Portfolio Optimization with Cryptocurrencies

Master's Thesis submitted

to

Prof. Dr. Wolfgang Karl Härdle

Dr. Alla Petukhina

Humboldt-Universität zu Berlin

School of Business and Economics

Ladislaus von Bortkiewicz

Chair of Statistics

by

Dinesh Sivagourou

(586622)

in partial fulfillment of the requirements

for the degree of

Master of Statistics

Berlin, August 28, 2018

Acknowledgement

I would like to thank my supervisor Professor Härdle for his useful advice and guidance during this master's degree and the master thesis. His support helped me to understand and discover the field of crypto-currencies. I would like to thank him especially for the trip in Frankfurt where we had the chance to visit the ECB, it was a memorable experience. I would like also to thank Alla Petukhina my second supervisor who helped me to find my master thesis subject and helped me all along the writing of my master thesis. I want to thank Marius Sterling who helped me all along with my master's degree. His advice and technical skills helped me a lot to achieve my different projects, Bruno Spilak and Victor Cluzel my classmates who helped me and gave me advice during the writing of this master thesis. My gratitude also to Dr. Simon Trimborn who provided me the data.
Overall I would like to thank the members of the Ladislaus von Bortkiewicz Chair of Statistics and also ENSAI and HU for giving me the opportunity to study in Germany and to do this double-master's programm.

Abstract

Investors, asset managers are constantly looking for financial instruments that could increase the expected return on their investment while minimizing the potential risk. In practice Asset managers will diversify their asset allocation by chosing among different asset classes : stocks, bonds, commodities, etc. Since the last few years a new category of assets has emerged : Cryptocurrencies. The first one called Bitcoin became very famous because of his technology : decentralized, secure and fast electronic money.

This paper focus on the performance of core-satellite management with those new digital currencies as satellite. The challenge is finding out those crypto-currencies that could compensate downward trends and give better returns to the investor and limiting the risk. To select those crypto-currencies we need to study the correlation structure and find crypto-currencies that are moving adversely. To do so we will use TEDAS - Tail Event Driven ASset allocation an active selection technique to select our cryptocurrencies. This method study the dependence of cryptocurrencies at different quantiles in the left tail of the ditribution. We will compare different srategies based on TEDAS to select those cryptos .

Contents

List of Abbreviations

Cryptos	Crypto-currencies
ecdf	Empirical distribution function
TEDAS	Tail Event Driven ASset allocation
QR	Quantile Regression

List of Figures

List of Tables

1 Introduction

When one is investing, the main goal is to increase the return on this investment. But the increase of the return can't be achieved without an increase of risk. The new theory of modern portfollio introduced by Harry Markowitz help us to solve this problem by using the effect of diversification. He explained that a smart combination of assets in a portfolio can reduce the risk for a given expected return.

The stake is to find those assets. The performance relatively low of bonds or stocks leads us to think for alternatives such as hedge funds ,precious metals funds or High Yield Bonds. The basic idea of chosing the satellites is potential to deliver higher returns. Research papers advises Hedge funds as a good alternative because they can stabilize the portfolio in terms of risk and increase the return.

Since a few years a new category of financial assets emerged : Crypto-currencies. Those digital currencies attracts more and more investor because of their nature. Crypto-currencies are digital money using the decentralized technology to be traded. The main advantage are security and transaction costs. What is also interesting is that for some like Bitcoin, not only the market capitalization explode but also the price. It was exchanging at the begining at 0.008 $ now it has reached 10000 $. Since lots of new digital currencies emerged.

The increasing popularity of those new currencies also attracts investors to use them as alternatives. For example the biggest asset manager BlackRock is exploring this technology in order to include them in their portofolio in the furute. Several paper studied the portfolio diversification with Bitcoin. The Bitcoin as alternative exhibits a high volatility and a high return.

In this study we decided to focus on asset allocation strategies using different crypto-currencies. To deal with this issue we will use core-satellite investment strategy with cryptos as satellites.The challenge is to see if by including cryptos in a particular investment strategy, it could perform as good as or better than hedge-funds, mutual funds or bonds. The first section motivates the use of cryptos in our investment strategy. First we will develop the different methods used with TEDAS to find the cryptos for diversification. We will show the results for the different methods and analyse them. Then we will see what parameters we could modify in order to improve the performance of the different strategies.

2 Method

2.1 Investment Core-Satellite

Basically, the core-satellite approach is a an investment approach that combines the Index funds - low cost, good diversification, and attenuated volatility - to actively managed funds or other alternatives of direct investments with potential for excess returns. The core-satellite approach gives additional stability to an investment portfolio .

There are three steps to make Core-Satellite investment :

1. Determination of risk profile and asset allocation

   Analysis of the risk profile is the first step in the construction of a portfolio. Then the distribution of the assets between stocks, fixed-income securities, currencies and other investments.

2. Distribution core / satellite

   For each asset class, determine how much will be allocated to index management, and active management. This depends on the degree of risk the investor is willing to accept

3. Find the core and satelites

   After selecting the core you need to find the assets that will be as satelites. Ideally, assets should show a low or negative correlation with the core .

In our study we will just select as core major indices like SP500, FTSE100 ... As it is the main goal of our study we will use cryptos as satellites.

2.2 Selection of Cryptos

2.2.1 Quantile Regression

Conversly to the simple regression , which study the effects of different factors on the mean, quantile regression give us the impact on different quantiles of the response variable. So QR make possible to study the impact of different factors on the whole distribution of the variable of interest.

First we will consider $\{(Y_i, X_i) \text{ with } i \in [\![1 \ ; \ n]\!]\}$ and $X_i \in \mathbb{R}^p, Y_i \in \mathbb{R}$.

Let ρ_τ a test function define as follow :

$$\rho_\tau(u) = u\{\tau - \mathbf{I}(u < 0)\}$$

The conditional quantile function is then given by :

$$q_\tau(Y|X = x) = x^\top \beta(\tau) = \arg \min_{\beta \in \mathbb{R}^p} \mathrm{E}_{Y|X=x}\, \rho_\tau\{Y - X^\top \beta\},$$

The estimator of the quantile regression is given by :

$$\hat{\beta}_\tau = \arg \min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(Y_i - X_i^\top \beta) \tag{1}$$

There is no explicit solution to (1), so we have to solve this program numerically. One problem is that the objective function is not differentiable (the function ρ_τ is not differenciable in 0 nor strictly convex). Standard algorithms such as Newton Raphson can not be used here. We won't develop the computation here but we used a very complete R package was developed by R. Koenker: quantreg.

2.2.2 TEDAS

TEDAS is a method that allows us to find assets moving adversly than the core at different level of lower tail. We will deal only we quantiles lower than the median because we are assuming that the median return of the crypto is positive. As we don't have the problem of high dimensionality, we won't use ALQR but only a simple QR.

First we will define some notation and hypothesis :

- Initial Wealth $W_0 = 1\$$
- n = number of observations and p = number of assets
- l the moving window length and $t = l, ..., n$,
- $Y \in \mathbb{R}^n$ core log-returns; $X \in \mathbb{R}^{n \times p}$ satellites' log-returns, $n > p$
- $\tau_{j=1,..,5} = (0.05, 0.15, 0.25, 0.35, 0.50)$ a set of quantile indices
- We define $\hat{F}_n(x)$ the ecdf of the core Y
- $\hat{q}_\tau = \hat{F}_n(\tau)$ the empirical quantile function of the core

Here are the steps for TEDAS :

1. Determine core asset return Y_t and set $\tau_t = \hat{F}_n(Y_t)$ with $\tau_{j=1,..,5} = (0.05, 0.15, 0.25, 0.35, 0.50)$
2. Select τ_j, t according to the right side in , $Y_t \leq \hat{q}_{\tau_1,t}$ or $\hat{q}_{\tau_1,t} < Y_t \leq \hat{q}_{\tau_j,t}$
3. Quantile regression between $Y \in \mathbb{R}^{t-l+1,..,t}$ and $X \in \mathbb{R}^{(t-l+1,..,t)\times p}$
4. Choose adversely moving assets with the core Y
5. Apply on of the TEDAS strategies to determine assets weights $\hat{w}_t$
6. Derive the realized portfolio wealth at t+1 :

   $W_{t+1} = W_t + \hat{w}_t^\top X_{t+1}$

After getting weights the question now is what is the decision to make. There are three different posibilities :

1. One of the equalities in step 2 holds :
   - sell the core portfolio and buy satellites (step 4) with estimated weights
   - stay "in cash" if there are no adversely moving satellites (step 4)
2. No one of the equalities in step 2 holds : invest in the Core

For the step 5 we have different strategies to get the weights. Those will be explained in the following sections.

2.2.3 TEDAS Naive

TEDAS naive is just simply that every cryptos receives an equal portfolio weight.

$$\forall\, i, j \in [\![1\ ;\ p]\!] \quad w_i = w_j$$

The main advantage of this strategy is the simplicity in terms of implementation and calculation. But normaly portfolio presents some correlation. This portfolio does not take into account volatilities and cross-correlations. As we can see it is not a very developped method but this will give a basic idea if we are just interested on selecting adversly moving cryptos on performance.

2.2.4 TEDAS equally weighted optimisation

In this strategy we are weighting the return by the corresponding cryptos volume.

$$\forall\, i \in [\![1\ ;\ p]\!] \quad w_i = \frac{m_i}{\sum_{i=1}^{n} m_i}$$

with m_i the volume of i_h crypto

This is like the previous method in terms of calculation and implementation . It has also the same disadvantages of not taking into account the correlation structure among cryptos.

2.2.5 ERC Optimization (Equally Risk weighted)

The Equally-weighted Risk Contributions takes into account the interactions between the different cryptos likely to be selected in the portfolio. That's why we will take as a paramater the covariance to explain the exposure to the risk allocation.

Let Σ the covariance matrix of the selected cryptos and $\sigma(w) = \sqrt{w^\top \Sigma w}$. Euler decomposition:

$$\sigma(w) \stackrel{\text{def}}{=} \sum_{i=1}^{n} \sigma_i(w) = \sum_{i=1}^{n} w_i \frac{\sigma(w)}{\partial w_i}$$

where $\frac{\sigma(w)}{\partial w_i}$ is the marginal risk contribution and $\sigma_i(w) = w_i \frac{\sigma(w)}{\partial w_i}$ the risk contribution of i-th asset. The idea of ERC strategy is to find risk balanced porfolio, such that:

$$\sigma_i(w) = \sigma_j(w)$$

i.e. the risk contribution is the same for all assets of the portfolio

Conversly to the naive method ERC takes into account not only the volatility of each asset but also their correlation (The covariance matrix is deduced from these two indicators). ERC approach has the advantage of optimizing level and uses the decorrelation potential of each cryptos that is included in the portfolio. Moreover, this technique uses directly the covariance matrix and not its reverse. Therefore perfectly it meets our expectations in terms of stability. The disadvantage is the computation of the weights. They are solution of more complex equation than the previous method.

2.2.6 TEDAS Mean-Variance optimization (Tedas Hybrid)

The Mean-Variance optimization strategy is a method that minimize the variance of the portfolio for a target return r_T.A variation of the Mean-Variance optimization is to maximize the expected return gor a given level of risk.

$$
\begin{aligned}
\underset{w \in \mathbb{R}^d}{\text{minimize}} \quad & w^\top \Sigma w \\
\text{subject to} \quad & w^\top \overline{r} = r_T, \\
& \sum_{i=1}^{d} w_i = 1, \\
& w_i \geq 0
\end{aligned}
$$

where w_i, $i = 1, \ldots, p$ are weights, $\Sigma \in \mathbb{R}^{d \times d}$ is the covariance matrix for p portfolio cryptos returns $\overline{r}_i$, r_T is the "target" return for the portfolio.

In our case we will use a compromise between risk and return by maximizing the Sharpe Ratio. The Sharpe Ratio is defined by :

$$
Sharperatio = \frac{\mu_p - R_f}{\sigma_p}
$$

where μ_p is the return of the p portfolio cryptos and σ_p the covariance matrix. R_f is the return of an riskless asset.

To do so we will construct the efficient frontier. We call efficient portfolio any portfolio that offers the highest expected return for a given level volatility. And then we will calculate the sharpe ratio for each efficient portfolio. We will select the portfolio that maximizes the Sharpe ratio.

2.2.7 Minimum-Variance optimization

This strategy responds the question : Which efficient portfolio offers the lowest level of risk? This consistis of to optimizing the following quadratic program:

$$
\begin{aligned}
\underset{w \in \mathbb{R}^d}{\text{minimize}} \quad & w^\top \Sigma w \\
\text{subject to} \quad & \sum_{i=1}^{d} w_i = 1,
\end{aligned}
$$

3 Data

3.1 Data processing

The data has been provided by the Chair of Statistics. It's coming from the CRIX database. The CRIX index is a real time benchmark computed by the Chair of Statisitics of Humboldt University. Like other references benchmarks SP500, DAX30, the CRIX is an index for crypto-currencies. The challenge was to create a new indice that could represnents the crypto-currencies market like the usual indices reprensents financial markets segments. That is how the chair of Statistics along with other partners come up with a new index. For more information see : http://thecrix.de/.

Here is the plot of the current CRIX indice :

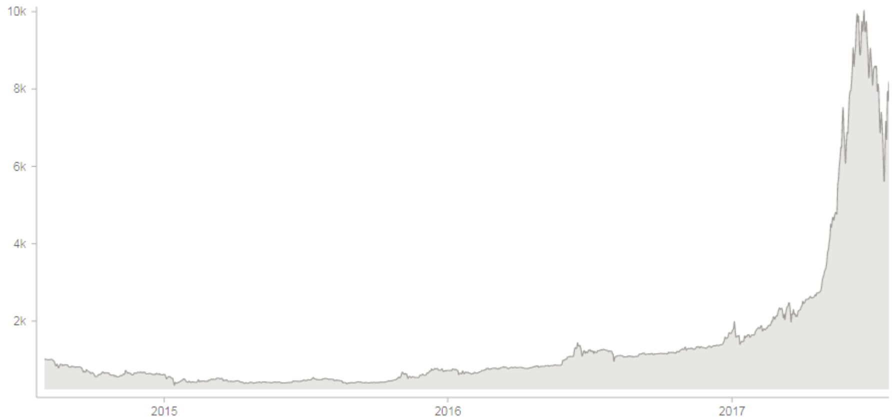


Figure 1: CRypto IndeX Source : crix.hu-berlin.de

The data set has been provided mid-May 2017 and had 475 cryptos. The oldest value is 2013-04-28 and the latest 2017-05-14. That corresponds to around 3 years of daily observations. The data was almost clean. There wasn't any missing values. The data had 259 802 observations with 11 variables described as follows :

- crypto_symbol : Code of the cryptos (btc = bitcoin).
- price_usd : Price in USD of the crypto at a certain time.
- total_volume_usd : Volume of the crypto at a certain time.
- market_cap_usd : share price multiplied by the number of shares outstanding in USD
- price_btc : Price in USD of the crypto relative to the price of bitcoin at a certain time.

- total_volume_btc : Volume of the crypto relative to the volume of bitcoin at a certain time.
- market_cap_btc : share price multiplied by the number of shares outstanding in btc
- date

We noticed that cryptos period were not matching. Some cryptos life span were very small compared to others. So we decided to find a period were we could get the maximum cryptos on daily observations. After working on the data the framework of our study will be from **2015-02-04 to 2017-04-25** . All results are done with log returns. In the rest of the study we will just use the word return.

3.2 Descriptive statistics

In the previous section we explained how we cleaned the data, now we will use the processed data to show first basic statistics.

We first show in Figure 2 the plot of all cumulative cores with an initial investment of 1 dollar. One can see that we have 4 cores with same general tendancies : SP500, NASDAQ, FTSE100 and DAX30. The Nikkei 225 has the same movement at the begining and then changes adversly compared to other cores. That could be interseting to challenge our models to see how models are behaving when core are slighty different in tendancies but also totally like Nikkei 225.

Then we checked the normality of the return of our cryptos. Some details of the results are given in the Appendix A.1 Descriptive Statistics on Cryptos. None of our cryptos has a normal return because the Shapiro normality test has been rejected for all of them at a significance level of 5% .

We looked to the correlation structure of the cryptos with our core indices.The Figure 3 depicts the correlation between the core Nasdaq computer and all the cryptos. We can see for all cryptos correlation are less than 0.1 in absolute value. See the plots also for the other cores in Figure 11. We can also notice that there are more cryptos with negative correlations than positive. That could be potentially be interesting for selecting adversly moving satellites. The fact that correlations are close to 0 is interesting for us. First this mean that cryptos are not highly positvely correlated with the core. Then we have also there are also slighlty negatively correlated cryptos. So we could use them to hedge or outperform during decline of the core .

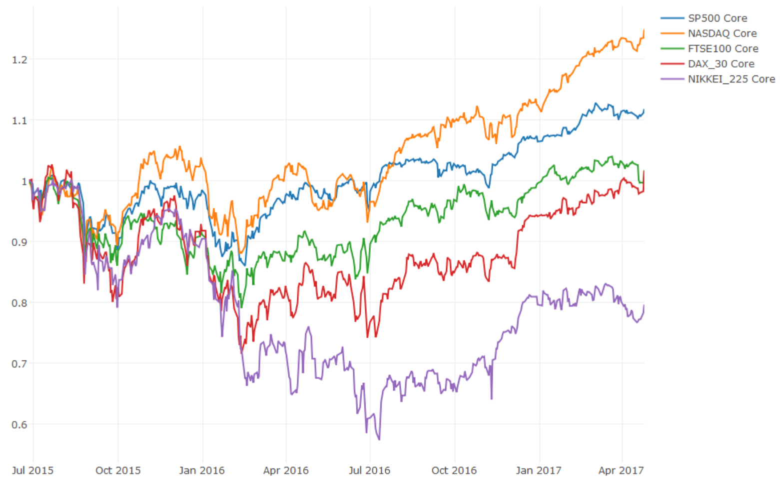


Figure 2: Cumulative return for the different cores

CRIXcoreWealth

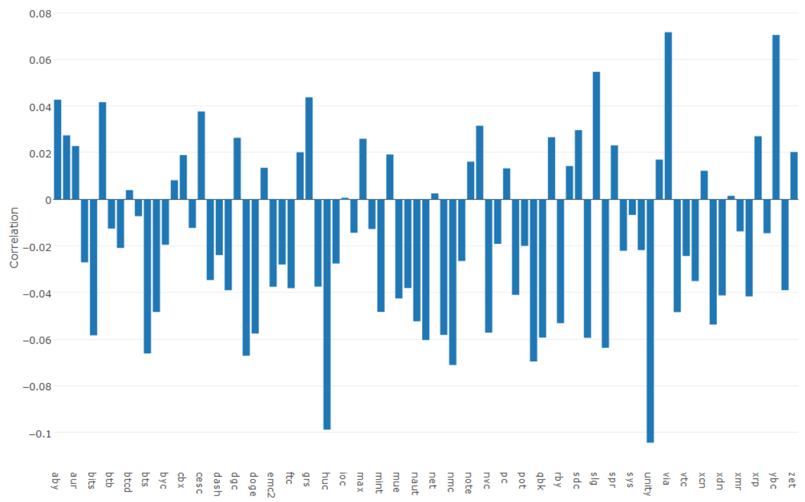


Figure 3: Correlation beetween SP500 and cryptos

CRIXtedasCorrelationCoreCryptos

4 Results

4.1 Analysis of the different strategies

In this section we will review the behaviour of the different strategies compared to the different cores.

We will proceed in two steps. First we give a first insight of the strategies by looking at graphical representation of all strategies with SP500 as core. This approach will allows us to better understand those strategies. In a second time we give a grid of several metrics for all strategies and cores. Metrics will try to give us a better indications to select among strategies. The analysis is conducted by calculating the cumulative return (Strategy wealth) at each date until the end of the period for each strategies for a fixed window length. Then we compare the strategy's wealth with the core wealth. As explained in the methodology every strategy depends on the window length parameter. We start the study with a fixed window length of 100 days. The parameter has been chosen to get enough data to run the strategy correctly and to not get the dimensionality issue. We will come on further section on the choice of this parameter.

4.1.1 Analysis of the different strategies with SP500 core

Figure 3 is splitted in two parts. At the begining, from the end of June 2015 until the begining of January, the naive TEDAS doesn't seem to select correctly the adversly moving cryptos. Then we could see that the strategy capture correctly the cryptos to outperform the core portfolio. We could see in the second part there is quite an increasing cumulative return until the end. We have also to keep in mind that in general for cryptos, there have been a positive cumulative return in the second period due to massive investment from investors (see Figure 1).

Figure 4 shows us the Hybrid strategy. This strategy is working less better than the Naive. We have to wait a longer period of time to perform better than the core portfolio. In addition we are getting a very long downward return at the begining. The selection of cryptos doesn't work well with this strategy.

The equally risk weighted strategy (Figure 5) offers better performance. As we can see the excess cumulative return on this strategy is almost positive all over the considered period. We still have some issues at the begining to capture the adversly moving cryptos. In fact we still have some downward movement.

The volume weighted strategy doesn't perform well also during the first period (June 2015 - End January 2016). The overall cumulative return is less stable compared to other strategies . This strategy will give more importance obviously to cryptos with larger market caps. Some cryptos like bitcoin have very large market cap and some others are negligeable. Due to this fact some weights should be close to zero if small market cap cryptos have been selected along with larger marketcap cryptos. As a consequence this strategy will be mainly driven by larger market cap cryptos. For example if the quantile regression is capturing some large market cap cryptos and they have a downward movement at the next step, this will very negatively

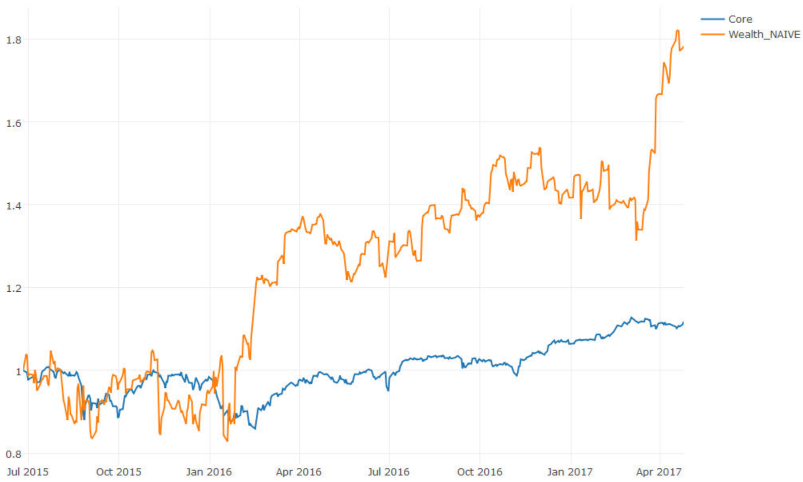


Figure 4: Cumulative return for the Core SP500 and TEDAS Naive

CRIXtedasNaive

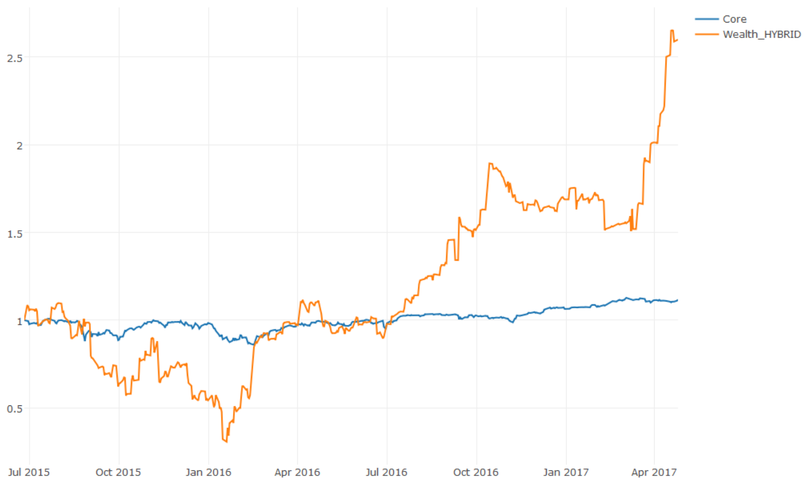


Figure 5: Cumulative return for the Core SP500 and TEDAS Hybrid

CRIXtedasHybrid

impact our portfolio.

The Minimum variance (Figure 7) strategy gives really good performance overall compared to other strategies but juste before the end of the considered period there is a huge drop in return.

Overall it's complicated to conclude that one strategy is outperforming the other, some are behaving well but not all the period. We will see how these strategies work with other cores.

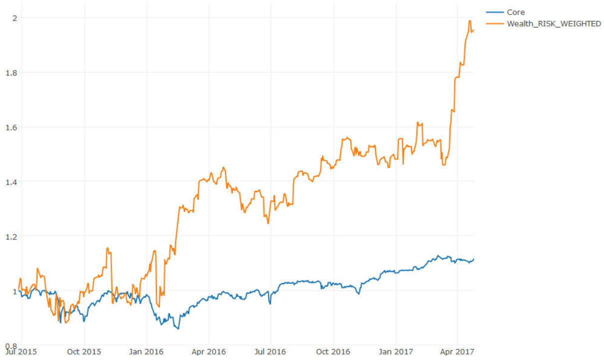


Figure 6: Cumulative return for the Core SP500 and TEDAS Risk Weighted

CRIXequallyRiskWeighted

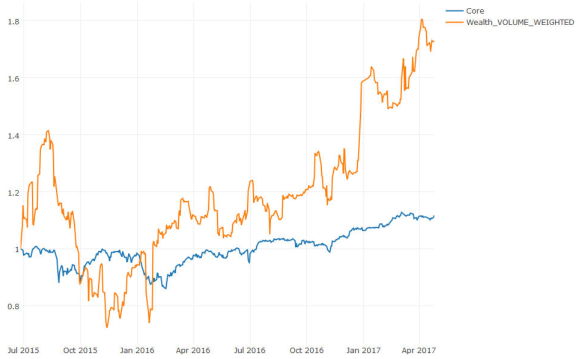


Figure 7: Cumulative return for the Core SP500 and TEDAS volume weighted

CRIXvolumeWeighted

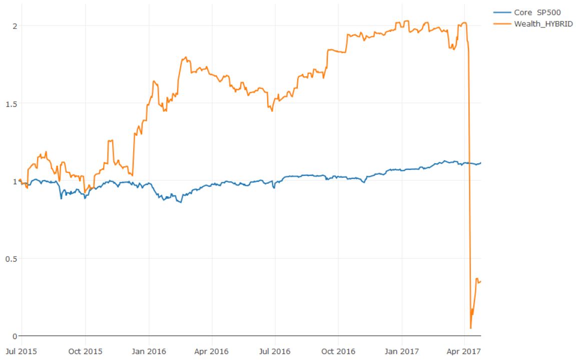


Figure 8: Cumulative return for the Core SP500 and TEDAS Minimum Variance

CRIXminimumVariance

4.1.2 Strategies' Performance

For this section we dropped the volume weighted strategy. As we explained in the previous section, this strategy leads to weights close to 0 when we are selecting with TEDAS large market cap coins like bitcoin or ethereum. This could have huge impact if we didn't predict correctly and in the next step for example if we tedas has selected the bitcoin and plummet.

We also need to keep in mind when we write period it refers to the period **2015-02-04 to 2017-04-25**.

Here we give some indicators that could help us to compare those strategies. We also plot the cumulative return for all strategies for each cores. See A.2 Performance of the different strategies.

We define the **Excess return** as the difference between the strategy weatlh and the core wealth. The **negative and positive cumulative excess return** represents the sum of the negative or null and positive cumulative excess return respectively. The **negative and positive excess return freq** represents the number of times the excess return is negative or null and positive respectively. The **Final excess return** is the excess return at the end of the period. For example in Table 1 , for the Naive strategy at the end of the period we would have 109.82 % of excess return against an investment on the core SP500 only.The **Maximum drawdown** represents the maximum loss recorded by our portfolio over the period.

Strategies	Negative cumulative excess return	Negative excess return freq	Positive cumulative excess return	Positive excess return freq	Final excess return	Maximum drawdown
Naive	-4,53	82	114,34	378	66%	0,20
Risk weighted	-1,07	38	141,87	422	84%	0,21
Hybrid	-37,00	195	120,27	265	136%	0,75
Minimum-Variance	-9,41	17	256,92	443	77%	1,03

Table 1: Metrics for the SP500

Strategies	Negative cumulative excess return	Negative excess return freq	Positive cumulative excess return	Positive excess return freq	Final excess return	Maximum drawdown
Naive	-56,77	287	11,60	173	0.42%	0,48
Risk weighted	-46,53	275	14,64	185	4.04%	0,44
Hybrid	-496,48	451	0,30	9	-122.15%	1,60
Minimum-Variance	-12,39	123	41,75	337	16.31%	0,45

Table 2: Metrics for the FTSE100

Strategies	Negative cumulative excess return	Negative excess return freq	Positive cumulative excess return	Positive excess return freq	Final excess return	Maximum drawdown
Naive	-0,28	16	121,38	444	40.02%	0,23
Risk weighted	-2,77	49	103,00	411	41,59%	0,24
Hybrid	-134,59	416	5,91	44	-31.03%	0,74
Minimum-Variance	-3,38	64	79,69	396	40.29%	0,30

Table 3: Metrics for NASDAQ

Strategies	Negative cumulative excess return	Negative excess return freq	Positive cumulative excess return	Positive excess return freq	Final excess return	Maximum drawdown
Naive	-49,77	351	8,69	109	-13.90%	0,54
Risk weighted	-84,08	418	2,75	42	-12.14%	0,54
Hybrid	-562,10	440	0,89	20	-140.71%	2,00
Minimum-Variance	-69,03	362	7,11	98	-62.42%	0,88

Table 4: Metrics for DAX 30

Strategies	Negative cumulative excess return	Negative excess return freq	Positive cumulative excess return	Positive excess return freq	Final excess return	Maximum drawdown
Naive	-8,36	90	75,22	370	42.36%	0,42
Risk weighted	-4,30	57	114,76	403	61.64%	0,36
Hybrid	-73,23	264	32,20	196	10.48%	0,94
Minimum-Variance	-0,16	5	173,27	455	64.78%	0,32

Table 5: Metrics for the NIKKEI 225 core

As can be seen from the results in the different tables there is not one strategy that outperforms the others. Overall the different strategies works better with some cores and underperform with other cores.

If we take a look on the Minimum-Variance strategy, it is performing well with Nikkei

225 and NASDAQ and underperform with the others. For Nikkei, the excess return is only negative 5 times at the begining of the period (See Figure 16). However if you see this strategy for SP500 core, the excess return is almost positive until the end of the period but we have huge decline just before the end of the period.

The Risk weighted strategy offers also some good performances with Nikkei, NASDAQ and in particular SP500. With the latter (See Figure 4), the wealth is always positive after a period of stabilization and then there is an upward trend until the end of the period.

For the naive strategy, we have similar results like the equally risk weighted strategy on SP500 and NASDAQ. After a period of stabilization we have an upward trend until the end of the period. It's intersting to see that one strategy that uses equal weights ond the other one which takes the correlation and volatility could have similar pattern. This could be explained that cryptos that have selected have same structure.

The Hybrid strategy offers the worst performance compared to others. Especially with FTSE100 and Dax 30 where we could see in the tables and A.2 Performance of the different strategies that the excess return is almost always negative.

We also see that for every core there is at least one strategy that shows high financial risk. We have for those strategies a maximum dropdown of at least 100 %. Meaning a potential loss of 100 % during the period.

Obviously, strategies behaves very differently with the different cores. However we set the window length at 100 days and at the moment we don't know if this is the optimal setting. We selected 100 days first to get enough data and not the dimensionality issue. So we need to check the robustness of the different strategies when this parameter changes. We will tune in the next section the window length and see how the different methods adapt.

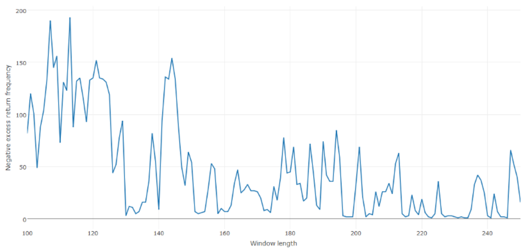
4.2 Optimization of the window

In this section, we perform a tuning of the window length to see the evolution of the different strategies' wealth. The window length should allow the strategy to capture the relevant information of the correlation structure between core and the cryptos. A small window wouldn't allow us to capture all the relevant information , but a higher window would probably overfit the correlation structure. So the idea is to find a compromise . We proceed by splitting the data into two parts: one part for training (400 days) and the remaining (159 days) for testing. We are running multiple simulations by changing the window length in the range from 100 days to 250 days. At the end we are selecting the best window length in terms of excess return. We will select windows for which we have the least amount of negative excess return and with the best amount positive cumulated excess return. Then we are testing it on the remaining period to see if the chosen window is still optimized for the testing data.

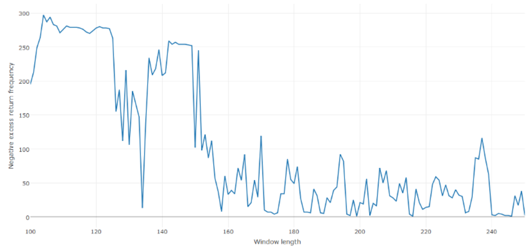
4.2.1 Grid Search for the optimal window

In the Figure 9, we plot the negative excess return frequency with respect to the window length. This will show us how many times the excess return is negative when changing the window length. Nonetheless minimizing the excess return frequency doesn't mean that we will have a better return, we then have to find which window give the best performance at the end of the training period.

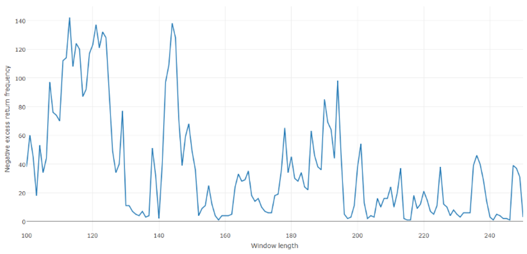
As can be seen from the results in Figure 9 (see also A.3 Performance of the strategies when window length changes), we observe that when we increase the window length, the negative excess return frequency decreases for all cores. But it is not decreasing linearly. The negative excess return frequency is very sensitive to the window length. Indeed we have huge variations when changing the window length of 1 day. This could be explained by the volatility of cryptos. Indeed, even though we are selecting adversly moving cryptos at a certain time t, due to their volatility, cryptos could have negative return at time t+1.



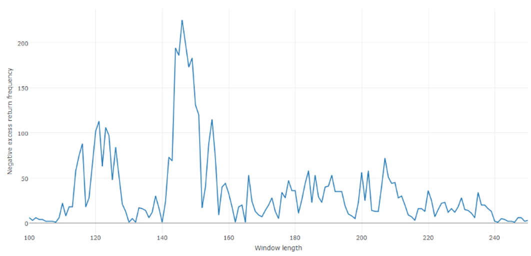
(a) TEDAS Naive



(b) TEDAS Hybrid



(c) Equally Risk Weighted



(d) Minimum-Variance

Figure 9: Negative excess return frequency of the different Strategies on SP500

CRIXtedasWindowlengthGrid

Optimal window length

Now the grid search has been processed, we have selected the windows with the least negative excess return frequency. Then we had several times different windows with the same negative excess return frequency. So we selected the window with the higher positive excess return if there was equality.

Strategies	SP500	FTSE100	NASDAQ	DAX30	NIKKEI225
Naive	231	213	137	198	154
Hybrid	199	175	213	242	154
Risk Weighted	158	213	137	198	154
Minimum-Variance	108	213	116	209	172

Table 6: Optimal window length of the cores and the different strategies

In Table 6, we observe that we have very different optimal window lengths depending on the cores and strategies. But it is interesting to see that the window length stay in a certain range. For example for the FTSE 100 and DAX 30 the optimal window length is relatively higher than for the NASDAQ and Nikkei 225. For the SP500 we have very different window length according to the strategies. We see here that the window length most probably needs to suit to the strategies and the benchmark to outperform..

4.2.2 Backtest of the different strategies with the optimal window length

We now investigate the performance of the different strategies on the test data. As explained before we trained the different strategies on 400 days. What we have done is we initialize each strategy/core with their optimal window length so that the first return on the different strategies will be achieved on the day 401. By doing so we could compare the performance of each strategies and do correct Backtesting.

By configuring every strategies/cores with their respective optimal window, we should expect only positive excess return overall during backtesting until the end of the period of testing. Unfortunately, results are varying accross strategies/cores. For example, if we look at SP500 (a) Figure 10, we have a positive excess return only for the Naive strategy. For the MV strategy, we have to wait for a period of stabilization until it is positive. And for the RW and Hybrid it remains negative during all the considered period.
If we look at DAX 30 Table 7 , we have positive excess return at the end of the period for

Strategies	SP500	FTSE100	NASDAQ	DAX30	NIKKEI225
Naive	12%	-33%	44%	32%	76%
Hybrid	-32%	45%	-13%	2%	31%
Risk Weighted	-8%	20%	79%	10%	58%
Minimum-Variance	48%	-59%	36%	29%	81%

Table 7: Cumulative return for the optimal window length of the cores and the different strategies

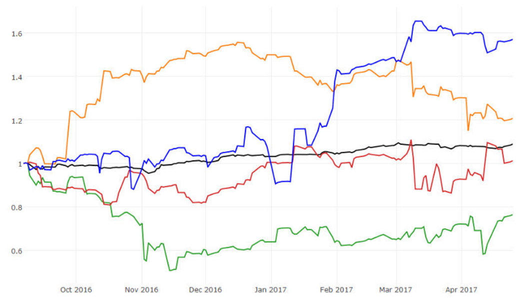
Strategies	SP500	FTSE100	NASDAQ	DAX30	NIKKEI225
Naive	0,26	0,35	0,22	0,20	0,15
Hybrid	0,50	0,35	0,34	0,25	0,14
Risk Weighted	0,22	0,57	0,26	0,25	0,13
Minimum-Variance	0,22	0,20	0,18	0,15	0,11

Table 8: Overview of the Maximum drawdown for each core and strategies

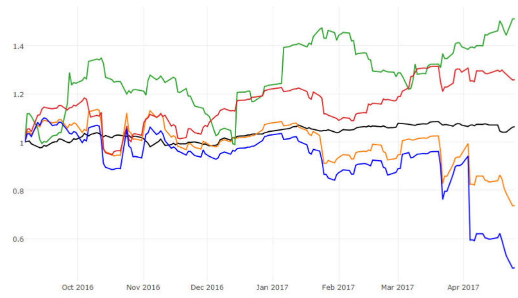
all strategies. But when looking at his graph (d) Figure 10, we see that strategies' wealth are volatile and the cumulative return encounter some huge drawdowns.

We find also that the performance of all strategies works better on NIKKEI 225. We have an increasing trend for all strategies until the end of the period. Additionally the different strategies performs with a lower financial risk compared to other cores.

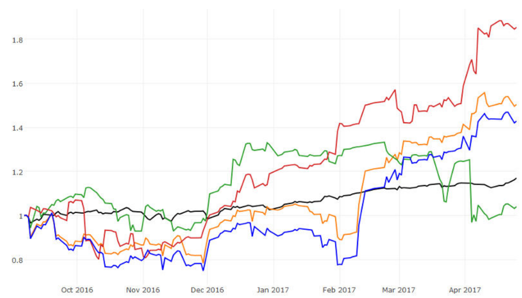
As can been seen in Table 7, none of the strategies has a positive excess return for all cores. After the backtesting we couldn't say that one strategy outperform the other. Overall for all strategies we encountered a high financial risk even if we obtain for some strategies at the end of the period a positive excess return. Those strategies can't meet the expectation of conservative investors. These latter investors would look for investment less volatile.



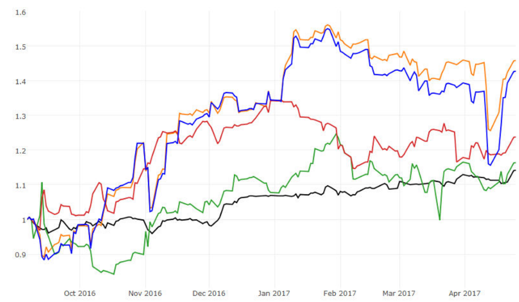
(a) SP500



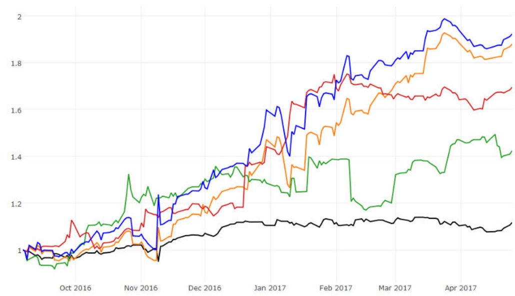
(b) FTSE 100



(c) NASDAQ



(d) DAX 30



(e) NIKKEI 225

Figure 10: Overview of the different strategies on the different cores .**Core**(black), TEDAS Naive (orange), TEDAS Hybrid (green), Minimum-Variance (red), Equally risk weighted (blue) CRIXtedasBacktesting

5 Discussion

Data

In this paper, we used for the different experiments a sample of 559 working days. The sample is most probably not well adapted to wealth management due to the cryptos' financial market. We know that cryptos are very volatile, so to capture correctly the strucuture and information we need to reduce the time stamp from a daily basis to a less than 10 min time stamp. In addition the problem is that cryptos are traded every day and every time unlike the cores that traded only during market hours. As a consequence we need definitely to take into consideration if we are trading with a small time interval like 5 or 10 minutes. At the moment the cryptocurrency market has just emerged, we haven't been able get to more data especially to backtest.

Discussion on the method and Results

In this paper we studied core-satelite management combined with TEDAS method. As explained in the methodology section, we run quantile regression at different quantiles. More precisely TEDAS employs 5%, 10%, 25%, 35% and 50% tail events. We have used only this five events but we could have used mored tail events, i.e. broaden the tail events but staying betwenn 5% and 50%.

We also saw when tuning the window length by doing a grid search the excess return has been varying a lot when expanding or reducing the window length of just one day. But this can"t be explained only by the method used, it could be explained also as we said before by the choice of our sample. The second explanation could be the unpredictable character of cryptos.

The Grid Search has been done by first selecting windows with the least negative excess return frequency and then we selected windows with the highest positive cumulative return (sum of all strictly positive cumulative return). We first optimize along one metric and then the second metric. The limit of this optimization axis by axis, sequentially, is that we will select most of the time a higher window. A higher window is not necessary the optimal window compared to a smaller one with the same negative excess return frequency. To improve this method we need most probably consider more metrics like maximum drawdown, volatility and other metrics. Then we could try to optimize globally these metrics and not sequentially. This approach will lead us to select with more consistency the optimal window.

Improvement and Future works

We used all along in this study a static moving window and cryptos are definiteley not stationary processes. Meaning that the distribution of each cryptos is evolving over time. By using a static moving window in this study, we don't keep in memory long term effect that could be potentially important to predict the future. So we need to keep learning but also keep consistent information in memory. One of the idea would be to use LSTM models. LSTM-Long short-term memory are coming from the field of Machine Learning. This model could be useful to solve our problem. LSTM architecture was designed to model temporal sequences and their long-range dependencies more accurately than the traditional neural networks. So we could try to use and adapt this method to predict correlation structure.

Another field we could look into would be the bayesian statistics. As the distribution of cryptos are not well known and keep changing , bayesian inference could help us to refine our not well known distribution over time.

6 Conclusions

We presented in this study the behaviour of an active management technique called core-satellite with Cryptocurrencies. Adding cryptos in a traditional core-investment strategy was a challenge due to the nature of the asset. To determine the active set of portfolio elements we implemented a technique called TEDAS - Tail Event Driven ASset allocation already used in a previous study with Mutual funds and Hedge Funds as satellite. Different variant of TEDAS have been implemented to challenge the models such as Hybrid or ERC.

First We showed that the window length is one important parameter to tune in order to maximize the wealth of our different strategies. We also saw that performance of the different strategies are very different according to the cores. Overall it was hard to select one strategy the outperform the other strategies. We have also seen even after tuning the window we still don't get always a stabilized positive excess return. The non-stationarity of the cryptos' processes leads probably to not capture properly adversly moving cryptos.

The results need to be taken carefully because of our initial dataset. Since during the writing of the master thesis we were just at the begining of the emergence of cryptos. We didn't have enough historical data which could have been helpful not only for learning but also for backtesting.

The next step would be to do this study again with additional historical data and by reducing the time stamp for a smaller period of time. Indeed as we saw cryptos are very volatile and reducing the time interval could bring us more information. We could also use techniques coming from other field like LSTM in Machine Learning or Bayesian statistics to update cryptos distribution over time.

References

Cizek Pavel, Härdle Wolfgang Karl, Weron Rafal (2011). *Statistical Tools for Finance and Insurance*, Springer.

Eisl. A, Gasser. S, M, Weinmayer K (2015): Caveat Emptor: Does Bitcoin Improve Portfolio Diversification? .

Francesco Cesarone, Andrea Scozzari. and Fabio Tardella (2010): Portfolio selection problems in practice: a comparison between linear and quadratic optimization models.

George A Christodoulakis (2002): Bayesian Optimal Portfolio Selection: the Black-Litterman Approach .

Härdle, W. K.,Nasekin Sergey, Lee David Kuo Chuen, Phoon Kok Fai. (2014). TEDAS – Tail Event Driven ASset Allocation, *SFB 649 Econmic Risk*, 2014-032.

Härdle, Wolfgang Karl, Lee David Kuo Chuen, Nasekin Sergey, Ni Xinwen, and Petukina Alla. Tail Event Driven Asset Allocation: Evidence from Equity and Mutual Funds Markets. (2015), *Singapore Economic Review Conference 2015 August 5-7.* , 1-25.

Jan Hauke, Tomasz Kossowski (2011). Comparison of values of pearson´ s and spearman´ s correlation coefficients on the same set of data, 1–7.

Jörg Osterrieder, Julian Lorenz (2016). A statistical risk assessment of Bitcoin and its extreme tail behaviour.

Markowitz, H.(Mar. 1952). Portfolio selection, *The journal of finance*, 77–91.

Noël Amenc, Philippe Malaise. and Lionel Martellini (2012): Revisiting Core-Satellite Investing - A Dynamic Model of Relative Risk Management, 1–18.

Pauline Givord, Xavier D´ Haultfœuille (2013). La régression quantile en pratique, 5–44.

Satoshi Nakamoto (2008). Bitcoin: A Peer-to-Peer Electronic Cash System.

Stephen Chan, Jeffrey Chu, Saralees Nadarajah, and Joerg Osterrieder (2017): A Statistical Analysis of Cryptocurrencies.*J. Risk Financial Manag.* 2017, 10, 12.

Trimborn, S. and Härdle, W. K. (2016). Crix an index for blockchain based currencies, *SFB 649 Econmic Risk, revise and resubmit Journal for Empirical Finances*, 2016-021.

Trimborn, S., Li, M. and Härdle, W. (2017). Crix an index for blockchain based currencies, *Investing with cryptocurrencies - a liquidity constrained investment approach, SFB 649 Discussion Paper*, 2017-014.

Yanuar Andrianto,Yoda Diputra (2018): The Effect of Cryptocurrency on Investment Portfolio Effectiveness , *Journal of Finance and Accounting.*

X. Y. Zhou and D. Li (2000): Continuous-Time Mean-Variance Portfolio Selection:A Stochastic LQ Framework.

A Appendix

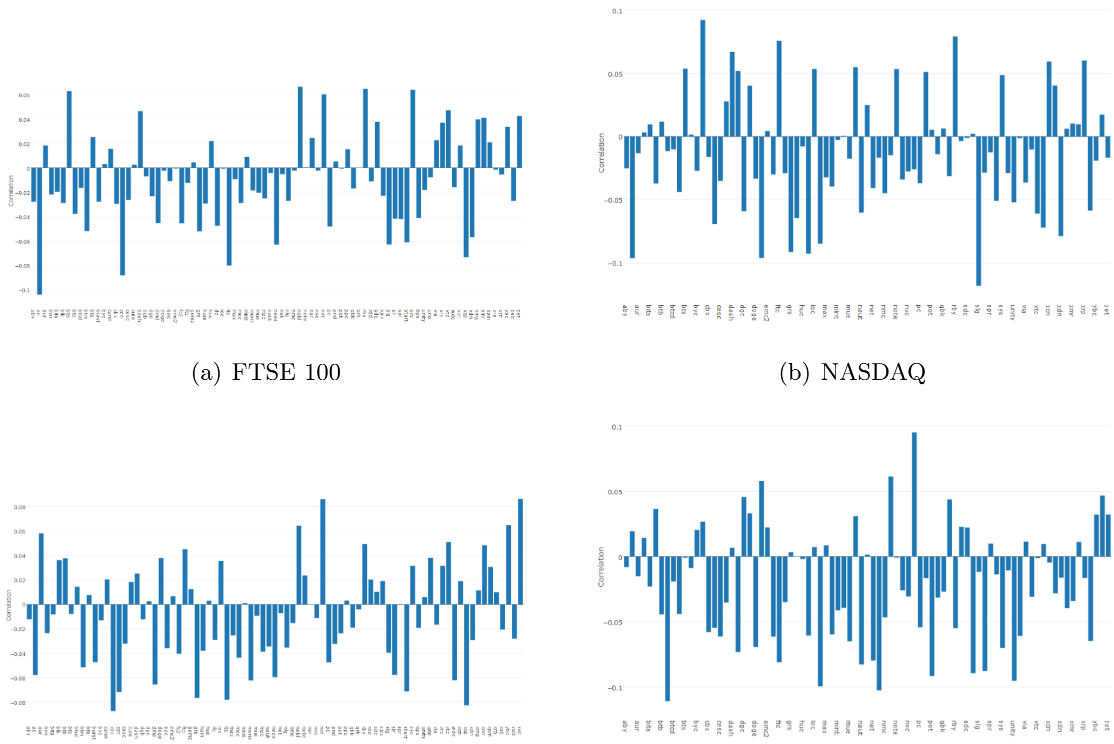
A.1 Descriptive statistics on cryptos

Here we represent the Shapiro normality test on some cryptos. We didn't represent for all cryptos but for all cryptos the null hyptothesis of a normally distributed sample has been rejected at a level of significance of 5%.

Cryptos	Shapiro statisitc	p-value
aby	0.95	2.55E-13
ac	0.63	7.45E-33
aur	0.91	5.29E-18
bcn	0.69	1.15E-30
bits	0.84	1.78E-23
blk	0.82	1.87E-24
btb	0.64	1.05E-32
btc	0.85	2.06E-22
btcd	0.60	8.03E-34
btm	0.92	4.53E-17
bts	0.90	7.99E-19
burst	0.86	1.42E-21
byc	0.92	3.56E-16
cann	0.93	1.16E-15
cbx	0.74	1.67E-28
ccn	0.87	7.60E-21
cesc	0.83	5.21E-24

Table 9: Normality test of the cryptos return

We plot the correlation of the different cores against cryptos.



(a) FTSE 100

(b) NASDAQ

(c) NIKKEI 225

(d) NIKKEI 225

Figure 11: Correlation beetween the core and cryptos

CRIXtedasCorrelationCoreCryptos

A.2 Performance of the different strategies

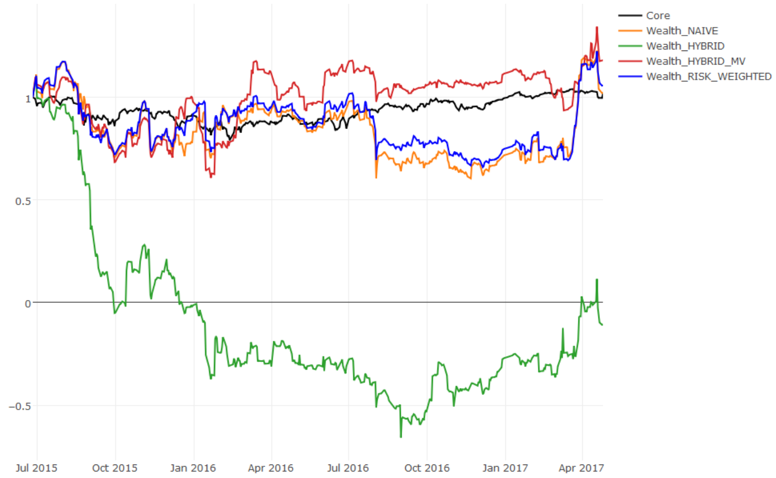


Figure 12: Cumulative return for the **Core FTSE 100 (black)** ,TEDAS Naive ,TEDAS Hybrid ,TEDAS Minimum-Variance and TEDAS Risk weighted

CRIXtedasStrategies

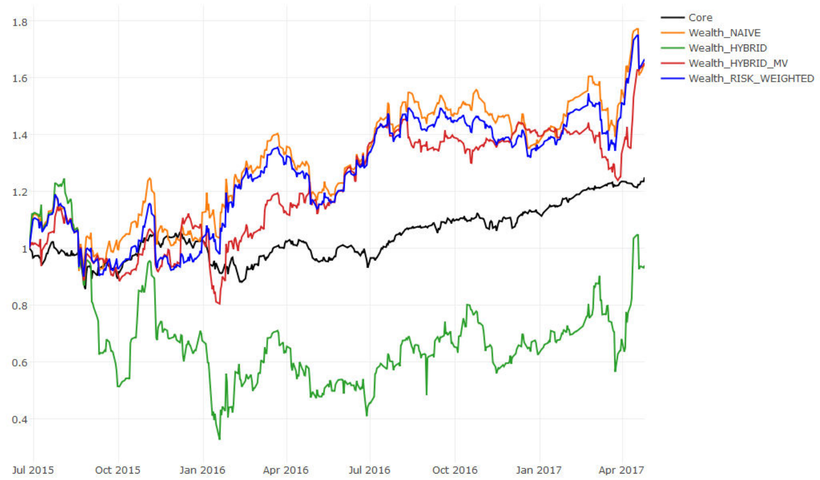


Figure 13: Cumulative return for the **Core NASDAQ (black)** ,TEDAS Naive ,TEDAS Hybrid ,TEDAS Minimum-Variance and TEDAS Risk weighted

CRIXtedasStrategies

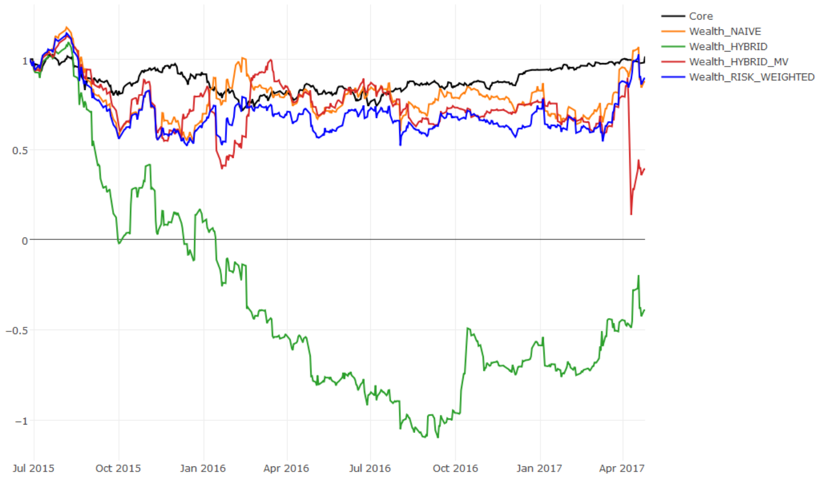


Figure 14: Cumulative return for the **Core DAX 30 (black)** ,TEDAS Naive ,TEDAS Hybrid ,TEDAS Minimum-Variance and TEDAS Risk weighted

CRIXtedasStrategies

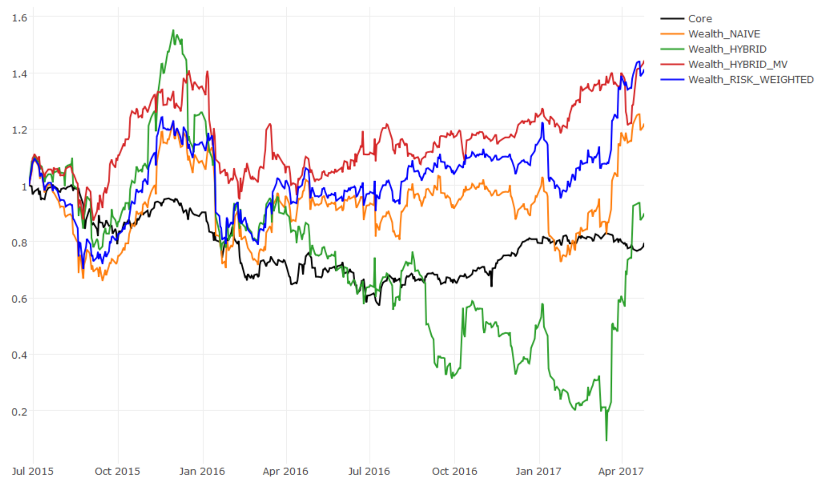
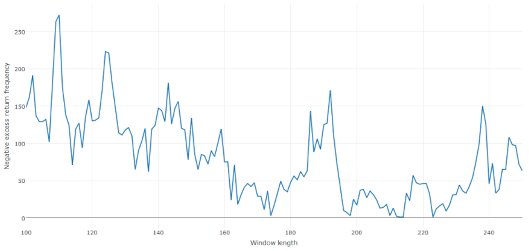


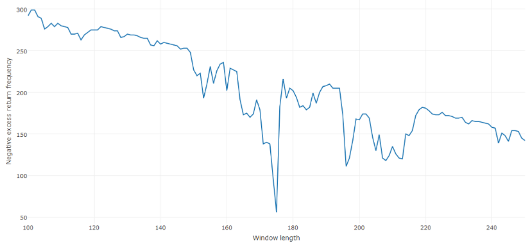
Figure 15: Cumulative return for the **Core NIKKEI 225 (black)** ,TEDAS Naive ,TEDAS Hybrid ,TEDAS Minimum-Variance and TEDAS Risk weighted

CRIXtedasStrategies

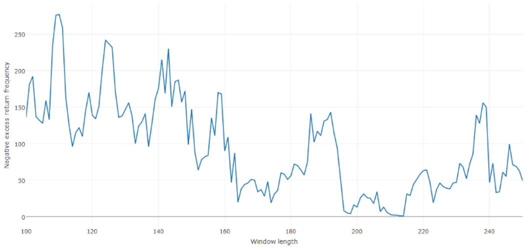
A.3 Performance of the strategies when window length changes



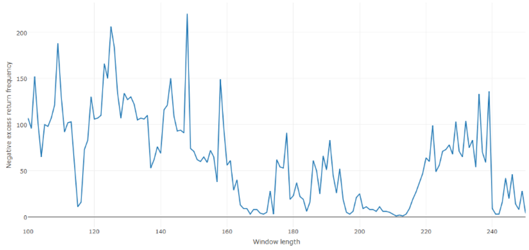
(a) TEDAS Naive



(b) TEDAS Hybrid



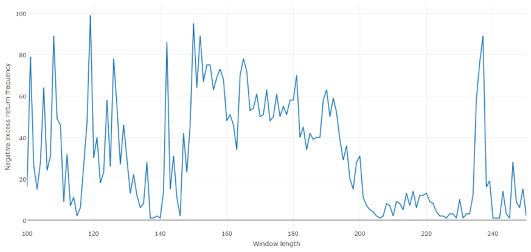
(c) Risk Weighted



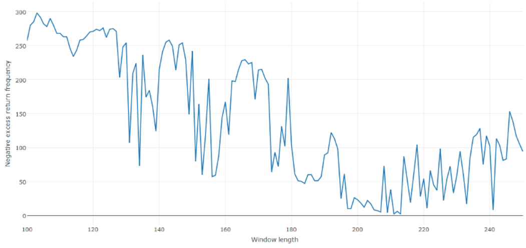
(d) Minimum-Variance

Figure 16: Negative excess return frequency of the different Strategies on FTSE 100

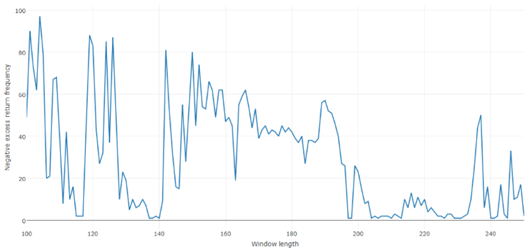
CRIXtedasWindowlengthGrid



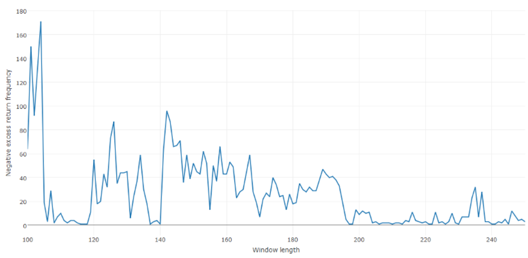
(a) TEDAS Naive



(b) TEDAS Hybrid



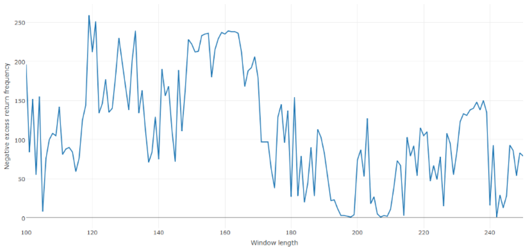
(c) Risk Weighted



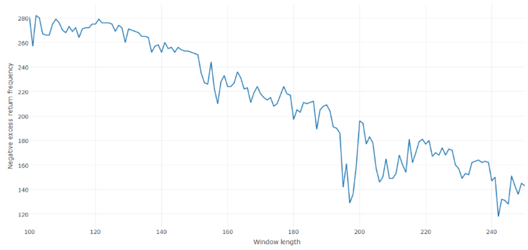
(d) Minimum-Variance

Figure 17: Negative excess return frequency of the different Strategies on NASDAQ

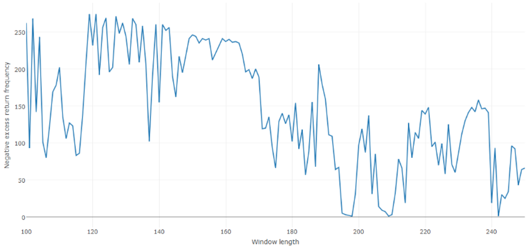
CRIXtedasWindowlengthGrid



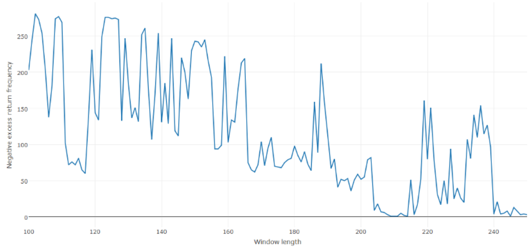
(a) TEDAS Naive



(b) TEDAS Hybrid

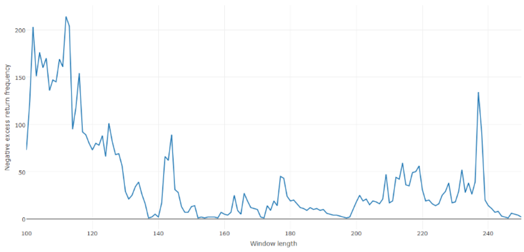


(c) Risk Weighted

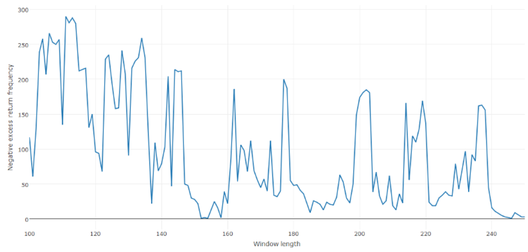


(d) Minimum-Variance

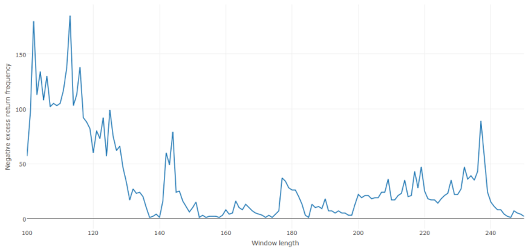
Figure 18: Negative excess return frequency of the different Strategies on DAX30 CRIXtedasWindowlengthGrid



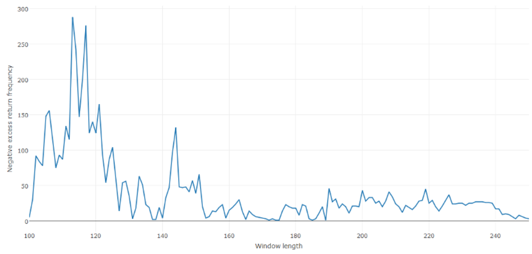
(a) TEDAS Naive



(b) TEDAS Hybrid



(c) Risk Weighted



(d) Minimum-Variance

Figure 19: Negative excess return frequency of the different Strategies on NIKKEI 225 CRIXtedasWindowlengthGrid

Declaration of Authorship

I hereby confirm that I have authored this Master thesis independently and without use of others than the indicated sources. All passages which are literally or in general matter taken out of publications or other sources are marked as such.

Berlin, August 28, 2019

Dinesh.