View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Library of National Technical University "Kharkiv Polytechnic Institute"

UDC 621.9

V. N. BURLAYENKO, T. SADOWSKI, D. PIETRAS

A NUMERICAL ANALYSIS OF NEAR TIP FIELDS IN A BENDING MOMENT-LOADED DOUBLE CANTILEVER SANDWICH BEAM FRACTURE SPECIMEN

The paper presents an interfacial crack problem adopted for studying fracture toughness and debonding tolerance of sandwich composite materials. A specific example of the fracture sandwich specimens such as a double cantilever sandwich beam subjected to uneven bending moments (DCB-UBM) is considered. A finite element modelling of this test method is carried out using the ABAQUS™ code. A two-dimensional (2-D) model has been developed to highlight the distribution of stress and displacement fields and to calculate the energy release rate (ERR) and the phase angle at the interface crack between two dissimilar orthotropic materials. The J – integral approach built-in ABAQUS code and the crack surface displacement method programmed as an add-on subroutine within the Matlab® environment are used for computing those fracture parameters. The influence of different moment ratios on the near crack tip stress state, the ERR, and phase angle fracture parameters is estimated.

Key words: double cantilever sandwich beam, bi-material interface, fracture parameters, finite element method, ABAQUS™.

В. М. БУРЛАЄНКО, Т. САДОВСЬКИЙ, Д. ПЕТРАС

ЧИСЕЛЬНИЙ АНАЛІЗ ПОЛІВ В ОКОЛІ ВЕРШИНИ ТРІЩИНИ У ДВОКОНСОЛЬНОМУ БАЛОЧНОМУ ТРИШАРОВОМУ ЗРАЗКУ, ЩО НАВАНТАЖЕНИЙ ЗГИНАЛЬНИМИ МОМЕНТАМИ

Представлено проблему міжфазної тріщини, за допомогою якої вивчаються в'язкість руйнування та стійкість до відшарування у тришарових композиційних матеріалах. Розглянуто конкретний приклад тришарового зразка на руйнування – двохконсольна тришарова балка, яка навантажена згинальними моментами. Скінчено елементне моделювання цього тесту здійснюється за допомогою програми ABAQUS™. Двовимірна модель була розроблена для виявлення розподілу полів напружень та переміщень, а також для розрахунку швидкості вивільнення енергії руйнування та фазового кута міжфазної тріщини поміж двома різними ортотропними матеріалами. J – інтеграл підхід, який є вбудованою опцією у ABAQUS, та метод відносних зміщень на поверхнях тріщини, який запрограмований у середовищі Matlab® окремою програмою, використовуються для обчислення цих параметрів руйнування. Оцінюється вплив різних співвідношень згинальних моментів на напружений стан в околі вершини тріщини, швидкості вивільнення енергії руйнування і фазовий кут.

Ключові слова: двохконсольний балочний тришаровий зразок, інтерфейс біматеріалу, параметри руйнування, метод скінчених елементів, ABAQUS™.

В. Н. БУРЛАЕНКО, Т. САДОВСКИЙ, Д. ПЕТРАС

ЧИСЛЕННЫЙ АНАЛИЗ ПОЛЕЙ В ОКРЕСТНОСТИ ВЕРШИНЫ ТРЕЩИНЫ В ДВУХКОНСОЛЬНОМ БАЛОЧНОМ ТРЁХСЛОЙНОМ ОБРАЗЦЕ, НАГРУЖЕННОМ ИЗГИБАЩИМИ МОМЕНТАМИ

Представлена задача межфазной трещины, с помощью которой изучается вязкость разрушения и устойчивость к отслоению в трехслойных композиционных материалах. Рассмотрен конкретный пример трехслойного образца на разрушение – двухконсольная трехслойная балка, нагруженная изгибающими моментами. Кончено элементное моделирование этого теста осуществляется с помощью программы ABAQUS™. Двухмерная модель была разработана для моделирования распределения полей напряжений и перемещений, а также для расчета скорости высвобождения энергии разрушения и фазового угла межфазной трещины между двумя различными ортотропными материалами. J – интеграл подход, который является встроенной опцией в ABAQUS, и метод относительных смещений на поверхностях трещины, который запрограммирован в среде Matlab® отдельной программой, используются для вычисления этих параметров разрушения. Оценивается влияние различных соотношений изгибающих моментов на напряженное состояние в окрестности вершины трещины, скорости высвобождения энергии разрушения и фазовый угол.

Ключевые слова: двухконсольний балочный трехслойный образец, интерфейс биматериала, параметры разрушения, метод конечных элементов, ABAQUS™.

Introduction. New materials such as sandwich composites have been developed to provide the strength of the structure at minimum its weight. Such materials being considered as an assemblage of two stiff and tough face sheets (skins) bonded to a soft and light core have become much used in a wide range of engineering fields. With the growth of the application of sandwich materials, their strength and damage tolerance should be well predicted. Particularly, the problem of interface crack in the face sheet-to-core interface or so-called debonding is often encountered failure mode of sandwich structural components [1]. Because the sandwich material interface is, in essence, a bi-material system, stress concentration takes place there, as a result the crack originates mainly from the interface. Once such a crack occurs, the load bearing capacity of the sandwich structure is significantly reduced, and the integrity is compromised because of the imminent risk of debonding propagation. Thus, proper evaluation of interface fracture parameters is important.

A standard approach to provide the resistance of the sandwich material against the debonding is based on an appropriate test method with the assumptions of linear fracture mechanics defining the fracture toughness value. In doing so, the interface flaw is to be treated as a crack between dissimilar materials. Moreover, it should be taken into account that such interface cracks propagate mostly in mixed mode conditions, in contrast to cracks developing in homogeneous materials in pure mode I load. Hence, the interface fracture toughness is dependent on the loading phase angle and can't be presented by a single quantity but rather is a function of the phase angle [2].

A variety of sandwich specimens for measuring interfacial fracture toughness has been proposed [3]. An effective and relatively simple testing method among many others is a double cantilever beam (DCB) sandwich specimen. Also, if this specimen is subjected to uneven bending moments (DCB-UBM) it will allow for testing over a large range of mode-

© V. N. Burlayenko, T. Sadowski, D. Pietras, 2018

mixities by varying direction and magnitude of moments only. Moreover, this type of loading produces a stable crack growth, since the crack loading does not change with crack length [4]. However, although all the specimens are able to characterize fracture toughness of the sandwich material, they cannot present a distribution of near tip stress and displacement fields. The latter is of importance to understand the fracture behaviour of sandwich structures and, also, to develop analytical or numerical methods for debonding growth predictions.

The objective of this paper is to develop an accurate finite element model for predictions of near tip stress and displacement fields in a particular sandwich fracture test called a DCB-UBM specimen, which is used for experimental measurements of interfacial fracture toughness. Moreover, we aim to present a method for calculations of fracture parameters such as the energy release rate (ERR) and the phase angle in the DCB-UBM sandwich specimen consisting of dissimilar orthotropic constitutive material layers. Also, we study the dependence of these parameters on the loading phase angle.

Fundamentals of interface fracture. Let us consider a crack along the interface between two linearly elastic, homogeneous, anisotropic materials. The materials are defined by the material tensors $C_{ij}^{(\alpha)}$, which are a contracted notation of the fourth order elastic constant tensors $C_{ijkl}^{(\alpha)}$, respectively. The subscripts $\alpha = \#1, \#2$ refer to material $\#1$, above the interface, and material $\#2$, below the interface, as shown in Fig. 1.

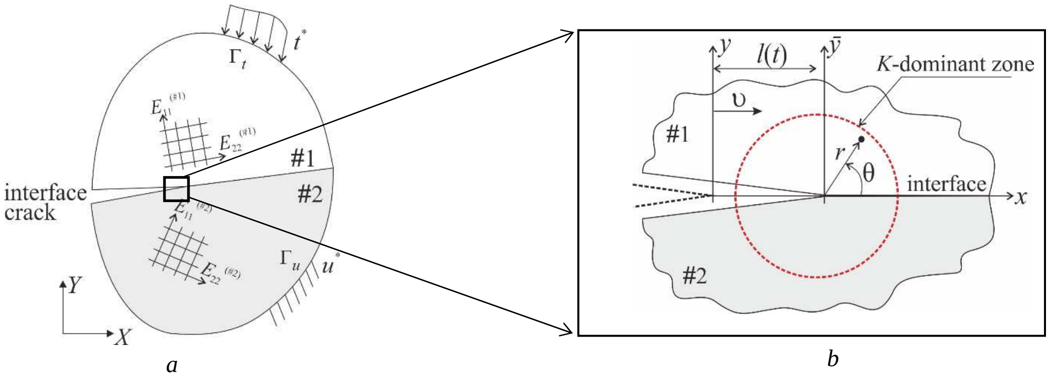


Fig. 1 – A general model of the interface crack: *a* – an interface crack between orthotropic materials; *b* – a schematic representation of *K* – dominant zone for bi-material interface crack growth.

The structure of the stress and displacement fields at the tip of either stationary crack or a crack propagating dynamically with speed υ follows from the solution of the eigenvalue problem induced by the traction free boundary conditions on the crack flanks (Fig. 1, *b*) and can be described as follows [5]:

$$\bar{\boldsymbol{H}}\boldsymbol{w} = e^{2\pi\varepsilon}\boldsymbol{H}\boldsymbol{w} . \qquad (1)$$

Here $\boldsymbol{w}$ is the normalized unit eigenvector; $\boldsymbol{H}$ is a positive defined compliance-like Hermitian matrix involving the bi-material elastic constants $C_{ijkl}^{(\alpha)}$ and $\bar{\boldsymbol{H}}$ is the complex conjugate matrix; ε is the oscillation index given by $\varepsilon = 1/2\pi \ln(1-\beta)/(1+\beta)$, where $\beta = -\sqrt{-1/2tr\left\{\left(\Im m\left[\boldsymbol{H}\right]/\Re e\left[\boldsymbol{H}\right]\right)^2\right\}}$ is one of the two Dundurs parameters. The three eigenpairs of (1) have the form $(\varepsilon, \boldsymbol{w})$, $(-\varepsilon, \bar{\boldsymbol{w}})$ and $(0, w_3)$, where $\boldsymbol{w}$, $\bar{\boldsymbol{w}}$ and w_3 are complex, complex conjugate and real eigenvectors, respectively.

The near tip stress fields are a linear combination of two types of singularities such as a coupled oscillatory field scaled by a complex stress intensity factor (SIF) $\boldsymbol{K} = K_1 + iK_2$ and a non-oscillatory filed scaled by a real factor K_3 [6]:

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}}\left\{\Re e\left[\boldsymbol{K}r^{i\varepsilon}\right]\sum\nolimits_{ij}^{(1)}(\theta) + \Im m\left[\boldsymbol{K}r^{i\varepsilon}\right]\sum\nolimits_{ij}^{(2)}(\theta) + K_3\sum\nolimits_{ij}^{(3)}(\theta)\right\}, \qquad (2)$$

where (r, θ) are polar coordinates and $\sum_{ij}^{(k)}(\theta)$ with $k = 1, 2, 3$ are the angular functions corresponding to in-plane opening and shearing and anti-plane shearing tractions across the bi-material interface, Fig. 1, *b*. Herewith, the components of the complex stress intensity factor K_1 and K_2 are no more individual stress amplitudes for respectively mode I and II.

Projecting the actual stress state (2) onto the eigenvectors $\boldsymbol{w}$, $\bar{\boldsymbol{w}}$ and w_3 mentioned earlier, the generalized components of the stress vector $\boldsymbol{t} = \left\{\sigma_{2j}\right\}$ at the crack tip ($\theta = 0$) as $r \to 0$ asymptote to

$$\boldsymbol{t}(r) = \frac{1}{2\sqrt{2\pi r}}\left\{\boldsymbol{K}\boldsymbol{w}\left(\frac{r}{\hat{l}}\right)^{i\varepsilon} + \bar{\boldsymbol{K}}\bar{\boldsymbol{w}}\left(\frac{r}{\hat{l}}\right)^{-i\varepsilon} + 2K_3 w_3\right\}, \qquad (3)$$

where $\hat{l}$ is a characteristic length of the problem under consideration (a specific distance from the crack tip used to avoid the dependence of dimension of the complex SIF on the oscillation index ε).

Similarly, the relative crack flank displacements at a distance r behind the crack tip $(\theta = \pm\pi)$ are defined as

$$\delta(r) = \sqrt{\frac{r}{2\pi}} \left\{ \frac{\cosh \pi\varepsilon}{1+2i\varepsilon} \boldsymbol{K}\boldsymbol{v} \left(\frac{r}{\hat{l}} \right)^{i\varepsilon} + \frac{\cosh \pi\varepsilon}{1-2i\varepsilon} \bar{\boldsymbol{K}}\bar{\boldsymbol{v}} \left(\frac{r}{\hat{l}} \right)^{-i\varepsilon} + K_3 v_3 \right\}, \tag{4}$$

where the auxiliary vectors $\boldsymbol{v}$ and v_3 are introduced for convenience as follows: $\left(\boldsymbol{H}^{-1} + \bar{\boldsymbol{H}}^{-1}\right)\boldsymbol{v} = \boldsymbol{w}$ and $\left(\boldsymbol{H}^{-1} + \bar{\boldsymbol{H}}^{-1}\right)v_3 = w_3$, respectively.

The ERR and the phase angle are related to the SIFs as follows:

$$g = \frac{\bar{\boldsymbol{w}}^T \left(\boldsymbol{H} + \bar{\boldsymbol{H}}\right)\boldsymbol{w}}{4\cosh^2 \pi\varepsilon} |\boldsymbol{K}|^2 + \frac{1}{8} w_3^T \left(\boldsymbol{H} + \bar{\boldsymbol{H}}\right) w_3 K_3^2 \tag{5}$$

and

$$\psi = \tan^{-1}\left(\frac{\Im m\left[\boldsymbol{K}\hat{l}^{i\varepsilon}\right]}{\Re e\left[\boldsymbol{K}\hat{l}^{i\varepsilon}\right]} \right). \tag{6}$$

Determination of face sheet-to-core interface fracture parameters. The face sheet-to-core interface of modern sandwich materials usually is a bi-material system of two highly dissimilar orthotropic materials. In the case of two orthotropic materials with their axes of symmetry aligned along the material face sheet-to-core interface, the matrix $\boldsymbol{H}$ takes the form [7]:

$$H_{11} = \left[2n\lambda^{1/4}\sqrt{s_{11}s_{22}}\right]_{\#1} + \left[2n\lambda^{1/4}\sqrt{s_{11}s_{22}}\right]_{\#2},\ H_{22} = \left[2n\lambda^{-1/4}\sqrt{s_{11}s_{22}}\right]_{\#1} + \left[2n\lambda^{-1/4}\sqrt{s_{11}s_{22}}\right]_{\#2},$$
$$H_{12} = \bar{H}_{21} = i\left[\sqrt{s_{11}s_{22}} + s_{12}\right]_{\#1} - \left[\sqrt{s_{11}s_{22}} + s_{12}\right]_{\#2},$$
$$H_{33} = \left[\sqrt{s_{44}s_{55}}\right]_{\#1} + \left[\sqrt{s_{44}s_{55}}\right]_{\#2},\ H_{13} = H_{31} = H_{23} = H_{32} = 0, \tag{7}$$

where $s_{11} = 1/E_1$, $s_{22} = 1/E_2$, $s_{12} = s_{21} = -\nu_{12}/E_1 = -\nu_{21}/E_2$, $s_{44} = 1/G_{13}$, $s_{55} = 1/G_{23}$, $s_{66} = 1/G_{12}$ are constants in plane stress and they are transformed in plane strain as $\tilde{s}_{ij} = s_{ij} - s_{i3}s_{3j}/s_{33}$, $\lambda = s_{11}/s_{22} = E_2/E_1$ and $\rho = (2s_{12} + s_{66})/2\sqrt{s_{11}s_{22}} = \sqrt{E_1E_2}/2G_{12} - \sqrt{\nu_{12}\nu_{21}}$ are parameters of anisotropy; and $n = \sqrt{(1+\rho)/2}$.

Moreover, one can express the eigenvectors in (1) as $w = \left\{-i/2, 1/2\sqrt{H_{11}/H_{22}}, 0\right\}$ and $w_3 = \{0, 0, 1\}$. The Dundurs parameter via the components of the matrix $\boldsymbol{H}$ in (7) takes the form: $\beta = iH_{12}/\sqrt{H_{11}H_{22}}$.

The stress vector (3) and the relative displacements vector (4) for a 2-D state can also be determined via the complex SIFs and the components of the matrix $\boldsymbol{H}$ as follows:

$$\sqrt{\frac{H_{22}}{H_{11}}}\sigma_{yy} + i\sigma_{xy} = \frac{\boldsymbol{K}}{\sqrt{2\pi r}}\left(\frac{r}{\hat{l}}\right)^{i\varepsilon} \tag{8}$$

and

$$\sqrt{\frac{H_{11}}{H_{22}}}\delta_y + i\delta_x = \frac{2H_{11}\boldsymbol{K}}{\sqrt{\boldsymbol{2\pi}}(1+2i\varepsilon)\cosh \pi\varepsilon}\left(\frac{r}{\hat{l}}\right)^{\frac{1}{2}+i\varepsilon}, \tag{9}$$

where σ_{yy} and σ_{xy} are transverse normal and shear stress tensor components in front of the crack tip; δ_y and δ_x are the opening and sliding relative displacement of the crack flanks.

In turn, the displacement field stated in (9) is used to express the ERR (5) and the mode-mixity (6) in terms of the relative crack flank displacements as follows:

$$g = \frac{H_{11}|\boldsymbol{K}|^2}{4\cosh^2 \pi\varepsilon} = \frac{\pi\left(1+4\varepsilon^2\right)}{8H_{11}\left(r/\hat{l}\right)}\left(\frac{H_{11}}{H_{22}}\delta_y^2 + \delta_x^2\right) \tag{10}$$

and

$$\psi = \tan^{-1}\left(\sqrt{\frac{H_{11}}{H_{22}}}\frac{\delta_x}{\delta_y}\right) - \varepsilon \ln\left(\frac{r}{\hat{l}}\right) + \tan^{-1}(2\varepsilon). \tag{11}$$

In the case of steady state conditions, the energy release rate defined by (5) or (10) can be calculated using the domain $J-$*integral formula* suitable for using the finite element solution to calculate the ERR for the interface crack [8]:

$$g = J = \sum\nolimits_{k=\#1,\#2} \int_{A^{(k)}} \left[\left(\boldsymbol{\sigma}^{(k)} \cdot \frac{\partial \boldsymbol{u}^{(k)}}{\partial x} \right) \frac{\partial q}{\partial \mathbf{x}} - W^{(k)} \frac{\partial q}{\partial x} \right] dA, \tag{12}$$

where A is the domain enclosed by an arbitrary contour, Γ, surrounding a crack tip and crack surfaces; the weighting parameter q is a smooth function of $\mathbf{x} = \{x, y\}^T$, it takes values from zero on the Γ contour to unity at the crack tip; $\boldsymbol{\sigma}$ and $\mathbf{u}$ are the stress tensor and the displacement vector at a material point $\mathbf{x}$; W is the strain energy density at the point $\mathbf{x}$.

The Crack Surface Displacement (CSD) method [9, 10] fits classical bi-material interface theory solutions (5), (6) into a FE analysis framework by Eqs. (10), (11) to directly calculate the energy release rate and mode mixity of a bi-material crack. In accordance with this method, values of the ERR obtained from the different nodal relative displacements along the crack flanks by (10) are linearly extrapolated into $r \to 0$. Then, the extrapolated value of the ERR is compared with the value of $|\boldsymbol{K}|$ defined by the J – integral method by (12), as a result a critical distance r_c, at which the two values of the ERR are equal within a given tolerance, is used to compute the phase angle by (11). Moreover, using the values for the ERR and the phase angle, the components of the complex SIF, K_1 and K_2 can be also found.

Finite element modelling. The finite element (FE) model of a double cantilever beam sandwich specimen (Fig. 2, *a*) subjected to uneven bending moments (DCB-UBM) has been developed in the commercial FE package ABAQUS [11]. The eight-node isoparametric plane strain finite elements were used for creating the 2-D model. The FE mesh contained a refinement near the crack-tip region, as shown in Fig. 2, *b*.

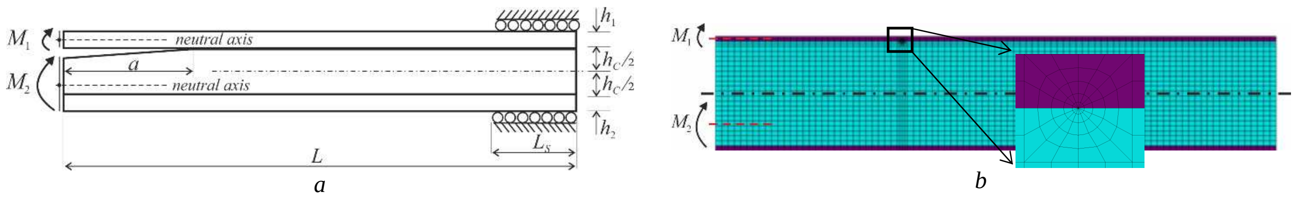


Fig. 2 – A DCB-UBM sandwich specimen:
a – a schematic representation of the specimen; *b* – a FE model of the specimen with mesh refinement around the crack tip.

The debonded region of the specimen was modelled using a double set of nodes with coinciding coordinates along the face sheet-to-core interface. A rosette of quarter-point singular finite elements has been inserted into the mesh around the crack tip to reproduce the square root singularity at the crack front. Moreover, when both applied moments were rotating the arms of the specimen in same directions (co-rotated), contact conditions were imposed on the surfaces of the finite elements defining the debonded region in order to avoid non-physical interpenetration between them. The frictionless hard contact model within the penalty contact algorithm available in ABAQUS [12, 13] was used in the simulations.

The energy release rate g was determined from the FE solution by using both the relative nodal pair displacements along the crack flanks by (10) and the J – integral by (12). In the former case, an add-on subroutine was developed in Matlab® environment. The subroutine extracted the finite element displacements at given nodal sets from the database file of the ABAQUS' static analysis. The J – integral method is a built-in option in ABAQUS. In those calculations, the value of g was averaged over at least five contours chosen around the interface crack tip. The phase angle ψ was also determined from the finite element analysis using the Matlab subroutine, which post-processed the results in accordance with the CSD method outlined in the previous section.

Numerical results and discussions. The modelled sandwich beam specimen consisted of laminated composite face sheets made of graphite fibre reinforced plastic (GFRP) bonded to a 100 kg/m^3 (H 100) PVC foam core. The elastic properties used in the model for the laminated composite face sheets and PVC foam core (both considered orthotropic) are listed in Table 1.

Table 1 – Material properties of the DCB – UBM specimen

Constituent	Material constants
1. GFRP face sheet	$E_x = E_z = 16.5$ GPa, $E_y = 3.8$ GPa, $G_{xy} = G_{xz} = 1.8$ GPa, $G_{yz} = 6.6$ GPa, $v_{xy} = 0.05$, $v_{xz} = v_{yz} = 0.25$, $\rho = 1650$ kgm^{-3}
2. PVC H 100 foam core	$E_x = E_y = E_z = 105$ MPa, $G_{xy} = G_{xz} = G_{yz} = 78$ MPa, $v_{xy} = v_{xz} = v_{yz} = 0.325$, $\rho = 100$ kgm^{-3}

The moment-ratio (M_R) is defined as the ratio of the moments applied at the debonded face sheet (M_1) to the substrate (M_2) i.e. $M_R = M_1 / M_2$ (see Fig. 2). Different moment ratios, where the bending moments were either co-rotated or rotated in opposite directions but induced nearly the same value of the ERR have been examined in the studies. The finite element predictions of the ERR and the phase angle regarding M_R are presented in Table 2.

The contour plots of the stress tensor components associated with the ratios M_R in Table 2 are illustrated in Fig. 3, where the first row of the images corresponds to σ_{xx}, the second one to σ_{yy}, and the last one to σ_{xy}. A complicated nature of the near-tip stress field is clearly observed there. One can see that the shear stress exists in the vicinity of crack regardless of the loading case as seen in the third row of Fig. 3. This clearly emphasizes conditions of mode mixity being expected in a sandwich type structure.

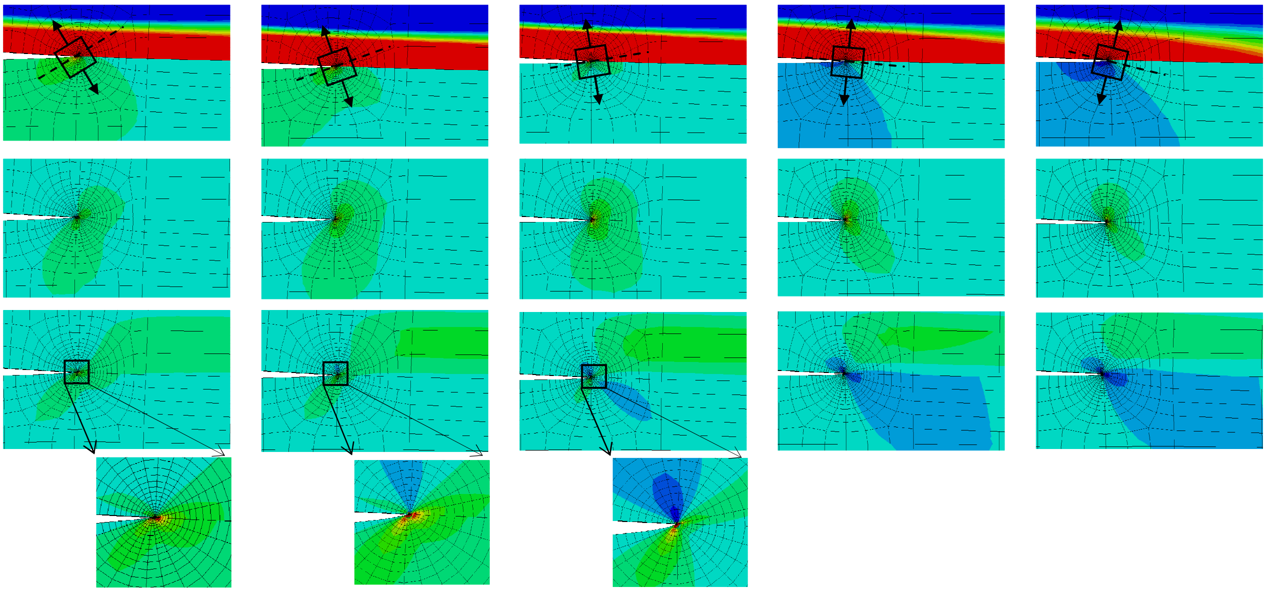
Table 2 – The energy release rate and the phase angle with respect to the moment ratios, M_R

M_1, N mm	75.6	103.42	123.4	104.13	73.8
M_2, N mm	-1512.2	-1034.2	-123.4	1041.3	1476
M_R / *Fracture parameters*	-0.05	-0.1	-1	0.1	0.05
ERR, g, N mm^2	0.399	0.399	0.403	0.399	0.377
Phase angle, ψ, deg.	30.26	11.31	-17.16	-51.87	-70.71

Table 3 – The energy release rate mode I component and the angle of the principal plane with respect to the moment ratios, M_R

M_R / *Parameters*	-0.05	-0.1	-1	0.1	0.05
ERR, g_I, %	66.2	84.8	78.4	48.5	33.1
Angle, ϑ, deg.	42.32	37.72	29.28	-7.26	-12.68

Moreover, the sign of the shear stress ahead crack may be used to define a favourable direction of interface crack propagation if the crack growth in the bi-material structure is postulated by the plane of maximum principal stresses [14]. For the sake of estimation, the contribution of mode I component into the total ERR and the angle of the principal plane, ϑ with respect to the value of M_R has been evaluated. The values of these computed parameters are shown in Table 3. A preferred direction of the crack growth predicted by the angle ϑ is demonstrated in Fig. 3 as well. From these outcomes one can conclude that the positive shear stress at the crack tip results in upward crack growth along the face sheet-to-core interface, but the negative one causes a downward crack growth into the core. Herewith, the latter case is featured by the dominant mode II stress state as shown in Table 3 and Fig. 3 for M_R equal to 0.1 and 0.05.



a *b* *c* *d* *e*

Fig. 3 – Contour plots of the stress tensor components at the crack tip of the DCM-UBM specimen w.r.t. the moment ratio M_R: *a* – $M_R = -0.05$; *b* – $M_R = -0.1$; *c* – $M_R = -1$; *d* – $M_R = 0.1$; *e* – $M_R = 0.05$.

Conclusions. In this research, the problem of stationary debonding in sandwich materials consisting of orthotropic material layers is considered and the theoretical background of the problem is highlighted. A two-dimensional finite element model for predicting the energy release rate, phase angle and stress state in the DCB-UBM sandwich specimen by tools available in the ABAQUS code and with additional procedures programmed within Matlab for post-processing the finite element solution in accordance with the Crack Surface Displacement method is presented.

The simulations showed that regardless of the loading case defined by the ratio of bending moments, the mixed mode conditions occur in the sandwich specimen. This is due to the bi-material nature of the interface crack. In doing so, the shear stress exciting in the vicinity of crack may be used as an important parameter to predict a preferred interface crack growth direction.

Acknowledgements. The results presented in this paper were obtained within the framework of the research grant No. UMO/2016/21/B/ST8/01027 of the National Science Centre, Poland, which is co-funded by the European Union's Horizon 2020 programme under the Marie Skłodowska-Curie grant agreement No. 665778.

Bibliography

1. *Burlayenko V. N., Sadowski T.* Linear and nonlinear dynamic analyses of sandwich panels with face sheet-to-core debonding // Shock and Vibration. – 2018. – Т. 2018. – 26 с. doi: 10.1155/2018/5715863.
2. *Szekrényes A.* Delamination of composite specimens : Ph.D. thesis. – Hungary, 2005. – 130 p.
3. *Shivakumar K. N., Smith S. A.* In situ fracture toughness testing of core materials in sandwich panels // Journal of Composite Materials. – 2004. – Т. 38. – С. 655 – 658. doi: 10.1177/0021998304042392.
4. *Sorensen B. F., Jorgensen K., Jacobsen T. K., Ostregaard R. C.* DCB-specimen loaded with uneven bending moments // International Journal of Fracture. – 2006. – Т. 141. – С. 163 – 176. doi: 10.1007/s10704-006-0071-x.
5. *Yang W., Suo Z., Shih C. F.* Mechanics of dynamic debonding // Proceedings of the Royal Society London A. – 1991. – Т. 433. – С. 679 – 697. doi: 10.1098/rspa.1991.0070.
6. *Hutchinson J. W., Suo Z.* Mixed mode cracking in layered materials // Journal of Applied Mechanics. – 1992. – Т. 29. – С. 63 – 191. doi: 10.1016/S0065-2156(08)70164-9.
7. *Wang T. C., Shih C. F., Suo Z.* Crack extension and kinking in laminates and bicrystals // International Journal of Solids and Structures. – 1992. – Т. 29. – С. 327 – 344. doi: 10.1016/0020-7683(92)90203-6.
8. *Atluri S. N.* Computational methods in the mechanics of fracture. – New York, Elsevier Science Ltd., 1986. – 424 c.
9. *Smelser R. E.* Evaluation of stress intensity factors for bimaterial bodies using numerical crack flank displacement data // International Journal of Fracture. – 1979. – Т. 15. – № 2. – С. 135 – 142. doi: 10.1007/BF00037829.
10. *Matos P. P. L., McMeeking R. M., Charalambides P. G., Drory M. D.* A method for calculating stress intensities in bimaterial fracture // International Journal of Fracture. – 1989. – Т. 40. – С. 235 – 254. doi: 10.1007/BF00963659.
11. *ABAQUS User's manual Version 2016*. – Dassault Systémes Simulia Corp., 2016.
12. *Бурлаєнко В. М., Морачковський О. К.* Застосування кінематичного контактного алгоритму з явною схемою інтегрування за часом у скінченноелементних задачах динаміки тіла з тріщиною // Вісник НТУ «ХПІ». Сер. : Математичне моделювання в техніці та технологіях. – 2012. – № 54. – Вип. 960. – С. 19 – 25.
13. *Бурлаєнко В. М., Морачковський О. К.* Скінченноелементна модель нелінійної динаміки тіла з тріщиною, границі якої контактують // Вісник НТУ «ХПІ». Сер. : Математичне моделювання в техніці та технологіях. – 2012. – № 2. – С. 44 – 51.
14. *Adams D. O., Kessler J. A., Kuramoto B., Bluth J., Weaver C., Gill A.* Development and evaluation of fracture mechanics test methods for sandwich composites // Proceedings of the FAA JAMS Technical Review Meeting (July 21–22, 2010 p., USA). – Wichita : KS, 2010. – 29 c.

References (transliterated)

1. Burlayenko V. N., Sadowski T. Linear and nonlinear dynamic analyses of sandwich panels with face sheet-to-core debonding. *Shock and Vibration*. 2018, vol. 2018, 26 p. doi: 10.1155/2018/5715863.
2. Szekrényes A. *Delamination of composite specimens* : Ph.D. thesis. Hungary, 2005. 130 p.
3. Shivakumar K. N., Smith S. A. In situ fracture toughness testing of core materials in sandwich panels. *Journal of Composite Materials.* 2004, vol. 38, pp. 655–658. doi: 10.1177/0021998304042392.
4. Sorensen B. F., Jorgensen K., Jacobsen T. K., Ostregaard R. C. DCB-specimen loaded with uneven bending moments. *International Journal of Fracture*. 2006, vol. 141, pp. 163–176. doi: 10.1007/s10704-006-0071-x.
5. Yang W., Suo Z., Shih C. F. Mechanics of dynamic debonding. *Proceedings of the Royal Society London A*. 1991, vol. 433, pp. 679–697. doi: 10.1098/rspa.1991.0070.
6. Hutchinson J. W., Suo Z. Mixed mode cracking in layered materials. *Journal of Applied Mechanics*. 1992, vol. 29, pp. 63–191. doi: 10.1016/S0065-2156(08)70164-9.
7. Wang T. C., Shih C. F., Suo Z. Crack extension and kinking in laminates and bicrystals. *International Journal of Solids and Structures*. 1992, vol. 29. pp. 327–344. doi: 10.1016/0020-7683(92)90203-6.
8. Atluri S. N. *Computational methods in the mechanics of fracture*. New York, Elsevier Science Ltd., 1986, 424 p.
9. Smelser R. E. Evaluation of stress intensity factors for bimaterial bodies using numerical crack flank displacement data. *International Journal of Fracture.* 1979, vol. 15, no. 2, pp. 135–142. doi: 10.1007/BF00037829.
10. Matos P. P. L., McMeeking R. M., Charalambides P. G., Drory M. D. A method for calculating stress intensities in bimaterial fracture. *International Journal of Fracture*. 1989, vol. 40, pp. 235–254. doi: 10.1007/BF00963659.
11. *ABAQUS User's manual Version 2016*. Dassault Systémes Simulia Corp., 2016.
12. Burlayenko V. N., Morachkovskiy O. K. Zastosuvannya kinematychnogo kontaktnogo algorytmu z yavnoyu skkhemoyu integruvannya za chasom u skinchennoelementnykh zadachakh dynamiky tila z trishchynoyu [An application of a kinematic contact algorithm within an explicit time-integration scheme in finite element problems for dynamics of a body with a crack]. *Visnyk NTU "KhPI". Seriya: Matematychne modelyuvannya v tekhnitsi ta tekhnolohiyakh* [Bulletin of NTU "KhPI". Series: Mathematical modelling in engineering and technology]. 2012, no. 54, issue 960, pp. 19–25.
13. Burlayenko V. N., Morachkovskiy O. K. Skinchennoelementna model' neliniynoyi dynamiky tila z trishchynoyu, granytsi yakoyi kontaktuyut'. [Finite element model of nonlinear dynamics of a body with a crack whose boundaries are in contact]. *Visnyk NTU "KhPI". Seriya: Matematychne modelyuvannya v tekhnitsi ta tekhnolohiyakh* [Bulletin of NTU "KhPI". Series: Mathematical modelling in engineering and technology]. 2012, no. 2, pp. 44–51.
14. Adams D. O., Kessler J. A., Kuramoto B., Bluth J., Weaver C., Gill A. *Development and evaluation of fracture mechanics test methods for sandwich composites*. Proceedings of the FAA JAMS Technical Review Meeting. Wichita, KS, 2010. 29 p.

Received (надійшла) 16. 02.2018

Відомості про авторів / Сведения об авторах / Information about authors

Burlayenko Vyacheslav Nikolayevich (Бурлаєнко В'ячеслав Миколайович, Бурлаенко Вячеслав Николаевич) – кандидат технічних наук, доцент, Національний технічний університет «Харківський політехнічний інститут», м. Харків, Люблінський технологічний університет, м. Люблін; тел.: (057) 707-60-33; e-mail: burlayenko@yahoo.com.

Sadowski Tomasz (Садовський Томаш, Садовский Томаш) **–** доктор технічних наук, професор, Люблінський технологічний університет, м. Люблін; тел.: +48 (81) 53-84-386; e-mail: t.sadowslki@pollub.pl.

Pietras Daniel (Петрас Даніель, Петрас Даниэль) – кандидат технічних наук, асистент, Люблінський технологічний університет, м. Люблін; тел.: +48 (81) 53-84-449; e-mail: d.pietras@pollub.pl.