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NOMOGRAMS FOR USE WITH BARR AND STROUD DENDROMETER FP.15

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CONTENTS

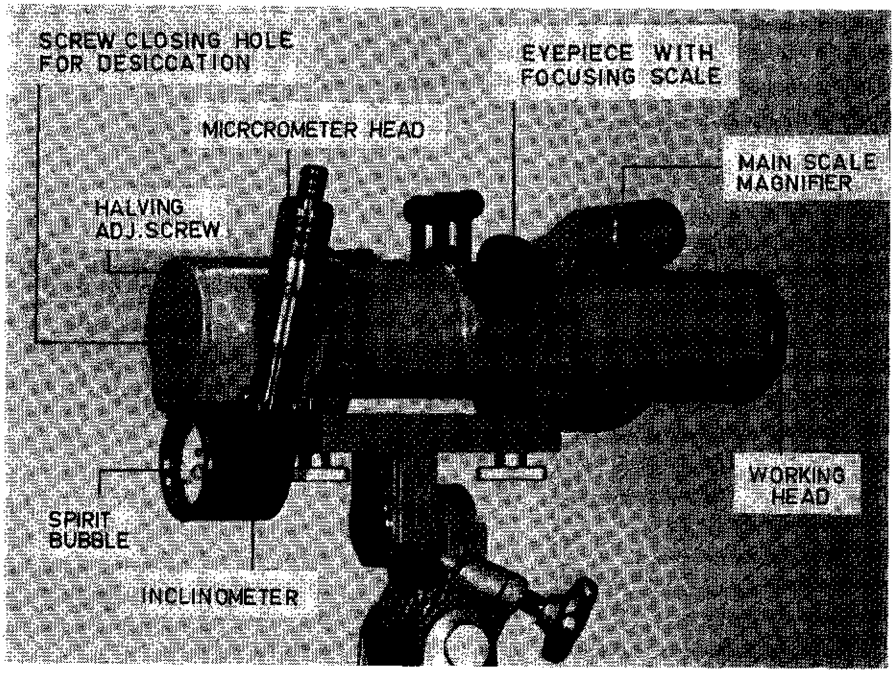


PHOTO 1. Rear view of Barr and Stroud Dendrometer type FP.15. *H. A. Hendrikx phot.*

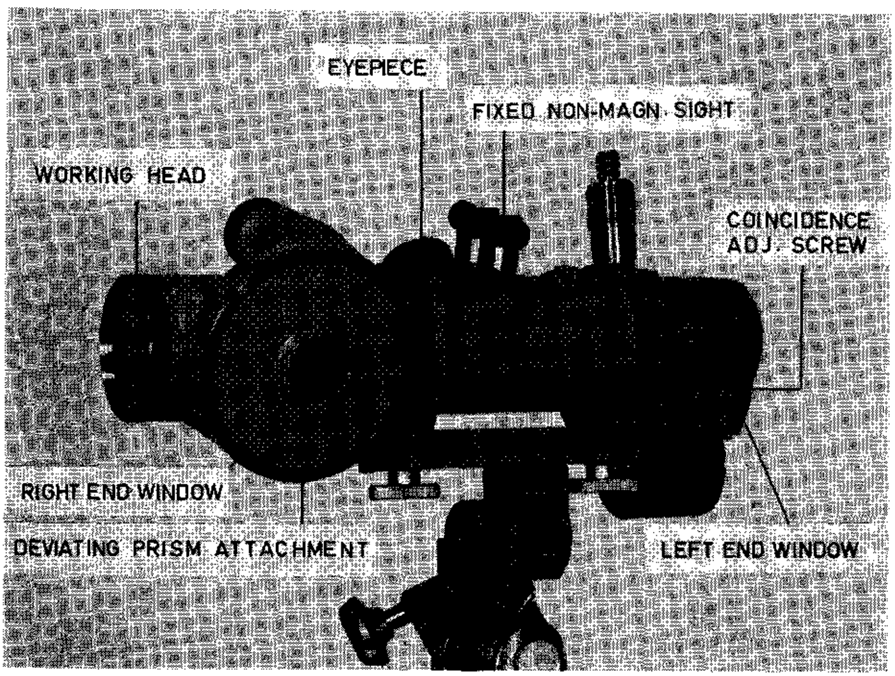


PHOTO 2. Front view of Barr and Stroud Dendrometer type FP.15. *H. A. Hendrikx phot.*

1. Introduction

The Barr and Stroud Dendrometer FP.15 (phot. 1 and 2) is the latest (1968) type of a series (e.g. FP.7, 9, 12) of similar dendrometers manufactured by Messrs. Barr and Stroud Ltd., Glasgow, Great Britain.

These dendrometers are, with a description from Mesavage (1964): 'short-base, split-image, coincident-type, magnifying rangefinders adapted for estimating out-of-reach stem diameters'.

To the FP.15 a spirit bubble inclinometer is attached, the graduations of which are the sine values of angles of elevation and depression between $+60°$ and $-60°$, which is an extended range in comparison with preceding types. With the aid of the graduated micrometer head these sine values can be read to three decimals. Contrary to earlier types and with the purpose of avoiding confusion between readings of elevations and depressions afterwards, the inclinometer reading is 1.000 with horizontal line of sight, and readings at an elevation or depression of $\alpha°$ are the values $(1.000 + \sin \alpha)$ and $(1.000 - \sin \alpha)$ respectively.

Hence with this instrument ranges and their inclination to out-of-reach stem diameters, as well as the widths of the latter can be measured, and from these data a stem volume estimate of a standing tree can be derived.

Further improvements in respect of earlier types are: 1. a fixed non-magnifying sight providing a means of quick approximate aiming at the desired point; 2. a better illumination of the main scale, achieved by mounting the scale magnifier in a sanded transparent plastic barrel. (A suggestion is herewith made to the manufacturers to try illumination with a tritium lamp); 3. a vertical black reference line in the centre of the upper part of the field of view, as an aid to bring a target point accurately on a line of sight emerging at right angles with the base line from the left window.

2. Technical specifications as supplied by the manufacturers

In the Manual, supplied with the instrument, the following technical specifications are given:
Base length 20.3 cm ($\pm$ 0.1 %)
Magnification 5.5 $\times$
Type of field: coincidence with both images erect
Angular field 7° circular
Exit pupil 2 mm diameter
Ranges:
as a dendrometer 11–110 m
as a rangefinder 11–620 m
Tree diameters 3.8–500 cm
Angular limits of inclinometer $-60°$ to $+60°$
Weight of dendrometer 2.3 kg.

Approximate uncertainty of observations:
diameters 3.8–25.4 cm: $\pm$ 2.5 mm
diameters 25.4–508 cm: $\pm$ 1%
heights: $\pm$ 1.5% for all heights above 10° elevation
ranges: the complete table is not repeated here. Uncertainty is given as 0.19% for a range of 13.72 m, and increases to 1.27% for a range of 91.4 m, being 6.8% at 617 m.

3. Principle and operation

Principle and method of operation of this type of dendrometer have been described by various authors in lesser or greater detail (JEFFERS, 1956; GROSENBAUGH, 1963; MESAVAGE, 1964).

Because of the scant information supplied by the manufacturers (the manual e.g. does not contain a scheme or satisfactory description of the instrument's interior), the following rough attempt is made to elucidate the principle on which the instrument is based.

A ray of light emerging from point p (fig. 1) of a (for simplicity's sake) flat vertical target of width D (parallel to B), and passing through the left end l of, and at right angles with a base line lr of fixed length B, is reflected by a fixed surface[1] m11 into a direction parallel to lr, after which the likewise fixed surface m12 reflects this ray into a direction perpendicular to lr. The image of p is p′ (The optical system by which real images like p′ are produced is omitted in the sketch). In that situation the image of q is formed in q′.

A ray emerging from p in the direction of r hits a system s of two counter-rotating prisms. The axis of counter-rotation is in the plane of the figure.

By counter-rotating the prisms the deflective power of the system may be varied continuously from a maximum deviation, via zero, to a maximum deviation in the opposite direction.

Counter -rotation is achieved by turning the instrument's working head, and the angle z by which the head is turned can be read on the main scale or drum with a vernier in tenths of centesimal degrees (grads).

By counter-rotating the prisms the incident ray from p can be deflected into a direction perpendicular to lr, after which it is reflected by the fixed surface m21 into a direction parallel to the base. Reflection by the likewise fixed surface m22 puts the image of p in p″. In that situation the right-hand optics produce the image q″ of q.

Both images p′q′ and p″q″ are observed through a magnifying lens 0. Optical provisions are such that p′q′ is seen in the upper, and p″q″ in the lower part of the field of view.

The situation described above, in which p′ coincides with p″, is named 'true

[1] Actually the 'reflecting fixed surfaces' are pentaprisms.

coincidence' or 'No. I setting'. The corresponding reading on the main scale in grads is named z_t. From the figure it is evident that the angle of total deviation in case of true coincidence equals the angle of convergence lpr $= \delta_t$.

The angle of deviation caused by the entire optical system depends on:
1. the angle by which the prisms are counter-rotated. Zero counterrotation is when the prisms completely neutralize each other's deflection, which occurs at drum setting 100.0. At drum settings unequal to 100.0 the prisms are in a counter-rotated position. Maximum 'positive' counter-rotation is at setting 0.0: the prisms then cooperate to produce a maximum positive (i.e. clockwise) counter-rotational deflection. Extreme 'negative' counter-rotation is at setting 156.0: the prisms then cooperate to produce a 'maximum' negative (i.e. anti-clockwise) deflection.

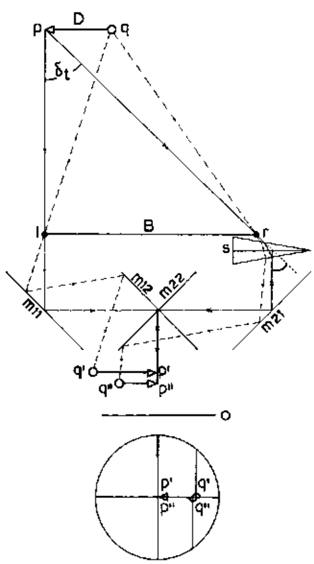


FIG. 1. True coincidence (schematically).

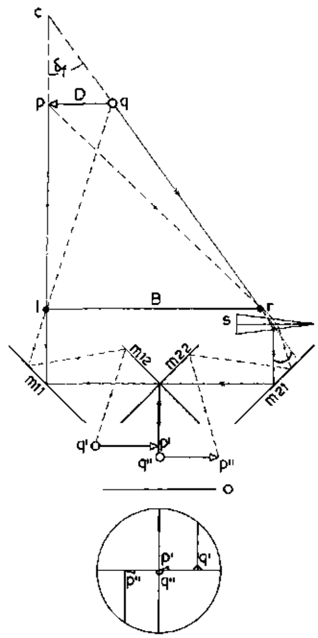


FIG. 2. False coincidence (schematically)

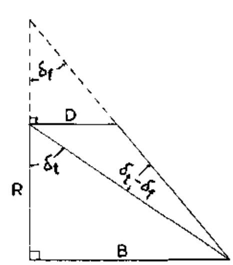


FIG. 3. Approximate determination of R and D.

The angle of counter-rotation depends on the drum setting and on instrument constants.

2. prism setting, which e.g. may be setting for minimum-deviation in any situation, or otherwise. Prism setting is a constructional instrument feature.
3. a constant negative bias deflection, caused by the pentaprisms, of M radians, which is also present in the situation of zero counterrotation.

Zero total system deviation (parallelism) occurs at drum setting 64.5. In that situation the positive counter-rotational deflection is exactly compensated by the sum of biases of the system.

Given a drum setting of z_t and the relevant instrument constants, total deviation of the system in case of true coincidence can be calculated, and consequently δ_t is known.

In the situation of true coincidence the ray from q in the direction of r is not deflected perpendicularly to lr. In order to obtain the latter, i.e. to make q″ coincide with p′ (fig. 2), counter-rotation has to be decreased, as a result of which counterrotational deflection also decreases, i.e. the drum setting must become greater than z_t.

The situation in which q″ coincides with p′ is named 'false coincidence' or 'No. II setting'. The corresponding reading on the main scale in grads is named z_f, and $z_f > z_t$.

The angle of total deviation in case of false coincidence equals the angle of 'back convergence' lcr $= \delta_f$, and δ_f can be calculated from z_f by the relations indicated above.

Once δ_t and δ_f are known in radians, the range R and target width D (fig. 3) follow from:

$$R \simeq B/\delta_t \tag{1}$$

$$D \simeq R(\delta_t - \delta_f) = B(1 - (\delta_f/\delta_t)) \tag{2}$$

In the manual supplied with the instrument in January 1971 formula (2) erroneously contains a division sign instead of the correct minus sign. The manufacturer's attention was drawn to this error by the author.

The above outline has been given for target widths $D < B$. For $D = B$ (parallelism) the main scale reading in case of false coincidence is 64.5 grads, and total deviation, δ_f, is zero, which is in accordance with (2).

Counting angles of convergence, δ, positive in an anti-clockwise direction from the direction pl, the angles δ_f become negative for targets $D > B$. Hence (2) is generally valid.

As tree diameters are not flat targets, and as in case of inclined line of sight the plane through base B and the fixed line of sight pl has an elliptical intersection with the stem, the exact formulae for R and D are rather complex, and the same holds for the formula for the exact height H of the target above or below the horizontal. The latter generally is approximated by:

$$H \simeq R \sin \alpha = (B \sin \alpha)/\delta_t \tag{3}$$

where α is the angle of inclination (elevation or depression).

The complex formulae for R, D and H (GROSENBAUGH, 1963) call for a computer program which was supplied in Fortran by the latter author, together with the general theory.

With the normally occurring angles of convergence of only a few degrees, the approximations (1) to (3) however, will do for most practical purposes. In the manual supplied by the manufacturers, tables are given, derived from (1) and (2).

4. Constants and tables

In the Manual instrument constants necessary for deriving angles of convergence δ from drum readings z are supplied:

$B = 20.32$ cm

$H = \arctan 11/216 \simeq 0.050\ 881\ 20$ radians

$K = \arctan 1/54 \simeq 0.018\ 515\ 03$ radians

From these, two other constants are derived, viz.:

$L = (H + K)/(1 + \cos 40°) \simeq 0.039\ 294\ 71$

$M = (H - K.\cos 40°)/(1 + \cos 40°) \simeq 0.020\ 779\ 68$

The angles of convergence then can be found as:

$$\delta_{t,f} = L.Z_{t,f} - M \text{ radians} \tag{4}$$

where

$$Z_t = \cos 0.9\, z_t \text{ and } Z_f = \cos 0.9\, z_f$$

the factor 0.9 being the conversion factor from grads to sexagesimal degrees.

Then R and D can be computed by (1) and (2), and if the angle of inclination is known, H follows from (3).

From (4) it is easily found that for $\delta_f = 0$ (parallelism) the drum reading is $z_f = 64.5$ grads.

The following tables are supplied with the instrument:

1. two pages of values of R (calculated by (1) and (4)) for No. I settings from $z_t = 0.0$ grads ($R = 10.97$ m) to $z_t = 63.9$ grads ($R = 618.8$ m);
2. two pages in which 'X' – values for No. I settings from $z_t = 0.0$ to $z_t = 60.9$ can be looked up;
3. two pages in which 'Y' – values for No. II settings on 'small trees' can be looked up at values from $z_t = 0.0$ to $z_f = 64.5$ grads;
4. three pages in which at the values 'X + Y' (from tables 2 and 3) for 'small trees' the diameter D in mm can be looked up;
5. three pages in which 'Y' – values for No. II settings on 'large trees' can be looked up at values from $z_f = 64.6$ to $z_f = 156.0$ grads;
6. eight pages in which at the values 'X + Y' (from tables 2 and 5) the diameter D in mm can be looked up for 'large trees'.

Altogether 20 pages of tables with over 4.500 numbers.

By the author the manufacturers' attention has been drawn to the fact that the headings 'small trees' and 'large trees' are inadequate and may easily lead to incorrect use of the tables. The criterion is not the size of the tree, but each individual diameter in a tree. Within a tree these diameters may range from 'small' to 'large', hence a certain tree may require the use of the tables for 'small trees' as well as those for 'large trees'. The manufacturers were advised to provide the tables with headings giving the proper criterion. Those for 'small trees' should instead have the heading: 'No. II setting $<$ 64.5 grads' (i.e. diameters smaller than 20.32 cm), and those for 'large trees' should have the heading: 'No. II setting $>$ 64.5 grads (i.e. diameters larger than 20.32 cm).

The Manual contains no tables for converting inclinometer readings and No. I settings into heights.

5. Alternatives to tabular conversion procedures

Determination of tree diameters from instrument readings at No. I and No. II settings by means of five different tables requires much time, the more so as also addition of two generally 4-digit numbers (X and Y) is involved.

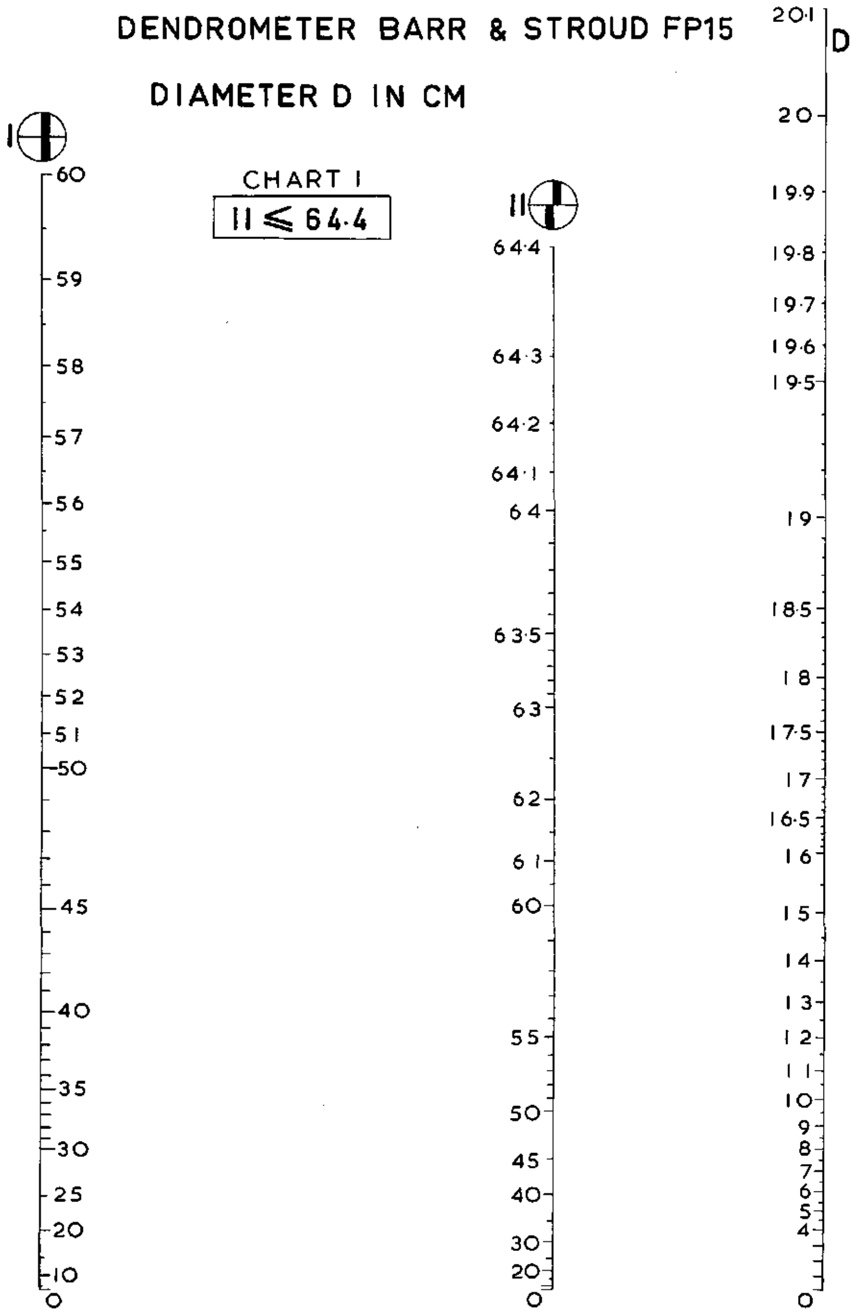
By using the tables of course the proper approximate values are found. These values however, are subject to an uncertainty specified before, as $\pm$ ca. 1% for diameters and between $\pm$ ca. 0.2% and $\pm$ ca. 0.8% for reasonable ranges. The uncertainty of heights above 10° elevation is given as $\pm$ 1.5%.

If a computer is available, as it should be when using this instrument in larger-scale mensuration projects, the tables are not necessary and the exact dimensions can be calculated (Grosenbaugh, 1963).

As in the field even reasonably careful observers will record erroneous measurements as often as once in ten times (Bruce, 1967), and as these errors cannot be corrected in the office afterwards, several means have been developed by which measurements can be quickly checked in the field. Mesavage (1964) published a field chart consisting of 57 lines for checking diameters. Grosenbaugh (1963) suggested a slide rule, Bruce (1967) constructed it and Mesavage (1968) improved it.

Both means are said to give sufficiently accurate results for field checking.

In this article attention is drawn to nomograms (figs. 4 to 7) which on a small photographic scale also can be used for ready field checking. If no computer is available some of them, on a large scale, even might serve for office work of almost tabular accuracy, thereby reducing conversion time to a fraction of that needed when using tables, and most probably also the risk of making errors.


DENDROMETER BARR & STROUD FP15
DIAMETER D IN CM
CHART I
II ≤ 64·4
I
II
D
60
59
58
57
56
55
54
53
52
51
50
45
40
35
30
25
20
10
0
64·4
64·3
64·2
64·1
64
63·5
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62
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0

FIG. 4.

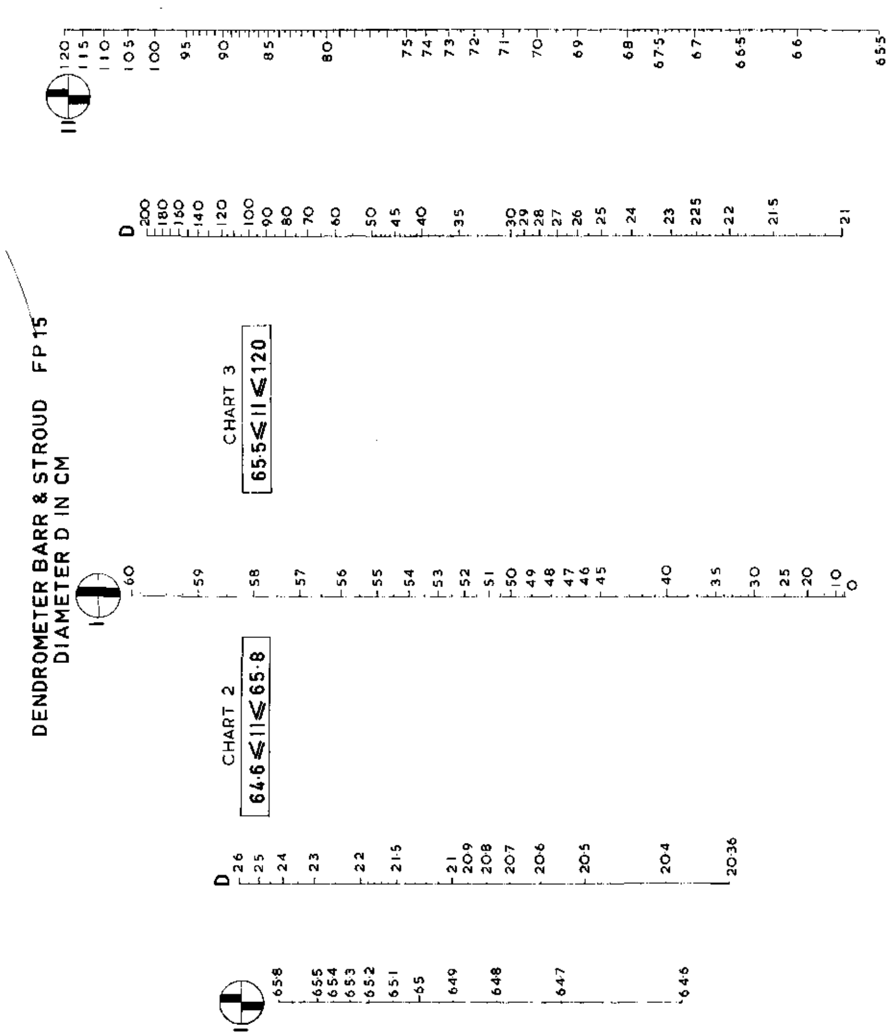
DENDROMETER BARR & STROUD FP 15
DIAMETER D IN CM
CHART 2
64.6 ≤ II ≤ 65.8
CHART 3
65.5 ≤ II ≤ 120
I
II
D

FIG. 5.

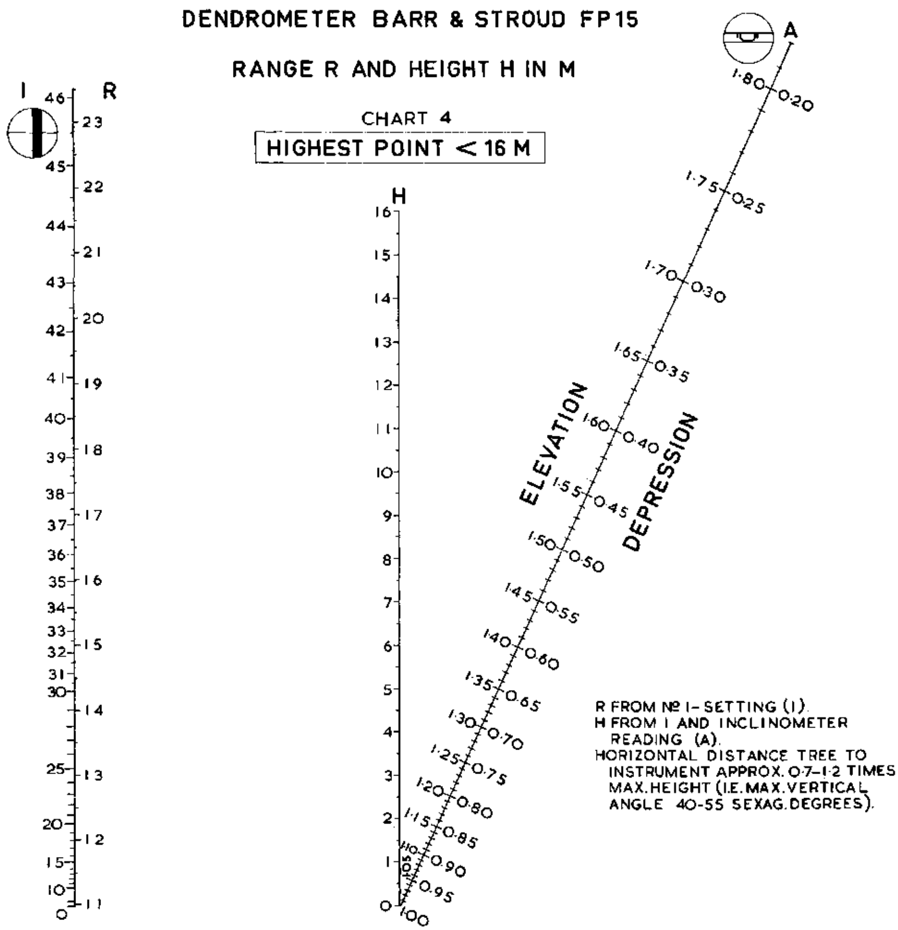
DENDROMETER BARR & STROUD FP15
RANGE R AND HEIGHT H IN M
CHART 4
HIGHEST POINT < 16 M
ELEVATION
DEPRESSION
R FROM Nº I- SETTING (I).
H FROM I AND INCLINOMETER READING (A).
HORIZONTAL DISTANCE TREE TO INSTRUMENT APPROX. 0·7–1·2 TIMES MAX. HEIGHT (I.E. MAX. VERTICAL ANGLE 40-55 SEXAG. DEGREES).

FIG. 6.

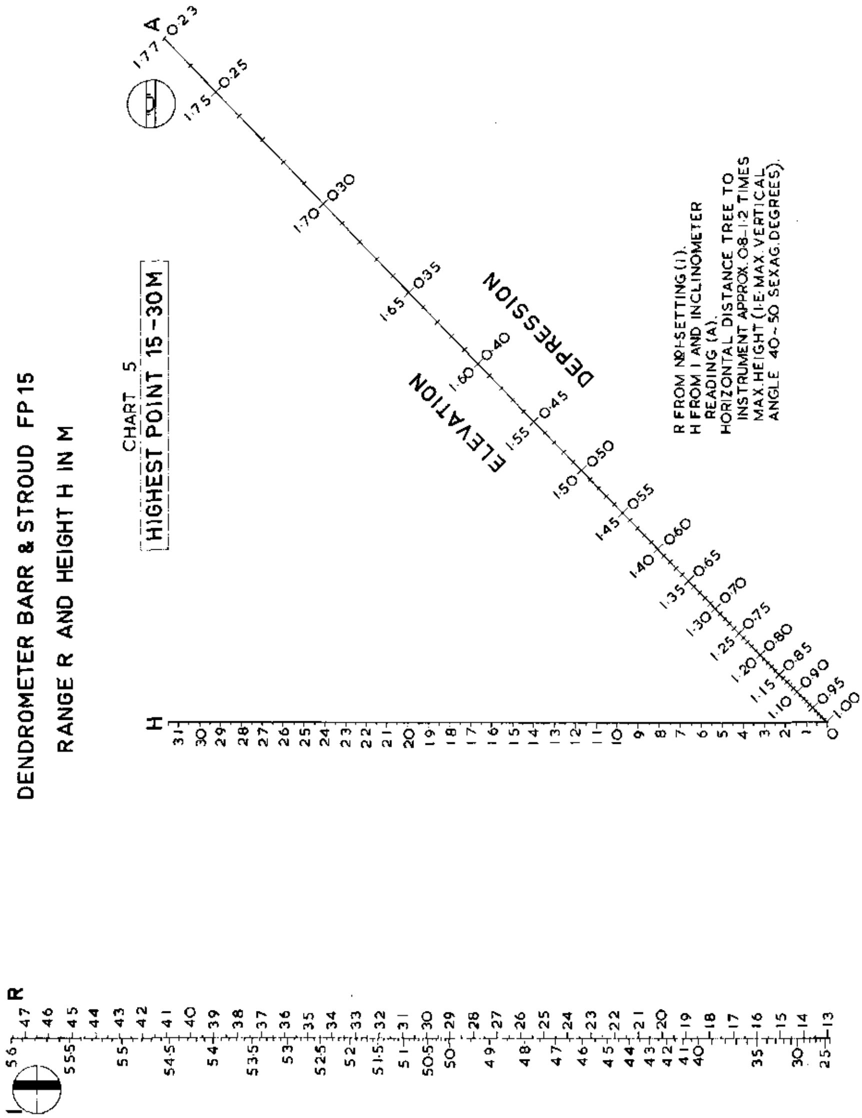
DENDROMETER BARR & STROUD FP 15
RANGE R AND HEIGHT H IN M
CHART 5
HIGHEST POINT 15 - 30 M
ELEVATION
DEPRESSION
R FROM No1-SETTING (I).
H FROM I AND INCLINOMETER READING (A).
HORIZONTAL DISTANCE TREE TO INSTRUMENT APPROX. 0·8-1·2 TIMES MAX. HEIGHT (I.E. MAX. VERTICAL ANGLE 40-50 SEXAG. DEGREES).
I
R
H
A

FIG. 7.

6. NOMOGRAMS

Charts 1, 2 and 3 (figs. 4, 5) are nomograms of the following relations:
Chart 1 for No. II settings (II-axis) of $z_f \leqslant 64.4$:

$$\log[\cos 0.9\, z_t - M/L] + (-\log[\cos 0.9\, z_f - M/L]) + \log(B-D)/B = 0$$

Charts 2 and 3 for No. II settings of $z_f \geqslant 64.6$:

$$\log[\cos 0.9\, z_t - M/L] + (-\log[-\cos 0.9\, z_f + M/L]) + \log(D-B)/B = 0$$

which relations easily follow from (2) and (4), remembering that δ_f changes sign when $z_f > 64.5$ grads.

In using chart 1 it should be noted that:
if $z_f = 64.5$ grads, $D = 20.3$ cm;
if $z_f = 64.4$ grads, and z_t (I-axis) is such that the reading on the D-axis would be at a point at least as far above point 20.1 as point 20 is below, one may put $D =$ 20.2 cm. Extension of the D-axis beyond the 20.1 – point results in a nomogram of less efficient size.

The region of No. II settings with $z_f > 64.5$ has been split into two nomograms, charts 2 and 3, in order to avoid too narrow a calibration of the axes.

Chart 2 is for z_f – values between 64.6 and 65.8 grads and chart 3 for z_f – values beyond 65.5 grads, so both nomograms have a slight overlap.
The two nomograms have the I-axis (z_t – values) in common.

The D-axis in chart 3 has not been extended beyond 200 cm, as this seems sufficient for normal situations. A D of 200 cm may be observed from a range between 87.7 m ($z_t = 60$) and 54.3 m ($z_t = 57.1$). For observations of very large diameters at closer ranges either the tables can be used or separate nomograms constructed.

The length of the I-axis in the original drawings was made 50 cm; readings in these full-size nomograms agreed very closely with corresponding tabular values. By slight photographic reduction, which will not markedly affect reading accuracy, they already can be put to manageable size for office use in case of lacking computer facilities. Further reduction, e.g. to half or one third of the original linear size still provides nomograms of sufficient accuracy for rapid field checking.

Charts 4 and 5 (figs. 6, 7) serve a dual purpose, viz.:
1. determination of R from reading at No. I setting, on the I, R-axis, which is based on the relation:

$$R = (B/L)/(\cos 0.9\, z_t - M/L)$$

2. determination of height of target above or below the horizontal, by the relation:

$H = (A - 1). [(B/L)/(\cos 0.9\, z_t - M/L)]$ for elevations, where A is the inclinometer reading. In case of depressions, $(1 - A)$ is substituted for $(A - 1)$ in the above expression.

In order to avoid too close a calibration of the axes, chart 4 was constructed for a maximum target height of about 16 m, and chart 5 for maximum target heights between 15 and 30 m, which seems sufficient for many practical situations. The two charts have a small overlap. Of course nomograms for other maximum target heights can be constructed.

Chart 4 is based on the assumption that the angle of inclination to the highest target on a tree is between 40° and about 55° which means that the instrument should be placed at a horizontal distance of about 0.7–1.2 times this maximum target height from the tree.

Chart 5 is based on the assumption of a minimum horizontal range of 13 m, and on an angle of elevation of the line of sight to a highest target point of between 40° and about 50°. This slightly reduces the interval in which the instrument should be placed, viz. to distances between 0.8–1.2 times maximum target height.

Charts 4 and 5 permit of slightly less accurate estimates of H. On the original scale (length of I-axis ca. 50 cm) the accuracy of reading was $\pm$ 5 cm, which should be compared with the uncertainty of $\pm$ 1.5% of heights arrived at by using tables. A 2-fold linear reduction still leaves these charts usable for field checks.

In considering the use of large-scale versions of charts 4 and 5 for office work if no better means are available, one should not forget that rounding errors in nomogram readings tend to compensate in differences between readings (sectional lengths). On the other hand neither the determination of R from the manufacturers' 2-page table, nor the calculation of the sine values from inclinometer readings are much time consuming, so that in less favourable situations still much can be done with a desk calculator.

SUMMARY

After a brief description of the properties of the Barr and Stroud dendrometer type FP.15 some attention is paid to the optical principle and to the geometry by which approximate values for ranges, diameters and heights are derived.

Suggestions are made for proper headings of the conversion tables supplied with the instrument.

Attention is drawn to the possibility of using nomograms of appropriate size for field checking instrument readings, or even for office use in case of lacking computer facilities, as an alternative to slide rules or elaborate conversion tables respectively. Four examples of nomograms on a very small scale are added.

Acknowledgment

For their kind permission to reproduce data from the 'Manual', the author is indebted to Messrs. Barr and Stroud Ltd..

Samenvatting

Een korte beschrijving van de eigenschappen van de Barr en Stroud dendrometer type FP.15 wordt gevolgd door een schets van het optische principe en de meetkundige grondslag van afstand-, diameter- en hoogtebepaling.

Gewezen wordt op de noodzaak, de bij het instrument verstrekte tabellen van de juiste titels te voorzien.

De aandacht wordt gevestigd op de mogelijkheid van het gebruik van nomogrammen, zowel voor snelle controle op de metingen in het veld, als voor meer nauwkeurig kantoorwerk indien geen electronisch rekentuig beschikbaar is. Zulks als alternatief voor resp. het gebruik van speciale rekenlinealen en omvangrijke tabellen. Vier sterk verkleinde voorbeelden van nomogrammen worden afgebeeld.

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