DANUBE: Law and Economics Review, 4 (2), 97–114
DOI: 10.2478/danb-2013-0005

DOES HIGHER TAX MORALE IMPLY HIGHER OPTIMAL LABOR INCOME TAX RATE?

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Abstract

We analyze the impact of tax morale on optimal progressive labor income taxation. Only universal basic income is financed from a linear tax and the financing of public goods is neglected. Each individual supplies labor and (un)declares earning, depending on his labor disutility and tax morale. Limiting the utilitarianism to the poorer parts of the population (defined by the *inclusion share*), the optimal tax rate is an increasing function of the tax morale and a decreasing function of the inclusion share, provided that the average wage of those included is higher than 0.54 times the average wage.

Keywords

Tax Morale, Progressive Income Tax, Undeclared Earning, Income Redistribution, Labor Supply

I. Introduction

Between 1930 and 1970, the ratio of government tax and social security revenues to GDP rose sharply and has remained at a high level since the 1970s in the developed world. It is a commonplace that rising labor income (and other) tax rates may diminish labor supply and increase tax evasion, therefore relative labor supply and tax evasion may have been increasing. But comparing different countries, it becomes evident that the impact of taxation on economic activity also depends on so-called "tax morale" (or morality). This concept refers to the propensity to pay taxes or captures "the readiness with which individuals leave the official economy and enter the illegitimate (untaxed) hidden economy" (Frey and Weck-Hannemann, 1984; see also Lago-Penas and Lago-Penas (2010) for its determinants). We should distinguish between *exogenous* and *endogenous* individual tax morales: the former is a given parameter of the utility function, the latter depends on the exogenous tax morale as well as on the observed behavior of the individual's neighborhood.

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In the present paper, we analyze the impact of the exogenous tax morale on the tax rate in a very simple *static* model, where a *flat-rate labor income tax* finances a universal basic income (transfer), neglecting the fiscal demand of providing public goods. Every individual chooses his labor supply and reports his earnings to maximize his utility, depending on his consumption, labor- and moral disutility. In this model, the existence of tax morale makes monitoring and punishing tax evasion superfluous. In the traditional approach, the government takes the tax revenue as given and then the usual wisdom prevails: the higher the tax morale, the lower the (lower) balancing tax rate.

In contrast, we consider a government maximizing social welfare (cf. Cowell and Gordon, 1988). Following earlier trials (Simonovits, 2010 and 2011), we create a model and answer our title question in the affirmative: *higher tax morale implies higher optimal flat tax rate* under a qualification soon to be explained.

If one works with the usual, strictly concave utility functions (even avoiding *design* problems), then the calculations soon become excessively complex. One makes a lot of assumptions on the underlying utility functions, and even then often must rely on numerical illustrations with parametric functions. However, we adopt the linear-quadratic utility function of Doerrenberg et al. (2012), yielding simple linear decision functions. The optimal labor supply is a linear function of the tax rate (with a negative coefficient, whose absolute value increases in the tax morale) and the optimal share of undeclared earning is equal to the inverse tax morale.[2]

But the linearity of the consumption utility makes income redistribution superfluous under a purely utilitarian social welfare function and therefore we must introduce generalized utilitarian social welfare functions. To preserve analytical simplicity, we define a *truncated (or generalized Rawlsian) social welfare function* as the average of the first J lowest utilities out of I utilities, $J < I$, J being the *cutoff index*, and $\nu = J/I$ being the *inclusion share*. It is obvious that the lower the J (or equivalently, ν), the more progressive the social welfare function. For any truncated or *exclusive* social welfare function, we were able to determine explicitly the *socially optimal tax rate* and show that it *is a relatively simple increasing function of the tax morale and a decreasing function of the inclusion share* (Theorems 2 and 3), provided that the average wage of those included is higher than 0.54 times the average wage. (Note, however, that if the pre-tax wage rates of certain types are too low, fixed costs prevent these types from working at all.) In an earlier model (Simonovits, 2011), we have also investigated *numerically* this dependence for inclusive CRRA social welfare functions, and we invariably received qualitatively the same schedule.[3]

In this framework, the intuition behind the major result appears to be relatively simple. The government's objective is to maximize a progressive social welfare function. Recall that raising the tax rate initially increases the transfer received by the poorest but diminishes the labor supply and thus the total output. For any realistically given tax rate, the higher

[2] In Simonovits (2011), under a simpler although less appropriate assumption, the optimal undeclared earning was equal to the ratio of the wage rate to the tax morale, yielding an unconditionally affirmative answer.

[3] This confirms Ravaillon's (1997, p. 359) observation: "the theoretical distinction [between exclusive and inclusive social welfare functions, A.S.] can sometimes be of very little practical consequence".

the tax morale, the lower the optimal undeclared earning, making taxing less costly. Due to the specification of the model, however, for any given positive tax rate, the optimal labor supply is also a diminishing function of the tax morale! Therefore our intuitive argument is not watertight and we need to give the conditions under which the statement holds, namely that the average wage of the included population with respect to the total average wage is high enough. Our general (nonparametric) model highlights the complexities arising in the proof of our conjecture.[4]

Despite the artificial specification of the utility functions and the exclusion of consumption and social security taxes, Doerrenberg et al. (2012) report empirical verification. To relate our highly theoretical observation to the real world, a very stylized table is presented, describing various combinations of tax morales (lower and higher) and tax shares (low, medium, high), with the latter defined as the ratio of tax (and pension) revenues to GDP.[5] The tax morale can be approximated by the corruption index (10 minus the traditional one). We tentatively interpret Table 1 as showing that a medium tax share may be *socially optimal* for a country with lower tax morale (e.g. the Czech Republic versus Slovakia or Hungary with an approximately common corruption index value 5), while a high tax share may be optimal for a country with higher tax morale (like Sweden versus the US or Germany with corruption index values below 2.5).

Table 1: Tax shares, tax morales and ranking of social welfare

Tax share	Low cc. 30%		Medium cc. 40%		High cc. 50%
Lower morale	Slovakia	$<$	Czech Rep.	$>$	Hungary
Higher morale	USA	$<$	Germany	$<$	Sweden

At this stage, a short review of the literature is given. In his pioneering paper, Mirrlees (1971) solved the theoretical problem of designing socially optimal labor income taxation, when labor supply is flexible but productivity is private information. Sheshinski (1972) simplified the analysis by confining his attention to linear taxes (cf. Feldstein, 1973). One limitation of these papers is that they did not consider tax evasion. Taking the opposite extreme position, Allingham and Sandmo (1972) analyzed income tax evasion, neglecting the flexibility of labor supply. In a sequel to that paper, Sandmo (1981) extended the research on tax evasion into the direction of social welfare maximization with flexible labor supply and raised a weaker form of the basic result of the present paper (p. 279): "a natural question to ask is whether . . . the marginal tax rate in some sense ought to be lower than otherwise have been because of the presence of tax evasion."[6]

[4] It is to be hoped that the results remain valid in more general settings (cf. Simonovits (2010) with strictly concave utility functions and public expenditures).

[5] The tax share index – somewhat unreliable but still characteristic – refers to the pre-crisis era and contains many things directly *not* related to our problem (budget deficits, interest payments, different public pension systems, etc.).

[6] Later on (p. 282) he gave alternative sufficient conditions, namely either "regular income is now a less reliable indicator of economic welfare" or "the numerical value of the compensated supply derivative in the regular market is increased" but did not commit himself to their validity. Cremer and Gavhari (1996) were also agnostic

In their survey, Andreoni et al. (1998) extended the narrow neoclassical model and introduced soft but relevant concepts like moral sentiments and the satisfaction of the taxpayer with the provision of public goods and services. From our point of view, they made three important observations: (i) the morally more sensitive citizens declare a higher share of their true (pre-tax) incomes; (ii) the more unfair the tax-and-transfer system is in the eyes of citizens, the less income they declare; (iii) the less satisfied the taxpayers are with the provision of public goods and services, the less income they declare.[7]

Traxler (2010) extended the analysis from exogenous to endogenous tax morale, where the individual tax morale depends on the observed degree of tax evasion. Combining the two approaches, Garay, Simonovits and Tóth (2012) and Méder, Simonovits and Vincze (2012) investigated the *dynamics* of the tax evasion process. Using the framework of Simonovits (2010), both papers neglected income redistribution and studied the dynamics of the declared (or taxable) incomes underlying financing the provision of public goods, just the opposite of the present paper's approach.

In the paper already mentioned, Doerrenberg et al. (2012) considered differentiated taxation among different groups in different countries. Using econometric techniques, they found that typically in any country, "nice guys finish last: people with higher tax morale are taxed more heavily".[8]

Romer (1975) also obtained interesting results concerning the impact of *majority voting* on linear income taxes – an alternative to welfare analysis. In such a political economy framework, Meltzer and Richard (1981) proved an interesting intuitively appealing result: the greater the pre-tax income inequality, the greater redistribution will be chosen by the median voter. (By the way, Theorem 2* of the present paper reproduces this result, also preserving the influence of tax morale.)

Comparing two countries, say the US and Sweden, an apparent anomaly can be found (e.g. Alesina and Angelitos, 2005). Although the US inequality of the pre-tax incomes is greater than the Swedish, the US personal income tax is less progressive than the Swedish. The foregoing authors created a model with country-specific beliefs on the role of luck in the determination of individual pre-tax earnings. Their major result was as follows: the stronger the presumed role of luck, the greater income redistribution is selected by the median voter. In contrast, in our social welfare maximization framework, this anomaly can be explained by the difference between the countries' social welfare functions: the impact of the lower inclusion share overrules the impact of the lower pre-tax income inequality, implying a higher optimal tax rate.

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about the impact of tax evasion on the optimal tax rate (see also Sandmo, 2012, pp. 20–21).

[7] From Feldstein (1999) to Chetty (2009) and Saez et al. (2009), a great number of papers studied tax avoidance and the deadweight loss due to the income tax in a more direct way. These papers put the concept of *elasticity of taxable income* to the center of the analysis, eliminating any distinction between restrained labor supply and underreported earnings.

[8] Making the individual utilities dependent on others' utilities, Doerrenberg and Peichl (2013) discussed the opposite causality and found that greater tax progressivity implies higher tax morale.

DOI: 10.2478/danb-2013-0005

The structure of the remainder of the paper is as follows: Section 2 presents the model and Section 3 displays the illustrations. Section 4 concludes. An Appendix contains the proofs.

II. The model

First, we shall sketch the general framework, then we parameterize it to obtain explicit formulas and definite results.

General framework

There are $I(>1)$ types in the population, indexed as $i = 1, \ldots, I$. Type i's *labor supply* is l_i, $0 < l_i \leq 1$, his pre-tax *wage rate* (independently of the tax system) is $w_i > 0$, both reals, thus his *earning* is $w_i l_i$. To achieve income redistribution, the government collects a linear tax with a *flat (marginal) tax rate* τ, $0 \leq \tau \leq 1$ and transfers a *basic income* $\beta \geq 0$. Type i *undeclares* $e_i \geq 0$ from his earning, i.e. he evades tax τe_i, therefore his *declared (or taxable) earning* is $y_i = w_i l_i - e_i$ and his *net tax* is equal to $\tau y_i - \beta = \tau(w_i l_i - e_i) - \beta$. Consequently, his *consumption* is given by $c_i = (1-\tau)w_i l_i + \tau e_i + \beta$. Note that for any positive basic income $\beta > 0$, even the flat-rate tax is progressive in the sense that the average net tax rate $t_i = (\tau y_i - \beta)/y_i$ is increasing in the declared earning y_i. To derive the dual choice of labor supply and undeclared earning from individual utility maximization, we must assume individual objective functions.

First, we use a general utility function $u_i(c_i, l_i, e_i)$. Of course, the value of the basic income β as well as of the consumption c_i depends on the decisions of all the workers (see below). Since the impact of any single worker on β can be neglected, our workers neglect it. Therefore type i maximizes the reduced utility function $v_i(l_i, e_i) = u_i((1-\tau)w_i l_i + \tau e_i + \beta, l_i, e_i)$ without explicitly considering the basic income. Taking the partial derivatives of this concave function with respect to l_i and e_i and equating them to zero, his optimal decisions are respectively l_i^* and e_i^*, satisfying the conditions

$$v'_{i,l}(l_i, e_i) = u'_{i,c}(c_i, l_i, e_i)(1-\tau)w_i + u'_{i,l}(c_i, l_i, e_i) = 0$$

and

$$v'_{i,e}(l_i, e_i) = u'_{i,c}(c_i, l_i, e_i)\tau + u'_{i,e}(c_i, l_i, e_i) = 0,$$

where the partial derivatives have the usual signs:

$$u'_{i,c} > 0 > u'_{i,l}, u'_{i,e}.$$

Let f_i denote the frequency of type i in the population, $f_i > 0$ and $\sum_{i=1}^{I} f_i = 1$. Then the expected output is $Z^* = \sum_{i=1}^{I} f_i w_i^* l_i^*$ and the expected evasion is $E^* = \sum_{i=1}^{I} f_i e_i^*$. For the sake of simplicity, we assume that the total (or average) net tax is zero (neglecting the provision of public goods), i.e. we end up with the following budget constraint taken at the individual optima: $\beta^* = \tau(Z^* - E^*)$.

At this point we introduce our main concept, the exogenous tax morale μ. It is a parameter, represented by a real number. We assume that the individual i's utility function depends on

μ in the following way: in addition to c_i, l_i and e_i, $u_i(\cdot,\cdot,\cdot,\mu)$ as well as $v_i(\cdot,\cdot,\mu)$ depends also on μ. Assuming the usual smoothness conditions, and taking the partial derivatives with respect to the tax morale μ, yields the usual equations:

$$v''_{i,ll}l^{*\prime} + v''_{i,le}e^{*\prime} + v''_{i,l\mu} = 0 \quad \text{and} \quad v''_{i,el}l^{*\prime} + v''_{i,ee}e^{*\prime} + v''_{i,e\mu} = 0.$$

To have a meaningful model, we must ensure that the share of the unreported income in the true income, $e_i^*(\mu)/[w_i l_i^*(\mu)]$ is an increasing function.
Let ψ be a concave and weakly increasing function of v_i^*s. Then we can introduce a social welfare function $V(\mu,\tau) = \sum_{i=1}^I f_i\psi(v_i^*(\mu,\tau))$ and we can formulate our major if somewhat empty claim.

Theorem 0. *Let us assume that $V(\mu,\tau)$ satisfies the usual smoothness and concavity conditions, moreover, conditions $V''_{\tau\tau} < 0 < V''_{\tau\mu}$ also hold. Then the welfare maximizing tax rate $\tau(\mu)$ is an increasing function of the tax morale μ and*

$$\tau'(\mu) = -\frac{V''_{\tau\mu}}{V''_{\tau\tau}} > 0.$$

Proof. Taking the total derivative of the social optimality condition $V'_\tau(\mu,\tau) = 0$ with respect to μ yields $V''_{\tau\mu} + V''_{\tau\tau}\tau'(\mu) = 0$.

The more concave ψ is, the more progressive is the social welfare function, probably yielding a higher socially optimal tax rate.
We have made a lot of assumptions which are hard to check. We shall now turn to a parameterized model where every *ad hoc* assumption can be derived. We shall see, however, that the negative impact of the tax morale on the labor supply can be so strong that the optimal tax rate is a decreasing function for high enough morales.

Parameterized model

Following Doerrenberg et al. (2012), we shall rely on linear–quadratic utility functions and obtain explicit formulas. In addition to a usual linear *consumption utility* $2c_i$ and a quadratic *labor disutility* function $-\alpha w_i l_i^2$, ($\alpha > 0$ being the coefficient of labor disutility), we introduce a quadratic *moral disutility* function of tax evasion $-\mu\tau w_i l_i[e_i/(w_i l_i)]^2$, ($\mu > 0$ being the coefficient of tax morale, for short, the tax morale).[9] In sum, type i's utility function is

$$u_i = 2c_i - \alpha w_i l_i^2 - \mu\tau w_i^{-1} l_i^{-1} e_i^2.$$

Inserting formula for c_i into formula for u_i, we receive a *reduced* utility function

$$v_i(l_i,e_i) = 2(1-\tau)w_i l_i + 2\tau e_i + 2\beta - \alpha w_i l_i^2 - \mu\tau w_i^{-1} l_i^{-1} e_i^2.$$

[9] We shall see that factor $\tau w_i l_i$ equalizes the optimal share of undeclared earning to the inverse tax morale and factor w_i makes the optimal labor supply independent of the type-specific wage. Note that we imitate Yitzaki (1974), who made the penalty proportional to the evaded tax rather than the undeclared earning. Finally, by doubling the consumption in the utility function, the occurrence of fractions is minimized. Replacing labor disutility by $\alpha w_i(l_i - T)^2$ would enhance labor supply.

Neglecting β, and introducing notation $\delta = 1 - \mu^{-1}/2 < 1$, we obtain the individual optimum (see the Appendix):

$$l_i^* = \alpha^{-1}(1 - \delta\tau) = \lambda \quad \text{and} \quad e_i^* = \mu^{-1} w_i l_i^*.$$

Note the simple meaning of these rules: the uniform optimal labor supply $l_i^* = \lambda$ is a diminishing linear function of τ, where the proportionality coefficient is the reciprocal of α and the coefficient of τ is $-\delta$ (reflecting that the effective tax rate is increasing with morality); the optimal undeclared earning e_i^* is proportional to the wage earned $w_i l_i^*$, where the proportionality coefficient is the reciprocal of μ. In a *white economy* (studied by Mirrlees), $\mu = \infty$, $\delta = 1$, $l_i = \alpha^{-1}(1 - \tau)$ and $e_i^* = 0$.
To obtain a feasible labor supply for any tax rate, it is appropriate to assume $\alpha \geq 1$, in the limit: $\alpha = 1$. It is also logical to assume, that at the optimum, the undeclared earning is less than the true earning, i.e. $\mu > 1$. We can make the following observation: the reported earning

$$y_i^* = w_i l_i^* - e_i^* = (1 - \mu^{-1})\lambda w_i$$

is an increasing function of the tax morale and a decreasing function of the tax rate if $\tau < 2/3$, which is an innocent restriction.
Turning from individual to aggregate behavior, without loss of generality, we assume that the weight of type i in the population is uniform, i.e. $f_i = 1/I$. We shall need the average wage rate to be normalized to unity:

$$W = \frac{1}{I}\sum_{i=1}^{I} w_i = 1.$$

Three more averages are introduced: average labor supply, average earning and average undeclared earning, respectively:

$$L = \frac{1}{I}\sum_{i=1}^{I} l_i, \quad Z = \frac{1}{I}\sum_{i=1}^{I} w_i l_i \quad \text{and} \quad E = \frac{1}{I}\sum_{i=1}^{I} e_i.$$

At the optimum, they are equal to $L^* = \lambda = Z^*$ and $E^* = \mu^{-1}\lambda$, respectively.
Now the budget constrains gives the basic income:

$$\beta^* = \tau(1 - \mu^{-1})\lambda.$$

The government of a traditional economist takes the tax revenue β^* as given and looks for a balancing tax rate τ. We make the usual assumption that the fixed tax revenue is feasible: $0 < \beta < \bar{\beta}$ and the chosen balancing tax rate is to the left rather than to the right from the maximizing one: $0 < \tau < \bar{\tau}$ (see the literature on the Laffer-curve) and prove the traditional view.

Theorem 1. *Suppose that the basic income β^* is not too high:*

$$0 < \beta^* < \bar{\beta} = \frac{1-\mu^{-1}}{4\delta(\mu)\alpha}.$$

Then there are two balancing tax rates and the lower is given by

$$\tau_{\beta^*}[\mu] = \frac{1-\sqrt{1-4\delta\beta^*\alpha/(1-\mu^{-1})}}{2\delta(\mu)} < \bar{\tau} = \frac{1}{2\delta(\mu)}.$$

Moreover, the balancing tax rate is a decreasing function of the tax morale.

Remark. Due to the assumed homogeneity of the disutility parameters, this result is independent of the wage rate distribution.

Social welfare

We move now to the main field of interest, namely to social welfare maximization. To find the socially optimal tax rate and the corresponding basic income, it is worth expressing the optimal reduced utilities as *indirect* utility functions (see Appendix):

$$u_i^* = v_i(l_i^*, e_i^*) = 2\lambda w_i(1-\tau) + \mu^{-1} w_i \lambda \tau + 2\tau(1-\mu^{-1})\lambda - \alpha w_i \lambda^2.$$

Note that, contrary to Simonovits (2010), we cannot use a *purely utilitarian* social welfare function, because the individual utility is a linear function of the individual consumption – making any income redistribution not only useless but counterproductive. Rather we look for a family of generalized social welfare functions which preserve the simplicity of the purely utilitarian one but do not exclude redistribution. We shall introduce *truncated* utilitarian social welfare functions, defined as the average of the J lowest utilities, J being the cutoff index. (Later on we shall work with the relative index $\nu = J/I$, to be called *inclusion share*.) Note that, in the present model, these indirect utilities are increasing linear functions of the wage rates.[10] If we index the latter as $w_1 < w_2 < \cdots < w_{I-1} < w_I$, then $u_1^* < u_2^* < \cdots < u_{I-1}^* < u_I^*$. Hence the definition of the J-truncated social welfare function is simple:[11]

$$U_J = \frac{1}{J}\sum_{i=1}^{J} u_i^*, \qquad J = 1, 2, \ldots, I.$$

The higher the cutoff index J, the more indifferent the social planner to the utility differences. We display the two limit cases.
The purely utilitarian case:

$$U_I = \frac{1}{I}\sum_{i=1}^{I} u_i^*.$$

[10] In the Appendix it is shown that $u_i^* = \alpha\lambda^2 w_i + B$, where B is a constant.
[11] For practical reasons, the untruncated U_I is also included.

DOI: 10.2478/danb-2013-0005

The Rawlsian case:

$$U_1 = u_1^*.$$

Note that the social welfare functions $U_1, \ldots, U_{I-1}$ fail to depend on all the utilities but they are simple and approximate well the much more complex CRRA social welfare functions, therefore we rely on them.
Before announcing our main theorem, as a counterpart to U_J, we shall define the average wage rate of the J lowest types (for short, J-minimum average wage rate):

$$W_J = \frac{1}{J}\sum_{i=1}^{J} w_i, \quad j = 1, 2, \ldots, I.$$

Because w_is are increasing, so do W_Js:
$w_1 = W_1 < W_2 < \cdots < W_{I-1} < W_I = W = 1.$
Here is our major result.

Theorem 2. *a) Let us choose a cutoff index $J < I$. Then for the J-truncated social welfare function, the J-optimal tax rate is equal to*

$$\tau_J(\mu) = \frac{2 - W_J - 1/(1-\mu^{-1}/2)}{2(1-\mu^{-1}) - W_J} > 0;$$

provided the tax morale is higher than the J-critical value:[12]

$$\mu > \mu_J = \frac{2 - W_J}{2(1 - W_J)} \geq 1.$$

b) The tax rate–tax morale function $\tau_J(\mu)$ is increasing in the tax morale μ if the J-average wage rate is sufficiently high, namely

$$W_J > 4 - 2\sqrt{3} \approx 0.54.$$

Remarks. 1. Under the assumptions of Theorem 2, the upper limit of the tax rate is achieved in the white economy ($\mu = \infty$):

$$\tau_J(\infty) = \frac{1 - W_J}{2 - W_J} \leq \frac{1}{2}.$$

2. To see the importance of our lower limit on W_J, let us consider the Rawlsian optimal tax rate and assume that the worst-paid workers earn zero: $w_1 = 0$.[13] Then the Rawlsian tax rate is

$$\tau_1(\mu) = \frac{1}{2 - \mu^{-1}}$$

[12] Note that, for inclusive social welfare function, when $J = I$, the I-critical tax morale is infinite!
[13] This is a rather good approximation to very low wage rates.

which is clearly a diminishing rather than increasing function of the tax morale. In this somewhat paradoxical case, the negative impact of the tax morale on labor supply dominates its positive impact on the reported wage, making the optimal tax rate diminishing.

3. The literature has concentrated on the dependence of the tax rate on the labor disutility. In a surprising way, here the optimal tax rate is independent of α.
If we give up the uniformity of the tax morales and of the labor disutilities, then the ordering of the indirect utilities becomes cumbersome.
We turn now to the dependence of the optimal tax rate on the cutoff index J, measuring the extent of exclusion of the richer groups. Intuitively, we expect that the lower the cutoff index, i.e. the more progressive is the social welfare function, the higher is the optimal tax rate. Indeed, this is the case.

Theorem 3. *Let K be a nonnegative integer such that the tax morale μ lies between μ_K and μ_{K+1} (with $\mu_0 = 1$). Then the socially optimal positive tax rates are decreasing in the cutoff index J: $\tau_1 > \tau_2 > \cdots > \tau_K > \tau_{K+1} = \cdots = \tau_I = 0$.*

As is usual in welfare economics, for any J, it is worth calculating the *degree of the suboptimality* of the presumed tax morale-specific optimum $\tau_J[\hat{\mu}]$ in an economy with a true tax morale μ: $\hat{\mu} \neq \mu$. Fixing the proportions of pre-tax wage rates, we look for that average wage rate θ_J, for which $\tau_J[\mu, \hat{\mu}]$ yields the same welfare as the original unit average wage rate and $\tau_J(\mu)$ do. It is obvious that θ_J is optimal for $\hat{\mu} = \mu$.
At this point, we make a short *detour* into the realm of political economy. Let M be a positive integer. Assuming that $I = 2M - 1$, denote the median wage rate by $w_M < 1$. Then every worker's indirect utility satisfies the single-peaked condition and the optimal tax rate corresponds to that of Theorem 2, only the M-minimal average wage rate W_M is replaced by the median one w_M, and the critical tax morale μ_M with its counterpart μ^*. We have

Theorem 2.* *a) Assume that the tax morale is higher than the critical value μ^*:*

$$\mu > \mu^* = \frac{2 - w_M}{2(1 - w_M)} \geq 1.$$

Then the median voter's preferred tax rate is positive and is given by

$$\tau^*(\mu) = \frac{2 - w_M - 1/(1 - \mu^{-1}/2)}{2(1 - \mu^{-1}) - w_M}$$

b) If $4 - 2\sqrt{3} < w_M (< 1)$, then the median voter's tax rate is an increasing function of the tax morale.

Remarks. 1. In accordance with Meltzer and Richard (1981), in our model the greater the pre-tax earning inequality, here measured by the difference between the average and the median wage rates $1 - w_M$, the greater redistribution will be chosen by the median voter.

DOI: 10.2478/danb-2013-0005

2. Since the M-minimal average wage rate is generally lower than the median wage rate: $W_M < w_M$, therefore $0 < \tau^*(\mu) < \tau_M(\mu)$, i.e. the M-optimum tax rate is higher than the median voter's. Also, the bounds mean much stronger restrictions in the political economy model than in the welfare maximization model. If the cutoff index J is high, so $W_J \approx w_M$, then the J-optimal solution is close to the median voter's one.

III. Numerical illustrations

To give a sense of the magnitudes, we rely on numerical illustrations. We shall work with extremely simple specifications, for example, $\alpha = 1$.
First, we illustrate the traditional view formulated in Theorem 1. We choose the basic income as one tenth of the average wage rate, i.e. $\beta^* = 0.1$ and run the tax morale μ from 4 to 12 to ∞. Then the balancing tax rate drops from 0.27 to 0.22 and then to 0.19.

Table 2: The impact of tax morale on balancing tax rate

Tax morale μ	Balancing tax rate $\tau_{0.1}[\mu]$
4	0.267
6	0.240
8	0.229
10	0.222
12	0.218
...	...
∞	0.192

Remark. $\beta^* = 0.1$.

However, our government maximizes its social welfare function rather than fixes the basic income. We shall use an arbitrary but realistic wage rate distribution with quintiles ($I = 5$), and normalize its expected value to 1 (see the first column of Table 3 below). Since the J-minimum average wage rates W_J are also important, we display them in the second column. To make our presentation less dependent on the number of types, we shall work with the relative share of preferred workers in the population, namely the inclusion share: $\nu = J/I$ rather than their absolute numbers or the cutoff index J. We start the illustrations with index $J = 3$ or rather with the inclusion share $\nu = 0.6$.
First we display a simple run with tax morale $\mu = 4$, i.e. workers undeclare 1/4 of their wages, with the optimal tax rate being equal to 0.267. Table 3 produces sensible results and presumably can be used for further calculations. The redistribution is quite spectacular: the signed net transfers paid by the workers being equal to $T_i^* = \tau(w_i l_i^* - e_i^*) - \beta^*$, the poorest quintile receives 23% of its potential earnings and the richest quintile pays about 8% of its potential earnings.

Table 3: The individual optimal outcomes for 3-quintile optimum

Wage rate w_i	J-minimal wage rate W_J	Undeclared earning e_i^*	Transfer paid T_i^*	Consumption c_i^*
0.4	0.4	0.077	−0.091	0.399
0.6	0.5	0.115	−0.061	0.522
0.8	0.6	0.154	−0.030	0.646
1.2	0.75	0.231	0.030	0.893
2.0	1	0.385	0.152	1.386

Remarks. $\nu = 0.6$, $\mu = 4$, $\tau^{0.6} = 0.264$, $L^* = 0.769$. We use notation τ^ν rather than τ_ν, to distinguish the utilitarian optimum τ^1 from the Rawlsian optimum τ_1.

Next, we move on to studying the impact of tax morale on optimal average outcomes, and usually drop the adjective *average*. In Table 4, the tax morale runs from 2 to 5 to infinity (white economy) and see the quantitative side of Theorem 2: the optimal tax rate rises from 0.121 to 0.273 to 0.286. Note that the corresponding $W_3 > 0.54$, guaranteeing monotonicity. At the same time, the optimal basic income is an increasing function of the tax morale: it runs from 0.055 to 0.165 to 0.204 in terms of the potential average wage. (The low optimal value of basic income precludes the existence of balancing tax rate for $\mu = 2$ and $\beta = 0.1$ in Table 2.) Note that, even in the white economy, the lowest consumption is way below the average: $0.408 < 0.71$. It is quite disturbing that the poorest's welfare is only increasing because the lost labor supply is made up by redistribution.
At this point, we want to obtain an estimation of the welfare loss due to using the tax morale coefficient $\hat{\mu} = \infty$ rather than the true one. The last column of Table 4 displays the value of the scalar θ by which multiplying wage rates w_i of the appropriately taxed economy, the resulting J-welfare becomes equal to that of the falsely taxed economy. For example, in an economy with true tax morale 4 (italicized row), the idealistically chosen $\hat{\tau} = 0.286$ leads to $\theta = 0.923$, i.e. a relative loss about 8% with respect to the realistic optimum $\tau(\mu) = 0.264$.

Table 4: The impact of tax morale on optimal outcomes for 3-quintiles

Tax morale μ	Optimal tax rate $\tau^{0.6}(\mu)$	Aggregate labor $\lambda(\mu)$	Basic income $\beta^{0.6}(\mu)$	Lowest consumption c_1	Inefficiency due to error $\theta^{0.6}(\mu, \infty)$
2	0.121	0.909	0.055	0.397	0.848
3	0.240	0.800	0.128	0.397	0.896
4	*0.264*	*0.769*	*0.152*	*0.399*	*0.923*
5	0.273	0.755	0.165	0.401	0.939
...					...
∞	0.286	0.714	0.204	0.408	1

Remark. $\nu = 0.6$.

Including only the single poorest quintile rather than the three poorest quintiles, the impact of the tax morale on the socially optimal tax rate ceases to be monotone, moreover, the tax rate remains uniformly high but stagnating around 0.38. Note that the corresponding $W_1 < 0.54$, leaving room for decreasing tax rate for $\mu > 3.33$. The negative impact of tax morale on labor supply dominates the scene: the labor supply drops from 0.714 with $\mu = 2$ to 0.625 in the white economy. Therefore the increasing redistribution via the increasing basic income is counterbalanced by the dropping labor supply, thus the poorest quintile's consumption remains lower than in the less Rawlsian redistribution.

Table 5: The impact of tax morale on optimal outcomes for 1-quintile

Tax morale μ	Optimal tax rate $\tau^{0.2}(\mu)$	Aggregate labor $\lambda(\mu)$	Basic income $\beta^{0.2}(\mu)$	Lowest consumption c_1
2	0.381	0.714	0.136	0.367
3	0.400	0.667	0.178	0.373
4	0.398	0.652	0.194	0.378
5	0.394	0.645	0.203	0.380
...				...
∞	0.375	0.625	0.234	0.391

As a detour, we illustrate the political economy equilibrium. To distinguish the third quintile from the three lowest quintiles, we shall use subindex [0.5] rather than superscript 0.6. For any tax morale, the political economy equilibrium is much lower than the 3-quintile optimum, it remains zero for $\mu = 2, 3$ and only rises from 0.07 for $\mu = 4$ to 0.167 in the white economy. Correspondingly, the redistribution is also lower, but the average labor supply is higher than in the welfare model.

Table 6: The impact of tax morale in political economy

Tax morale μ	Optimal tax rate $\tau_{[0.5]}[\mu]$	Aggregate labor $\lambda[\mu]$	Basic income $\beta_{[0.5]}[\mu]$	Median consumption c_M
3	0	1	0	0.800
4	0.071	0.938	0.050	0.760
5	0.101	0.909	0.073	0.742
10	0.142	0.865	0.110	0.714
...				...
∞	0.167	0.833	0.139	0.694

Finally, to explain the anomaly found by Alesina and Angelitos (2005) with our framework, we display two countries with two inclusion shares: $\nu_L = 0.2$ (low) and $\nu_H = 0.6$ (high) and two pre-tax wage rate inequality setups. For the sake of simplicity, we keep the original wage rates of Table 3 for the high-inequality set up, and create a low-inequality set up by scaling down the deviations from the mean: $w_i(\omega) = 1 + \omega(w_i - 1)$ with $\omega = 0.8$.

Since the (dis)utility parameters are uniform, the pre- and post-tax indicators are the same. Comparing the economies represented by rows 2 and 3 of Table 7, higher inclusion share 0.6 (vs. 0.2) overrules the impact of higher earning inequality and leads to a lower optimal tax rate 0.26 (vs. 0.32).

Table 7: Lower inclusion share may overrule lower earning inequality

Earning inequality ω	Inclusion share ν	Optimal tax rate $\tau^\nu(4)$	Basic income $\beta^\nu(4)$
1	0.2	0.398	0.194
1	0.6	0.264	0.152
0.8	0.2	0.323	0.174
0.8	0.6	0.196	0.122

Remark. $\mu = 4$.

We could continue the numerical exploration without any difficulty but, for our purposes, this seems to be sufficient to show the basic idea of the model: in addition to the much studied elasticity of labor supply and inclusion share (or the indifference index), the tax morale also plays an important role in the choice of optimal income taxation. This observation is also supported by my previous paper (Simonovits, 2010), where a distinctly different specification of the problem (with logarithmic utility functions, fixed labor supply and purely utilitarian social welfare function) gave qualitatively similar results.

IV. Conclusion

In this very simple toy model, we were able to study the impact of the exogenous tax morale on the socially optimal tax rate. Under certain assumptions (uniform linear-quadratic utilities), first we demonstrated with pencil and paper the traditional view: for a given tax revenue, the higher the tax morale, the lower the balancing tax rate (Theorem 1). Furthermore, adding truncated (or exclusive) social welfare functions, we proved analytically Theorems 2 and 3: higher morale and lower inclusion share imply higher socially optimal tax rate, provided that the included average wage is higher than 0.54 times the average. We can add a third observation: higher earning inequality implies a higher optimal tax rate, but this can be reversed by a higher inclusion share. Incidentally, political economy considerations generates similar results: a higher tax morale implies a higher equilibrium tax rate (Theorem 2*). Further work should be done to check the robustness of our results, i.e. extend Theorems 2 and 3 to other utility functions, social welfare functions and heterogeneous tax morales. More importantly, the exogenous tax morales and simple labor disutility functions should be replaced by endogenous tax morales and sophisticated labor disutilities implying realistic labor supplies. The provision of public goods and its efficiency also require attention.

Acknowledgement

A previous version of this paper without a question mark in the title appeared as Simonovits (2011) with a less appropriate formulation of the moral disutility of underreporting wage. I express my debt to P. Benczúr, H. Fehr, A. Gáspár, L. Halpern, B. Kőszegi, M. Lackó, M. Maffezzoli, B. Menyhért, Gy. Molnár, B. Muraközy, A. Peichl, A. Tasnádi, Gy. I. Tóth and anonymous referees of previous versions of Simonovits (2010) for their useful comments, especially for convincing me to include flexible labor supply. I am solely responsible for the content of the paper. I acknowledge the generous financial support from OTKA K 81483.

Appendix: Proofs

Balancing tax rate

The definition of the basic income β^* yields an implicit function:

$$F(\mu) = \delta(\mu)\tau^2 - \tau + \frac{\alpha\beta^*}{1-\mu^{-1}} = 0.$$

Using the formula for the solution of the quadratic equation yields $\tau_{\beta^*}[\mu]$ in Theorem 1. Elementary reasoning establishes that $\tau_{\beta^*}[\mu]$ is a decreasing function.

Optimal labor supply and unreported wage

Take the partial derivatives with respect to l_i and e_i, then make them to zero:

$$0 = v'_{i,l}(l_i, e_i) = 2w_i(1-\tau) - 2\alpha w_i l_i + \mu\tau w_i^{-1} l_i^{-2} e_i^2$$

and

$$0 = v'_{i,e}(l_i, e_i) = 2\tau - 2\mu\tau w_i^{-1} l_i^{-1} e_i.$$

Rearranging the second equation we obtain $\hat{e}_i = \mu^{-1} w_i l_i$, for $\mu > 1$.
Although the first equation appears to be cubic in l_i, after substitution of $\hat{e}_i$ it also becomes linear:

$$0 = v'_{i,l}(l_i, \hat{e}_i) = 2w_i(1-\tau) - 2\alpha w_i l_i + \mu\tau w_i^{-1} l_i^{-2} \mu^{-2} w_i^2 l_i^2.$$

After rearrangement: $0 = 2(1-\tau) - 2\alpha l_i + \tau\mu^{-1}$ and recalling notations

$$\lambda = \lambda(\mu, \tau) = \alpha^{-1}(1-\delta\tau) \quad \text{and} \quad \delta = 1 - \frac{1}{2\mu},$$

the optimal labor supply and unreported wage are respectively

$$l_i^* = \lambda \quad \text{and} \quad e_i^* = \mu^{-1} w_i \lambda.$$

To have a true maximum, we must check concavity in the domain. We need the four second-order derivatives:

$$v''_{i,ll}(l_i, e_i) = -2\alpha w_i - 2\mu\tau w_i^{-1} l_i^{-3} e_i^2 < 0, \quad v''_{i,le}(l_i, e_i) = v_{i,el}(l_i, e_i) = 2\mu\tau w_i^{-1} l_i^{-2} e_i$$

and

$$v''_{i,ee}(l_i, e_i) = -2\mu\tau w_i^{-1} l_i^{-1} < 0.$$

The negativity conditions hold, and the determinant condition

$$\Delta = v''_{i,ll}(l_i, e_i) v''_{i,ee}(l_i, e_i) - v''_{i,le}(l_i, e_i)^2 > 0$$

also holds:

$$\frac{1}{4}\Delta = (\alpha w_i + \mu\tau w_i^{-1} l_i^{-3} e_i^2)\mu\tau w_i^{-1} l_i^{-1} - (\mu\tau w_i^{-1} l_i^{-2} e_i)^2 = \alpha w_i \mu\tau w_i^{-1} l_i^{-1} > 0.$$

For $\alpha \geq 1$ and $0 < \tau < 1$, $0 < l_i^* < 1$.

Optimal tax rate

Inserting β^* into the indirect utility functions,

$$u_i^* = v_i(l_i^*, e_i^*) = 2\lambda w_i(1-\tau) + 2\mu^{-1} w_i \lambda\tau + 2\tau(1-\mu^{-1})\lambda - \alpha w_i \lambda^2 - \mu^{-1}\tau w_i \lambda.$$

To have compact formulas

$$u_i^* = A(\mu,\tau) w_i + B(\mu,\tau) \quad \text{and} \quad U_J^*(\mu,\tau) = A(\mu,\tau) W_J + B(\mu,\tau),$$

we substitute δ and λ:

$$A(\mu,\tau) = 2\lambda(1-\tau) + \mu^{-1}\lambda\tau - \alpha\lambda^2 = \alpha\lambda^2 \quad \text{and} \quad B(\mu,\tau) = 2\tau(1-\mu^{-1})\lambda = 2(2\delta-1)\tau\lambda.$$

The socially optimal tax rate is given by the first-order necessary condition:

$$U_{J,\tau}^{*}{}'(\mu,\tau) = A'_\tau(\mu,\tau) W_J + B'_\tau(\mu,\tau) = 0.$$

Executing the calculations:

$$0 = -\delta W_J + \delta^2 W_J \tau + 2\delta - 1 - 2(2\delta-1)\delta\tau$$

yields the socially J-optimal tax rate:

$$\tau_J(\mu) = \frac{2 - 1/\delta - W_J}{2(2\delta-1) - \delta W_J}.$$

To have a positive tax rate, we must have $1 - \mu^{-1}/2 = \delta > 1/(2 - W_J)$, i.e.

$$\mu > \mu_J = \frac{1}{2[1 - 1/(2 - W_J)]} = \frac{2 - W_J}{2(1 - W_J)}.$$

To have an increasing tax rate–tax morale function, $\tau'_J(\mu) > 0$ must hold. Replacing μ by δ, introducing $z = 2 - W_J$ and dropping the square of the denominator, we have

$$0 < \tau'_J[\delta] \approx z(4\delta^2 - z\delta) - (z\delta - 1)8\delta.$$

Rearranging this inequality results in

$$\frac{1}{2} < \delta < \bar{\delta} = \frac{8 - z^2}{4z} = \frac{4 + 4W_J - W_J^2}{4(2 - W_J)}.$$

Since $\delta(\mu)$'s range is [1/2, 1], a simple calculation shows that $W_J = 4 - 2\sqrt{3}$ pushes $\bar{\delta}$ to 1, implying $\tau_J'(\mu) > 0$ for any μ. For $W_J < 4 - 2\sqrt{3}$ and $\mu > \mu_c$, $\tau_J'(\mu) < 0$.
Finally, we show that $\tau_J(\mu)$ is a decreasing function of W_J. Taking its partial derivative with respect to W_J and dropping the denominator:

$$-[2(2\delta - 1 - W_J] + 2 - W_J - \delta^{-1} = -4\delta + 4 - \delta^{-1} = -(2\delta - 1)^2\delta^{-1} < 0.$$

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