# **Application of Space-Time Diversity/Coding For Power Line Channels**

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# **ABSTRACT**

The purpose of the present work is to evaluate the application of space-time block codes to the transmission of digital data over the power-line communication channel (PLC).

Data transmitted over the power-line channel is usually corrupted by impulsive noise. In this work we analyse the performance of space-time block codes in this type of environment and show that a significant performance gain can be achieved at almost no processing expense.

# **INTRODUCTION**

Theoretical studies of communication links employing multiple transmit and receive antennas have shown great potential [1]-[4] for providing highly spectrally efficient wireless transmissions. The wireless channel suffers attenuation due to destructive addition of multi-paths in the propagation media and to interference from other users. The channel statistic is significantly often Rayleigh, which makes it difficult for the receiver to reliably determine the transmitted signal unless some less attenuated replica of the signal is provided to the receiver. This technique is called diversity, which can be provided using temporal, frequency, polarisation, and spatial resources. In some applications, the only practical means of achieving diversity is the deployment of antenna arrays at the transmitter and/or the receiver. Space-time trellis coding has been proposed [3], which combines signal processing at the receiver with coding techniques appropriate to multiple transmitting antennas. However, when the number of transmitting antennas is fixed, the decoding complexity of space-time trellis codes (measured by the number of trellis states in the decoder) increases exponentially with the transmission rate.

In addressing the issue of decoding complexity, Alamouti [5] recently discovered a remarkable scheme for transmission using two transmitting antennas. This scheme is much less complex than space-time trellis coding for the same number of antennas but there is a loss in performance compared to space-time trellis coding. Despite this performance penalty, Alamouti's scheme is still appealing in terms of simplicity and performance. Space-time block coding (STBC), introduced in [6], generalises the transmission scheme discovered by Alamouti to an arbitrary number of transmitting antennas and is able to achieve the full

diversity promised by the transmitting and receiving antennas.

#### THE TRANSMISSION MODEL

To begin with, we must take into account that in the ST wireless channel the received signal at antenna j (j = 1,...,m), at any time t, is the result of the combination of the signals emitted by the n transmitting antennas, which is not the case of the power line channel where each phase provides a completely isolated path to the transmitted signal. Hence, in order to extend the concepts of wireless STC/STD techniques to the power line environment, we have to use a set of decoding equations that is different from the one used in [6].

We consider a communication system with n emitting points at the transmitter and n receiving points at the receiver (see **Figure 1**). Here, we use the terms *emitting points* and *receiving points* instead of the terms *transmitting antennas* and *receiving antennas* used in wireless communications.

At each time slot t, signals  $C_t^i$ ,  $i=1, 2, \ldots, n$  are transmitted simultaneously from the n emitting points. The channel is assumed to be a flat fading channel and the path gain from emitting point i to receiving point i is defined to be  $\alpha_{i,i}$ . The path gains are modelled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension. The communication channel is assumed to be quasi-static so that the path gains are constant over a frame of length l and vary from one frame to another.

At time t the signal  $r_t^i$ , received at the receiving point i, is given by

$$r_t^i = \alpha_{i,i} c_t^i + \eta_t^i \tag{1}$$

where the samples  $\eta_t^i$  of additive white Class A noise (AWCN) are independently identically distributed (i.i.d.) complex random variables according to Middleton's Class A noise model [7]. The Class A pdf is given by

$$p_{n\kappa}(x) = \sum_{m=0}^{\infty} \frac{\alpha_m}{2\pi\sigma_m^2} \exp\left(-\frac{|x|^2}{2\sigma_m^2}\right) \text{ with}$$

$$\alpha_m = e^{-A} \frac{A^m}{m!}$$
(2)

with the complex valued argument x. The variance  $\sigma_m^2$  is defined as

$$\sigma_m^2 = \sigma^2 \frac{(m/A) + T}{1 + T} \tag{3}$$

where  $\sigma^2$  is the variance of the Class A noise. The Class A noise model combines an additive white Gaussian noise component  $g_{\kappa}$ , with variance  $\sigma_g^2$  and an additive impulsive noise component  $i_{\kappa}$ , with variance  $\sigma_i^2$  [8]. Therefore, the parameter  $T = \sigma_g^2/\sigma_i^2$  is used in the Class A noise model. The second model parameter A is called the impulsive index. For small A, say A = 0.1, we get highly structured (impulsive) noise whereas for  $A \to \infty$  the pdf becomes Gaussian [8].

The variance  $\sigma_m^2$  of x is determined by the channel state  $m=0,1,2,3,\ldots$  using equation (3). Since the Class A noise is memoryless, the states are taken independently for every noise sample with probability  $P(m)=\alpha_m$ , which can be interpreted as a worst-case scenario to model impulsive noise on a power-line channel [8]. The channel state is unknown to the observer of the process and therefore its pdf is given by the expectation over all states.

The average energy of the symbols transmitted from each emitting point at the transmitter side is normalised to be one, so that the average power of the received signal at each receiving point is n and the signal-to-noise ratio is SNR.

Assuming perfect channel state information is available, the receiver computes the decision metric

$$\sum_{t=1}^{l} \sum_{i=1}^{n} \left| r_{t}^{i} - \alpha_{i,i} c_{t}^{i} \right|^{2}$$
 (4)

over all code words

$$c_1^1 c_1^2 ... c_1^n c_2^1 c_2^2 ... c_2^n ... ... c_1^1 c_1^2 ... c_1^n$$

and decides in favour of the code word that minimises the sum.

### SPACE-TIME BLOCK CODES

**Encoding Algorithm** 

A space-time block code (STBC) is defined by a  $p \times n$  transmission matrix G. The entries of the matrix G are linear combinations of the variables  $x_1, x_2, ..., x_k$  for the case of *real orthogonal designs*, or linear combinations of the variables  $x_1, x_2, ..., x_k$  and their conjugates for the case of *complex orthogonal designs* [6]. We can use a

real orthogonal design when transmission at the baseband employs a real signal constellation such as M-PAM. However, for complex constellations like M-QAM or M-PSK, we must use a complex orthogonal design. For example,  $G_c^2$  represents a code that utilises two emitting points and is based on a complex orthogonal design. This is the Alamouti's scheme [5] and is defined by

$$G_c^2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$
 (5)

Similarly,  $G_r^3$  represents a space-time code based on a real orthogonal design that utilises three emitting points.

$$G_r^3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{bmatrix}$$
 (6)

Assuming that transmission at the baseband employs a signal constellation A with  $2^b$  elements, at time slot 1 kb bits arrive at the encoder and select constellation signals  $s_1, s_2, \dots, s_k$ . Setting  $x_i = s_i$  for  $i = 1, 2, \dots, k$  in G, we arrive at a matrix C with entries linear combinations of  $s_1, s_2, ..., s_k$  for the case of real orthogonal designs, or of  $s_1, s_2, ..., s_k$  and their conjugates for the case of complex orthogonal designs. So, while G contains indeterminate variables  $x_1, x_2,...,x_k$ , C contains specific constellation symbols (or their linear combinations), which are transmitted from n emitting points for each kb bits as follows: if  $c^i_t$  represents the element in the th row and the *i*th column of C, the entries  $c_t^i$ , i = 1, 2, ..., n are transmitted simultaneously from emitting points 1, 2, ... , *n* at each time slot t = 1, 2, ..., p. Hence, the *i*th column of C represents the transmitted symbols from the ith emitting point and the tth row of C represents the transmitted symbols at time slot t.

Since p time slots are used to transmit k symbols, we define the rate R of the code to be R = k/p. For example, the rate of  $G_c^2$  in equation (5) is one. Note that for complex constellations, Alamouti's scheme is the only case where we can achieve rate one. For more than two emitting points, complex orthogonal designs allow for rates less than one. On the other hand, for real signal constellations (M-PAM), STBCs with transmission rate 1 can be constructed [6].

#### The Decoding Algorithm

Maximum likelihood decoding of any space-time block code can be achieved using only linear processing at the receiver. We shall illustrate the ML decoding for a 3-phase PLC/STBC communication system, using a real signal constellation with an alphabet of four symbols (4-PAM). This is the scheme used in our computer simulation.

The space-time block code  $G_r^3$  uses the transmission matrix in (6). Suppose that there are  $2^b$  signals in the constellation (b=2 for 4-PAM). At the first time slot kb bits arrive at the encoder and select k=M=4 real symbols  $s_1$ ,  $s_2$ ,  $s_3$ , and  $s_4$ . Then the encoder populates the transmission matrix and at time slots t=1, 2, 3, and 4, the signals  $G_{t1}$ ,  $G_{t2}$ ,  $G_{t3}$  are transmitted simultaneously from emitting points 1, 2, 3. For example, at t=1,  $G_{11}=s_1$ ,  $G_{12}=s_2$ ,  $G_{13}=s_3$ . At t=2,  $G_{21}=-s_2$ ,  $G_{22}=s_1$ ,  $G_{23}=-s_4$ , and so on.

Then maximum-likelihood detection amounts to minimizing the decision metric

$$\left(\left|r_{1}^{1}-\alpha_{1,1}s_{1}\right|^{2}+\left|r_{2}^{1}+\alpha_{1,1}s_{2}\right|^{2}+\left|r_{3}^{1}+\alpha_{1,1}s_{3}\right|^{2}+\left|r_{4}^{1}+\alpha_{1,1}s_{4}\right|^{2}+\right. \\
+\left|r_{1}^{2}-\alpha_{2,2}s_{2}\right|^{2}+\left|r_{2}^{2}-\alpha_{2,2}s_{1}\right|^{2}+\left|r_{3}^{2}-\alpha_{2,2}s_{4}\right|^{2}+\left|r_{4}^{2}+\alpha_{2,2}s_{3}\right|^{2}+\right. \\
+\left|r_{1}^{3}-\alpha_{3,3}s_{3}\right|^{2}+\left|r_{2}^{3}+\alpha_{3,3}s_{4}\right|^{2}+\left|r_{3}^{3}-\alpha_{3,3}s_{1}\right|^{2}+\left|r_{4}^{3}-\alpha_{3,3}s_{2}\right|^{2}\right) \tag{8}$$

which takes into account only three signal paths.

Expanding the above metric and deleting the terms that are independent of the code words, the maximum-likelihood detection rule amounts to form the decision variables

$$R_1 = \left(r_1^1 \alpha_{1,1}^* + r_2^2 \alpha_{2,2}^* + r_3^3 \alpha_{3,3}^*\right) \tag{9}$$

$$R_2 = \left(-r_2^1 \alpha_{1,1}^* + r_1^2 \alpha_{2,2}^* + r_4^3 \alpha_{3,3}^*\right) \tag{10}$$

$$R_3 = \left(-r_3^1 \alpha_{1,1}^* - r_4^2 \alpha_{2,2}^* + r_1^3 \alpha_{3,3}^*\right) \tag{11}$$

$$R_4 = \left(-r_4^1 \alpha_{1,1}^* + r_3^2 \alpha_{2,2}^* - r_2^3 \alpha_{3,3}^*\right) \tag{12}$$

and decide in favour of  $s_i$  among all the constellation symbols s if

$$s_{i} = \underset{s \in A}{\operatorname{argmin}} |R_{i} - s|^{2} + \left(-1 + \left|\alpha_{1,1}\right|^{2} + \left|\alpha_{2,2}\right|^{2} + \left|\alpha_{3,3}\right|^{2}\right) s^{2}$$
 (13)

# SIMULATION RESULTS

In this section we provide computer simulation results for the performance of the space-time code  $G_r^3$  discussed in the previous section. We have chosen an Mary PAM system, with M=4, which transmit over three phases simultaneously, to demonstrate the potential of space-time codes for efficient data transmission over the power-line channel because this baseband digital modulation scheme is particularly sensitive to noise impairments in the channel.

M-ary PAM signals can be represented geometrically as

$$S_m = A_m \sqrt{E_g} \tag{14}$$

where  $E_g$  is the energy of the basic pulse  $g_T(t)$ , and T is the symbol interval. We assume that the amplitudes take the form

$$A_m = 2m - 1 - M$$
,  $m = 1, 2, ..., M$  (15)

so that the distance between adjacent signals is

$$\sqrt{(s_m - s_{m-1})^2} = \sqrt{E_g [2m - 2(m-1)]^2} = 2\sqrt{E_g}$$
 (16)

Assuming equally likely symbols, the average energy for this *real* signal constellation is

$$E_{av} = \frac{E_g \left( M^2 - 1 \right)}{3} \tag{17}$$

In our computer simulation, we used *Gray encoding* for the mapping of groups of b bits on to the  $2^b$  amplitude signals in the 4-PAM constellation, where  $b = \log_2 M$ .

The average energy of the signal constellation in (17) was scaled so that the average energy of the constellation points is one. Therefore, the variance  $\sigma^2$  of the Class A noise at the input of each receiving point is 1/2SNR (see equation (3)).

The path coefficients  $\alpha_{i,i}$  were modelled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension so that the total average power of this distribution is one. This yields a normalized Rayleigh distribution for the magnitude of the complex coefficients.

The inter arrival times of impulse noise were generated using a homogeneous Poisson process, with the state occurrence probabilities given by  $\alpha_m$ . The variance  $\sigma_m^2$  for each possible channel state m is calculated using equation (3). We used a truncated version of (2) with states m=0,1,2,3, which provides a very good approximation, and the class A noise parameters A=0.1 and  $T=10^{-3}$ .

The performance results of the space-time code  $G_r^3$  were compared with those obtained using a conventional single-input single-output (SISO) 4-PAM system working under the same channel conditions but using only one phase for data transmission. The simulation for this scheme was implemented in a separated computer program, where we used an extension of the STBC's ML decision rule that is different from the conventional detection scheme used in M-PAM systems. The decision rule for the SISO case is

$$\hat{s} = \underset{s \in A}{\operatorname{argmin}} |R_1 - s|^2 + \left(-1 + \left|\alpha_{1,1}\right|^2\right) s^2 \tag{18}$$

where

$$R_1 = r^1 \alpha_{1,1}^* \tag{19}$$

Figure 2 shows the values of bit error rate (BER) obtained by the multiple-input multiple-output (MIMO) PLC system, with three emitting points and three receiving points, and by the SISO system using only one emitting point and one receiving point, under the combined effects of multiplicative Rayleigh fading and additive white Class A noise (AWCN). For the sake of comparison, the values obtained with a wireless 4-PAM STBC system have also been included. It is seen that at the bit error rate of  $10^{-2}$  the PLC 4-PAM scheme with the space-time code  $G_r^3$  gives about 7 dB gain over the use of an uncoded system. The better performance of the wireless system is attributable to its higher diversity order with respect to the PLC scheme.

Figure 3 provides simulation results for the case of Rayleigh fading plus additive white Gaussian noise (AWGN), which is the usual channel representation for wireless systems.

As expected, the performances of the two coded systems improve with respect to the case of an AWCN channel.

Finally, Figure 4 and Figure 5 provide simulation results for AWCN and AWGN channels, respectively, in the absence of Rayleigh fading. That is, assuming equal and constant attenuation for every different path and time frame.

# **CONCLUSIONS**

STBC is a new coding/modulation technique for multiple-antenna wireless systems. We described both its encoding and decoding algorithms and proposed its use for data communication over the power-line channel. Simulation results were provided to demonstrate that significant gains are achieved over the use of single-input single-output (SISO) systems.

Further work in this field will be based on power-line point-to-point and point-to-multipoint data communications systems for marine applications, where the design of new specific STBC's transmission matrices will be required [9].

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# **FIGURES**

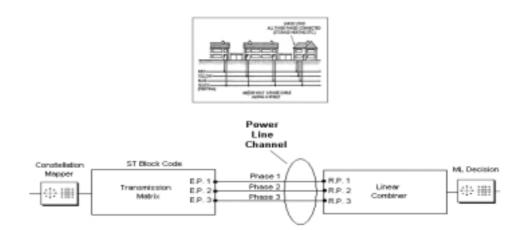
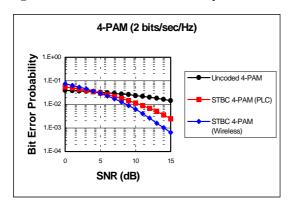
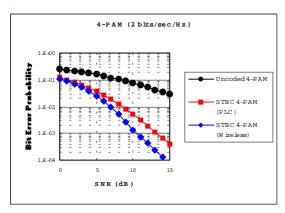


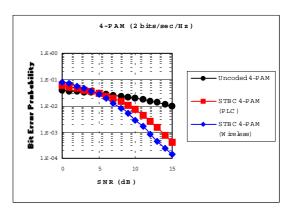
Figure 1 Power line communication system with ST block coding.



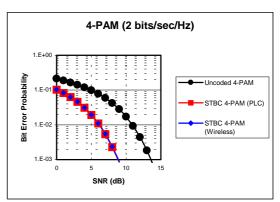
**Figure 2** Bit error probability versus SNR in the presence of Rayleigh fading and additive white Class A noise (AWCN).



**Figure 3** Bit error probability versus SNR in the presence of Rayleigh fading and additive white Gaussian noise (AWGN).



**Figure 4** Bit error probability versus SNR for an additive white Class A noise (AWCN) channel model, in the absence of Rayleigh fading.



**Figure 5** Bit error probability versus SNR for an additive white Gaussian noise (AWGN) channel model, in the absence of Rayleigh fading.