

# MATURITY EFFECTS IN CONCRETE DAMS

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## Abstract

Model equations for determining the coupled heat, moisture and maturity changes within a concrete block are introduced and briefly examined. Preliminary results are obtained for the heat exchange between concrete slabs in contact driven by maturity differences.

## 1 Introduction

To allow shrinkage movement to take place, dam walls are constructed sequentially in discrete blocks with a second layer poured on top of the first layer weeks after the first layer is set. A better understanding of this process may lead to better construction practices. It has been observed that hydration heat transferred across the interface from the upper block causes enhanced hydration heat release within the first block, see Figure 1, from Ballim and Graham (2003). The effect is primarily due to the strong dependence of the hydration rate on temperature (an Arrhenius effect). To understand the underlying processes Ballim and Graham (2003) have introduced a maturity state variable to be evaluated at locations and time throughout the concrete. The present study sets up a slightly more complete (but not necessarily better) model to determine the maturity, temperature and moisture changes within maturing concrete under a variety of conditions. It is

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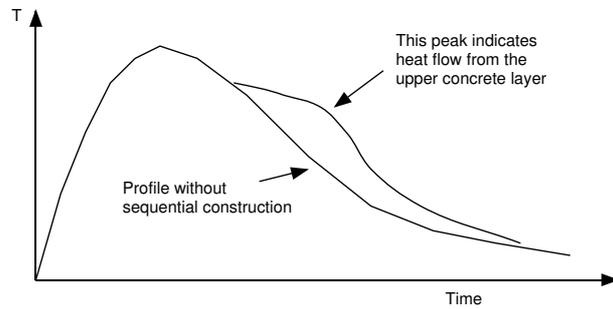


Figure 1: Temperature at a point in the lower layer of concrete.

hoped that the model will provide a better understanding of the underlying processes.

## 2 The model

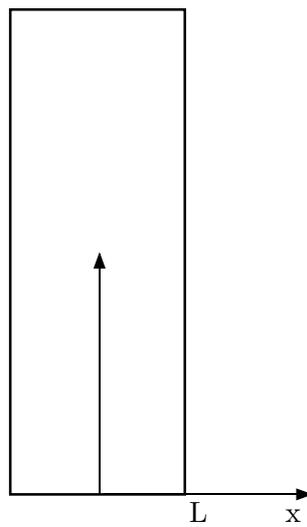


Figure 2: The concrete slab.

We examine the thermal and moisture exchange within a one dimensional concrete slab of thickness  $2L$  subjected to environmental moisture and heat exchanges, see Figure 2. In this report we mainly focus our attention on processes within the concrete, leaving the detailed modelling of surface ex-

changes to another report. It is the liberation of heat within the concrete due to the exothermic cement water (hydration) reaction that drives the thermal exchanges of interest. The hydration rate is initially relatively large and then continually decreases. The initial large hydration rate stage occurs over periods of weeks, but significant hydration is still occurring even after 15 years.<sup>1</sup> Following Ballim and Graham (2003) we introduce a maturity (or degree of hydration) state function of position and time  $0 \leq m(\mathbf{r}, t) \leq 1$ . This can for example be measured by chemical analysis or X-ray quantitative analysis.

The relevant variables are the temperature  $T(x, t)$ , moisture content  $0 \leq \theta(x, t) \leq \theta_0$ , where  $\theta_0$  is the initial concentration of water in the concrete, and the maturity  $m(x, t)$ . The conservation and reaction equations are:

Heat conservation:

$$\rho_c c_c \frac{\partial T}{\partial t} = k_T \frac{\partial^2 T}{\partial x^2} + \gamma \frac{\partial m}{\partial t}, \quad (1)$$

where  $k_t, \rho_c, \rho_c$  are the thermal heat parameters for concrete and  $\gamma \frac{\partial m}{\partial t}$  is the heat release rate due to hydration.

Hydration chemistry:

$$\frac{\partial m}{\partial t} = \mu(1 - m)\theta e^{-E/RT}, \quad (2)$$

where the constant  $E$  is the apparent activation energy of the reaction,  $R$  the gas constant,  $T$  the absolute temperature, and  $\mu$  the reaction rate. The reaction is assumed to occur at a rate proportional to the product of concentrations of the reacting components; free water and 'unreacted' concrete measured in terms of the level of maturity. Also the reaction is strongly temperature dependent; an Arrhenius temperature dependence is assumed.

Water conservation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left( D_m(\theta) \frac{\partial \theta}{\partial x} \right) - \eta \frac{\partial m}{\partial t}, \quad (3)$$

where  $D_m(\theta)$  is the moisture diffusivity of concrete. The reaction removes free water from the concrete;  $\eta$  is the Stoichiometric ratio for the hydration reaction, assumed constant.

The water conservation equation is generally not invoked in such studies, presumably because water is normally present in abundance and so does not limit maturation. This is likely to be the case throughout most of the slab,

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<sup>1</sup>but with relatively little heat release

because of the small moisture diffusivity, but is unlikely to be the case at the surface under dry conditions; certainly if the surface quality of concrete is important this is an important issue. The moisture diffusivity  $D_m$  is strongly dependent on the moisture level<sup>2</sup>, so that moisture fronts are to be expected. Such effects could significantly alter the global moisture balance.

Initially we assume uniform conditions prevail:

$$m(x, 0) = 0, \quad \theta(x, 0) = \theta_0, \quad T(x, 0) = T_0.$$

The surface temperature is assumed to be prescribed. Except for a constant average temperature, daily and monthly fluctuations will dominate other Fourier components, so

$$T(L, t) = \bar{T}_0 + T_d \sin \omega_d t + T_m \sin \omega_m t, \quad (4)$$

is a useful model. A Robin condition

$$-M_m \frac{\partial \theta(1, t)}{\partial x} = \xi(\bar{\theta} - \theta(1, t)), \quad (5)$$

with the exchange coefficient  $\xi$  constant, can be used to describe environmentally driven moisture fluxes to and from the concrete surface, where  $\bar{\theta}(t)$  is the effective moisture content of the air in contact with the surface. As noted earlier the aim of the present work is not to understand surface exchanges, so the above boundary conditions simply represent sensible conditions to use for understanding aspects of the physics. Symmetry requirements at the centre of the slab give

$$\frac{\partial \theta(0, t)}{\partial x} = \frac{\partial T(0, t)}{\partial x} = 0.$$

## Scaling

There are a variety of time scales in the problem with associated length scales:

- surface heating: daily temperature variations with a penetration depth of about 10 cm,
- seasonal temperature variations with associated penetration depth 2m,
- the diffusion time associated with the slab thickness, typically 10m with associated time scale 15 years.

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<sup>2</sup>varying by a factor of  $10^3$  over the moisture range of interest

Of engineering concern is the stresses set up during construction<sup>3</sup> and specifically during, and soon after, the pouring of the second slab on top of the first. With this in mind we choose a time scale  $t_0$  of 1 month; the typical time span between the pouring of the two slabs. Based on this time scale the sensible scaling for the problem is given by:

$$x = Lx', \quad T = \bar{T} + \mathcal{T}T', \quad \text{with } \mathcal{T} = \frac{\gamma\dot{m}_0 t_0}{\rho c}, \quad \theta = \theta_0\theta', \quad (6)$$

in terms of which the problem reduces to (after dropping primes)

$$\frac{\partial T}{\partial t} = \kappa_L \frac{\partial^2 T}{\partial x^2} + \frac{\partial m}{\partial t}, \quad (7)$$

$$\frac{\partial m}{\partial t} = \mu(1 - m)\theta e^{\xi(T-1)}, \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \kappa_m \frac{\partial}{\partial x} (k_m(\theta) \frac{\partial \theta}{\partial x}) - \mathcal{S}\dot{m}. \quad (9)$$

The dimensionless groups are

$$\kappa_L = \frac{k_T t_0}{L^2 \rho c}, \quad \mu = \nu t_0 \theta_0 e^{-E/R\bar{T}}, \quad \xi = \frac{E}{R\bar{T}}, \quad (10)$$

and the scaled Stochiometric ratio  $\mathcal{S}$ . The surface boundary condition reduces to

$$T(1, t) = 1 + \alpha_d \sin \omega_d t + \alpha_m \sin t, \quad (11)$$

with  $\omega_d \approx 30 \gg 1$ . Symmetry and initial conditions complete the specification of the problem.

### Suggested parameter values

Over the time scales of interest the heat and moisture diffusion length scales will be relatively small compared with the slab thickness so that  $\kappa_L, \kappa_M \ll 1$  (for computational purposes we'll use  $\kappa_L = \kappa_M = 0.001$ ). The reaction parameter  $\mu$  will be of unit order, reflecting the observation that significant hydration heat is liberated over the adopted time scale of a month ( $\mu = 1$  is used). A significant modification of the reaction rate is observed due to the second slab which indicates  $\xi$  is of order 1 ( $\xi = 2$  is used). The moisture diffusivity is known to vary rapidly with  $\theta$ ,  $\kappa_m(\theta) = e^{3\theta}$  used. The Stochiometric number is expected to be of order 1 (we will use  $\mathcal{S} = 2$ ).

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<sup>3</sup>Such stresses generate undesirable cracking

### 3 Simulations

#### A simple slab on slab model

An in-depth investigation of this system is well beyond the scope of the MISG, however a few simulations have been performed to illustrate some features of the physics. Of special interest is the effect of a second slab on the thermal history on the first slab. We will model the effect of the introduced second slab as a constant temperature boundary condition,  $T(1, t) = T_1$ , prescribed. Whilst crude this condition reflects the fact that the newly placed slab is more ‘active’ in a heat production sense than the other slab. This condition allows heat to enter the first slab across the boundary located at  $x = 1$ . Also to *greatly* simplify matters we will assume both slabs are equally moist (so  $\theta = 1$ ); near the interface between the slabs this is likely to be the case, unless the interface has dried out before the second slab is poured. (A more accurate model would include both slabs with continuity conditions across the interface and different initial maturity levels in the two slabs. The present model simply represents the simplest sensible model.) Under the above circumstances the equations reduce to

$$\frac{\partial T}{\partial t} = \kappa_L \frac{\partial^2 T}{\partial x^2} + \frac{\partial m}{\partial t}, \quad (12)$$

$$\frac{\partial m}{\partial t} = \mu(1 - m)e^{\xi(T-1)}, \quad (13)$$

with

$$\frac{\partial T(0, t)}{\partial x} = 0, \quad T(1, t) = T_1, \quad T(x, 0) = 0, \quad m(x, 0) = 0. \quad (14)$$

Strictly speaking we should specify the boundary condition at the other face,  $x = -1$ , of the slab but we will retain the symmetry condition  $\frac{\partial T(0, t)}{\partial x} = 0$ ; the results of interest are not sensitive to this condition.

#### The $\xi = 0$ case

Firstly note that in the  $\xi = 0$  case there is no coupling between the maturation equation and the heat equation, so that  $m(x, t) \equiv m(t)$  is uniform in  $x$  and approaches  $m = 1$  in a time  $t$  of order 1. The temperature increases uniformly in  $t$  due to the hydration heat release, except close to the interface  $x = 1$ . Additional heat diffuses from the second slab across the interface at  $x = 1$  effecting a change in temperature over a distance of order  $\sqrt{\kappa_L t}$ ; i.e. initially slowly but then slowing down rapidly, see Figure 3.

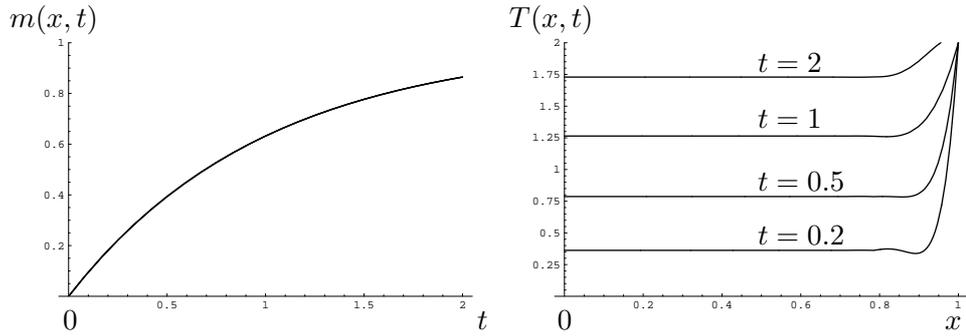


Figure 3: Maturity  $m(x,t)$  and temperature  $T(x,t)$  profiles in the  $\xi = 0$  case with  $\kappa_L = 0.001$  and  $\mu = 1$ .

### The $\xi > 0$ case

In the  $\xi > 0$  case, see Figure 4, the heat inflow from the second slab enhances the maturation in the first slab so that  $m$  increases more rapidly to 1 close to the interface; Figures 3 and 4 are *very* different. Also there is additional hydration heat (due to the Arrhenius factor) released near the surface, so that the temperature rise is larger than would be expected if Arrhenius effects were ignored. Such near surface effects will only be felt within the diffusion distance from the interface. Note that  $m$  reaches 1 immediately at the interface because the higher temperature at the surface enhances maturation. Note, however, that the temperature throughout the slab is increasing due to hydration so that enhanced maturation will occur throughout the slab; the reaction in fact completes by time  $t = 1$  compared with  $t \approx 2$  in the  $\xi = 0$  case, see Figure 4.

The difference between the cases is most easily seen in Figure 5 which displays spatial variations in maturity; compare with Figure 3.

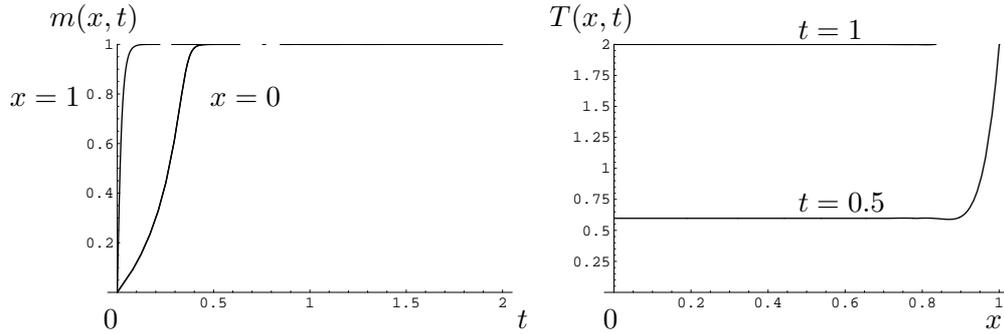


Figure 4: Maturity  $m(x, t)$  and temperature  $T(x, t)$  profiles in the  $\xi = 2$  case with  $\kappa_L = 0.001$  and  $\mu = 1$ .

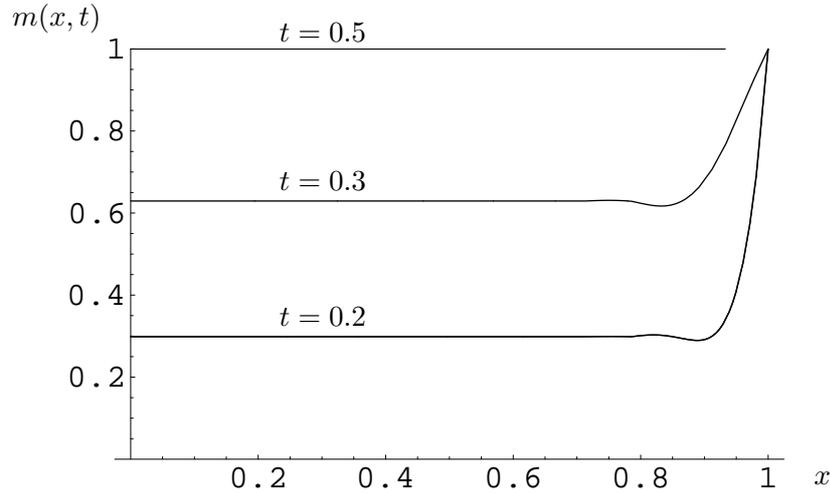


Figure 5: Maturity  $m(x, t)$  profiles for fixed values of  $t$ , in the  $\xi = 2$  case ( $\kappa_L = 0.001, \mu = 1$ ).

#### 4 Surface effects

Because of the low conductivity of concrete most of the interesting behaviour occurs on or close to interfaces; the surface is particularly important. For the same reason moisture transport effects are confined to the surface or interfaces. This suggests that a semi-infinite slab model is appropriate for

investigations, and similarity solutions could be useful for understanding the fronts moving in from the interfaces.

## **5 Recommendations and conclusions**

Evidently the above just scrapes the surface of this problem, but at least the simulations performed illustrate the phenomenon under investigation. The indicated work requires the careful investigation and time input that would be best achieved through postgraduate work.

## **References**

- Ballim, Y. and Graham, P. C. (2003). A maturity approach to the rate of heat evolution in concrete. *Magazine of Concrete Research, Thomas Telford*, **55**, 249-256.
- Carslaw, H. S. and Jaeger, J. C. (1959). *Conduction of Heat in Solids*, Oxford University Press, Oxford.