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CREAM SKIMMING, DREGS SKIMMING, AND POOLING: On the Dynamics of Competitive Screening.

Diderik Lund **Tore Nilssen**

Department of Economics - Copenhagen Business School
Solbjerg Plads 3, DK-2000 Frederiksberg

Cream Skimming, Dregs Skimming, and Pooling: On the Dynamics of Competitive Screening*

Diderik Lund
Department of Economics
University of Oslo
diderik.lund@econ.uio.no

Tore Nilssen†
Department of Economics
University of Oslo
tore.nilssen@econ.uio.no

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Abstract

We discuss the existence of a pooling equilibrium in a two-period model of an insurance market with asymmetric information. We solve the model numerically. We pay particular attention to the reasons for non-existence in cases where no pooling equilibrium exists. In addition to the phenomenon of cream skimming emphasized in earlier literature, we here point to the the importance of the opposite: dregs skimming, whereby high-risk consumers are profitably detracted from the candidate pooling contract.

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†Correspondence: Tore Nilssen, Department of Economics, University of Oslo, P.O.Box 1095 Blindern, N-0317 Oslo, Norway. Phone: 47 22 85 49 36. Fax: 47 22 85 50 35.

1 Introduction

It is by now well recognized that cream skimming is a serious impediment to workable competition. Cream skimming occurs when one or more firms take advantage of other firms' offers in the market in order to attract the most profitable customers, the "cream". The threat of cream skimming invariably makes cross-subsidization impossible. In markets with asymmetric information, such as credit and insurance markets, the impossibility of cross-subsidization may result in non-existence of any equilibrium in pure strategies (Rothschild and Stiglitz, 1976).

While most models of such markets are static ones, we will in this paper discuss a dynamic model of a market with asymmetric information where insurers are unable to commit to long-term contracts. In particular, we analyze a two-period version of the Rothschild-Stiglitz (1976) model.[1] Like in the original model, insurers offer state-contingent contracts to consumers who initially have private information on their accident probabilities. Although consumers require insurance in both of two periods, neither insurers nor consumers are able to enter long-term contracts covering both periods. Furthermore, any accident that occurs in the first period is observed only by the consumer having the accident and his insurer. Thus, at the start of the second period, there is asymmetric information among the insurers about consumers' accident histories.

In such a two-period setting, cream skimming is much less prevalent than in the single-period one. In fact, as shown by Nilssen (2000), in contrast to the one-period case, pooling *may* occur in equilibrium in this two-period model. In the present paper, we continue this line of research and discuss the *prevalence* of the pooling outcome. In addition, and interestingly, we draw attention to the reasons for non-existence in cases where no pooling equilibrium exists. While the cross-subsidization in a pooling equilibrium may break down because of the profitability of cream skimming, we find that, in many cases in our two-period model, it breaks down because it rather becomes profitable to attract the *least* profitable customers. As a counterpart to the concept of cream skimming, we dub this phenomenon *dregs skimming*.

A number of authors, starting with Freixas, et al. (1985), have shown how, in the single-principal, or monopoly, case, the introduction of multiple periods creates a scope for pooling. This happens also in a competitive market, but for different reasons. In particular, it is the weakened profitability of skimming, whether it is the cream or the dregs, that makes pooling a viable proposition. In contrast, skimming is not an issue in the single-principal case.

The dynamics of competitive screening is not a well researched topic. One reason for this may be the complexity of the problem. Below, we resort to numerical analysis in order to solve the model. Although this does not give a complete picture of the model, our view is that it is helpful in indicating the prevalence of pooling on one hand and of profitable cream and dregs skimming on the other. While the literature on the dynamics of competitive screening

[1] This two-period version was first studied by Nilssen (2000).

is thin, our analysis should be compared with that of Parigi (1994), who highlights the reduced profitability of cream skimming following the introduction of multiple periods in a competitive market with asymmetric information. However, Parigi fails to take into consideration the possibility of profitable dregs skimming, as we do here.

In Section 2, we present the two-period insurance-market model. In Section 3, we discuss the occurrence of a pooling equilibrium and how, in order to be viable, a pooling contract will have to be robust with respect to both cream-skimming and dregs-skimming offers. The analysis is carried out numerically, and while the details of our procedure are given in an appendix, the results of our numerical analysis are discussed in Section 4. Section 5 offers a few concluding remarks.

2 A two-period insurance market

Here, we present our two-period version of the Rothschild-Stiglitz (1976) model of an insurance market with asymmetric information.

On the demand side of the market, there is a continuum of individuals. Each individual faces, in each of two periods, two possible states of nature: In the good state 1, no accident occurs and his endowment is w_1^0. In the bad state 2, an accident *does* occur and his endowment is w_2^0, with $\infty > w_1^0 > w_2^0 > 0$. All individuals are identical, except for the probability of an accident occuring in a period. The high-risk (H) type has accident probability p^H, while the low-risk (L) type has probability p^L, with $0 < p^L < p^H < 1$. The fraction of high-risks in the population is φ^0, which also is the *ex-ante* probability that an individual is high-risk.

On the supply side, insurance is provided by the firms in the set $J := \{1, \ldots, n\}$. *Buying insurance* from one of these firms means trading the state-contingent endowment $w^0 = (w_1^0, w_2^0)$ for another endowment $w = (w_1, w_2) \gg 0$.[2] The set of feasible contracts is: $W := \{(w_1, w_2) : w_1 \geqslant w_2 > 0\}$. Firms can only offer short-term, or single-period, contracts.[3] No other restrictions on contracts are made. However, each consumer is restricted to buying insurance from only one firm in each period.

Consumers are risk averse. A consumer of type $\theta \in \{H, L\}$ evaluates a

[2] We use the following notation for vector inequalities: $s \gg t$ if and only if $s_i > t_i, \forall\ i$; $s \geqq t$ if and only if $s_i \geqslant t_i$.

[3] Restricting insurer to offer single-period contracts is meant to capture the notion that insurers have limited abilities to commit to future contract specifications. An alternative between no commitment, as we assume here, and full commitment would be a situation where insurers are able to commit to long-term contracts but at the same time are unable to commit not to renegotiate such contracts when new information about the insurees become available. The analysis differs depending on whether the renegotiation can take place only after one period has passed or immediately after a contract has been signed. See Dionne and Doherty (1994) for an analysis of the former case and Asheim and Nilssen (1996) for an analysis of the latter case.

contract $w \in W$ according to the expected utility

$$u^{\theta}(w) := \left(1 - p^{\theta}\right) v(w_1) + p^{\theta} v(w_2), \tag{1}$$

where v is, in general, a strictly increasing, twice continuously differentiable, and strictly concave von Neumann-Morgenstern (vN-M) utility function. When we turn to the numerical analysis, we will restrict ourselves to utility functions exhibiting constant relative risk aversion (CRRA);[4] *i.e.*, we will make use of the following class of specific vN-M utility functions v:

$$v(w) = \begin{cases} \frac{1}{1-k} w^{1-k}, \text{ if } k \neq 1, \\ \ln w, \text{ if } k = 1, \end{cases} \tag{2}$$

where $k > 0$ is the measure of (constant) relative risk aversion.

Suppliers, on the other hand, are risk neutral. The expected profit from selling the contract $w \in W$ to an individual who is believed to be high-risk with probability φ is

$$\pi(w, \varphi) := R(\varphi) - C(w, \varphi), \tag{3}$$

where

$$R(\varphi) := \left[\varphi\left(1 - p^H\right) + (1 - \varphi)\left(1 - p^L\right)\right] w_1^0 + \left[\varphi p^H + (1 - \varphi) p^L\right] w_2^0 \tag{4}$$

is the expected (gross) revenue from taking over the no-insurance endowment w^0, and

$$C(w, \varphi) := \left[\varphi\left(1 - p^H\right) + (1 - \varphi)\left(1 - p^L\right)\right] w_1 + \left[\varphi p^H + (1 - \varphi) p^L\right] w_2 \tag{5}$$

is the expected cost of providing the endowment w.

Both consumers and firms discount the future with a discount factor $\delta > 0$.

The insurance market is open for two periods. The game in this two-period model is as follows:

In *Stage 1*, each firm $j \in J$ offers a menu $M_j^1 \in M := W \times W$ of contracts for the first period, one for each consumer type. If a firm's stage-1 offer is a pooling contract, then its menu is degenerate, containing two identical contracts. All the menus offered in this stage are immediately observed by all firms and consumers.

In *Stage 2*, each consumer chooses one of the contracts offered in Stage 1. The consumers' choices are immediately observed by all firms.

In *Stage 3*, each consumer and the consumer's insurer - but no-one else - observe whether or not an accident occurs for this consumer in the first period; and first-period contracts are fulfilled.

In *Stage 4*, each firm offers a second-period menu $M_j^{2U} \in M$ to consumers on whom it has no accident information, *i.e.*, consumers who were with another firm in the first period. The offered menus are observed immediately by all firms and consumers.

[4] According to Szpiro (1986a, 1986b), a hypothesis of constant relative risk aversion fits well with consumers' purchases of property/liability insurance in a number of countries.

In *Stage 5*, each firm $j \in J$ offers second-period menus to consumers on whom it does have accident information from the first period, *i.e.*, the firm's old customers. It offers the menu $M_j^{2A} \in M$ to old customers with a first-period accident and the menu $M_j^{2N} \in M$ to old costumers without one. The offered menus are immediately observed by all consumers.

In *Stage 6*, each consumer chooses one of the contracts offered to him in Stages 4 and 5.

In *Stage 7*, accidents are observed and second-period contracts fulfilled.

There are two features of this set-up that deserve comments. First, we assume that a consumer's accident record is private information to his present insurer. This creates scope for such accident records to have a value for insurers, so that they may be willing to compete hard in the first period in order to have sole access to them later on.[5]

Secondly, firms offer second-period contracts in a sequential manner. A consumer first receives offers from other insurers (in Stage 4) before he receives an offer also from his previous insurer. In the simultaneous-move alternative, there is a possibility for non-existence of a pure-strategy equilibrium in the second-period game. Introducing sequential moves creates, here as in other games with such existence problems, a possibility for coordination that ensures the existence of a pure-strategy equilibrium. Among the two available sequential-move structures, we choose the most reasonable, with a consumer's current insurer being able to respond to the offer being made to this consumer in the general market.

We restrict attention to *symmetric* equilibria and can therefore save on firm-specific subscripts. A symmetric equilibrium is a vector $\left(M^1, M^{2U}, M^{2A}, M^{2N}\right) = \left(\left(w^{1H}, w^{1L}\right), \left(w^{UH}, w^{UL}\right), \left(w^{AH}, w^{AL}\right), \left(w^{NH}, w^{NL}\right)\right)$. An equilibrium is *separating* if the first-period menu is separating, *i.e.*, if $w^{1H} \neq w^{1L}$ and consumers choose among these contracts according to type. An equilibrium is *pooling* if the first-period menu is pooling, *i.e.*, if $w^{1H} = w^{1L}$.

In analyzing this model, we will concentrate on the question whether a pooling equilibrium exists and, if not, what the reason is.[6]

[5] The issue of accident-record value, and the resulting scope for informational consumer lock-in, is the main focus of Nilssen (2000). Although our concerns are different, we keep the assumption, because we think it is a realistic description of insurance markets; see, *e.g.*, the empirical support provided by Cohen (2002). It should be noted that making accident records public would neither simplify nor complicate the analysis.

[6] The existence of a pure-strategy separating equilibrium in this model is discussed in Nilssen (2000, Sec. 3). In addition to pooling and separating equilibria, there may also exist hybrid, or semi-pooling, equilibria. An example of a hybrid equilibrium would be one where, say, low-risks buy the low-risk contract, but where some high-risk consumers buy the high-risk contract and the rest of them join the low-risks at the low-risk contract. While our attention here is restricted to equilibria with full pooling, it is likely that the same questions as we raise presently come into consideration when it comes to the possible existence of a semi-pooling equilibrium.

3 Pooling, Cream Skimming, and Dregs Skimming

There is a fundamental tension in an insurance market with asymmetric information: High-risk consumers are the ones most eager to buy insurance, and therefore firms, in designing their insurance contracts, must pay attention to these consumers' incentive-compatibility constraints. At the same time, low-risk consumers are the ones most profitable to the firms and the ones they are fighting over. Thus, fighting for the low-risks while adhering to the incentives of the high-risks describes well the lives of the insurers in such a market.

Figure 1 illustrates the viability of a pooling first-period contract in the present two-period version of the Rothschild and Stiglitz (1976) model. The Figure depicts the contract space, W, with full-insurance endowments along the 45° line. The two straight lines emanating from the no-insurance point w^0 depict contracts that are actuarially fair, *i.e.*, zero-profit, when traded with high-risks, respectively low-risks. Let w^P be the candidate pooling contract, represented by $\times$ in Figure 1; its precise definition is provided below.

< FIGURE 1 >

The contract w^P is vulnerable to *cream skimming* if, in the area vertically hatched in Figure 1, there exist contracts that are profitable when sold to low-risks, the "cream" of the consumer population. A contract in this area, which is denoted $S^C\left(w^P\right)$ and defined precisely below, has two properties. On one hand, a low-risk consumer would rather buy it, reveal his type, and get full insurance under full information in the second period, than be pooled together with the high-risks at w^P. *I.e.*, the contract must be above the low-risk utility level u_C^L in Figure 1; this utility level is strictly below w^P because of the low-risks' benefit of full information, compared to continuing asymmetric information, in period 2. On the other hand, a high-risk consumer would rather reveal his type at w^P, when the low-risks are skimmed away, than buy this contract in $S^C\left(w^P\right)$ and be considered mistakenly by insurers as a low-risk. *I.e.*, the contract must be below the high-risk utility level u_C^H; this utility level is also strictly below w^P, because the consumer would gain from being considered low-risk rather than high-risk in period 2.

In general, the set $S^C(w)$ of cream-skimming contracts is detached from the contract w that these contracts cream-skim because of consumers' rational expectation about the gain of being considered low-risk rather than (perhaps) high-risk in the future. In a one-period model, such as the original Rothschild-Stiglitz (1976) one, there is no future to consider, and any candidate pooling contract is therefore connected to its corresponding set of contracts cream-skimming it. One condition for a pooling contract to be viable in equilibrium is that it yields a non-negative profit. This must imply a cross-subsidization from low-risks to high-risks: Insurers offer the pooling contract only because they earn at least as much on the low-risks buying the contract as they lose on the high-risks buying it. But if the pooling contract is profitable when sold to low-risks, then, in the single-period case, so must also some contracts that cream-skim it be profitable when sold to low-risks, since, by the connectedness, there exist contracts arbitrarily close to the pooling contract that cream-skim it. Thus, in the single-period case, a pooling contract cannot be both profitable and cream-skimming proof.

While, in the present two-period framework, a pooling contract is not necessarily deemed non-viable because of cream skimming, we need to consider the possibility that also high-risks can be profitably detracted from a candidate pooling contract; it is this phenomenon that we dub *dregs skimming*. The contract w^P is vulnerable to dregs skimming if, in the area horizontally hatched in Figure 1, there exist contracts that are profitable when sold to high-risks, the "dregs" of the consumer population. A contract in this area, which is denoted $S^D\left(w^P\right)$ and also defined precisely below, has two properties. On one hand, a high-risk consumer would rather buy it and reveal his type than be pooled together with the low-risks at w^P. *I.e.*, the contract must be above high-risk utility level u_D^H in Figure 1; this utility level is strictly above w^P because of the high-risks' loss from full information, compared to continuing asymmetric information, in period 2. On the other hand, a low-risk consumer would rather reveal his type at w^P, when the high-risks are skimmed away, than buy this contract in $S^D\left(w^P\right)$ and be considered mistakenly by insurers as a high-risk. *I.e.*, the contract must be below the low-risk utility level u_D^L; this utility level is also strictly above w^P, because the consumer would gain from being considered low-risk rather than high-risk in period 2.

Following a separation of consumers by type in period 1, either in a separating equilibrium or after an out-of-equilibrium cream skimming or dregs skimming, there will be full information about consumer types in period 2 among all firms. In the case of full information, all consumers are fully insured and firms earn zero profit [Rothschild and Stiglitz (1976, Sec I.5)]. Define W_F as the set of full-insurance contracts, *i.e.*, $W_F := \{w \in W : w_1 = w_2\}$. The two contracts offered to high-risks and low-risks, respectively, in case of full information, are denoted w_{FI}^H and w_{FI}^L and defined as follows:

$$R(1) = C\left(w_{FI}^H, 1\right) \tag{6a}$$

$$R(0) = C\left(w_{FI}^L, 0\right) \tag{6b}$$

$$w_{FI}^H, w_{FI}^L \in W_F \tag{6c}$$

Following a pooling contract in period 1, there exists a period-2 equilibrium in which firms, in Stage 4, offer the Rothschild-Stiglitz (R-S) contracts, *i.e.*, the same zero-profit pair of separating, incentive-compatible contracts that constitute the equilibrium contract menu in the single-period model when such an equilibrium exists (in pure strategies) [Nilssen (2000, Prop. 3)]. We denote this pair of contracts $\left(w_{RS}^H, w_{RS}^L\right)$. While the high-risk R-S contract coincides with its full-insurance equivalent, *i.e.*, $w_{RS}^H = w_{FI}^H$, the low-risk R-S contract is defined by:

$$R(0) = C\left(w_{RS}^L, 0\right) \tag{7a}$$

$$u^H\left(w_{RS}^L\right) = u^H\left(w_{FI}^H\right) \tag{7b}$$

I.e., the low-risk R-S contract is that zero-profit low-risk contract which exactly balances the high-risk consumers' incentives to buy it instead of the full-insurance contract assigned to them.

Following a first-period pooling contract, firms' beliefs about consumers at the start of period 2 can be described by the vector $\left(\varphi^U, \varphi^A, \varphi^N\right)$, describing their subjective probabilities that a consumer is high-risk: $\varphi = \varphi^U$ when a firm is uninformed about a consumer's accident record; $\varphi = \varphi^A$ when the firm knows the consumer had an accident in period 1; and $\varphi = \varphi^N$ when the firm knows the consumer did not have an accident. An uninformed firm does not update its prior belief, so $\varphi^U = \varphi^0$. An informed firm updates its belief according to Bayes' Rule, taking into account the accident record:

$$\varphi^A = \frac{\varphi^0 p^H}{\varphi^0 p^H + (1-\varphi^0)\, p^L} \tag{8a}$$

$$\varphi^N = \frac{\varphi^0 \left(1-p^H\right)}{\varphi^0 (1-p^H) + (1-\varphi^0)(1-p^L)} \tag{8b}$$

In equilibrium, consumers do not switch to another insurer in the second period. Thus, according to whether they have a first-period accident or not, after a first-period pooling contract, consumers will purchase period-2 contracts from one of the lists $\left(w^{AH}, w^{AL}\right)$ and $\left(w^{NH}, w^{NL}\right)$ of contracts offered by insurers to old customers. These contracts are found by solving a maximization problem similar to the one facing an insurance monopolist [Stiglitz (1977), Kreps (1990, Sec. 18.1)], except that the incumbent insurer's constraints are not consumers' option to self-insure but old customers' option to go to other insurers. For each of the two groups of old customers with a first-period accident ($\alpha = A$) and those without one ($\alpha = N$), insurers find their second-period contract menu as the solution to the maximization problem

$$\left(w^{\alpha H}, w^{\alpha L}\right) = \arg \max_{(w^H, w^L)\in M} \left[\varphi^\alpha \pi\left(w^H, 1\right) + (1-\varphi^\alpha)\, \pi\left(w^L, 0\right)\right],\ \alpha \in \{A, N\}, \tag{9a}$$

subject to:

$$w^H \in W_F, \tag{9b}$$

$$u^H\left(w^H\right) = u^H\left(w^L\right) \tag{9c}$$

$$u^L\left(w^L\right) = u^L\left(w^L_{RS}\right) \tag{9d}$$

$$u^H\left(w^H\right) \geq u^H\left(w^H_{FI}\right) \tag{9e}$$

Here, the first restriction is not really a restriction but rather just a property of the optimum menu: high-risks receive full insurance because low-risk incentive-compatibility is not a binding constraint. Furthermore, the second restriction is the high-risk incentive-compatibility constraint; the third restriction is the participation constraint for the low-risks; and the fourth restriction, which may or may not be binding, is the participation constraint for high-risks.

Since $\varphi^A > \varphi^N$, an insurer is more interested in cross-subsidization among old customers without a first-period accident than among those with one. Thus, while

$$u^L\left(w^{NL}\right) = u^L\left(w^{AL}\right) = u^L\left(w^L_{RS}\right),$$

we have

$$u^H\left(w^{NH}\right) \geq u^H\left(w^{AH}\right) \geq u^H\left(w^H_{FI}\right),$$

where the first inequality is strict if the second one is, and where these inequalities are strict for a sufficiently low fraction φ^0 of high-risks in the population [Nilssen (2000, Props. 4 and 5)].

The pooling contract that is going to be the candidate equilibrium contract in a pooling equilibrium is the one that survives in competition with other pooling contracts. This is that pooling contract which maximizes low-risk first-period expected utility subject to a non-negativity constraint on firms' overall profit when consumers divide themselves evenly among firms so that each firm gets a representative set. Therefore, the candidate equilibrium pooling contract is defined as:

$$w^P := \quad \arg\max_{w \in W} \; u^L(w), \text{ subject to:} \tag{10a}$$

$$\pi\left(w, \varphi^0\right) + \delta\{\varphi^0\left[p^H \pi\left(w^{AH}, 1\right) + \left(1 - p^H\right)\pi\left(w^{NH}, 1\right)\right] + \left(1 - \varphi^0\right)\left[p^L \pi\left(w^{AL}, 0\right) + \left(1 - p^L\right)\pi\left(w^{NL}, 0\right)\right]\} \geq 0 \tag{10b}$$

Given any contract w, the set of contracts that cream-skim it is defined as:

$$S^C(w) := \{w' \in W \setminus \{w\}:$$

$$u^H(w') + \delta u^H\left(w^L_{FI}\right) \leq u^H(w) + \delta u^H\left(w^H_{FI}\right), \text{ and} \tag{11a}$$

$$u^L(w') + \delta u^L\left(w^L_{FI}\right) \geq u^L(w) + \delta u^L\left(w^L_{RS}\right)\} \tag{11b}$$

Condition (11a) is an incentive-compatibility constraint for high-risk consumers: With this condition satisfied, a high-risk consumer would not choose a cream-skimming contract in $S^C(w)$ even if, by so doing, he would mistakenly be considered a low-risk in period 2. Condition (11b) is a participation constraint for low-risk consumers: With this condition satisfied, a low-risk consumer would prefer revealing his type, by choosing a cream-skimming contract in $S^C(w)$, to staying at the contract w and be pooled together with the high-risks.

Given any contract w, the set of contracts that dregs-skim it is defined as:

$$S^D(w) := \{w' \in W \setminus \{w\}:$$

$$u^L(w') + \delta u^L\left(w^H_{FI}\right) \leq u^L(w) + \delta u^L\left(w^L_{FI}\right), \text{ and} \tag{12a}$$

$$u^H(w') + \delta u^H\left(w^H_{FI}\right) \geq u^H(w) + \delta\left[p^H u^H\left(w^{AH}\right) + \left(1 - p^H\right)u^H\left(w^{NH}\right)\right]\} \tag{12b}$$

Corresponding to the previous definition, condition (12a) is an incentive-compatibility condition for low-risk consumers: When this condition is satisfied, a low-risk consumer would not choose a dregs-skimming contract in $S^D(w)$ if, by so doing, he would mistakenly be considered a high-risk in period 2. Condition (12b) is, likewise, a participation constraint for high-risk consumers: When this condition is satisfied, a high-risk consumer would prefer revealing he is high-risk, by choosing a dregs-skimming contract in $S^D(w)$, to staying at the contract w, even if this means being pooled together with the low-risks. Note how condition (12b) takes into account the uncertainty regarding which period-2 offer high-risks will obtain from their period-1 insurers: This offer, in contrast to what the low-risks are offered, may vary, in terms of high-risk expected utility, according to whether a consumer has a first-period accident or not.

Both cream skimming and dregs skimming are *single-contract deviations*: the deviating firm offers a single contract that detracts the low-risks and the high-risks, respectively. In order for the analysis to be complete, however, we also need to consider the possibility of a *menu deviation*, *i.e.*, a deviation detracting both low-risks and high-risks by way of a menu consisting of one contract for the low-risks and one contract for the high-risks.[7] In a sense, such a menu deviation is a combination of cream skimming and dregs skimming, since both types are detracted. But it is more fruitful to consider it as a variation of cream skimming: In order to make cream skimming profitable, it may be necessary to invite the high-risks to buy a contract more attractive than the candidate pooling contract that is on the table and in so doing relax the high-risk incentive-compatibility constraint to such an extent that what is spent on attracting the high-risks this way is more than regained on a more profitable low-risk contract. Since this menu deviation entails a cross-subsidization between the two types, we dub it *cross skimming*. Given any contract w, the set of menus that cross-skim it is defined as:

$$M^X(w) := \left\{ \left(w^{H\prime}, w^{L\prime}\right) \in M \setminus \{w, w\} : \right.$$

$$u^H\left(w^{L\prime}\right) + \delta u^H\left(w^L_{FI}\right) \le u^H\left(w^{H\prime}\right) + \delta u^H\left(w^H_{FI}\right), \tag{13a}$$

$$u^L\left(w^{L\prime}\right) + \delta u^L\left(w^L_{FI}\right) \ge u^L(w) + \delta u^L\left(w^L_{RS}\right), \text{ and} \tag{13b}$$

$$\left. u^H\left(w^{H\prime}\right) \ge u^H(w) \right\} \tag{13c}$$

Condition (13a) parallels condition (11a) and is an incentive compatibility

[7] Taking into consideration such a menu deviation is an improvement relative to the analysis in Nilssen (2000), where only single-contract deviations are considered. It should be noted, though, that the pooling equilibrium claimed to exist in Nilssen's numerical example does survive also any menu deviations.

The menu deviaton we consider here consists of a pair of fully separating contracts. Also the single-contract deviations, cream and dregs skimming, are based on full separation. A question arises, then, whether the profitability of a semi-pooling deviation needs attention. A semi-pooling deviation would involve a contract that attracts some, but not all, consumers of a particular type. Consider, then, a candidate semi-pooling deviation from the pooling contract. The consumers of the type in question would have to be indifferent between the terms of the contract and the terms of the pooling contract, account taken of the consequences for the second period of accepting each. But then, by continuity, the insurer offering the deviating contract could do even better by making the contract slightly better for the consumers and attracting all consumers of that type. Thus, there is no need to consider semi-pooling deviations. We are grateful to an anonymous referee for raising this question.

constraint on the high-risks: When this condition is satisfied, a high-risk consumer is happy to choose the high-risk contract $w^{H\prime}$ rather than the low-risk contract $w^{L\prime}$, even if the latter choice would have insurers mistakenly believe him to be a low-risk in period 2. Condition (13b) similarly parallels condition (11b) and is a participation constraint on the low-risks: When this condition is satisfied, revealing his type by choosing the contract $w^{L\prime}$ is better for a low-risk consumer than staying at the pooling contract and be pooled together with the high-risks. Condition (13c) is a participation constraint on the high-risks that is necessary in order to ensure that the high-risks are detracted from the pooling contract to this cross-skimming deviation.

In determining whether or not cream skimming or dregs skimming is profitable, it suffices to assess the profitability of the most profitable contract in each set. We define:

$$w^C(w) := \arg\sup \pi(w', 0), \text{ subject to: } w' \in S^C(w), \tag{14}$$

as the most profitable cream-skimming contract, when sold to low-risks. This contract is the unique contract for which both the constraints defining $S^C(w)$ are satisfied,[8] *i.e.*, the contract is characterized by:

$$u^H\left(w^C(w)\right) + \delta u^H\left(w^L_{FI}\right) = u^H(w) + \delta u^H\left(w^H_{FI}\right), \text{ and} \tag{15a}$$

$$u^L\left(w^C(w)\right) + \delta u^L\left(w^L_{FI}\right) = u^L(w) + u^L\left(w^L_{RS}\right). \tag{15b}$$

Furthermore, we define:

$$w^D(w) := \arg\sup \pi(w', 1), \text{ subject to: } w' \in S^D(w), \tag{16}$$

as the most profitable dregs-skimming contract when sold to high-risks. The low-risk incentive-compatibility constraint delineating the set of dregs-skimming contracts poses clearly no restriction on the contracts a dregs-skimming insurer would want to offer. The most profitable dregs-skimming contract $w^D(w)$ is therefore defined as the full-insurance contract that exactly satisfies the high-risk participation constraint for dregs skimming, *i.e.*, it is given by the following two conditions:

$$w^D(w) \in W_F, \text{ and} \tag{17a}$$

$$u^H\left(w^D(w)\right) + \delta u^H\left(w^H_{FI}\right) = u^H(w) + \delta\left[p^H u^H\left(w^{AH}\right) + \left(1 - p^H\right) u^H\left(w^{NH}\right)\right] \tag{17b}$$

In contrast to the two single-contract deviations, there does not exist any simple characterization of the optimum cross-skimming deviation, which we denote $\left\{w^{XH}(w), w^{XL}(w)\right\}$, apart from $w^{XH}(w) \in W_F$, parallelling (9b) above.

[8] Of the two constraints defining $S^C(w)$, the low-risk participation constraint (11b) is clearly binding. And among contracts satisfying this constraint with equality, a risk-neutral insurer profits from offering one as close to full insurance as possible, since insurees are risk averse. Thus, also the high-risk incentive-compatibility constraint (11a) will be binding.

However, in determining whether or not cross skimming is profitable, it is sufficient to assess whether there exists a menu $\left(w^H, w^L\right) \in M^X(w)$, such that[9] $w^H \in W_F$, and

$$\left[\varphi^0 \pi\left(w^H, 1\right) + \left(1-\varphi^0\right) \pi\left(w^L, 0\right)\right] > 0. \tag{18}$$

A pooling equilibrium exists in this model if, for the candidate equilibrium pooling contract w^P, neither cream skimming, dregs skimming, nor cross skimming is profitable, *i.e.*, if both $\pi\left(w^C\left(w^P\right), 0\right) \leq 0$, $\pi\left(w^D\left(w^P\right), 1\right) \leq 0$, and $\left[\varphi^0 \pi\left(w^{XH}\left(w^P\right), 1\right) + \left(1-\varphi^0\right) \pi\left(w^{XL}\left(w^P\right), 0\right)\right] \leq 0$.

In the analysis below, we distinguish between the following cases:

P - a pooling equilibrium exists;

C - a cream-skimming deviation is profitable;

D - a dregs-skimming deviation is profitable;

X - a cross-skimming deviation is profitable;

B - a dregs-skimming deviation is profitable, but so is also at least one of the other two kinds of deviations.

4 Analysis

Under the assumption of constant relative risk aversion, this model has a total of seven parameters, or exogenous variables: w_1^0 - consumers' endowment without an accident; w_2^0 - consumers' endowment with an accident; φ^0 - the fraction of high-risks in the population; p^H - the accident probability of a high-risk consumer; p^L - the accident probability of a low-risk consumer; δ - the discount factor; and k - the measure of consumers' relative risk aversion. For any allowed combination of these seven variables, we are able to determine whether an equilibrium (in pure strategies) exists, and if so, the type of equilibrium. In particular, we determine the relevant one of the cases P, C, D, X, and B. Details about the calculations are in the Appendix.

We have no theorems giving conditions for the existence of a pooling equilibrium, or for its non-existence due to profitable dregs skimming and/or cream skimming. We have, however, run the computer through a large number of parameter combinations and have found that pooling is a prevalent phenomenon, and that, when pooling is not viable in equilibrium, profitable dregs skimming is a major reason for this. Instead of a report of all computations we have done, we organize it around a reasonable base case and sensitivity analyses of it.

We believe a reasonable, albeit stylized, picture of an insurance market is one where the probability of a considerable accident is moderate for a huge majority of the consumers, while a small minority of the consumers contaminate the market by having a much higher accident probability. Therefore, our base case is one where the fraction of high-risks φ^0 as well as the low-risk accident probability

[9] In the numerical analysis, we solve a problem that is even (slightly) easier: We look for a cross-skimming menu that gives zero profits, given that other insurers offer the candidate pooling contract. If such a menu exists and is not at the endpoints of the constraints, then, by continuity and differentiability of the profit function, there will also exist a menu giving positive profits and satisfying the constraints.

p^L are rather small; where the high-risk accident probability p^H is much larger than p^L; and where the accident damage $\left(w_1^0 - w_2^0\right)$ is considerable relative to the initial endowment w_1^0. In particular, our base case has: $w^0 = (10, 4)$, $\varphi^0 = 0.10$, $p^L = 0.08$, and $p^H = 0.30$. Furthermore, we use $k = 0.9$ and set the discount factor δ equal to 0.9.

There exists a pooling equilibrium in this base case. This equilibrium is the one illustrated in Figure 1 above. In Figures 2-5 below, we report graphically the results of our sensitivity analyses. In each graph, we vary two of the parameters to see how the candidate pooling contract fares against cream and dregs skimming, while the other five parameters are kept at their base-case values. In each Figure, the base case is encircled.

In Figure 2, we vary the fraction of high-risks, φ^0, together with w_2^0, consumers' wealth in case of an accident. In particular, φ^0 varies from 0.02 to 0.34, while w_2^0 varies between 1 and 9. The picture we get is quite typical: Pooling is wide-spread. And when it is not viable, profitable dregs skimming is a major reason for it. Although we insist that a low φ^0 is more reasonable than a high one, the picture indicates that such a low fraction of high-risk consumers is important for the occurrence of pooling in equilibrium. Note that the lower w_2^0 is, the larger is the damage that an accident causes. Interestingly, cream skimming is only viable in cases where the damage is large, while the opposite is true for dregs skimming. Thus, there is scope for a pooling equilibrium in cases of a damage of medium size, even in cases where φ^0 is not very low.

In Figure 3, we let the low-risk accident probability p^L vary between 0.02 and 0.18 and the high-risk probability p^H between 0.05 and 0.45, but in such a way that $p^H > p^L$. We see that pooling again is prevalent and particularly so when both probabilities are high. Dregs skimming is viable when the difference between the two probabilites is particulary high.

In Figure 4, we vary the fraction of high-risks, φ^0, between 0.05 and 0.45 and the risk-aversion parameter k between 0.3 and 2.7. We see that, for low and moderate degrees of risk aversion, it is dregs skimming that eventually destroys the viability of the pooling equilibrium as φ^0 increases. For higher values of k, on the other hand, it is either cream skimming or cross skimming that makes pooling non-viable.

In Figure 5, we picture variations in the discount factor together with variations in the low-risk accident probability. We let δ vary from 0.4 to 1.2; values of δ above 1 may be interpreted as the second period having a longer duration than the first period, for example as a representation of "the future". In this Figure, p^L varies between 0.02 and 0.26. We see that a low discount factor leads to cream skimming of the candidate pooling contract and that dregs skimming has but a minor role to play here. But we also see that pooling may occur for quite low discount factors. In particular, we get pooling for discount factors as low as 0.8. This is in contrast to similar studies done earlier for the monopoly case, *i.e.*, where one principal offers single-period contracts to agents in two periods. For example, Dionne and Fluet (2000), in their analysis of the model of Laffont and Tirole (1993), do not report full pooling for any discount factor below 1.0.

< FIGURES 2-5>

Our results are not conclusive in a strict sense, since we only report a few computer runs, although they are carefully chosen. One should, therefore, be careful in interpreting them. The picture we get, however, besides the prevalence of pooling and dregs skimming, is that pooling occurs when the discount factor is high; when the fraction of high-risks is low; when accident probabilities are high; when the accident damage is considerable; and when the degree of risk aversion is moderate.

The effect of the discount factor is straightforward: A low discount factor means consumers do not care much for the next period, implying, in terms of Figure 1, that the two sets of cream-skimming and dregs-skimming contracts are closer to the skimmed contract w^P than when the discount factor is high. While this does not necessarily affect very much the profitability of dregs skimming, it has a positive effect on the profitability of cream skimming. For a sufficiently low discount factor, therefore, cream skimming is profitable and pooling becomes non-viable.

When the fraction of high-risks is low, and provided there was a pooling contract on the market in period 1, insurers find it profitable to offer cross-subsidizing contract menus to their old customers in period 2, particularly those consumers without a first-period accident. Thus, high-risk consumers may have something to gain, through this cross-subsidization, by sticking to the pooling contract in period 1. This implies that, as the fraction of high-risks decreases, the high-risk participation constraint for dregs-skimming contracts gets stricter and the profitability of dregs skimming deteriorates. Thus, pooling is viable for a low fraction of high-risks, whereas an increase in this fraction implies that dregs skimming becomes profitable and, thus, pooling non-viable.

The effect of an increase in the degree of risk aversion is to make indifference curves more curved. Thus, in cases of a low fraction of high-risks, which we focus on here, a decrease in consumers' risk aversion has the effect that the candidate pooling contract w^P moves downwards in Figure 1, *i.e.*, an increase in k decreases w_2^P with little effect on w_1^P. As w^P moves downwards, so does the sets of dregs- and cream-skimming contracts that correspond to it. While this has little effect on the profitability of cream skimming, it enhances that of dregs skimming. Thus, when consumers' risk aversion is small, dregs skimming becomes profitable, as Figure 4 illustrates.

A similar mechanism is at work as one varies the size of the accident damage. Varying w_2^0 from high (small damage) to low (large damage) has little effect on the candidate pooling contract and, therefore, little effect on the sets of dregs- and cream-skimming contracts. Thus, an increase in w_2^0 moves the high-risk zero-profit line in Figure 1 upwards so that, in the end, dregs skimming becomes profitable. Thus, dregs skimming tends to be profitable when the damage is low, as Figure 2 indicates.

5 Concluding remarks

This paper has shown, through numerical analysis of a two-period insurance market with asymmetric information, how the performance of such a market is dependent on the viability of a pooling equilibrium, and how this viability in turn depends not only on whether cream skimming is profitable but also, and often more importantly, on whether dregs skimming, the detraction of high-risk consumers from the candidate equilibrium pooling contract, is profitable.

Our results indicate not only that pooling may occur, as Nilssen (2000) showed, but that pooling is actually widespread. In particular, we have found that markets with a low fraction of high-risk consumers is conducive to pooling. This is interesting in light of the prediction of the single-period model of Rothschild and Stiglitz (1976) for this case: Whereas, in the single-period model, few high-risks mean non-existence of a pure-strategy equilibrium and, therefore, a prediction of an unstable market, we have found a theoretical basis for predicting not only a stable market, but one where there is no separation, in cases where most consumers are low-risks.

We also believe it interesting, and something that should be intriguing for future research, that the profitability of dregs skimming, rather than of cream skimming, for such a large sets of parameters is the reason for pooling not to survive in equilibrium. As indicated above, this occurs particularly when the fraction of high-risks is low. But this is a situation we believe is prevalent: a market being contaminated by a small fraction of low-value consumers. It seems wise, therefore, to continue exploring the dregs-skimming phenomenon that we have pointed to in the present work.

6 Appendix: Numerical analysis

In this Appendix, we provide details of the numerical analysis behind the results discussed in Section 4. The calculations are done in the following sequence:[10]

1. The pair $\left(w^H_{FI}, w^L_{FI}\right)$ of contracts offered under full information is found directly from (6).

2. We calculate the pair $\left(w^H_{RS}, w^L_{RS}\right)$ of contracts offered when there is a separating equlibrium in the single-period case, and also by uninformed insurers in period 2 in the present two-period model, in case a pooling contract is offered in period 1. We have $w^H_{RS} = w^H_{FI}$, while w^L_{RS} is found by solving (7) numerically.

3. We calculate the pairs $\left(w^{AH}, w^{AL}\right)$ and $\left(w^{NH}, w^{NL}\right)$ of contracts offered by informed insurers in period 2 to old customers with and without a first-period accident, respectively, in case a pooling contract is offered in period 1, defined in (9) above. To do this, we first need to distinguish

[10] See http://folk.uio.no/dilund/dregs/ for detailed information about the Gauss programs that we developed for this research.

between the three cases defined in Proposition 4 in Nilssen (2000). For a given vector of exogenous variables, the informed firm offers either the RS menu for any accident history, or the RS menu to those with accidents and a CS (cross-subsidizing) menu to the others, or a CS menu to both types. The distinguishing inequalities in that Proposition are calculated and the relevant case is determined. If this is the second or the third case, the CS menu is calculated by numerically solving a system of three equations for each of the menu's two elements. Each of the two equation systems has only three scalar unknowns, $w_i^{\alpha H}, w_1^{\alpha L}, w_2^{\alpha L}, \alpha \in \{A, N\}$, since $w_1^{\alpha H} = w_2^{\alpha H}$ by (9b). Each equation system consists of (9c), (9d), and the first-order condition for (9a), with the relevant φ^{α} taken from (8).

4. We calculate the pooling contract w^P that is offered by all insurers in period 1 if the equilibrium is pooling. This is defined in (10), which gives two equations in two scalar unknowns, w_1^P and w_2^P. The first equation is the first-order condition for (10a). Since we have a formula for the inverse of the v' function, that equation gives us w_2^P as a function of w_1^P. Next, we observe that (10b) must be satisfied with equality and solve that equation numerically for w_1^P.

5. We check whether cream skimming is profitable, thus destroying the pooling equilibrium. We must calculate the profit $\pi\left(w^C\left(w^P\right), 0\right)$ that can be earned from cream-skimming the pooling contract, with $w^C(w)$ defined in (15) above. First, we determine whether $S^C(w^P)$ is empty. This may occur if $k < 1$, in which case u (expected utility) values are positive, and any indifference curve intersects the horizontal axis at $v^{-1}(u^{\theta}/(1-p^{\theta}))$, where u^{θ} is the utility level of that curve, $\theta \in \{H, L\}$. For $k < 1$, one (for u^H) or both of the two indifference curves delimiting $S^C(w^P)$ may be non-existing if the right-hand sides of (11a) and (11b) have low values. The utility levels defining the two indifference curves, if they exist, are found by rearranging the two inequalities as two equations with $u^H(w)$ and $u^L(w)$ on the left hand sides, respectively. We know that $u^H(w) < u^L(w)$ when both are positive. The existence of the u^H indifference curve is checked by checking that the corresponding right-hand side is positive. Its intersection with the u^L indifference curve within the feasible set W is checked by checking that $u^H(w)/(1-p^H) > u^L(w)/(1-p^L)$, so that the intersections with the horizontal axis (in Figure 1) occur in the opposite order of the intersections with the $w_2 = w_1$ line. Next, if $S^C(w^P)$ is non-empty, we calculate cream-skimming profits. We solve for the intersection of the two indifference curves by solving (15) numerically. If profit at this point is positive, then cream skimming destroys the pooling equlibrium.

6. We check whether dregs skimming is profitable, thus destroying the pooling equilibrium. We must calculate the profit $\pi\left(w^D\left(w^P\right), 1\right)$ that can be earned from dregs-skimming it, with $w^D\left(w^P\right)$ defined in (17) above. First we determine whether $S^D(w^P)$ is empty. This may occur if $k > 1$, in which case u values are negative, and each indifference curve lies to the northeast

of its asymptotes $w_1 = v^{-1}(u^\theta/(1-p^\theta))$ and $w_2 = v^{-1}(u^\theta/p^\theta)$. If $S^D(w^P)$ is non-empty, its profit-maximizing element is the solution to (17). We only need to solve for a scalar, since $w_1^D(w) = w_2^D(w)$ by (17a). We solve for $u^H(w^D(w^P))$ analytically from (17b) and then check whether it has the same sign as $1-k$. In that case, $S^D(w^P)$ is non-empty, and the profit-maximizing element (or rather, the scalar) is given by $v^{-1}(u^H(w^D(w^P)))$. If profit at this point is positive, then dregs skimming destroys the pooling equlibrium.

7. If neither cream skimming nor dregs skimming is profitable, we check whether cross skimming is profitable, thus destroying the pooling equilibrium. Cross skimming is defined in (13), with $w = w^P$, the candidate pooling equilibrium. One could attempt to do this by solving for the maximum profit attainable through cross skimming. But that problem is complicated by the shifting constraints: We do not know whether (13a) or (13c) will be binding. Computationally, it is more straightforward to solve for a menu which makes expected profits, given in (18), equal to zero. As long as this menu is not located at the endpoints of the constraints, it will be possible, by continuity and differentiability of the profit function, to find a slightly different menu which satisfies the constraints and yields strictly positive expected profits. Since $w^{H\prime} \in W_F$, the number of scalar unknowns is reduced to three, $w_1^{H\prime}, w_1^{L\prime}, w_2^{L\prime}$. We also assume that (13b) is binding, cf. equation (15b), illustrated as u_C^L in Figure 1. The three unknowns are determined by (18) set equal to zero, and the equality versions of (13b) and either (13a) or (13c). Either a solution (not at the endpoints) is found for the first or the second of these sets of equations, implying that cross skimming destroys the pooling equilibrium, or it is concluded that profitable cross skimming is impossible.

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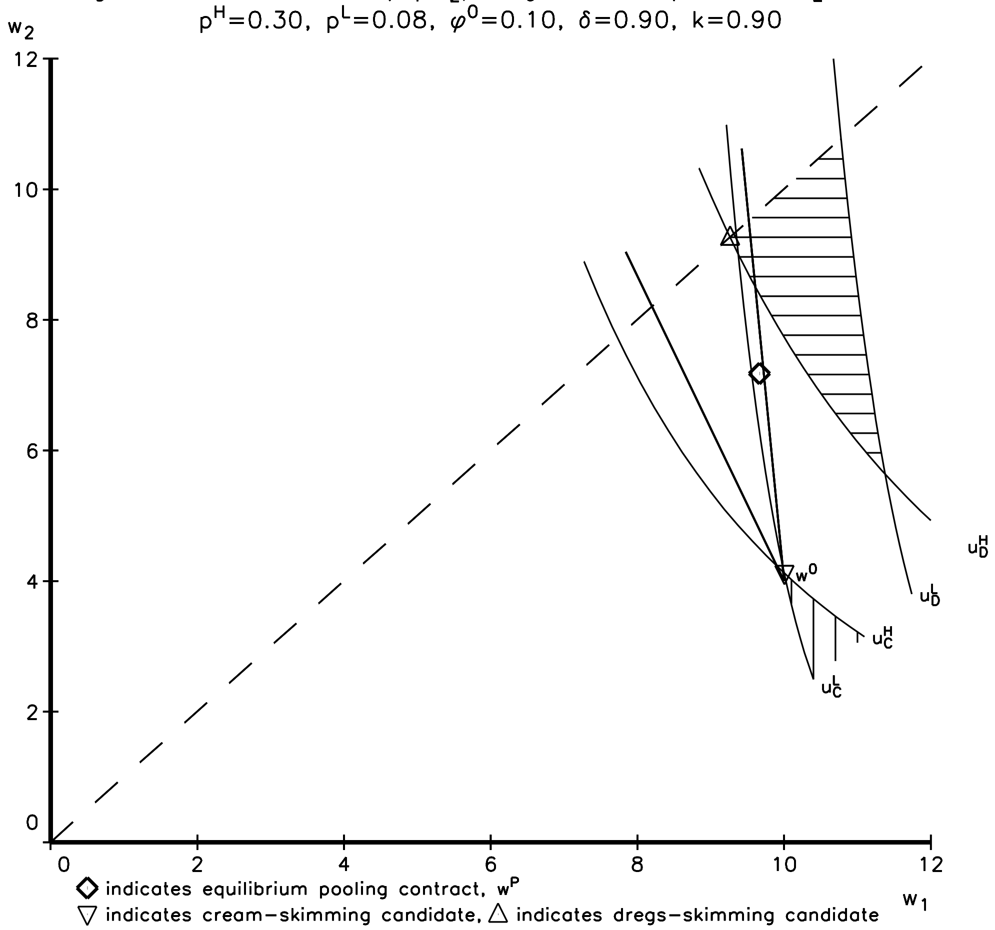


Figure 1: Contracts in (w_1, w_2) diagram for $w_1^0=10.00$, $w_2^0=4.00$, $p^H=0.30$, $p^L=0.08$, $\varphi^0=0.10$, $\delta=0.90$, $k=0.90$

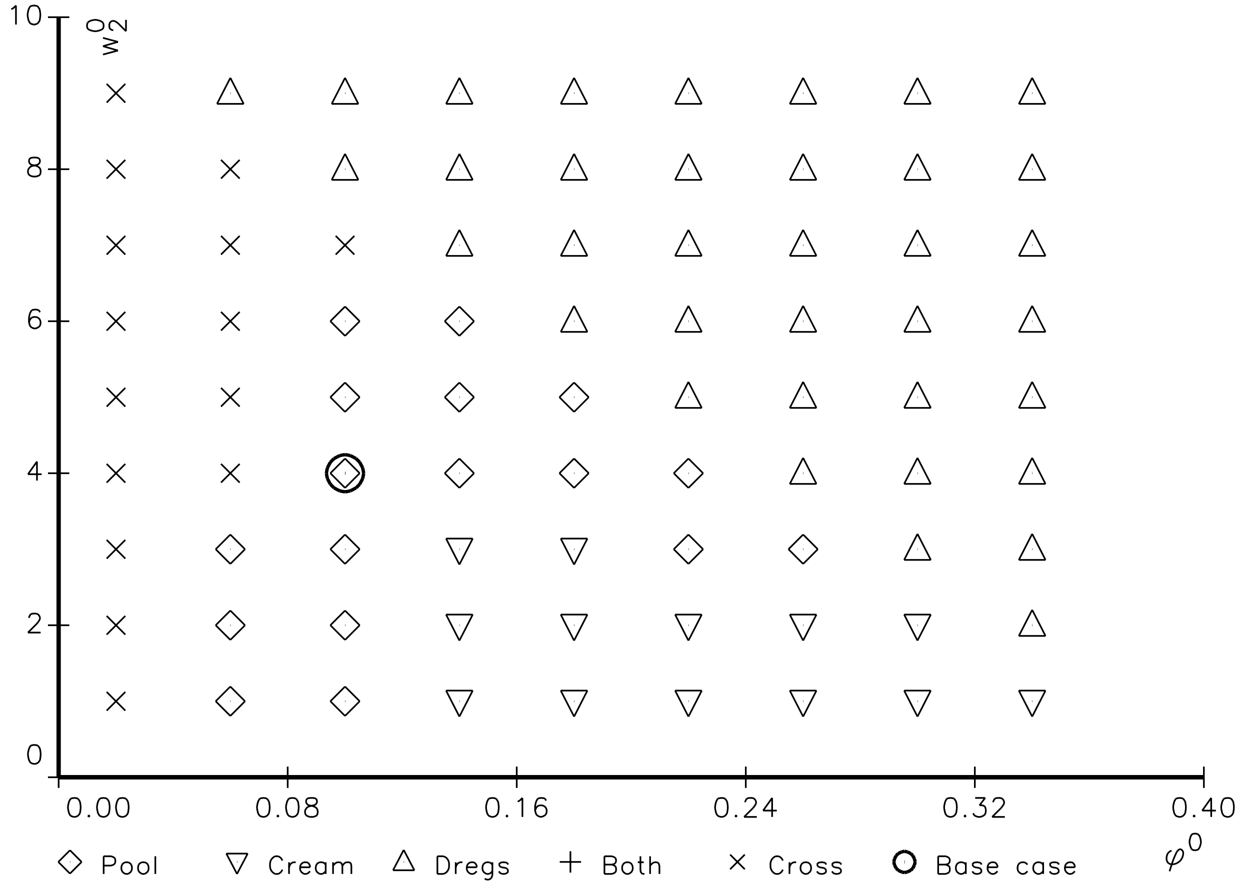
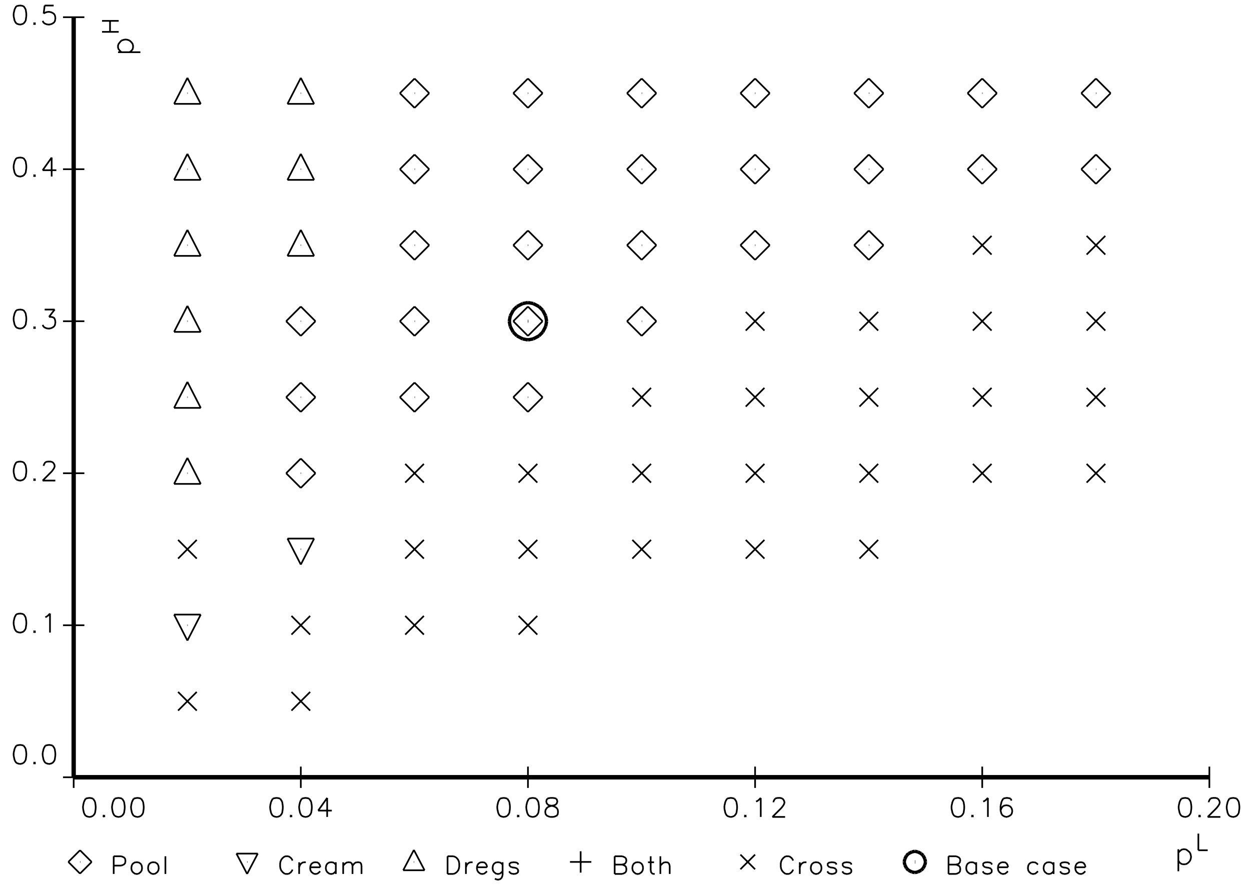


Figure 2: Equilibrium type as function of φ^0 and w_2^0 for w_1^0=10.00, p^H=0.30, p^L=0.08, δ=0.90, k=0.90

Figure 3: Equilibrium type as function of p^L and p^H for w_1^0=10.00, w_2^0=4.00, φ^0=0.10, δ=0.90, k=0.90



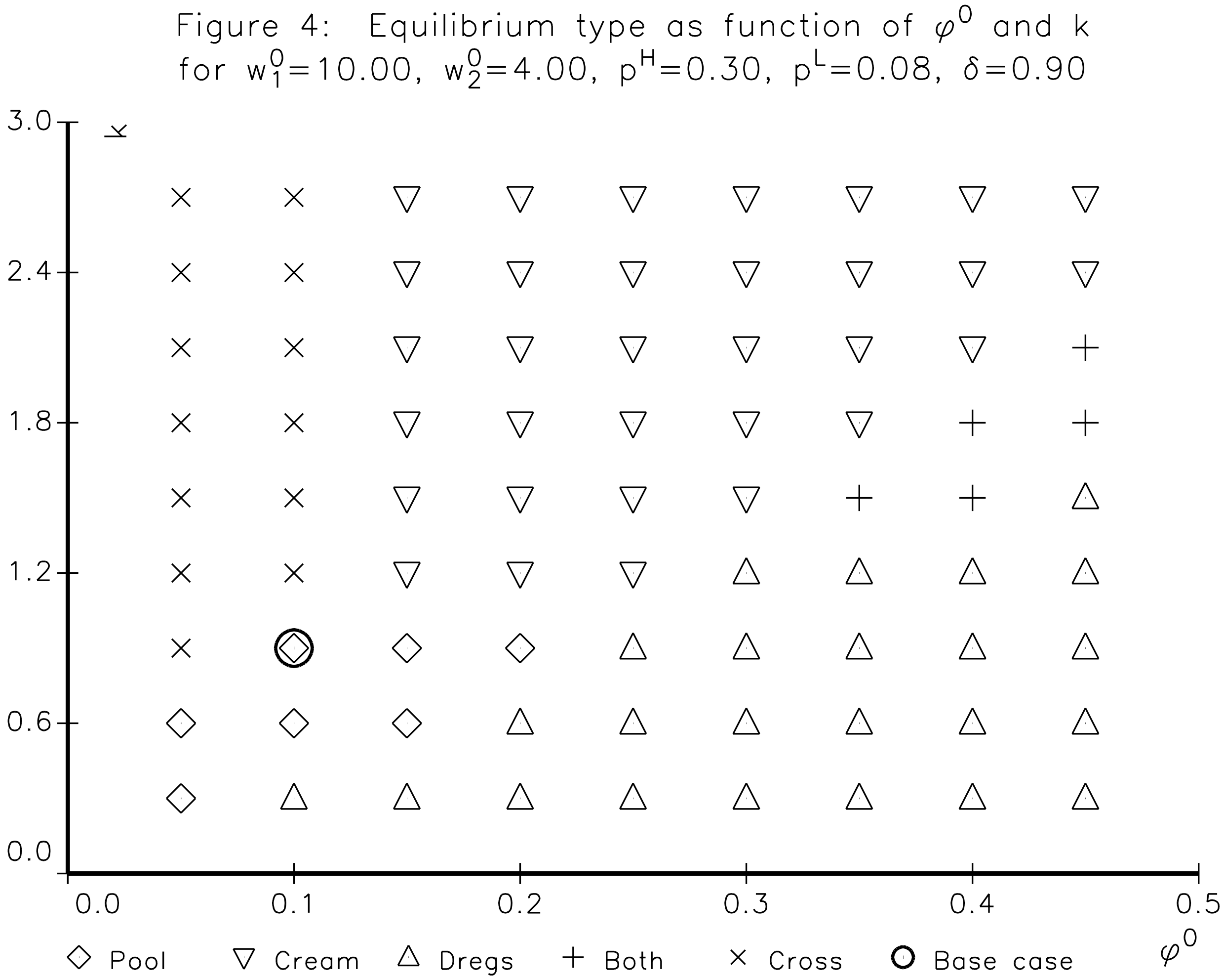


Figure 4: Equilibrium type as function of φ0 and k
for w10=10.00, w20=4.00, pH=0.30, pL=0.08, δ=0.90
k
3.0
2.4
1.8
1.2
0.6
0.0
0.0
0.1
0.2
0.3
0.4
0.5
φ0
Pool
Cream
Dregs
Both
Cross
Base case

Figure 5: Equilibrium type as function of p^L and δ
for w_1^0=10.00, w_2^0=4.00, p^H=0.30, φ^0=0.10, k=0.90