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Working paper 17-2004

SKILL LEVEL, COGNITIVE ABILITY, UNEMPLOYMENT AND WELFARE

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September 16, 2004

Abstract

This paper examines the implications of that workers may not be able to estimate their true costs of acquiring skills. Consequently, too few workers may acquire skills. This allows for the possibility that subsidizing education is welfare improving. Furthermore, if the presence of skill-biased technological shocks increase unemployment, this may explain why the market it-self cannot respond to this by making it sufficiently attractive to acquire skills. Consequently, the trade-off in-between subsidizing education and thereby reducing unemployment and optimizing welfare may be eliminated. We analyse this issue in a simple educational model and next in a search equilibrium model including a skill choice decision.

Keywords: Education, subsidies, efficiency, unemployment.
JEL codes: I20, J64.

1 Introduction

In recent years, the focus on skill-biased technological changes potentially causing increasing unemployment for unskilled workers has increased the attention towards education. Education is meant to be important as a cure against this, by shifting workers into the skilled labour force. This has been used as an argument for subsidizing education.

However, if the demand for skilled workers increases relatively to the demand for unskilled workers we should observe that more workers find it profitable to acquire skills. No educational policy should be needed in the long run as the market takes care of the problem itself. This should only be the case if positive externalities associated with skill acquisition or capital market imperfections exist. Several papers do not support this view. Papers by Heckman and Klenow (1997), Krueger and Lindahl (1999), Acemoglu and Angrist (1999)

*Comments from particapants at the Lower conference, London 2004, and from John Hassler are gratefully acknowledged.

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do not find empirical evidence for the positive externalities and papers by Shea (2000), Cameron and Taber (2000), questions the importance of capital market imperfections. Why then subsidize education?

The novelty of the present paper is that we assume that workers may not be able to estimate their true costs of acquiring skills. We assume that costs of acquiring skills is related to the ability of the worker. The higher the worker ability, the lower the costs of acquiring skills. Worker ability is observed with an error. Consequently, skill acquisition costs are observed with an error. This allows for the possibility that subsidizing education increases welfare even when no capital market imperfections or externalities are present. We should here emphasize that we do not need to assume that the error goes in one particular direction in order to obtain the result. That is, we do not assume that everyone underestimate their true ability levels. Furthermore, we consider the relation between a biased estimate of ability and the relation in-between welfare and unemployment.

There exists other papers disregarding externalities associated with education and still obtaining a possible positive impacts from subsidizing education. However, these papers usually consider welfare functions with redistributional considerations. See for example, the paper by Bovenberg and Jacobs (2001) where they include another disturbance, namely policy aiming at redistribution which calls for education subsidies in order to motivate workers to undertake an education. Furthermore, concerning the absence of complete rationality is considered in Becker (1962), where he assumes randomness in individual behaviour and by Akerlof and Dickens (1982) where they consider the impact of individual's tendency to disregard past information when making their choice. However, the present paper considers the macroeconomic impacts in a model where workers cannot correctly estimate educational costs.

So would limited cognitive ability be an important issue to consider? Should it have any significant impact? There is no reason to believe that there is any change in human being's ability to correctly estimate their educational costs. Or is there? In case skill biased technological progress urges a larger part of the population to acquire skills than previously, more workers may potentially move from a different background which may be associated with it being more difficult to correctly estimate ones true ability costs. Alternatively, technological progress happening at a higher speed, may imply that new skills are called for and thereby in general reduce the workers' ability to correctly estimate the worker's ability to acquire the necessary training for a given work.

We propose a simple model to examine the direct welfare impacts resulting from limited cognitive ability acknowledging the worker's skill choice decision. A similar simple search equilibrium model is chosen in order to show how government policy may be optimal in an economy with skill-biased technological progress and educational choice but without any capital market imperfections or educational externalities. In this model, instead, we allow for labour market imperfections due to search frictions, in order to generate unemployment. We show that search frictions provide an interaction between job supply and skill level which affects welfare. Hence, either this mechanism or the presence of limited cognitive ability, is a channel through which a subsidy can be welfare improving.

The paper is organised as follows. Section 2 describes a two sector model with no other imperfections than limited cognitive ability. The following section considers welfare in this model. Section 4 provides some examples and in Section 5 a simple equilibrium model for the labour market is set up and the impact of skill biased technological progress and subsidies are considered. Finally, section 6 concludes.

2 The Model

The economy consists of a continuum of workers normalized to one. When the worker enters the labour market he or she decides whether to acquire skills or to remain unskilled. The labour force is heterogenous, in the sense that some workers are more able than others to acquire and retain skills,[1] whereby obtaining skills is less costly for those workers. We let a denote worker ability and let workers be uniformly distributed between zero and one, $a \in [0,1]$. However, workers do not observe their true ability level.[2] Instead they observe $\tilde{a} = a + \varepsilon$, where ε is uniformly distributed, $\varepsilon \in [-\frac{1}{2}, \frac{1}{2}]$ and hence $E(\varepsilon) = 0$.

The true costs of acquiring and retaining skills are $c(a)$, where $c'(a) < 0$. Workers observe the costs $c(\tilde{a})$ where these costs may be larger or smaller than the true costs. The observed costs are important for the individual worker's skill choice decision and thereby the skill level in the economy. When skills are acquired, workers learn about their true costs and hence true costs are important for welfare. The larger the error, the larger the difference in-between the true costs and the observed costs.

An unskilled worker has productivity y_l, whereas a highly skilled worker has productivity y_h, where $y_h > y_l$. We assume that workers receive a wage equal to their productivity. This assumption is relaxed later in the paper.

2.1 Skill Acquisition

Workers choose to acquire skills if the gain associated with this is higher than the costs. That is, worker i acquires skills if:

$$y_h - y_l > c(\tilde{a}_i).$$

When considering the impact of imposing a subsidy, s, we can distinguish in-between 3 different cases. **In case** A,the condition for skill acquisition is the following:

$$y_h - y_l > c(\tilde{a}_i).$$

[1] We assume that skilled workers keep paying the costs after they have become skilled. This is only a simplying assumption and is not important for the results.

[2] This is similar to the assumption made in Gilles Saint-Paul, 2002, Cognitive ability and Paternalism.

Even without a subsidy worker i acquires skills. **Case** B reads:

$$\begin{aligned} y_h - y_l &< c(\tilde{a}_i), \\ y_h - y_l + s &> c(\tilde{a}_i). \end{aligned}$$

Only with a subsidy worker i acquires skills. And finally, **case** C :

$$\begin{aligned} y_h - y_l &< c(\tilde{a}_i), \\ y_h - y_l + s &< c(\tilde{a}_i). \end{aligned}$$

In the last case, the implied costs are higher than the gain from acquiring skills and hence in case C the worker does not acquire skills. We thereby have the non-surprising result that including the subsidy increases the number of workers acquiring skills, that is the skill level in the economy increases.

For simplicity, we assume that educational costs are linear and given by the function, $c(a) = 1 - a$. The marginal worker is just indifferent in-between acquiring skills or remaining unskilled. We therefore have the condition:

$$y_h - y_l + s = 1 - \tilde{a}^*,$$

where $\tilde{a}^*$ is the estimated ability level of the marginal worker acquiring skills. Without limited cognitive ability, workers with ability $a < a^*$ constitutes the unskilled labour force and workers with ability $a \geq a^*$ belong to the skilled labour force. With limited cognitive ability, $\varepsilon^* = 1 - a - (y_h - y_l) - s$ denotes the marginal worker acquiring skills and hence the number of skilled workers is $\int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} d\varepsilon da$ and the number of unskilled workers is derived as $\int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} d\varepsilon da$.

However, welfare may increase or decrease as a result of including a subsidy. A worker who has acquired skills and observed educational costs higher than the true ability costs, experiences skill acquisition costs higher than the gain in productivity. Welfare is reduced for this worker. Similarly, welfare increases for the worker who has correctly estimated or underestimated his or her ability and thereby correctly estimated or overestimated skill acquisition costs.

3 Welfare

This section considers the marginal impact on welfare from a subsidy when workers are subject to limited cognitive ability. For expositional reasons, the case where workers observe their true ability level and thereby their true educational costs is first examined. Next, we examine the impact on welfare from increasing the subsidy if ability is observed with an error.

3.1 Agents without limited ability

We first consider the case where the worker can correctly estimate his or her ability. Taking expectations we have $E(\tilde{a}) = a + E(\varepsilon) = a$. Welfare is determined as

$$W(s) = (1 - (y_h - y_l + s))\, y_l + \int_{1-(y_h-y_l+s)}^{1} (y_h + s - (1-a))\, da.$$

Welfare increases with the subsidy as more workers acquire skills: $W'(s) = (y_h + s - y_l) > 0$. However, the subsidy has to be financed. In order to focus on any potential inefficiency caused by the subsidy and not the tax system per se, we let all workers pay a lump-sum tax. The government budget constraint is equal to

$$\int_{1-(y_h-y_l+s)}^{1} s(1+\mu)\, da = \int_{0}^{1-(y_h-y_l+s)} t da + \int_{1-(y_h-y_l+s)}^{1} t da = t, \tag{1}$$

where μ is administrative costs associated with the subsidy. Including financing of the subsidy we obtain the following welfare function:

$$\begin{aligned} W(s) &= \int_{0}^{1-(y_h-y_l+s)} (y_l - t)\, da + \int_{1-(y_h-y_l+s)}^{1} (y_h + s - (1-a) - t)\, da \\ &= \int_{0}^{1-(y_h-y_l+s)} y_l da + \int_{1-(y_h-y_l+s)}^{1} (y_h - (1-a))\, da - \int_{1-(y_h-y_l+s)}^{1} s\mu da. \end{aligned}$$

Proposition 1 *Without limited cognitive ability, the optimal subsidy is $s = 0$.*

Proof. Differentiating with respect to the subsidy delivers

$$(y_h - y_l) - (y_h - y_l + s) - (y_h - y_l + 2s)\, \mu = -s - (y_h - y_l + 2s)\, \mu < 0.$$

Hence, a subsidy always reduces welfare. ■

Welfare decreases with s. Without any externalities the market provides the best outcome. Thus, a subsidy obviously increases the skilled labour force but welfare decreases. However, if the worker does not observe his or her true ability level this result may not hold. We now turn to this issue.

3.2 Agents with limited cognitive ability

When the worker observes ability with an error the government budget constraint is modified accordingly:

$$\int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} s(1+\mu)\, d\varepsilon da = \int_0^1 t da = t. \tag{2}$$

Welfare with limited cognitive ability reads

$$\begin{aligned} W(s) &= \int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} (y_l - t)\, d\varepsilon da + \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} (y_h + s - (1-a) - t)\, d\varepsilon da. \qquad (3) \\ &= \int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} y_l d\varepsilon da + \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} (y_h - (1-a))\, d\varepsilon da - \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} s\mu d\varepsilon da, \end{aligned}$$

where $\varepsilon^* = 1 - a - (y_h - y_l + s)$ denotes the marginal worker acquiring skills. The impact on welfare from a higher subsidy is then given by

$$W'(s) = (y_h - y_l - (1 - a)) - \mu (y_h - y_l + 2s) = (y_h - y_l)(1 - \mu) - \frac{1}{2} - \mu 2s. \tag{4}$$

Hence, welfare increases if

$$y_h - y_l > \frac{1}{2} + \mu (y_h - y_l + 2s). \tag{5}$$

On the right hand side of equation (5) is the net gain from one more worker acquiring skills. On the right hand side we have expected costs associated with the marginal worker acquiring skills, which are educational costs, $\frac{1}{2}$, plus administrative costs $\mu (y_h - y_l + 2s)$. The government being able to estimate the expected costs associated with increasing the subsidy should then do so if the expected benefits are higher than the expected costs.

Proposition 2 *The optimal subsidy level s^o is equal to $s^o = \frac{(y_h - y_l)(1-\mu) - \frac{1}{2}}{2\mu}$.*

Proof. Let the derivative $W'(s)$, given by equation (4), equal to 0 and solve for the optimal subsidy level s^o. Derive the second order condition by differentiating $W'(s)$ with respect to s and obtain $W''(s) < 0$. ■

The optimal subsidy increases in the productivity difference and decreases in expected education costs and administrative costs.

4 Examples

In order to increase the transparency of the present problem we now consider some examples. We assume there are two different ability levels in the economy, a_l and a_h where $a_h > a_l$. Let half of workers have ability a_h and the remaining part of the labour force have ability a_l. For simplicity, the administrative waste cost are excluded and the optimal subsidy levels are therefore not considered.

4.1 Agents without limited ability

We first consider the case without limited cognitive ability. The agents correctly estimate the educational costs which are given by low costs associated with the high ability worker, $c_l = 1 - a_h$ and high costs associated with the low ability worker $c_h = 1 - a_l$ where $c_h > c_l$. Suppose $c_h > y_h - y_l > c_l$. Then only high ability workers acquire skills. The welfare function reads

$$W = \frac{1}{2}(y_l + y_h - c_l) > 0.$$

Introducing a subsidy ensuring $y_h + s - y_l > c_h$, implies that all workers acquire skills and the welfare function is modified to

$$W(s) = \frac{1}{2}(y_h + s - c_h + y_h - c_l) - t = \frac{1}{2}(y_h - c_h + y_h - c_l),$$

given the government budget constraint, which is given by $\frac{1}{2}s = t$. Welfare including a a subsidy is larger than welfare without if $W(s) > W$ if and only if

$$y_h - y_l - c_h > 0.$$

This can never hold as $c_h > y_h - y_l$. Consequently, introducing the subsidy leads to welfare reduction.

4.2 Agents with limited cognitive ability

Let a fraction of the population observe their ability with an error, hence $\frac{1}{2}p_l$ of the high ability workers observe ability $\tilde{a}_l = \frac{a_h + a_l}{2}$ and similarly for the fraction $\frac{1}{2}p_l$ of the low ability workers. Hence, observed costs for $\frac{1}{2}p_h$ and $\frac{1}{2}p_l$ low ability workers is $\tilde{c} = 1 - \frac{a_h + a_l}{2}$. Then $c_h > \tilde{c} > c_l$. We consider two different examples.

4.2.1 Example one, $c_h > y_h - y_l > \tilde{c}$

In the first example we have that $c_h > y_h - y_l > \tilde{c}$ whereby all high ability workers acquire skills as well as the $\frac{1}{2}p_l$ low ability workers observing a higher ability level than they actually have. The low ability workers, correctly observing their ability, do not acquire skills as educational costs are too high. Welfare without a subsidy then becomes:

$$\begin{aligned} W &= \frac{1}{2}y_l(1-p_l) + \frac{1}{2}(y_h - (1-a_h)) + \frac{1}{2}p_l(y_h - (1-a_l)) \\ &= \frac{1}{2}(y_l + (y_h - c_l) + p_l(y_h - y_l - c_h)), \end{aligned}$$

where the last term is negative. Suppose a subsidy is introduced such that $y_h + s - y_l > c_h$, whereby everyone acquire skills. In this case welfare including the subsidy and using the government budget restriction is:

$$W(s) = \frac{1}{2}(y_h + s - c_l + y_h + s - c_h) - t = y_h - \frac{1}{2}c_l - \frac{1}{2}c_h. \quad (6)$$

Introducing a subsidy increases welfare if $W(s) > W \Leftrightarrow (1-p_l)(y_h - y_l - c_h) > 0$, which does not hold. It follows that welfare falls. The condition can be rewritten as

$$\frac{1}{2}(1-p_l)(y_h - y_l) > \frac{1}{2}(1-p_l)c_h,$$

where the left hand side is the expected marginal increase in welfare and the right hands side is the expected marginal increase in costs.

4.2.2 Example two, $\tilde{c} > y_h - y_l > c_l$

Example two considers the case where $\tilde{c} > y_h - y_l > c_l$. Then only the fraction $1 - p_h$ of the high ability workers acquire skills. Welfare without a subsidy is

$$W = \frac{1}{2}(y_l + p_h y_l + (1-p_h)(y_h - c_l)) > 0.$$

Now, consider two different subsidies. In the first case, the subsidy is set such that $c_h > y_h + s - y_l > \tilde{c}$. This implies that all but the $\frac{1}{2}(1-p_l)$ workers acquire skills. Welfare yields

$$\begin{aligned} W(s) &= \frac{1}{2}\left(y_h + s - c_l + p_l\left(y_h + s - c_h\right) + (1-p_l)\, y_l\right) - s\left(\frac{1}{2} + \frac{1}{2}p_l\right), \\ &= \frac{1}{2}\left(y_h - c_l + p_l\left(y_h + s - y_l - c_h\right) + y_l\right), \end{aligned}$$

Introducing a subsidy implies a welfare improvement if $W(s) > W$, which reduces to

$$p_h\left(y_h - y_l - c_l\right) + p_l\left(y_h - y_l - c_h\right) > 0.$$

Welfare increases if this condition holds. The first term is positive and the second term is negative. If $p_l = p_h$ we have $2p_l\left(y_h - y_l - \tilde{c}\right) > 0$,which is not satisfied. Hence in order for the condition to be satisfied we need that $p_h > p_l$, that is, more high ability types observe their true ability with an error than low ability types. The condition may be rewritten as

$$\frac{1}{2}\left(p_h + p_l\right)\left(y_h - y_l\right) > \frac{1}{2}p_h c_l + \frac{1}{2}p_l c_h.$$

Again, the left hand side is the expected marginal increase in welfare and the right hands side is the expected marginal increase in costs.

Consider another example where the subsidy is set such that $y_h + s - y_l > c_h$ then all workers would acquire skills and welfare is given by (6). Welfare increases with the subsidy if

$$y_h - y_l - c_h + p_h\left(y_h - y_l - c_l\right) > 0.$$

The first term is negative and the second term is positive. Hence welfare may decrease or increase. The higher the fraction of high ability workers observing their true ability with an error the more likely it is that the condition is satisfied and welfare increases. The condition can be rewritten as $\left(\frac{1}{2} + p_h\right)\left(y_h - y_l\right) > \frac{1}{2}c_h + \frac{1}{2}p_h c_l$.

In general, welfare increases if the expected benefits are higher than the expected costs for the workers choosing to acquire skills given the subsidy. If the government can perform this calculation in order to determine whether or not to introduce a subsidy, the possibility of increasing welfare exists.

5 A model of the Labour Market

We have just seen that, even without any capital market imperfections or educational externalities, there may a scope for subsidizing education if workers observe their true ability with an error. What does this imply for unemployment? One argument behind subsidizing education is that skill-biased technological shocks may increase unemployment if workers do not optimally react to the economic changes. Consider the situation where the economy is subject to skill-biased technological changes. A feature of a long run model should capture that unemployment is independent of productivity. However, the argument behind skill-biased technological progress and their impact on unemployment does not capture this. Our model can capture this feature by noting that if skill-biased technological changes should be a reason for high unemployment in the last couple of decades in many OECD countries, then this implicitly assumes that we have not seen skill-biased technological changes to a large extend previously. This section considers a slightly different labour market model in which we are able to consider the impact of skill-biased technological progress. Here we present a model where we have some interaction in-between labour market tightness and the skill choice decision. We do this by proposing a model where firms supply vacancies in an undirected way.[3] Hence, the firm employs a highly skilled worker if it is matched with one and a low skilled worker if it meets up with such a worker. The firm pays low skilled and high skilled workers different wages. The matching functions is given by: $x = x(u, v)$, where all workers and firm compete, that is u is the total unemployment rate and v is the total vacancy rate. The matching function has positive first order derivatives, negative second order derivatives, positive cross partial derivatives, and is homogenous of degree one. The worker's transition rate can be expressed as $f = \frac{x}{u} = x(\theta)$, where θ is labour market tightness. The transition rate facing firms is $q = \frac{x}{v} = x\left(\frac{1}{\theta}\right)$. In order to model that skill-biased technological changes may increase unemployment, we assume that skill-biased technological changes increase relative productivity but also increase the separation rate of the low skilled worker.

Employed workers receive wages w_j, h, l. All workers receive a lump sum transfer, R. Let r be the discount rate and d the continuous flow into and out of the labour force. The values of employment, $E_j, j = h, l$ and unemployment, $U_j, j = h, l$ as highly educated and low skilled workers are therefore determined by the arbitrage equations:

$$\begin{aligned} rE_h &= R + w_h - t + \lambda(U_h - E_h) - dE_h, \quad (7) \\ rE_l &= R + w_l - t + \lambda(p)(U_l - E_l) - dE_l, \quad (8) \end{aligned}$$

and

$$rU_j = R - t + f(E_j - U_j) - dU_j, \; j = h, l, \quad (9)$$

where the separation rate for firms employing a low skilled worker is an increasing function of the skill-biased productivity parameter $p, \lambda'(p) > 0$. Furthermore we assume that $\lambda(p) > \lambda$ captured by the specific form, $\lambda(p) = p\lambda$.

Sector j firms employ workers with the marginal productivity py and y. The value of having a filled job, $J_j, j = h, l$ solve the equations:

$$\begin{aligned} rJ_h &= py - w_h + \lambda(V - J_h) - dJ_h, \quad (10) \\ rJ_l &= y - w_l + \lambda(p)(V - J_l) - dJ_l, \quad (11) \end{aligned}$$

[3] A model where firms search for workers in a direct manner is available upon request. This gives a simple segmented model, but without any direct interaction in-between skill level and unemployment.

where the productivity of a high skilled workers is larger than the productivity of a low skilled worker, $p > 1$. The equation determining the value of a vacancy is

$$rV = q((1 - \tilde{a}^*)(J_h - V) + \tilde{a}^*(J_l - V)) - ky - dV. \tag{12}$$

At the rate $(1 - \tilde{a}^*)$ the firm obtains a skilled worker and at the rate $\tilde{a}^*$ the firm faces an unskilled worker. Different valuations are then attached to employing these workers. The term ky is vacancy costs. The firm chooses to employ one of these workers if it meets up with either of them as the value of a vacancy, by free entry, is equal to zero, $V = 0$, and wages are determined through bargaining, as shown below.

Wages, w_j, are determined by Nash Bargaining with the workers' bargaining power equal to $\frac{1}{2}$. The first order condition yields:

$$E_j - U_j = J_j - V, \ j = h, l. \tag{13}$$

We can solve for the bargained wage by using equations (7),(8)-(13), assuming free entry, $V = 0$ and a symmetric equilibrium. Solving, we get the following wage rules:

$$w_h = y\frac{(p + \theta k)}{2}, w_l = y\frac{(1 + \theta k)}{2}. \tag{14}$$

Labour market tightness θ, can be derived from equations (10), (11) and (12), using the free entry condition and the wage rule in (13):

$$2\frac{k}{q} = \frac{(1 - \tilde{a}^*)\, p}{r + d + \lambda} + \frac{\tilde{a}^*}{r + d + \lambda\,(p)} - \theta k. \tag{15}$$

Labour market tightness is a function of $\tilde{a}^*$ and parameters, $\theta\,(\tilde{a}^*)$. We observe that general productivity, y does not appear in this equation. That is, a general unbiased increase in productivity has no direct impact on labour market tightness. If labour market tightness is not affected, then unemployment is unchanged. This is accordance with the usual result that unemployment is independent of general productivity changes.

As the productivity of a low skilled worker is lower than the productivity of a high skilled worker and the separation rate of low skilled workers is higher than the separation rate of high skilled workers, it follows that labour market tightness increases with the number of skilled workers. Hence, a partial equilibrium result is that labour market tightness falls with $\tilde{a}^*$. However, an increase in p both has a positive and a negative impact on labour market tightness as a higher p both raises the productivity of a high skilled worker and the separation rate of a low productivity worker.

5.1 Skill Level

When workers decide whether to acquire higher education or remain a manual worker, they compare the value of unemployment as an educated worker to the value of unemployment as a manual worker. In section (2.1) we analysed the problem in a static model. Here, we obtain the condition giving the marginal worker's ability level, $\tilde{a}^* = (a+\varepsilon)^*$, which makes him just indifferent between acquiring higher education and remaining an unskilled worker. We can write the condition determining the ability level of the marginal worker as:

$$rU_h + s - rU_l = 1 - \tilde{a}^*.$$

Using equations (7)-(9) gives

$$\left(\frac{fw_h}{r+d+\lambda+f} - \frac{fw_l}{r+d+\lambda(p)+f}\right) = 1 - \tilde{a}^* - s, \tag{16}$$

Equation (16) gives $\tilde{a}^*$ as a function of the subsidy and productivity. Workers with $\tilde{a} \leq \tilde{a}^*$, choose not to acquire education, whereas workers with $\tilde{a} > \tilde{a}^*$ acquire education. Hence, $\tilde{a}^*$ and $1-\tilde{a}^*$ resolve the uneducated and highly educated labour forces, respectively. Using the wage equation gives the marginal ability level as a function of labour market tightness, $\tilde{a}^*(\theta)$:

$$\eta(\theta, p, y) = 1 - \tilde{a}^* - s, \tag{17}$$

where $\eta(\theta, p, y) = \frac{1}{2}y\left(\frac{f(\theta)(p+\theta k)}{r+d+\lambda+f(\theta)} - \frac{f(\theta)(1+\theta k)}{r+d+\lambda(p)+f(\theta)}\right)$. The partial equilibrium results are the following. The number of workers acquiring skills are increasing in general productivity, $\frac{\partial\eta}{\partial y} > 0$, increasing in skill-biased technological changes, $\frac{\partial\eta}{\partial p} > 0$ and the number of workers acquiring skills increases in labour market tightness, that is, $\tilde{a}^*$ is decreasing in $\theta, \frac{\partial\eta}{\partial\theta} > 0$. Solving for ε^* we obtain

$$\varepsilon^* = 1 - a - s - \eta(\theta, p, y).$$

For $r \to 0$ we get an educational choice equation which allows for a very intuitive interpretation:

$$e_h w_h - e_l w_l + s = 1 - \tilde{a}^*. \tag{18}$$

On the right hand side is the number of workers acquiring skills. On the left hand side of equation (18) we have the following. The wage rate received as a highly educated weighted by the employment rate tends to increase the number of workers acquiring skills, whereas the opposite holds in terms of wages and employment rate for the low skilled workers. The subsidy also rises the number of skilled workers for given labour market tightness.

Illustrating the two curves, $\theta(\tilde{a}^*), \tilde{a}^*(\theta)$, in a $\tilde{a}^*, \theta$ space then delivers two downward sloping curves. The slopes are

$$\begin{aligned}
\frac{d\tilde{a}^*}{d\theta}|_{\theta(\tilde{a}^*)} &= -\frac{k\left(-\frac{2}{q^2}\frac{\partial q}{\partial\theta}+1\right)}{\frac{p}{r+d+\lambda} - \frac{1}{r+d+\lambda(p)}} = -\gamma_\theta(p) < 0, \\
\frac{d\tilde{a}^*}{d\theta}|_{\tilde{a}^*(\theta)} &= -\frac{\partial\eta}{\partial\theta} = -\gamma_a(p) < 0.
\end{aligned}$$

Which slope is the greater depends on the productivity of high productivity workers relatively to low productivity workers. For relative low values of p the skill acquisition curve is the steeper one. See Figure 1. The reverse holds for high values of p. This implies the following result holds. To ensure existence of a stable equilibrium we need that $-\gamma_a(p) > -\gamma_\theta(p)$.[4] Consequently, a stable equilibrium exists for $p < \hat{p}$ where $\hat{p}$ solves, $-\gamma_\theta(\hat{p}) = -\gamma_a(\hat{p})$. We only consider stable equilibria, that is, we assume that $p < \hat{p}$. That is, we consider equilibria where productivity differences in-between the two types of workers are not too high.

— Figure 1 about here —

5.2 Unemployment

Unemployment and employment in the two sectors are determined by inflow equal to outflow. We obtain that the unemployment rates are equal to

$$u_h = \frac{\lambda + d}{f + \lambda + d}, u_l = \frac{\lambda(p) + d}{f + \lambda(p) + d}. \tag{19}$$

Unemployment for skilled workers is higher than unemployment for unskilled workers as the separation rate for unskilled workers is higher than the separation rate for skilled workers, $\lambda(p) > \lambda$, and the workers' transition rates are equivalent. Total unemployment is given as

$$u = \int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} u_l d\varepsilon da + \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} u_h d\varepsilon da = u_l + (\eta(\theta, p, y) + s)(u_h - u_l). \tag{20}$$

5.3 Welfare

The social welfare function is the sum of utilities deducted educational costs:

$$W = \int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} \tilde{W}_l d\varepsilon da + \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} \left(\tilde{W}_h + s\right) d\varepsilon da - \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} (1-a)\, d\varepsilon da,$$

where $\tilde{W}_j = u_j r U_j + (1-u_j) r E_j + n_j r J_j + v_j r V_j, j = h, l$.

By making use of the asset equations for workers and firms in the two sectors, imposing the flow equilibrium conditions as well as the government budget restriction which is given by equation (2) and considering the case of no discounting, i.e., $r \to 0$, the welfare function simplifies to:

$$W(s) = \int_0^1 \int_{-\frac{1}{2}}^{\varepsilon^*} W_l d\varepsilon da + \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} W_h d\varepsilon da - \int_0^1 \int_{\varepsilon^*}^{\frac{1}{2}} (1 - a + s\mu)\, d\varepsilon da, \tag{21}$$

[4] The slope $\gamma_\theta(p)$ tends towards a fixed number for $\theta \to 0$ and $\theta \to \infty$ whereas $\gamma_a(p)$ tends towards ∞ for $\theta \to 0$ and $\gamma_a(p) \to 0$ for $\theta \to \infty$.

where

$$\begin{aligned} W_h &= ((1-u_h)\,p - vk)\,y = e_h \frac{1}{2}\,(p+\theta k)\,y = e_h w_h, \\ W_l &= ((1-u_l) - vk)\,y = e_l \frac{1}{2}\,(1+\theta k)\,y = e_l w_l. \end{aligned}$$

High productivity increases welfare whereas vacancy costs and educational costs reduce welfare. Another interpretation is that welfare increases with employment and wages, but falls with educational costs.

5.4 Skill-biased technological changes

Skill-biased technological changes in the form of a higher p tend to increase labour market tightness due to the higher expected productivity. However, the implication of the a higher separation rate for unskilled workers is a reduction in labour market tightness. We have the following result:

Proposition 3 *Skill-biased technological progress increases the number of skilled workers,* $\frac{d\tilde{a}^*}{dp} < 0$ *and labour market tightness,* $\frac{d\theta}{dp} > 0$.

Proof. Differentiating equation (15) and (17) with respect to p gives

$$\begin{aligned} \frac{d\theta}{dp} &= \frac{-\left(\frac{p}{r+d+\lambda} - \frac{1}{r+d+\lambda(p)}\right)\frac{\partial\eta}{\partial p} - \frac{1-\tilde{a}^*}{r+d+\lambda}}{D} > 0 \\ \frac{d\tilde{a}^*}{dp} &= \frac{\left(-2\frac{k}{q^2}\frac{\partial q}{\partial\theta} + k\right)\frac{\partial\eta}{\partial p} + \frac{\partial\eta}{\partial\theta}\frac{1-\tilde{a}^*}{r+d+\lambda}}{D} < 0, \end{aligned}$$

where $D = \left(\frac{p}{r+d+\lambda} - \frac{1}{r+d+\lambda(p)}\right)\left(\gamma_a(p) - \gamma_\theta(p)\right) < 0.$ ■

The impact on labour market tightness resulting from higher productivity is larger than the negative effect from the separation rate causing labour market to increase. The increase in labour market tightness improves relative employment perspectives for skilled workers and thereby the number of skilled workers increases. More skilled workers increase labour market tightness further due to their relative higher productivity. This causes another increase in the number of skilled workers. Given stability, the impact dampens until a new equilibrium is reached.

By inspection of equation (14), we note that the increase in labour market tightness increases wages for both high skilled and low skilled workers as the workers' transition rate increases which improve workers' bargaining position. This, in turn, tends to reduce wage dispersion as the impact on high skilled worker wages is smaller than the impact on low skilled worker wages as the former wages are the higher. In addition, there is a direct positive impact on high skilled workers' wages, the first term in the numerator of equation (22), increasing wage dispersion. The total impact is ambiguous.

Proposition 4 *The impact on wage dispersion, $w_D = \frac{w_h - w_l}{w_l}$, from skill-biased technological progress is ambiguous.*

Proof. Differentiating wage dispersion with respect to p gives

$$\frac{dw_D}{dp} = \frac{1 + \theta k - (p-1)\frac{\partial \theta}{\partial p}k}{(1+\theta k)^2}, \tag{22}$$

which has ambiguous sign. ■

If skill-biased technological changes increases unemployment, the reason is that the unemployment rate for low skilled workers increases. Unemployment rates are affected in the following way:

Proposition 5 *Skill-biased technological progress decrease the unemployment rate for high skilled workers and increase the unemployment for low skilled workers if the elasticity of the separation rate of low skilled workers with respect to $p, 1$, is larger than the elasticity of the transition rate with respect to p, $\frac{p}{f(\theta)}\frac{\partial f}{\partial p}$.*

Proof. Differentiating the unemployment rates, (19), with respect to p gives

$$\begin{aligned}
\frac{\partial u_h}{\partial p} &= -\frac{\lambda \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial p}}{(f+\lambda)^2} < 0, \\
\frac{\partial u_l}{\partial p} &= \frac{\frac{\partial \lambda(p)}{\partial p} f(\theta) - \lambda(p)\frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial p}}{(f+\lambda(p))^2} = \frac{u_l}{p} f(\theta) \frac{1 - \frac{p}{f(\theta)}\frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial p}}{(f+\lambda(p))},
\end{aligned}$$

where, for $\lambda(p) = \lambda p$, $\frac{p}{\lambda(p)}\frac{\partial \lambda(p)}{\partial p} = 1$, the sign of $\frac{\partial u_l}{\partial p}$ is positive if $1 - \frac{p}{f(\theta)}\frac{\partial f}{\partial p} > 0$. ■

Unemployment facing high skilled workers unambiguously falls as a consequence of a higher labour market tightness. Hence, in terms of employment opportunities skilled workers are better off.

Technological progress has a positive impact on low skilled unemployment as more low skilled workers become redundant. However, technological progress also raises the workers' transition rate through the increase in labour market tightness. The general impact on unemployment is, in general, indeterminate. In addition, the impact on total unemployment is ambiguous as the direct reduction in high skilled unemployment and the reallocation of workers from the low skilled to the high skilled labour force both tend to reduce unemployment, whereas the potential increase in unemployment facing low skilled workers has a positive impact on unemployment.

Proposition 6 *The impact of skill-biased technological progress on total unemployment is ambiguous.*

Proof. Differentiating unemployment with respect to p gives

$$\begin{aligned}
\frac{\partial u_{TOT}}{\partial p} &= \frac{\partial u_l}{\partial p} + (\eta(\theta,p,y) + s)\left(\frac{\partial u_h}{\partial p} - \frac{\partial u_l}{\partial p}\right) + \frac{d\eta(\theta,p,y)}{dp}(u_h - u_l), \\
&= \frac{\partial u_l}{\partial p}\tilde{a}^* + (1-\tilde{a}^*)\frac{\partial u_h}{\partial p} + \frac{\partial \tilde{a}^*}{\partial p}(u_h - u_l),
\end{aligned}$$

where we have used equation (17). The first term has ambiguous sign, the second and last term are negative as $u_h < u_l$. ■

On the one hand, in case unemployment for low skilled workers increases, this tends to increase total unemployment. On the other hand, the decrease in high skilled unemployment and the reallocation of workers towards the high skilled labour force, have a negative impact on unemployment. If skill-biased technological shocks causes unemployment to increases, there may be scope for subsidizing education. However, this may only be the case if limited cognitive ability has a certain importance. We turn to this issue in the following section.

5.5 The impact of a higher subsidy

In this section we consider the impact of an educational subsidy on labour market tightness, skill level, wage dispersion, unemployment and welfare. First we examine how labour market tightness and the skill level are affected and obtain the result:

Proposition 7 *An increase in the subsidy increases labour market tightness and the number of skilled workers.*

Proof. Differentiating the system of equations $\theta(\tilde{a}^*), \tilde{a}^*(\theta)$ and s with respect to $\theta, \tilde{a}^*$ and s to obtain:

$$\begin{aligned} \frac{\partial \theta}{\partial s} &= -\frac{\frac{p}{r+d+\lambda} - \frac{1}{r+d+\lambda(p)}}{D} > 0, \\ \frac{\partial \tilde{a}^*}{\partial s} &= \frac{-2\frac{k}{q^2}\frac{\partial q}{\partial \theta} + k}{D} < 0. \end{aligned}$$

■

This policy experiment is illustrated in figure 2. A higher subsidy increases the skill level for given labour market tightness, that is, the curve representing $\tilde{a}^*$ as a function of labour market tightness shifts inwards. Labour market tightness increases, due to the better estimated perspectives of opening a vacancy. This, in turn, increases the skill level further and thereby labour market tightness. The impacts become smaller and smaller until the new equilibrium is reached.

– Figure 2 about here —

Both wages increase with the subsidy as labour market tightness increases, $\frac{\partial w_h}{\partial s} = \frac{\partial w_l}{\partial s} = \frac{1}{2}yk\frac{\partial \theta}{\partial s} > 0$. Concerning wage dispersion the result is clear cut.

Proposition 8 *An increase in the subsidy reduces wages dispersion,* $\frac{\partial w_D}{\partial s} < 0$.

Proof. Differentiating wage dispersion with respect to s gives

$$\frac{\partial w_D}{\partial s} = -\frac{(p-1)\,k\frac{\partial \theta}{\partial s}}{(1+\theta k)^2} < 0.$$

■

The increase in labour market tightness increases wages for both high skilled and low skilled workers as the workers' transition rate increases which improve their bargaining position. This causes a reduction in wage dispersion as the impact on high skilled worker wages is smaller than the impact on low skilled worker wages as the former wages are the higher. The reduced wage dispersion, implying relatively better perspectives in the unskilled sector, therefore modify the positive impact on the skill level.

In terms of the direct impact on employment and thereby unemployment, both unemployment rates fall following the increase in labour market tightness, $\frac{\partial u_l}{\partial \theta}\frac{\partial \theta}{\partial s} < 0$, $\frac{\partial u_h}{\partial \theta}\frac{\partial \theta}{\partial s} < 0$. It follows that employment rates increase for the two groups of workers. The impact on high skilled unemployment is larger than the impact on unemployment facing low skilled workers. Thereby dispersion, in terms of unemployment rates, falls. Considering total unemployment, the impact on unemployment from the subsidy is negative.

Proposition 9 *Unemployment decreases with the subsidy,* $\frac{\partial u_{TOT}}{\partial s} < 0$.

Proof. Differentiating the unemployment equation (20) gives

$$\frac{\partial u_{TOT}}{\partial s} = \left(\frac{\partial u_l}{\partial \theta}\tilde{a}^* + (1-\tilde{a}^*)\frac{\partial u_h}{\partial \theta}\right)\frac{\partial \theta}{\partial s} - \frac{\partial \tilde{a}^*}{\partial s}(u_h - u_l) < 0,$$

which is negative as $\frac{\partial \theta}{\partial s} > 0$ and $u_h < u_l$. ■

As the subsidy implies that more workers acquire skills and as the unemployment rate for skilled workers is lower than unemployment facing unskilled workers, unemployment falls. Acknowledging that workers perceive limited cognitive ability with respect to educational costs would not affect the policy conclusions in terms of unemployment reductions. However, without limited cognitive ability, there may be a trade-off in-between reducing unemployment and increasing welfare as workers have responded optimally and correctly to the productivity shock. In order to see this we first consider the impact on welfare without limited cognitive ability.

In this case, welfare reads

$$W = \int_0^{a^*} W_l da + \int_{a^*}^1 W_h da - \int_{a^*}^1 s\mu da - \int_{a^*}^1 (1-a)\, da,$$

and the number of skilled workers is determined by $1 - a^* = W_h - W_l + s$.

The impact on welfare from a higher subsidy is given by

$$\begin{aligned}
\frac{\partial W}{\partial s} &= \frac{\partial W}{\partial a^*}|_s \frac{\partial a^*}{\partial s} + \frac{\partial W}{\partial s}|_{a^*} \\
&\quad where \\
\frac{\partial W}{\partial a^*}|_s \frac{\partial a^*}{\partial s} &= -\left(W_h - W_l - s\mu - (1-a^*)\right)\frac{\partial a^*}{\partial s} = s\mu\frac{\partial a^*}{\partial s} < 0, \\
\frac{\partial W}{\partial s}|_{a^*} &= \frac{\partial W_l}{\partial s} + \left(\frac{\partial W_h}{\partial s} - \frac{\partial W_l}{\partial s}\right)(W_h - W_l - s\mu).
\end{aligned}$$

The impact on welfare through more workers acquiring skills, $\frac{\partial W}{\partial a^*}|_s \frac{\partial a^*}{\partial s}$, is negative, as this number is optimally determined through individual skill acquisition and the only impact is therefore the negative effect through administrative costs. The direct impact on welfare, $\frac{\partial W}{\partial s}|_{a^*}$ is positive given the sufficient condition, $W_h - W_l - s\mu > 0$, as a higher subsidy causes employment and wages to increase, and the total impact is the higher for highly educated workers than for low skilled workers, $\frac{\partial W_h}{\partial s} - \frac{\partial W_l}{\partial s} > 0$. The total impact on welfare is positive if $\frac{\partial W}{\partial s}|_{a^*}$ is positive and larger than the absolute value of $\frac{\partial W}{\partial a^*}|_s \frac{\partial a^*}{\partial s}$. Hence, without limited cognitive ability, welfare may increase with the subsidy due to labour market imperfections providing an additional impact on the economy through relatively more vacancies supplied and associated with this, a higher skill level.

With limited cognitive ability, then a subsidy may be welfare improving even if the impact on welfare through labour market imperfections is insignificant. The impact on welfare resulting from a higher subsidy is:

$$
\begin{aligned}
\frac{\partial W}{\partial s} &= \frac{\partial W}{\partial \varepsilon^*}|_s \frac{d\varepsilon^*}{ds} + \frac{\partial W}{\partial s}|_{\varepsilon^*} \\
& \quad \textit{where} \\
\frac{\partial W}{\partial \varepsilon^*}|_s \frac{d\varepsilon^*}{ds} &= \left(\eta_0 - \frac{1}{2} - s\mu\frac{1}{2}\right)\left(-\frac{d\varepsilon^*}{ds}\right), \\
\frac{\partial W}{\partial s}|_{\varepsilon^*} &= \int_0^1 \frac{dW_l}{ds}\left(\varepsilon^* + \frac{1}{2}\right) da + \int_0^1 \frac{dW_h}{ds}\left(\frac{1}{2} - \varepsilon^*\right) da - \int_0^1 ds\mu\left(\frac{1}{2} - \varepsilon^*\right) da,
\end{aligned}
$$

where $\eta_0 = \eta$ for $r = 0$, that is $\eta_0 = e_h w_h - e_l w_l$ and $-\frac{d\varepsilon^*}{ds} = 1 + \frac{\partial \eta_0}{\partial s} = 1 + \frac{\partial W_h - \partial W_l}{\partial s} > 0$. The first three terms, inside the first parentheses capture that the subsidy increase the skill level, $\left(-\frac{d\varepsilon^*}{ds}\right) > 0$ which has a direct positive impact on welfare as employment perspectives in the highly educated sector, $e_h w_h$ are higher than in the unskilled sector, $e_l w_l$, that is, $\eta_0 > 0$. However, more skilled workers reduce welfare, due to the implied educational costs, $\frac{1}{2}$, and administrative costs, $s\mu\frac{1}{2}$. As in the simple model presented in the first sections of the paper, this has a positive impact on welfare, that is $\frac{\partial W}{\partial \varepsilon^*}|_s \frac{d\varepsilon^*}{ds} > 0$, given the net gain from more workers acquiring skills is higher than the expected costs associated with the marginal worker acquiring skills, plus administrative costs.

The two following terms capture that welfare, both for low educated workers and highly educated, increase, $\frac{dW_l}{ds} > 0$ and $\frac{dW_h}{ds} > 0$, and the relative importance of these increases depends on how many low skilled and highly educated workers there are in the economy. Finally, the last term is the reduction in welfare, due to administrative costs, corresponding to all highly educated workers, as they all receive the subsidy.

The following two propositions summarize the result.

Proposition 10 *With limited cognitive ability an optimal subsidy, $s^o > 0$ exists.*

Proof. The optimal subsidy, $s^o > 0$, solves $W'(s^o) = 0$ assuming the second order condition is fulfilled. ■

If skill-biased technological progress provide an increase in unemployment, then the existence of limited cognitive ability eliminates the trade-off in-between reducing unemployment and increasing welfare.

Proposition 11 *With limited cognitive ability, the trade-off in-between reducing unemployment, reducing wage dispersion and increasing welfare does not exist for* $s \leq s^o$.

The result follows immediately from proposition (10),(8), (7) and (9).

6 Conclusion

This paper has focused on the case where workers may not correctly observe their own ability levels, which we denoted limited cognitive ability. Workers are therefore not able to correctly estimate their true educational costs providing an inefficient outcome in terms of skill level in the economy. A subsidy therefore potentially increases welfare even without any capital market imperfections or educational externalities. It is worth noting that we do not need to assume that individuals are underestimating their ability levels, in order obtain this result.

We then considered this feature of limited cognitive ability in relation to skill-biased technological progress. In case skill-biased technological progress result in higher unemployment, at least for the low skilled group of workers, absence of capital market imperfections or educational externalities leaves not immediately any room for policy measures if efficiency should be retained. Each individual worker should, at least, in the medium run, respond to the changed economic environment acquiring skills if that is profitable. If it is not profitable for each individual worker, welfare is reduced if workers are provided with an educational subsidy. Hence, there may exist a trade-off between reducing unemployment and increasing welfare.

However, if due to labour market imperfections or if workers are subject to limited cognitive ability, it may the case that the trade-off in-between reducing unemployment and increasing welfare cease to exist. A subsidy serves to shift workers into the skilled labour force, and may be welfare improving if with limited cognitive ability the market solution implies that too few workers acquire skills. Consequently a nontrivial optimal subsidy level exists.

It remains to be consolidated that limited cognitive ability is important. The present paper shows that limited cognitive ability indeed is a potential explanation that Europe for decades still experiences high rates of unemployment among low skilled workers even though capital market imperfections may be absent, which otherwise would be an explanation why workers do not respond to skill-biased productivity shocks. Experimental studies could shed light on the correlation in-between a person's own perception on his or her abilities and the persons true abilities. Future work is here called for.

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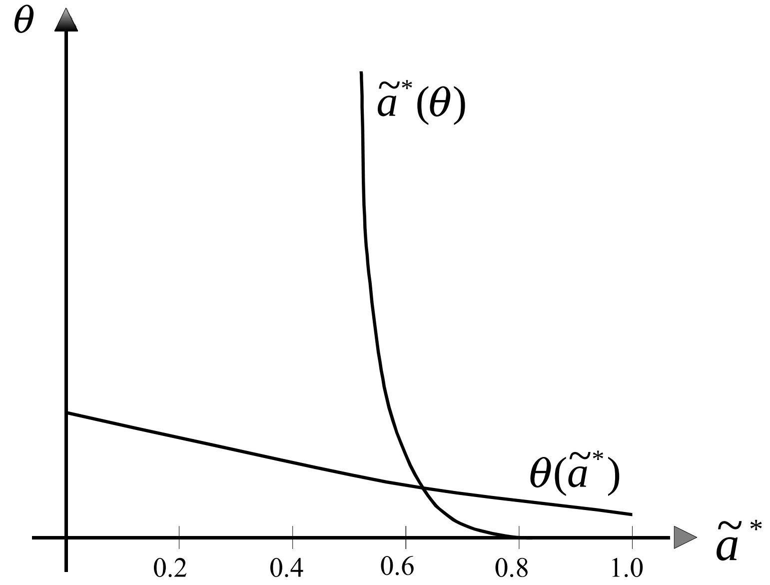
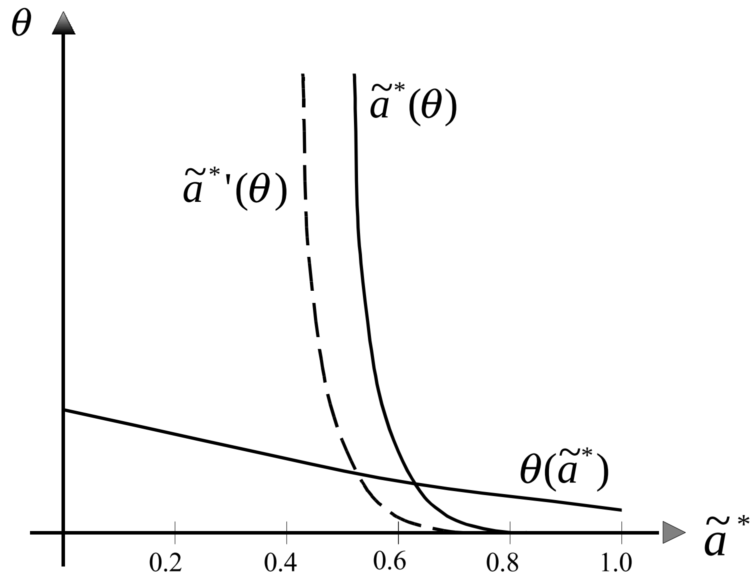
Figure 1



Equilibrium

Figure 2



The impact of a higher subsidy