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ON THE ADVANTAGES OF PIECEMEAL INTEGRATION: A model of trade with several countries where local integration benefits all

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On The Advantages of Piecemeal Integration:

A model of trade with several countries where local integration benefits all

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Abstract

For the study of economic integration, it is costumary to use a three country world, where two of the countries may introduce forms of closer economic cooperation. In the present model, we follow this tradition but put special emphasis on the role of credit and entrepreneurship. Our model is of the standard neoclassical type, with the addition that production takes time and is subject to uncertainty. Also, firms must use the financial system in order to buy inputs; the cost of credit may differ among countries and industries, reflecting their basic patterns of uncertainty.

Following the Newbery-Stiglitz approach, we show that in such model we may exhibit cases of Pareto inferior trade and, in particular, Pareto inferior economic integration. More specifically, we show that integrating countries of very different economic size may give rise to adverse effects on welfare, whereas integration of countries with a more similar economic structure and size tends to have beneficial effects for the parties.

Keywords: trade, uncertainty, Pareto inferior trade, regional integration.

JEL classification: F11, F15, F34

1. Introduction

In recent years, economic integration has been a main theme of both research and policy, and new instances of economic integration of countries into free trade areas or costums unions are presented each year, whether in connection with the expansion of the European Union or the formation of free trade areas in America.

While initially economic integration was largely a question of abolishing or diminishing tariffs within the cooperating family of countries (see the classical contributions of e.g. Viner (1953) and Vanek (1965)), the ongoing process of trade liberalization has to some extent shifted focus to other aspects of economic integration. This is supported by the experience that there is limit to what trade liberalization can do for the countries, as seen both in the unification of the two Germanies and after the formation of NAFTA. Consequently, there is a need for a better understanding of what is actually being achieved in the cause of economic integration. Some of the important aspects of economic integration, namely those related to credit and investment, have been less intensively studied than those related directly to commodity trading, and the role of the institutions which are used for these purposes (financial markets, intermediaries) have only been objects of theoretical studies in later years. Therefore, adding these features to models of international trade will often produce surprising results, such as the case of Pareto inferior trade presented by Newbery and Stiglitz (1984).

In order to have financial markets in the model, it must contain a time dimension, running over at least two periods of time. And if the credit institutions are to be of non-trivial nature, there should be some form of uncertainty in the first period about outcomes in the second period. This means that we need a model of international trade under uncertainty, and presumably with less than full information; as it emerges, information asymmetry is not needed to obtain the conclusions, which follow from the structural properties of the model alone.

Models of international trade under uncertainty is of course no novelty, they have been used actively at least since the seminal work by Helpman and Razin (1978). However, in the model to be developed the emphasis is on the financial markets and their impacts on commodity allocation. In particular, we consider situations where financial markets prevent small (in a sense to be made precise) countries from reaping benefits of economic integration.

The model follows the tradition of international trade theory in dealing with a world where two commodities are produced from two inputs, using a technology which is known to everybody. However, we depart slightly from tradition in allowing for entrepreneurship, even if in a very crude way: There is a given number of firms or entrepreneurs, but an entrepreneur may choose to produce either of the two commodities, and, in connection with an economic integration, to operate the firm in another country; thus the distribution of firms producing one or the other commodity is an equilibrium outcome of the model. The point of this way of introducing entrepreneurship is that the progress expected from economic integration would be that domestic firms get a larger share of world production, rather than that foreign firms establish branches in the country to exploit cheap production factors.

We study equilibria under different assumptions on the rules for international cooperation. Throughout the paper, we assume that there is free international trade in commodities, while factors of production cannot be traded between countries; as mentioned above, firms of one country may however operate in another country, at least under suitable regimes of economic integration, thereby achieving results which are similar to moving the factors of production. In the definition of equilibria, we assume that consumers choose savings under assumptions on the (random) yield of bonds which are rational in the sense that they are confirmed by actual performance.

The markets for bonds issued by firms of a given country play an important role, and since the performance is determined by the outcome of the firms which belong to the country, their repayment rates are expressions of the country risk which again has to do with the economic size of the country; since output shocks are independent, the variance of output produced by firms of that country will be smaller, the larger its number of firms, and the lower will be the repayment rate at which the bond will be demanded by the consumers. Thus, country risk is connected with economic size or strength, and an economic integration which allows for operation of one country's firms in another country, while retaining country-specific bond markets, may result in a situation where the smaller country's firms close down while its factors of production are employed in firms belonging to the larger country. This is the case of disadvantageous integration; it does not arise when the participating countries are of comparable size.

The paper is organized as follows: In Section 2, we introduce the model, which is a standard simple trade model over two periods and with uncertainty. The uncertainty comes from independent and identically distributed output shocks in the firms, and it affects second period output and consumption as well as the payoff of the bonds used to finance the investment in production factors needed to produce the outcome. In the following section, we consider the special case of constant returns to scale in production; here equilibrium profits are zero, and the workings of the model become more transparent, but on the other hand it fails to illustrate the role of the entrepreneur, which does not then play an essential role. In Section 4, we treat the more general case of nonincreasing returns to scale, opening up for nonzero profits as well as a choice of industry by entrepreneurs. Having treated the basic features of the model, we may then turn to what was its purpose, namely the study of economic integration, which is done in Section 5; here we show that in certain cases (namely such where all other, trade-related, effects of integration are netted out) integration between economically strong (in the sense of having many firms) and weak (few firms) countries will have unfavorable effects for the latter, whereas economic integration by more evenly situated countries will be welfare improving for both. Section 6 contains some concluding comments; the proof of the equilibrium existence result stated in Section 3 is given separately in an appendix.

2. The model

We consider a model of trade based on the classical Heckscher-Ohlin-Samuelson model with two produced commodities and two productive factors. We shall assume that there are *three* (and not two, as is usual in trade models) countries, since we shall consider a situation where two countries integrate while leaving the third outside. The countries are assumed to be identical with respect to producers' and consumers' characteristics but they may differ in size. Although our model is deliberately kept simple, it still contains several specific details, which are introduced below, where we proceed from producers over consumers to a description of the financial institutions in each country.

Production. Each commodity y_i, $i = 1, 2$, is produced separately in a constant-returns-to-scale technology using the two production factors z_1 and z_2 as inputs; the technology of a firm producing commodity i (available to producers in all countries) is given by production functions

$$y_i = \tau_i f_i(z_{1i}, z_{2i});$$

here τ_i is a random variable which for simplicity is assumed to take only two values, namely $\tau_i = 1$ ("success") or $\tau_i = 0$ ("failure"). The production functions f_i are assumed to satisfy the usual conditions of monotonicity, continuity and quasi-concavity.

We assume that there is a continuum of firms in each country. The firms choosing to operate in industry i are connected in chunks or "conglomerates" of size 1, for which the random variables are perfectly correlated, whereas the random shocks are uncorrelated between conglomerates. We may think of the conglomerates as industrial centers which are subject to the same random disturbance; the construction is made to allow for choice of technique without introducing nonconvexities which may prevent existence of equilibria. The size of the productive sector in country h is $n^h \in \mathbb{N}$, $h = 1, 2, 3$, representing the endowment with entrepreneurial skills of this country; individual firms are indiced by the letter ν.

Production is assumed to be time-consuming, so that inputs are chosen at time $t = 0$, while output is obtained at time $t = 1$. Firms are owned by consumer-entrepreneurs in the country who hold no other endowments, so that the purchase of inputs at $t = 0$ has to be financed by credits, to which we return shortly. We assume that the choice of the entrepreneur also involves the industry in which to produce; thus, in our model, firms may engage in production of any of the two commodities; however, the total number of firms in the country, n_h, is assumed fixed, determined by the availability of entrepreneurial skills in the country.

Consumption. Consumers that are not entrepreneurs will be referred to in the sequel as "factor owners"; they have an endowment $\omega = (\omega_1, \omega_2)$ (the same for all) of factors of production. Consumption takes place only at $t = 1$, so consumers sell their endowment to the firms at $t = 0$ and save the income using the available financial institutions. We assume that all consumers (entrepreneurs as well as factor owners) have identical von Neumann-Morgenstern utility functions $u(x_1, x_2)$ depending on the consumption of the two commodities (at $t = 1$), and homogeneous of degree one. At $t = 1$, consumers dispose of their incomes, which are either derived from profits of firms or are repayments of the savings, in order to buy the two commodities at (state dependent) prices (p_1, p_2) which are the same in all countries (free trade of commodities is assumed throughout). Since factor owners in any country differ only in endowments, and since they have homogeneous utility functions, by a classical aggregation result, the aggregate behavior is the same as if there was only one factor owner in each country, which we assume in the sequel.

Financial intermediation. In order to link consumer savings with firms' borrowing, we need a financial intermediary. We assume that these financial intermediaries take the form of coalitions of borrowers (firms), issuing bonds at repayment rate R_i^h, depending on country h and industry i; thus, bonds can be distinguished only by these characteristics, not by the individual debtors.

Since we deal with a model where production is subject to random shocks, the individual borrower may at $t = 1$ be unable to fulfill the obligations contracted at $t = 0$. Since we restrict ourselves to a two-period model, where the financial intermediaries cease to exist at $t = 1$, we assume that the shocks are reflected in the payoffs to the holders of bonds, so that if only a fraction e_i^h of the total number of firms in country h, industry i, succeed in producing nonzero output, then the return on the bond is $e_i^h R_i^h$. This feature makes the bonds of our model look somewhat unusual, but basically we are only taking into account the possibility of default on bonds.

Equilibrium. We have now outlined our model and may proceed to describe equilibria in this model. The equilibrium conditions must specify individual optimizing behaviour by consumers and firms (or, equivalently, by factor owners and entrepreneurs), together with clearing of markets for factors, commodities, and bonds; finally, there is an additional condition related to the number of firms in any industry and country. We start with a more informal description of the details and then give the precise definition.

By the nature of the uncertainty in our model, there is a finite set of possible states of nature at $t = 0$, characterized by the value of τ_i at each chunk $[0, 1],]1, 2], \ldots,]n^h - 1, n^h]$ of potential enterprises in industry i of country h, $i = 1, 2$, $h = 1, 2, 3$. The actual distribution of enterprises among industries is denoted $n = (n_1^h, n_2^h)_{h=1}^3$. Given this structure, the factor owner faces the problem of choosing an optimal portfolio of bonds, so that expected utility of the bundles bought using the bond yield is maximal. If factor prices at $t = 0$ are (q_1^h, q_2^h) and commodity prices at $t = 1$ and in state s are $(p_1(s), p_2(s))$, then the problem to be solved by the factor owner in country h is to choose consumption bundles $(x_1(s), x_2(s))$, for all $s \in S$, portfolio weights θ_i^h for bonds of type (h, i), and savings s^h such that

$$\max \mathsf{E}u(x_1^h(s), x_2^h(s))$$

subject to

$$p_1(s)x_1(s) + p_2(s)x_2(s) \le s^h \sum_{h=1}^{3} \sum_{i=1}^{2} \theta_i^h [e_i^h(s) R_i^h],\ s \in S, \tag{1}$$

$$s^h \le q_1^h \omega_1^h + q_2^h \omega_2^h.$$

For the entrepreneur, the problem to be solved is similar to (1), only here the income used for buying commodities at $t = 1$ and in state s is obtained not from bond yields but from the profits of operating the firm. Thus, the entrepreneur operating firm ν of industry i in country h has to choose inputs (z_{i1}^h, z_{i2}^h) at time $t = 0$ and consumption bundles $(x_{i1}^h(s), x_{i2}^h(s))$ solving

$$\begin{aligned} &\max \mathsf{E}u(x_{i1}^h(s), x_{i2}^h(s)) \\ &\text{subject to} \\ &p_1(s)x_{i1}(s) + p_2(s)x_{i2}(s) \leq \\ &\qquad\qquad \max\{0, \tau_{i\nu} f_i(z_{i1}^h, z_{i2}^h) - R_i^h(q_1^h z_{i1}^h + q_2^h z_{i2}^h)\}, \end{aligned} \tag{2.i}$$

where the numbering (2.i) has been used to stress that the problem described relates to industry i. It should be noticed that the problem to be solved is the same for all firms in industry i and the solution is therefore independent of ν; we denote by U_i^h the optimal value of the objective function in (2.i).

Given factor supply in each country as described by (1), commodity demand at $t = 1$ and $s \in S$ as defined by (1) and (2), and commodity supply as determined by factor inputs and random shocks, there is equilibrium if (a) supply equals demand in each market (factor markets at $t = 0$ and commodity markets at $t = 1$, $s \in S$), and, furthermore if (b) the industry chosen by each firm is the best possible, that is the one which gives the maximal expected utility of final consumption as described in (2.i).

Now we may summarize the above discussion in the following definition.

DEFINITION 1. *An array*

$$(n, (x^h(s))_{s\in S}, (z_i^h, (x_i^h(s))_{s\in S})_{i=1}^2, q^h, \pi^h)_{h=1}^3, (p(s))_{s\in S})$$

is an equilibrium if the following conditions are fulfilled:

(1) for the factor owner of country h, $(x^h(s))_{s\in S}$ solves the problem (1),

(2) for each entrepreneur in industry i of country h, $(z_i^h, (x_i^h(s))_{s\in S})$ solves the problem (2.i), yielding the optimal value U_i^h,

(3) for each country h,

$$\sum_{i=1}^2 n_i^h z_i^h = \omega^h$$

and for each state $s \in S$,

$$\sum_{h=1}^3 [x_i^h(s) + n_i^h x_i^h(s)] = \sum_{h=1}^3 \int_0^{n_i^h} \tau_{i\nu} f_i(z_i^h)\, d\nu,\ i = 1, 2,$$

(4) for each h, (n_1^h, n_2^h) maximizes $\lambda_1 U_1^h + \lambda_2 U_2^h$ over all $(\lambda_1, \lambda_2) \in \mathbb{R}_+^2$ with $\lambda_1 + \lambda_2 = n^h$.

Since the equilibrium even in this model, even with our simple structure of identical consumers and producers, is quite complex, the existence problem is nontrivial and needs to be addressed. We present the existence result below; its proof is given in an appendix to the paper.

THEOREM 1. *Assume that in the model introduced above, the following assumptions are satisfied:*

(1) the common von Neumann-Morgenstern utility function u is continuous, monotonous, quasi-concave, and homogeneous of degree one,

(2) the production functions f_i are continuous, monotonous, and concave.
Then there exists an equilibrium.

Moreover, if both industries operate in each country, then their bond repayment rates are equal.

The proof of Theorem 1 is given in the appendix. Since the model is constructed using standard assumptions, the crucial assumptions are those which take care of the particular features introduced, in particular the perfect foresight of future prices, which to be meaningful presupposes uniqueness of equilibrium prices in each period 2 state, which also figures among our assumptions.

Since bond repayment rates depend only on countries and not on industries, we may suppress reference to industry in the sequel.

We pause briefly at this point to consider the case of constant returns to scale in production. In that case, expected profits are 0 in equilibrium, and as a consequence the structure of trade will resemble that of the classical Heckscher-Ohlin model.

3. The case of constant returns to scale in production

If production functions exhibit constant returns to scale, then the producers' choices of optimal levels of inputs – and hence of credits – becomes somewhat simpler to analyze. It seems natural that this case is used as a starting point, even if – as it will emerge from the analysis – the conclusions are not as immediately appealing as with the decreasing returns to scale.

The key to the results which we obtain under this assumption on the technologies is given by the following simple observation, formulated as a lemma.

LEMMA 1. *Assume that the production functions f_1, f_2 are positively homogeneous of degree one. Then in equilibrium, ex ante expected profits are 0 for all active firms (that is all firms choosing nonzero input levels).*

PROOF: Clearly negative expected profit implies negative expected utility, so that the firm would prefer inaction to production. Assume that ex ante profits were positive in a firm of type i in country h, that is

$$\mathsf{E}\left[\tau_{i\nu} f_i(z_{i1}^h, z_{i2}^h) - R_i^h(q_1^h z_{i1}^h + q_2^h z_{i2}^h)\right] > 0,$$

where expectation is taken over all states of nature s (which influence not only production in the firm but also period 1 prices). Then the entrepreneur might obtain higher expected utility (see expression (2.i) above) by increasing inputs, thus contradicting equilibrium. Consequently, expected profits must be 0 under constant returns to scale. □

While firms have identical profits, namely 0, in both industries, this does not imply that the industry bonds have the same payoff; intuitively, the repayment rate of the country bond of industry i reflects not only expected payoff in the industry but also the risk. On the other hand, the case of identical repayment rates inside each country is sufficiently interesting as a benchmark situation, since in this case the model will show strong resemblance to the classical Heckscher-Ohlin-Samuelson model of trade, displaying factor price equalization under the usual assumptions on production technologies. We formulate this as a proposition.

PROPOSITION 1. *Assume that the production functions f_1, f_2 exhibit constant returns to scale and satisfy the assumption of "no factor intensity reversal". Assume further that $R_1^h = R_2^h$ for each h, that $n_1 > n_2 \geq n_3$, and that both commodities are produced in country 1. Then*

(i) Profits are zero,

(ii) Relative factor prices are the same in all countries, and

(iii) Per capita savings in the countries are inversely proportionate to the country bond repayment rates .

PROOF: Part (i) was proved in Lemma 1. Let p_1, p_2 be equilibrium commodity prices, and let R^1 be the equilibrium repayment rate of country 1 bonds. For $i = 1, 2$, let

$$\phi_i(q_1, q_2) = \max\{f_i(z_1, z_2) \mid q_1 z_1 + q_2 z_2 = 1\}$$

be the maximal production obtainable in industry i (before shock) using inputs of total value 1. Since both commodities are produced in country 1, we have

$$\mathsf{E}\left[p_1 \tau_{1\nu} \phi_1(q_1^1, q_2^1)\right] = \mathsf{E}\left[p_2 \tau_{2\nu} \phi_2(q_1^1, q_2^1)\right] = R^1, \tag{3}$$

where q_1^1, q_2^1 are the equilibrium factor prices. As in the standard proof of the factor equalization theorem, we have that there is a unique price ratio q_1^1/q_2^1 such that (3) can be satisfied.

Consider now a country $h > 1$. We claim that if there is a market for country h bonds, then $R^h > R^1$. Indeed, since country h has fewer firms than country 1, the variance of the average output value in country 1 is smaller than that of country h, and since the risk averse savers buy both bonds, the repayment of the high-variance bond must be higher, so that $R^h > R^1$. As country h produces both commodities at factor prices (q_1^h, q_2^h), we get an expression similar to (3),

$$\mathsf{E}\left[p_1 \tau_{1\nu} \phi_1(q_1^h, q_2^h)\right] = \mathsf{E}\left[p_2 \tau_{2\nu} \phi_2(q_1^h, q_2^h)\right] = R^h. \tag{4}$$

Since there can be only one value of relative factor prices such that the first equality in (4) is satisfied, namely the ratio q_1^1/q_2^1, we get statement (ii) in the theorem.

From the second equality we get that

$$q_1^h = \frac{R^1}{R^h} q_1^1,\ q_2^h = \frac{R^1}{R^h} q_2^1.$$

We conclude that the value of savings in country h satisfies

$$q_1^h \omega_1^h + q_2^h \omega_2^h = \frac{R^h}{R^1}(q_1^1 \omega_1^h + q_2^1 \omega_2^h),$$

which is the third part of the theorem. □

The theorem shows that under constant returns to scale – together with some standard assumption of well-behavedness – our model yields equilibrium allocations which are not too different from those of the classical trade models. There are, however, differences, some of which may be rather far-reaching. For example, while the equilibria with factor price equalization in the classical case are no different from equilibra in an integrated world economy (cf. e.g. Helpman and Krugman, 1986), this is not the case here; as a matter of fact, such equilibria may even not be efficient, in the sense that world allocation may be changed in a way increasing the utility of everybody.

We show this in the example below; here the riskiness of production differs between industries, so that a country which has many resources but few firms would benefit from specializing in producing the low-risk output. This feature is however not sustained in equilibrium.

EXAMPLE 1. Assume that the production functions are given by

$$f_1(z_1, z_2) = z_1^{\frac{1}{3}} z_2^{\frac{2}{3}},\ f_2(z_1, z_2) = z_1^{\frac{2}{3}} z_2^{\frac{1}{3}},$$

and that the three countries have aggregate endowments $\omega^1 = (2, 2)$ $\omega^2 = \omega^3 = (1, 1)$. Period 1 utility is

$$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}.$$

There is a single firm of each type in country 3, whereas there are many firms in both country 1 and 2.

It may easily be verified that in this symmetric model, in the absence of shocks there is an equilibrium where each country inserts $1/3$ of its endowment of the first factor and $2/3$

of the second in production of commodity 1, and the rest in the production of commodity 2. Without shocks, world production is $(4a, 4a)$ with $a = \frac{1}{3}2^{\frac{2}{3}} \sim 0.53$.

We now add assumption on the random shocks in production, namely that they occur only in the production of commodity 1; to simplify computations, we assume that the production function has the form

$$y_1 = \tau 2 z_1^{\frac{1}{3}} z_2^{\frac{2}{3}}$$

where τ takes the values 0 and 1 with probability $1/2$ each. Since there are many firms in countries 1 and 2, we may assume that the production function is the same as considered above in the deterministic setup. In country 3, production of commodity 1 is either 0 or twice the amount produced in the other countries with the same input combination.

Using the zero profit condition, we get that the input allocation considered above is still sustained in an equilibrium. Since the variance of country 3 investment is higher than that of the two other countries, factor remuneration is lower, even though relative factor prices remain unchanged.

The allocation is not ex ante efficient, however. It may be improved if country 3 withdraws completely from producing commodity 1, specializing in the production of the riskfree commodity 2. This will eliminate production risk, obviously at the cost of some production efficiency. However, if consumers are sufficiently risk averse, there will be an overall gain from this change.

In our model, the new allocation, with complete specialization of e.g. countries 2 and 3 in the production of commodities 1 and 2, is also an equilibrium allocation, now sustained by relative factor prices which differ significantly among countries. Moving somewhat ahead of our story, we note however that in our particular case, the inefficiency comes from the lack of firms in country 3 as compared to the factor endowment; if country 2 firms can establish themselves in country 3 – possibly as a result of economic integration opening up for direct foreign investment – then the original allocation where every country produces the commodities in the same ratio will be efficient, thus providing an example of an advantageous integration (between two of the three countries).

4. The case of decreasing returns to scale

In the previous section, we assumed constant returns to scale in production; this case is somewhat simpler to analyze than the general case of non-increasing returns to scale, but it does not produce all the features that we are interested in, since incomes are derived only from ownership of factors of production, not from entrepreneurship. To have positive profits in firms, we need non-increasing returns to scale. Therefore we assume from now on that each f_i is twice continuously differentiable and strictly concave.

Under decreasing returns to scale, entrepreneurial profits are non-zero in equilibrium if there is non-zero production; indeed, at any level of production, marginal expected revenue exceeds average expected revenue. Once firms earn non-zero profits, we have an additional source of income.

EXAMPLE 2. As in Example 1, we assume that countries have proportional factor endowments (no traditional comparative advantages) but differ in the characteristics affecting country risk; the only new feature of the present case is that production functions are concave, with the form

$$f_1(z_1, z_2) = v\left(z_1^{\frac{1}{6}} z_2^{\frac{1}{3}}\right),\ f_2(z_1, z_2) = v\left(z_1^{\frac{1}{3}} z_2^{\frac{1}{6}}\right),$$

where $v : \mathbb{R}_+ \to \mathbb{R}_+$ is an increasing concave transformation, that is v is C^2 with $v(0) = 0$, $0 \leq v' < 1$ and $v'' < 0$. We assume again that the random shock affects only one of the industries, and that the probability of 0 outcome in any firm is $1/2$. Similarly, we assume that there are many firms in country 1 and 2, and few in country 3; in the present case, we need however to specify the actual numbers, which are set to 40 in country 1, 20 in country 2, and 2 in country 3.

We start by considering the case of no random disturbances, where classical trade theory applies. Then factor proportions are 1:3 in the first and 3:1 in the second industry (independent of the level of activity); lowest cost of production in each country is obtained by dividing the relevant input combination equally among firms of each type, and due to the symmetry of the example, there should be the same number of firms operating in each industry. Thus, in countries 1 and 2, each firm in indutry 1 uses $1/30$ and $2/30$ of the total country endowment of factor 1 and 2, respectively (since there are 20 firms in this industry sharing endowment $(2/3, 4/3)$ in country 1, 10 firms sharing $(1/3, 2/3)$ in country 2), wheras firms in industry 2 use $2/30$ and $1/30$. Output per firm is $v(\frac{1}{10}\alpha)$, where as before

$$\alpha = \frac{1}{3} 2^{\frac{2}{3}},$$

and total output in the two countries is found by multiplying by the number of firms in each industry. In country 3 with only one firm, total output is $v(\alpha)$.

We note that due to decreasing returns to scale, total output could be increased either if production factors could be moved from country 3 to country 1 or 2, or if firms from these countries could operate in country 3 instead of the home country. This type of inefficiency on world scale is caused by the restrictions on movements of factors and firms and will occur also in traditional trade models; it is not related to the presence of uncertainty.

Since the symmetric allocation described above is efficient (under the given restrictions), it may be achieved as an equilibrium for suitable consumer preferences, for example if their von Neumann-Morgenstern utility functions are symmetric in the two consumption goods. In this case, expected profits will be equal in the two industries so that no firm has an incentive to change its industry of operation.

Adding now random shocks in the production of commodity 1, we get as in Example 1 that country 3 bonds must have a higher repayment rate to offset the difference in variance as compared to the bonds of the other countries. However, the idea of improving world allocation by specialization of country 3 in the production of commodity 2 needs some qualification, since now the decreased volatility of output has to be balanced against the loss of productivity, since the single firm in country 3, which was already "too big" in the initial equilibrium, has now doubled its inputs. Thus, the advantages of integration have to be balanced against disadvantages arising from other sources.

Under decreasing returns to scale, entrepreneurial profits play a role, and their relative size will depend on the level of activity in the firms of the country. The presence of nonzero entrepreneurial remunerations is a feature which will play an important role when we turn to the effects of economic integration.

5. Advantageous and disadvantageous integrations

Having developed the model in the course of the two previous sections, we may now put it to the use for which it was designed, as outlined in the introduction, namely that of economic integration, partial or full. In our model with unrestricted free trade in output commodities, economic integration must take one of the following forms:

(1) integrated factor markets ("unrestricted movement of factors of production"),

(2) integrated financial markets (one common bond, and consequently one repayment rate, for the countries involved), and

(3) right of establishing a firm in any of the countries involved.

While the first two aspects of economic integration may suggest themselves from the very logic of this (and other) models of international trade, the third form of economic integration may indeed be much closer to real world phenomena. Indeed, the typical result of closer economic cooperation, as experienced e.g. in the European Union, is that production is moved from countries with high factor prices to low cost countries, while the basic structure of the firm, including its methods of obtaining finance for investments, is retained. In the following, we concentrate on this type of economic integration which is little investigated in the literature and which also produces some rather striking results. Thus, in the following we mean integration of type (3) whenever we speak about economic integration.

Formally, if countries h and k belong to a group of countries having chosen to integrate their economies, then any firm of country h may choose to operate in country k (and vice versa). Given such a choice, the firm will *use the bond market of its country of origin* and the factor market in the country of operation. Consequently, we use double index hk for the variables pertaining to such a firm.

Economic integration understood as perfect firm mobility makes it possible for a firm to exploit differences in credit availability *and* in factor price levels: If its home country has a large number of firms, and consequently small variance in output, its firms have access to credit with low rate of repayment. Moving to a country with fewer firms and more expensive credits, it can offer the factor owners of that country better prices than the domestic firms. This will wipe out the domestic firms, so that entrepreneurship and the profits which are derived from it is transferred from the small country to the large one, producing a case of disadvantageous economic integration.

PROPOSITION 2. *Assume that the large country 1 and one of the small countries h are integrated. If the probability of failure is non-zero in both industries and the ratio n^1/n^h is sufficiently large, then in the integrated equilibrium, the following holds:*

(i) $(z_1^{hk}, z_2^{hk}) = 0$ for $k = 1, h$ (firms with origin in country h do not produce),

(ii) $(q_1^h, q_2^h) = (q_1^1, q_2^1)$ (factor prices are equalized between the two countries),

(iii) if countries have proportional factor endowments, then expected world production has decreased and its volatility has increased.

PROOF: If a firm belonging to country h operates in country h, then its marginal cost at $(0,0)$ equals expected marginal revenue. Clearly, marginal cost of the firm with origin in h exceeds that of the firm from country 1 by a factor R_i^h/R_i^1. Since this factor depends on the relative volatility of output in the respective countries, it grows beyond limits as n^1/n^h increases. Since the marginal productivities are upper bounded at 0, we get that marginal cost will exceed marginal expected revenue for large enough n^1/n^h, and (i) follows.

To see that (ii) holds, we need only notice that the firms of country 1 now have access to the aggregate factor endowment of country 1 and h, and since n^1 is large, there is no problem of indivisibility of firms, meaning that the firms may choose to establish in countries 1 and h in such a way as to realize a cost minimum for producing total output, meaning that production and consumption takes place as if the countries were fully integrated into one. As a consequence, factor prices are indeed equal.

For (iii), we notice that since integration results in fewer firms, equilibrium input per firm has increased, meaning that expected output decreases due to decreasing returns to scale. The increased volatility is also an obvious (but possibly surprising) consequence of the reduction of the number of operating firms. □

The welfare reducing effects of integration obtained in the Proposition should not be generalized beyond the setup for which it was derived; we considered a case where countries were similar in the traditional sense of no comparative advantages in factor endowments. Such comparative advantages might well lead to different results, since the integrated use of ressources might outweigh the decrease in output and the increase in risk due to smaller number of productive units.

Even in the case covered by the Proposition, we cannot be sure whether the gains or losses are spread evenly or unevenly among countries, since the loss of entrepreneurial income in country h is partially offset by better remuneration of factors. That the latter effect is not always big enough can be seen from the next example.

EXAMPLE 3. We use here the model considered in Example 2, with random shocks now affecting the production of both commodities in the same way, and with consumers having utility functions that are symmetric in the consumption of the two goods. We consider an economic integration between countries 1 and 3; this means that firms of both countries can establish production in any of the two countries. By Proposition 1 we obtain that country 3 firms must be inactive, so that all incomes in country 3 are derived from sale of factor endowment.

Whether country 3 as a whole has become better off as a result of the arrangement depends therefore on the increase in factor prices. We can check what happens in our simple model; here we assume a very specific functional form of v, namely

$$v(r) = \min\{a + br, cr\},$$

where $a, b, c > 0$, $b < 1$, $c > 1$, and $a < \frac{1}{10}\alpha$ (here α is the parameter used in Examples 2 and 3). Thus, the concave transformation v is the minimum of two affine functions, having a kink at the value $\bar{r} = (c - b)/a$.

Since the two commodities enter the utility function in symmetrical way and their production is open to the same kind of random shocks, the allocation of factors of production will be the same as with no uncertainty, with the difference that there are now 10 firms operating in each country (all of which have their origin in country 1). Each of these produce $v\left(\frac{1}{10}\alpha\right)$ of the relevant commodity; if $\bar{r}$ is small enough, more specifically if $\bar{r} \leq \frac{1}{10}\alpha$, then marginal products and hence factor prices remain constant at input levels equal to and above those of the equilibrium, including the level corresponding to pre-integration production in the firms of country 3. Consequently, factor remuneration does not change, whereas entrepreneurial profits disappear in country 3 after integration, and it follows that the country has become worse off.

While in our examples disadvantageous integration occurs due to differences in country risk, we may similarly encounter cases of advantageous integration when country risk is of comparable size.

EXAMPLE 4. To give an example of advantageous integration, we modify the previous example by setting the number of firms of each type to 1, so that countries 2 and 3 become absolutely identical. However, integration makes specialization possible: Each country may specialize in a particular industry, in the sense that its firms produce only one commodity, with one of them operating in the other country in order to use the factor endowments efficiently. This will reduce country risk in each industry, allowing for cheaper credit, larger production, and a greater profit share in both countries.

Summing up, in the model considered the preconditions for successful integration (where integration is considered as free choice of localization of the firms of the countries involved) is that the partners must not be too unequal, since the economically stronger part – where strength is measured as availability of entrepreneurial skills – will reap the benefits while the weaker part may suffer a loss. On the other hand, integration of the small countries may be beneficial to all countries involved.

If the final goal is a fully integrated world economy, it may be argued that the path towards this goal, whether achieved through integrating weak countries with the strong country one by one or integrating the weak countries first and then proceed to the final full integration, must be of minor importance, and indeed this is the case in our model, where the equilibria in the fully integrated economy are independent of the way in which the integration was achieved. On the other hand, only small changes are needed for this path to make a difference: If we assume that firms which do not produce cannot be opened again, then indeed the approach to world integration will matter very much, giving preference to the integration of countries of equal economic development rather than of countries with differing economic potential. A full treatment of a model where the number of firms in each country is subject to change is however beyond the scope of this paper.

6. Concluding comments

In the model considered in the previous sections, we have analyzed the patterns of trade and the results of economic integration in a simple world with three countries producing two commodities from two factors. The initial situation was one of liberalized commodity trade. This does not however mean that there is no scope for further integration of economic activity. Once we add uncertainty, even in its simplest possible form, namely as an output shock acting independently on each firm, we may consider more or less liberalized regimes of investment financing, and we may consider the possibility of a firm operating in another country, the latter being one of the most widespread real world features of economic integration, encountered in the movement of several industries from North to South in the European Union as well as the *maquiladores* industries in Mexico following after the NAFTA agreement.

The paper sets out to analyze the workings of this type of integration in a context of standard trade theory, minimally extended so that questions of financing investment and change of localization of firms make sense. It should of course. Also, an underlying assumption is that all investment must be financed via the bond market, with no possibility of state-dependent contracting.

Given these shortcomings, it is however worth noticing that the results of the models are quite different from those of standard trade theory, since welfare gains are to be expected when countries are not too different and losses, at least for the weaker part, in the case where the countries are very different. This may be reassuring in view of the stylized facts about trade which seems to be flourishing more between equals than between very different countries, cf. the early contribution by Burenstam Linder (1961), but it should be remembered that we are considering gains and losses relative to a situation of perfectly liberalized commodity trade.

The impacts for current international economic cooperation would be that this should be encouraged particularly for similar countries, while there is reason for some scepticism as to the benefits of integration between unequal partners. Whether this recommendation is valid in a more general, and therefore more realistic setting, is however something which has yet to be investigated.

7. Mathematical appendix: Proof of Theorem 1

In this section, we give a proof of the existence of equilibria in our basic model, as stated in Theorem 1.

PROOF OF THEOREM 1: Let $\mathcal{N}$ be the set of all arrays $n = (n_1^h, n_2^h)_{h=1}^3$ with $n_i^h \in \mathbb{Z}_+$ and $n_1^h + n_2^h = n^h$. The set of allocations $(x^h(s))_{s\in S}, (z_i^h, (x_i^h(s))_{s\in S})_{i=1}^2)$ which may occur in an equilibrium is bounded, so performing a standard truncation operation on feasible sets we may assume that each factor owner chooses final consumption from the set $X_1 = ([0, M]^2)^S$ and each entrepreneur chooses from $X_2 = ([0, M]^2 \times X_1$. We denote by Θ the set of all portfolio weights $(\theta_1^h, \theta_2^h)_{h=1}^3$. We construct a family of correspondences as follows:

For each country h, the demand correspondence of the factor owner $\xi_0^h : \mathbb{R}_+^2 \times [0, M]^6 \times (\mathbb{R}_+^2)^S \to X_1 \times \Theta$ assigns to each array $(q^h, (R_1^h, R_2^h)_{h=1}^3, (p(s))_{s\in S})$ of factor prices, bond rates, and state-dependent commodity prices the solution to the consumer's problem (1). This correspondence is upper hemicontinuous with nonempty, compact and convex values.

For each country h and industry i we similarly define the demand correspondence ξ_{ij}^h : $\mathbb{R}_+^2 \times [0, M]^6 \times (\mathbb{R}_+^2)^S \to X_2$ by assigning to each array $(q^h, (R_1^h, R_2^h)_{h=1}^3, (p(s))_{s\in S})$ the solution of problem (2.i). Again, each ϕ_i^h is upper hemicontinuous with compact and convex values.

To obtain market clearing prices, we define price correspondences ψ^h for each country h and commodity price correspondences $\psi[s]$ for each state s as follows: For each allocation $\xi = ((x^h(s))_{s\in S}, (z_i^h, (x_i^h(s))_{s\in S})_{i=1}^2)_{h=1}^3$, let

$$\psi(\xi) = \Big\{ ((\tilde{q}^h)_{h=1}^3, \tilde{p}(s)_{s\in S}) \in \mathbb{R}_+^6 \times (\mathbb{R}_+^2)^S \,\Big|\, \tilde{q}^h \cdot \Big[\sum_{i=1}^2 n_i^h z_i^h - \omega^h \Big] > 0, h = 1, 2, 3,$$

$$\tilde{p}(s) \cdot \sum_{h=1}^3 \Big[x^h + (n_1^h x_1^h, n_2^h x_2^h) - \Big(\int_0^{n_1^h} \tau_{1\nu} f_1(z_1^h)\, d\nu, \int_0^{n_2^h} \tau_{2\nu} f_2(z_2^h)\, d\nu \Big) \Big] > 0,\ s \in S,$$

$$\sum_{i=1}^3 (\tilde{q}_1^h + \tilde{q}_2^h) + \sum_{s\in S} (p_1(s) + p_2(s)) = 1, \Big\}$$

then ψ has open graph and convex (possibly empty) values.

The next correspondence serves to set the repayment rates R_i^h; define α as the correspondence taking portfolio weights $\theta = (\theta_i^h)$ and allocations into possible bond repayments,

$$\alpha(\xi, \theta) = \left\{ (\tilde{R}_1^h, \tilde{R}_2^h)_{h=1}^3 \,\Big|\, \sum_{h=1}^{3} \sum_{i=1}^{2} \tilde{R}_i^h [\theta_i^h s^h - q^h \cdot (n_1^h z_1^h, n_2^h z_2^h)] > 0 \right\}.$$

Again, α has open graph and convex, possibly empty values.

The final correspondence is used for obtaining the equilibrium choice of n: For each allocation ξ, denote by $U_1^h(\xi)$ and $U_2^h(\xi)$ the expected utility of the factor owner and the entrepreneur in country h, and let $\beta^h(n, \xi)$ be given by

$$\beta^h(n, \xi) = \{(\lambda_1^h, \lambda_2^h) \in \mathbb{R}_+^3 \mid \lambda_1^h U_1^h + \lambda_2^h U_2^h > n_1^h U_1^h + n_2^h U_2^h, \lambda_1^h + \lambda_2^h = n^h\}.$$

As before, β has open graph and convex, possibly empty values.

Now we collect all the above correspondences into one correspondence ψ which takes allocation-price pairs

$$\chi = (((x^h(s))_{s \in S}, \theta^h, (z_i^h, (x_i^h(s))_{s \in S})_{i=1}^2, q^h, (R_1^h, R_2^h))_{h=1}^3, (p(s))_{s \in S})$$

into themselves; here the allocation components of ϕ are given by the correspondences ψ_0^h for commodity and portfolio demand of factor owners, ψ_i^h for the demand for factors and commodities of entrepreneurs, ψ^h for factor prices, $\psi[s]$ for commodity prices in state s, α for bond repayment rates, and β for . The components of ψ_n are either upper hemicontinuous with closed, convex and nonempty values, or they have open graph and convex values.

We now apply the fixed point theorem used in Gale and Mas-Colell (1975) to give the existence of an allocation-price pair

$$\hat{\chi}_n = (((\hat{x}^h(s))_{s \in S}, \hat{\theta}^h, (\hat{z}_i^h, (\hat{x}_i^h(s))_{s \in S})_{i=1}^2, \hat{q}^h, (\hat{R}_1^h, \hat{R}_2^h))_{h=1}^3, (\hat{p}(s))_{s \in S}, \hat{n})$$

such that

- for each h, the choice of the factor owner in country h, $(\hat{x}^h(s))_{s \in S}, \theta^h)$, belongs to $\psi_0^h(\hat{\chi}_n)$, meaning that it satisfies (1),
- for each country h, industry i and firm j, $(\hat{z}_{ij}^h, (\hat{x}_{ij}^h(s))_{s \in S})$ belongs to $\phi_{ij}^h(\hat{\xi}_n)$, so that it satisfies (2.i); it follows now that
- since the solutions to (1) and (2.i) satisfy Walras' law, we have that

  $$((\hat{q}^h)_{h=1}^3, (\hat{p}(s))_{s \in S}) \notin \psi(\hat{\chi}_n),$$

  so that $\psi(\hat{\chi}_n) = \emptyset$, which gives that demand does not exceed supply in any factor or commodity market;

– similarly we have that $\alpha(\hat{\chi}_n) = \emptyset$, so that loans in country h, industry i do not exceed demand for bonds of this type, and
– $\beta(\hat{n}, \hat{\chi}) = \emptyset$.

It is easily checked that the allocation-price pair with all these properties is an equilibrium, and we have thereby showed that equilibria exist. □

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