

Modelling of plasmas used for etching semiconductors

Problem presented by

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Problem statement

A process used by Trikon to etch a semiconductor comprises a plasma chamber in which radicals and electrons are produced through ionisation in a plasma. These then migrate onto a silicon wafer, leading to etching through chemical action. Power for this process comes from an external radio-frequency source which causes skin heating of the gas inside the chamber. The gas in the device is primarily oxygen, with a flow rate that can be controlled as part of the process. The main questions posed to the Study Group were how the process depends on the gas flow, the RF heating power and the gas pressure.

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1 Introduction

A process used by Trikon to etch a semi-conductor comprises a plasma chamber in which radicals and electrons are produced through ionisation in a plasma. These then migrate onto a silicon wafer, leading to etching through chemical action. Power for this process comes from an external radio-frequency source which causes skin heating of the gas inside the chamber. The resulting heat is then transferred to the plasma through conduction, leading to an electron temperature T_e . The gas in the devices studied is primarily oxygen O_2 which has a flow rate λ that can be controlled as part of the process. The heating of this gas in its plasma state leads both to ionisation, forming electrons e and positive ions O^+ , and bond breaking of the molecules leading to neutral oxygen O radicals, which can then recombine. (There are many other related reactions as well.) It is the electrons e and radicals O that migrate to the wafer, which can be regarded as a sink of charged particles. More precisely we can identify **sources** of ions, radicals and electrons through the following processes:

- *Bond breaking:* $O_2 + e \rightarrow O + O + e$
- *Thermal ionisation:* $O_2 + e \rightarrow O_2^+ + e + e$; $O + e \rightarrow O^+ + e + e$
- Other species ... *e.g.* -ve ions (not considered)

and **sinks** via

- *Recombination:* $O + O \rightarrow O_2$; $O_2^+ + e \rightarrow O_2$; $O^+ + e \rightarrow O$
- Boundary losses at the wafer
- Pumping of the gas out of the container.

We illustrate these processes in Figure 1.

The main questions that Trikon wanted answering in the Study Group were how the process depends on

- the gas flow λ ,
- the RF heating power P ,
- the gas pressure, which is directly related to the available number of oxygen molecules for ionisation.

Our investigation comprised two parts. Firstly we looked at some detailed models of the transport of the electrons and the oxygen ions and their associated field E within the plasma. Secondly, we considered a simplified diffusion model of this process to approximate the global distribution of the electrons and oxygen radicals in a simple one-dimensional approximation to the plasma chamber.

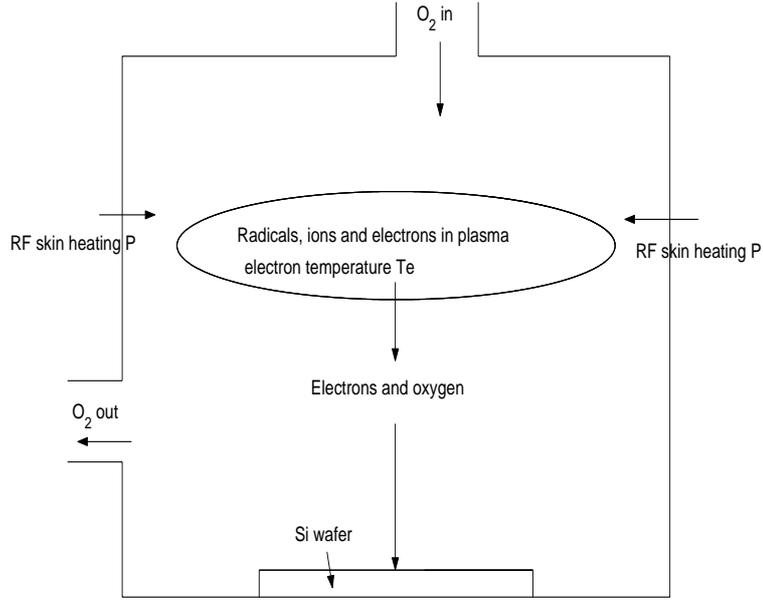


Figure 1: A diagram of the plasma chamber

2 Modelling the transport of ions and electrons in the plasma

In this section we write down a simple model for the transport of oxygen ions (O^+) and electrons inside the chamber (the model is easily generalised to include other charged species). Since the plasma is fairly weakly ionised (about 1%) we neglect ‘coulomb collisions’, whereby charged particles interact over short ranges via their electric fields, and assume that the dominant collisional interactions are with the uncharged oxygen molecules (which form about 99% of the plasma). This assumption leads us to model a scenario in which ions (and electrons) diffuse around the plasma chamber being scattered off oxygen molecules but have a net drift due to the action of the ‘averaged’ electric field. Here the electric field is averaged over many charged particles to smooth out the small scale variations that give rise to coulomb collisions.

In the modelling process we shall use the following variables and parameters:

- n_I : number density of oxygen ions,
- n_e : number density of electrons,
- v_I : velocity of ions in the absence of diffusion,
- v_e : velocity of electrons in the absence of diffusion,
- D_I : diffusion coefficient of the ions,
- D_e : diffusion coefficient of the electrons,
- F : reaction rate for the creation of ions and electrons,
- T : absolute temperature of the plasma,
- T_e : electron temperature,

x : distance from the electrode (silicon wafer),
 E : magnitude of the ‘averaged’ electric field,
 ε_0 : the permittivity of free space,
 q : the charge on an O^+ ion,
 m^+ : the mass of an O^+ ion.

We start by writing down equations for the conservation of oxygen ions and electrons:

$$-F + \frac{\partial n_I}{\partial t} = \frac{\partial}{\partial x} \left(D_I \frac{\partial n_I}{\partial x} \right) - \frac{\partial}{\partial x} (v_I n_I), \quad (1)$$

$$-F + \frac{\partial n_e}{\partial t} = \frac{\partial}{\partial x} \left(D_e \frac{\partial n_e}{\partial x} \right) - \frac{\partial}{\partial x} (v_e n_e). \quad (2)$$

These are coupled to Poisson’s equation for the electric field,

$$\frac{\partial}{\partial x} (\varepsilon_0 E) = q(n_I - n_e), \quad (3)$$

and equations which give the ion and electron drift velocities in terms of the electric field E . It is usual to equate the force on an ion (or electron) with a linear drag term such that $\mu_I v_I = qE$, where μ_I is the mobility coefficient. However, the derivation of this linear drag law assumes that collisions between an ion and the neutral gas molecules occur with average frequency ν independent of the drift velocity. This is equivalent to assuming that the thermal velocity of the oxygen molecules is much higher than the drift velocity of the ion. A rough estimate for this mobility coefficient based on an ion uniformly accelerating in an electric field and losing all its momentum in collisions at uniform frequency ν gives $\mu_I = C m_I \nu$, where m_I is the mass of the ion and C an order one constant. At the other extreme, if we assume that the drift velocity of the ion is much greater than the thermal velocity of the oxygen molecules, then it follows that the average distance d_I between collisions is independent of the drift velocity of the ions. This leads to a drag law of the form $\gamma_I v_I |v_I| = qE$. The coefficient γ_I may be estimated (roughly) by looking at the motion of an ion accelerating in an electric field and losing all its momentum in collisions separated by distance d_I . This leads to the result $\gamma_I = B m_I / d_I$, where B is an order one dimensionless constant. An order of magnitude estimate indicates that the thermal velocity of an oxygen molecule is comparable with the drift velocity of an ion and so we opt to model the drag on an ion by the law

$$\mu_I v_I + \gamma_I v_I |v_I| = qE. \quad (4)$$

Similarly in the case of an electron we obtain a law of the form

$$\mu_e v_e + \gamma_e v_e |v_e| = -qE. \quad (5)$$

The preceding arguments then suggest that $\mu_e \approx \mu_I m_e / m_I$ and $\gamma_e \approx \gamma_I m_e / m_I$ since d and ν will be approximately the same for electrons and ions.

In a similar vein it is worth noting that the diffusion coefficients D_I and D_e cannot be derived using the Einstein relation because the thermal velocity of the ions is comparable to, and the thermal velocity of the electrons is much greater than, that of the oxygen

molecules. Thus for an electron the mean free path d_e is determined by the density of oxygen molecules through the relation

$$d_e = \frac{1}{\sigma_c n_{O_2}} \quad (6)$$

where σ_c is the collision cross section of an oxygen molecule (roughly the area a molecule presents for collisions) and n_{O_2} is the number density of oxygen molecules in the gas (see The Feynman Lectures on Physics [1] Vol 1 formula (43.12)). This is in turn related to the diffusion coefficient for electrons D_e by

$$D_e = \frac{1}{3} d_e V_e \quad \text{where} \quad \frac{1}{2} m_e V_e^2 = \frac{3}{2} k T_e. \quad (7)$$

Here V_e is the thermal velocity of the electrons and k is Boltzmann's constant (see Feynman [1] Vol 1 formula (43.27)).

2.1 Nondimensionalisation of the model

We nondimensionalise the model comprising equations (1)–(5) as follows:

$$\begin{aligned} t = \tau t^*, \quad x = L x^*, \quad n_e = I_0 n_e^*, \quad n_I = I_0 n_I^*, \quad F = F_0 F^*, \\ v_e = \frac{D_e}{L} v_e^*, \quad v_I = \frac{D_e}{L} v_I^*, \quad E = \frac{\gamma_e D_e^2}{L^2 q} E^*, \end{aligned} \quad (8)$$

where τ is the typical timescale for the etching process, $2L$ is the height of the plasma chamber, I_0 is the typical number density of ions and F_0 is the typical reaction rate. We then write

$$v_I^* = \left(\frac{\gamma_e}{\gamma_I} \right)^{1/2} \bar{v}_I. \quad (9)$$

The dimensionless model is

$$-\lambda F^* + \delta \frac{\partial n_e^*}{\partial t^*} = \frac{\partial^2 n_e^*}{\partial x^{*2}} - \frac{\partial}{\partial x^*} (v_e^* n_e^*), \quad (10)$$

$$|v_e^*| v_e^* + \frac{\varepsilon^2}{\epsilon} \Gamma v_e^* = -E^*, \quad (11)$$

$$-\frac{\lambda}{\epsilon} F^* + \frac{\delta}{\epsilon} \frac{\partial n_I^*}{\partial t^*} = \frac{1}{\alpha \epsilon} \frac{\partial^2 n_I^*}{\partial x^{*2}} - \frac{\partial}{\partial x^*} (\bar{v}_I^* n_I^*), \quad (12)$$

$$\bar{v}_I^* |\bar{v}_I^*| + \Gamma \bar{v}_I^* = E^*, \quad (13)$$

$$\frac{\partial E^*}{\partial x^*} = \Lambda (n_I^* - n_e^*), \quad (14)$$

where we make use of the following dimensionless parameters:

$$\lambda = \frac{F_0 L^2}{D_e I_0}, \quad \delta = \frac{L^2}{D_e \tau}, \quad \Lambda = \frac{q^2 L^3 I_0}{\varepsilon_0 \gamma_e D_e^2}, \quad \varepsilon = \left(\frac{\mu_e}{\mu_I} \right)^{1/2},$$

$$\epsilon = \left(\frac{\gamma_e}{\gamma_I}\right)^{1/2}, \quad \Gamma = \frac{\mu_I L}{(\gamma_e \gamma_I)^{1/2} D_e}, \quad \alpha = \frac{D_e}{D_I}. \quad (15)$$

A crude estimation of the dimensionless parameters, based on a pressure of 1 Nm^{-2} , a gas temperature of 1000 K , a lengthscale of 10^{-1} m and an electron temperature of 5 eV , gives

$$\Gamma \sim O(1), \epsilon \sim O(10^{-3}), \varepsilon \sim O(10^{-3}), \delta \sim O(10^{-7}), \Lambda \sim O(10^{12}), \alpha \sim O(10^4). \quad (16)$$

This would suggest that we retain the following terms in the model:

$$\frac{\partial^2 n_e}{\partial x^2} - \frac{\partial}{\partial x} (v_e n_e) = 0 \quad (17)$$

$$v_e |v_e| = -E, \quad (18)$$

$$\frac{1}{\alpha \epsilon} \frac{\partial^2 n_I}{\partial x^2} - \frac{\partial}{\partial x} (\bar{v}_I n_I) = -\frac{\lambda}{\epsilon} F, \quad (19)$$

$$|\bar{v}_I| \bar{v}_I + \Gamma \bar{v}_I = E, \quad (20)$$

$$n_I = n_e, \quad (21)$$

where we drop the stars. Here we have ended up with a quasistationary model. However we have lost the highest derivative in E and to satisfy all the boundary conditions on the model we must match the solutions of the simplified model to an inner region (or Debye layer) in which the $\partial E / \partial x$ term in (14) again becomes important. In order to solve the simplified model we substitute $n_I = n_e$ into (19) and then eliminate $\partial^2 n_e / \partial x^2$ between (17) and (19); this gives an equation for E which can be solved. The solution for E is then substituted back into (18), which is then solved for n_e .

3 Global models of electron and radical transport

3.1 A simple model

For the remainder of this report we will consider a simplification of the model proposed above for the motion of the electrons, together with a model for the motion of the oxygen radicals. In this simplification we make the following assumption of a diffusion-dominated plasma in which rapid motions of electrons due to local field imbalances are neglected, and we look only at the relatively slow motion of the electrons, ions and radicals as they move together. Explicitly, we assume

1. The local distributions of the positive and negative charges balance, leading to a locally zero field, with all motion of the particles due to diffusion.
2. The electrons follow the positive ions and diffuse (macroscopically) at a similar rate $2D_I$.
3. The density n_{O_2} of oxygen molecules remains (approximately) constant and is a function of the gas pressure.

Accordingly we consider a diffusion model in which free electrons and neutral radicals are created by ionisation and bondbreaking, and are lost through recombination and the effects of pumping and boundary condition. This leads to the following system of equations for the densities of electrons, radicals and molecules given by n_e , n_O and n_{O_2} .

Motion of the oxygen radicals

$$-D_O n''_O = B_O(T_e) n_{O_2} n_e - k_{OO} n_O^2 - K_2(T_e) n_O n_e - \lambda n_O \quad (22)$$

Motion of the electrons

$$-2D_I n''_e = K_1(T_e) n_{O_2} n_e - k_{ee} n_e^2 + K_2(T_e) n_O n_e. \quad (23)$$

Note that the effect of the motion of the oxygen ions has been absorbed into the equation for the motion of the electrons via the enhanced diffusion term. In this system the right hand side gives the various source and recombination terms. In particular we have

- Bond-breaking: $B_O(T_e) n_{O_2} n_e$.
- Thermal ionisation: $K_1(T_e) n_{O_2} n_e$ and $K_2(T_e) n_O n_e$.
- Recombination: $k_{ee} n_e^2$ and $k_{OO} n_O^2$ (since the ion density is taken equal to the electron density, the ion-electron recombination rate is proportional to n_e^2).
- Pumping: λn_O .

In these expressions the various ionisation coefficients $B_O(T_e)$, $K_1(T_e)$ *etc.* are Arrhenius functions of the electron temperature T_e , so that they are very low if T_e is lower than some threshold value (around 5 eV) and then rise rapidly to a saturated value above this threshold.

3.2 Boundary conditions

The boundary conditions in this model are important and represent the dominant cause of electron depletion in the plasma. Very close to the boundary, in the so-called ‘dark space’ region, a complicated set of physical processes take place as electrons move rapidly. However, this occurs only in a thin boundary layer, away from which we have the behaviour described in the previous section, where the electrons move with the ions. At the edge of the boundary layer the plasma obeys the following set of derived boundary conditions called the Bohm conditions:

$$-D_I n'_e = \sqrt{\frac{kT_e}{m^+}} n_e \equiv V_{B_e} n_e \quad \text{and} \quad -D_O n'_O = \sqrt{\frac{kT_g}{m^+}} n_O \equiv V_{B_g} n_O. \quad (24)$$

Here the Bohm velocity is given by $V_{B_e} \gg V_{B_g}$ and $T_e \gg T_g$.

3.3 Some scalings

To make progress with this simple model we introduce some scaling based on the observed physical values of certain of the terms. In particular we have the following:

- The dimensions of the chamber are given by $[-L, L]$ where L is approximately 20 cm.
- The Bohm electron velocity at the boundary is $V_{B_e} \approx 10^6 \text{ cm s}^{-1} \gg V_{B_g}$.
- The diffusion coefficients are $D_O \approx D_I \approx 3 \times 10^6 \text{ cm}^2 \text{ s}^{-1}$.
- The pumping rate is given by $\lambda \approx 100\Lambda$ where Λ is of order unity.
- The electron temperature is given by $T_e \approx 5 \text{ eV}$.
- The observed relative sizes of the molecular, radical and electron densities are

$$n_{O_2} \approx 10^{14-15} \text{ cm}^{-3}, \quad n_O \approx 10^{12} \text{ cm}^{-3}, \quad n_e \approx 10^{11} \text{ cm}^{-3}.$$

We can also obtain estimates for the sizes of the various Arrhenius factors which control the *ionisation* and *recombination* effects. These are given by

Ionisation

- $B_O(T_e) = 10^{-11} \alpha_0 e^{-5/T_e}$ where α_0 is of order unity.
- $K_1 \approx K_2 \approx 10^{-11} \alpha_1 e^{-10/T_e}$ where α_1 is of order unity.

These are illustrated in Figure 2.

Recombination

It is observed that in the absence of pumping (so that $\Lambda = 0$) the n_O ions have a half-life of 10^{-3} s. Solving the equation

$$n'_O = -k_{OO} n_O^2$$

which describes the loss due to recombination, we have that an initial density of $n_O(0)$ decreases by half in a time τ given by $\tau = 1/(k_{OO} n_O(0))$. Accordingly we may estimate k_{OO} by $1/(\tau n_O(0))$ so that $k_{OO} \approx 10^3 \cdot 10^{-12}$ and $k_{ee} \ll k_{OO}$.

Using these values we may rescale the various coefficients. In particular we set

$$n_{O_2} = 10^{15} N_{O_2}, \quad n_e = 10^{11} N_e, \quad n_O = 10^{12} N_O. \quad (25)$$

This then leads to the rescaled equations

$$-D_O N''_O = 10^3 \alpha_0 e^{-5/T_e} N_e - \alpha_1 e^{-10/T_e} N_e N_O - 100 \Lambda N_O - 10^3 N_O^2 \quad (26)$$

$$-2D_I N''_e = 10^4 \alpha_1 e^{-10/T_e} N_e + 10 \alpha_1 e^{-10/T_e} N_e N_O - (\text{small}). \quad (27)$$

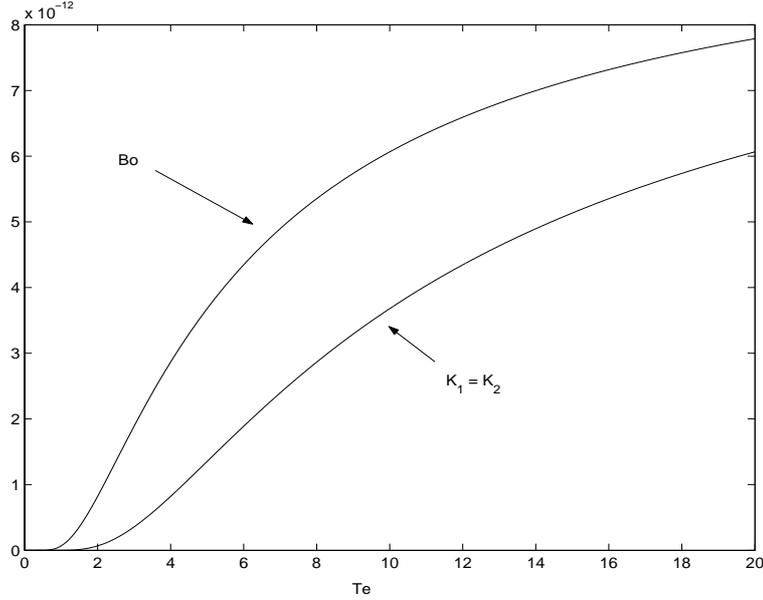


Figure 2: The ionisation coefficients B_O and $K_1 = K_2$ plotted as functions of T_e in electron volts.

3.4 A simplified equation for electrons and radicals

Consider first averaging the equation for the electron density. Integrating over the length of the cavity gives

$$[-2D_I N'_e]_L^L \equiv 2V_{B_e} N_e = L \left[10^4 \alpha_1 \overline{e^{-10/T} N_e} + \overline{10 \alpha_1 N_e N_O e^{-10/T_e}} \right]. \quad (28)$$

Looking at the relative sizes of the terms, the effect of the boundary conditions is of the order of $10^6 N_e$ which is comparable in size to the number of electrons created in ionisation and related processes. In contrast the number of electrons lost through recombination and pumping is around $10^3 N_e$ which is much smaller. The dominant loss of the electrons created in the plasma is thus due to the effects of the boundary terms, *i.e.* most of the created electrons are arriving at the work-piece.

In contrast we can also consider averaging the equation for the radical concentration N_O . This gives

$$[-2D_O N'_O]_L^L \equiv 2V_{B_g} N_O = L \left[10^3 \alpha_0 \overline{e^{-5/T} N_e} - 10^2 \Lambda \bar{N}_O - 10^3 \bar{N}_O^2 \right]. \quad (29)$$

Estimating the size of the coefficients in this equation we see that the effect of the boundary losses is *small*, and that there is a dominant balance between the number of radicals created within the plasma and the various internal sinks. In this case rather fewer radicals reach the work-piece. Balancing the source and sink terms throughout the plasma gives a simple relation between the electrons and the radicals:

$$N_O \approx \frac{\alpha_0 e^{-5/T_e} N_e}{N_O + \Lambda/10}. \quad (30)$$

The radical concentration can thus be estimated once we have solved the (scaled)

differential equation for the electron densities. Keeping only the dominant terms this equation is given by

$$\begin{aligned} -2D_I N_e'' &\approx 10^4 \alpha_1 e^{-10/T_e} N_e & \text{for } -L < x < L \\ -D_I N_e' &= V_{B_e} N_e & \text{at } x = \pm L \end{aligned} \quad (31)$$

Observe that the diffusive length scale is given by $\sqrt{10^4 \alpha_1 e^{-10/T_e} / 2D_I}$. For a value of $D_I = 3 \times 10^6$ and T_e of order 5 eV this gives a diffusive length scale of around 10 cm, which is comparable to L .

3.5 Temperature

We now consider the equation for the electron temperature T_e (measured in electron volts), noting that it is not always easy to define a temperature for a rarefied plasma! (see Lieberman and Lichtenberg [2] for a discussion of this). We consider there to be a generation of thermal energy through ohmic heating P within the body of the plasma and heat loss through conduction and ionisation. There is an additional source of heat which arises from the radical terms which are neglected in the one-dimensional approximation. Averaging the temperature profile over the radius of the plasma this leads to a source term proportional to T_e arising from boundary effects. In Lieberman and Lichtenberg [2] it is proposed that there is an additional source term $5T_e D_I n_e''$ due to variations in the electron density.

Putting this together and looking for a thermal balance we have

$$-kT_e'' = hT_e + P - (5\alpha_0 e^{-5/T_e} + 10\alpha_1 e^{-10/T_e}) \times 10^{-11} n_{O_2} n_e + 5T_e D_I n_e'' \quad (32)$$

If we consider the relative sizes of these different terms we have from experimental measurements that

$$P \approx 10^{15} Q \text{ eVcm}^{-3}, \quad \text{where } Q = O(1). \quad (33)$$

Also

$$n_e n_{O_2} \times 10^{-11} \approx 10^{15} N_e N_{O_2} \quad (34)$$

and, from the differential equation for N_e ,

$$-5D_I n_e'' = -2.5 \times 10^{11} D_I N_e'' \approx -2.5 \times 10^{15} \alpha_1 e^{-10/T_e} N_e, \quad (35)$$

so that the contributions to the thermal balance equation due to electron diffusion and due to ionisation via the term $N_e N_{O_2}$ are of order 10^{15} and roughly balance. If we substitute the scaled values of the various components and consider N_{O_2} to be unity we have

$$-kT_e'' = hT_e + 10^{15} \left[Q - \left(5\alpha_0 e^{-5/T_e} + 10\alpha_1 e^{-10/T_e} \right) N_e - 2.5\alpha_1 e^{-10/T_e} N_e T_e \right] \quad (36)$$

In practice T_e is constant or slowly varying and k is small, so that the diffusive term in the above can be neglected. Combining the various equations on this assumption, it follows that T_e, N_e, N_{O_2} satisfy the following differential algebraic equation system

$$0 = 10^{-15} hT_e + Q - \left[(5\alpha_0 e^{-5/T_e} + 10\alpha_1 e^{-10/T_e}) + 2.5\alpha_1 e^{-10/T_e} T_e \right] N_e \quad (37)$$

$$-2D_I N_e'' = 10^4 \alpha_1 e^{-10/T_e} N_e, \quad (38)$$

with boundary condition

$$-D_I N_e' = V_{B_e} N_e \quad \text{at} \quad x = -L, L \quad (39)$$

and an algebraic equation for \bar{N}_O given by

$$N_O = \frac{\alpha_0 e^{-5/T_e} N_e}{N_O + \Lambda/10}. \quad (40)$$

We may solve the differential algebraic system numerically (for example by using the MATLAB code `ode15s`) for general coefficients and we will give some figures for this shortly. However, we can make some rough analysis as follows. If we set $N_e = Q \hat{N}_e$ then

$$-2D_I \hat{N}_e'' = 10^4 \alpha_1 e^{-10/T_e} \hat{N}_e, \quad -D_I \hat{N}_e' = V_{B_e} \hat{N}_e \quad \text{at} \quad x = -L, L. \quad (41)$$

If we consider T_e to be constant then \hat{N}_e (which is symmetric about $x = 0$) is given by

$$\hat{N}_e(x) = \hat{N}_e(0) \cos \left(\sqrt{\frac{10^4 \alpha_1 e^{-10/T_e}}{D_I}} x \right). \quad (42)$$

From the boundary conditions we then find that T_e must satisfy the eigenvalue equation

$$D_I \beta \tan(\beta) = V_{B_e}, \quad (43)$$

where

$$\beta^2 = 10^4 \alpha_1 e^{-10/T_e} / D_I. \quad (44)$$

This condition fixes the value of T_e which, to this level of approximation, is independent of Q . We then have further that for a given exponential function $g(T_e)$ of T_e

$$\hat{N}_e g(T_e) = 1 + 10^{-15} h T_e / Q. \quad (45)$$

If h is zero and N_e does not vary too much from a constant, we thus have that

- $N_e = Q \hat{N}_e$, \hat{N}_e independent of Q to first order;
- T_e is independent of Q to first order;
- $\bar{N}_O = (\alpha_0 e^{-5/T_e} \hat{N}_e \cdot Q) / (N_O + \Lambda/10)$.

Thus N_e and N_O both *increase* with Q and N_O *decreases* with Λ . These observations are both in agreement with experiment.

3.6 Numerical results

To supplement this very rough analysis we now present a solution of the differential algebraic system in the case of (large) $h = 10^{14}$ with $\alpha_0 = \alpha_1 = D_I/V_{B_e} = 1$ and $L = 20$. This was calculated numerically using the Matlab code `ode15s`. The results of the various calculations are presented below. In these we see that the cosine approximation is a good one and that the predictions from this of the variations of N_e , N_O and T_e with Q and Λ are all consistent with the numerical calculations.

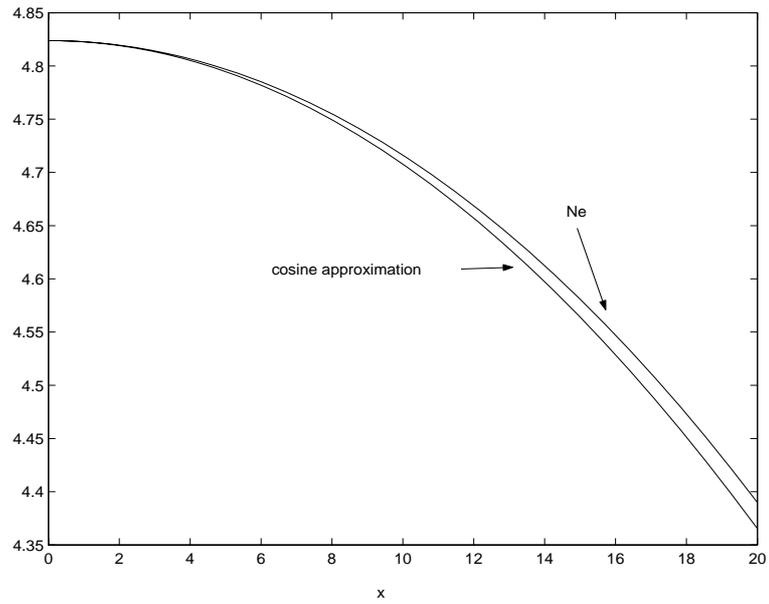


Figure 3: The variation of N_e with x comparing it to the cosine approximation

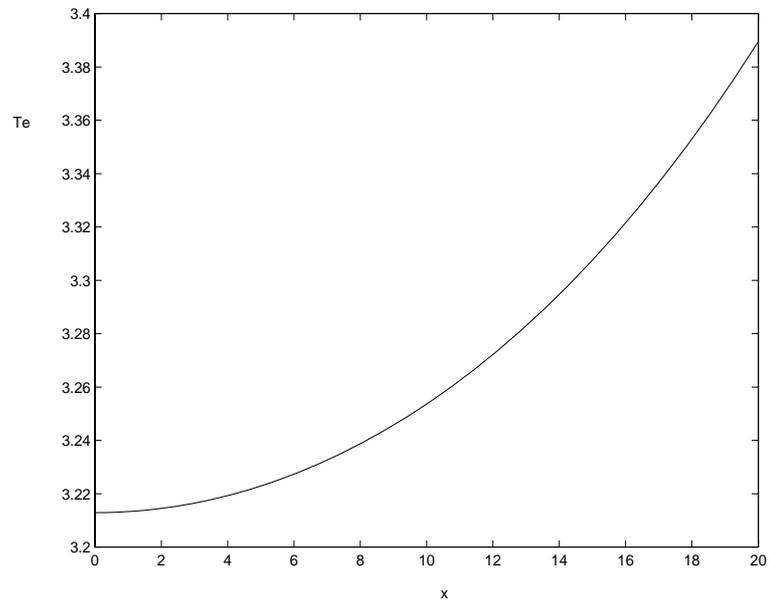


Figure 4: The variation of T_e with x . Note that T_e is close to constant but increases slightly towards the boundary.

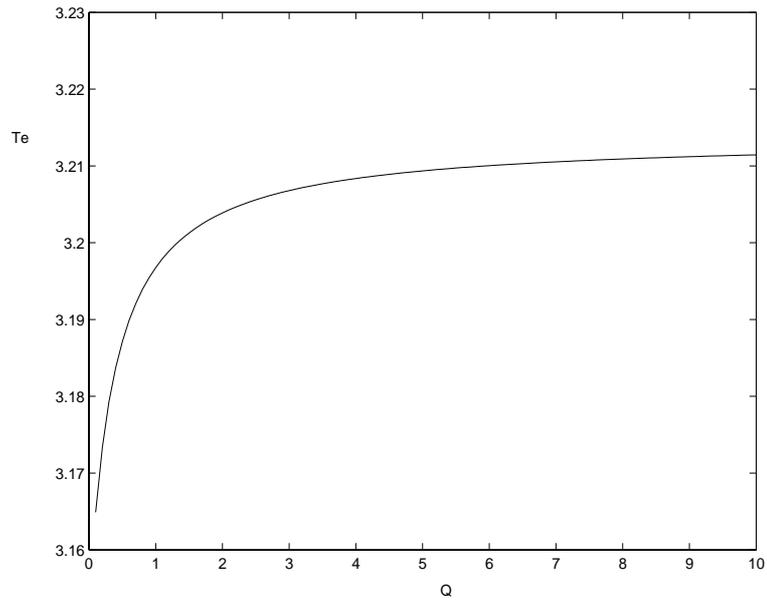


Figure 5: The variation of the maximum value of T_e with Q . Note that for moderately large values of Q the temperature T_e is close to being constant.

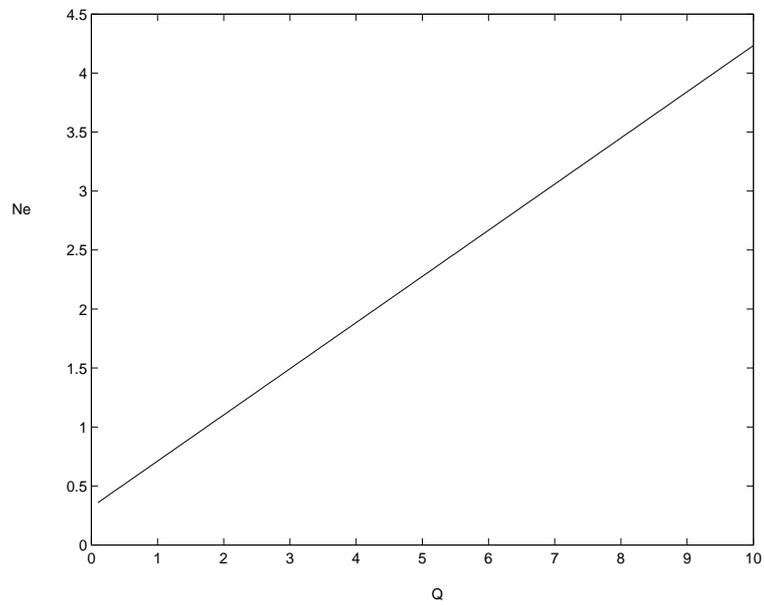


Figure 6: The variation of the maximum value of N_e with Q . Note that N_e is proportional to Q .

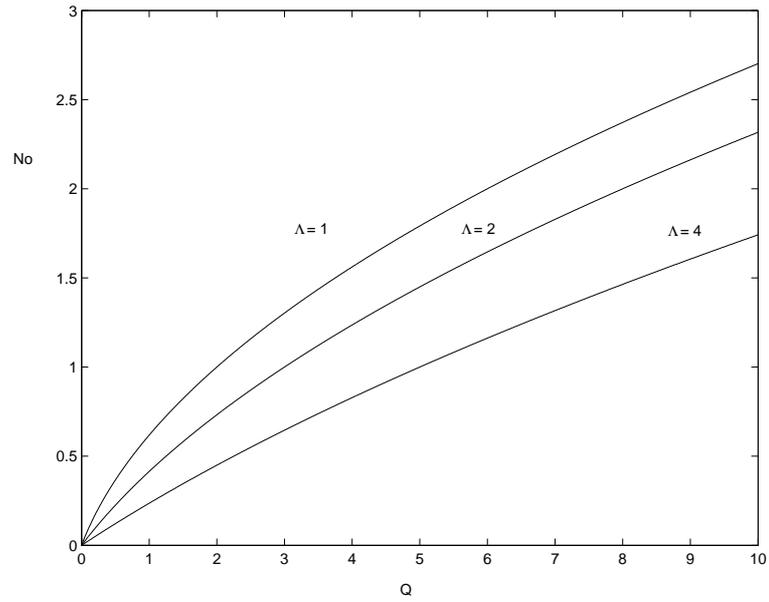


Figure 7: The variation of N_O with Q and Λ , showing that it increases with Q and decreases with Λ .

References

- [1] The Feynman Lectures on Physics. Richard P. Feynman. Addison-Wesley, 1963–65.
- [2] Principles of Plasma Discharges and Materials Processing. Michael A. Lieberman and Allan J. Lichtenberg. Wiley, New York, 1994.