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Application of Vibration Correlation Technique for Open Hole Cylinders

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Abstract
As non-destructive method for axial buckling load determination - Vibration Correlation Technique (VCT) showed major advantages for a range of industrial application. Particular technique for validation of structural limit state in accordance to numerical model prediction for large (true) scale structures are getting the required momentum. The Vibration Correlation Technique (VCT) allows to correlate the ultimate load or instability point with rapid decrement of self-frequency response. Nevertheless this technique is still under development for thin-walled shells and plates. The current research discusses an experimental verification of extended approach, applying vibration correlation technique, for the prediction of actual buckling loads on unstiffened isotropic cylindrical shells with circular cut-outs, loaded in axial compression. Validation study include several aluminium cylinders which were manufactured and repeatedly loaded up to instability point. In order to characterize a correlation with the applied load, several initial natural frequencies and mode shapes were measured during tests by 3D laser scanner. Results demonstrate that proposed vibration correlation technique allows one to predict the experimental buckling load with high reliability, by loading up to ¾ of ultimate load Additional experimental tests including geometric imperfections from initial manufacturing and postbuckling mode shape are currently under development to further validation of proposed approach.

Keywords
Vibration correlation technique, buckling, thin-walled structures, cylindrical shells

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Introduction

Stability is one of most important failure modes of thin-walled structures subjected to compressive loading. In practice it is sufficiently difficult to experimentally determine the buckling load of thin-walled structures by exploiting static test methods. Since even the smallest amount of imperfection of the specimen or boundary conditions can have an apparent impact on the buckling behaviour, thus diverging obtained results from theoretical estimates. There is a need for an alternative approach in order to determine a buckling load for such structures. One of the major aspects of non-destructive testing techniques is ability to accurately predict the buckling loads without damaging the tested specimen itself. In present study the Vibration Correlation Technique (VCT) was explored to derive the buckling load of thin-walled structures.

A research by Lurie (1) showed that for columns in the form of rigid rectangular frames the relationship between the square of the frequency and the load is almost linear, and that the extrapolated load corresponding to a zero frequency coincides with the buckling load. In the case of flat plate, tests showed that such linear behaviour may not be achieved in physical tests because of plate equations are not valid due to initial curvatures of actual plates. More recently (2) studied the analytical relationship between the buckling and vibration behaviour of thin plate. The relationship between applied in-plane load and the natural frequency of plates were derived from the differential governing equations of both problems. The relationship (the square of the natural frequency is linearly related to the applied load) was verified by applying the Ritz method. Research has been extended (3) in the form of modal test series by employing VCT. A set of rectangular plates has been tested for natural frequencies by utilizing an impact test method. The measured buckling load was determined from the plot of square of the measured natural frequency versus an in plane load (tensile). The buckling loads from the measured vibration data come to good agreement with the numerical solutions. Many investigators (4, 5, 6) have extended the VCT approach to thin wall shell structures. Singer, Arbocz and Weller provided the most detailed descriptions about the application of VCT for non-destructive buckling testing of the thin-walled cylindrical structures.

1. Vibration Correlation Techniques

The VCT method is aimed to reduce the scatter in prediction of the buckling loads of thin-walled shells. This approach determines the real natural frequencies of the in-situ shell as a function of the applied axial load. Nevertheless it hasn`t been fully exploited so far as it requires implementation of both destructive and non-destructive testing means as well as high – fidelity measurements.

Both theoretical and experimental research reported in the literature had shown that in some cases a linear descending relationship may be observed. Thus this relationship can be expressed as

$$\frac{P}{P_{cr}}+\left(\frac{f_m}{f_o}\right)^2=1 \tag{1}$$

where P is the applied load, P_{cr} is the buckling load, f_0 is the frequency without any load applied, and f_m is the frequency at load pre-stress state P. The above equation states that for structure reaching the buckling load level the natural frequency of the structure (f_m) would become zero (see Fig. 1a).

Recent efforts to improve the work done so far on the VCT field are presented by Abramovich (5), where new semi-analytical methods was experimentally verified for shells, considering both the non-linear effect of the static state and the nonlinear effect of the geometric imperfections. The current research will present and discuss an experimental verification of a modified VCT approach originally presented by Arbelo [7]. This approach is based on the observations made by Souza. The original approach proposed by Souza is a linear fit between $(1\text{-}p)^2$ versus $(1\text{-}f^{\,4})$, where $p=P/P_{cr}$, $f=f_m/f_0$; P is the applied axial load, P_{cr} is the critical buckling load for a perfect shell, f_m is the measured frequency at P load and f_0 is the natural frequency of the unloaded shell. Souza states that the value of $(1\text{-}p)^2$ corresponding to $(1\text{-}f^{\,4})=1$ would represent the square of the drop of the load carrying capacity(ξ^2), due to the initial imperfections. However, if this approach is applied on unstiffened cylindrical shells the results will be negative values of the drop of the load carrying capacity (ξ^2), which doesn't have any physical meaning (8).

Therefore in modified VCT approach instead of plotting $(1\text{-}p)^2$ versus$(1\text{-}f^{\,4})$, Arbelo proposed to plot $(1\text{-}p)^2$ versus $(1\text{-}f^{\,2})$ and represented the points by a second order fitting curve (which could be further improved). Moreover, the minimum value of $(1\text{-}p)^2$ obtained using this approximation represents the square of the drop of the load carrying capacity(ξ^2), for unstiffened cylindrical shells, due to the initial imperfections. Then, the buckling load can be estimate by Eq. 2 and see Fig. 1b:

$$P_{imperfect}=P_{cr}\left(1-\sqrt{\zeta^2}\right) \tag{2}$$

The main goal on this research is to compare the predicted buckling load versus the real buckling load measured on samples with different materials, geometries (radius, thickness, height) and fabrication technologies. More details about each study case are given in the following section.

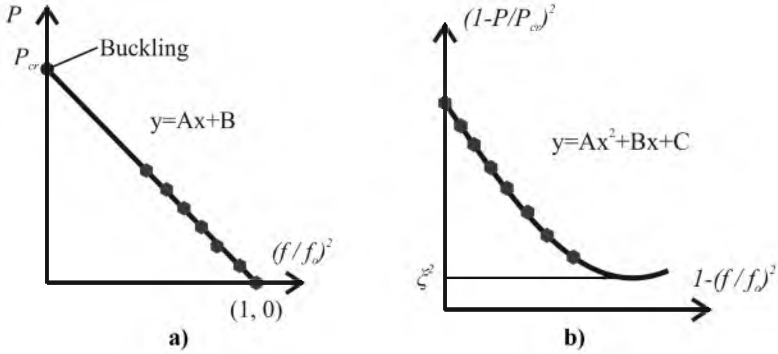


Figure 1. Vibration Correlation Technique: a) VCT method; b) modified VCT

2. Experimental test: materials and methods

2.1. Test specimen: Overview

Five identical aluminium cylindrical shells of nominal diameter D=500 mm and free height H=460 mm was manufactured and tested. Shells R33, R35 and R36, features 50mm, 80 mm and 30 mm diameter, cut-outs, while R37 and R39, features additional ply reinforcement in shape of 20 mm wide ring for R37 and 10 mm wide ring for R39 with corresponding cut-out diameters (30 mm, 80 mm). Cylinders were prototyped at RTU premises, from 0.5 mm thick EN AW 6082 T6 aluminium alloy sheet. Pieces of shells were cut and machined manually to be bonded by 25 mm overlap longitudinal joint with Araldite® 2011 universal 2-component epoxy structural adhesive see Figure 2 and cured in room temperature. Both opposite edges were unsymmetrically taper machined to facilitate as uniform as possible the thickness distribution across the 25 mm wide overlap joint of the aluminium sheet. Cut-outs were machined afterwards on build cylinders. Shells reinforced with rings, adhesively bonded by the same Araldite® 2011 adhesive as used for shell bonding, were processed the same way to obtain the same cut-out diameter as reinforcement ring.

Table 1. Material properties of AW 6082 T6 aluminium tensile tests

Parameters	Value	Standard Deviation
E-Modulus [GPa]	69.7	0.908
Tensile stress at Yield point (Offset 0.2%) [MPa]	276.6	2.805

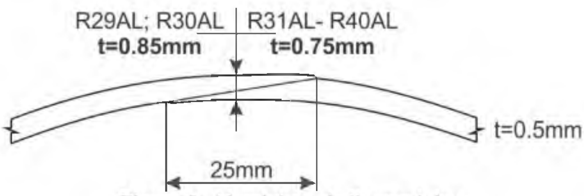


Figure 2. Aluminium shell taper joint.

2.2. Experimental test setup and boundary conditions

The universal quasi static testing machine Zwick 100 was used to apply axial compressive load on the cylinder. Starting from zero – unloaded structure, the compressive load was gradually increased at 2 kN step up to 85% of predetermined buckling load. During each increment the natural frequencies and vibration modes were scanned using Polytec laser vibrometer on a grid of points distributed along a small area of the cylinder (white area figure 3a). For structural excitation a loudspeaker placed 180° opposite the measured area was used and measurement conducted in range from 200 to 400 Hz. Measured sector of cylinder was approximately 72 deg wide which correspond to 1/5 of cylinder. The scanned area consisted of 300 grid points which was found as a good trade-off between the scanning time and the level of detail of modal response.

In general, the top and bottom cylinder edges were clamped by a resin potting (20 mm x 20 mm) on outer surface while internal - metal ring (20 mm x 30 mm) was placed inside the shell, as shown in Figure 3b. For testing, each cylinder is placed between two metallic loading plates and the narrow gap in-between was filled with reinforced epoxy resin (see Figure 3b). Thus it was considered that clamped boundary conditions was realized with 20 mm clamped zone at each end of the shell.

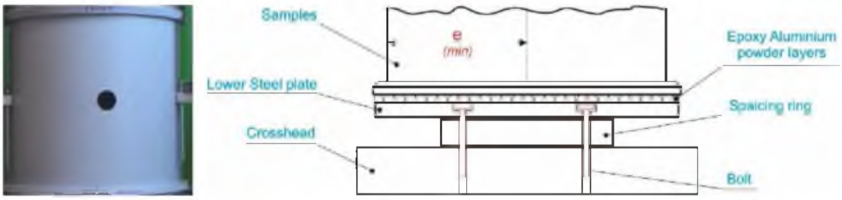


Figure 3. (a) Boundary conditions on top and bottom edges; (b) Experimental test setup for compressive loading.

2.3. Experimental results

In order to evaluate the manufacturing signature initial geometrical imperfection measurements for five shells were carried out by internal laser scanner. Figure 4, shows unfiltered imperfection pattern for R33AL shell, captured before and after cut-out. It should be noted that cut-out was externally covered in order to avoid out of range readings for laser, nevertheless one may see that both cut-out and adhesive joint are giving most severe imperfections.

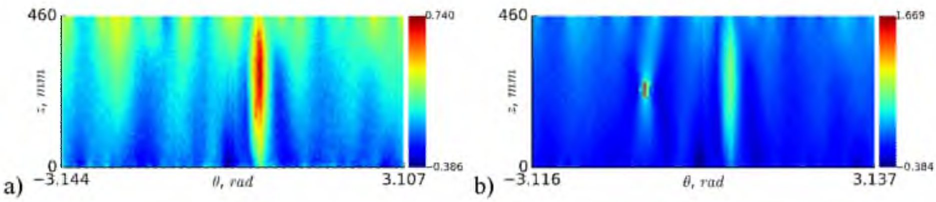


Figure 4. Imperfection scan from internal laser scanner for R36AL shell: a) w/o hole, b) with hole.

The test load/shortening results are summarised in Figure 5. Where one may see that experimental results are much lower than ones predicting by finite element analysis with clamped boundary conditions. Nevertheless if more detailed analysis is conducted and boundary conditions are integrated in model as 3D elements including mass, rigidity and contact then obtained numerical prediction is almost matching the physical test. Furthermore a special attention should be given to local buckle development before the buckling of the structure. The load corresponding that particular load level will be indicatively given in brackets.

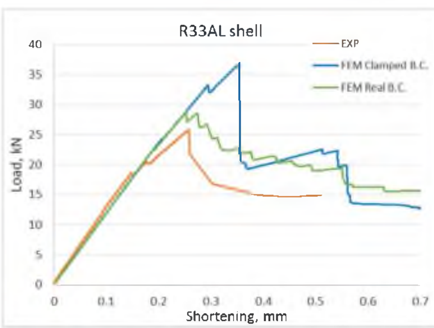
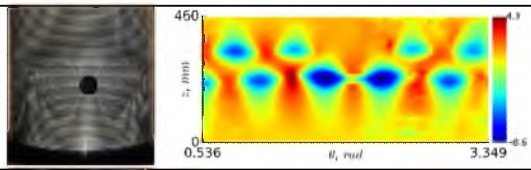
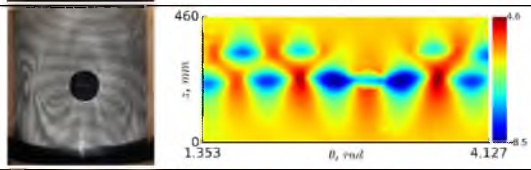
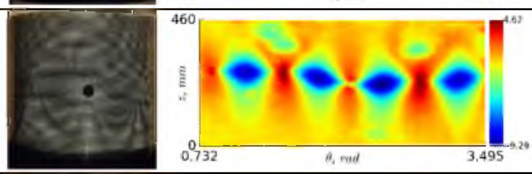
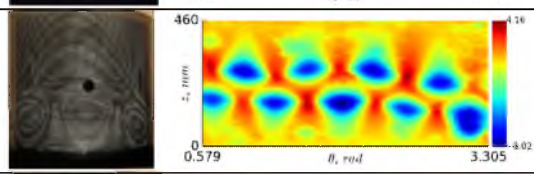
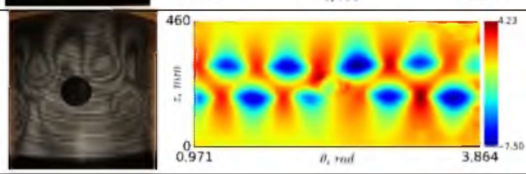


Figure 5. R33AL physical versus numerical results.

Both obtained buckling loads and mode shapes for each tested cylinder is summarized in Table 2. Where load values given in brackets indicate of initial local buckling load level. If specimen has no additional reinforcement then buckling load for specimens with 30 to 80 mm are almost similar to 25kN. While 10mm wide reinforcement added only 10% additional strength in contrary to 20mm wide reinforcement patch which elevated buckling load above 70%.

Table 2. Experimental buckling load of tested cylinder and mode shape

Cylinder	Experimental buckling load [kN]	Experimental buckling mode
R33AL D=50mm	(21.2) 25.85	
R35AL D=80mm	(21.5) 25.05	
R36AL D=30mm	25.39	
R37AL D=30mm D_o=50mm	43.79	
R39AL D=80mm D_o=90mm	(27.7) 28.12	

A frequency response graph summarised in Figure 6 (a) indicate that natural frequency of specimens with no pre-stress state are scattered, where once specimen has axial force acting on it the frequency response becomes more robust and narrowing the scatter. At the same time the dependency between natural frequency and applied compression load is shown in Figure 6 (b). The response of cylinder with reinforced cut-out (20 mm) R37AL is given as a reference.

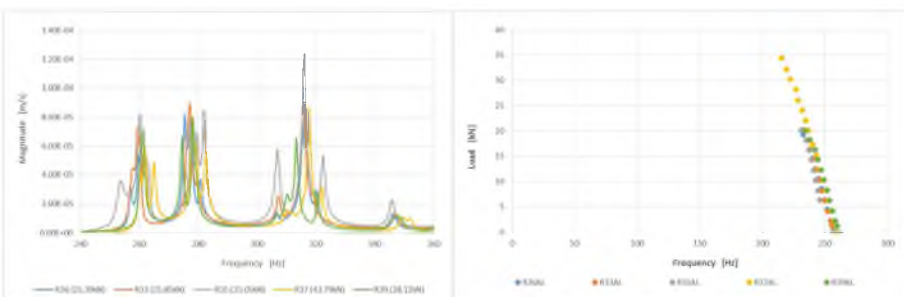


Figure 6. (a)Natural frequency response of cylinder wit not loading; (b) Decrement of natural frequency due axial compression.

3. Verification of the proposed VCT approach

As a next step in order to predict the onset of buckling following the modified VCT approach, presented by Arbelo (7, 8), it is required to follow these steps:

a) Throughout the tests the evolution of the first natural frequency and mode for each load step was recorded.

b) Numerical determination of reference value - linear buckling load (eigenbuckling) of the perfect cylinder (P_{cr}). This value can be obtained through a finite element model. An eigenvalue analysis of a perfect cylindrical shell is fast, simple and provides such an estimate. Even though there is little correlation with experimental results thus knock down factors should be added. For this study, a finite element model (FEM) is implemented by commercial software ANSYS [9]. The critical buckling loads obtained for a perfect cylindrical shell with the corresponding dimensions, materials properties and boundary conditions used as a reference was 66.74 kN.

The proposed VCT approach has been plotted in function of the dimensionless parameters (f) versus (P) or plot of $(1\text{-}p)^2$ versus $(1\text{-}f^2)$ for modified approach as shown in Figure 7. Furthermore result assessment in table 3 confirms conformity with theoretical assumptions.

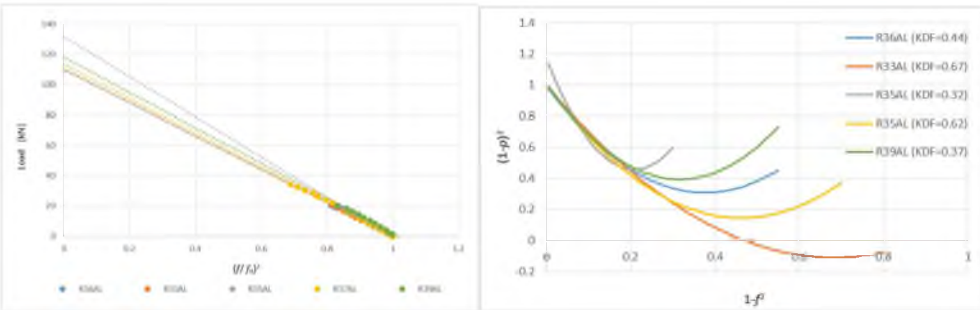


Figure 7. a) Plot of (P) versus $(f / f_0)^2$ by stepwise axial load increment; b) Plot of $(1\text{-}p)^2$ versus $(1\text{-}f^2)$ by stepwise axial load increment.

Classical - linear VCT has a tendency of overestimation while modified VCT approach gives more relative values which correlate with approach implemented in NASA (10) guidelines of knock down factors KDF. Nevertheless modified approach is struggling to set an appropriate amount of test points required for robust second order polynomial approximation function. If the obtained buckling load level could be subdivided in three parts then if approximation made from initial tests will have buckling overestimation by up to 20% while if the test goes up to 80% of buckling load level (Figure 8.), the predicted value are conservative by approximately 10%.

Table 3. Buckling load prediction using the VCT approach for the different studied cases.

Cylinder	Experimental buckling load [kN]	Predicted buckling load VCT [kN]	Deviation from VCT and experimental results [%]	Predicted buckling load by modified VCT [kN]	Deviation from modified VCT and experimental results [%]
R33AL	(21.2) 25.85	110.49	327.43	44.72	73.00
R35AL	(21.5)25.05	131.83	426.27	21.66	-13.53
R36AL	25.39	109.73	332.31	29.51	16.23
R37AL	43.79	113.41	159.01	41.59	-5.02
R39AL	(27.2) 28.12	118.16	320.20	24.94	-11.31

From these results one can conclude that the present modified vibration correlation approach could be applied as an experimental non-destructive method to estimate the buckling load on unstiffened cylindrical shells loaded in compression. This indicates that modified VCT procedure show potential capacity to estimate ultimate load carrying capacity of real structures in real loading conditions nevertheless there may be more appropriate mathematical function to be more robust for estimation of the buckling load level.

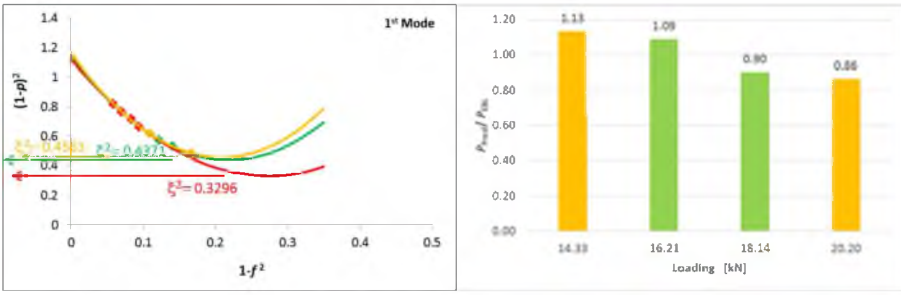


Figure 8. Step by step predicted buckling load by modified VCT by cylinder R35AL

Conclusions

Compared to the reference test of prototyped cylinder the specimens with cut-outs cause up to 50% reduction of load carrying capacity. If the cut out is reinforced with 10mm wide ring this may have only slight – 12% strength increase, while if the cu out is reinforced with 20mm width ring the buckling load is only 10% lower compared to the reference specimen.

The statistical amount of tested specimens with cut-outs indicate that classical VCT where correlation between load and natural frequencies are assessed by linear dependency is not valid as it may lead to 300% overestimation. Such approach is not helpful nor potentially viable for any technical application. While modified approach by second order polynomial function with 20% average error margin has some robustness issues, which should be overtaken by search of alternative mathematical function. If further elaborate modified approach it there should be more strict criteria for minimum number of conducted tests which in principle may lead to safety issues if applied in real world testing environment.

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