendations are followed, such as having human subjects play against a computer program that always follows recommendations. Recommendations are typically followed even more often in these variations.

[7] For example, their instructions state "[y]ou should follow the recommendation given by the computer, because as long as the person you are paired with also follows his or her recommendation then you earn more on average by following the recommendation" and "[t]o reiterate: you always earn more by following your recommendation as long as the participant you are paired with also follows his or her recommendation"; see http://www.krannert.purdue.edu/faculty/cason/papers/corr-inst.pdf.

for each player. We also consider two correlated equilibria that are not convex combinations of Nash equilibria. In our "good–recommendations" treatment, the recommended outcomes (D, C), (C, D), and (C, C) are each selected with probability one–third, and (D, D) is selected with probability zero. These probabilities satisfy the conditions for a correlated equilibrium, and yield payoffs of $6\frac{1}{3}$ for each player—more than any point in the convex hull of Nash equilibrium payoff pairs. In addition to the good–recommendations treatment, however, we also consider a "bad–recommendations" treatment, in which the recommended outcomes (D, C) and (C, D) are each selected with probability 0.4, and (D, D) is selected with probability 0.2, so that (C, C) is selected with probability zero. These probabilities also satisfy the conditions for a correlated equilibrium, but result in payoffs of only 4.8 for each player—less than any point in the convex hull of Nash equilibrium payoff pairs. As far as we know, there are no existing experimental studies of correlated equilibria that are payoff–reducing relative to Nash equilibrium.

Finally, as an even stronger test of the correlated equilibrium concept, we consider one distribution of recommended outcomes that is *not* a correlated equilibrium. In our "very–good–recommendations" treatment, the recommended outcome (C, C) is selected with probability 0.8, (D, C) and (C, D) are each selected with probability 0.1, and (D, D) is selected with probability zero. Given these probabilities, a player receiving a D recommendation will prefer to follow it—assuming she believes her opponent will also follow recommendations—but a player receiving a C recommendation will not, instead preferring to choose D. If recommendations are followed, however, payoffs are 6.8 for each player—higher than in any of three correlated equilibria discussed above.

Some features of the four recommended outcome distributions we use, as well as the mixed–strategy Nash equilibrium, are shown in Table 1. The expected payoffs from following these distributions of recommended outcomes are also shown in Figure 2 (as plus signs).

Table 1: Outcome frequencies imposed in the experiment

	Probability of (D,D) outcome	Probability of (D,C) outcome	Probability of (C,D) outcome	Probability of (C,C) outcome	Probability of C choice	Expected payoffs
Good recommendations	0.000	0.333	0.333	0.333	0.667	(6.333,6.333)
Bad recommendations	0.200	0.400	0.400	0.000	0.400	(4.8,4.8)
Nash recommendations	0.000	0.500	0.500	0.000	0.500	(6,6)
Very good recommendations	0.000	0.100	0.100	0.800	0.900	(6.8,6.8)
Mixed–strategy NE	0.160	0.240	0.240	0.360	0.600	(5.4,5.4)

3 Experimental procedures

Besides varying the type of recommendations that were given to subjects (that is, the probability distribution over outcomes), we varied whether recommendations were given at all. All experimental sessions lasted for 40 rounds: 20 rounds with recommendations and 20 rounds without recommendations. We also varied the order of these; in half of our sessions, the 20 rounds without recommendations came first, and in the other half, the 20 rounds with recommendations came first. (Thus, whether or not recommendations were given was varied within–subject, while the type of recommendations and the ordering between recommendations and no recommendations were varied between–subjects.) Each experimental session involved 12 subjects. Subjects were primarily undergraduate students from University of Pittsburgh, and were recruited by newspaper advertisements and email. No one took part in more than one session of this experiment.

At the beginning of a session, subjects were seated in a single room and given a set of written instructions for the first twenty rounds.[8] They were told at this time that there would be a second part to the session, but details of the second part were not announced until after the first part had ended. The instructions for the first part were read aloud to subjects, in an attempt to make the rules of the game common knowledge. After the instructions were read, subjects were given a short quiz to ensure that they understood the instructions. After subjects' quizzes were completed, they were graded anonymously. If any question was answered incorrectly, the experimenter went over the question and answer out loud for the benefit of all subjects (without identifying which subject had answered incorrectly). After any incorrect answers were discussed, the first round of play began. After the twentieth round of play was completed, each subject was given a copy of the instructions for the remaining twenty rounds. These were also read aloud, after which another (shorter) quiz was given out, before the final twenty rounds were played.

In the instructions, we strove to use neutral terminology. Instead of relatively loaded terms such as "opponent" or "partner", we used phrases such as "the player matched with you". Also, in our discussion of recommendations, we never went so far as to instruct subjects to follow recommendations, or even to point out that following recommendations might lead to higher payoffs; rather, we merely explained how one's recommendation may or may not convey information about the recommendation given to the other player.[9]

The experiment was run on networked computers, using the z–Tree experiment software package (Fischbacher (2007)). Subjects were asked not to communicate directly with one another, so the only interactions were via the computer program. Subjects were paired using a round–robin matching format, in an attempt to minimize incentives for reputation building and other supergame effects; for the same reason, subjects were not given identifying information about their opponents in any round.

A round of the game in which there were no recommendations (either rounds 1–20 or rounds 21–40, depending on the cell) began by prompting subjects to choose one of the two available actions. (In the instructions and during the session, the actions were named X and Y instead of D and C, respectively.) After the action choices were entered, each subject was shown the following information: own action,

[8]The set of instructions given to subjects—as well as additional materials given to them (quizzes and record sheets)—from one of our cells can be found at http://www.abdn.ac.uk/~pec214/papers/corr_instructions.pdf. Materials used in the other cells and screenshots of the computer interface seen by subjects, as well as the raw data from the experiment, are available from the corresponding author upon request.

[9]One passage from our instructions states, "These recommendations are optional; it is up to you whether or not to follow them. Notice that your recommendation gives you information about the recommendation that was given to the person matched to you." To further emphasize this point, one of the questions in the quiz given to subjects after reading the instructions was, "You are required to follow the recommendations shown on your computer screen (circle one): TRUE FALSE"—to which the correct answer was FALSE. We acknowledge the possibility that our use of the term "recommendations" itself might have influenced subjects to follow them to some extent.

opponent action, own payoff, and opponent payoff. In a round of the game with recommendations, the sequence of play was similar except for the recommendations. Specifically, subjects would first be shown their "recommended actions", which were randomly drawn from the appropriate outcome distribution. Then, they were prompted to choose an action. After these choices were entered, each subject was shown the following information: own recommendation, own action, opponent recommendation, opponent action, own payoff, and opponent payoff. In all treatments, subjects were not given information about the results of any other pairs of subjects, either individually or in aggregate. At the end of the round, subjects were asked to observe their result, write the information from that round down onto a record sheet, and then click a button to continue to the next round.

At the end of round 40 of any treatment, the experimental session ended. One of the first twenty rounds and one of the last twenty rounds were randomly chosen, and each subject received his/her earnings from these two rounds, at an exchange rate of $1 per point. Additionally, all subjects received a $5 show–up fee. Total earnings for subjects participating in a session averaged about $15, and sessions typically lasted between 45 and 60 minutes.

4 Experimental results

A total of 16 sessions were conducted—four of each treatment—with 12 subjects per session, for a total of 192 subjects. We first look at whether (and how) recommendations affected aggregate behavior. In the following section, we look at how subjects responded to the recommendations given to them.

4.1 Effect of recommendations on population aggregates

The first question we ask is whether the recommendations given to subjects have any effect at all. Table 2 provides strong evidence that they do. This table reports the relative frequencies over 20 rounds of each of the three possible outcomes: both choosing D, exactly one choosing D (the pure–strategy Nash equilibrium), and both choosing C, conditional on the type of recommendations given to subjects (none, good, bad, Nash, very good).[10]

Table 2: Aggregate observed outcome frequencies

Outcome	Recommendations				
	None	Good	Bad	Nash	Very good
(D,D)	0.159	0.140	0.192	0.112	0.169
(C,D) or (D,C)	0.497	0.579	0.481	0.565	0.469
(C,C)	0.343	0.281	0.327	0.323	0.363
Average payoff	5.387	5.444	4.902	5.648	5.350

[10]In this table and elsewhere, we combine the data from sessions with recommendations in the first 20 rounds with the data from sessions with recommendations in the last 20 rounds. We will see later (in Table 4 and surrounding discussion) that pooling the data in this way is justifiable.

In the rounds with no recommendations, the aggregate outcome frequencies were very close to the mixed–strategy Nash equilibrium prediction of 16% (D,D) outcomes, 48% (C,D) and (D,C) outcomes, and 36% (C,C) outcomes. On the other hand, outcome frequencies are significantly different when subjects are given recommendations, and the difference is in the direction predicted by correlated equilibrium (following recommendations), in each of the cases where recommendations are consistent with a correlated equilibrium. Both good recommendations and Nash recommendations increase the likelihood of a pure–strategy Nash equilibrium outcome, from 49.7% without recommendations to 56.5% with Nash recommendations and 57.9% with good recommendations, though this likelihood decreases slightly in the game with bad recommendations—to 48.1%—and with very good recommendations, to the lowest frequency of 46.9%. Also, the Pareto–dominated (D,D) outcome becomes more likely under bad or very good recommendations (19.2% and 16.9% of the time respectively, versus 15.9% when no recommendations are given) and less likely under good recommendations (14.0%) or Nash recommendations (11.3%). For some of the treatments, this last result might be expected in light of the outcome probabilities we attempted to impose: 20% chance of (D,D) in the bad–recommendations treatment and 0% in the good– and Nash–recommendations treatment as compared with 16% in the mixed–strategy Nash equilibrium. However, this does not hold for the very–good–recommendations treatment, as the frequency of (D,D) recommended outcomes was 0% in this treatment as well.

One–sample chi–square tests imply that the distribution of outcomes in the good–recommendations treatment is significantly different from the mixed–strategy Nash equilibrium distribution ($\chi^2 = 38.71$, $d.f. = 2$, $p < 0.001$), as are those in the bad–recommendations treatment ($\chi^2 = 8.91$, $d.f. = 2$, $p \approx 0.012$) and the Nash–recommendations treatment ($\chi^2 = 31.51$, $d.f. = 2$, $p < 0.001$).[11] We do not find significant differences between the very–good–recommendations treatment and the mixed–strategy Nash equilibrium ($\chi^2 = 0.729$, $d.f. = 2$, $p \approx 0.695$), suggesting that on average, subjects ignore recommendations in this case. Similarly, two–sample chi–square tests find significant differences in the distribution of outcomes under no recommendations and under good, bad, or Nash recommendations ($p < 0.05$ for each), but not between no recommendations and very good recommendations ($\chi^2 = 2.522$, $d.f. = 2$, $p \approx 0.283$). The finding of no difference between no recommendations and very good recommendations is striking: it suggests that subjects will not blindly follow just *any* recommendations, but rather will follow them only if they are consistent with implementation of a correlated equilibrium. Furthermore, we note that in the cases where recommendations *were* consistent with implementation of a correlated equilibrium, aggregate outcome frequencies—while different from the point predictions of Table 1—typically move in the direction predicted by correlated equilibrium relative to the mixed–strategy Nash equilibrium value. For example, if subjects were always to follow recommendations in the good–, bad– and Nash–recommendations treatments, the resulting frequency of (C,C) outcomes would be lower than in the mixed–strategy Nash equilibrium. As Table 2 shows, the frequencies of (C,C) outcomes in these cases are indeed lower than in mixed–strategy Nash equilibrium. By contrast, in the very–good–recommendations case, if subjects followed recommendations, the predicted frequency of (D,D) outcomes would be lower than in the mixed Nash equilibrium (0

[11] See Siegel and Castellan (1988) for descriptions of the nonparametric tests used in this paper. We note that in each treatment, observed frequencies are significantly different from any of the *correlated*–equilibrium predictions, as each of the latter predicts zero probability of at least one outcome that occurs with positive frequency in the experimental data.

versus 0.16), but Table 2 shows that the observed frequency is actually higher. Finally, two–sample chi–square tests nearly always find significant differences in the distribution of outcomes between any two of the recommendations treatments ($\chi^2 = 5.73$, $d.f. = 2$, $p \approx 0.057$ between the good– and Nash–recommendations treatments, $p < 0.05$ for any other pair of treatments).

Summarizing, we have:

Result 1 *When no recommendations are given, aggregate outcome frequencies are similar to mixed–strategy Nash equilibrium frequencies. When recommendations are given, they lead to significant differences in aggregate outcome frequencies, compared with the no–recommendations case, if and only if the recommendations come from a correlated equilibrium. Also, there are significant differences in aggregate outcomes across the treatments with recommendations. When recommendations come from a correlated equilibrium, the effect on aggregate outcome frequencies is consistent with the directional predictions—though usually not the point predictions—of the corresponding correlated equilibrium.*

We next consider the effects of recommendations on average payoffs. If recommendations are always followed, then good recommendations should lead to higher payoffs than in mixed–strategy Nash equilibrium, while bad recommendations should lead to lower payoffs. Also, our Nash recommendations should lead to payoffs in between good and bad recommendations, though still higher than mixed–strategy Nash equilibrium payoffs, while very good recommendations should lead to the highest payoffs of all the treatments.

In fact, average payoffs in rounds without recommendations are approximately 5.387, very close to the mixed–strategy Nash equilibrium level of 5.4. In rounds with good recommendations, average payoffs do increase, but only slightly—to 5.444 (compared with a prediction of 6.333 in the good correlated equilibrium). In rounds with Nash recommendations, average payoffs increase even more (despite the correlated–equilibrium prediction that payoffs in this treatment should be lower than in the good–recommendations treatment), to 5.648, though this is still below the corresponding correlated–equilibrium prediction of 6.[12]

In rounds with bad recommendations, average payoffs do decrease relative to rounds without recommendations, but again, not by as much as predicted—to 4.902 (compared with a prediction of 4.8 in the bad correlated equilibrium). In rounds with very good recommendations, average payoffs also decrease, to 5.350 (compared with an increase to 6.8 if subjects always followed their recommendations), and lower than when subjects were not given recommendations.

Despite these many payoff differences, a nonparametric Kruskal–Wallis one–way analysis of variance fails to reject the null hypothesis that average payoffs are the same in all four recommendations

[12]The success of the Nash–recommendations treatment—relative to the good–recommendations treatment—in achieving higher payoffs may reflect the greater transparency of recommendations in the former. Under Nash recommendations, low–payoff (D,D) outcomes are the result of a player following a D recommendation while her opponent chooses D despite having received a C recommendation. However, the latter player, after receiving a C recommendation, knows *with certainty* his opponent received a D recommendation, and thus that choosing D rather than C is likely to result in a low–payoff outcome. On the other hand, a player receiving a C recommendation in the good–recommendations treatment knows that his opponent might have gotten either a C or a D recommendation. If his opponent did also receive a C recommendation, then he himself could choose D in the (reasonable) hope that his opponent will choose C. The resulting greater temptation to choose D in the good–recommendations treatment might be a factor in the higher frequency of (D,D) outcomes, and lower average payoffs, in that treatment than under Nash recommendations.

treatments (session–level data, $p > 0.10$), and robust rank–order tests find no significant differences in pairwise comparisons between treatments (session–level data, $p > 0.10$ in all cases). This lack of significance in the payoff dimension is likely owing to the relatively small differences in predicted expected payoffs amongst the various correlated equilibria, combined with the inherent conservatism of nonparametric tests using session–level data.

4.2 Effects of recommendations on individual behavior

Having shown that aggregate play with recommendations is usually different from aggregate play without recommendations—and that this difference depends on which recommendations are given—we next consider how subjects treat the particular recommendations they receive. Table 3 shows the frequencies with which recommendations are followed in each treatment (over all twenty rounds) as well as for the last five rounds of each treatment (after subjects have had time to gain experience with the strategic environment). For the sake of comparison, the table also shows the corresponding predicted frequencies according to mixed–strategy Nash equilibrium. (Note that the predictions in the last two columns depend on the actual frequencies of C versus D recommendations given in the experiment, so these will vary across treatments and rounds).

Table 3: Frequencies of followed recommendations—all subjects, all rounds

Treatment		Frequency of followed D recommendations	Frequency of followed C recommendations	Frequency of followed recommendations (overall)	Frequency of followed recommendations (pairs)
Good	Observed (all rounds)	0.735*	0.732*	0.733*	0.531*
	Observed (rnds 16–20)	0.750*	0.770*	0.762*	0.583*
	Mixed NE prediction	0.400	0.600	0.531	0.277
Bad	Observed (all rounds)	0.477	0.631	0.541	0.269*
	Observed (rnds 16–20)	0.529*	0.530	0.529*	0.300*
	Mixed NE prediction	0.400	0.600	0.483	0.226
Nash	Observed (all rounds)	0.567*	0.777*	0.672*	0.454*
	Observed (rnds 16–20)	0.608*	0.792*	0.700*	0.517*
	Mixed NE prediction	0.400	0.600	0.500	0.240
Very good	Observed (all rounds)	0.511*	0.608	0.599	0.381
	Observed (rnds 16–20)	0.227	0.537	0.508	0.308
	Mixed NE prediction	0.400	0.600	0.580	0.336

*: *Significantly different from corresponding mixed–strategy prediction (sign test, session–level data, p=0.0625)*

Here we see more differences across treatments. In the good– and Nash–recommendations treatments, subjects are substantially more likely to follow recommendations than would be predicted by the mixed–strategy Nash equilibrium. They follow D recommendations 73.5% of the time in the good–recommendations treatment and 56.7% of the time in the Nash–recommendations treatment, compared to a prediction of 40%, and they follow C recommendations 73.2% of the time in the good–recommendations treatment and 77.7% of the time in the Nash–recommendations treatment, compared to a prediction of 60%. As the table shows, these frequencies are even higher if we concentrate on the last five rounds of the treatment, and all of these differences are significant (one–tailed sign test, session–level data, $p = 0.0625$).

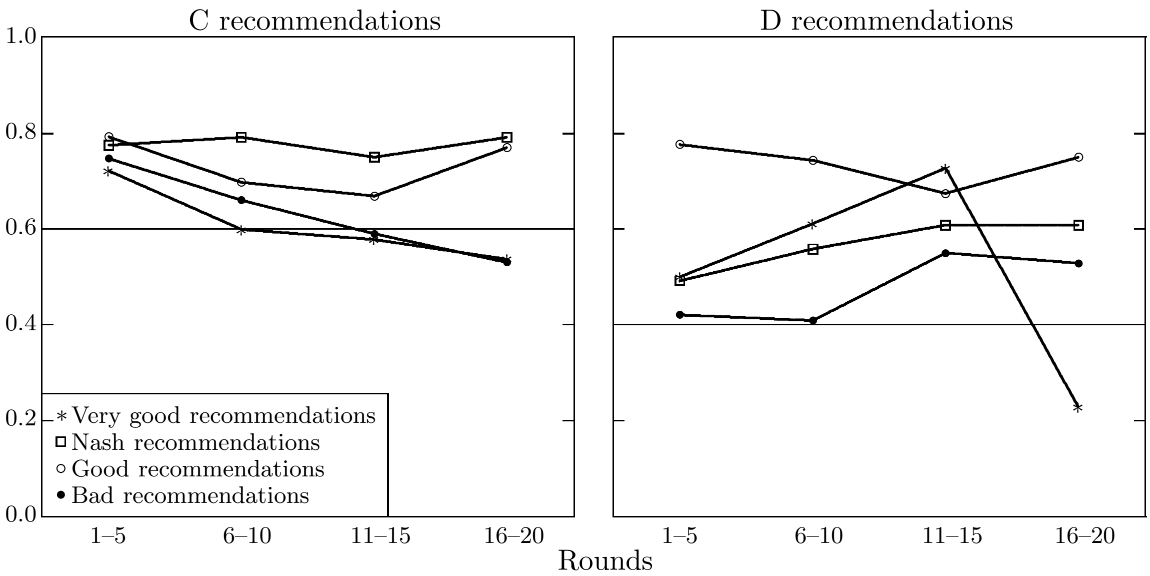
In the bad– and very–good–recommendations treatments, evidence that subjects follow recommendations is weaker. If we look at frequencies for the entire bad–recommendations treatment, subjects are not significantly more likely to follow either type of recommendation than predicted by mixed–strategy equilibrium ($p > 0.10$), though the frequencies are slightly higher than predicted (47.7% for D recommendations and 63.1% for C recommendations, versus predictions of 40% and 60%). However, if we focus on the last five rounds, we find a higher frequency of following D recommendations, and this frequency is significantly higher than the mixed–strategy equilibrium prediction ($p = 0.0625$), though we also see that subjects actually become less likely to follow C recommendations in the last 5 rounds. Subjects in the very–good–recommendations treatment are not significantly more likely to follow C recommendations than mixed–strategy equilibrium predicts ($p > 0.10$), either over all rounds or in the last five. They are more likely to follow D recommendations over all rounds (51.1% versus a predicted 40%), and this difference is significant ($p = 0.0625$), but this frequency drops sharply in the last five rounds to below one–fourth, and in those rounds is not significantly different from the mixed–strategy equilibrium prediction ($p > 0.10$).

Given how these results vary according to the treatment, it should not be surprising that we find significant differences across treatments in how often recommendations are followed. A Kruskal–Wallis one–way analysis of variance rejects the null hypothesis that the likelihood of following a C recommendation is the same across the four treatments ($p < 0.05$), and similarly for the case of a D recommendation ($p < 0.05$). The null of equal frequencies across treatments of following recommendations overall is also rejected ($p < 0.05$), but we should note that mixed–strategy Nash equilibrium does not imply equal frequencies in this case.

We next examine how subjects' willingness to follow recommendations changes over time. Figure 4 shows the frequency with which recommendations are followed in each five–round block, disaggregated according to which correlated equilibrium was being implemented, and which action was recommended. For C recommendations, there are no obvious time trends in the good– and Nash–recommendations treatments, while subjects in the bad– and very–good–recommendations treatments become less likely over time to follow these (falling from about 75% to just over 50% in both). The frequency of following D recommendations stays roughly constant over time in the good–recommendations treatment and rises slightly in the bad– and Nash–recommendations treatment. In the very–good–recommendations treatment, sample sizes for D recommendations are small (since only one–tenth of recommendations is for a D choice), but their frequency of being followed rises somewhat from the first to the third five–round block, before plummeting in the last five–round block.

Further evidence of the effects of recommendations on individual subject choices can be found in Table 4, which reports the results of several probit regressions with the subject's choice of action as the dependent variable. (To be precise, the dependent variable is an indicator for a C choice.) Our main independent variables are two indicators for recommendations given to subjects—one for a C recommendation (viz., taking on the value of one if a C recommendation was made, and zero

Figure 4: Frequency of followed recommendations



otherwise) and one for a D recommendation. (To avoid perfect collinearity, we do not include an indicator for no recommendation.) We also include variables for the products of these indicators with the round number, to capture any time–varying effect of recommendations that exists. Additionally, we include a variable for the round number itself, as well as an indicator variable that takes the value one in sessions in which recommendations were given in the first twenty rounds rather than the last twenty (to capture any order effects).

We estimate coefficients separately for the four treatments. For each treatment, we estimate one set of coefficients using individual–session fixed effects, and one set without these. All of the regressions were performed using Stata (version 10) and incorporate individual–subject random effects. The results are shown in Table 4, which shows the coefficient and standard error for each variable in our eight model specifications. Also shown is the absolute value of the log–likelihood, as well as a pseudo–R^2, for each model specification.[13]

Before discussing the main results of these regressions, we note that there do not seem to be substantial order effects between rounds with recommendations and rounds without recommendations in any of the treatments, as the "Order" variable is never significant. Also, coefficient estimates are quite robust to whether session fixed effects are used. On the other hand, there is some nonstationarity in the data, as shown by the negative and significant coefficient on the round number t in three of the four treatments (the lone exception being the good–recommendations treatment).

The significance of the recommendation variables varies substantially across treatments. In the good and Nash–recommendations treatments, the C–recommendations and D–recommendations indicators are both significant, but their products with the round number are not, and each has the sign associated with subjects' following recommendations: positive for C and negative for D. In the bad–recommendations treatment, the D–recommendations indicator is insignificant, but its product

[13]The pseudo–R^2 values were computed by rescaling the log–likelihoods into [0,1], such that a model with no right–hand–side variables other than the constant term maps to zero, and a perfect fit maps to one.

Table 4: Results of probit regressions with random effects (std. errors in parentheses)

Dependent variable: cooperative action chosen in round t	Good–recommendations treatment ($N = 1920$)		Bad–recommendations treatment ($N = 1920$)		Nash–recommendations treatment ($N = 1920$)		Very–good–recommendations treatment ($N = 1920$)	
Session fixed effects?	No	Yes	No	Yes	No	Yes	No	Yes
constant	0.372	0.411	0.464	0.295	0.727***	0.854**	0.956***	0.898***
	(0.237)	(0.320)	(0.295)	(0.395)	(0.250)	(0.340)	(0.241)	(0.324)
Order (indicator	0.046	–0.256	0.232	0.825	–0.097	–0.137	–0.440	–0.252
for order effects)	(0.312)	(0.434)	(0.392)	(0.539)	(0.327)	(0.462)	(0.314)	(0.437)
t (round number)	0.001	0.001	−0.022**	−0.022**	−0.027***	−0.027***	−0.048***	−0.048***
	(0.008)	(0.008)	(0.009)	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)
Drec (D recom–	−1.098***	−1.097***	–0.075	–0.076	−0.463***	−0.463***	−1.313***	−1.315***
mendation given)	(0.207)	(0.207)	(0.181)	(0.171)	(0.172)	(0.172)	(0.299)	(0.299)
Drec $\cdot\, t$	–0.001	–0.001	−0.022***	−0.022***	–0.009	–0.009	0.088***	0.088***
	(0.017)	(0.017)	(0.014)	(0.014)	(0.015)	(0.015)	(0.025)	(0.025)
Crec (C recom–	0.593***	0.592***	0.338*	0.338*	0.472**	0.472**	0.086	0.086
mendation given)	(0.162)	(0.162)	(0.200)	(0.200)	(0.184)	(0.184)	(0.150)	(0.150)
Crec $\cdot\, t$	–0.014	–0.014	–0.021	–0.021	0.021	0.022	0.002	0.002
	(0.013)	(0.013)	(0.016)	(0.016)	(0.015)	(0.015)	(0.012)	(0.012)
–ln(L)	970.789	970.058	898.429	897.032	950.705	950.485	971.544	971.325
pseudo–R^2	0.106	0.107	0.027	0.028	0.086	0.087	0.037	0.037

* (**,***): *Coefficient significantly different from zero at the 10% (5%, 1%) level.*

with the round number is significant; the coefficient of the C–recommendations indicator is barely significantly different from zero, while that of its product with the round number is insignificant. In the very–good–recommendations treatment, both the C–recommendations indicator and its product with the round number are significant, with the former negative and the latter positive, but neither of the D–recommendations variables are significant.

Next, in order to examine the overall significance of C and D recommendations in our four treatments, we estimate the effect of a recommendation versus no recommendation on the subject's subsequent action. To do this, we first note that the total effect in round t of a C recommendation, instead of no recommendation at all, on the argument of the normal c.d.f. used in the probit model is given by $\beta_{\text{Crec}} + \beta_{\text{Crec}\cdot\text{Round}} \cdot t$ (where β_Y is the coefficient of the variable Y). So, the incremental effect (the analog to a marginal effect, for a discrete variable) of a C recommendation rather than no recommendation in round t has the form

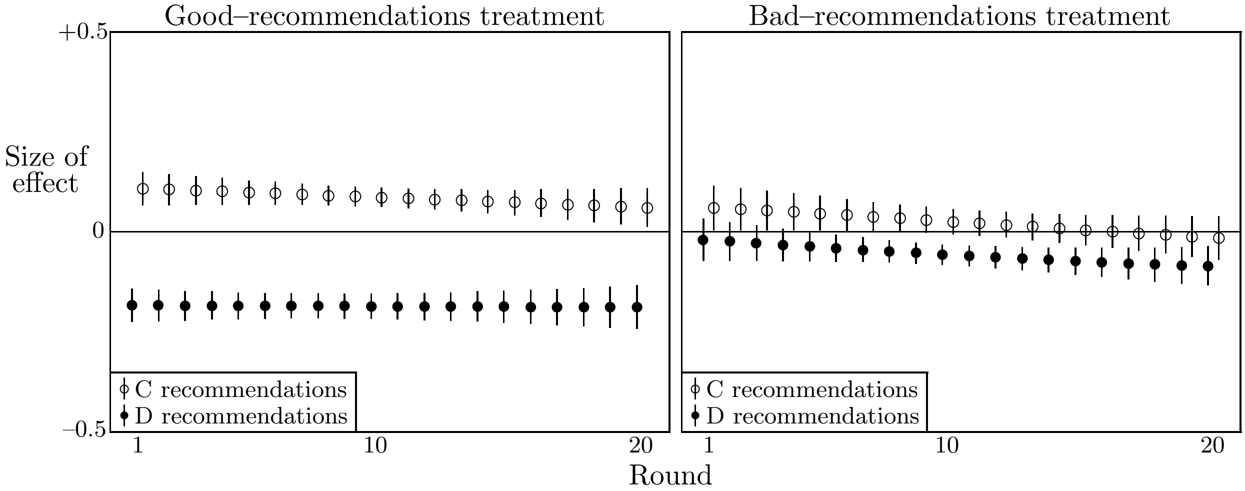
$$\Phi\left(\bar{X} \cdot B + \beta_{\text{Crec}} + \beta_{\text{Crec}\cdot\text{Round}} \cdot t\right) - \Phi\left(\bar{X} \cdot B\right), \tag{1}$$

where $\bar{X}$ is the row vector of the other right–hand–side variables' values, and B is the column vector of their coefficients. The incremental effect of a D recommendation has a similar form.

In Figures 5 and 6, we graph all eight versions of this incremental effect (versions of Equation 1 for C and D recommendations and for good–, bad–, Nash–, and very–good–recommendations treatments). These figures show, for each expression, the corresponding point estimates and 90% confidence intervals for each value of t (the round number) from 1 to 20.[14] Consistent with what we've seen already, Figure 5

[14]We use 90% confidence intervals rather than the more common 95% confidence intervals in order to give us 5% rejection regions for each tail; this corresponds to using one–tailed hypothesis tests at the 5% level.

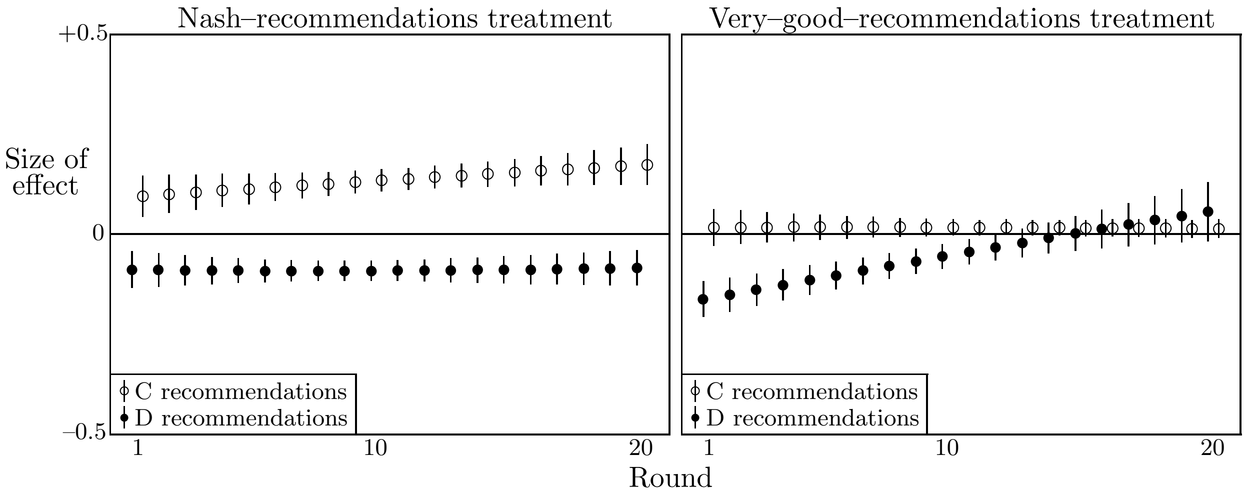
Figure 5: Estimated incremental effect of recommendation on C choice probability based on Table 4 results *(Circles represent point estimates; line segments represent 90% confidence intervals)*



shows that in the good–recommendations treatment, both types of recommendation have a significant effect: C recommendations increase the subject's likelihood of choosing C, and D recommendations increase the likelihood of a D choice, in every round. In the bad–recommendations treatment, a C recommendation has no significant effect on the likelihood of a C choice except in the first few rounds. The effect of a D recommendation, conversely, is initially insignificant, but by round 6 becomes significant and negative, and remains so for the remainder of the session.

Figure 6 shows that in the Nash–recommendations treatment, as in the good–recommendations treatment, both types of recommendation have effects that are significant and in the predicted direction in every round: C recommendations increase the likelihood of choosing C, and D recommendations decrease it. In the very–good–recommendations treatment, a C recommendation has no significant

Figure 6: Estimated incremental effect of recommendation on C choice probability based on Table 4 results *(Circles represent point estimates; line segments represent 90% confidence intervals)*



effect on the likelihood of a C choice in any round. A D recommendation significantly decreases the likelihood of a C choice—that is, increases the likelihood of a D choice—in early rounds, but this effect decreases over time, becoming insignificant by the end of the session.

Based on the results in this section, we conclude:

Result 2 *There are significant differences across treatments in how subjects use their recommendations. Subjects are most likely to follow recommendations in the good– and Nash–recommendations treatments. In the bad–recommendations treatment, recommendations have less effect on behavior. In the very–good–recommendations treatment, subjects either don't follow recommendations at all, or learn over time not to follow them.*

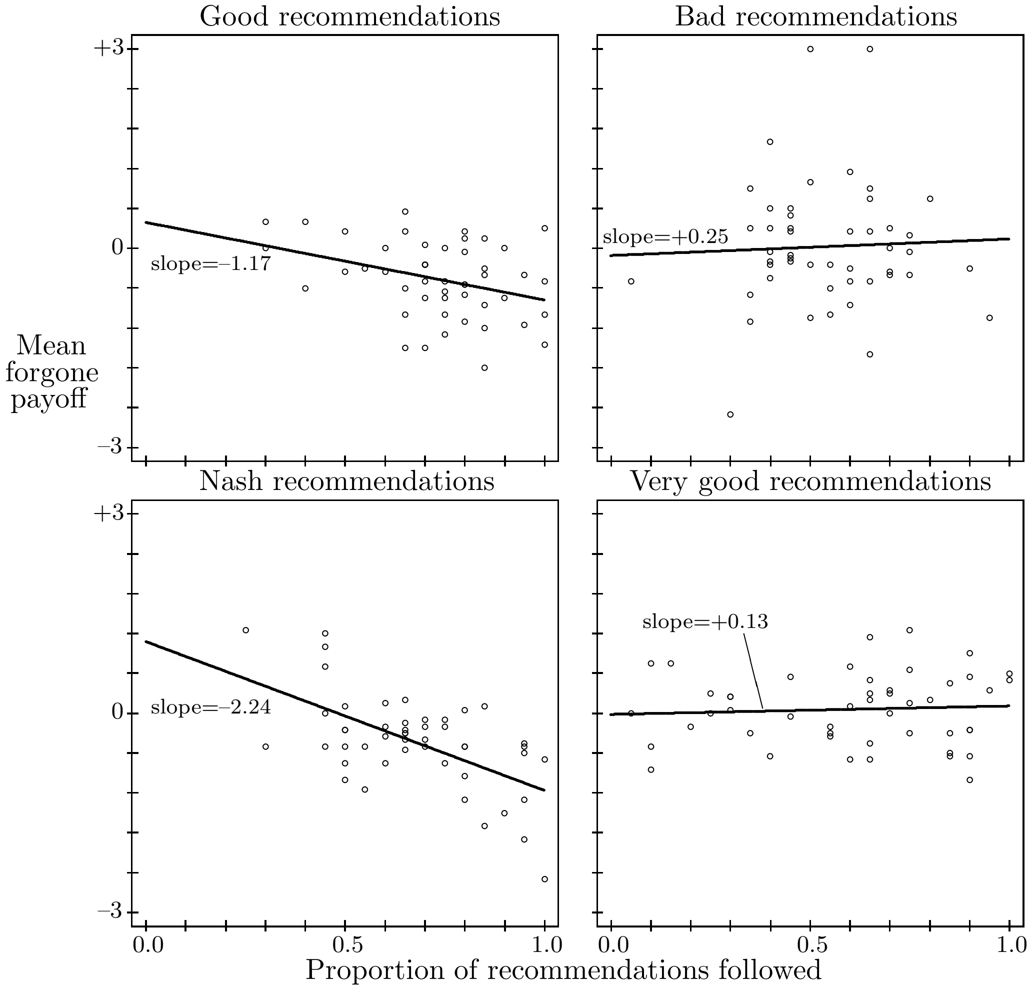
A natural next question to ask is whether subjects who fail to follow recommendations suffer (monetarily) due to this failure. To examine this question, we consider subjects' *forgone payoffs*: the payoff a subject would have gotten from choosing the other action, minus the payoff the subject actually got. (Thus, a negative forgone payoff means the subject chose a best response.)

Overall, in rounds with no recommendations, forgone payoffs averaged –0.079 points per round, meaning that on average, subjects earned higher payoffs with the actions they chose than they would have by choosing the opposite action. Forgone payoffs averaged –0.467 points per round in rounds with good recommendations, +0.026 points per round in rounds with bad recommendations, –0.428 points per round in rounds with Nash recommendations, and +0.059 points per round in rounds with very good recommendations, suggesting that subjects in the good– and Nash–recommendations treatments by and large made correct choices, while subjects in the other two treatments did not. Since the good– and Nash–recommendations treatments were also the ones where subjects were most likely to follow recommendations, the implication is that following recommendations is indeed positively associated with better outcomes for the individual subject, at least on average. However, we are interested less in these treatment–wide aggregates than in how forgone payoffs are associated with how often subjects followed the recommendations they were given.

In Figure 7, we present scatterplots showing, for each individual subject, the proportion of recommendations that were followed (on the horizontal axis) and the subject's mean forgone payoff (on the vertical axis). Also shown are least–squares lines for each scatterplot, as well as the corresponding slopes. For subjects in the good– and Nash–recommendations treatments, there is a visible negative correlation between following recommendations and forgone payoffs, while no such correlation is apparent for subjects in the bad– and very–good–recommendations treatments. Spearman rank–order correlation tests provide further, quantitative, evidence of this result. The Spearman correlation coefficient between frequency of followed recommendations and mean forgone payoff is approximately –0.312 in the good–recommendations treatment and –0.469 in the Nash–recommendations treatment, both of which are significantly different from zero ($p \approx 0.02$ for the former and $p < 0.001$ for the latter), suggesting that following recommendations more often was associated with better payoffs for individual subjects. In the bad– and very–good–recommendations treatments, on the other hand, the Spearman coefficients are approximately +0.003 and +0.092 respectively, neither of which is significantly different from zero ($p \approx 0.98$ and $p \approx 0.25$, respectively), suggesting that subjects in these treatments did not do better by following recommendations than by ignoring them. The trend lines

Figure 7: Relationship between followed recommendations and

forgone payoffs *(Individual subjects, all rounds)*



give additional evidence of these relationships; their slopes are negative and significantly different from zero in the good– and Nash–recommendations treatments ($p < 0.01$ for both treatments, using robust standard errors adjusted for clustering by session), and are not significantly different from zero in the bad– and very–good–recommendations treatments ($p > 0.20$ for both treatments).

Finally, disaggregating by round and according to the recommended action tells a more detailed, but similar, story. Linear panel–data regressions with individual–subject random effects, either with or without session fixed effects, show that a subject's following either type of recommendation in either the good–recommendations treatment or the Nash–recommendations treatment is associated with significant decreases in forgone profit, as is following a C recommendation in the bad–recommendations treatment. In contrast, there is no significant association between forgone payoffs and either following D recommendations in the bad–recommendations treatment or following C recommendations in the very–good–recommendations treatment. Finally, following D recommendations is actually positively correlated with foregone payoffs in that treatment; that is, following D recommendations actually lowers the player's payoff.[15]

We thus conclude:

Result 3 *In the good and Nash–recommendations treatments, it pays subjects (individually) to follow either type of recommendation. In the bad–recommendations treatment, it pays subjects to follow C recommendations, but there is no statistically significant relationship between following D recommendations and payoffs. In the very–good–recommendations treatment, there is no significant relationship between following C recommendations and payoffs, and it pays subjects not to follow D recommendations.*

5 Summary and discussion

The aim of this paper was to assess the empirical validity of correlated equilibria, a generalization of the Nash equilibrium concept which Hart and Mas-Colell (2000, p. 1128) characterize as perhaps the "most relevant noncooperative solution concept." Specifically, we have explored whether subjects would make use of known distributions of private recommendations as a coordination device in the game of Chicken, the simplest game with which to study a wide variety of correlated equilibria. The treatments in our experiment differ in the distributions of third–party recommendations. Three of our four treatments use distributions that form correlated equilibria; two of these yield symmetric payoffs that are outside the convex hull of Nash equilibrium payoff vectors. In our "good" correlated equilibrium, payoffs are better than any symmetric payoff in the convex hull of Nash equilibrium payoff vectors, while in our "bad" correlated equilibrium, payoffs are worse than any symmetric payoff in the convex hull of Nash equilibrium payoff vectors. A third, "Nash" treatment uses a correlated equilibrium with payoffs in the convex hull of Nash equilibrium payoff vectors, and a fourth, "very good" treatment uses an outcome distribution yielding high payoffs, but which is not a correlated equilibrium.

We find that when subjects do not receive recommendations, their choices can be described fairly well by mixed–strategy Nash equilibrium. This result suggests that theoretical rationales for correlated equilibria that do not rely on extrinsic, third–party recommendations (or some other "external event space" in the terminology of Vanderschraaf (2001)) might be difficult to observe in practice—though we acknowledge the possibility that if subjects had interacted in fixed pairings rather than under the random matching protocol we adopted, spontaneously arising correlated equilibrium might have been more likely to have been observed.

By contrast, giving subjects recommendations nearly always has an effect on behavior, but the effect depends on what recommendations are given. When recommendations are based on an under-

[15]These regressions used the subset of the data in which recommendations were given. The dependent variable is forgone payoff, and independent variables are C recommendation, D recommendation, followed C recommendation, followed D recommendation. No constant term was used. In the results, p–values were below 0.001 for both types of recommendation in the good– and Nash–recommendations treatment, approximately 0.043 for C recommendations in the bad–recommendations treatment, and approximately 0.77 for D recommendations in the bad–recommendations treatment. In the very–good–recommendations treatment, the p–value was approximately 0.78 for C recommendations and 0.030 for D recommendations, but the coefficient for the latter was positive. Adding session fixed effects to these regressions had little qualitative effect.

lying correlated equilibrium, subjects follow them more often than the mixed–strategy equilibrium predicts, though far less than 100% of the time. When recommendations are not based on a correlated equilibrium, subjects learn to ignore them.

As in previous efforts to experimentally implement correlated equilibria in the laboratory, our results cast some doubt on the usefulness of this solution concept as a descriptive notion, as the correlated equilibrium point predictions are not observed. On the other hand, our study reveals several new and important empirical findings about the correlated equilibrium concept. First, the lesson of our very–good–recommendations treatment is that correlated equilibrium is likely a necessary condition for recommendations to be followed. In particular, we found that, consistent with the theoretical prediction, subjects were not blindly following recommendations in this treatment (as they would have if they were, for example, simply trying to please the experimenters, or choosing high–payoff outcomes irrespective of the outcomes' strategic properties). Second, we found that average payoffs were highest in our Nash–recommendations treatment, even though correlated equilibrium payoffs were predicted to be highest in our good–recommendations treatment. This finding suggests that *simpler* correlated equilibria—those involving randomizations over pure Nash equilibria—might be the more likely outcome of agent learning dynamics than more complicated correlated equilibria yielding payoffs outside the convex hull of Nash equilibrium payoffs. Finally, our bad–recommendations treatment shows that it may not be possible to induce subjects to follow recommendations based on correlated equilibria that are Pareto *inferior* to the available Nash equilibria. This last finding would seem to greatly limit the class of empirically relevant correlated equilibria to those that Pareto improve upon the set of Nash equilibria.

Future theoretical and empirical work on the topic of correlated equilibria might relax the assumption that recommendations arise from a non–strategic third party according to deterministic (and commonly known) probabilities. In place of this construct, a self–interested "monitor" player might repeatedly choose recommendations to make to the players of the stage game. In such an environment, the monitor's payoff could be based on the payoffs earned by the stage–game players: for example, it might be proportional to their average payoff. In this setting, the researcher could explore whether the monitor's frequencies of recommendations to players were consistent with any correlated equilibrium, and if so, which one: good, bad, Nash, or some other one.

A second useful extension would be to consider some "language" issues. For instance, one might wonder whether the form of recommendations matters: for example, whether subjects are told, "It is recommended that you play C", as in our design, or they simply see the message "C" on their screens. The salience and literal meanings of recommendations are also of interest: must the message space for recommendations correspond precisely to the action space, or might it be larger (for example, including also "no message"), or consist of a set of messages with no clear mapping to the action space (such as the message space {@, & })?[16]

We leave these extensions to future research.

[16]To this last question, we note that the recommendations in this setup are a special case of games with cheap talk. See Crawford (1998) for a survey of games with cheap talk, including a discussion of the effects of literal meanings in messages.

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