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A Safety Stock Adjustment Procedure to Enable Target Service Levels in Simulation of Generic Inventory Systems

Connie Køhler Gudum[1]

Copenhagen Business School, Denmark

connie@cbs.dk

Ton G. de Kok

Eindhoven University of Technology, The Netherlands

a.g.d.kok@tm.tue.nl

Abstract

The aim of this paper is to present a technique, called the safety stock adjustment procedure (SSAP), which enables the determination of safety stocks that ensure target service levels in simulation studies of inventory systems. The technique is based on a netting procedure constructed so that the net requirement process and the replenishment process are independent of the safety stock and that the inventory process satisfies an invariance relation. The procedure is presented for three kinds of service measures; namely the cycle service level, the fill rate and the ready rate. In a numerical example the benefits of using the safety stock adjustment procedure are shown. In this example three well-known lot size models are compared assuming stochastic and time-varying demand. Moreover, we propose the safety stock adjustment procedure to be used in practical situations to set safety stock levels in companies for instance when demand is non-stationary.

Keywords: Simulation, Inventory, Safety Stocks, Service Levels, Lot Sizing.

[1] Corresponding author: Connie Køhler Gudum, Department of Management Science and Statistics, Copenhagen Business School, Solbjerg Plads 3, DK-2000 Frederiksberg, Denmark. Tel: +45 3815 3509. Fax: +45 3815 3500. Email: connie@cbs.dk

1. Introduction

When incorporating uncertainty into simulation studies of inventory systems stockouts can occur. Therefore the evaluation of performance should include the impact of stockouts on customer service. One way of dealing with this is to assume some penalty cost for shortages. Penalty costs, however, are hard to determine. Another way is to compare different inventory management scenarios under the constraint of some target customer service level.

A number of approaches to obtain a certain specified service level in simulation studies are suggested in previous studies. Eilon and Elmaleh (1968) perform simulations, which give a number of exchange curves with different service levels and different costs. To compare under similar service levels the final costs from the exchange curve are interpolated. Since this kind of exchange curve is non-linear the interpolations have to be based on co-ordinates relatively close to each other. The drawback of this approach, therefore, is that it requires a possibly large number of co-ordinates of service and cost on the exchange curve.

In the studies of Callarman and Hamrin (1979, 1984) cost comparisons are made by introducing a safety stock at each run to keep service levels at 95% and 98% respectively. The authors determine the necessary amount of safety stock by using a so-called Service Level Decision Rule (SLDR), which has been developed by Callarman and Mabert (1978). The SLDR is based on linear regression analysis on simulated values of a specific set of experimental factors, which are the forecast error, the coefficient of variation of demand and the expected time between orders. To achieve the desired service level the SLDR is used with a search routine.

Wemmerlöv and Whybark (1984) calculate net requirements based on allowing backorders and Wemmerlöv (1986) calculates net requirements based on lost sales. In both studies cost comparisons are made with a service level of at least 99.999%. To determine the necessary amount of safety stock, a search routine is used by repeating the simulation, until the target service level is reached. The service measure used in these two studies is the fraction of demand satisfied directly from stock, which later in this paper will be referred to as the P_2 - service measure.

These approaches however demand either a large number of simulations to get enough data to make accurate interpolations, the use of a search routine or a regression analysis on simulated values of a specific combination of experimental factors, which all complicates the simulation. Moreover, none of these papers exploit or show the fact that the netting procedure proposed in this paper leads to safety stock independence.

This paper therefore proposes a straightforward technique, called the safety stock adjustment procedure (SSAP), to obtain target service levels in simulation studies. The technique is based on the assumption that a Time Phased Order Point (TPOP) policy is applied (Orlicky (1975)) to derive an unconstrained Master Production Schedule meaning that lot sizing, material and capacity constraints are not yet taken into account. By applying some algorithm, the Master Production Schedule (MPS) is then modified taking into account all relevant constraints.

The general idea is that the forecasting procedure is independent of the netting procedure, which is independent of the lot sizing procedure. The information flow in the TPOP policy can be represented by figure 1.

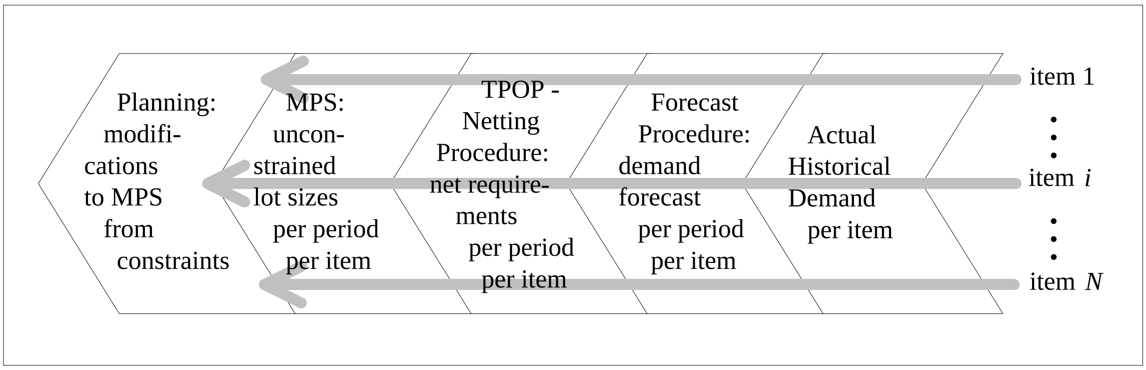

Figure 1. Information flow

This upstream information is determined for all items in the system, however the procedures work on item level until information is gathered in the planning engine, which then accounts for capacity, lot sizing and other restrictions. Hence, the seemingly complicated problem of setting safety stocks under stochastic demand for multiple end items that interact, due to usage of common materials and resources, is converted into single item problems. Therefore the notation in the remainder of the paper will not include a subscript for the item number.

Furthermore, the technique is based on discrete event simulation and exploits an invariance relation derived from a sample path analysis. There need not be any knowledge about the theoretical demand distribution or any specific forecasting model. The proposed TPOP netting procedure ensures that the net requirement process and the replenishment process are independent of the safety stock, and also that the inventory process is invariant to the safety stock. Based on this independence and the invariance relation, a technique to adjust the safety stock has been developed which ensures that comparison of performance of different planning and control scenarios is made under identical service levels. An essential assumption of the approach is that all excess demand is backordered, otherwise the important property of safety stock independence can not be maintained. The technique is very generic, though, and it can be applied to any rolling schedule concept including finite capacity constraints, variable lead times, multi-product and multi-echelon inventory analysis. Moreover, since the approach does not assume any specific demand distribution it can be applied in practice when the theoretical demand distribution is either unknown or difficult to adapt to traditional inventory models. To illustrate the benefits, the technique is applied to a simulation study of a lot-sizing problem under stochastic and non-stationary demand. In this example the lot sizing techniques compared are the EOQ model, the Silver-Meal heuristic and the Wagner-Whitin algorithm.

2. The Netting Procedure

As already mentioned, it is assumed that end-item inventories are controlled according to a Time Phased Order Point (TPOP) policy (Orlicky (1975)), which basically means that ordering decisions are made periodically based on information on so-called net requirements. Prior to the simulation, initial levels of the safety stock and of the net stock are specified. In the simulation model the activities take place as follows: At the beginning of each period demand is forecasted over the forecast horizon. Then based on outstanding orders, the current net stock and forecasted demand, net requirements are calculated. Net requirements are similar to those in Wemmerlöv and Whybark (1984) and Wemmerlöv

(1986). The latter, however, is assuming lost sales where we assume backorders, so the approach is different. Here, net requirements are based on a netting procedure, which works as follows: Within the lead time, the net requirements are zero. Then during the lead time planned net stock below the safety stock is accumulated and added to the net requirement in the first period in which an order can arrive (i.e. current period plus lead time). If planned net stock is above the safety stock after the lead time, net requirements are still zero. Otherwise net requirements are determined as the difference between the safety stock and the net stock, and the planned net stock is set equal to the safety stock. Outstanding orders will be received within the lead time and an order released in the current period will arrive after the lead time. The order released in the current period is therefore based on the net requirements in the periods from after the lead time until the end of the forecast horizon. The size of the order will moreover depend on the lot sizing procedure and cost parameters.

Define t+z as the first period after the lead time, where the planned net stock is below (or equal to) the safety stock, then $z = \arg\min\{m \geq L \mid \hat{X}_{t,t+m} \leq \Psi\}$. The net requirement in period t+i (calculated at time t) can then be determined as

$$R_{t,t+i} = \begin{cases} 0 & \text{for} \quad i = 0,...,z-1 \\ \sum_{j=1}^{t+L-1} Q_{j-L,j} - \sum_{j=1}^{t-1} d_j - \sum_{j=0}^{i} f_{t,t+j} - (X_0 - \Psi) & \text{for} \quad i = z \\ f_{t,t+i} & \text{for} \quad i = z+1,...,T-1 \end{cases} \tag{2.1}$$

where

t	is the period number (t = 1,…, L_R, where L_R is the run length).
T	is the length of the forecasting horizon.
i	is the forecast horizon index (i = 0,…, T-1).
L	is the fixed lead time.
$\hat{X}_{t,t+i}$	is the planned net stock at the end of period t+i (determined at period t).
X_t	is the actual net stock at the end of period t.
X_0	is the initial net stock.
Ψ	is the safety stock.
$f_{t,t+i}$	is the forecast made at the beginning of period t for period t+i.

d_t is the actual demand in period t.

$Q_{t,t+L}$ is the replenishment order placed at the beginning of period t arriving in the beginning of period $t+L$.

For the case where $z = L$ and where there is one outstanding order arriving in the beginning of period $t+2$, the netting procedure is explained graphically in figure 2.

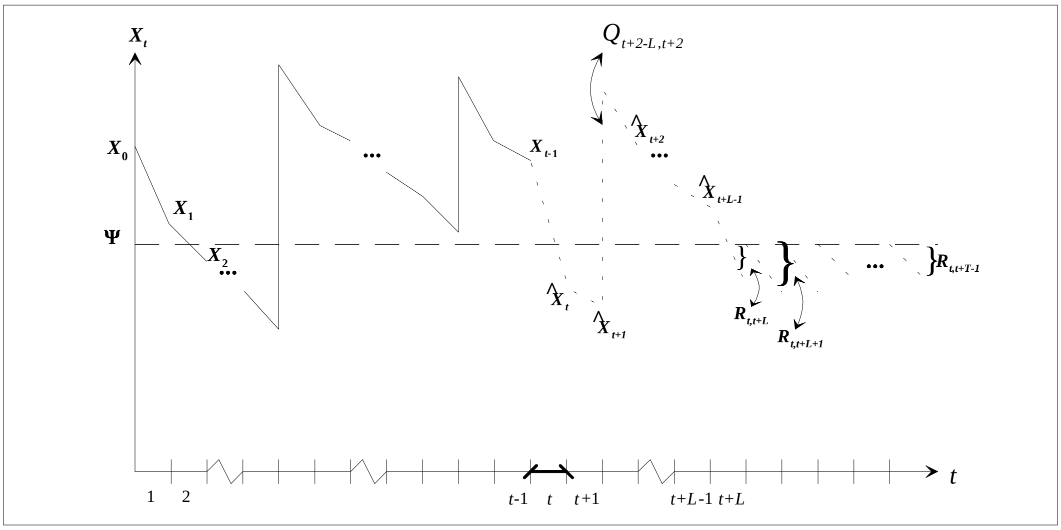

Figure 2. The Netting Procedure

In (2.1) and in figure 2 the lead time is assumed constant, however the procedure can easily be extended to variable lead times. Then, the netting procedure uses the expected lead time instead of the constant lead time. Also the actual net stock is a kind of inventory position including the order in the period, where it was expected to arrive even though it did not, if that order is delayed.

3. Safety stock independence

The safety stock adjustment procedure proposed in this paper is based on a safety stock independence property. Through three lemmas it will therefore be shown that net requirements and order sizes are independent of the safety stock and that the net stock is translation invariant. These lemmas however, only hold under a specific set of assumptions, which are:

1. The system is assumed ergodic meaning that

$$\lim_{t\to\infty} P\{X_t \le x \mid X_0 = a\} = \lim_{t\to\infty} P\{X_t \le x \mid X_0 = b\} \quad \forall a \ne b\,.$$

   This is a general and important assumption that is usually required in other studies as well, however seldom mentioned.
2. The initial condition, where $\delta = X_0 - \Psi$ is fixed, has to be met independently of the value of Ψ. This assumption, however is not restrictive for the technique developed in this paper due to assumption number 1, however to replicate the exact same replenishment process this assumption is included. When assumption 1 holds, this assumption can therefore be omitted with respect to the comparison of different safety stock scenario's and or different planning concepts.
3. Actual and forecasted demands are assumed given from external procedures that do not depend on the netting procedure or the choice of the safety stock.
4. Orders that have either been received in the past or are outstanding are independent of Ψ.
5. All unfilled demand is backordered.

The future planned orders are determined first by the Master Production Schedule and then modified by the Planning Engine. The order sizes determined in the current period depend on the future net requirements also determined in the current period and some exogenous variables (ξ), such as cost functions or state variables that are independent of the safety stock. Hence, $Q_{t+i,t+L+i} = \Xi_i(R_{t,t},\dots,R_{t,t+T-1},\xi) \quad i = 0,\dots,T-1$, where Ξ_i is determined by the Planning Engine. Examples of Ξ_i could be the Wagner-Whitin algorithm, the Silver-Meal heuristic, the EOQ model or some multilevel lot sizing rule. Since, only the order size in the current period is implemented in a rolling horizon environment the main interest is $Q_{t,t+L}$.

Lemma 1. $R_{t,t+i}(X_0, \Psi) = R_{t,t+i}(X_0 + \Delta, \Psi + \Delta)$, where $i = 0,\ldots,T$-1.

Proof. Since past and outstanding orders, historical demands and forecasted demand are independent of Ψ, then

$$R_{t,t+i}(X_0+\Delta,\Psi+\Delta) = \begin{cases} 0 & \text{for} \quad i = 0,\ldots, z-1 \\ \sum_{j=1}^{t+L-1} Q_{j-L,j} - \sum_{j=1}^{t-1} d_j - \sum_{j=0}^{i} f_{t,t+j} - (X_0 + \Delta - \Psi - \Delta) & \text{for} \quad i = z \\ f_{t,t+i} & \text{for} \quad i = z+1,\ldots,T-1 \end{cases}$$

$$= R_{t,t+i}(X_0, \Psi)$$

and $\delta = X_0 + \Delta - (\Psi + \Delta) = X_0 - \Psi$.

Lemma 1 implies that net requirements are independent of Ψ as long as $\delta = X_0 - \Psi$ is fixed.

Lemma 2. $Q_{t,t+L}$ is independent of Ψ.

Proof. This follows immediately from Lemma 1 and $Q_{t,t+L} = \Xi(R_{t,t},\ldots,R_{t,t+T-1},\xi)$. Since net requirements and the exogenous variables are independent of Ψ, as long as $\delta = X_0 - \Psi$ is fixed, then so is $Q_{t,t+L}$.

Lemma 3. The net stock process, X_t, is translation invariant, meaning that

$$X_t(X_0, \Psi) + \Delta = X_t(X_0 + \Delta, \Psi + \Delta).$$

Proof. The following relation defines the net stock process as the ending net stock of period t:

$$X_t(X_0,\Psi) = X_0 + \sum_{j=1}^{t} Q_{j-L,j} - \sum_{j=1}^{t} d_j .$$

By adding the same constant to X_0 and Ψ, then, due to Lemma 2, the replenishment process, $Q_{1,L+1},\ldots,Q_{t-L,t}$, is the same, affecting the net stock process in the following way:

$$X_t(X_0+\Delta,\Psi+\Delta) = X_0 + \Delta + \sum_{j=1}^{t} Q_{j-L,j} - \sum_{j=1}^{t} d_j = X_t(X_0,\Psi) + \Delta,$$

meaning that the net stock process is translation invariant.

Lemma 3 implies that when using the netting procedure proposed here, the safety stock only has influence on the service level and not on the development of the inventory process itself.

4. The Safety Stock Adjustment Procedure

The invariance relation derived above enables us to determine the required safety stocks for different end products to achieve the target customer service levels. When doing a simulation study using the proposed procedure, the simulation is carried out in two steps. The first step of the simulation determines the maximum and minimum value of the net stock during those periods. This represents an interval for which the probability that the net stock is within this interval is close to 1. The second step determines the frequency function of the net stock process leading to a discrete probability distribution. Based on this probability distribution the safety stock is adjusted to ensure the specified target service level. Based on the adjusted safety stock and the probability distribution the performance measures can be calculated. To verify the results a third step in the simulation could be carried out. The third step is based on the adjusted safety stock. If the exact same replenishment process is to be replicated under the adjusted safety stock, the initial net stock must be adjusted as well, and also the same random numbers must be used, since the steps must be based on the same random demand process.

The initial net stock value is omitted from the net stock expression in the remainder of the paper. Let Ψ_0 denote an arbitrary initial choice of the safety stock. From the first two steps of the simulation an approximation to the empirical probability distribution of $X_t(\Psi_0)$ can be determined, where X_t is the net stock at the end of a period. Hence, $p_k = P\{X_t(\Psi_0) \le x_k\}$ is determined for $k = 0,\ldots, K$, where K+1 is the chosen number of probabilities, x_0 represents the minimum recorded net stock value and x_K represents the maximum recorded net stock value in the inventory process $X_t(\Psi_0)$ determined during step 1 of the simulation. Then x_k is determined as

$$x_k = x_0 + \frac{k}{K}\left(x_K - x_0\right), \qquad 1 \le k \le K - 1.$$

Hence, the whole simulation does not determine the maximum and minimum net stocks. In stead, determining these throughout step 1 and choosing the size of K is a part of the approximation of the empirical probability distribution. It should be noted here, that it is not strictly necessary to approximate the empirical distribution. The probability distribution can also be based on the sorted set of the individual values of net stock process, however it will demand a rather big storage of variables during the simulation.

The netting procedure is not depending on the choice of the service measure. However the service measure is relevant for the value of the safety stock and thus for the Safety Stock Adjustment Procedure (SSAP). The SSAP will now be presented for three kinds of service measures; namely the ready rate, the cycle service level and the fill rate.

The ready rate (*P_3*)

The ready rate service measure, here denoted by P_3, is defined as the fraction of time during which the system has positive net stock, which is the same as the probability of no stockout at the end of an arbitrary period. From actual historical data or from simulated data, the P_3 measure can be calculated as

$$P_3 = P\{X_t \geq 0\} = 1 - N_B/N_P,$$

where N_B is the number of periods with additional backordered demand and N_P is the total number of time periods considered. Hence, the P_3 service measure represents a time dimension of demand satisfied without backorders.

Let γ be the target service level based on P_3 and let Ψ^* be the safety stock that satisfies the target service level. From the definition of the P_3 service measure we know that $1-\gamma = P\{X_t(\Psi^*) \leq 0\}$, which is the probability of stockout at any time period.

Define $x_{1\text{-}\gamma}$ by $1-\gamma = P\{X_t(\Psi_0) \leq x_{1-\gamma}\}$. Since $X_t(\Psi_0)$ is translation invariant this equation can be reformulated to:

$$\begin{aligned}
&1-\gamma = P\{X_t(\Psi_0) \leq x_{1-\gamma}\} \Leftrightarrow \\
&1-\gamma = P\{X_t(\Psi_0) - x_{1-\gamma} \leq 0\} \Leftrightarrow \\
&1-\gamma = P\{X_t(\Psi_0 - x_{1-\gamma}) \leq 0\} \Leftrightarrow \\
&\Psi^* = \Psi_0 - x_{1-\gamma}.
\end{aligned}$$

Hence, from the known p_k-values we must find the τ satisfying $p_{\tau-1} \le 1-\gamma \le p_\tau$ with the corresponding $x_{\tau-1}$- and x_τ-values leading to $x_{\tau-1} \le x_{1-\gamma} \le x_\tau$. Consequently $x_{1-\gamma}$, which is called the safety stock adjustment quantity, can be found by linear interpolation:

$$x_{1-\gamma} = \frac{\left((1-\gamma) - p_{\tau-1}\right)x_\tau + \left(p_\tau - (1-\gamma)\right)x_{\tau-1}}{p_\tau - p_{\tau-1}}$$

and thus

$$\Psi^* = \Psi_0 - \frac{\left((1-\gamma) - p_{\tau-1}\right)x_\tau + \left(p_\tau - (1-\gamma)\right)x_{\tau-1}}{p_\tau - p_{\tau-1}}.$$

The safety stock adjustment procedure for the P_3 service measure is illustrated in figure 3.

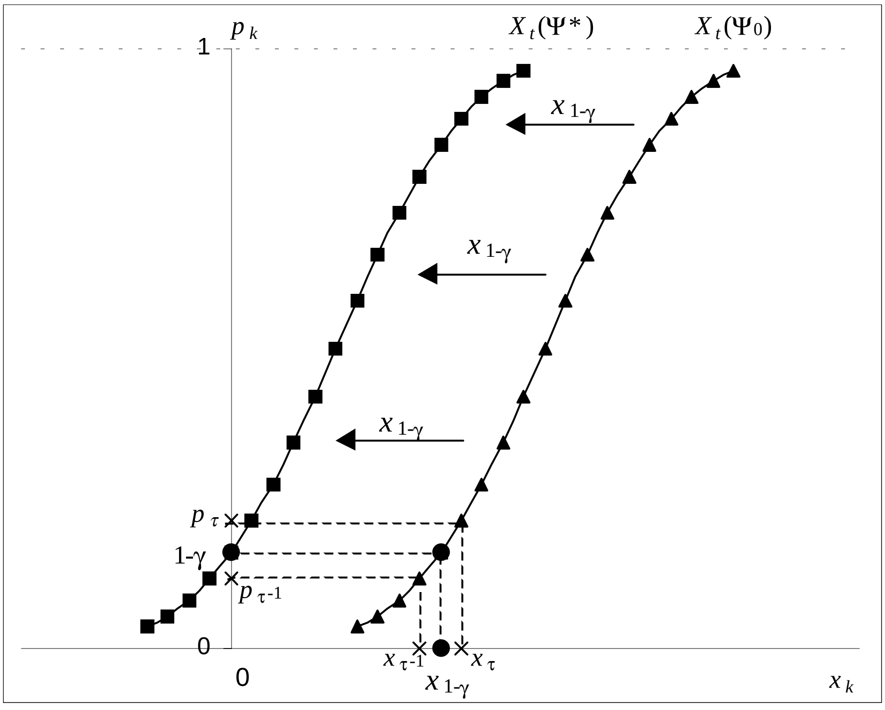

Figure 3. The safety stock adjustment procedure for P_3

The cycle service level (P_1)

The cycle service level, here denoted by P_1, is defined as the probability of no stockout during a replenishment cycle. For the P_1 measure only a specific subsequence of the (ending) net stock process is used; namely the net stock values in the periods just prior to the arrival of a replenishment order. Let Y_t denote this subsequence of the net stock process called the cycle stock process. Hence during the simulation the probability distribution will not be based on all periods but only the periods immediately prior to the arrival of an order. Hence, $q_k = P\{Y_t(\Psi_0) \le y_k\}$ is also determined for $k = 0,\ldots, K$. From actual historical data or from simulated data, the P_1 measure is calculated as

$$P_1 = P\{Y_t \ge 0\} = 1 - M_B/N_{RC},$$

where M_B is the number of replenishment cycles with additional backorders and N_{RC} is the total number of replenishment cycles considered.

Let α be the target service level based on P_1 and let Ψ^* be the safety stock, which satisfies the target service level. From the definition of the P_1 service measure we know that $1-\alpha = P\{Y_t(\Psi^*) \le 0\}$, which is the probability of a stockout during a replenishment cycle.

Define $y_{1\text{-}\alpha}$ by $1-\alpha = P\{Y_t(\Psi_0) \le y_{1-\alpha}\}$. Find the τ satisfying $q_{\tau-1} \le 1-\alpha \le q_\tau$. Then, since Y_t is also translation invariant (being a fixed subsequence of X_t), by substituting X_t with Y_t, x_i with y_i, p_i with q_i and γ with α in the SSAP for P_3, both the SSAP and figure 3 are the same for P_1 and P_3.

The fill rate (P_2)

The fill rate service measure, here denoted by P_2, is defined as the long-run fraction of demand satisfied directly from stock. From actual historical data or from simulated data, the exact P_2 measure is calculated as

$$P_2 = 1 - \overline{B}/\overline{D},$$

where $\overline{B}$ is the average backorder per period and $\overline{D}$ is the average demand per period. $\overline{B}$ has to be adjusted for double-counts of backorders that are carried over from one replenishment cycle to another. Hence, the P_2 service measure represents a quantity dimension of demand satisfied without backorders.

Let β be the target service level based on P_2 and let Ψ^* be the safety stock, which satisfies the target service level β. From the definition of the service measure we know that $1-\beta = \overline{B}/\overline{D}$.

Define $Z_t(\Psi_0)$ as the net stock at the *beginning* of an arbitrary period, immediately after (possible) arrival of a replenishment. Using the fact that $Z_t(\Psi_0) \geq X_t(\Psi_0)$, then from step 1 and 2 of the simulation $r_k = P\{Z_t(\Psi_0) \leq x_k\}$ is also determined for $k = 0,\ldots, K$, where x_0 is defined as the minimum recorded value of $X_t(\Psi_0)$ and x_K is defined as the maximum recorded value of $Z_t(\Psi_0)$.

We need to find the amount of average backorders that satisfies the target service level β. From the probability distribution of $X_t(\Psi_0)$ and $Z_t(\Psi_0)$ the average backorder per period, if x_k was the adjustment quantity, can be calculated as

$$\overline{B}(x_k) = E_1 - E_2 = \sum_{i=0}^{k-1}(x_{i+1} - x_i)(p_i - r_i) \quad \geq 0 \tag{4.1}$$

where

$$E_1 = E\left[(X_t(\Psi_0) - x_k)^-\right] = \sum_{i=0}^{k-1}(x_{i+1} - x_i)\,p_i \quad \geq 0$$

$$E_2 = E\left[(Z_t(\Psi_0) - x_k)^-\right] = \sum_{i=0}^{k-1}(x_{i+1} - x_i)r_i \quad \geq 0$$

and $E_1 \geq E_2$.

The average demand, $\overline{D}$, is known from the simulation step 2 (or from historical data). Define $x_{1\text{-}\beta}$ by $(1-\beta)\overline{D} = \overline{B}(x_{1-\beta})$. Hence, from the known p_k- and r_k-values we must find the τ satisfying $\overline{B}(x_{\tau-1}) \leq (1-\beta)\overline{D} \leq \overline{B}(x_\tau)$ with the corresponding $x_{\tau\text{-}1}$- and x_τ-values, leading to $x_{\tau-1} \leq x_{1-\beta} \leq x_\tau$. Consequently, the adjustment quantity in the case of P_2, $x_{1\text{-}\beta}$, can be found by linear interpolation:

$$x_{1-\beta} = \frac{\left((1-\beta)\overline{D} - \overline{B}(x_{\tau-1})\right)x_\tau + \left(\overline{B}(x_\tau) - (1-\beta)\overline{D}\right)x_{\tau-1}}{\overline{B}(x_\tau) - \overline{B}(x_{\tau-1})}$$

and, since X_t and Z_t are translation invariant

$$\Psi^* = \Psi_0 - \frac{\left((1-\beta)\overline{D} - \overline{B}(x_{\tau-1})\right)x_\tau + \left(\overline{B}(x_\tau) - (1-\beta)\overline{D}\right)x_{\tau-1}}{\overline{B}(x_\tau) - \overline{B}(x_{\tau-1})}.$$

The safety stock adjustment procedure for the P_2 service measure is illustrated in figure 4.

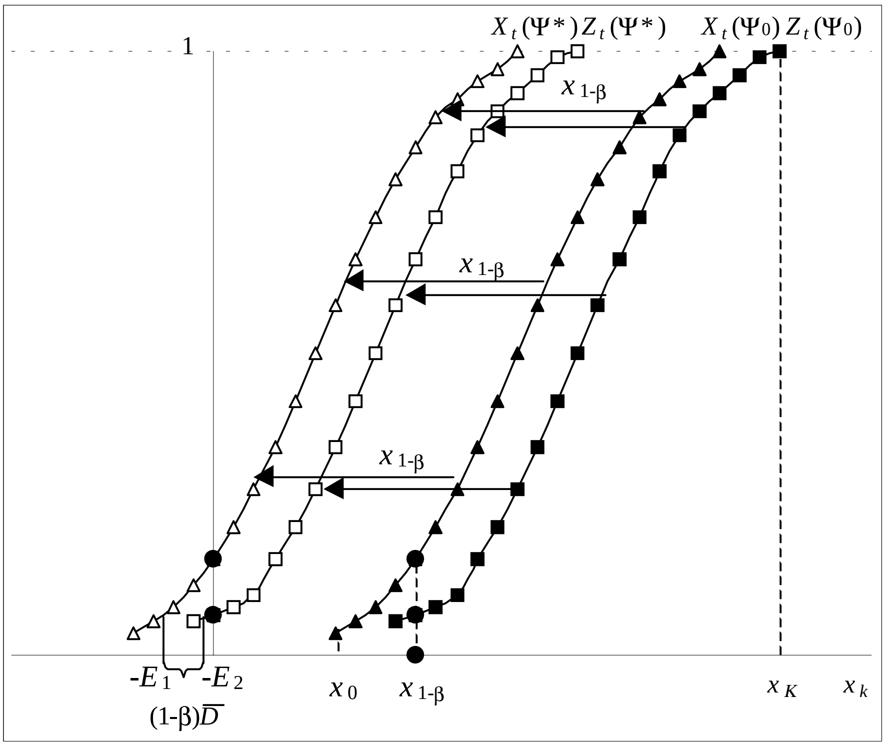

Figure 4. The safety stock adjustment procedure for P_2

Performance measures

After the safety stock adjustment quantity has been calculated for a given scenario, step 1 and 2 can be replicated in a third step with the adjusted safety stock in order to verify the procedure and to calculate the performance measures under study. However, the performance measures can also be calculated directly from the known values of the probability distribution functions of the various net stock processes which are determined from step 1 and 2 and from additional data already collected during step 2. Formulas for a number of performance measures are presented in appendix A.

When calculating the performance measures directly after step 2, without completing with step 3 to verify the results, it is important to carry out a sensitivity analysis of the chosen number of probabilities. If the size of K is too low the accuracy of the calculated performance measures may also be too low. A sensitivity analysis is included in the example in the next paragraph to show the importance of accuracy and verification.

The safety stock adjustment procedure can now be summarised to consist of the following steps:

1. Run step 1 of a discrete event simulation of the system with an arbitrary choice of the safety stock, Ψ_0 and of the initial net stock. Record the minimum ending net stock (x_0=y_0), the maximum beginning net stock (x_K), and the maximum ending cycle stock (y_K) of those periods.
2. Continue with step 2 of the simulation. Record relevant data.
3. From the discrete event simulation compute the approximate empirical distribution functions of the ending net stock, $p_k = P\{X_t(\Psi_0) \le x_k\}$, of the beginning net stock, $r_k = P\{Z_t(\Psi_0) \le x_k\}$, and of the cycle stock, $q_k = P\{Y_t(\Psi_0) \le y_k\}$ for k = 0,…, K.
4. Given these empirical distributions and given the choice of service measure compute the adjustment quantity and the adjusted safety stock, Ψ*, such that the required end-item service level is achieved.
5. Calculate performance measures directly from recorded data (see appendix A) or run another simulation (step 3 of the simulation) with Ψ* to compute and verify performance measures.

5. A Numerical Example

A simulation experiment has been designed in order to show the difference between a traditional simulation analysis and the analysis based on the safety stock adjustment procedure. The design of the experiment is relatively simple. Assume that the objective of the simulation is to compare three lot sizing techniques in terms of total cost and service level in a single stage inventory system under stochastic and seasonal demand. The service measure used in this simulation study is the P_3 service measure.

Experimental design and data

Order quantities are determined on a rolling horizon basis, which basically means that forecasts and order quantities are computed for a fixed number of time periods given by the forecast horizon, however only the decision related to the current period is implemented. The order quantities in this experiment are determined from three well-known lot sizing techniques. These are the Economic Order Quantity (EOQ), the Silver-Meal (SM) heuristic and the Wagner-Whitin (WW) algorithm. For a presentation of these techniques see for example Silver *et al*. (1998).

The system works in an environment where customer demand is assumed uncertain. Actual and historical demands are generated from a normal distribution with mean 100 units per period. The standard deviation is varied as one of the experimental input factors taking on values 10, 25 and 50 respectively corresponding to a coefficient of variation ranging from 0.1 to 0.5. Negative actual demands are truncated to 0. The demand is assumed to be time varying with a seasonal pattern and no trend is assumed present.

A forecast model is applied based on the following exponential smoothing procedure proposed by Silver *et al*. (1998, p. 99).

$$f_{t,t+i} = \hat{a}_{t+i}\hat{F}_{t+i} \quad \forall i \quad \text{where} \quad \hat{a}_{t+i} = \hat{a}_{t+i-1} + \alpha_{sc}\left(\frac{d_{t-1}}{\hat{F}_{t+i-P}} - \hat{a}_{t+i-1}\right)$$

$$\text{and} \quad \hat{F}_{t+i} = \gamma_{sc}\frac{d_{t-1}}{\hat{a}_{t+i}} + (1-\gamma_{sc})\hat{F}_{t+i-P}$$

where

t	is the current time period.
i	is the forecast horizon index.
$f_{t,t+i}$	is the forecast made at the beginning of period t for period $t+i$.
d_{t-1}	is the actual demand in period t-1 (the most recent actual demand).
$\hat{a}_{t+i}$	is the estimated level component for period $t+i$.
$\hat{F}_{t+i}$	is the estimated seasonal component for period $t+i$.
P	is the number of seasons in a cycle.
α_{sc} and γ_{sc}	are the smoothing constants of level and season.

Demand is forecasted T periods ahead and the forecasts are made at the beginning of each period, i.e. before the demand of the current period is known. Prior to running the simulation experiment historical demand data are generated to initialise the forecast model. After each period the forecasting model is updated with new information on the latest actual demand. The seasonal cycle consists of 4 seasons and the mean demand in each season is adjusted according to the indices I = (1, 0.5, 1, 1.5) respectively. The smoothing constants of level and season used in the forecasting procedure are set to α_{sc} = 0.2 and γ_{sc} = 0.3 and the forecast horizon is set to 12 periods.

In the simulation the cost ratio between the ordering and the holding costs is also one of the experimental factors with three values implying different average time between orders. The holding cost per unit per period is fixed to 1 and the ordering cost per order is varied. Backorders are allowed and lead time is constant. Three values of the constant lead time are used. The three lot sizing procedures also represent an experimental factor. Hence the experimental design has 3x3x3x3 = 81 factor level combinations. The models will be compared under a target P_3 service level of 90%. The constant input data are summarised in table 1, and experimental factors and their levels are summarised in table 2.

Table 1
Summary of constant input data

Constant Input	Notation and Value
Mean demand	E(D) = 100
Indices for seasonal demand	I = (1, 0.5, 1, 1.5)
Smoothing constant, level	α_{sc} = 0.2
Smoothing constant, season	γ_{sc} = 0.3
Forecast horizon	T = 12
Holding cost per unit per period	h = 1
Target P_3 – service level	γ = 0.90

Table 2
Summary of experimental factors

Experimental Factor	Levels	Number of levels
Lot size technique	{EOQ, SM, WW}	3
Standard deviation on demand, $\sigma(D)$	{10, 25, 50}	3
Lead time, L	{0, 4, 8}	3
Cost ratio, A/h	{100, 333, 500}	3

Before the main experiment is completed, the length of the start up period, the run length and the number of replications are determined from pilot studies. The pilot studies are resulting in the simulation data given in table 3.

Table 3
Simulation data

Simulation data	Value
Run length	20,000

Warm-up period	2,000
Number of replications of each factor level combination	10

The warm-up period consists of step 1 of the simulation, where only the minimum and maximum net stock values are recorded. Note that we do not determine the maximum and minimum net stock of the whole simulation. However determining these throughout the warm-up period is a part of the approximation of the empirical distribution as already mentioned earlier. The remainder of the run length consists of 18,000 periods, constituting the simulation process corresponding to step 2, where data for the probability distribution are collected along with data for calculating the performance measures. For verification issues the simulation is replicated in a third step, where no data are collected during the warm-up period.

The performance measures of this illustrative study are: The total relevant cost, C_T, as average per period based on the sum of inventory holding costs and ordering costs (average per period), the deviation from optimal cost, $\Delta\%$, and the simulated service level, P_3. The service level represents the probability of no stockout at the end of an arbitrary time period.

Initial safety stock and initial net stock value

For stationary demand, the safety stock is traditionally determined as $\Psi_0 = k * \sqrt{L} * \sigma(D)$, where k is the safety factor depending on the choice of service measure, L is the lead time and $\sigma(D)$ is the standard deviation of demand. Let ϕ denote the standard normal density function and let Φ denote the standard normal distribution function. Let s denote the reorder level and Q the order size, then in the (s, Q)-model with normal distributed demand k is determined the following way for each of the service measures [Silver *et al*. (1998)]. For the P_1 service measure, $k = \Phi^{-1}(P_1)$. For the approximative P_2 service measure, k is chosen to satisfy

$\phi(k) - k(1-\Phi(k)) = \frac{Q}{\sqrt{L}\,\sigma(D)}(1-P_2)$, where Q could be determined by the EOQ formula.

From Sahin (1990) it can be deducted that the value of the P_3 service measure is close to the value of the P_2 service measure. Therefore we approximate k for the P_3 service measure. Note that $\sigma(D)$ should be interpreted as the forecast error, since cases with non-stationary demand are considered here.

In the safety stock adjustment procedure the necessary amount of safety stock is determined by adding the adjustment quantity to the initial safety stock. The initial safety stock value determined prior to the analysis can be any arbitrary value. However, to make the outcome of the first two steps of the simulation as close to the target safety stock as possible, the initial value in this experiment is determined from the safety stock value given above for the stationary situation. Without loss of generality the initial net stock value prior to step 1 is set equal to the initial safety stock value. As a consequence, to replicate exactly the same replenishment process, the initial net stock value in step 3 has to be adjusted with the same quantity as the safety stock.

Results and interpretations

The results of the simulation experiment are given in appendix B. Table B.1 corresponds to a traditional analysis and shows the direct results of step 1 and 2 before the safety stock is adjusted. As can be seen the simulated service levels differ even though the safety stock for each scenario is based on a target service level of 90%. Table B.2 shows the results after using the SSAP. In table B.2 total costs are calculated directly from the net stock distribution function following the formulas in appendix A. To show the importance of accuracy a small sensitivity analysis has been carried out, where the total costs are based on different values of K, where K+1 is the chosen number of probabilities in the approximate net stock distribution function. For verification issues the total costs and the identical service levels are determined by replicating the simulation in a third step using the adjusted safety stock for each scenario. This is shown in table B.3. The total costs based on step 3 are also included in table B.2 as a benchmark for the sensitivity analysis.

As seen from table B.1 it is possible to compare the performance of the models for some of the scenarios. For example, in scenario 11 all three models lead to the same service level, hence the EOQ leading to lowest cost is the best. For scenario 10, 13 and 19, the Silver-Meal has the highest service level and the lowest cost, so the Silver-Meal heuristic outperforms both of the other two models in terms of both performance measures. In a scenario like scenario 9, however, the comparison is more difficult. EOQ gives the best service level, however it also gives the highest costs, whereas the Wagner-Whitin model gives the lowest costs but also the lowest service level. When taking into account several performance measures, it is therefore not possible to identify the optimal model.

Moreover, it is very difficult to interpret the effects of the experimental factors. Comparing scenario 4, 5 and 6 in table B.1 one concludes that an increase in demand uncertainty leads to higher costs and higher service level, however it is not clear whether an increase in costs is due to a higher service level or the increased demand uncertainty. (This is made clear in table B.3 where an increase in demand uncertainty leads to increased cost at a fixed service level). Also, from scenario 1, 2 and 3 in table B.1 one concludes that a higher demand uncertainty leads to higher costs and a lower service level. In these cases it would be much easier to compare scenarios and models if the service level was the same for all experiments.

Since the service levels are identical in table B.3 all comparisons and analyses are based on costs only. Therefore, it is possible to use the cost deviation measure to identify the optimal model for each scenario. Moreover, it will be relatively easy to analyse the impact and interactions of the experimental factors. Hence, by comparing the results of the traditional line of thought with the results based on the SSAP the benefits of using the SSAP are obvious.

Examples of earlier studies that compare several performance measures of different planning scenarios under non-identical service levels are Biggs and Campion (1982) and De Bodt and Van Wassenhove (1983). The latter seems to be aware of the pitfall discussed above since they eventually compare the results through a trade off analysis. However, the study of Biggs and Campion (1982) shows difficulties in dealing with this comparison, which may even have led to misinterpretations of the results.

As can be seen from table B.2, there is a gap between the cost values for the different values of K. This means that the accuracy of the performance measures is very sensitive to changes in K. Comparing the costs when K=300 with the benchmarking cost from step 3 shows that K=300 for this specific simulation study is an adequate size. Each kind of simulation study, however, must make its own sensitivity analysis to determine the adequate size of K.

6. Using SSAP in practice

Traditionally the safety stock and the order size are determined “a priori” from inventory models. Often the safety stock is determined as $\Psi = k\sigma_L$, where σ_L is the standard deviation of demand during lead time. The problem with this is that it requires knowledge about the theoretical demand distribution, as do other traditional inventory models that are usually presented in textbooks; see for instance Silver *et al*. (1998). Moreover, for some service measures it may be a very complex matter to find the optimal value of the safety factor, k.

Advanced forecasting techniques in the field of time series analysis have been developed to estimate non-stationarity; see for instance Box *et al*. (1994) or Clements and Hendry (2001). However, incorporating this non-stationarity into inventory models and production planning techniques is very difficult both in theory and in practice. Thus, if there exist no exact models or if demand is either non-stationary or its distribution is unknown there is a problem with using the traditional inventory models. Hence, there is a need for models and techniques that can deal with these problems in order to determine appropriate safety stocks in practice.

Therefore we propose a new approach based on the SSAP technique developed in this paper, which can be applied as a means of optimisation or determination of parameters for control policies. The idea is that the safety stock is determined “a posteori” meaning that the safety stock is adjusted retrospectively based on a sample path analysis of the historical data assuming that the demand process and the lot sizing decisions will be similar in the future.

The advantage of the SSAP is that it can be used even if there is no knowledge about the theoretical demand distribution. The only assumption made is that the historical demand pattern, or more general, the forecast error pattern, in a stochastic sense represents the pattern to be expected in the (near) future, for which we have to decide on the safety stocks and lot sizing rules to be used. Hence, the data need not be derived from some statistical model. In fact the historical data incorporate the possible combination of statistical forecasting and human judgement. The main constraint of using the SSAP procedure is that it requires the company to use a TPOP netting procedure to calculate net requirements and that unfilled demand needs to be backordered.

7. Conclusion

In this paper we have developed a technique that ensures target end-item service levels in simulation studies of inventory systems. The technique can be used for any multi-product multi-echelon inventory problem and there need not be any assumptions on the demand process or the forecasting procedure. Moreover, lead times may vary and there may be capacity or lot sizing constraints as long as unfilled demand is backordered.

The technique is based on a TPOP netting procedure from which net requirements, that are independent of the safety stock, are calculated. This safety stock independence also implies that the replenishment process is independent of the safety stock and furthermore that the net stock process is translation invariant. Using the properties of safety stock independence and the invariance relation, the safety stock adjustment procedure (SSAP) has been developed for three service measures, namely the cycle service level (P_1), the fill rate (P_2) and the ready rate (P_3). SSAP is based on an approximation of the empirical distribution of the inventory process, which, based on an arbitrary value of the safety stock, is generated by simulation. Based on this distribution a formula for the adjustment quantity has been derived for each type of service measure. This adjustment quantity is used to adjust the safety stock for each scenario in a simulation study to achieve the target service level. Performance measures can then either be calculated directly from data collected during the simulation, or the simulation can be replicated with the adjusted safety stock.

An example was presented showing the benefits of applying SSAP as opposed to the traditional comparison studies where the comparison is complicated with the presence of different service levels. Additionally, the example showed that it is important either to perform a sensitivity analysis of the number of probabilities in the net stock distribution function or to replicate the simulation with the adjusted safety stock.

By using the SSAP, comparison of scenarios and models in inventory simulation studies becomes much easier. Since the application goes further than the numerical example presented here, the SSAP is useful for a wide range of inventory simulation studies and even for determining safety stocks in practice for instance when demand is non-stationary or its distribution is unknown.

On-going research applying this line of thought comprises an analysis of the variability of the replenishment process done by the authors of this paper and analysis of mathematical programming models for supply chain planning.

Based on the procedure developed here we only determine safety stocks for end-items. It is however possible to determine safety stocks for intermediate items with external demand as well. Moreover, it may be worthwhile to set safety stocks for other intermediate items to cope with dependent demand uncertainty, however this is open for further research.

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Appendix A

Calculation of Performance Measures

Data that are independent of the safety stock are collected during step 2 of the simulation. The most common ones are listed in the following table:

Table A.1.
Measures, independent of the safety stock, recorded from simulation step 2

Description of data	Notation
Simulation period index	t
The period number when recording starts	L_S
The period number when recording stops (here equal to the run length of the simulation)	L_R
Number of periods recorded	$(L_R\text{-}L_S+1)$
Total demand during simulation (sum of demand over all recorded periods)	D_T
Mean demand per period	$\overline{D} = D_T/(L_R\text{-}L_S+1)$
Ordering Costs (average per period)	C_o
Order sizes (at each period, if any)	$Q_{t,t+L}$, where $t = L_S,\ldots,L_R$

Data for other performance measures of interest, which are independent of the safety stock, like time between orders, number of orders, the coefficient of variation of order sizes and nervousness measures can also be collected during simulation step 2.

From step 2 also the probability distributions of $X_t(\Psi_0)$, $Y_t(\Psi_0)$ and $Z_t(\Psi_0)$ are determined. Thereby we have determined $p_k = P\{X_t(\Psi_0) \le x_k\}$, $q_k = P\{Y_t(\Psi_0) \le y_k\}$ and $r_k = P\{Z_t(\Psi_0) \le x_k\}$ for $k = 0,\ldots, K$, where K+1 is the chosen number of probabilities. Furthermore, let h denote the inventory holding cost per unit per period.

The relevant performance measures can now be calculated from these probabilities and from the collected data shown above. All calculations take place after the adjustment quantity has been calculated. The calculations of the most common performance measures are given in table A.2, where numbers in parentheses refer to equation numbers below the table.

Table A.2.
Performance measures in the adjusted process meeting target service levels.

Performance Measure	Nota-tion	P_1 service measure	P_2 service measure	P_3 service measure
Adjustment quantity		$y_{1-\alpha}$ (A.1)	$x_{1-\beta}$ (A.2)	$x_{1-\gamma}$ (A.3)
Safety stock	Ψ^*	$\Psi_0 - y_{1-\alpha}$	$\Psi_0 - x_{1-\beta}$	$\Psi_0 - x_{1-\gamma}$
τ - position	τ	$\arg\min\{k \mid 1-\alpha \le q_k\}$	$\arg\min\{k \mid (1-\beta)\overline{D} \le \overline{B}(x_k)\}$	$\arg\min\{k \mid 1-\gamma \le p_k\}$
ω - position	ω	$\arg\min\{k \mid y_{1-\alpha} \le x_k\}$	$\arg\min\{k \mid x_{1-\beta} \le y_k\}$	$\arg\min\{k \mid x_{1-\gamma} \le y_k\}$
Expected backorder per period	$\overline{B}$	$\overline{B}(y_{1-\alpha})$ (A.4)	$\overline{B}(x_{1-\beta}) = (1-\beta)\overline{D}$	$\overline{B}(x_{1-\gamma})$ (A.4)
Cycle service level	P_1	α (target)	$\alpha(\beta)$ (A.5)	$\alpha(\gamma)$ (A.6)
Fill rate	P_2	$\beta(\alpha)$ (A.7)	β (target)	$\beta(\gamma)$ (A.8)
Ready rate	P_3	$\gamma(\alpha)$ (A.9)	$\gamma(\beta)$ (A.10)	γ (target)
Average positive net stock (of adjusted process)	$E(X_t^+)$ (≥ 0)	(A.11)	(A.12)	(A.13)
Average negative net stock	$E(X_t^-)$ (≥ 0)	(A.14)	(A.15)	(A.16)
Average net stock	EX_t	$EX_t^+ - EX_t^-$	$EX_t^+ - EX_t^-$	$EX_t^+ - EX_t^-$
Inventory holding costs (average per period)	C_h	$h^* EX_t^+$	$h^* EX_t^+$	$h^* EX_t^+$
Total Costs (average per period)	C_T	$C_o + C_h$	$C_o + C_h$	$C_o + C_h$

Note here that, since τ does not take the same value for P_1, P_2 and P_3, also x_τ, $x_{\tau-1}$, p_τ, $p_{\tau-1}$ are not the same for P_2 and P_3.

$$y_{1-\alpha} = \frac{\left((1-\alpha) - q_{\tau-1}\right)y_\tau + \left(q_\tau - (1-\alpha)\right)y_{\tau-1}}{q_\tau - q_{\tau-1}} \qquad \text{(A.1)}$$

$$x_{1-\beta} = \frac{\left((1-\beta)\overline{D} - \overline{B}(x_{\tau-1})\right)x_\tau + \left(\overline{B}(x_\tau) - (1-\beta)\overline{D}\right)x_{\tau-1}}{\overline{B}(x_\tau) - \overline{B}(x_{\tau-1})}$$, where $\overline{B}(\cdot)$ is given by (4.1) (A.2)

$$x_{1-\gamma} = \frac{\left((1-\gamma) - p_{\tau-1}\right)x_\tau + \left(p_\tau - (1-\gamma)\right)x_{\tau-1}}{p_\tau - p_{\tau-1}} \qquad \text{(A.3)}$$

$$\bar{B}(a) = E_1 - E_2 = \sum_{i=0}^{\omega-2}(x_{i+1} - x_i)(p_i - r_i) + (a - x_{\omega-1})(p_{\omega-1} - r_{\omega-1}) \geq 0 \text{, where} \tag{A.4}$$

$$E_1 = E\left[(X_t(\Psi_0) - a)^-\right] = \sum_{i=0}^{\omega-2}(x_{i+1} - x_i)p_i + (a - x_{\omega-1})p_{\omega-1} \geq 0$$

$$E_2 = E\left[(Z_t(\Psi_0) - a)^-\right] = \sum_{i=0}^{\omega-2}(x_{i+1} - x_i)r_i + (a - x_{\omega-1})r_{\omega-1} \geq 0$$

$$\alpha(\beta) = 1 - \frac{(x_{1-\beta} - y_{\omega-1})q_\omega + (y_\omega - x_{1-\beta})q_{\omega-1}}{y_\omega - y_{\omega-1}} \tag{A.5}$$

$$\alpha(\gamma) = 1 - \frac{(x_{1-\gamma} - y_{\omega-1})q_\omega + (y_\omega - x_{1-\gamma})q_{\omega-1}}{y_\omega - y_{\omega-1}} \tag{A.6}$$

$$\beta(\alpha) = 1 - \frac{\bar{B}(y_{1-\alpha})}{\bar{D}} \tag{A.7}$$

$$\beta(\gamma) = 1 - \frac{\bar{B}(x_{1-\gamma})}{\bar{D}} \tag{A.8}$$

$$\gamma(\alpha) = 1 - \frac{(y_{1-\alpha} - x_{\omega-1})p_\omega + (x_\omega - y_{1-\alpha})p_{\omega-1}}{x_\omega - x_{\omega-1}} \tag{A.9}$$

$$\gamma(\beta) = 1 - \frac{(x_{1-\beta} - x_{\tau-1})p_\tau + (x_\tau - x_{1-\beta})p_{\tau-1}}{x_\tau - x_{\tau-1}} \tag{A.10}$$

$$E(X_t^+) = (x_\omega - y_{1-\alpha})(1 - \gamma(\alpha)) + \sum_{i=\omega}^{K-1}(x_{i+1} - x_i)p_i \tag{A.11}$$

$$E(X_t^+) = (x_\tau - x_{1-\beta})(1 - \gamma(\beta)) + \sum_{i=\tau}^{K-1}(x_{i+1} - x_i)p_i \tag{A.12}$$

$$E(X_t^+) = (x_\tau - x_{1-\gamma})(1 - \gamma) + \sum_{i=\tau}^{K-1}(x_{i+1} - x_i)p_i \tag{A.13}$$

$$E(X_t^-) = \sum_{i=0}^{\omega-2}(x_{i+1} - x_i)p_i + (y_{1-\alpha} - x_{\omega-1})p_{\omega-1} \tag{A.14}$$

$$E(X_t^-) = \sum_{i=0}^{\tau-2}(x_{i+1} - x_i)p_i + (x_{1-\beta} - x_{\tau-1})p_{\tau-1} \tag{A.15}$$

$$E(X_t^-) = \sum_{i=0}^{\tau-2}(x_{i+1} - x_i)p_i + (x_{1-\gamma} - x_{\tau-1})p_{\tau-1} \tag{A.16}$$

Appendix B

Numerical Results

Table B.1.
Results, when the safety stock for each scenario is determined "the traditional way".

Scenario	Experimental Factors			EOQ	EOQ	EOQ	SM	SM	SM	WW	WW	WW
$S\{i\}$	A/h	$\sigma(D)$	L	C_T	Δ	P_3	C_T	Δ	P_3	C_T	Δ	P_3
1	100	10	0	117	0.09	0.81	114	0.05	0.8	108	0	0.71
2	100	25	0	121	0.05	0.79	119	0.04	0.78	115	0	0.71
3	100	50	0	130	0.03	0.75	131	0.03	0.76	127	0	0.71
4	100	10	4	114	0.04	0.69	112	0.02	0.7	110	0	0.6
5	100	25	4	150	0.01	0.74	151	0.02	0.76	148	0	0.7
6	100	50	4	246	0.01	0.81	248	0.02	0.82	243	0	0.8
7	100	10	8	216	0.77	0.88	123	0.01	0.72	123	0	0.62
8	100	25	8	327	0.68	0.85	197	0.01	0.75	194	0	0.71
9	100	50	8	606	0.47	0.9	415	0.01	0.85	411	0	0.84
10	333	10	0	258	0.03	0.96	251	0	0.97	256	0.02	0.96
11	333	25	0	259	0	0.94	262	0.01	0.94	261	0.01	0.94
12	333	50	0	272	0	0.9	280	0.03	0.91	272	0	0.91
13	333	10	4	247	0.05	0.83	235	0	0.83	241	0.02	0.82
14	333	25	4	270	0.02	0.8	275	0.03	0.82	266	0	0.82
15	333	50	4	359	0	0.83	377	0.05	0.85	358	0	0.84
16	333	10	8	294	0.21	0.85	246	0.01	0.8	243	0	0.83
17	333	25	8	387	0.25	0.83	325	0.05	0.79	309	0	0.79
18	333	50	8	708	0.38	0.9	550	0.07	0.87	514	0	0.86
19	500	10	0	316	0.04	0.97	305	0	0.98	305	0	0.97
20	500	25	0	316	0	0.95	315	0	0.95	317	0.01	0.96
21	500	50	0	331	0	0.92	339	0.02	0.93	339	0.02	0.92
22	500	10	4	300	0.07	0.82	284	0.02	0.85	279	0	0.86
23	500	25	4	323	0.02	0.8	321	0.01	0.83	317	0	0.83
24	500	50	4	410	0	0.83	433	0.06	0.86	412	0.01	0.85
25	500	10	8	341	0.18	0.86	295	0.02	0.81	289	0	0.85
26	500	25	8	425	0.16	0.83	372	0.01	0.81	367	0	0.83
27	500	50	8	710	0.23	0.89	616	0.07	0.88	577	0	0.88

Table B.2.
Total costs for different values of K, when the safety stock for each scenario is determined from SSAP and performance measures are calculated analytically from formulas in table 2.

Scena-rio	Experimental Factors			EOQ	EOQ	EOQ	EOQ	SM	SM	SM	SM	WW	WW	WW	WW
S{i}	*A*/*h*	σ(*D*)	*L*	*K=50*	*K=100*	*K=300*	*Step 3*	*K=50*	*K=100*	*K=300*	*Step 3*	*K=50*	*K=100*	*K=300*	*Step 3*
1	100	10	0	124.11	123.28	122.75	122.64	121.47	120.46	119.81	119.66	118.12	117.31	116.81	116.74
2	100	25	0	138.17	136.86	136.01	135.78	137.79	136.32	135.34	135.12	137.97	136.75	135.92	135.73
3	100	50	0	168.08	165.05	163.08	163.18	167.67	164.62	162.72	162.89	169.97	166.88	165.01	164.91
4	100	10	4	143.51	142.08	141.16	141.05	141.4	139.88	138.89	138.74	146.83	145.41	144.49	144.45
5	100	25	4	205.26	201.92	199.78	199.95	202.08	198.85	196.74	196.90	211.91	208.69	206.54	206.40
6	100	50	4	321.25	310.82	304.3	306.00	318.53	308.19	301.78	303.47	323.04	312.57	306.08	308.01
7	100	10	8	235.38	230.93	228.1	227.56	168.83	166.46	164.91	164.64	181.95	179.58	178.04	177.28
8	100	25	8	384.46	375.09	369.07	369.83	290.03	284.06	280.19	279.21	299.04	293.19	289.38	287.79
9	100	50	8	639.13	616.71	601.9	607.82	501.03	481.31	468.92	474.87	504.88	486.5	474.24	478.55
10	333	10	0	250.72	249.12	248.08	247.66	241.19	239.29	238.07	237.74	248.1	246.4	245.28	245.06
11	333	25	0	250.95	248.66	247.2	246.68	254.47	251.51	249.62	249.05	250.4	248.3	246.93	246.55
12	333	50	0	277.09	273.35	270.95	270.66	285.56	280.67	277.59	276.63	277.2	273.41	271.04	270.73
13	333	10	4	269.79	267.46	265.97	265.41	257.69	254.82	252.97	252.56	263.31	261.38	260.13	259.86
14	333	25	4	321.49	317.22	314.35	313.56	323	317.73	314.34	313.58	310.98	306.8	304.09	303.78
15	333	50	4	433.18	422.59	416.02	415.80	436.89	425.64	418.69	418.94	422.38	412.11	405.61	406.10
16	333	10	8	323.82	318.13	314.52	313.61	287.32	283.17	280.62	280.31	273.77	270.76	268.79	268.63
17	333	25	8	461.46	450.66	443.83	442.16	412.66	405.02	400.27	398.89	397.47	390.94	386.5	385.07
18	333	50	8	745.83	719.84	703.04	711.38	614.74	596.04	584.04	586.97	589.28	569.8	557.9	562.27
19	500	10	0	302.42	300.53	299.29	298.8	289.26	286.93	285.44	285.08	291.22	289.37	288.16	287.9
20	500	25	0	301.38	298.8	297.14	296.96	300.96	298.06	296.19	295.81	300.51	297.69	295.89	295.5
21	500	50	0	326.86	322.53	319.64	319.05	334.76	329.26	325.67	324.99	335.44	331.15	328.34	327.75
22	500	10	4	324.22	321.69	320.14	319.75	303.45	300.6	298.65	298.12	297.28	294.63	292.92	292.49
23	500	25	4	373.57	369.26	366.47	365.67	364.93	359.53	355.97	355.62	358.73	354.49	351.63	351.3
24	500	50	4	480.01	469.08	461.99	462.73	487.4	474.5	466.2	465.15	473.05	462.23	455.42	454.98
25	500	10	8	371.48	365.93	362.33	361.42	335.62	331.61	329.02	328.64	316.07	312.08	309.44	309.13
26	500	25	8	501.08	489.7	482.59	480.96	456.14	447.62	442	441.14	434.05	427.19	422.67	421.79
27	500	50	8	756.12	729.8	713.03	725.01	671.83	651.59	638.83	643.96	629.85	610.01	597.73	599.83
Max absolute deviation				5.15%	1.90%	1.65%		5.51%	2.01%	1.25%		5.50%	1.88%	0.90%	
Mean absolute deviation				2.88%	1.10%	0.31%		2.96%	1.16%	0.26%		2.71%	1.07%	0.23%	

Table B.3.
Results, when the safety stock for each scenario is determined from SSAP and performance measures are computed from simulation step 3.

Scenario	Experimental Factors			EOQ	EOQ	EOQ	SM	SM	SM	WW	WW	WW
S\{i\}	A/h	$\sigma(D)$	L	C_T	Δ	P_3	C_T	Δ	P_3	C_T	Δ	P_3
1	100	10	0	123	0.05	**0.90**	120	0.03	**0.90**	117	0	**0.90**
2	100	25	0	136	<0.01	**0.90**	135	0	**0.90**	136	<0.01	**0.90**
3	100	50	0	163	<0.01	**0.90**	163	0	**0.90**	165	0.01	**0.90**
4	100	10	4	141	0.02	**0.90**	139	0	**0.90**	144	0.04	**0.90**
5	100	25	4	200	0.02	**0.90**	197	0	**0.90**	206	0.05	**0.90**
6	100	50	4	306	<0.01	**0.90**	303	0	**0.90**	308	0.02	**0.90**
7	100	10	8	228	0.38	**0.90**	165	0	**0.90**	177	0.08	**0.90**
8	100	25	8	370	0.32	**0.90**	279	0	**0.90**	288	0.03	**0.90**
9	100	50	8	608	0.28	**0.90**	475	0	**0.90**	479	<0.01	**0.90**
10	333	10	0	248	0.04	**0.90**	238	0	**0.90**	245	0.03	**0.90**
11	333	25	0	247	<0.01	**0.90**	249	0.01	**0.90**	247	0	**0.90**
12	333	50	0	271	0	**0.90**	277	0.02	**0.90**	271	<0.01	**0.90**
13	333	10	4	265	0.05	**0.90**	253	0	**0.90**	260	0.03	**0.90**
14	333	25	4	314	0.03	**0.90**	314	0.03	**0.90**	304	0	**0.90**
15	333	50	4	416	0.02	**0.90**	419	0.03	**0.90**	406	0	**0.90**
16	333	10	8	314	0.17	**0.90**	280	0.04	**0.90**	269	0	**0.90**
17	333	25	8	442	0.15	**0.90**	399	0.04	**0.90**	385	0	**0.90**
18	333	50	8	711	0.27	**0.90**	587	0.04	**0.90**	562	0	**0.90**
19	500	10	0	299	0.05	**0.90**	285	0	**0.90**	288	0.01	**0.90**
20	500	25	0	297	<0.01	**0.90**	296	<0.01	**0.90**	296	0	**0.90**
21	500	50	0	319	0	**0.90**	325	0.02	**0.90**	328	0.03	**0.90**
22	500	10	4	320	0.09	**0.90**	298	0.02	**0.90**	292	0	**0.90**
23	500	25	4	366	0.04	**0.90**	356	0.01	**0.90**	351	0	**0.90**
24	500	50	4	463	0.02	**0.90**	465	0.02	**0.90**	455	0	**0.90**
25	500	10	8	361	0.17	**0.90**	329	0.06	**0.90**	309	0	**0.90**
26	500	25	8	481	0.14	**0.90**	441	0.05	**0.90**	422	0	**0.90**
27	500	50	8	725	0.21	**0.90**	644	0.07	**0.90**	600	0	**0.90**