Nerlovian Revenue Inefficiency in a Bank Production context: Evidence from Shinkin Banks

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Abstract: The paper further advances the contemporary methodological and empirical research on bank efficiency. We introduce a bank two-stage network revenue decomposition analysis based on the slack-based directional inefficiency measurement framework, by building upon the approaches of Epure and Lafuente (2015), Fukuyama and Weber (2009, 2010), Fukuyama and Matousek (2017) and Färe, Fukuyama, Grosskopf and Zelenyuk (2015, 2016). We apply the model on the segment of Japanese financial institutions – *Shinkin* banks. We examine the efficiency performance of *Shinkin* banks during the period from March 2007 to March 2015. This covers the recent years after the turmoil in the US subprime residential mortgage market which started early 2007. The applied innovative framework is particularly suitable for this type of banks since the financing of their business activities is mainly dependent on the different types of deposits that the proposed model takes into consideration.

Keywords: Data envelopment analysis (DEA); revenue efficiency; directional distance function; slack-based technical inefficiency, banking
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1. Introduction

The Japanese banking system has been under severe financial distress for almost three decades. The initial financial crisis can be dated back to the early 1990s. The peak of the crisis occurred in the late 1990s when several banks failed or were on the brink of closure. These significant structural changes and ongoing underperformance of the Japanese banking system have become an interesting subject for empirical and methodological research. The systemic crisis in the segment of the large city banks spilt over to Regional Banks and *Shinkin* Banks (Credit Cooperatives). As Assaf et al. (2011) argue, this group of the Japanese banks has not been the focus of recent banking studies even though *Shinkin* banks have a specific and important position in the regional financial markets (Satake and Tsutsui, 2002; Hosono et al., 2006). *Shinkin* banks are key financial intermediaries for small and medium-sized companies and households in Japan. The restricted number of empirical studies on *Shinkin* banks is mainly because of the availability of financial data for this type of analysis.

Shinkin banks are cooperative financial institutions that have a special place in the Japanese financial market. They provide financial services to local communities through supporting the business activities within the individual prefectures. The main distinctive features compared to other banks are that the membership is limited to small and medium companies and individuals residing in a determined geographical area. There are also restrictions in terms of their geographical activities. *Shinkin* banks can provide financial services (loanable funds) only to small and medium-sized enterprises that do the business activities within the geographical service area can borrow funds. Despite their small size, they are important players on deposit market. The deposits outstanding at 267 *Shinkin* banks achieved $1,244 bn. The protracted economic downturn forces *Shinkin* banks to search for other business activities. They become important financial intermediaries in financing the regional infrastructure projects. The business activities of *Shinkin* banks have also been affected by lack of capital and the cumulated non-performing loans (NPLs). These specific characteristics of *Shinkin* banks are important determinants for being selected for our specifically designed model that is underpinned by the assumption that bank deposits play an important role in bank activities.

Our study aims to further expand methodological and empirical research on bank efficiency in general and the Japanese banking system in particular. We advance the current research in several ways. The paper contributes to contemporary research on bank efficiency in general by introducing a bank network revenue decomposition analysis based on the directional slack-based inefficiency measurement framework. The present study develops an additive revenue efficiency decomposition that is consistent to the strongly efficient output-frontier with a network structure. Our decomposition allows us to directly compare the decomposition presented by Fukuyama and Matousek (2017).

This new model is underpinned by studies published by Fukuyama and Weber (2009, 2010), Fukuyama and Matousek (2017) and Färe, Fukuyama, Grosskopf and Zelenyuk (2015, 2016). The main difference from the previous studies notably by Fukuyama and Weber (2013, 2015, 2017, 2016) and Fukuyama and Matousek (2017) is that they use a directional distance framework for the efficiency measurement. In this case the output technical efficiency is computed relative to the weakly efficient output-frontier in the directional distance framework. By contrast, in the present paper we compute the output technical efficiency relative to the Pareto-Koopmans (strongly efficient) output-frontier. Therefore, we do not overestimate output technical efficiency. Our slack-based measure allows for non-proportional movement. In the case of the directional distance function, if all the inputs and/or outputs cannot be improved proportionately along the predetermined directional vector then the bank is on the weak or strong efficient frontier. However, in our model as far as we have slack we can improve bank efficiency.

Furthermore, we do not employ a dynamic setting, even though it is an important future research topic. This is because we would currently face the problem of deciding the time span before carrying out the dynamic optimization problem where the choice of the span affects the optimal path and hence efficiency estimates. Next, it would require to define the dynamic technologies, which impose various theoretical and empirical considerations. For a further discussion see, e.g., Emrouznejad and Thanassoulis (2005), Fallah-Fini, Triantis, and Johnson (2014), Fukuyama and Weber (2015, 2016, 2017) and Fukuyama and Matousek (2017). We focus on the static setting in this study.

As for the empirical contribution, we apply our method to study the efficiency performance *Shinkin* banks during the period from March 2007 to March 2015, which correspond to fiscal years[1] (FY) 2006 and 2014. This covers the recent years after the turmoil in the US subprime residential mortgage market which started early 2007. The applied

[1] Japan's fiscal year starts on the first day of April and end at the last day of March, and hence the fiscal year differs from the calendar year.

innovative framework is particularly suitable for this type of banks since the financing of their business activities is mainly dependent on the different types of deposits that the proposed model takes into consideration.

The structure of the paper is as follows: Section 2 overviews empirical and methodological research on bank efficiency with the special focus on the Japanese banking sector. Section 3 explains the methodological concept of the develop model. Section 4 analyses and discuss the obtained results. Section 5 then concludes.

2. Literature Review

2.1. Empirical Research Bank Efficiency in Japan: An Overview

There has been an extensive empirical research on Japanese banking. Both empirical and methodological studies on the performance of Japanese banks try to capture their specific characteristics. In addition, the research studies also reflect on the deteriorated economic situation in Japan and the impact on bank performance. Research sheds light on the important policy oriented questions that include impact of bank size on efficiency, impact of consolidation activities on efficiency, Mergers and Acquisitions (M&As) and bank efficiency. Namely these studies explored how bad assets affect bank performance. Pioneering studies on Japanese bank efficiency include Fukuyama’s (1993, 1995) papers that show that the Japanese banks underperform due to technical inefficiencies. Altunbas et al. (2000) confirms that NPLs had a deteriorated effect on bank performance that was measured by efficiency scores. A further study by Drake and Hall (2003) conclude that bank efficiency cannot be improved without resolving the accumulated NPLs on bank balance sheets. Fukuyama and Weber (2008b) in their methodological paper clearly show that NPLs should be considered in studies that analyze bank efficiency in Japan. They explain that NPLs should be a by-product of loan production. Therefore, bad loans should be treated as an undesirable output. Barros et al. (2012) confirm previous findings and provide further evidence that NPLs are a key factor that affects bank efficiency and these findings have important implications for policy makers.

The recent studies by Fukuyama and Weber (2013, 2015) develop a dynamic two-stage network model of the production process. They propose that in the first stage of bank production process banks use three inputs (labour, physical capital, and equity capital). These inputs serve to produce intermediate outputs that are deposits and other raised funds. Finally, banks produce outputs: loans and securities. The model proposes NPLs as an additional so-called undesirable output. The applied dynamic framework has the advantages of allowing to allocate inputs through the evaluated period. That enables banks to maximize the volume of desirable outputs. At the same time, it minimizes the production of NPLs in this particular case. Fukuyama and Weber (2017) then develop a dynamic network Luenberger productivity indicator.

Despite the fact *Shinkin* banks are important financial institutions within the Japanese financial markets, there are a limited number of studies on productivity and efficiency that explore their performance. Fukuyama (1996) investigates the technical and scale efficiency of credit banks and Fukuyama et al. (1999) then examine *Shinkin* efficiency and productivity growth during the period from 1992 to 1996. Hosono et al. (2006) overview the rational and the impact of the consolidation programme applied for *Shinkin* banks. They look at the performance measures such as bank profitability and efficiency of those banks that merged during the investigated period that spans from 1984 to 2002). Fukuyama and Weber (2008b, 2009) analyze differences in the organizational structure of *Shinkin* banks and Regional Banks I and II. Their research objective was to identify whether the organizational forms affect technical efficiency. The paper also contributes to contemporary methodological issues by testing whether slack causes bank inefficiency. They conclude that Regional banks are less efficient than *Shinkin* banks. Their paper also indicates that banks with larger volume of NPLs are less efficient. Barros et al. (2011) explore the productivity and efficiency of *Shinkin* banks in Japan, over the period from 2000 to 2006. Their findings show that the efficiency growth and productivity growth of *Shinkin* banks did not improve over the analyzed period.

2.2. Methodological issues

A brief overview about current empirical research studies on bank efficiency that apply DEA indicates that these studies are predominantly focused on bank technical efficiency and allocative efficiency. Fethi and Pasourias (2010) further point out that there is a lack of studies that analyze profit and/or revenue efficiency using the DEA. Fukuyama and Matousek (2017) highlight the potential reasons behind the lack of studies that consider profit/revenue. Those include a shortage of a good quality of output prices but most importantly to resolve the methodological issue linked with decomposing profit efficiency into technical efficiency and allocative efficiency. A further challenge that can be observed in current methodological studies is to model true bank business activities. That means to construct a model that adequately defines the use of inputs and outputs. Most empirical papers apply the intermediation approach.

A number of recent research studies propose to apply a two-stage approach in which deposits are used as an intermediary output. These studies include Fukuyama and Weber (2010), Fukuyama and Matousek (2011) and Holod and Lewis (2011) including the most recent paper by Fukuyama and Matousek (2017). The application of these models is rapidly advancing. Fukuyama and Weber (2005), for example, propose an indirect production approach to examine indirect input allocative efficiency. Liu and Tone (2008) propose a three-stage non-parametric approach to estimate bank efficiency. Drake et al. (2009) argue that the application of different methodological approaches give different results. The most recent study by Pham and Zelenyuk (2018) extends the slack-based directional distance function introduced by Färe and Grosskopf (2010). They measure efficiency in the presence of bad outputs and illustrate it by an application to data on Vietnamese commercial banks.

In the case of Japan, several studies apply a dynamic network structure at a bank level. As stated in Section 3.1, Fukuyama and Weber (2013, 2015, 2017) introduced a dynamic network DEA framework into Japanese banking efficiency research. Fukuyama and Weber's (2013, 2015, 2017) framework builds on the two stage static US commercial bank efficiency model of Seiford and Zhu[2] (1998) by incorporating nonperforming loans and carryover. While Seiford and Zhu's (1998) two stages are profitability and marketability, Fukuyama and Weber's (2013, 2015, 2017) two-stage framework, which the present study adopts, is that a bank converts the exogenous inputs such as labor and physical capital to generate deposits in stage 1 and utilizes the deposits to produce loans and securities investments with some portion of the amount of loans becoming nonperforming.

In Fukuyama and Weber (2013, 2015, 2017), the dynamic aspect is implemented using carryover assets, which are calculated using time deposits etc. The present static model does not require this variable but requires the prices of outputs because we estimate the bank revenue function directly. The good output prices are calculated like Fukuyama and Weber (2008a, b). Regarding the price of nonperforming loans, we follow Fukuyama and Matousek

[2] For some extensions and further results of this two stage model, see Liang, Cook, and Zhu (2008) and Chen, Cook and Zhu (2010).

(2017) and define the price of the output as the ratio of the reserves of possible loan losses to the amount of nonperforming loans.

2.3. Gaps in the Literature and the Proposed Solutions

It is important to emphasize that while we exploit various models documented in the literature for our estimation, the most direct references are Fukuyama and Matousek (2017) and Färe et al. (2015, 2016). Fukuyama and Matousek (2017) introduced a bank revenue function by estimating the price of nonperforming loans and the corresponding additive Nerlovian revenue (in)efficiency decomposition, one of whose components is the technical inefficiency computed relative to a weakly (not strongly) efficient output-frontier. We identify several gaps in the most recent literature. In particular we identify the limitation in Fukuyama and Matousek (2017) that introduce a Nerlovian revenue inefficiency decomposition analysis with both directly and indirectly linked variables. The technical inefficiency is measured relative to the weakly efficient output-frontier. The key limitation of Fukuyama and Matousek's (2017) model is due to the use of the weakly efficient output-frontier, which means that possible existing slacks are not incorporated in the decomposition and the technical efficiency tends to be overestimated.

In order to cope with this limitation, we turn to the strongly efficient frontier-based multiplicative decompositions by Färe et al. (2015, 2016). Since the Nerlovian revenue inefficiency decomposition by Fukuyama and Matousek is additive with the use of directional output distance function, we develop an additive adaptation of Färe et al.'s (2015, 2016) multiplicative decomposition. The decomposition proposed in this paper has primarily three advantages and desired characteristics compared to those of Fukuyama and Matousek (2017) and Färe at al. (2015, 2016).

First, the decomposition uses the slack-based inefficiency measure (rather than the directional output distance function) and hence it does not overestimate the technical efficiency. Second, our decomposition is composed of the slack-based technical and allocative inefficiency components that are expressed with respect to comparable value terms (e.g., dollars), and the two inefficiency components are compared directly. Third, while the decision making units with a zero slack-based inefficiency score (even if there exists allocative inefficiency) are not used in Färe et al's (2015,16) decomposition method, such DMUs are incorporated in the proposed decomposition. Furthermore Fukuyama and Matousek (2017) chose average values but the present paper employed the price dependent directional vector.

3. Notation and Methodology

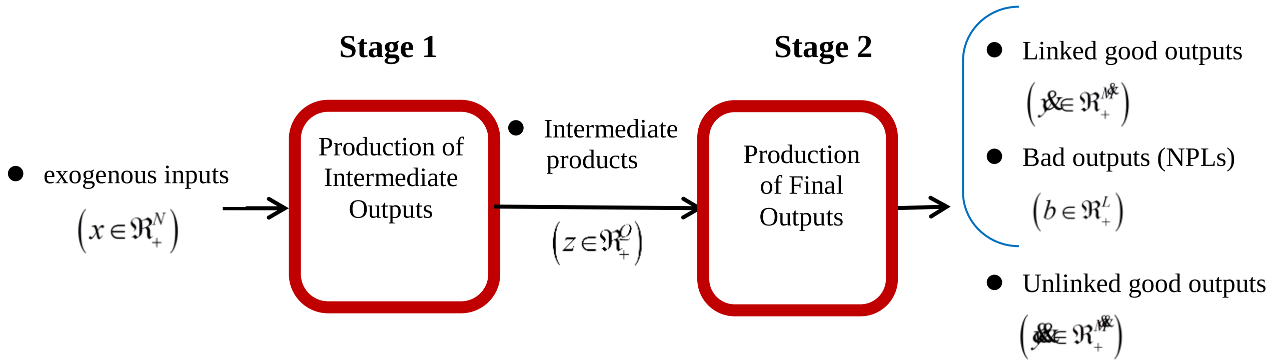
3.1. Basics: Directional Distance Function and Nerlovian Inefficiency decomposition

We start with a two-stage network DEA (data envelopment analysis) framework with two types of good outputs: good outputs linked directly and good outputs unlinked to bad outputs. Network DEA due to Färe and Grosskopf (1996, 2000) is a sophisticated extension of the traditional black-box DEA model originating from Charnes, Cooper and Rhodes (1978). Let $x \in \Re_+^N$ denote an N-dimensional input column vector, $z \in \Re_+^Q$ a Q-dimensional intermediate product column vector, and $b \in \Re_+^L$ an L-dimensional bad output vector, where $\Re_+^N$ is the nonnegative orthant of N-dimensional column vector. Linked good outputs, $\dot{y} \in \Re_+^{\dot{M}}$, and unlinked good outputs, $\ddot{y} \in \Re_+^{\ddot{M}}$, are, respectively, $\dot{M}-$ and $\ddot{M}-$ dimensional column vectors. That is, each $\dot{y}_{\dot{m}}$ $(\dot{m} = 1,...,\dot{M})$ is directly linked to its corresponding to b_l $(l = 1,...,L)$ but $\ddot{y}_{\ddot{m}}$ $(\ddot{m} = 1,...,\ddot{M})$ is not directly linked to any bad outputs. The term "linked" indicates that bad outputs must be increased in order to increase the linked good outputs. See Epure and Lafuente (2015), Fukuyama and Weber (2017) and Fukuyama and Matousek (2017) for linked and unlinked good and bad outputs in banking efficiency settings. The first and second stage sub-technologies are, respectively, defined as

$$T^1 = \left\{(x,z) \in \Re_+^N \times \Re_+^Q \mid z \text{ is produced from } x\right\} \quad \text{and}$$
$$T^2 = \left\{(z,\dot{y},\ddot{y},b) \in \Re_+^Q \times \Re_+^{\dot{M}} \times \Re_+^{\ddot{M}} \times \Re_+^L \mid (\dot{y},\ddot{y},b) \text{ is produced from } z\right\}. \tag{1}$$

Figure 1 visualizes a two-stage network bank production process with good outputs linked and unlinked to bad outputs (notably nonperforming loans).

Figure 1: Two-Stage network Process



Note: NPLs: nonperforming loans.

Combining two sub-technologies, T^1 and T^2, a bank network technology is represented by the bank production possibility set defined as

$$T = \left\{ (x, \tilde{y}, \tilde{\tilde{y}}, b) \in \Re_+^N \times \Re_+^{\tilde{M}} \times \Re_+^{\tilde{\tilde{M}}} \times \Re_+^L \,\middle|\, (x, z) \in T^1 \text{ and } (z, \tilde{y}, \tilde{\tilde{y}}, b) \in T^2 \right\} \qquad (2)$$

where we assume that intermediate products are endogenously determined. See, among others, Kao and Hwang (2008), Tone and Tsutsui (2009) and Lozano, Gutierrez and Moreno (2012) for DEA models which treat intermediate products as endogenous. The bank production possibility set T is assumed to be a nonempty, closed set satisfying no free lunch. Moreover, the output possibility set, denoted as

$$P(x) = \left\{ (\tilde{y}, \tilde{\tilde{y}}, b) \in \Re_+^{\tilde{M}} \times \Re_+^{\tilde{\tilde{M}}} \times \Re_+^L \,\middle|\, (x, \tilde{y}, \tilde{\tilde{y}}, b) \in T \right\}, \quad x \in \Re_+^N \qquad (3)$$

is assumed to be bounded for x, where

$$(x, \tilde{y}, \tilde{\tilde{y}}, b) \in T \quad \Leftrightarrow \quad (\tilde{y}, \tilde{\tilde{y}}, b) \in P(x), \quad x \in \Re_+^N .$$

The output possibility set is an alternative of production possibility set (2). We also assume strong (free) disposability of inputs (SD^x) and strong disposability of good outputs (SD^y):

$$\begin{aligned} &\mathrm{SD}^x: \quad x' \geq x \in \Re_+^N \quad \Rightarrow \quad P(x) \subseteq P(x') \\ &\mathrm{SD}^{\tilde{y}\tilde{\tilde{y}}}: \quad (\tilde{y}', \tilde{\tilde{y}}') \leq (\tilde{y}, \tilde{\tilde{y}}) \in \Re_+^{\tilde{M}+\tilde{\tilde{M}}} \text{ and } (\tilde{y}, \tilde{\tilde{y}}, b) \in P(x) \text{ for } x \in \Re_+^N \quad \Rightarrow \quad (\tilde{y}', \tilde{\tilde{y}}', b) \in P(x). \end{aligned} \qquad (4)$$

The SD^x property shows that if the inputs are increased from x to x', then the resulting bank output possibility set $P(x')$ is not smaller than the original bank output possibility set $P(x)$. The $\mathrm{SD}^{\tilde{y}\tilde{\tilde{y}}}$ property states that, for a given level of exogenous inputs and bad outputs, if a vector of linked and unlinked good outputs is technologically feasible, then any vectors smaller than the original vector is also feasible. Furthermore, we assume that joint weak disposability property of linked good outputs and bad outputs, $\text{JWD}^{\dot{y}b}$, as follows:

$$\text{JWD}^{\dot{y}b}: \quad 1 \geq \theta \geq 0 \text{ and } (\dot{y}, \ddot{y}, b) \in P(x) \quad \Rightarrow \quad (\theta \dot{y}, \ddot{y}, \theta b) \in P(x). \tag{5}$$

The $\text{JWD}^{\dot{y}b}$ property states that, if the observed linked good outputs and bad outputs are feasible then the proportional reduction of these vectors is still feasible. The property implements the idea that the reduction of observed bad outputs is costly with respect to linked good outputs. Denoting $\mathbf{g} = \left(\dot{g}, \ddot{g}, g \right) \in \Re_{+}^{\dot{M}} \times \Re_{+}^{\ddot{M}} \times \Re_{+}^{L}$ a vector of pre-assigned nonnegative direction, the bank directional output distance function is defined as

$$\vec{D}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = \sup_{\beta} \left\{ \beta \,\middle|\, (\dot{y} + \beta \dot{g}, \ddot{y} + \beta \ddot{g}, b - \beta g) \in P(x) \right\}, \ x \in \Re_{+}^{N}. \tag{6}$$

For various directional distance functions, see Luenberger (1995, 1992) and Chambers et al. (1996, 1998). Suppose the inner products, $\dot{p}\dot{y}$ and $\ddot{p}\ddot{y}$, represent the revenues from linked and unlinked good outputs, respectively, where $\dot{p} = (\dot{p}_1, \dot{p}_2, ..., \dot{p}_{\dot{M}}) \in \Re_{++}^{\dot{M}}$ and $\ddot{p} = (\ddot{p}_1, \ddot{p}_2, ..., \ddot{p}_{\ddot{M}}) \in \Re_{++}^{\ddot{M}}$ are positive price row vectors[3]. Similarly, $v \in \Re_{++}^{\dot{M}}$ is a positive bad output price row vector and the inner product vb represents the cost associated with bad outputs. Using (3) as the technology, the bank revenue function is denoted as

$$R(x, \dot{p}, \ddot{p}, v) = \max_{\dot{y}, \ddot{y}, b} \left\{ \dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \,\middle|\, (\dot{y}, \ddot{y}, b) \in P(x) \right\}, \ (x, \dot{p}, \ddot{p}, v) \in \Re_{+}^{N} \times \Re_{++}^{\dot{M}} \times \Re_{++}^{\ddot{M}} \tag{7}$$

where the objective $\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb$ of (7) is interpreted as an effective net benefit and can be thought of as a bank's revenue function extension of Färe, Grosskopf and Weber's (2006) environmental (ecological) revenue function. See Fukuyama and Matousek (2017). Due to the dual relationship[4] between the bank directional output distance function and the bank revenue function, the latter function (7) can be derived from the bank directional output distance function by optimally selecting the values of $\dot{y}$, $\ddot{y}$ and b, as

$$R(x, \dot{p}, \ddot{p}, v) = \sup_{\dot{y} \geq 0, \ddot{y} \geq 0, b \geq 0} \left\{ \dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \,\middle|\, \vec{D}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) \geq 0 \right\}. \tag{8}$$

A bank's revenue function, which uses nonperforming loans as a bad output, was given in

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[3] Throughout this paper quantities and prices are shown in column and row vectors, respectively.

[4] For standard duality theory between the cost and the radial input distance function, see Shephard (1953) and Färe and Primont (1995).

Fukuyama and Weber (2008a, 2008b). Duality theory based on the directional output distance function also tells us that Equation (6) can be recovered from the bank revenue function by choosing the optimal values of $\dot{p}$, $\ddot{p}$ and v, as follows:

$$\vec{D}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = \inf_{\dot{p}\geq 0, \ddot{p}\geq 0, v\geq 0} \left\{ \frac{R(x, \dot{p}, \ddot{p}, v) - \left(\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \right)}{\dot{p}\dot{g} + \ddot{p}\ddot{g} + vg} \right\}, \quad \dot{p}\dot{g} + \ddot{p}\ddot{g} + vg > 0 \tag{9}$$

The term $\dot{p}\dot{g} + \ddot{p}\ddot{g} + vg$ is a normalization to be used to connect the revenue inefficiency and the bank directional output distance function. Note that if the maximization problem (8) is formulated as the Lagrangian problem, then the normalization factor turns out to be the Lagrange multiplier. Equation (9) implies that

$$NRI \geq \vec{D}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) \tag{10}$$

where

$$NRI = \frac{R(x, \dot{p}, \ddot{p}, v) - \left(\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \right)}{\dot{p}\dot{g} + \ddot{p}\ddot{g} + vg} \tag{11}$$

is the bank's Nerlovian revenue inefficiency. Defining the difference between the left-hand-side and the right-hand-side of (10) as the directional allocative inefficiency DAI, we obtain the following revenue inefficiency decomposition:

$$NRI = DAI + DTI\,. \tag{12}$$

where $DTI = \vec{D}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g})$ is technical inefficiency which indicates how far the observation $(\dot{y}, \ddot{y}, b)$ can be projected along the direction $\mathbf{g}$ to the output boundary of $P(x, z)$. DAI is a bank's output allocative inefficiency version of Balk, Färe and Grosskopf's (2004) input-oriented directional allocative inefficiency measure. Fukuyama and Weber (2008c) provided a Luenberger productivity indicator extension of Balk et al.'s (2004) directional allocative inefficiency measure.

However, DAI of Equation (12) may include positive output slacks because DTI is the radial measure obtained relative to the translated origin. To deal with this limitation, we consider a slack-based allocative inefficiency measure in the next section. For the standard black-box Nerlovian revenue inefficiency measure, see Färe and Grosskopf (2004).

3.2 Multiplicative decomposition: Slack-based decomposition and allocative inefficiency

Adapting the non-oriented slack-based inefficiency measure[5] (Fukuyama and Weber 2009), we define an output-oriented slack-based inefficiency measure as

$$\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = \max_{\dot{s} \geq 0,\, \ddot{s} \geq 0,\, s \geq 0} \left\{ \frac{1}{3}\left(\frac{1}{\dot{M}} \sum_{\dot{m}=1}^{\dot{M}} \frac{\dot{s}_{\dot{m}}}{\dot{g}_{\dot{m}}} + \frac{1}{\ddot{M}} \sum_{\ddot{m}=1}^{\ddot{M}} \frac{\ddot{s}_{\ddot{m}}}{\ddot{g}_{\ddot{m}}} + \frac{1}{L} \sum_{l=1}^{L} \frac{s_l}{g_l} \right) \middle| \begin{pmatrix} \dot{y} + \dot{s} \\ \ddot{y} + \ddot{s} \\ b - s \end{pmatrix} \in P(x) \right\}. \quad (13)$$

If $(\dot{y}, \ddot{y}, b) \notin P(x)$, then $\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = -\infty$. The objective of (13) maximizes the average of three slack components that consist of the average linked good output shortfalls, the average unlinked good output shortfalls and the average bad output surpluses. Note that the number "3" appears in (13) because we have three kinds of output slacks, which correspond to the kinds of slacks associated with linked outputs, unlinked outputs and bad outputs. For $(\dot{y}, \ddot{y}, b) \in P(x, z)$, we have the indication property (Russell 1998):

$$\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = 0 \quad \text{if and only if} \quad (\dot{y}, \ddot{y}, b) \in EF(P(x))$$

where $EF(P(x))$ is the strongly efficient output-frontier of $P(x)$. Since $(\dot{y} + \dot{s}^*, \ddot{y} + \ddot{s}^*, b - s^*) \in P(x)$, we have the revenue inequality

$$R(x, \dot{p}, \ddot{p}, v) \geq \dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \quad \text{for all } (\dot{y}, \ddot{y}, b) \in P(x)$$

yielding

$$R(x, \dot{p}, \ddot{p}, v) - \left(\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb \right) \geq \dot{p}\dot{s}^* + \ddot{p}\ddot{s}^* + vs^*. \quad (14)$$

Now assume that $(\dot{y}, \ddot{y}, b) \notin EF(P(x))$. Then we have

$$\begin{aligned} &\dot{p}\dot{s}^* + \ddot{p}\ddot{s}^* + vs^* \\ &= \vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) \times \left(\dot{p}\dot{g} + \ddot{p}\ddot{g} + vg \right) \times \frac{\sum_{\dot{m}=1}^{\dot{M}} \dot{\alpha}_{\dot{m}} \dot{s}^*_{\dot{m}} + \sum_{\ddot{m}=1}^{\ddot{M}} \ddot{\alpha}_{\ddot{m}} \ddot{s}^*_{\ddot{m}} + \sum_{l=1}^{L} \alpha_l s^*_l}{\underbrace{\frac{1}{3}\left(\frac{1}{\dot{M}} \sum_{\dot{m}=1}^{\dot{M}} \frac{\dot{s}^*_{\dot{m}}}{\dot{g}_{\dot{m}}} + \frac{1}{\ddot{M}} \sum_{\ddot{m}=1}^{\ddot{M}} \frac{\ddot{s}^*_{\ddot{m}}}{\ddot{g}_{\ddot{m}}} + \frac{1}{L} \sum_{l=1}^{L} \frac{s^*_l}{g_l} \right)}_{\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g})}} \end{aligned} \quad (15)$$

where

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[5] While (13) can be thought of as a weighted additive model (Charnes et al. 1985), the formulation is based on a directional distance function framework or equivalently the graph version of Russell measures of Färe and Lovell 1978. See Fukuyama and Weber (2009) for the relationships among various (in)efficiency measures.

$$\tilde{\alpha}_{\tilde{m}} = \frac{\tilde{p}_{\tilde{m}}}{\tilde{p}\tilde{g} + \hat{p}\hat{g} + vg}, \quad \hat{\alpha}_{\hat{m}} = \frac{\hat{p}_{\hat{m}}}{\tilde{p}\tilde{g} + \hat{p}\hat{g} + vg}, \quad \alpha_l = \frac{v_l}{\tilde{p}\tilde{g} + \hat{p}\hat{g} + vg}. \tag{16}$$

which are the weighted price shares of linked good outputs, unlinked good outputs and bad outputs, respectively. Note that $\sum_{\tilde{m}=1}^{\tilde{M}} \tilde{\alpha}_{\tilde{m}} \tilde{g}_{\tilde{m}} + \sum_{\hat{m}=1}^{\hat{M}} \hat{\alpha}_{\hat{m}} \hat{g}_{\hat{m}} + \sum_{l=1}^{L} \alpha_l g_l = 1$. Considering (15) and using (13), we define slack-based technical inefficiency (SBTI) as

$$SBTI = \vec{S}_O(x, \tilde{y}, \hat{y}, b; \mathbf{g}) \times \overbrace{\left(\frac{\sum_{\tilde{m}=1}^{\tilde{M}} \tilde{\alpha}_{\tilde{m}} \tilde{s}_{\tilde{m}}^{*} + \sum_{\hat{m}=1}^{\hat{M}} \hat{\alpha}_{\hat{m}} \hat{s}_{\hat{m}}^{*} + \sum_{l=1}^{L} \alpha_l s_l^{*}}{\frac{1}{3}\left(\frac{1}{\tilde{M}} \sum_{\tilde{m}=1}^{\tilde{M}} \frac{\tilde{s}_{\tilde{m}}^{*}}{\tilde{g}_{\tilde{m}}} + \frac{1}{\hat{M}} \sum_{\hat{m}=1}^{M} \frac{\hat{s}_{\hat{m}}^{*}}{\hat{g}_{\hat{m}}} + \frac{1}{L} \sum_{l=1}^{L} \frac{s_l^{*}}{g_l} \right)} \right)}^{\text{adjustment factor}}. \tag{17}$$

Using (15) and closing the inequality gap in (14), we define the slack-based allocative inefficiency *SBAI* as

$$SBAI = NRI - SBTI \tag{18}$$

Consequently, we obtain the following additive slack-based decomposition:

$$NRI = SBTI + SBAI\ . \tag{19}$$

which will be used for our empirical analysis. Alternative to (18), we can define the multiplicative slack-based allocative inefficiency *MSBAI* as

$$MSBAI = NRI \times \left(SBTI\right)^{-1} \tag{20}$$

whose corresponding multiplicative slack-based decomposition is expressed as follows:

$$NRI = SBTI \times MSBAI\ . \tag{21}$$

Slack-based decomposition (21), which differs from the conventional Farrell efficiency-based decomposition of revenue efficiency, was developed by Färe et al. (2015, 2016) for the profit and cost inefficiency case where the directional vector is the vector of ones[6]. Our slack-based decompositions[7] (19) and (21) are more general in the sense that the directional vector is not defined as a unitary vector. For a technically inefficient bank, the decomposition based on SBI can be used to identify the sources of slack-based revenue

[6] Note that if we not only delete bad outputs and have only one kind of outputs but we also set $\left(\tilde{g}, \hat{g}\right) = \left(1/(2\tilde{M}), 1/(2\hat{M})\right)$ where $M = \tilde{M} + \hat{M}$, then we obtain the slack-based revenue inefficiency decomposition version of Fare et al. (2015, 2016).

[7] An alternative decomposition was provided by Cooper, Pastor, Aparicio and Borras (2011) based on an additive Fenchel-Mahler inequality.

inefficiency.

For our empirical analysis, we employ the following direction:

$$\mathbf{g} = (g_{\dot{y}}, g_{\ddot{y}}, g) = \left(\frac{1}{3\dot{M}\dot{p}}, \frac{1}{3\ddot{M}\ddot{p}}, \frac{1}{3Lv} \right), \text{ where} \tag{22}$$

$$g_{\dot{y}} = \frac{1}{3\dot{M}\dot{p}} = \left(\frac{1}{3\dot{M}\dot{p}_1}, \ldots, \frac{1}{3\dot{M}\dot{p}_{\dot{M}}} \right), \quad g_{\ddot{y}} = \frac{1}{3\ddot{M}\ddot{p}} = \left(\frac{1}{3\ddot{M}\ddot{p}_1}, \ldots, \frac{1}{3\ddot{M}\ddot{p}_{\ddot{M}}} \right), \quad g = \frac{1}{3Lv} = \left(\frac{1}{3Lv_1}, \ldots, \frac{1}{3Lv_L} \right). \tag{23}$$

Under the assumption of (22), the slack-based technical inefficiency (13) becomes

$$\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g}) = \vec{S}_O\left(x, \dot{y}, \ddot{y}, b; \frac{1}{3\dot{M}\dot{p}}, \frac{1}{3\ddot{M}\ddot{p}}, \frac{1}{3Lv} \right) = \sum_{\dot{m}=1}^{\dot{M}} \dot{p}_{\dot{m}} \dot{s}_{\dot{m}}^* + \sum_{\ddot{m}=1}^{M} \ddot{p}_{\ddot{m}} \ddot{s}_{\ddot{m}}^* + \sum_{l=1}^{L} v_l s_l^* \tag{24}$$

For the derivation of (24), see Appendix A. We have chosen the direction (22) because the slack-based inefficiency (24) has a nice interpretation, i.e., $\vec{S}_O(x, \dot{y}, \ddot{y}, b; \mathbf{g})$ is the total value of output short-falls or output technical inefficiencies.

In our bank efficiency application, we utilize a non-parametric linear programming approach called data envelopment analysis (DEA) due to Farrell (1957) and Charnes, Cooper and Rhodes (1978), Banker, Charnes and Cooper (1984) and Färe, Grosskopf and Lovell (1985). Appendix B shows a two-stage network implementation.

4. Data and Empirical Results

4.1.Data description

In this section, we apply our proposed model to *Shinkin* banks. The data were collected from Nikkei Financial Quest and the sample contains *Shinkin* banks observed over 9 recent years from March 2007 to March 2015 (corresponding to FY2006 and FY2014).

Regarding outputs, the data set consists of two good outputs: linked output ($\dot{y}$) and unlinked output ($\ddot{y}$) representing performing loans and securities investments, respectively, as well as one bad output (b) of nonperforming loans. The number of employees is used as a proxy for labor (x_1) and the sum of tangible and intangible fixed assets are used as the proxy for physical capital (x_2). In addition, members' equity in the balance sheet is treated as the third input (x_3). Since our approach incorporates the fund-raising process as an intermediate system, we need to define intermediate products. We have chosen two kinds: one is deposits

(z_1) and the other is other raised funds (z_2). The construction of our variables is explained in Table 1.

Table 1: The list of the Applied Variables in the Model

x_1	Labor	=Number of workers
x_2	Physical capital	= tangible fixed assets + intangible fixed assets
z_1	Intermediate product	=deposits
z_2	Intermediate product	=other raised funds
$\dot{y}$	Performing loans linked to NPLs	= total loans - NPLs
$\ddot{y}$	Good output not linked to bads	= securities investment
b	Bad output	= Nonperforming loans
$\dot{p}$	Price of $\dot{y}$	= (interest income) / $\dot{y}$
$\ddot{p}$	Price of $\ddot{y}$	= (non-interest income) / $\ddot{y}$
v	Price of b	= (Reserve for possible loan losses)/NPLs

There have been disputes about the appropriate definition of inputs and outputs for modelling the bank production process. We may discern three main ways of how to describe the production process in banking industry, i.e., the asset approach, the cost user approach and production (value added) approaches. As Sealey and Lindley (1977) argue, banks are seen as financial intermediaries that transform liability to bank loans. Thus, loans and other assets are perceived as bank outputs. Inputs are then composed by deposits and other liabilities. On the other hand, the user cost approach classifies inputs/outputs on the basis of its net contribution to bank revenue, see, for more detailed discussion, for example, Berger and Humphrey (1992). The production approach (value added), takes into account physical inputs that include labour and capital to produce deposits and other types assets that are expressed in terms of the number of deposit and loan accounts. Berger and Humphrey (1997) suggest the asset value and the user cost approach are best suited for evaluating bank efficiency, whereas the production approach is appropriate for evaluating the efficiency of bank branches. In our model we adopt the amended asset approach to capture bank production process. This is in line with other recent studies, for example, Fujii, Managi and Matousek (2014), Fukuyama and Matousek (2011, 2017) among others. We treat deposits as an intermediary output. In other words, the deposits are an output in the first stage of production and an input in the second stage in the two-stage network model. As for NPLs, we assume that NPLs are the output in the production process.

In Table 2, we provide the descriptive statistics of inputs, outputs and intermediate products. When we estimated optimal values for performing loans $\dot{y}$ and non-performing loans b , where $\dot{y}$ and b are singletons for our data some observations had zero values. As an efficiency target, this is not acceptable because a commercial bank's main activity is lending.

Table 2: Descriptive Statistics of Inputs, outputs, intermediate products, FY2006-FY2014

	$\dot{y}$	$\ddot{y}$	b	x_1	x_2	x_3
Mean	231645	139555	16204	407	7032	25474
Stdev	308661	170725	20877	410	9543	30718
Min	9033	232	739	31	198	1350
Max	2338388	1679023	201533	2663	67012	232472
	z_1	z_2	$\dot{p}$	$\ddot{p}$	v	
Mean	466420	4506	0.02632	0.01289	0.31814	
Stdev	568330	10479	0.00582	0.00421	0.14774	
Min	36356	68	0.01197	0.00058	0.02894	
Max	4644009	185462	0.08420	0.06631	0.97988	

Note: monetary values are in millions of yen and the labor input is in terms of numbers.

4.2. Results

The proposed innovative methodological approach allows us to quantify the gap between optimal revenue and actual revenue. Fukuyama and Matousek (2017) discuss the interpretation of optimal values that are derived from the bank revenue function with NPLs and are part of a bank's revenue. We can also exactly estimate what measures and business strategy banks should take to match their actual revenue level with the optimal one. We first discuss the results for *Shinkin* banks and then we look at the differences across the individual prefectures.

From Table 3 we may see by how much actual revenue and outputs are below the optimal levels. In the case of NPLs we try to reduce the actual levels. We report the changes for the observed period that is from 2006 to 2014 individual years. The presented results are rather alarming and significantly lower than for Regional banks as reported by Fukuyama and Matousek (2017). In terms of revenue changes, we can trace up a gradual but rather marginal improvement over the analyzed period. The difference between optimal and actual revenue was reduced from 70% to 62% in 2014. In other words, the average actual revenue was 62% lower than optimal in 2014. In 2008 at the ousted of the Global Financial Crisis (GFC), the actual revenue was down to 63% of optimal revenue. The actual revenue is well below the level that *Shinkin* banks could achieve. Since then the gap deepened again and there was just a marginal improvement in 2012.

If we look at the individual outputs, then we see that *Shinkin* banks significantly reduced the gap in terms of the securities (y2). The initial gap of 65% was reduced over the examined period to 2% only. This can be explained by the fact that *Shinkin* banks revised their business activities and because of the lack of investment opportunities they increased the holding of government and municipal bonds on their balance sheets. This change can also be linked with the changes in economic environment introduced by the Japanese government. As for the volume of loans (y1), we show that in 2009 the volume of actual loans was even higher than the optimal level. However, we further observe that over the analyzed period the actual volume of loans has been declining with respect to the optimal volume. It indicates that *Shinkin* banks reduced their lending activities. This reflects the view of economic analysts, who argue that banks reshuffled their portfolio in favour of securities (IMF, 2017). These changes further reflect the actual volume of NPLs with respect to the estimated optimal volume of NPLs. The actual NPLs were 166% higher than the estimated optimal volume. After then we observe a gradual decline but the volume of actual NPLs remained unsustainably high even in 2014. The actual NPLs were 100% higher than the estimated optimal volume. The output mix of y_1/y_2 has been reduced as the results of the changes in banks business strategy but we can see significant discrepancies between the actual mix and optimal one. These changes had a negative effect on bank revenue.

Table 3: Optimal vs Actual levels (% changes - averages)

Year	opt-act (Rev)	opt-act (y1)	opt-act (y2)	opt-act (bad)	opt-act (y1/y2)
2014	0.62	0.29	0.02	-1.00	2.03
2013	0.65	0.29	0.08	-1.01	1.97
2012	0.65	0.28	0.03	-1.40	2.04
2011	0.66	0.18	0.37	-1.55	1.23
2010	0.65	0.09	0.53	-1.60	0.89
2009	0.67	-0.15	0.71	-1.66	0.44
2008	0.63	0.15	0.50	-1.14	1.10
2007	0.64	0.18	0.53	-0.70	1.04
2006	0.70	0.07	0.65	-0.91	0.69

We report and analyze the gaps across the individual regions. From Table 4.1 and 4.2 we observe that the best performing region is Kanto. Kanto shows the lowest value in terms of the change of optimal vs. actual revenue throughout the analyzed period. In 2014, the actual revenue was 55% lower than the optimal revenue. The worst region is Tohoku where the actual revenue is 100% lower than optimal revenue. These differences can be explained by the fact that Kanto is one of the richest districts in Japan compared to Tohoku that is significantly poorer in terms of regional GDP.

A further analysis of our results shows that the economic development is reflected in bank performance as we also discussed below for bank efficiency. We can also see that there is a positive trend in the individual regions from 2006 to 2014. As we have already mentioned, Shinkin banks increased the volume of securities over the time period. We observe that in FY 2014 and FY 2013 there is several regions where the volume of actual volume of securities was higher than the optimal one. In FY 2014 a half of the region indicates a shift to securities. Shikoku region, for example, indicates that the volume of actual securities was 23% higher than the estimated optimal volume. On the other hand, the volume of declined over the analyzed period across all the regions. In FY 2006- 2009, there were regions where Shinkin banks exhibited the higher actual volume of loans (y1) than the estimated optimal.

Table 4.1: Regions Optimal vs Actual levels (% changes)

Region	Year	**opt-act (Rev)**	**opt-act (y1)**	**opt-act (y2)**	**opt-act (bad)**	**opt-act (y1/y2)**
Chubu	FY2014	0.72	0.31	-0.08	-1.05	1.23
Chugoku	FY2014	0.91	0.31	0.20	-1.52	1.66
Hokkaido	FY2014	0.76	0.10	0.31	-1.22	-1.08
1.66Kanto	FY2014	0.55	0.31	-0.12	-0.68	1.74
Kinki	FY2014	0.60	0.21	-0.11	-1.43	1.54
Kyushu	FY2014	0.77	0.34	0.50	-0.74	1.99
Shikoku	FY2014	0.83	0.48	-0.23	-0.72	0.71
Tohoku	FY2014	0.99	0.35	0.46	-1.59	1.51
Chubu	FY2013	0.75	0.31	0.01	-1.06	1.29
Chugoku	FY2013	0.93	0.37	0.17	-1.17	1.73
Hokkaido	FY2013	0.77	0.09	0.34	-1.20	1.06
Kanto	FY2013	0.61	0.29	0.00	-0.68	1.77
Kinki	FY2013	0.70	0.21	-0.06	-1.67	1.60
Kyushu	FY2013	0.82	0.41	0.45	-0.65	2.13
Shikoku	FY2013	0.89	0.43	-0.04	0.82	0.74
Tohoku	FY2013	1.04	0.42	0.42	-1.19	1.50
Chubu	FY2012	0.74	0.27	0.02	-1.66	1.33
Chugoku	FY2012	0.99	0.30	0.33	-1.98	1.79
Hokkaido	FY2012	0.77	0.32	0.06	-1.08	1.07
Kanto	FY2012	0.57	0.26	-0.09	-1.01	1.75
Kinki	FY2012	0.64	0.24	-0.16	-1.82	1.55
Kyushu	FY2012	0.81	0.34	0.52	-1.01	2.21
Shikoku	FY2012	0.84	0.47	-0.17	-0.85	0.83
Tohoku	FY2012	1.17	0.41	0.47	-1.94	1.53
Chubu	FY2011	0.76	0.12	0.41	-1.93	1.40
Chugoku	FY2011	0.95	0.24	0.51	1.96	1.87
Hokkaido	FY2011	0.74	-0.35	0.60	-1.86	1.11
Kanto	FY2011	0.59	0.21	0.22	-1.15	1.87
Kinki	FY2011	0.64	0.11	0.34	-1.78	1.66
Kyushu	FY2011	0.85	0.37	0.56	-1.18	2.40
Shikoku	FY2011	0.84	0.32	0.16	-1.14	0.75
Tohoku	FY2011	1.19	0.48	0.26	-1.76	1.75
Chubu	FY2010	0.79	0.04	0.53	-1.70	1.46
Chugoku	FY2010	0.86	0.08	0.69	-1.98	1.95
Hokkaido	FY2010	0.74	-0.42	0.64	-2.44	1.20
Kanto	FY2010	0.62	0.12	0.45	-1.32	2.07
Kinki	FY2010	0.67	0.09	0.44	-1.69	1.83
Kyushu	FY2010	0.87	0.22	0.75	-1.55	2.61
Shikoku	FY2010	0.82	0.23	0.61	-1.42	1.52
Tohoku	FY2010	1.10	0.24	0.73	2.60	1.96

Table 4.2: Regions Optimal vs Actual levels (% changes)

Region	Year	**opt-act (Rev)**	**opt-act (y1)**	**opt-act (y2)**	**opt-act (bad)**	**opt-act (y1/y2)**
Chubu	FY2009	0.80	-0.24	0.71	-1.68	1.50
Chugoku	FY2009	0.89	-0.28	0.81	-2.07	2.04
Hokkaido	FY2009	0.78	-1.61	0.77	-2.34	1.27
Kanto	FY2009	0.65	-0.02	0.62	-1.45	1.99
Kinki	FY2009	0.70	-0.11	0.66	-1.81	1.88
Kyushu	FY2009	0.88	-0.20	0.88	-1.56	2.80
Shikoku	FY2009	0.72	-0.01	0.62	-0.52	0.78
Tohoku	FY2009	1.18	0.26	0.86	-2.65	2.01
Chubu	FY2008	0.78	0.13	0.49	-1.21	1.61
Chugoku	FY2008	0.86	0.15	0.69	-1.43	2.08
Hokkaido	FY2008	0.70	0.01	0.56	-1.19	1.37
Kanto	FY2008	0.62	0.17	0.39	-0.93	2.13
Kinki	FY2008	0.68	0.13	0.41	1.29	2.02
Kyushu	FY2008	0.88	0.30	0.74	-1.27	2.84
Shikoku	FY2008	0.68	0.25	0.44	-0.25	0.88
Tohoku	FY2008	1.03	0.21	0.78	-1.81	2.20
Chubu	FY2007	0.79	0.17	0.51	-0.66	1.57
Chugoku	FY2007	0.90	0.13	0.69	-1.28	1.97
Hokkaido	FY2007	0.71	-0.12	0.63	-0.92	1.37
Kanto	FY2007	0.69	0.20	0.45	-0.49	1.96
Kinki	FY2007	0.69	0.16	0.42	-0.87	1.96
Kyushu	FY2007	0.90	0.28	0.76	-1.08	2.73
Shikoku	FY2007	0.73	0.32	0.37	0.26	0.81
Tohoku	FY2007	0.97	0.08	0.82	-1.51	2.21
Chubu	FY2006	0.82	0.03	0.64	0.90	1.58
Chugoku	FY2006	1.00	0.05	0.74	-1.78	1.95
Hokkaido	FY2006	0.74	-0.89	0.75	-1.44	1.37
Kanto	FY2006	0.70	0.17	0.56	-0.68	2.09
Kinki	FY2006	0.80	0.06	0.55	-0.98	2.01
Kyushu	FY2006	0.93	0.09	0.86	-1.31	2.79
Shikoku	FY2006	0.76	0.22	0.54	-0.43	0.82
Tohoku	FY2006	0.93	-0.02	0.85	-1.33	2.10

However, this trend has been changed after 2009 and banks revised their portfolio. For example, in FY 2009 Shinkin banks in Hokkaido had the actual volume of loans by 161% higher than the estimated optimal level. In FY 2014, Shinkin banks in the same region had the actual of volume of loans 10% below the estimated optimal volume. We can also see this trend through the ratio y1/y2.

These changes are explained also by analyzing the differences between actual and optimal NPLs. Hokkaido, Chugoku and Tohoku regions have been the worst regions in terms of the changes between the actual and optimal volume of NPLs. The highest values were reached in 2009, when for example, Shinkin banks in Tohoku reported the change of actual to the optimal volume of NPLs 245%. This has been reduced to 159% in FY2015. The analysis of the difference across the regions provide fuller picture about the structural problems in the segment of Shinkin banks.

Next, we turn our attention to the estimated bank efficiency scores. Table 5 reports the overview of the descriptive statistics of the analyzed efficiency scores. We report the following inefficiency scores: Nerlovian Revenue Inefficiency (NRI) that is the sum of Directional Allocative Inefficiency (DAI) and Directional Technical Inefficiency (DTI). Slack-based technical inefficiency (SBTI) that represents the total value of inefficiencies related to linked good outputs, unlinked good outputs and bad outputs. Slack-based Allocative Inefficiency (SBAI) that is the ratio of revenue inefficiency to total slack-based inefficiency. We also measure NRI by decomposing into DTI and DAI. DAI is defined as the difference between revenue inefficiency minus the directional distance function.

Table 5: Descriptive Statistics of Estimates under VRS (Excluding DMUs with Nerlovian revenue efficient)

	NRI	SBTI	DTI	SBAI	DAI
Mean	5683	5513	3657	172	2027
Stdev	4761	4700	3158	399	3336
Min	123	55	0	0	1
Max	46651	46476	41297	4014	46651

Tables 6.1 and 6.2 provide the average efficiency scores when NRI is decomposed into SBTI and SBAI inefficiency and DTI and DAI. If we analyze the individual component of NRI, we see that the main source of NRI is from SBTI. SBTI reached the maximum in FY 2009. Since then we can see the gradual improvement in terms of SBTI. From FY2009 to FY 2014 we observe that SBTI decline by 27%. This seems to be the current trend across *Shinkin* banks. As we have discussed early, this reflects the fact that *Shinkin* banks reduced the share of loans and this reduction was compensated by the increased volume of securities that is composed by the government and municipalities bonds. These structural changes in bank

balance sheets eased the request on reserves and provision for NPLs. As for the second component that is SBAI, we observe the similar trend. SBAI levels peak in FY2009. Since then there was an improvement in the inefficiency levels by 50%. Despite these improvements in terms of SBAI levels the overall NRI scores remain very high.

Table: 6.1 Nerlovian Revenue Efficiency, Slack-based technical inefficiency and Slack-based Allocative Inefficiency

Year		**NRI**	**SBTI**	**SBAI**
2014	Average	4721	4592	129
	Stdev	4138	4088	273
2013	Average	5070	4956	114
	Stdev	4170	4049	316
2012	Average	5715	5503	147
	Stdev	4170	4049	508
2011	Average	5715	5503	213
	Stdev	4625	4549	454
2010	Average	5811	5679	132
	Stdev	4755	4710	296
2009	Average	6675	6234	441
	Stdev	5143	5016	645
2008	Average	5741	5654	87
	Stdev	4991	4891	296
2007	Average	5817	5706	110
	stdev	4992	4995	206
2006	average	6322	6150	172
	stdev	5389	5402	283

In Table 6.2 we present the results of the decomposition into Directoral Techincal Inefficiency (DTI) and Directoral Alocative Inefficiency (DAI). We see that the average NRI peaked in FY 2009. The increase in *Shinkin* bank inefficiency can reflect the problems caused by the deteriorated economic environment in Japan but also the impact of the Global Financial Crisis (GFC). From FY 2009 till FY2014, NRI improved by 29.3%. It is also important to note that STD deviation over the analyzed period was very high. The source of the NRI comes from DTI.

Table 6.2 Nerlovian Revenue Efficiency, Directional technical inefficiency and Directional Allocative Inefficiency

Year		**NRI**	**DTI**	**DAI**
2014	average	4721	2885	1836
	stdev	4138	2420	3050
2013	average	5070	3136	1934
	stdev	4143	2495	2981
2012	average	5194	3030	2164
	stdev	4170	2374	3105
2011	average	5715	3629	2087
	stdev	4625	3012	3306
2010	average	5811	3980	1831
	stdev	4755	3232	3280
2009	average	6675	4160	2515
	stdev	5143	3332	3612
2008	average	5741	3914	1827
	stdev	4991	3181	3849
2007	average	5817	4054	1762
	stdev	4992	3784	2908
2006	average	6322	4046	2275
	stdev	5389	3837	3726

We further analyse the trajectories of bank efficiency across the regions in Japan. We observe from Tables 7.1 and 7.2 that there are relatively large discrepancies across the regions in terms of NRI. *Shinkin* banks in Hokkaido show over the analyzed period the lowest level of bank inefficiencies. On the contrary *Shinkin* banks in Kinki had the NRI scores 2.1 times higher than in the Kinki regions. If we compare SBTI and SBAI, we identify that SBAI scores are, in this case, almost three times higher than *Shinkin* banks in Hokkaido report. Despite the high degree of differences in bank performances across the regions we observe that inefficiency levels gradually decrease. We also identify FY2009 as a period in which the level of inefficiencies is the highest one. For example, *Shinkin* banks in Kinki have improved their NRI from FY2009 to FY2014 by almost 25%. Banks in Hokkaido that is the one of the best performing regions report the improvement of almost 75% over the same period. These large changes in inefficiency levels indicate that bank managers can cope with the challenges and adapt to shocks that have become integral part of the economic environment in Japan. The main challenge remains how to improve SBTI levels that we identify as the main contributor to NRI. *Shinkin* banks in some regions, for example, Chubu and Chugoku report very low levels of SBAI compared to other regions in FY2014. In other way, the results reveal large discrepancies in terms of how to manage banks performance.

Table 7.1 Nerlovian Revenue Efficiency, Slack-based technical inefficiency and Slack-based Allocative Inefficiency

Region	Year		NRI	SBTI	SBTI
Chubu	FY2014	average	4,835.48	4,705.75	129.73
Chugoku	FY2014	average	4,048.56	3,990.27	58.29
Hokkaido	FY2014	average	2,870.20	2,751.39	118.82
Kanto	FY2014	average	5,949.69	5,445.26	504.44
Kinki	FY2014	average	6,031.67	5,631.86	399.81
Kyushu	FY2014	average	3,007.41	2,987.82	19.59
Shikoku	FY2014	average	5,037.42	3,439.78	1,597.63
Tohoku	FY2014	average	3,701.42	3,629.06	72.36
Chubu	FY2013	average	5,221.92	5,090.08	131.84
Chugoku	FY2013	average	4,412.94	4,401.96	10.98
Hokkaido	FY2013	average	3,079.92	2,955.04	124.88
Kanto	FY2013	average	6,191.02	5,778.57	412.45
Kinki	FY2013	average	6,339.60	5,898.89	440.71
Kyushu	FY2013	average	3,520.55	3,478.64	41.91
Shikoku	FY2013	average	5,798.43	3,877.10	1,921.33
Tohoku	FY2013	average	4,198.33	4,122.40	75.92
Chubu	FY2012	average	5,230.70	5,075.80	154.90
Chugoku	FY2012	average	4,870.84	4,870.85	-0.01
Hokkaido	FY2012	average	3,182.46	3,057.83	124.64
Kanto	FY2012	average	6,258.58	5,820.63	437.95
Kinki	FY2012	average	6,261.80	5,819.85	441.95
Kyushu	FY2012	average	3,361.44	3,331.59	29.86
Shikoku	FY2012	average	5,318.63	3,713.36	1,605.27
Tohoku	FY2012	average	4,559.19	4,540.08	19.11
Chubu	FY2011	average	5,903.77	5,627.63	276.14
Chugoku	FY2011	average	4,938.17	4,893.16	45.01
Hokkaido	FY2011	average	3,631.79	3,346.41	285.38
Kanto	FY2011	average	6,956.89	6,663.65	293.25
Kinki	FY2011	average	6,534.70	5,956.32	578.37
Kyushu	FY2011	average	3,919.33	3,823.29	96.04
Shikoku	FY2011	average	5,534.04	4,289.55	1,244.49
Tohoku	FY2011	average	4,984.92	4,878.50	106.42
Chubu	FY2010	average	6,108.65	5,944.12	164.53
Chugoku	FY2010	average	4,891.28	4,811.71	79.57
Hokkaido	FY2010	average	3,678.55	3,511.01	167.54
Kanto	FY2010	average	7,116.96	6,916.28	200.69
Kinki	FY2010	average	6,804.09	6,335.53	468.57
Kyushu	FY2010	average	4,124.86	4,052.76	72.10
Shikoku	FY2010	average	4,336.13	4,044.27	291.86
Tohoku	FY2010	average	5,000.11	4,935.20	64.91

Table 7.2: Nerlovian Revenue Efficiency, Slack-based technical inefficiency and Slack-based Allocative Inefficiency

Region	Year		NRI	SBTI	SBTI
Chubu	FY2009	average	7,140.28	6,548.95	591.33
Chugoku	FY2009	average	5,789.07	5,551.87	237.20
Hokkaido	FY2009	average	4,902.04	3,944.29	957.75
Kanto	FY2009	average	7,625.85	7,261.89	363.96
Kinki	FY2009	average	7,940.47	6,965.52	974.94
Kyushu	FY2009	average	4,720.29	4,472.97	247.32
Shikoku	FY2009	average	5,180.29	4,079.02	1,101.27
Tohoku	FY2009	average	5,808.65	5,613.10	195.55
Chubu	FY2008	average	5,938.62	5,808.83	129.78
Chugoku	FY2008	average	4,771.91	4,720.31	51.60
Hokkaido	FY2008	average	3,234.56	3,132.95	101.62
Kanto	FY2008	average	7,346.41	7,154.59	191.83
Kyushu	FY2008	average	4,261.72	4,230.60	31.12
Shikoku	FY2008	average	4,356.49	3,427.59	928.91
Tohoku	FY2008	average	4,492.90	4,449.20	43.71
Chubu	FY2007	average	5,836.11	5,724.26	111.85
Chugoku	FY2007	average	4,922.70	4,824.19	98.51
Hokkaido	FY2007	average	3,314.53	3,127.84	186.69
Kanto	FY2007	average	6,931.68	6,778.93	162.55
Kinki	FY2007	average	6,905.97	6,451.50	454.47
Kyushu	FY2007	average	4,446.56	4,389.05	57.51
Shikoku	FY2007	average	5,058.65	3,569.26	1,489.39
Tohoku	FY2007	average	4,399.18	4,325.82	73.35
Chubu	FY2006	average	6,265.22	6,082.29	182.93
Chugoku	FY2006	average	5,567.42	5,411.87	155.55
Hokkaido	FY2006	average	3,843.86	3,432.68	411.17
Kanto	FY2006	average	8,150.56	8,022.62	127.95
Kinki	FY2006	average	7,878.93	7,071.89	807.04
Kyushu	FY2006	average	4,982.89	4,919.80	63.10
Shikoku	FY2006	average	5,532.86	3,945.25	1,587.61
Tohoku	FY2006	average	4,565.56	4,429.47	136.09

A further analysis is focused on bank inefficiency that is measured through the directional distance function across the regions. NRI is decomposed into DAI and DTI. From Table 8.1 and 8.2 we see the inefficiency scores of NRI, DTI and DAI. Our results indicate that the main source inefficiency is DTI. If we compare the contribution of the individual components on NRI, we may observe that the proportional share of DTI and DAI on NRI is more balanced than in the case when we estimated NRI by using SBTI and SBAI. We again confirm that the average NRI reached its peak in FY2009. During the period from FY2009 to FY2014 NRI improved on average by almost 30%. DTI during the same period dropped by more than 30% and DAI by 25%. These large changes indicate that managers can address the problem and improve bank inefficiency.

The analysis of the individual regions provides a similar picture, as in the case of SBTI and SBAI. DTI remains the main driver of bank inefficiency. However, a closer look reveals that *Shinkin* banks in some regions indicate that DAI is the main factor of bank inefficiency, for example, see *Shinkin* banks in Shikoku in FY2012. The average DAI scores are higher than for DTI. In addition, we find the high volatilities within *Shinkin* banks within the individual geographical regions.

Table 8.1 Nerlovian Revenue Efficiency, Directional technical inefficiency and Directional Allocative Inefficiency

Region	Year		NRI	DTI	DAI
Chubu	FY2014	average	4835.48	2768.05	2067.43
Chugoku	FY2014	average	4048.56	2733.12	1315.44
Hokkaido	FY2014	average	2870.20	1790.69	1079.51
Kanto	FY2014	average	5949.69	3243.17	2706.52
Kinki	FY2014	average	6031.67	3340.28	2691.39
Kyushu	FY2014	average	3007.41	2423.23	584.17
Shikoku	FY2014	average	5037.42	2233.78	2803.63
Tohoku	FY2014	average	3701.42	2628.16	1073.26
Chubu	FY2013	average	5221.92	3104.56	2117.36
Chugoku	FY2013	average	4412.94	2969.82	1443.12
Hokkaido	FY2013	average	3079.92	2070.32	1009.60
Kanto	FY2013	average	6191.02	3598.86	2592.17
Kinki	FY2013	average	6339.60	3177.17	3162.43
Kyushu	FY2013	average	3520.55	2786.10	734.45
Shikoku	FY2013	average	5798.43	2475.74	3322.69
Tohoku	FY2013	average	4198.33	2843.97	1354.36
Chubu	FY2012	average	5230.70	3024.95	2205.75
Chugoku	FY2012	average	4870.84	3034.16	1836.69
Hokkaido	FY2012	average	3182.46	1820.56	1361.90
Kanto	FY2012	average	6258.58	3470.79	2787.79
Kinki	FY2012	average	6261.80	3238.06	3023.74
Kyushu	FY2012	average	3361.44	2576.34	785.10
Shikoku	FY2012	average	5318.63	2242.48	3076.15
Tohoku	FY2012	average	4559.19	2574.92	1984.27
Chubu	FY2011	average	5903.77	3664.17	2239.60
Chugoku	FY2011	average	4938.17	3422.24	1515.93

Hokkaido	FY2011	average	3631.79	2301.91	1329.88
Kanto	FY2011	average	6956.89	4374.19	2582.70
Kinki	FY2011	average	6534.70	3740.31	2794.39
Kyushu	FY2011	average	3919.33	2971.47	947.87
Shikoku	FY2011	average	5534.04	2879.93	2654.11
Tohoku	FY2011	average	4984.92	2981.94	2002.97
Chubu	FY2010	average	6108.65	4023.45	2085.20
Chugoku	FY2010	average	4891.28	3675.93	1215.36
Hokkaido	FY2010	average	3678.55	2458.44	1220.12
Kanto	FY2010	average	7116.96	4911.58	2205.38
Kinki	FY2010	average	6804.09	4109.08	2695.01
Kyushu	FY2010	average	4124.86	3368.22	756.64
Shikoku	FY2010	average	4336.13	3336.87	999.27
Tohoku	FY2010	average	5000.11	3250.60	1749.51

Table 8.2 Nerlovian Revenue Efficiency, Directional technical inefficiency and Directional Allocative Inefficiency

Region	Year		NRI	DTI	DAI
Chubu	FY2009	average	7140.28	4263.48	2876.80
Chugoku	FY2009	average	5789.07	4121.92	1667.15
Hokkaido	FY2009	average	4902.04	2617.16	2284.88
Kanto	FY2009	average	7625.85	4854.95	2770.89
Kinki	FY2009	average	7940.47	4301.39	3639.08
Kyushu	FY2009	average	4720.29	3581.30	1138.98
Shikoku	FY2009	average	5180.29	2781.69	2398.60
Tohoku	FY2009	average	5808.65	3511.68	2296.97
Chubu	FY2008	average	5938.62	3912.00	2026.62
Chugoku	FY2008	average	4771.91	3737.88	1034.03
Hokkaido	FY2008	average	3234.56	2255.97	978.59
Kanto	FY2008	average	7346.41	4698.36	2648.06
Kyushu	FY2008	average	4261.72	3515.91	745.82
Shikoku	FY2008	average	4356.49	2286.77	2069.73
Tohoku	FY2008	average	4492.90	3236.41	1256.49
Chubu	FY2007	average	5836.11	4124.38	1711.73
Chugoku	FY2007	average	4922.70	3696.38	1226.32
Hokkaido	FY2007	average	3314.53	2212.90	1101.63
Kanto	FY2007	average	6931.68	4521.53	2439.91
Kinki	FY2007	average	6905.97	4478.44	2427.53
Kyushu	FY2007	average	4446.56	3539.42	907.14
Shikoku	FY2007	average	5058.65	2566.55	2492.10
Tohoku	FY2007	average	4399.18	3272.16	1127.01
Chubu	FY2006	average	6265.22	4112.23	2152.99
Chugoku	FY2006	average	5567.42	4032.67	1534.75
Hokkaido	FY2006	average	3843.86	2206.92	1636.94

Kanto	FY2006	average	8150.56	5190.83	2959.73
Kinki	FY2006	average	7878.93	3940.08	3938.85
Kyushu	FY2006	average	4982.89	3623.03	1359.86
Shikoku	FY2006	average	5532.86	2646.54	2886.31
Tohoku	FY2006	average	4565.56	3076.26	1489.30

In Table 9 we report also the results of sigma (σ) convergence of the estimated banks' inefficiency measures. According to Furceri (2005) and Young et al. (2008), there is sigma (σ) convergence, when the dispersion of banks' inefficiency levels reduces among the two examined periods (i.e. 2006-2014). A positive value indicates a sigma convergence of banks' inefficiencies whereas a negative indicates divergence. When we examine the inefficiencies at bank levels the results indicate a divergence of banks' inefficiencies. Moreover when we are using the per region aggregate data, the results verify the divergence of banks' inefficiency levels expect for the cases of DTI and SBTI.

Table 9: **Sigma-convergence of banks' inefficiency values**

per banks sigma convergence (σ)			
2006-2014 values	*Nerlovian Revenue Inefficiency (NRI)*	*Directional Allocative Inefficiency (DAI)*	*Directional Technical Inefficiency (DTI)*
	-0.0038	-0.0063	-0.0038
2006-2014 values	*Slack-based technical inefficiency (SBTI)*	*Slack-based Allocative Inefficiency (SBAI)*	
	-0.0038	-0.0038	
per regions sigma convergence (σ)			
2006-2014 values	*Nerlovian Revenue Inefficiency (NRI)*	*Directional Allocative Inefficiency (DAI)*	*Directional Technical Inefficiency (DTI)*
	-0.00551	-0.02636	0.02289
2006-2014 values	*Slack-based technical inefficiency (SBTI)*	*Slack-based Allocative Inefficiency (SBAI)*	
	0.00789	-0.08172	

In addition, we perform concordance analysis by adopting the concordance correlation coefficient (Lin 1989, 2000; Nickerson 1997) on banks different inefficiency measures. Table X provides the results of the concordance correlation among the years based on the different measures adopted. The results reveal that there is a poor concordance among the estimated inefficiency measures (i.e. below 0.40). This phenomenon is more pronounced when comparing the banks' inefficiency measures after and during the initiation of global financial crisis. It appears that banks performance has been heavily distorted by the GFC which reflected on the concordance analysis of the different estimated inefficiency measures.

5. Conclusion

This paper advances the contemporary methodological and empirical research on bank efficiency in general and in Japan in particular. We introduce an innovative two–stage network model that estimates bank efficiency in the segment of *Shinkin* banks. NRI is estimated in two ways. Firstly, we decompose NRI into slack-based technical inefficiency (multiplicative) and slack-based allocative efficiency. Secondly, a further decomposition of NRI is based on directional output distance function when we obtain DTI and DAI. We apply the model on the segment of Japanese financial institutions – *Shinkin* banks. This group of banks is appropriate for our analysis since the model allows us to simultaneously consider NPLs and bank revenue function. The role of NPLs is essential as we discuss in the literature review and reflect the burden on *Shinkin* bank balance sheets.

The paper clearly outlines the link with the previous research in particular Fukuyama and Matousek (2011, 2017) and Fukuyama and Weber (2015, 2017). As has been discussed in the previous Section, we compare optimal levels of revenue, NPLs and bank outputs with actual levels. In addition, we discuss inefficiency scores by using slack-based allocative and technical inefficiency and directorial technical and allocative inefficiency. In terms of its application, we shed light on the contemporary challenges that face the third most important group of banks in Japan. The paper also updates the previous empirical research on the issue lined to the geographical regions in Japan and financial institutions and/or Shinkin banks, see for example, Uchino (2014), Assaf et al. (2011), Kano and Tsutsui (2003) among others. Furthermore, we estimate the two-stage model that applies Nerlovian's revenue inefficiency indicator to estimate the revenue inefficiency of *Shinkin* banks. As we discuss above, our innovative approach does not only allow us to disaggregate bank inefficiency into directional technical inefficiency and allocative inefficiency but also into slack-based technical and allocative inefficiencies.

In the paper, we firstly present the results as for the optimal volume of loans and securities, and NPLs. We report that *Shinkin* banks offset the declining volume of loans by securities on their balance sheets. The lowest volume measured in terms of changes between optimal and actual volume of loans is reported in FY2011 and FY2013. After then there are visible changes in terms of the reduction of the actual volume of loans with respect to the estimated optimal volume. We also discuss the remarkable differences across the regions in which banks operate. We show that, for example, in Toholu region this change is 48% in FY2011. Shikoku region then shows continuously a high volume of actual loans with respect to the estimated optimal volume that is above 40% during the period of FY2012-FY2014. We also confirm that the actual volume of NPLs is significantly higher than the estimated optimal volume of NPLs. As for the analysis of *Shinkin* banks in the regions, we find that *Shinkin* banks in all the regions have the actual volume of loans higher than the estimated optimal values in FY 2009. The banks then restructured their portfolios and by the end of FY2014, four out of eight regions have the actual volume of securities higher than the estimated optimal volume.

Secondly, *Shinkin* banks face over the entire analyzed period the challenge to reduce the volume of NPLs. This is completely different situation from Regional banks as Fukuyama and Matousek (2017) report. The remaining volume of NPLs with respect to the estimated optimal volume is high. This reflects the fact that the consolidation and restructuring programmes launched by the Japanese government in the late 1990s and the 2000s was not targeted at the group of *Shinkin* banks. We report that the largest gap change is in Tohoku regions that report the change of the optimal to actual NPLs over 260%.

Thirdly, the decomposition of NRI into SBTI and SBAI indicate that the main source of bank inefficiency comes from SBTI. Since FY2009 we have observed the decline in *Shinkin* bank inefficiency. The analysis of the individual regions also discloses that there are significant differences across the regions. In the case of DTI and DTA our results show that about 60% of DTI contributes to NRI. That is remarkably different if we compare SBTI and SBAI where about 95% of NRI is contributed to SBTI.

The observed time period allows us to examine the changes during GFC but also to see how Abenomics policy affected this segment of the financial institution. In terms of policy implications, one could explain the high technical and allocative inefficiencies and the actual structure of their assets on bank balance sheets through the current economic environment. The extremely low interest rates that have become a dominant feature of the Japanese financial market impose unprecedented challenges for *Shinkin* banks. This is pronounced even more in this segment of relatively small banks because of their dependence on retail deposits and limited risk low business opportunities. Shinkin banks have significant problems to cover their operational cost. The situation can deteriorate in the case that NPLs increase and banks will be forced to set aside reserves and provisions for this type of loans. It is important to mention that this scenario is very likely since Shinkin banks are encouraged to be involved in high yielding unsecured lending by the Japanese financial Services Agency. Such business activities blended with the current investment activities in domestic and fixed income and securities and investment trusts. A further negative impact on bank operations is fact that the 'old' loans with marginally higher interest rates will be fully repaid in the horizon of a decade and new issued loans will generate even lower cash flow because of even lower interest rates. In such an environment even a small negative shock can cause a systemic crisis in this segment of banks that have share of 5 per cent on the total Japanese market in terms of loans.

Finally, our analysis might be further deepened by analyzing the economic differences in the individual regions or even prefectures and link them with *Shinkin* bank performance. There is also a need to fully understand the trend in their business activities. In particular, to analyze the shrinkage of loans that are replaced by securities. An integral part of next research in the context of the regional differences should be the analysis of the market segmentation and competitiveness within the segment of *Shinkin* banks. Such a study would further extend Uchino's (2014) recent paper. It seems that the regional differences are the key factor of bank inefficiency. Finally, we have found that standard deviations of the reported inefficiency scores are extremely high. Therefore, one should shed light on the identification of the determinants of *Shinkin* banks as conducted for Regional Banks by Fukuyama and Matousek (2017).

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Appendix A: Choice of directional vector

We choose the following directional vector for $(\dot{y},\ddot{y},b)\in P(x,z)$. Letting the directional vector as (22)

$$\mathbf{g}=\left(g_{\dot{y}}\quad g_{\ddot{y}}\quad g\right)=\left(\frac{1}{3\dot{M}\dot{p}},\quad \frac{1}{3\ddot{M}\ddot{p}},\quad \frac{1}{3Lv}\right),\ \text{where}$$

$$g_{\dot{y}}=\frac{1}{3\dot{M}\dot{p}}=\left(\frac{1}{3\dot{M}\dot{p}_1},\dots,\frac{1}{3\dot{M}\dot{p}_{\dot{M}}}\right),\quad g_{\ddot{y}}=\frac{1}{3\ddot{M}\ddot{p}}=\left(\frac{1}{3\ddot{M}\ddot{p}_1},\dots,\frac{1}{3\ddot{M}\ddot{p}_{\ddot{M}}}\right),\quad g=\frac{1}{3Lv}=\left(\frac{1}{3Lv_1},\dots,\frac{1}{3Lv_L}\right).$$

Then the slack-based measure (13) becomes (24) and the normalization factor becomes:

$$\dot{p}g_{\dot{y}}+\ddot{p}g_{\ddot{y}}+vg=\sum_{\dot{m}=1}^{\dot{M}}\frac{\dot{p}_{\dot{m}}}{3\dot{M}\dot{p}_{\dot{m}}}+\sum_{\ddot{m}=1}^{\ddot{M}}\frac{\ddot{p}_{\ddot{m}}}{3\ddot{M}\ddot{p}_{\ddot{m}}}+\sum_{l=1}^{L}\frac{v_l}{3Lv_l}=\frac{\overbrace{1+\dots+1}^{\dot{M}\text{ terms}}}{3\dot{M}}+\frac{\overbrace{1+\dots+1}^{\ddot{M}\text{ terms}}}{3\ddot{M}}+\frac{\overbrace{1+\dots+1}^{L\text{ terms}}}{3L}=1$$

which implies

$$\dot{\alpha}_{\dot{m}}=\frac{\dot{p}_{\dot{m}}}{\dot{p}g_{\dot{y}}+\ddot{p}g_{\ddot{y}}+vg}=\dot{p}_{\dot{m}},\left(\forall\dot{m}\right);\ \ddot{\alpha}_{\ddot{m}}=\frac{\ddot{p}_{\ddot{m}}}{\dot{p}g_{\dot{y}}+\ddot{p}g_{\ddot{y}}+vg}=\ddot{p}_{\ddot{m}},\left(\forall\ddot{m}\right);\alpha_l=\frac{v_l}{\dot{p}g_{\dot{y}}+\ddot{p}g_{\ddot{y}}+vg}=v_l,\left(\forall l\right).$$

Therefore,

$$\sum_{\dot{m}=1}^{\dot{M}}\dot{\alpha}_{\dot{m}}\dot{s}_{\dot{m}}^{*}+\sum_{\ddot{m}=1}^{\ddot{M}}\ddot{\alpha}_{\ddot{m}}\ddot{s}_{\ddot{m}}^{*}+\sum_{l=1}^{L}\alpha_l s_l^{*}=\sum_{\dot{m}=1}^{\dot{M}}\dot{p}_{\dot{m}}\dot{s}_{\dot{m}}^{*}+\sum_{\ddot{m}=1}^{\ddot{M}}\ddot{p}_{\ddot{m}}\ddot{s}_{\ddot{m}}^{*}+\sum_{l=1}^{L}v_l s_l^{*}$$

which yields that SBTI is equal to

$$\text{SBTI}=\vec{S}_O\left(x,\dot{y},\ddot{y},b;\frac{1}{3\dot{M}\dot{p}},\frac{1}{3\ddot{M}\ddot{p}},\frac{1}{3Lv}\right).$$

Appendix B: Nonparametric two-stage network DEA implementation

To non-parametrically incorporate the two-stage internal bank production system given in Figure 1, define the intensity variables for the two stages as $\boldsymbol{\lambda}^1 \in \Re_+^J$ and $\boldsymbol{\lambda}^2 \in \Re_+^J$. Let $\left\{(x_{1j},...,x_{Nj}, z_{1j},...,z_{Qj}, \dot{y}_{1j},...,\dot{y}_{Mj}, \ddot{y}_{1j},...,\ddot{y}_{Mj}, b_{1j},...,b_{Lj}) \middle| j=1,2,\ldots,J\right\}$ be observations consisting of J banks. We assume that the observed amounts of inputs, the three types of outputs are positive throughout the empirical part. In (25) and (26), utilizing the abatement factor $\theta_j \in [0,1]$, the good outputs $\dot{y}$ are jointly produced with bad outputs *b*, (Kuosmanen 2005; Kuosmanen and Podinovski 2009; Podinovski and Kuosmanen 2011). We incorporate the recent study of Epure and Lafuente (2015), who distinguished between linked outputs and unlinked outputs, developed a black-box DEA credit risk (NPLs) model for a bank. An alternative approach is given among others by Färe and Grosskopf (2003, 2009).

The two-stage network output possibility set is constructed as

$$P(x) = \left\{ (\dot{y}, \ddot{y}, b) \middle| \begin{array}{l} x_n \ge \sum_{j=1}^{J} x_{nj}\lambda_j^1 \ (\forall n),\ \sum_{j=1}^{J} z_{qj}\lambda_j^1 \ge z_q \ (\forall q),\ \sum_{j=1}^{J}\lambda_j^1 = 1,\ \ \lambda_j^1 \ge 0 \ (\forall j) \\ \sum_{j=1}^{J} z_{qj}\lambda_j^2 \le z_q \ (\forall q),\ \dot{y}_m \le \sum_{j=1}^{J} \dot{y}_{mj}\theta_j\lambda_j^2 \ (\forall m),\ \ddot{y}_{\ddot{m}} \le \sum_{j=1}^{J} \ddot{y}_{\ddot{m}j}\lambda_j^2 \ (\forall \ddot{m}) \\ b_l = \sum_{j=1}^{J} b_{lj}\theta_j\lambda_j^2 \ (\forall l),\ \sum_{j=1}^{J}\lambda_j^2 = 1,\ \lambda_j^2 \ge 0 \ (\forall j),\ z_q \ge 0 \ (\forall q),\ 1 \ge \theta_j \ge 0 \ (\forall j). \end{array} \right\}$$

where $\sum_{j=1}^{J}\lambda_j^1 = 1$ and $\sum_{j=1}^{J}\lambda_j^2 = 1$ allow for variable returns to scale. Since $P(x)$ is of nonlinear structure, we set $\Lambda_j^1 = \lambda_j^1 \ (\forall j)$, $\gamma_j = (1-\theta_j)\lambda_j^2 \ (\forall j)$ and $\Lambda_j^2 = \theta_j\lambda_j^2 \ (\forall j)$, to obtain the following linear programming directional output distance function:

$$\vec{D}_O\left(x_o, \dot{y}_o, \ddot{y}_o, b_o; \mathbf{g}\right) = \max\left\{ \beta \middle| \begin{array}{l} x_{no} \ge \sum_{j=1}^{J} x_{nj}\Lambda_j^1 \ (\forall n),\ \sum_{j=1}^{J} z_{qj}\left(\Lambda_j^1 - \Lambda_j^2 - \gamma_j\right) \ge 0, \\ \dot{y}_{mo} + \beta g_{\dot{m}} \le \sum_{j=1}^{J} \dot{y}_{mj}\Lambda_j^2 \ (\forall m), \\ \ddot{y}_{\ddot{m}o} + \beta g_{\ddot{m}} \le \sum_{j=1}^{J} \ddot{y}_{\ddot{m}j}\left(\Lambda_j^2 + \gamma_j\right) \ (\forall \ddot{m}), \\ b_{lo} - \beta g_l = \sum_{j=1}^{J} b_{lj}\Lambda_j^2 \ (\forall l),\ \ \sum_{j=1}^{J}\Lambda_j^1 = 1,\ \Lambda_j^1 \ge 0, \\ \sum_{j=1}^{J}\left(\Lambda_j^2 + \gamma_j\right) = 1,\ \Lambda_j^2 \ge 0 \ (\forall j),\ \gamma_j \ge 0 \ (\forall j),\ \beta: \mathit{free} \end{array} \right\} \quad (25)$$

This type of conversion is due to Kuosmanen (2005). See Fukuyama and Weber (2016) and Fukuyama and Matousek (2017) for (25). In a similar fashion, the network revenue function takes the form:

$$
R(x_o, \dot{p}, \ddot{p}, v) = \max \left\{ \begin{array}{l} \sum_{\dot{m}=1}^{\dot{M}} \dot{p}_{\dot{m}} \dot{y}_{\dot{m}} \\ + \sum_{\ddot{m}=1}^{\ddot{M}} \ddot{p}_{\ddot{m}} \ddot{y}_{\ddot{m}} \\ - \sum_{l=1}^{L} v_l b_l \end{array} \middle| \begin{array}{l} x_{no} \geq \sum_{j=1}^{J} x_{nj} \Lambda_j^1 \ (\forall n), \ \sum_{j=1}^{J} z_{qj} \left(\Lambda_j^1 - \Lambda_j^2 - \gamma_j \right) \geq 0, \\ \dot{y}_{\dot{m}} + \beta \dot{g}_{\dot{m}} \leq \sum_{j=1}^{J} \dot{y}_{\dot{m}j} \Lambda_j^2 \ (\forall \dot{m}), \\ \ddot{y}_{\ddot{m}} \leq \sum_{j=1}^{J} \ddot{y}_{\ddot{m}j} \left(\Lambda_j^2 + \gamma_j \right) \ (\forall \ddot{m}), \\ b_l = \sum_{j=1}^{J} b_{lj} \Lambda_j^2 \ (\forall l), \quad \sum_{j=1}^{J} \Lambda_j^1 = 1, \quad \Lambda_j^1 \geq 0, \\ \sum_{j=1}^{J} \left(\Lambda_j^2 + \gamma_j \right) = 1, \ \Lambda_j^2 \geq 0 \ (\forall j), \quad \gamma_j \geq 0 \ (\forall j), \\ \dot{y}_{\dot{m}} \geq 0 \ (\forall \dot{m}), \ \ddot{y}_{\ddot{m}} \geq 0 \ (\forall \ddot{m}), \ b_l \geq 0 \ (\forall l) \end{array} \right\} \quad (26)
$$

which can be thought of as a two-stage network DEA version of Färe, Grosskopf and Weber's (2006) parametric revenue function. The SBI model takes the form:

$$
\vec{S}_O \left(x_o, \dot{y}_o, \ddot{y}_o, b_o; \mathbf{g} \right) = \max \left\{ \left(\begin{array}{l} \frac{1}{3\dot{M}} \sum_{n=1}^{N} \frac{\dot{s}_{\dot{m}}}{\dot{g}_{\dot{m}}} \\ + \frac{1}{3\ddot{M}} \sum_{m=1}^{M} \frac{\ddot{s}_{\ddot{m}}}{\ddot{g}_{\ddot{m}}} \\ + \frac{1}{3L} \sum_{l=1}^{L} \frac{s_l}{g_l} \end{array} \right) \middle| \begin{array}{l} x_{no} \geq \sum_{j=1}^{J} x_{nj} \Lambda_j^1 \ (\forall n), \ \sum_{j=1}^{J} z_{qj} \left(\Lambda_j^1 - \Lambda_j^2 - \gamma_j \right) \geq 0, \\ \dot{y}_{\dot{m}o} + \dot{s}_{\dot{m}} = \sum_{j=1}^{J} \dot{y}_{\dot{m}j} \Lambda_j^2 \ (\forall \dot{m}), \\ \ddot{y}_{\ddot{m}o} + \ddot{s}_{\ddot{m}} = \sum_{j=1}^{J} \ddot{y}_{\ddot{m}j} \left(\Lambda_j^2 + \gamma_j \right) \ (\forall \ddot{m}), \\ b_{lo} - s_l = \sum_{j=1}^{J} b_{lj} \Lambda_j^2 \ (\forall l), \ \sum_{j=1}^{J} \Lambda_j^1 = 1, \ \Lambda_j^1 \geq 0, \\ \sum_{j=1}^{J} \left(\Lambda_j^2 + \gamma_j \right) = 1, \ \Lambda_j^2 \geq 0 \ (\forall j), \ \gamma_j \geq 0 \ (\forall j), \\ \dot{y}_{\dot{m}} \geq 0 \ (\forall \dot{m}), \ \ddot{y}_{\ddot{m}} \geq 0 \ (\forall \ddot{m}), \ b_l \geq 0 \ (\forall l) \end{array} \right\}. \quad (27)
$$

We use $\dot{g}_{\dot{m}} = \frac{1}{3\dot{M}\dot{p}_{\dot{m}}} \ (\forall \dot{m})$, $\ddot{g}_{\ddot{m}} = \frac{1}{3\ddot{M}\ddot{p}_{\ddot{m}}} \ (\forall \ddot{m})$ and $g_l = \frac{1}{3Lv_l} \ (\forall l)$ for our empirical example. Under this directional vector, the objective function in (27) can be expressed as $\sum_{\dot{m}=1}^{\dot{M}} \dot{p}_{\dot{m}} \dot{s}_{\dot{m}} + \sum_{\ddot{m}=1}^{M} \ddot{p}_{\ddot{m}} \ddot{s}_{\ddot{m}} + \sum_{l=1}^{L} v_l s_l$ as shown in subsection 4.2.