



## From p+p to Pb+Pb Collisions: Wounded Nucleon versus Statistical Models

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System size dependence of hadron production properties is discussed within the Wounded Nucleon Model and the Statistical Model in the grand canonical, canonical and micro-canonical formulations. Similarities and differences between predictions of the models related to the treatment of conservation laws are exposed. A need for models which would combine a hydrodynamical-like expansion with conservation laws obeyed in individual collisions is stressed.

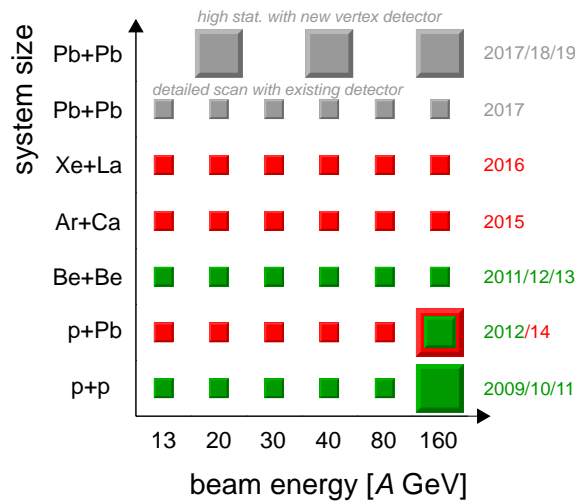
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\*Speaker.



**Figure 1:** The NA61/SHINE data taking schedule for the ion program. The already registered reactions are indicated by green squares, whereas the approved future data taking and the proposed extension of the program are shown in red and gray, respectively.

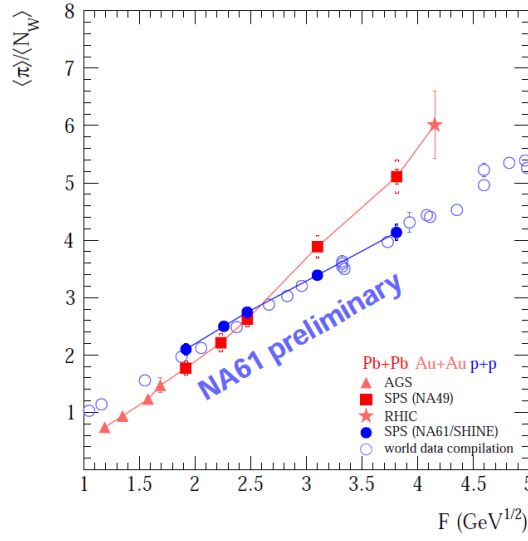
## 1. Introduction

This work is motivated by the NA61/SHINE ion program [1, 2] in which hadron production properties are studied as a function of size of colliding nuclei and their collision energy. The status and plans of the NA61/SHINE data taking within this program are presented in Fig. 1. The main goals of this program are the study of the onset of deconfinement and the search for the critical point of strongly interacting matter. They can be reached providing "trivial" phenomena which affect the system size dependence and collision energy dependence are sufficiently understood. In view of this I discuss here first the influence of fluctuations of the system size, and second the role of conservation laws on the system size dependence. Two models are selected for this purpose: the Wounded Nucleon Model [3] and the Statistical Model [4]. This is because they play a special role in physics of heavy ion collisions, that is they are forefathers of the currently most popular approaches, the hydrodynamical and string-hadronic models. Moreover, they continuously serve as the basic tools to interpret results on hadron production in high energy collisions. This is due to them being simple and approximately reproducing several basic properties of the data. They were formulated before the QCD era and their relation to QCD still remains unclear (for a brief historical review of multi-particle production in high energy collisions see Ref. [5]).

The paper is organized as follows. First the models are introduced, then the similarities and differences in their predictions concerning the system size dependence are discussed. Closing remarks conclude the paper.

## 2. The Wounded Nucleon Model

The Wounded Nucleon Model (WNM) was proposed by Bialas, Bleszynski and Czyz [3] in 1976 as a late child of the S-matrix period [5]. It assumes that particle production in nucleon-nucleon and nucleus-nucleus collisions is an incoherent superposition of particle production from



**Figure 2:** The ratio of mean pion multiplicity to the mean number of wounded nucleons for p+p interactions and central Pb+Pb collisions plotted as a function of collision energy [7] ( $F \approx s_{NN}^{1/4}$ ). In the SPS energy region shown in the plot, the dependences for central Pb+Pb collisions and p+p interactions cross each other at about 40A GeV. At this energy the prediction of the WNM (see text) is strictly valid.

wounded nucleons (nucleons which interacted inelastically and whose number is calculated using the Glauber approach). Properties of wounded nucleons are independent of the size of colliding nuclei, e.g. they are the same in p+p and Pb+Pb collisions at the same collision energy per nucleon.

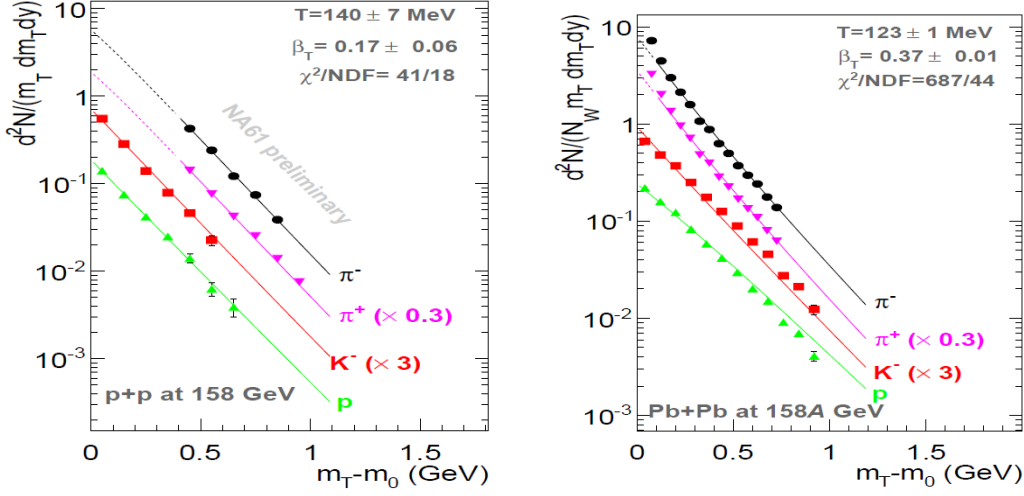
The most famous prediction of the model reads:

$$\langle A \rangle / \langle W \rangle = \langle A \rangle_{NN} / 2, \quad (2.1)$$

where  $\langle A \rangle$  and  $\langle A \rangle_{NN}$  is the mean multiplicity of hadron  $A$  in nucleus-nucleus collisions and nucleon-nucleon interactions, respectively, whereas  $\langle W \rangle$  and 2 are mean numbers of wounded nucleons in nucleus-nucleus and nucleon-nucleon collisions. This prediction is approximately valid for the most copiously produced hadrons, pions. This is demonstrated in Fig. 2, where the ratio of mean pion multiplicity to the mean number of wounded nucleons in p+p interactions and central Pb+Pb collisions is plotted as a function of collision energy. From the low SPS to the top RHIC (not shown in the plot) energies the ratio for p+p and Pb+Pb collisions differs "only" by about 30% [6]. This is why the WNM and its string-hadronic successors are still in use.

### 3. The Statistical Model

The Statistical Model of multi-particle production (SM) was initiated by Fermi in 1950 [4]. Its basic assumption states that all possible micro-states of the macroscopic system created in a collision are equally probable. For large enough systems (e.g. central Pb+Pb collisions) and abundantly produced hadron species (e.g. pions, kaons and protons) the grand canonical formulation of the model, SM(GCE), can be used for particle inclusive spectra and mean multiplicities. This leads to



**Figure 3:** Examples of transverse mass spectra of the most abundant hadrons produced in p+p interactions (*left*) and central Pb+Pb collisions (*right*) at 158A GeV [7]. In p+p interactions the spectra are approximately exponential with the inverse slope parameter, which is the same for different hadrons as predicted by the Statistical Model. The spectra in central Pb+Pb collisions deviate somewhat from the prediction, which is interpreted as due to non-equilibrium effects as the collective expansion of matter before freeze-out.

the most famous prediction of the model, namely:

$$d^3n/dp^3 \sim V \cdot e^{-E/T}, \quad (3.1)$$

where  $d^3n/dp^3$  is the particle momentum distribution and  $V$ ,  $T$  and  $E$  are system volume, its temperature and particle energy, respectively. This prediction agrees approximately with experimental data if spectra in the direction transverse to the collision axis are considered. Namely, distributions in the transverse mass of hadrons,  $m_T = \sqrt{p_T^2 + m^2}$ , are measured to be approximately exponential for  $m_T \leq 2$  GeV ( $p_T$  and  $m$  denote particle transverse momentum and mass, respectively). This is shown in Fig. 3, where examples of the transverse mass spectra in p+p interactions (*left*) and central Pb+Pb collisions (*right*) at 158A GeV are presented. This is why the SM(GCE) and its hydrodynamical successors are in use today.

#### 4. System size dependence: Similarities

In the WNM model hadrons are produced from "decays" of independent wounded nucleons. Similarly in the SM(GCE) model they are produced from "decays" of independent volume elements. This leads to almost identical predictions concerning the system size dependence of hadron production properties in both models. The detailed discussion of this conclusion is presented below using as an example multiplicities of two different types of hadrons,  $A$  and  $B$ . Two multiplicities are needed in order to express the model predictions on the system size dependence so that they are independent of the system size and its fluctuations. It obviously simplifies a comparison between the models themselves as well as between the latter and experimental data.

First let us consider mean multiplicities,  $\langle A \rangle$  and  $\langle B \rangle$ . In the WNM they are proportional to the number of wounded nucleons:

$$\langle A \rangle \sim W, \quad \langle B \rangle \sim W, \quad (4.1)$$

and in the SM(GCE) they are proportional to the system volume:

$$\langle A \rangle \sim V, \quad \langle B \rangle \sim V. \quad (4.2)$$

Obviously the ratio of mean multiplicities is independent of the system size parameter,  $W$  and  $V$ , in the WNM and the SM(GCE), respectively. Moreover, it is easy to show that it is also independent of the system size parameter fluctuations. Thus, the ratio  $\langle A \rangle / \langle B \rangle$  is independent of  $P(W)$  and  $P(V)$ , where  $P(W)$  and  $P(V)$  are probability (density) distributions of  $W$  and  $V$ , respectively, for the considered set of collisions. Quantities which have the latter property are called strongly intensive quantities [8]. Such quantities should be used to study the system size dependence as they eliminate influence of usually poorly known distributions of the system size parameters,  $W$  and  $V$ .

Second we consider multiplicity fluctuations characterized by second moments of multiplicity distributions. Two quantities of relevance are variance,  $Var[X] = \langle (X - \langle X \rangle)^2 \rangle$  and scaled variance,  $\omega[X] = Var[X] / \langle X \rangle$ , where  $X$  stands for  $A$  or  $B$ .

The scaled variances of  $A$  and  $B$  and the mixed second moment  $\langle AB \rangle$  calculated within the WNM read [8]:

$$\omega[A] = \omega^*[A] + \langle A \rangle / \langle W \rangle \cdot \omega[W], \quad (4.3)$$

$$\omega[B] = \omega^*[B] + \langle B \rangle / \langle W \rangle \cdot \omega[W], \quad (4.4)$$

$$\langle AB \rangle = \langle AB \rangle^* \langle W \rangle + \langle A \rangle \langle B \rangle \langle W \rangle^2 \cdot (\langle W^2 \rangle - \langle W \rangle^2). \quad (4.5)$$

As may be expected they have a similar form in the SM(GCE) [8]:

$$\omega[A] = \omega^*[A] + \langle A \rangle / \langle V \rangle \cdot \omega[V], \quad (4.6)$$

$$\omega[B] = \omega^*[B] + \langle B \rangle / \langle V \rangle \cdot \omega[V], \quad (4.7)$$

$$\langle AB \rangle = \langle AB \rangle^* \langle V \rangle + \langle A \rangle \langle B \rangle \langle V \rangle^2 \cdot (\langle V^2 \rangle - \langle V \rangle^2). \quad (4.8)$$

In Eqs. 4.3-4.8 quantities denoted by \* are quantities calculated for any fixed value of the system size parameter,  $W$  and  $V$ , within the WNM and the SM(GCE), respectively.

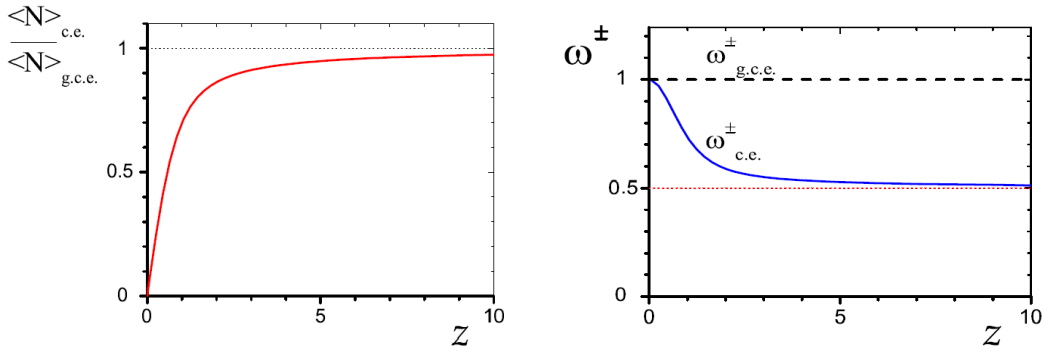
From Eqs. 4.3-4.5 and Eqs. 4.6-4.8 follows [8, 9] that properly constructed functions of the second moments, namely

$$\Delta[A, B] = (\langle B \rangle \omega[A] - \langle A \rangle \omega[B]) / (\langle B \rangle - \langle A \rangle) \quad (4.9)$$

and

$$\Sigma[A, B] = (\langle B \rangle \omega[A] + \langle A \rangle \omega[B] - 2(\langle AB \rangle - \langle A \rangle \langle B \rangle)) / (\langle B \rangle + \langle A \rangle) \quad (4.10)$$

are independent of  $P(W)$  and  $P(V)$  in the WNM and the SM(GCE), respectively. Thus  $\Delta[A, B]$  and  $\Sigma[A, B]$  are strongly intensive quantities which measure fluctuations, i.e. they are sensitive to second moments of  $A$  and  $B$  distributions. The  $\Sigma$  quantity is a reincarnation of the popular  $\phi$  measure of fluctuations [10]. As the quantities  $\Delta[A, B]$ ,  $\Sigma[A, B]$  and  $\phi$  are strongly intensive their



**Figure 4:** The ratio of mean multiplicities (*left*) and scaled variances (*right*) calculated within the SM(CE) and the SM(GCE) are plotted as a function of mean multiplicity  $z$  for the SM(GCE), the latter being proportional to the system volume. The ideal gas model of classical positively and negatively charged particles is used for calculations.

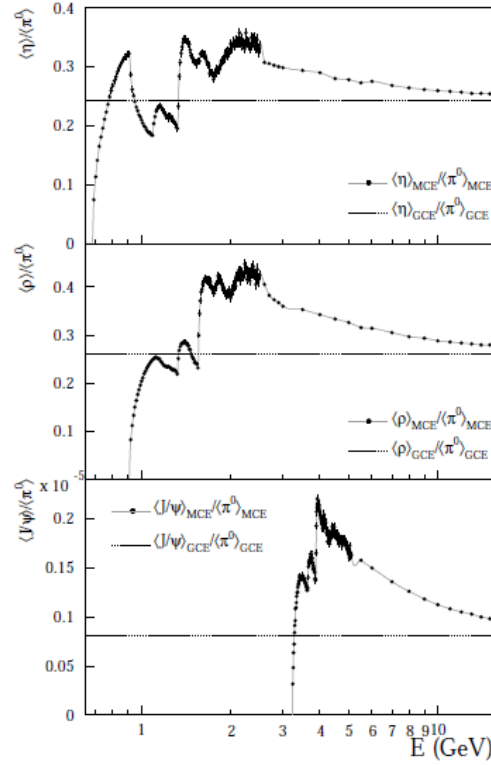
values in p+p and Pb+Pb collisions are equal. Of course, this is valid only within the WNM and the SM(GCE). At this conference preliminary experimental results on the  $\phi$  quantity in p+p and central Pb+Pb collisions at the CERN SPS energies were presented [11]. They concern three choices of  $A$  and  $B$  hadron multiplicities, namely  $[A, B] = [K, \pi]$ ,  $[p, \pi]$  and  $[K, p]$ , where  $K$ ,  $\pi$  and  $p$  denote multiplicities of charged kaons, pions and protons, respectively. Noticeably, the results for p+p interactions are close to the ones for central Pb+Pb collisions.

In summary, predictions of the WNM and the SM(GCE) concerning the system size dependence are very similar. In particular, the models predict that the mean multiplicity ratio as well as the  $\Delta$ ,  $\Sigma$  and  $\phi$  quantities are independent of the system size and its fluctuations. The number of wounded nucleons - the system size parameter in the WNM - is discrete, whereas the volume - the system size parameter in the SM(GCE) - is continuous. This may lead to somewhat different predictions for quantities which are dependent on the system size parameter distribution.

## 5. System size dependence: Differences

Predictions of the Statistical Model concerning the volume dependence change qualitatively when material and/or motional conservation laws are introduced, i.e. instead of the grand canonical ensemble, the canonical (CE) or micro-canonical (MCE) ensembles are used. The effect of conservation laws has been extensively studied for mean multiplicities since 1980 (see e.g. Refs. [12, 13, 14]) and for second moments of multiplicity distributions since 2004 (see e.g. Refs. [15, 16]). Three examples are presented below to illustrate the main results.

Figure 4 taken from Ref. [15] presents the results of calculations performed within the simplest model which allows to study the effect of the material conservation laws on mean multiplicity and scaled variance of the multiplicity distribution. In the model the ideal gas of classical positively and negatively charged particles is assumed. The ratio of the mean multiplicities calculated within the SM(CE) and the SM(GCE) is plotted in Fig. 4 (*left*) as a function of the mean multiplicity from the SM(GCE), the latter being proportional to the system volume. The ratio approaches one with increasing volume. Thus for sufficiently large systems mean multiplicities obtained within the



**Figure 5:** Ratios of mean multiplicities of neutral hadrons calculated within the MCE and GCE formulations of the hadron-resonance gas model are plotted as a function of the system energy, the latter being proportional to the system volume in the GCE.

SM(GCE) can be used instead of mean multiplicities from the SM(CE) and the SM(MCE) [16]. This is however not the case for the scaled variance as illustrated in Fig. 4 (right). The results for the SM(CE) and the SM(GCE) approach each other when the volume decreases to zero. Of course, the scaled variance in the SM(GCE) is one independent of volume. Different behaviour is observed for the scaled variance in the SM(CE), it decreases with increasing volume and for a sufficiently large volume approaches 0.5.

Finally, the comparison of mean hadron multiplicities calculated with the GCE and MCE formulations of the hadron resonance gas model [17] is shown in Fig. 5. For small energies of the system (in the GCE energy is proportional to volume) the multiplicity ratios strongly depend on system energy and volume. The observed peaks and dips are correlated with the thresholds for the production of various hadrons.

## 6. Closing remarks

1. I demonstrated that the predictions of the Wounded Nucleon Model and the Statistical Model in the grand canonical formulation concerning the system size dependence are almost identical. Differences may appear only because the system size parameter is discrete in the WNM (the number of wounded nucleons) and continuous in the SM(GCE) (the system volume).

The similarity of predictions explain why these models and their successors (string-hadronic and hydrodynamical models) can co-exist.

2. We all believe that material and motional conservation laws are obeyed in each high energy collision. Within the Statistical Model their influence can be studied by a comparison of the model predictions obtained using different ensembles, GCE, CE and MCE. The result is that the predictions are strongly dependent on conservation laws. For small systems mostly mean multiplicities are affected whereas for large systems scaled variances are modified. Thus the GCE formulation of the Statistical Model is neither valid for small nor for large systems when both mean multiplicities and fluctuations are of relevance.
3. Rich experimental data on hadron spectra in heavy ion collisions favor hydrodynamical models. In these models, similarly to the SM(GCE), conservation laws are obeyed only for averages over many collisions. This significantly limits their applicability. Thus, it seems to be urgent to develop models which would combine a hydrodynamical-like expansion with conservation laws obeyed in individual collisions. Fortunately this effort has already started [18].

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