Case study-based sensitivity analysis of scale estimates w.r.t. the shape of fuzzy data

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Abstract. For practical purposes, and to ease both the drawing and the computing processes, the fuzzy rating scale was originally introduced assuming values based on such a scale to be modeled by means of trapezoidal fuzzy numbers. In this paper, to know whether or not such an assumption is too restrictive, we are going to examine on the basis of a real-life example how statistical conclusions concerning location-based scale estimates are affected by the shape chosen to model imprecise data with fuzzy numbers. The discussion will be descriptive for the considered scale estimates, but for the Fréchet-type variance it will be also inferential. The study will lead us to conclude that statistical conclusions are scarcely influenced by data shape.

Keywords: fuzzy data, fuzzy shape, location-based scale estimates

1 Introduction

In previous papers we have discussed the influence of the shape of fuzzy data coming from a random process in some statistical conclusions about this process. Although the assumption of the trapezoidal shape is not at all mandatory to develop statistics with fuzzy data, such an assumption substantially eases computations. Moreover, several authors have provided with different arguments either to employ trapezoidal fuzzy numbers or to employ trapezoidal approximations of fuzzy numbers preserving some key features (like ambiguity, expected interval, etc.).

The already developed discussions concern location of the random processes generating fuzzy data (see Lubiano *et al.* [7,9]), and a few ones regard the Fréchet-type variance of these processes (see De la Rosa de Sáa *et al.* [2,3]).

This paper presents a discussion involving some scale estimates for fuzzy data sets that have been recently introduced (see [3]). The discussion is to be based on a case study and will include both, descriptive and inferential conclusions.

2 Preliminaries

By a (bounded) **fuzzy number** we mean a mapping $\widetilde{U} : \mathbb{R} \to [0,1]$ such that for all $\alpha \in [0,1]$, the α-level set $\widetilde{U}_\alpha = \{x \in \mathbb{R} : \widetilde{U}(x) \geq \alpha\}$ if $\alpha \in (0,1]$, $= \mathrm{cl}\{x \in \mathbb{R} : \widetilde{U}(x) > 0\}$ if $\alpha = 0$ (with 'cl' denoting the closure of the set) is a nonempty compact interval.

As frequently used examples of fuzzy numbers we can consider those in Figure 1, which are instances of the so-called LU-fuzzy numbers (see Stefanini *et al.* [13]).

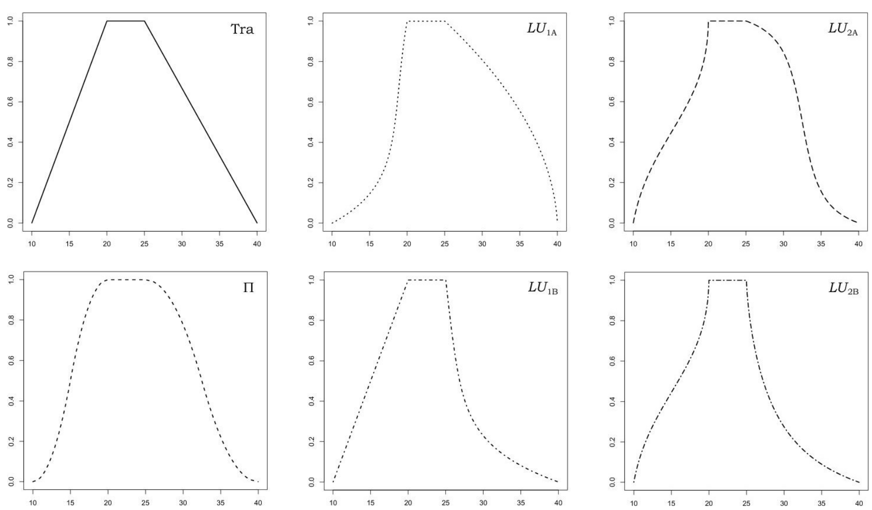

Fig. 1. Six types of fuzzy numbers sharing core $[20, 25]$ and support $(10, 40)$ and differing in shape. On the left, trapezoidal (top) and Π-curve (bottom), along with four different LU fuzzy numbers on the middle and the right

Random processes generating (intrinsically-valued) fuzzy data can be soundly formalized by means of **random fuzzy numbers** (for short RFN's), the one-dimensional convex version of fuzzy random variables, as defined by Puri and Ralescu [10] (i.e., a random fuzzy number is a fuzzy number-valued mapping $\mathcal{X}$ associated with a probability space and such that, for each α, the α-level interval-valued mapping is a random interval associated with the probability space).

Let $\mathcal{X}$ be an RFN associated with a probability space, and let $\widetilde{\mathbf{x}}_n = (\widetilde{x}_1, \ldots, \widetilde{x}_n)$ be a sample of observations from $\mathcal{X}$. The **sample Aumann-type mean** is the fuzzy number such that for each α

$$(\overline{\widetilde{\mathbf{x}}}_n)_\alpha = \left[\sum_{i=1}^{n} \inf(\widetilde{x}_i)_\alpha / n, \sum_{i=1}^{n} \sup(\widetilde{x}_i)_\alpha / n\right],$$

and the **sample 1-norm median** (Sinova *et al.* [12]) is the fuzzy number such that for each α

$$(\widehat{\mathrm{Me}}(\widetilde{\mathbf{x}}_n))_\alpha = [\mathrm{Me}_i \inf(\widetilde{x}_i)_\alpha, \mathrm{Me}_i \sup(\widetilde{x}_i)_\alpha].$$

3 Case study

The discussions in this paper will be based on the following case study.

Example 1. (Gil *et al.* [4]) This example is related to the well-known questionnaire TIMSS-PIRLS 2011 which is conducted on the **population** of Grade 4 students (i.e., nine to ten years old) and concerns their opinion and feeling on aspects regarding reading, math, and science. This questionnaire is rather standard and most of the involved questions have to be answered according to a 4-point Likert scale, responses being DISAGREE A LOT, DISAGREE A LITTLE, AGREE A LITTLE, and AGREE A LOT.

To get more expressive responses and informative conclusions, some items selected from the original **questionnaire form** (see Table 1) have been adapted to allow a double-type response: the original Likert and a fuzzy rating scale-based one with reference interval $[0, 10]$ (see Figure 2 for one of the items, and http://bellman.ciencias.uniovi.es/smire/Archivos/FormandDatasetFRS-TP.pdf for the full paper-and-pencil form, and Hesketh *et al.* [5] and Lubiano *et al.* [6,8]).

Table 1. Questions selected from the student questionnaire in Example 1

	READING IN SCHOOL
$R.1$	I like to read things that make me think
$R.2$	I learn a lot from reading
$R.3$	Reading is harder for me than any other subject
	MATHEMATICS IN SCHOOL
$M.1$	I like mathematics
$M.2$	My math teacher is easy to understand
$M.3$	Mathematics is harder for me than any other subject
	SCIENCE IN SCHOOL
$S.1$	My teacher taught me to discover science in daily life
$S.2$	I read about science in my spare time
$S.3$	Science is harder for me than any other subject

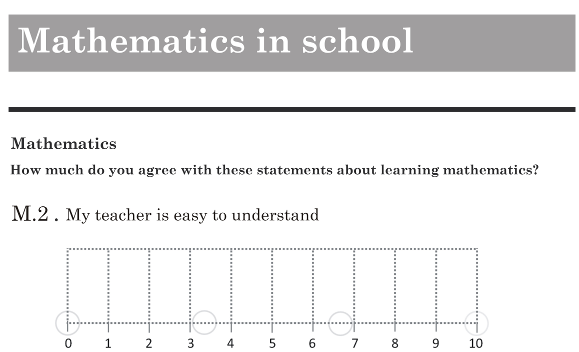

Fig. 2. Example of the response (paper-and-pencil) form to an item in Example 1

The questionnaire involving these double response questions has been conducted in 2014 on a **sample** of 69 fourth grade students from Colegio San Ignacio (Oviedo-Asturias, Spain). These students have been distributed in accordance with (their usual) three groups, so that the teachers have decided that the students in one of the three classrooms have to fill out the paper-and-pencil format and the students from the other two groups have to complete the computerized version.

The **training** of the students to let them know about the meaning and purpose of the case study, as well as the aim of the double response, has been carried out in up to 15 minutes, and three researchers from the Department of Statistics, OR and Math Teaching have been in charge of the explanation and conduction of the survey. At this point, it should be remarked that the students had no idea on the concept of real-valued functions and they have just learned that of a trapezium. With the guidelines enclosed in the form, the students have not had understanding problems, they have catched the philosophy behind and they have been able to provide us with quite coherent responses in most of the cases. Actually, for all the questions, the number of 'no response"s has been very small and smaller for the fuzzy rating than for the Likert scale. In summary, the training has been surprisingly much easier and more effective than we had expected.

Datasets associated with responses to this questionnaire can be found in http://bellman.ciencias.uniovi.es/smire/Archivos/FormandDatasetFRS-TP.pdf.

4 Metrics and scale measures for fuzzy data

Distances have been computed by considering two different metrics: the L^2 metric ρ_2 and the L^1 metric ρ_1 (see Diamond and Kloeden [1]), where for fuzzy numbers $\widetilde{U}, \widetilde{V}$ they are given by

$$\rho_2(\widetilde{U},\widetilde{V}) = \sqrt{\frac{1}{2}\int_{[0,1]} \left[(\inf \widetilde{U}_\alpha - \inf \widetilde{V}_\alpha)^2 + (\sup \widetilde{U}_\alpha - \sup \widetilde{V}_\alpha)^2\right]\, d\alpha},$$

$$\rho_1(\widetilde{U},\widetilde{V}) = \frac{1}{2}\int_{[0,1]} \left[|\inf \widetilde{U}_\alpha - \inf \widetilde{V}_\alpha| + |\sup \widetilde{U}_\alpha - \sup \widetilde{V}_\alpha|\right]\, d\alpha.$$

Let $\mathcal{X}$ be an RFN associated with the probability space $(\Omega, \mathcal{A}, P)$, $\widetilde{\mathbf{x}}_n = (\widetilde{x}_1, \dots, \widetilde{x}_n)$ a sample of observations from $\mathcal{X}$.

Then, the ***(sample) Fréchet-type ρ_2-Standard Deviation, ρ_1-Average Distance Deviation, ρ_2-Average Distance Deviation, ρ_2-Median Distance Deviation, ρ_1-Median Distance Deviation*** are given by

$$\rho_2\text{-SD}(\widetilde{\mathbf{x}}_n) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\rho_2(\widetilde{x}_i, \overline{\widetilde{\mathbf{x}}}_n)\right]^2},$$

$$\widehat{\rho_2\text{-ADD}}(\widetilde{\mathbf{x}}_n) = \frac{1}{n}\sum_{i=1}^{n}\rho_2(\widetilde{x}_i, \overline{\widetilde{\mathbf{x}}}_n),\ \ \widehat{\rho_1\text{-ADD}}(\widetilde{\mathbf{x}}_n) = \frac{1}{n}\sum_{i=1}^{n}\rho_1\left(\widetilde{x}_i, \widehat{\widetilde{\text{Me}}}(\widetilde{\mathbf{x}}_n)\right),$$

$$\widehat{\rho_2\text{-MDD}}(\widetilde{\mathbf{x}}_n) = \text{Me}_i\left\{\rho_2(\widetilde{x}_i, \overline{\widetilde{\mathbf{x}}}_n)\right\},\ \ \widehat{\rho_1\text{-MDD}}(\widetilde{\mathbf{x}}_n) = \text{Me}_i\left\{\rho_1\left(\widetilde{x}_i, \widehat{\widetilde{\text{Me}}}(\widetilde{\mathbf{x}}_n)\right)\right\}.$$

5 Case study-based descriptive discussion

A descriptive comparative study has been developed by computing the scale estimators in the last section over the samples of fuzzy-valued responses to Items in Table 1.

Table 2. Scale estimates values for the responses to Items $R.1$-$R.3$, concerning READING in Example 1, depending on the considered shape

$R.1$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.2609	2.2577	2.2578	2.2581	2.2573	2.2573	2.2329	2.2447
ρ_2-ADD	1.4413	1.4390	1.4689	1.3751	1.4284	1.3800	1.4330	1.5640
ρ_1-ADD	1.3683	1.3647	1.2640	1.3104	1.3658	1.3101	1.3364	1.4309
ρ_2-MDD	1.8205	1.8130	1.8176	1.8105	1.8131	1.8104	1.7645	1.7852
ρ_1-MDD	1.7189	1.7142	1.7113	1.7201	1.7121	1.7178	1.6944	1.7203

$R.2$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	1.8780	1.8733	1.8819	1.8957	1.8794	1.8983	1.8107	1.7554
ρ_2-ADD	1.4514	1.4476	1.4228	1.4082	1.4552	1.4291	1.3200	1.3448
ρ_1-ADD	1.3875	1.3835	1.3339	1.4234	1.3649	1.4037	1.3423	1.3351
ρ_2-MDD	1.6332	1.6237	1.6327	1.6543	1.6312	1.6570	1.5390	1.4835
ρ_1-MDD	1.4959	1.4932	1.4862	1.5323	1.4958	1.5317	1.4498	1.3834

$R.3$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.8987	2.8919	2.9193	2.8947	2.8950	2.8978	2.8535	2.8303
ρ_2-ADD	2.2743	2.2713	2.2857	2.2421	2.2667	2.2353	2.2446	2.0791
ρ_1-ADD	1.5844	1.6013	1.6006	1.5571	1.5652	1.5542	1.6123	1.6688
ρ_2-MDD	2.4180	2.4073	2.4457	2.4028	2.4124	2.4081	2.3459	2.3357
ρ_1-MDD	2.2435	2.2395	2.2807	2.2301	2.2458	2.2368	2.2198	2.1938

Table 3. Scale estimates values for the responses to Items $M.1$-$M.3$, concerning MATHEMATICS in Example 1, depending on the considered shape

$M.1$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.7000	2.6972	2.6866	2.7058	2.6961	2.7035	2.6378	2.6355
ρ_2-ADD	2.3100	2.2998	2.3368	2.4626	2.3166	2.4672	2.2321	2.0475
ρ_1-ADD	2.1719	2.1754	2.1970	2.3441	2.1809	2.3529	2.0469	2.0123
ρ_2-MDD	2.3777	2.3732	2.3642	2.3781	2.3720	2.3761	2.2803	2.2659
ρ_1-MDD	2.2741	2.2734	2.2565	2.2835	2.2706	2.2801	2.2163	2.2048

$M.2$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.3419	2.3380	2.3012	2.3785	2.3357	2.3722	2.2803	2.2165
ρ_2-ADD	1.7976	1.7954	1.7880	1.8233	1.7842	1.8017	1.7443	1.6821
ρ_1-ADD	1.0571	1.0564	0.9956	1.1065	1.0558	1.1030	1.1344	1.1188
ρ_2-MDD	1.9837	1.9788	1.9465	2.0164	1.9781	2.0116	1.8949	1.8272
ρ_1-MDD	1.7374	1.7371	1.6887	1.7951	1.7332	1.7870	1.7159	1.6497

$M.3$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	3.4820	3.4811	3.4686	3.4973	3.4803	3.4951	3.4744	3.4573
ρ_2-ADD	3.4066	3.4061	3.4307	3.3773	3.4142	3.3874	3.3268	3.3392
ρ_1-ADD	3.0357	3.0445	3.0127	3.0160	3.0447	3.0191	3.0068	2.9780
ρ_2-MDD	3.0942	3.0927	3.0844	3.1052	3.0928	3.1041	3.0829	3.0658
ρ_1-MDD	3.0276	3.0273	3.0169	3.0410	3.0268	3.0394	3.0278	3.0097

Table 4. Scale estimates values for the responses to Items $S.1$-$S.3$, concerning SCIENCE in Example 1, depending on the considered shape

$S.1$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.5932	2.5880	2.5909	2.5907	2.5928	2.5924	2.5419	2.5203
ρ_2-ADD	1.7080	1.7053	1.7131	1.7086	1.6889	1.6895	1.6893	1.7510
ρ_1-ADD	1.6899	1.6926	1.6245	1.7540	1.7098	1.7496	1.5653	1.6506
ρ_2-MDD	2.1580	2.1483	2.1427	2.1572	2.1564	2.1590	2.0765	2.0665
ρ_1-MDD	2.0268	2.0208	2.0209	2.0337	2.0233	2.0337	1.9951	1.9901
$S.2$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.3401	2.3316	2.3792	2.3127	2.3368	2.3192	2.2958	2.3077
ρ_2-ADD	1.6748	1.6716	1.7131	1.6096	1.6840	1.6268	1.5829	1.6988
ρ_1-ADD	1.6022	1.5897	1.6734	1.6384	1.5795	1.6343	1.5716	1.6289
ρ_2-MDD	1.9297	1.9182	1.9659	1.8889	1.9239	1.8960	1.8726	1.9007
ρ_1-MDD	1.8317	1.8267	1.8667	1.7972	1.8299	1.8024	1.8150	1.8452
$S.3$	Tra	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
ρ_2-SD	2.9307	2.9247	2.9530	2.9302	2.9286	2.9339	2.8818	2.8394
ρ_2-ADD	2.4098	2.4072	2.4812	2.3751	2.4132	2.3808	2.3283	2.2254
ρ_1-ADD	2.2448	2.2007	2.1720	2.2917	2.1704	2.2904	2.2457	2.1063
ρ_2-MDD	2.5827	2.5742	2.6057	2.5717	2.5790	2.5768	2.5068	2.4671
ρ_1-MDD	2.4685	2.4614	2.4941	2.4676	2.4666	2.4722	2.4379	2.3951

By considering the 4-tuples characterizing the fuzzy responses, we have built the six LU-fuzzy numbers in Figure 1, along with the triangular ones $\mathrm{Tri}(a,b,c,d) = \mathrm{Tra}(a,(b+c)/2),d)$, $\mathrm{TriS}(a,b,c,d) = \mathrm{Tra}(a,(a+d)/2),d)$. After computing the scale estimates we have obtained the outputs in Tables 2, 3, and 4. For each of the Items and scale estimates, one can conclude that the outputs scarcely differ.

6 Case study-based inferential discussion

In this section, we are going to examine, by means of an inferential analysis of the case study in Example 1, the influence of the shape of fuzzy data on the statistical conclusions.

The discussion is carried out on the basis of the test about the equality of variances with fuzzy data, the ***bootstrapped homoscedasticity test of k independent RFN's***, developed by Ramos-Guajardo and Lubiano [11], which is now algorithmically summarized for the two-sample case. If $\mathcal{X}_1, \mathcal{X}_2$ are independent RFN's, consider a sample of independent observations $\widetilde{\boldsymbol{x}}_{\boldsymbol{i}} = (\widetilde{x}_{i1}, \ldots, \widetilde{x}_{in_i})$ from $\mathcal{X}_i$, $i = 1,2$, the two samples being also independent, with $n = n_1 + n_2$. Denote $\overline{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}} = \frac{1}{n_i} \cdot (\widetilde{x}_{i1} + \ldots + \widetilde{x}_{in_i})$ the sample Aumann-type mean for $\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}$, $S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}} = \sum_{j=1}^{n_i} \left[\rho_2(\widetilde{x}_{ij}, \overline{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}})\right]^2 / n_i$ the sample Fréchet-type variance for $\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}$, and $\overline{S^2_{\widetilde{\boldsymbol{x}}}} = \sum_{i=1}^{2} n_i \cdot S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}} / n$.

Then, the bootstrapped algorithm to test the null hypothesis $H_0 : \sigma^2_{\mathcal{X}_1} = \sigma^2_{\mathcal{X}_2}$ (equality of the population Fréchet-type variances) proceeds as follows:

Step 1. Compute the value of the statistic

$$T_{n_1,n_2} = \frac{\displaystyle\sum_{i=1}^{2} n_i \left(S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}} - \overline{S^2_{\widetilde{\boldsymbol{x}}}}\right)^2}{\displaystyle\sum_{i=1}^{2} \frac{1}{n_i} \sum_{j=1}^{n_i} \left(\left[\rho_2(\widetilde{x}_{ij}, \overline{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}})\right]^2 - S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}}\right)^2}$$

Step 2. For each $i \in \{1,2\}$, obtain a bootstrap sample from $\left(\widetilde{x}_{i1} \cdot \sqrt{\overline{S^2_{\widetilde{\boldsymbol{x}}}}/S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}}}, \ldots, \widetilde{x}_{in_i} \cdot \sqrt{\overline{S^2_{\widetilde{\boldsymbol{x}}}}/S^2_{\widetilde{\boldsymbol{x}}_{\boldsymbol{i}}}}\right)$, $\widetilde{\boldsymbol{x}}^*_{\boldsymbol{i}} = (\widetilde{x}^*_{i1}, \ldots, \widetilde{x}^*_{in_i})$, and compute the value of the bootstrap statistic

$$T^*_{n_1,n_2} = \frac{\displaystyle\sum_{i=1}^{2} n_i \left(S^2_{\widetilde{\boldsymbol{x}}^*_{\boldsymbol{i}}} - \overline{S^2_{\widetilde{\boldsymbol{x}}^*}}\right)^2}{\displaystyle\sum_{i=1}^{2} \frac{1}{n_i} \sum_{j=1}^{n_i} \left(\left[\rho_2(\widetilde{x}^*_{ij}, \overline{\widetilde{\boldsymbol{x}}^*_{\boldsymbol{i}}})\right]^2 - S^2_{\widetilde{\boldsymbol{x}}^*_{\boldsymbol{i}}}\right)^2}$$

Step 3. *Step 2* should be repeated a large number B of times to get a set of estimates, denoted by $\{t^*_1, \ldots, t^*_B\}$.

Step 4. Compute the bootstrap p-value as the proportion of values in $\{t^*_1, \ldots, t^*_B\}$ being greater than T_{n_1,n_2}.

Table 5 collects the corresponding p-values (with $B = 10000$) for testing the equality of the population Fréchet-type variances for the trapezoidal RFN *vs* the other seven LU-valued RFN's.

Table 5. p-Values for the equality of population Fréchet's variances of trapezoidal *vs* other LU's responses for to Items $R.1$ to $S.3$ in Example 1

p-values Tra *vs* ...	Π	LU_{1A}	LU_{1B}	LU_{2A}	LU_{2B}	Tri	TriS
$R.1$	0.992	0.993	0.993	0.990	0.990	0.917	0.957
$R.2$	0.981	0.991	0.940	0.999	0.908	0.740	0.537
$R.3$	0.984	0.954	0.991	0.985	0.998	0.896	0.842
$M.1$	0.992	0.966	0.988	0.990	0.985	0.814	0.814
$M.2$	0.992	0.913	0.925	0.988	0.903	0.851	0.704
$M.3$	0.996	0.956	0.955	0.995	0.949	0.976	0.913
$S.1$	0.987	0.994	0.992	0.997	0.996	0.861	0.793
$S.2$	0.977	0.900	0.931	0.984	0.937	0.886	0.919
$S.3$	0.979	0.933	0.995	0.984	0.989	0.839	0.705

On the basis of the obtained p-values, one can immediately conclude that in computing Fréchet-type variance, data shape seems not to be significantly influential.

For all the usual significance levels one can consider, there are no significant differences between population Fréchet's variances for the seven developed comparisons, even for the triangular shaped data.

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