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A GENERAL THEORY ON LINE INTERSECT SAMPLING
WITH APPLICATION TO LOGGING RESIDUE INVENTORY

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CONTENTS

1. INTRODUCTION

As far as could be traced CANFIELD (1941) is the first to use a sampling technique that he describes as the 'line interception method' for sampling range vegetation. He rightly considers his technique as a streamlined version of earlier 'line transect sampling', a well-known design used on maps or aerial photographs and in the field, for estimating the (multinomially distributed) area proportions of strata. However, in the description of the extension of his method to assess forage utilization and forage volume, this author comes close to the principle of what more recently is denoted by 'line intersect sampling', a projection of the principle to other than range vegetation appraisals. More specifically line intersect sampling is propagated as a method to quickly obtain volume estimates of logging residue on clearfelled areas so as to have a check on forest fire danger, or on royalties due by pulpwood contractors because of incomplete removal of usable material. WARREN and OLSEN (1964) in New Zealand are the first to describe the method for the latter purpose; as far as could be traced the term 'line intersect method' comes to their credit. These authors were faced with the problem of annually assessing, with limited manpower, 2500 acres of clearfellings in *Pinus radiata* for low-priced pulpwood-size residue. Previous work by MITCHELL in New Zealand had shown that sampling with quarter-acre circular plots not only was too time consuming and expensive but also had to be very intensive in order to yield an acceptable precision. As most time went to the searching of plot areas for qualifying pieces, attention subsequently was given to long and narrow rectangular plots in which logs were encountered automatically, thus reducing searching time. Though these narrow plots seemed to sample the population more representatively, precision and time gain did not increase to such an extent as to make the inventory feasible under prevailing restrictions. The authors then conceived the idea of reducing plot width to a line, and pay attention only to the volume of the logs that intersected with that line, as this volume, they considered, should bear some relationship to total volume in a given area. On a semi-empirical basis they developed a formula for estimating volume of residue per acre in which, apart from the number n of intersecting logs and line length L, two empirical values appear, named α and I_c, of which the first depends on dimensional characteristics, and the second on the orientation of the log population relative to the sampling line. This orientation may be pronounced, as in skyline logging, or approximately random as in logging by tractor, but appeared unaffected by felling direction and log size. Field trials made in random and oriented populations by means of mutually perpendicular lines of 20 chains yielded good estimates as compared with plot sampling, and proved to be about five times quicker than the latter. The authors include a theoretical basis for their method and rules for the field procedure, but do not consider the case of logs showing multiple intersection.

A considerable improvement in the theoretical background of line intersect sampling is given by VAN WAGNER (1968) who measures log diameter at the point of intersection, and develops theoretically sound estimators for volume and weight of logging residue per acre. Moreover he computes maximum bias due to log orientation relative to the sampling line for three alternative applications, presents the results in a graph, and concludes that in unidirectionally oriented populations sampling with three lines under different angles is advisable in order to avoid the determination of a special bias coefficient as used by WARREN and OLSEN. A field test with nineteen 100-feet line sections in a 20-acre clearcut with stems bucked into 16-feet logs, randomly oriented, yielded a half 95%-confidence interval of 11% of the mean, but the result was not compared with true volume. The job implied diameter measurement at 680 intersections and was completed in 5 hours by two men. Further, the results of an office trial for estimating the number of elements per area unit are given, using various numbers of randomly distributed matches of uniform diameter and length. The latter author indicates a number of tallying rules; these include the advice to ignore any piece the central axis of which coincides with the sampling line, and further, to take diameter measurements at all points of intersection in case a log intersects more than once, though he expresses doubt as to the mathematical correctness of the latter procedure. This advice will be commented on in the following.

BROWN (1971) describes a planar intersect method for sampling fuel volume and surface area, based on VAN WAGNER's theoretical approach. The author discerns between cylindrical and rectangular parallelopiped shaped particles. Actually, ordinary line intersect sampling also is a planar intersect method, as in fact intersections with the line are evaluated by ocularly projecting pieces lying below or above the line onto the latter.

HOWARD and WARD (1972), though erroneously quoting the meaning of the symbol V for volume per acre in VAN WAGNER's estimator, describe an interesting field test in which 25 sampling lines, each having a length of 200 feet, are distributed in various patterns over clearcut areas of 40 acres, viz. randomly and in two systematic grid designs, one of which with straight lines, the other with symmetric L-shaped lines. Precision of the random pattern seems superior to that of the systematic one with L-shapes.

VAN WAGNER (1968) supplies the unbiased estimator for volume per acre and the closely related one for weight, as well as a very restricted one for number of pieces. BROWN (1972) adds the also closely related estimator for surface area of residue to it. However, for general purposes, including the ones the above authors are concerned with, but also phytosociological, ecological and range inventory activities, knowledge of various other population parameters might be of importance. Considering the assessment of logging residue once more, it might be of interest to be able to obtain from line intersect sampling estimations of e.g. average mid-diameter and average length per piece, total number of pieces per acre, etc. Forest entomologists might want to estimate total number of borer beetles in logs on an area, and mycologists total number of fructifications of wood destroying fungi. In range capacity assessment one would not only like to obtain from a line intersect sample (so not from a belt of certain width (CANFIELD, 1941) about it) an estimation of the area occupied by various herbaceous and ligneous species, but also data as to weight, growth etc. per species. Phytopathologists might be interested to obtain estimations of total number of host-attached plant parasites in a certain area, etc. Finally, just from a line through a stand or its aerial photograph, a forester might wish to obtain information on stand characteristics.

The author herewith presents a generalisation of the theory of line intersect sampling that might make possible all these estimations, and at the same time supplies an approximate estimation of the variance of the estimated quantity, even in case of a one-line sample.

2. BUFFON'S NEEDLE PROBLEM AND LINE INTERSECT SAMPLING

On a rectangular flat area of size WL (Fig. 1) through the center of which a line parallel to L has been drawn, a thin needle $e'e$ of length l_i is randomly thrown. Provisions are such that 1) $l_i \leqslant W$, 2) the needle's center M is always

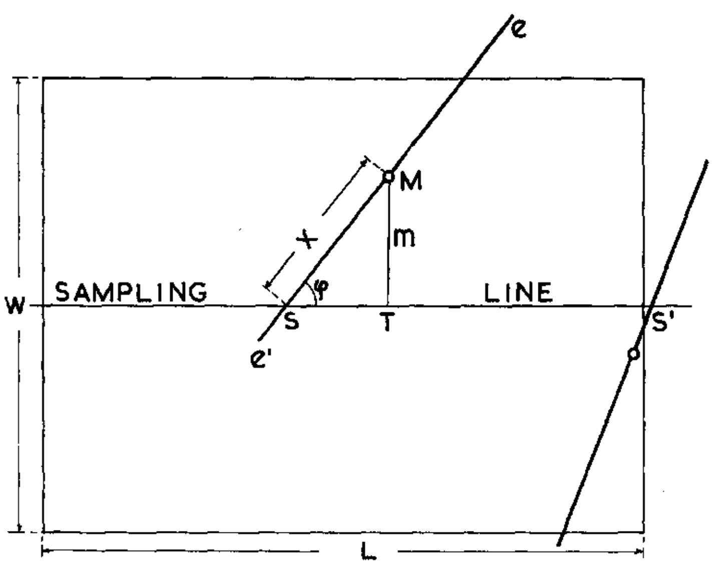

FIG. 1 The needle problem

within the boundaries of the area, 3) irrespective of the position of M the needle may point in any direction, and 4) the length L is sufficiently long relative to l_i to allow intersections of type S' to be neglected.

The question we put is: how large is the probability that the needle will intersect with the center line?

The position of the needle relative to the center line may be indicated by the perpendicular distance $m = MT$ of M to the center line, and the acute angle φ between the needle's direction and that of the center line. Obviously, within the area: $0 \leqslant m \leqslant W/2$ and $0 \leqslant \varphi \leqslant \pi/2$. It should be noted that we are neither interested in whether e points to 'the right' or to 'the left', nor whether M is located 'above' or 'below' the line.

All possible combinations of m and φ are equally probable, i.e. m and φ are stochastically independent and uniformly distributed, m on the interval $[0, W/2]$ and φ on the interval $[0, \pi/2]$. See Fig. 2.

In cases where needle and center line intersect in a point S at a distance x from M, the condition $x \leqslant l_i/2$ must be fulfilled, i.e.

$$m/\sin\varphi \leqslant l_i/2 \text{ or } m \leqslant (l_i/2)\sin\varphi \qquad (1)$$

In other words: all sets (φ, m) that satisfy (1) imply intersection. Such sets are found in the dotted region under the graph of $m = (l_i/2)\sin\varphi$ in Fig. 2; the area of this region is

$$\int_0^{\pi/2} (l_i/2)\sin\varphi \, d\varphi = l_i/2$$

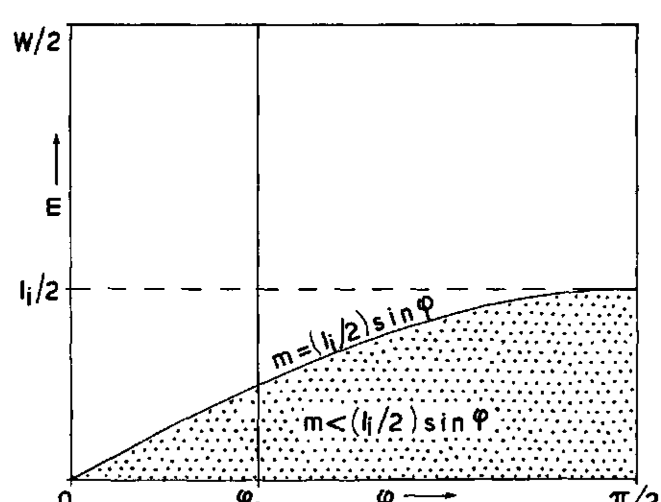

FIG. 2 Derivation of probability of intersection

The probability of intersection then is found as the quotient of the dotted area and that of the entire rectangle, i.e. as

$$p_i = (l_i/2)/(\pi\ W/4) = 2l_i/\pi\ W \tag{2}$$

The above is a slight modification of the solution published in 1777 by George Louis Leclerc, Comte de BUFFON (France, 1707–1788).

Note that the case $(\varphi, m) = (0,0)$ is also considered as an intersection. Further, if the needle has an inclination γ with respect to the flat plane, its effective length parallel to the plane is $l_i \cos \gamma$, so that in general the probability of intersection is

$$p_i = 2l_i \cos \gamma/\pi\ W \tag{3}$$

However, if one wishes to investigate whether the needle would have intersected if its position had been parallel to the plane, it should be rotated in its vertical plane and about its center of gravity M until it is parallel to the plane (Fig. 3).

If the needle is substituted by an arbitrary solid of revolution the axis of the latter can be identified with the former. A solid of revolution other than a cylinder will in general have its axis inclined under a non-zero angle to a flat plane on which it rests, so that (3) is applicable. This influence on p_i however may be neglected in the practical application to be considered. Under field circumstances we have no perfectly flat plane either, and a solid may show an appreciable inclination because of local topography. In order then to evaluate whether there occurs intersection, the method of Fig. 3 should be employed. For obliquely truncated solids l_i of course is the length of axis contained within the solid.

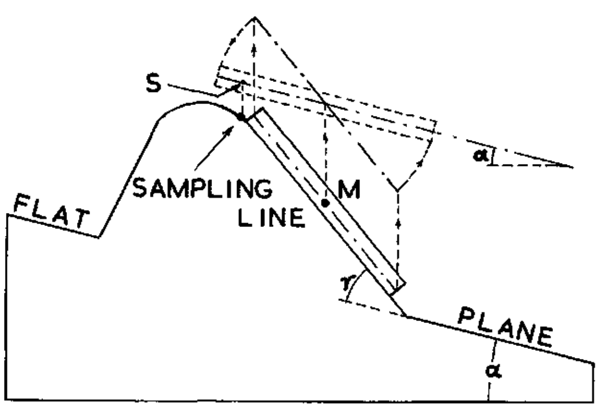

FIG. 3 Check on intersection of inclined solid with sampling line

3. Defining a random variable on a solid. Extension to n solids. General estimator for a total per unit area.

A solid i may have an attribute x (e.g. mid-diameter) of value x_i (e.g. d_i). With this attribute a stochastic (= random) variable $\underline{t}_i x_i$ may be associated [1] which takes the value x_i with probability p_i (2) in case of intersection ($\underline{t}_i = 1$), and the value 0 with probability $(1 - p_i)$ otherwise ($\underline{t}_i = 0$). Then the expected value of $\underline{t}_i x_i$ is:

$$\varepsilon\, \underline{t}_i x_i = x_i\, \varepsilon\, \underline{t}_i = x_i(1.p_i + 0\,(1 - p_i)) = x_i p_i = 2x_i l_i/\pi\, W \tag{4}$$

If the elements of a population of N solids, with different x_i and l_i are thrown randomly and independently into the area WL, and if n of these solids (constituting a sample of $\underline{n}$ out of N) intersect the center line, we have by (4):

$$\begin{aligned}&\varepsilon\, \Sigma^n\, x_i = \varepsilon\, \Sigma^N\, \underline{t}_i x_i = \Sigma^N\, x_i p_i = (2/\pi\, W)\, \Sigma^N\, x_i l_i \text{ or}\\ &(\pi/2L)\, \varepsilon\, \Sigma^n\, x_i = \Sigma^N\, x_i l_i/WL\end{aligned} \tag{5}$$

The right member of (5) is the mean quantity of $(x_i l_i)$ per unit area; obviously an unbiased estimator of the latter is given by:

$$(\pi/2L)\, \Sigma^n\, x_i \tag{6}$$

From (5,6) it follows that the total of any quantified solid attribute x_i per unit area, viz.:

$$X = \Sigma^N\, x_i/WL$$

can be estimated by:

$$\hat{X} = (\pi/2L)\, \Sigma^N\, \underline{t}_i x_i/l_i = (\pi/2L)\, \Sigma^n\, x_i/l_i \tag{7}$$

where $\Sigma^n\, x_i/l_i$ is the sum over the sample of n intersecting solids, of the quantified attributes x_i, each weighted with the inverse of the corresponding solid length l_i.

By proportional expansion, (7) yields an estimator of the population total for areas of arbitrary size.

Now, if an area of WL is randomly placed within a larger area already randomly strewn with solids, we intuitively are in the same situation as having thrown solids randomly onto a fixed area of WL. It should be noted that the magnitude of W does not appear in the estimator (7).

[1] Throughout the text a stochastic variable (like $\underline{t}_i$) will be underlined only when confusion with its realisation (t_i) might exist.

4.1 *Volume*

If the attribute x is defined as the volume of a solid, ${}_tx_i$ may take the values $v_i = \pi\, d_i^2 l_i/4$ or 0 with probabilities associated with intersection and non-intersection respectively, as above. From (7) it then follows that:

$$\hat{V} = (\pi^2/8L)\, \Sigma^n\, d_i^2$$

is an unbiased estimator of:

$$(1/WL)\, \Sigma^N\, \pi\, d_i^2 l_i/4 = (1/WL)\, \Sigma^N\, v_i = V'/WL = V$$

where v_i is the HUBER volume of a solid with mid-diameter d_i and length l_i, V' total volume of the N solids, and V is mean volume per area unit. If all measures are in feet, the estimation is in cft/sq.ft; conversion gives:

$$\hat{V}_d = (37.8125\pi^2/L)\, \Sigma^n\, d_i^2 \text{ cft/acre } (d_i \text{ in inches, } L \text{ in feet}) \qquad (8)$$

The metric equivalent is:

$$\hat{V}_m = (\pi^2/8L)\, \Sigma^n\, d_i^2 \text{ m}^3\text{/ha } (d_i \text{ in cm, } L \text{ in m})$$

The indices d and m stand for duodecimal and metric system respectively. It is noted that VAN WAGNER (1968) though by an other reasoning, arrives at the same estimator for volume per area unit.

4.2 *Weight*

The estimator for weight per unit area is readily derived from (8), putting S = specific gravity of solid substance relative to density of water, i.e. 62.4 lb/cft or 1000 kg/m^3:

$$\hat{Q}_d = (1.17975\pi^2\, S/L)\, \Sigma^n\, d_i^2 \text{ short tons/acre} \qquad (9)$$
$$\text{or } \hat{Q}_m = (\pi^2\, S/8L)\, \Sigma^n\, d_i^2 \text{ metric tons/ha}$$

with d_i and L in the same units as in (8).

4.3 *Mid-sectional area*

If the attribute x is defined as the mid-sectional area of a solid, ${}_tx_i$ may take the values $g_i = \pi\, d_i^2/4$ and 0. Then it follows from (7) that:

$$\hat{G} = (\pi^2/8L)\, \Sigma^n\, d^2/l_i$$

is an unbiased estimator of:

$$(1/WL)\, \Sigma^N\, \pi\, d_i^2/4 = (1/WL)\, \Sigma^N\, g_i = G'/WL = G$$

where g_i is the mid-sectional area of a solid, G' the sum of mid-sectional areas over all N solids, and G the mean quantity of mid-sectional area per unit area. Conversion of units gives:

$$\hat{G}_d = (37.8125\pi^2/L)\,\Sigma^n\, d_i^2/l_i \text{ sq.ft/acre } (d_i \text{ inches, } l_i, L \text{ feet}) \qquad (10)$$

or $$\hat{G}_m = (1.25\pi^2/L)\,\Sigma^n\, d_i^2/l_i \text{ m}^2\text{/ha } (d_i \text{ cm, } l_i \text{ dm, } L \text{ m})$$

4.4 *Number of solids*

Associating with each solid an attribute x of constant value $x_i = 1$, the stochastic variable $t_i x_i = t_i$ may take the values 1 and 0 in the cases of intersection and non-intersection respectively. It follows that an unbiased estimator of the number of solids per unit area is:

or $$\hat{N}_d = (21780\,\pi/L)\,\Sigma^n\, 1/l_i \text{ solids/acre } (l_i, L \text{ in feet}) \qquad (11)$$

$$\hat{N}_m = (5\pi\; 10^4/L)\,\Sigma^n\, 1/l_i \text{ solids/ha } (l_i \text{ in dm, } L \text{ in m})$$

It is noted that if all solids are of equal length l, (11) changes into the estimator for number of solids per area unit derived by VAN WAGNER (1968).

4.5 *Total length of solids*

Taking a solid's length as its attribute x, $t_i x_i$ may take the values l_i and 0. Hence $A = \Sigma^N\, l_i/WL$, i.e. total solid length per unit area, is estimated unbiasedly by $\pi\, n/2L$. In specified units:

$$\hat{A}_d = 21780\,\pi\, n/L \text{ feet/acre } (L \text{ in feet}) \qquad (12)$$

or $$\hat{A}_m = n\,\pi\; 10^5/2L \text{ dm/ha } (L \text{ in meters})$$

4.6 *Mean solid volume*

The estimator for mean volume per solid is, by (8, 11)

$$\hat{\bar{v}} = \hat{V}/\hat{N}$$

leading to:

$$\hat{\bar{v}}_d = (\pi/576)(\Sigma^n\, d_i^2)/(\Sigma^n\, 1/l_i) \text{ cft } (d_i \text{ inch, } l_i \text{ feet}) \qquad (13)$$

or $$\hat{\bar{v}}_m = (\pi/400)(\Sigma^n\, d_i^2)/(\Sigma^n\, 1/l_i) \text{ dm}^3 \; (d_i \text{ cm, } l_i \text{ dm})$$

4.7 *Mean mid-sectional area*

From (10, 11) the mean mid-sectional area per solid is estimated as

$$\hat{\bar{g}} = \hat{G}/\hat{N}$$

leading to:

$$\hat{\bar{g}}_d = (\pi/576)(\Sigma^n\, d_i^2/l_i)/(\Sigma^n\, 1/l_i) \text{ sq.ft } (d_i \text{ inch, } l_i \text{ feet}) \qquad (14)$$

or $$\hat{\bar{g}}_m = (\pi/4)(\Sigma^n\, d_i^2/l_i)/(\Sigma^n\, 1/l_i) \text{ cm}^2 \; (d_i \text{ cm, } l_i \text{ dm})$$

4.8 *Mean mid-diameter*

The estimator of the diameter D of the mean mid-sectional area satisfies:

$$\pi \hat{D}^2/4 = \hat{\bar{g}}$$

so that with (14) we have:

$$\hat{D}^2 = (\Sigma^n d_i^2/l_i)/(\Sigma^n 1/l_i) \text{ sq. inch or cm}^2 \qquad (15)$$

and $\hat{D} = \sqrt{\hat{D}^2}$ inch or cm.

Units as in (14).

4.9 *Mean solid length*

Taking λ_1 as the LOREY-type of average solid length, we have $\lambda_1 = V/G$, and obtain from (8, 10):

$$\hat{\lambda}_1 = \hat{V}/\hat{G} = (\Sigma^n d_i^2)/(\Sigma^n d_i^2/l_i) \text{ feet of dm} \qquad (16)$$

with units as in (14).

The estimator of the arithmetic mean solid length is:

$$\hat{\lambda}_2 = \hat{A}/\hat{N} = n/\Sigma^n 1/l_i \text{ ft; dm } (l_i \text{ in ft; dm}) \qquad (17)$$

4.10 *Total and mean of arbitrary attribute*

The unbiased estimators in sections 4.1–4.5 are only illustrations of the application of the general estimator (7). It is obvious that x_i may also stand for e.g. bark volume, surface area, cull percentage, number of borer beetles, number of knots or annual rings etc. in a solid (= log).

It can be shown (section 6) that the derived estimators (13–17) and similar ones related to arbitrary attributes are approximately unbiased if the number of intersections is large.

5. VARIANCE OF A QUANTITY PER UNIT AREA. REQUIRED LINE LENGTH

5.1 Within a given area of arbitrary size and shape a straight random sampling line that traverses the entire area may be chosen by first randomly selecting a point within the area, and then a random direction. So there is an infinite number of ways to select a random sampling line, and each of the corresponding sampling lines, the length of which may vary between zero (i.e. unspecified observation) and a maximum value, has the same probability of being chosen. By (7) an unbiased estimation of the same parameter X is associated with each sampling line. Supposing that all possible estimations are distributed about X with variance

$$\text{var } \hat{X} = S^2$$

and taking a random sample of k lines of length L_j ($j = 1 \ldots k$), we obtain k estimations with mean

$$\hat{X}_m = \Sigma^k \hat{X}_j / k,$$

from which S^2 can be estimated by:

$$s^2 = \Sigma^k (\hat{X}_j - \hat{X}_m)^2 / (k-1) \tag{18}$$

If we suppose a uniform random population, the estimations obtained from $j = 1 \ldots k$ disconnected sampling lines of length L_j are as good as the k estimations we would obtain if the lines were connected one behind the other to constitute one sampling line of length $\Sigma^k L_j$. The expected total number of intersections on the latter, $\varepsilon\, n$, may be expected to equal $\varepsilon\, \Sigma_j^k n_j$, and also: $\varepsilon\, \Sigma^n x_i / l_i = \varepsilon\, \Sigma_j^k \Sigma_i^{n_j} x_{ij} / l_{ij}$. Hence we may write for the estimation associated with the long line:

$$\varepsilon\, \hat{X} = \varepsilon(\pi / 2\Sigma^k L_j)\Sigma^n x_i / l_i = \varepsilon(1/\Sigma^k L_j)\Sigma_j^k L_j(\pi/2L_j)\Sigma_i^{n_j} x_{ij}/l_{ij} =$$
$$= \varepsilon\, \Sigma^k L_j \hat{X}_j / \Sigma^k L_j = \varepsilon\, \hat{X}_w$$

This suggests that an estimation should be weighted with the length L_j of the sampling line with which it is associated, and that in case k lines are used, the X for the entire area is represented best by:

$$\hat{X}_w = \Sigma^k L_j \hat{X}_j / \Sigma^k L_j \tag{19}$$

As in hypothetical repeated sampling with k random lines the L_j-series need not be the same, it is mathematically difficult to derive the estimator of the variance of (19). However, if we specify that in hypothetical repetitions the same L_j-series will be considered, we have the conditional expression:

$$\hat{\text{var}}\, \hat{X}_w | L_j (j = 1 \ldots k) = s^2\, \Sigma^k L_j^2 / (\Sigma^k L_j)^2 \tag{20}$$

where now s^2 is the estimation of the variance in the subpopulation of estimations (7) associated with a given set of line lengths $L_j (j = 1 \ldots k)$. The above also holds if the lines are given specified lengths a priori, without the condition that they have to traverse the entire area. For all L_j of equal length L, (20) becomes simply:

$$\hat{\text{var}}\, \hat{X}_w | L = s^2 / k \tag{21}$$

It follows that in line intersect sampling with k random lines we do not get around a conditional variance at present. But then there is still another aspect, which may also open the possibility of estimating a conditional variance in case of sampling with one line only.

In line intersect sampling the probability of finding a specified sample with n intersections on a line, i.e. the probability of obtaining a sample S_n is:

$$P(S_n) = \left(\prod_{i \in S_n} p_i\right)\left(\prod_{i \notin S_n} (1-p_i)\right) \tag{22}$$

From (22) it is seen that $P(S_n)$ depends on the p_i of the n solids that intersect the sample line and, as these probabilities in their turn depend on l_i, the $P(S_n)$ may be quite different even for samples of equal n. Actually, after a line intersect sample has been taken, we have randomly selected, without replacement, n units out of N by considering each of the N units separately, giving each a chance p_i of becoming included in the sample. This principle, basic to line intersect sampling, is named POISSON sampling by HÁJEK (1964); by its nature it produces samples of size n, where n itself is a stochastic variable.

So line intersect sampling is POISSON sampling. If we sample with one line of length L, we derive from (7):

$$\text{var } \hat{X} = (\pi/2L)^2 \, \Sigma^N (x_i/l_i)^2 \text{ var } t_i \tag{23}$$

with $\text{var } t_i = \varepsilon\, t_i^2 - (\varepsilon t_i)^2 = p_i(1-p_i) \simeq p_i$

as in practical cases the p_i may be considered small relative to one. It is noted that in (23) line length is supposed to be constant in hypothetical repetitions. Resubstitution of p_i from (2) in (23) gives:

$$\text{var } \hat{X} = (\pi/2L)\, \Sigma^N (x_i^2/l_i)/WL \tag{24}$$

and from the preceding theory it follows that this quantity can be estimated by:

$$\hat{\text{var}}\, \hat{X} = (\pi/2L)^2\, \Sigma^n (x_i/l_i)^2 = (\pi/2)^2 (n/L)(1/L) \Sigma^n (x_i/l_i)^2/n \tag{25}$$

which, it is reminded, is an approximate, conditional expression. For convenience (25) is written as:

$$\hat{\text{var}}\, \hat{X} = (\pi/2)^2 (n/L)(1/L) s^2 \tag{26}$$

with $s^2 = \Sigma^n (x_i/l_i)^2/n$

When k random lines, numbered $j = 1 \ldots k$ of length L_j are used, k independent estimates (7) with variances (24) are obtained. Combining the results of these k lines, the best estimate of X may be put at:

$$\hat{X}_{w'} = \Sigma^k\, w_j \hat{X}_j / \Sigma^k\, w_j \tag{27}$$

with $\text{var } \hat{X}_w = (\Sigma^k w_j)^{-1}$ and $w_j = 1/\text{var } \hat{X}_j$

which quantities can be estimated by putting

$$w_j = 1/\hat{\text{var}}\, \hat{X}_j.$$

It is seen that (26) decreases with increasing line length L_j, and increases with the number n_j/L_j of intersections per unit of L_j. In a uniform random population of solids, n_j/L_j and s_j^2 will not vary much, so that (26) will be approximately proportional to $1/L_j$. Then the estimate in (27) will be about equal to (19).

Using (25) the approximate estimated conditional variances of the estimators (8) through (12) can be written down directly. The results are given in the table below.

Metric and duodecimal estimators are indexed m and d respectively. In the metric system d_i is in centimeters, l_i in decimeters, L in meters, and S is specific gravity of solid material relative to the density of water, i.e. 1000 kg/m^3. In the duodecimal system d_i is in inches, l_i and L in feet, and S is specific gravity relative to the density of water, i.e. 62.4 lb/cft.

Estimated approximate conditional variances in line intersect sampling with one line.

Volume per unit area

$\hat{\text{var}}\ \hat{V}_m = (\pi^2/8L)^2\ \Sigma^n(d_i^2)^2$ (m^3/ha)^2

$\hat{\text{var}}\ \hat{V}_d = (302.5)^2$ (entire expression for $\hat{\text{var}}\ \hat{V}_m$) (cft/acre)^2

Weight per unit area

$\hat{\text{var}}\ \hat{Q}_m = S^2$ (entire expression for $\hat{\text{var}}\ \hat{V}_m$) (m.tons/ha)^2

$\hat{\text{var}}\ \hat{Q}_d = S^2$ (entire expression for $\hat{\text{var}}\ \hat{V}_d$) (short tons/acre)^2

Mid-sectional area per unit area

$\hat{\text{var}}\ \hat{G}_m = (10\pi^2/8L)^2\ \Sigma^n(d_i^2/l_i)^2$ (m^2/ha)^2

$\hat{\text{var}}\ \hat{G}_d = (30.25)^2$ (entire expression for $\hat{\text{var}}\ \hat{G}_m$) (sq.ft/acre)^2

Number of solids per unit area

$\hat{\text{var}}\ \hat{N}_m = (50000)^2(\pi/L)^2\ \Sigma^n(1/l_i)^2$ (no/ha)^2

$\hat{\text{var}}\ \hat{N}_d = (0.4356)^2$ (entire expression for $\hat{\text{var}}\ \hat{N}_m$) (no/acre)^2

Total length per unit area

$\hat{\text{var}}\ \hat{A}_m = (1/n)(n\,\pi\,10^5/2L)^2 = \hat{A}_m^2/n$ (dm/ha)^2

$\hat{\text{var}}\ \hat{A}_d = (1/n)(21780\ n\pi/L)^2 = \hat{A}_d^2/n$ (ft/acre)^2

Expression (26) may be used to approximately find the required line length in uniform random populations in case a specified sampling precision should be met. Assuming that (7) is normally distributed about X with (24), and requiring that half the 95% – confidence interval should not exceed a value H, we may put approximately:

$$2\sqrt{\hat{\text{var}}\ \hat{X}} \leqslant H$$

from which follows the estimated required sample line length as

$$\hat{L} \geqslant (n_p/L_p)\,(\pi\, s_p/H)^2 \tag{28}$$

where n_p/L_p and s_p are values from a pilot sampling line of length L_p.

5.2 For completeness' sake it should be mentioned that the theory of rejective sampling of size n, developed by HÁJEK (1964) as a special case of POISSON sampling, is also applicable to the problem of line intersect sampling. Rejective sampling of size n is accomplished by considering all elements of a population in succession, giving the i th element a chance p_i of becoming included in the sample, but only samples of final size n are given attention, all others are ignored.

In rejective sampling of size n, HÁJEK employs the unbiased HORVITZ-THOMPSON estimator:

$$\hat{X}_o = \Sigma^n x_i/\pi_i$$

for the population total X. Here π_i, the probability of inclusion of the i th unit in a sample, is in general a complicated function of the p_i of selection. However, if the p_i satisfy:

$$p_i = n.\, l_i/\Sigma^N l_i = n.z_i, \text{ i.e. } \Sigma^N p_i = n \tag{29}$$

where l_i is a measure of size of an element, HÁJEK proves that the π_i may be put approximately equal to the p_i.

Then the expression for the above estimator becomes identical to:

$$\hat{X}_{pps} = \Sigma^n(x_i/z_i)/n$$

i.e. the unbiased estimator for a population total X (COCHRAN, 1963) used in sampling with replacement with sample size n and selection probabilities z_i.

For rejective sampling of size n HÁJEK derives the expression for the population value of the variance which in our notation reads:

$$\text{var } \hat{X}_o \simeq \Sigma^N(x_i - R.l_i)^2(-n.l_i + \Sigma^N l_i)/n.l_i$$

with $R \simeq \Sigma^N x_i/\Sigma^N l_i$

Putting $z_i = l_i/\Sigma^N l_i$, it is easily seen that for small z_i:

$$\text{var } \hat{X}_o \simeq \Sigma^N z_i(x_i/z_i - \hat{X})^2/n \tag{30}$$

which is the expression for COCHRAN's (1963) estimator, and that HÁJEK's estimator vâr $\hat{X}_o$ likewise reduces to:

$$\text{vâr } X_o \simeq \text{vâr } \hat{X}_{pps} = \Sigma^n(x_i/z_i - \Sigma^n x_i/nz_i)^2/n(n-1) \tag{31}$$

Now if in line intersect sampling we take the sum of (2) over the N solids in the population, and require:

$$\Sigma^N p_i = (2/\pi\, W)\Sigma^N l_i = n \tag{32}$$

as in rejective sampling of size n, it follows that the requirement (see section 4.5):

$$A = \Sigma^N l_i/WL = \pi\, n/2L = \hat{A}$$

is implied, i.e. the identification of the estimate in 4.5 with its expected value A. In fact this is not unacceptable if only one sampling line is used. If, under this assumption, W from (32) is substituted in (2), we obtain (29), which implies that:

$$p_i = \pi_i = 2l_i/\pi W \quad \text{or} \quad \pi/2 = l_i/\pi_i W \tag{33}$$

holds in line intersect sampling with fixed sample size n. Substitution of $\pi/2$ from (33) in our estimator (7) gives:

$$\hat{X} = (1/L)\Sigma^n(\pi/2)(x_i/l_i) = (1/WL)\Sigma^n x_i/\pi_i = (1/WL)\,\hat{X}_o$$

which is of the HORVITZ-THOMPSON type. Consequently:

$$\hat{\text{var}}\,\hat{X} \simeq (1/WL)^2\ \hat{\text{var}}\,\hat{X}_o \text{ (see (31))}$$

supposing hypothetical repetitions are made with the same L. By substitution of W from (32) and of the expression for z_i, the latter formula changes into the approximate, conditional estimator:

$$\hat{\text{var}}\,\hat{X} \simeq (\pi/2)^2(n/L)(1/L)\Sigma^n(x_i/l_i - \Sigma^n x_i/nl_i)^2/(n-1) \tag{34}$$

which resembles (25) in many respects, and from which expressions for the estimated variances of estimated specific parameters can be derived in a similar way as before. However, in (34) restrictions are more severe than in (25) as, though in both formulae a constant L is assumed, the n in (25) is allowed to vary in repeated sampling, contrary to the n in (34). As we have the estimator (25), we can dispose of (34) the more so as the latter yields the value zero in a solid population of identical elements. Of course (25) produces larger values than (34).

6. APPROXIMATE VARIANCES OF MEANS PER SOLID

If we put

$$D^2 = \Sigma^N d_i^2/N = (\varepsilon\,\Sigma^n d_i^2/l_i)/(\varepsilon\,\Sigma^n 1/l_i) = R \text{ (population ratio)},$$

and in (15)

$$\hat{D}^2 = (\Sigma^n d_i^2/l_i)/(\Sigma^n 1/l_i) = \bar{u}/\bar{m} = \hat{R} \text{ (estimated population ratio)} \tag{35}$$

where $u_i = d_i^2/l_i$, $m_i = 1/l_i$, $\bar{u} = \Sigma^n u_i/n$ with analogon for $\bar{m}$, we have:

$$\begin{aligned}\hat{R}\text{-}R &= (1/\bar{m})(\bar{u} - R\bar{m}) \simeq (1/n\overline{M})\Sigma^n(u_i - Rm_i) = (1/n\overline{M})\Sigma^n(d_i^2 - R)/l_i = \\ &= (1/n\overline{M})\,\Sigma^n x_i/l_i = (1/n\overline{M})\,\Sigma^N t_i(x_i/l_i)\end{aligned} \tag{36}$$

where $\bar{m}$ has been put equal to

$$\overline{M} = \Sigma^N m_i/N$$

which is only allowable when n is large, and where further $x_i = d_i^2\text{-}R$ is the quantified attribute of a solid, $t_i(x_i/l_i)$ being the associated random variable as before, with $t_i = 1$ or 0 in case of intersection or non-intersection respectively. It is easily shown that the expected value of (36) is zero if n is large, so in that case (35) is an unbiased estimator of D^2. The same property holds for the estimators (14–17).

From (7) and (25), $\hat{\text{var}}\ \Sigma^n x_i/l_i$ can be found, and applying this result to (36) we obtain:

$$\hat{\text{var}}\ \hat{D}^2 = \hat{\text{var}}\ \hat{R} \simeq (\Sigma^n 1/l_i)^{-2}\ \Sigma^n((d_i^2 - \hat{D}^2)/l_i)^2\ \text{cm}^4;\ \text{inch}^4 \tag{37}$$

where $\bar{m}$ and (35) have been resubstituted for lack of knowledge of the population values. Units in (37) are as in (14).

It is noted that the same result is not found by applying the well-known estimator for the variance of an estimated ratio to (35):

$$\hat{\text{var}}\ \hat{R} = (1/\bar{m})^2(1/n)(s_u^2 + \hat{R}^2 s_m^2 - 2\,\hat{R} s_{um}) \tag{38}$$

In doing so we find the variance under the more restricted conditions of HÁJEK's rejective sampling of size n.

To find an approximate expression for the variance of the estimated mean mid-diameter we might expand the function

$$\text{f}(\hat{D}^2) = \sqrt{\hat{D}^2}$$

by a TAYLOR series, putting

$$\hat{D}^2 = D^2 + \delta,\ \text{where}\ \varepsilon\delta^2 = \text{var}\ \hat{D}^2.$$

We then obtain:

$$\hat{\text{var}}\ \hat{D} \simeq \hat{\text{var}}\ \hat{D}^2/4\hat{D}^2\ \text{cm}^2;\ \text{sq. inch} \tag{39}$$

In a similar way we find (cf. 13)

$$\hat{\text{var}}\ \hat{\bar{v}} \simeq (\Sigma^n 1/l_i)^{-2}\ \Sigma^n((v_i - \hat{\bar{v}})/l_i)^2\ \text{dm}^6;\ \text{cft}^2 \tag{40}$$

with units as in (13).

Further (cf. 16):

$$\hat{\text{var}}\ \hat{\lambda}_1 \simeq (\Sigma^n d_i^2/l_i)^{-2}\ \Sigma^n(d_i^2(l_i - \hat{\lambda}_1)/l_i)^2\ \text{dm}^2;\ \text{sq. ft} \tag{41}$$

with units as in (16), and finally (cf. 17):

$$\hat{\text{var}}\ \hat{\lambda}_2 \simeq (\Sigma^n 1/l_i)^{-2}\ \Sigma^n((l_i - \hat{\lambda}_2)/l_i)^2\ \text{dm}^2;\ \text{sq. ft} \tag{42}$$

with units as in (17).

From the above it will be clear that an expression for the conditional variance of similar mean values of other attributes can be derived analogously.

Formula (38), where $s_{um} = \hat{\rho}_{um} \cdot s_u \cdot s_m$ indicates that these variances will be smaller the more u and m are positively correlated. When many different mid-diameters and lengths are present in the population of solids, and if their values vary independently to a considerable extent, no high correlation coefficients can be expected however.

Using k random lines, k independent sets of estimates of a population mean and its variance are obtained, and weighted means may be constructed similar to (19) or (27).

7. BIAS DUE TO ORIENTATION OF SOLID AXES

The preceding formulae are based on a population of solids in random order, i.e. a population in which 1) the centers of the axes are randomly distributed over the area, 2) the orientation of the axes is random. If the orientation of the axes relative to the sampling line shows a pronounced trend in one direction, severely biased results may be obtained if the formulae for random populations are applied.

VAN WAGNER (1968) derived the values of maximum bias in expected value of estimated volume per unit area, for the theoretical case that all solids are oriented in the same direction, viz. under an angle φ with the sampling line. He computes this bias for three alternatives, viz. for 1) the result obtained in sampling with one line under a constant φ_o with the axes, 2) the average result in sampling with two perpendicular lines, one under an angle φ_o with the axes, and 3) the average result in sampling with three lines with mutual directional differences of 60°, one under an angle φ_o with the axes. The relation of this bias to the angle φ_o is easily explained by the needle model (Fig. 1) as follows: if φ_o is constant and $0 \leqslant m \leqslant W/2$, condition (1) becomes:

$$m \leqslant (l_i/2) \sin \varphi_o$$

so that (Fig. 2) the probability of intersection is found as the quotient of the line segments $(l_i/2) \sin \varphi_o$ and $W/2$, hence

$$p_i = (l_i \sin \varphi_o)/W \tag{43}$$

Using (43) in (4 etc) we obtain for one sample line:

$$\varepsilon \hat{X} = \varepsilon(\pi/2L) \Sigma^n x_i/l_i = (\pi/2) \sin \varphi_o \Sigma^N x_i/WL = (\pi/2) \sin \varphi_o.X$$

The procentual bias relative to X then is

$$B = 50(\pi \sin \varphi_o - 2)\ \% \tag{44}$$

Using 2 perpendicular sample lines, one under φ_o, so the other under $(90°-\varphi_o)$ with the axes, two values (7) are obtained with expected values $(\pi/2) \sin \varphi_o.X$ and $(\pi/2) \cos \varphi_o.X$ respectively. The procentual bias of the mean expected value then becomes:

$$B = 25\ [\pi(\sin \varphi_o + \cos \varphi_o) - 4]\ \% \tag{45}$$

Using 3 sampling lines as described above the mean of the expected values of the three values (7) then obtained differs for the ranges $0° \leqslant \varphi_o \leqslant 60°$ and $60° \leqslant \varphi_o \leqslant 90°$. The bias percentages for these two cases are respectively:

$$B = (100/6)\,[\pi(\sin\varphi_o + \sqrt{3}\cos\varphi_o) - 6]\ \% \tag{46}$$

and $$B = (100/3)\,[\pi\sin\varphi_o - 3]\ \%$$

Equations (44) through (46) correspond to the three graphs for maximum bias shown by VAN WAGNER (1968) for volume estimation. From the above it is obvious that this bias is the same for any estimated parameter in line intersect sampling. Severest bias, ranging, from –100% for $\varphi_o = 0°$, to +57% for $\varphi_o = 90°$ may occur when one line is used. With 2 lines bias ranges from –22% for $\varphi_o = 0°$ or 90°, to +11% for $\varphi_o = 45°$. With 3 lines the range is from –9% for $\varphi_o = 0°$ or 60°, to +5% for $\varphi_o = 30°$ and 90°.

The positive bias of about 15% in WARREN and OLSEN's (1964) figures for three compartments logged by Skyline/Skagit and sampled with randomly located clusters of 2 perpendicular lines, resulting in an average angle of $\varphi = 45°$, seems to correspond well with the theoretical value of 11%. The bias of 4.5% for areas logged by tractor should be regarded as random.

If there is only a slight unidirectional trend, expected bias will of course be less then the maximum values given by (44) to (46), but the latter may serve as a guide to the orders of magnitude. VAN WAGNER's (1968) proposal to use three lines in that case is well founded and should be given due attention. As these lines constitute a cluster, maybe the best method is to consider them as a whole and to derive only one estimate (7) and one (25) from them.

Finally the possibilities offered by the principle of stratification either with respect to the average number of solids per unit area, or with respect to the degree of orientation should not be forgotten.

8. APPLICATION TO LOGGING RESIDUE INVENTORY

It is obvious that the theory developed in the preceding sections in principle is applicable to the practical case of inventorying logging residue. However some field problems remain to be considered. Both WARREN and OLSEN (1964) and VAN WAGNER (1968) list a number of rules for the field procedure. From the theory described, a somewhat more complete list may be derived:

1. Locate one or more sampling lines randomly in the area. If there is bias in log orientation use one or more clusters of 3 lines with 60° mutual directional difference;
2. Verify whether the central axis of a log intersects with the sampling line, by the method indicated in Fig. 3. In obliquely trimmed logs consider only

intersection with the central axis as far as the latter is contained in wood;

3. Consider the (rare) case of a central axis exactly coinciding with the sampling line as a normal intersection;
4. If a volume estimate is required, measure only mid-diameter of each intersecting log. By measurement of mid-diameter d_i in favour of the diameter at the point of intersection (though the latter has an expected value equal to d_i) the introduction of an extra sampling error is avoided;
5. If estimates of other parameters are required, such as total log length per area unit, number of logs per area unit, mean mid-sectional diameter or mean log length) also measure the length l_i of each intersecting log. In crooked logs that intersect once, maybe the best thing to do is also to measure the length of the central axis and not a chord, in order to avoid systematic under or over estimation of certain parameters;
6. A crooked log that intersects twice or more should be considered as one part, yielding one d_i and one l_i (see Fig. 4). This procedure follows from the needle theory: if such a log is considered as two parts, the latter do not represent 'randomly thrown needles';
7. A forked log intersecting once or twice (Fig. 4) should be dealt with from the same point of view as under point 6. A representative diameter should be taken;
8. Reduce the estimates of quantities per unit area to the horizontal plane by dividing them by cos S in case of sampling on a slope with an angle of S degrees.

In order to check the formulae derived in the preceding sections, we will apply them to the field data given by VAN WAGNER (1968). As these data are incomplete from our point of view, some rough estimations will have to be made.

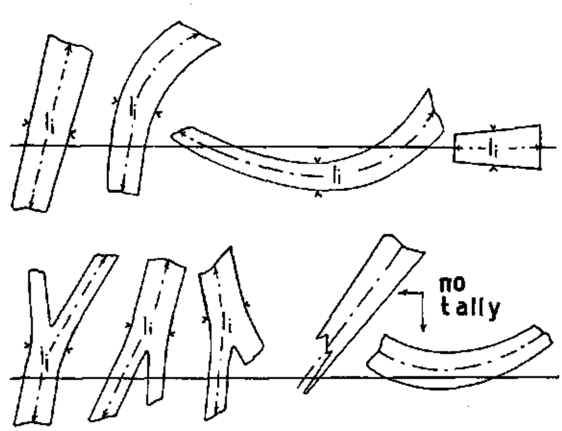

FIG. 4 Some types of intersection

VAN WAGNER describes a field trial in a 20 acre area in which a line intersect sample was taken of $k = 19$ lines of equal length $L = 100$ feet. On the area a population of logs of constant length $l_i = 16$ feet was randomly distributed. These logs originated from a clear felled mixed hardwood/conifer stand with diameters (at breast height = 4′6″) averaging about 6 inches, with a maximum of 15 inches, so average mid-diameter of these logs will have been about 5 inches. The average number of intersections per line was $n = 36$, and the average volume estimate found with VAN WAGNER's formula, which is identical to our (8), was 2701 cft/acre. This value is the mean of 19 observations of equal weight, as line length was constant, so its variance is given by (21) or (27). Substitution of the values for volume estimate and L in (8) gives $\Sigma^n d_i^2 = 724$, from which average mid-diameter is roughly estimated as $d_i = \sqrt{724/36} = 4.5$ inch, which agrees well with the above estimate of $d_i = 5$ inch.

Substitution of $d_i = 4.5$ inch, $L = 100$ feet in our formula (see table) for the variance of the volume estimate (weighted mean) gives a standard deviation of

$$\sqrt{\hat{\mathrm{var}}\ \hat{\mathrm{V}}_d/\mathrm{k}} = 103 \text{ cft/acre},$$

which as to order of magnitude compares well with the value of 141 cft/acre given by VAN WAGNER. The difference is most probably due to our calculations with the average $n = 36$.

As $\Sigma^n 1/l_i = 36/16$ on an average, the number of logs is estimated by (11) as 1539 per acre, with standard deviation of 59 logs/acre. From the estimated 2701 cft/acre and the stand data $l_i = 16$, $d_i = 5$, the number of logs is estimated on the other hand as $N = (4)\,(144)\,(2701)/\pi d_i^2 l_i) = 1238$ per acre, the difference with 1539 no doubt again to be ascribed to the same cause as above.

Total solid length is estimated by (12) as 24611 feet/acre, with standard deviation of

$$\sqrt{\hat{A}^2/kn} = 939 \text{ feet/acre}.$$

SUMMARY AND DISCUSSION

By applying the solution of BUFFON's slightly modified needle problem (which provides the probability of intersection of a randomly thrown thin needle with a straight line in a flat plane) to the similar case of throwing a solid of revolution the central axis of which is identified with a needle, and extending the theory to the throwing of N solids, a workable model for line intersect sampling is obtained.

After a stochastic variable has been defined on a solid, the model provides a general estimator (7) for the average total per unit area of any characteristic whatsoever, associated with the solid. This means a considerable extension of the potential of line intersect sampling, by which technique only estimates of volume, weight and surface area could be provided till now. Estimators both in the metric and in the duodecimal system are given for volume (8), weight (9), total mid-sectional area (10), number of solids (11) and aggregate solid length (12) on a unit area basis, for the case of sampling for logging residue. Moreover estimators for e.g. mean volume (13), diameter of mean mid-sectional area (15) and of mean solid length (16, 17) are provided.

It is made plausible that crooked or forked logs intersecting more than once with the sampling line should be measured only once, and that the rare case of the central log axis coinciding with the sampling line should, contrary to custom, be considered as an intersection.

It is indicated that line intersect sampling, where the number of intersections per line is a random variable, is in fact POISSON sampling as defined by HÁJEK (1964). The recognition of this basic principle allows the derivation of simple, approximate estimators for conditional variances of estimated quantities in line intersect sampling (25, 37 etc.). From this follow indications as to how to find the required line length to obtain a specified precision (28) and a weighted mean (27) in case more than one sampling line is used.

For completeness' sake the application of a special type of POISSON sampling, viz. HÁJEK's (1964) method of 'rejective sampling of size n' is considered also.

The influence of biased orientation of solid axes on the expected value of an estimator is considered, and formulae for its calculation are provided (44–46), corresponding to the graphs given by VAN WAGNER (1968).

Finally a number of rules for the field procedure in case of application of line intersect sampling to logging residue inventory is listed.

WARREN and OLSEN (1964) use the semi-empirical formula

$$V = (660)\,(66).\alpha.n/I_c.L \text{ cft/acre } (L \text{ in feet})$$

where α is a factor dependent on dimensional characteristics of the log population; it is set at 0.33 for pulpwood logging residue in *Pinus radiata* in New Zealand. Further n is the number of intersections counted on a line, L is line length, and I_c is a factor dependent on the orientation of the sampling line relative to the axes in the population; it is set at 0.67 for tractor (= random) skidding. Equalizing WARREN and OLSEN's formula for volume estimation and the duodecimal version of (8), where $\Sigma^n \pi d_i^2/4$ is substituted by $n.\bar{g}$ ($\bar{g}$ being the average mid-sectional area of the intersecting solids), it is found that the above empirical value $I_c = 0.67$ in tractor skidding indeed comes close to $2/\pi = 0.64$. The value $\alpha = 0.33$ then should equal $\bar{g}$ in square feet, from which follows an average mid-sectional diameter of 7.8 inch, a value that seems acceptable for the material in question.

CANFIELD's (1941) 'line interception technique' may now be extended to real line intersect sampling for forage weight etc. in range quality assessment, without using a belt of certain width along the line, and may also find application in phytosociological enumerations. As a mixed vegetation may roughly be considered to consist of roundish patches (stalks, tufts, shrub and tree crowns) the diameters l_i of which may be identified with a needle, it is easily seen that the probability of intersection is l_i/W (section 2). Then, if we put the material weight of an intersecting patch of a certain species equal to x_i, the expression $(\Sigma^n x_i/l_i)/L$ is an unbiased estimator of $(\Sigma^N x_i)/WL$, i.e. mean weight per area unit of that species. Of course x_i may also stand for the value associated with any other attribute (height, health, number of insects or fungus fructifications, necrotic leafspots etc.). In 'line intersect stand sampling' l_i may be crown diameter, measured in the field or on an aerial photograph.

The generalized theory of line intersect sampling may have numerous other applications in biological and technical fields, and might be extended to plane sampling in three dimensional space.

A paper by the author on line intersect subsampling is to be published shortly.

SAMENVATTING

In 1964 publiceerden WARREN en OLSEN, Nieuw Zeeland, over een nieuwe methode van steekproefsgewijze volumeschatting van na exploitatie op kapvlakten (ca. 1000 ha/jaar) achterblijvend hout. Zij noemden hun nog half-empirische methode 'line intersect sampling'. De volumeschatting werd verkregen door vaststelling van het aantal houtobjecten dat een aselect op het terrein uitgezette rechte lijn snijdt, welk aantal werd ingevuld in een formule, waarin voorts de lijnlengte en enige empirisch bepaalde waarden voorkomen. De methode bleek ca. 5 maal sneller en ook nauwkeuriger te zijn dan de traditionele met cirkelvormige of rechthoekige steekproefvlakten. VAN WAGNER (1968) ontwierp een op diametermeting ter plaatse van de snijpunten gebaseerde, wiskundig juiste schatter voor volume/ha, waaruit tevens die voor gewicht/ha volgt. BROWN (1971) voegde daaraan toe de zeer verwante schatter voor houtmantel-oppervlakte/ha, een grootheid met betrekking tot evaluatie van bosbrandgevaar.

Tot nu toe werd een algemene wiskundig-statistische basis voor deze steekproeftechniek niet onderkend; deze wordt in dit artikel gegeven en is gebaseerd op de reeds in 1777 door BUFFON gegeven oplossing van het zg. naaldprobleem.

Door deze generalisatie wordt het mogelijk om uit een 'lijnintersectie steekproef' een schatting te maken van populatie totaal en van populatiegemiddelde per element, voor willekeurige kwantificeerbare attributen der elementen. De ontwikkelde gedachte heeft, behalve in de bosbouw, wellicht toepassingsmogelijkheden op velerlei ander gebied, zoals b.v. bij plantensociologisch, entomologisch en fytopathologisch veldwerk, alsmede bij habitatinventarisaties t.b.v. faunabeheer. In principe kan deze steekproeftechniek ook op luchtfoto's worden toegepast.

Uitbreiding der theorie tot die voor een steekproef met een vlak in de driedimensionale ruimte lijkt logisch.

In het artikel wordt aangetoond dat de fundamentele achtergrond van 'line intersect sampling' wordt gevormd door de POISSON steekproef, zoals deze laatste door HÁJEK (1964) wordt gedefinieerd. Hierdoor is het mogelijk, uit slechts een lijn een benaderende schatter van de voorwaardelijke variantie van een geschatte grootheid te vinden. Tevens wordt een weg geopend tot het vinden van de gewenste lijnlengte bij a priori gestelde nauwkeurigheidseisen.

De theorie wordt geïllustreerd aan de hand van afleidingen van expressies voor schatters van parameters met betrekking tot houtresiduen op kapvlakten; deze zijn o.a. van belang voor controle op naleving van bepalingen in pulphoutcontracten, alsmede voor de evaluatie van bosbrandgevaar, ook in vergelijkbare situaties buiten kapvlakten, en tenslotte als onderdeel van biomassaschattingen in het ecosysteem bos. Een lijst van instructies voor veldwerk wordt toegevoegd. De nog schaarse literatuur wordt besproken en becommentarieerd.

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