Comparison of Magnetospheric MultiScale (MMS) Formation Design Algorithms

L. Mann[1], K. Parsay[1], T. Williams [2] and W. Yu [2]
[1]*a.i. solutions, Inc., Lanham, USA;* [2]*NASA GSFC, Greenbelt, USA*

1. Introduction

The MMS mission consists of four identical spinning spacecraft in a tetrahedron-like formation during predefined orbital regions-of-interest [1]. The MMS formation will fly about a highly eccentric orbit to study magnetic reconnection in the Earth's magnetosphere. The mission is divided in two main science phases of markedly different orbit size. Phase 1 is a 1.2 R_E x 12 R_E (R_E equals one Earth radius) orbit phase where science will be performed on the day-side of the magnetosphere, and Phase 2 is a 1.2 R_E x 25 R_E orbit phase where science will be performed on the night-side of the magnetosphere. For each phase, the MMS formation initially will fly a series of tetrahedron sizes ranging between 10-km and 400-km to assist the science team in determining the best value for the remainder of the phase. This paper presents a preliminary comparison of the algorithms currently available to determine desired MMS formation states. After a brief description of the MMS formation flying metrics and associated requirements, the five formation design algorithm combinations considered here are presented. Monte Carlo simulations determine the performance of each algorithm defined in terms of formation lifetime and fuel consumption. Preliminary results focusing on the smaller formation size for Phase 1 are discussed and the conclusions are presented along with indications for future work.

2. Formation Flying Metrics and Associated Requirements

The MMS formation flying is evaluated using various metrics including the formation lifetime (i.e. maneuver frequency) and fuel consumption metric. These two types of metrics are discussed below.

2.1 Formation Lifetime Metric

To minimize interruption when collecting scientific data and facilitate the overall operational concept, it is highly desirable to maximize the elapsed time between maneuvers. The desired MMS operational tempo is of 14 days. In this paper, the formation lifetime metric is defined as the time required for the formation states to degrade (referred to in this paper as degraded states) to the point where another maintenance maneuver is needed (T_M). The need for a maintenance maneuver is triggered by a violation of either the collision threshold (T_C) or a science threshold (T_F).

Science Maneuver Trigger - Four formation quality parameters that derive from science requirements are currently defined for the MMS mission. The first parameter is the instantaneous quality factor Q(t) which ranges from 0 to 1 and is a measure of the quality of the formation at any point in time in the orbit. This parameter as applied to MMS was discussed by Hughes and Guzman et. al.[3,4].

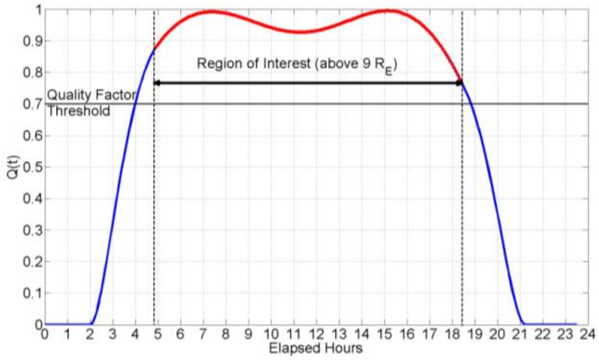

Figure 1. Q(t) versus elapsed hours (Phase 1).

Figure 1 shows an example of the evolution of the instantaneous quality factor over one Phase 1 orbit for a tetrahedral formation with an average 10-km side length at apogee.

The second parameter, is the average value of the instantaneous quality factor Q(t) over the predefined region-of-interest (RoI) about apogee. The third parameter Tq_{Rev} is the portion of time that Q(t) is above the 0.7 threshold in the RoI over a given orbital revolution. This parameter is currently used as a trigger for performing a formation maintenance maneuver. The time it takes for the formation Tq_{Rev} to fall below a Tq_{Rev} = 0.8 value is denoted by T_F. The fourth parameter Tq_{Avg} is the average Tq_{Rev} over all the orbits in a specific science phase. Figure 2 illustrates the evolution of and Tq_{Rev} in Phase 1 for a representative formation over 60 revolutions.

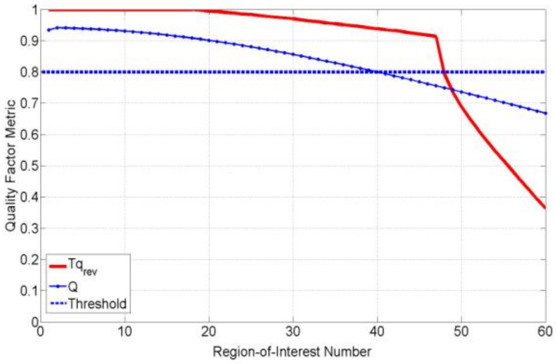

Figure 2. and Tq_{Rev} Evolution for a representative formation over 60 RoI (Phase 1).

Collision Maneuver Trigger - The minimum inter-spacecraft range over one orbit for each spacecraft pair is defined as the collision metric for the design of the MMS desired states. For a desired formation with an average side length of 10 km, the collision metric is required to remain above 6 km. Note that if this metric is predicted to go below 4 km at any time during the mission, a collision avoidance maneuver will be considered. The time it takes for a desired formation to violate the collision metric of 4 km is denoted by T_C.

2.2. Mass Consumption Metric

The second metric considered in this paper is the amount of fuel required to achieve the target state using a two-burn impulsive Lambert transfer. Note that the MMS formation flying maintenance concept is to maneuver only 3 spacecraft out of 4 at any time. The non-maneuvering spacecraft is called the reference spacecraft.

These metrics are taken into consideration in building the cost function for the formation design algorithms presented in the following section.

3. Formation Design Algorithm (FDA)

There are currently three algorithms used to define the MMS formation target states: maximum quality factor (*MaxQ*), minimum fuel (*MinFuel*) and the robust algorithm ($MaxT_F$)[2,3,5]. These formation design algorithms (FDAs) are presented below in turn.

<u>Maximum Quality Factor (*MaxQ*)</u> - The *MaxQ* formulation uses a constrained optimizer to maximize $\overline{Q}$ over a single orbit while enforcing a constraint on the collision metric and formation lifetime. The problem solved can be formulated as follow:

_ _ Eq.1

where is the number of steps taken in the RoI, is the RoI entrance time, _ is the state vector of a spacecraft (position and velocity), is the range between spacecraft m (m = 1,2,3,4) and spacecraft j (j = 1,2,3,4), is the reference spacecraft osculating semi-major axis, is the kth spacecraft osculating semi-major axis (k = 1,2,3) and is the reference spacecraft radius. Note that inter-spacecraft constraint is only checked when the reference spacecraft radius is below 5 R_E and that a design value of 6 km was chosen for the 10-km formation to allow for some margin against the 4 km collision avoidance maneuver threshold. This formulation does not take into account the mass consumption metric.

<u>Minimum Fuel (*MinFuel*)</u> - The *MinFuel* formulation also uses a constrained optimizer but it attempts to minimize the fuel expended by all spacecraft to reach the final states while enforcing a constraint on the , collision and formation lifetime metrics. This formulation shown below is also limited to a single orbit:

– – Eq.2

where is the two-burn impulsive Lambert value for spacecraft p (N = 3 since the reference spacecraft is not maneuvered). More information on this formulation can be found in [3].

<u>Robust</u> (*MaxT_F*) - The *MaxT_F* formulation is a multiple orbit formulation that uses an unconstrained optimizer to maximize the expected value of $\overline{Q}$ across multiple orbits including semi-major axis error due to navigation and maneuver execution errors. The *MaxT_F* formulation takes into account that the formation quality will degrade over time under two external perturbations: the natural orbital perturbation (mainly J_2) and the maneuver execution and navigation errors. This formulation shown below (due to Roscoe et al. [5]) does not include a direct constraint on either the collision metric or the fuel consumed:

— ——— for the k^{th} RoI Eq. 3

where is the number of orbits over which to optimize, is the 1x3 vector containing the i^{th} sample of semi-major axis error for all 3 maneuvering spacecraft, is the number of sample of the semi-major axis error, are the nominal differential elements for the j^{th} spacecraft, is the differential mean anomaly rate, is the differential argument of perigee rate, is the differential node rate and is the reference spacecraft inclination. More detail on this formulation can be found in [5].

4. FDA Comparison Simulation

The FDA simulation was designed to study and compare FDA performance without the high computational power required by the high fidelity MMS End-To-End (ETE) simulation process developed by the MMS mission design team for many of its studies and analyses [6]. The ETE simulation models the MMS mission from launch to reentry and includes the navigation errors and maneuver execution using both a prediction/planning process and an execution process.

For this comparison study, the maneuver execution and navigation errors were translated to equivalent SMA errors (δa). These differences in semi-major axis create differences in the orbital periods that cause the spacecraft to quickly drift apart in the along-track direction. SMA errors are the principal source of secular drift for formation flying with a much larger effect than the other orbital elements errors. For this reason, a key parameter in the MaxT_F formulation is the expected SMA error. The simplified FDA comparison simulation process shown in Figure 3 performs a Monte Carlo lifetime study for one degraded initial formation state.

Figure 3. FDA Monte Carlo Simulation Overview (Baseline).

Five degraded formations (i.e., degraded Tq_{Rev}) were selected randomly from previous ETE runs for this study. Each degraded formation state is input to each FDA algorithm combination (detailed in section 4.1) as an initial guess for determining the new formation target states (i.e., FDA solution). The FDA solution is then dispersed with mean SMA errors to simulate maneuver execution errors and propagated until either the Tq_{Rev} falls below 0.8 or the minimum inter-spacecraft range requirement (4 km) is violated. The spacecraft propagation force model included a 21 x 21 gravity model, along with solar and lunar gravity and solar radiation pressure perturbations. The SMA errors sampling is repeated for a total of 600 trials for each Monte Carlo simulation. A total of 25 Monte Carlo simulations were run (five algorithms combinations were tested per initial degraded state).

To determine the appropriate number of Monte Carlo runs for a given FDA solution, a trade study between the number of cases and average formation lifetime (T_M) was performed. Figure 4 shows the average formation lifetime as a function of the number of Monte Carlo trials. It takes about 600 trials for the statistic to converge to its final value.

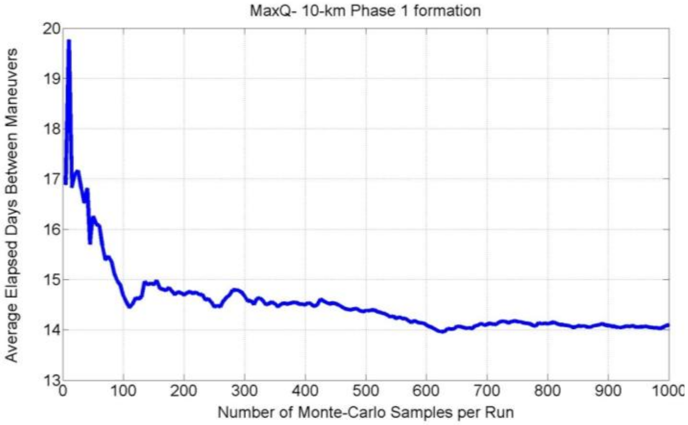

Figure 4. Average Formation Lifetime versus Monte Carlo Trials

4.1. FDA Combinations

Five FDA combinations were evaluated: (1) MaxQ algorithm only, (2) MaxQ and MaxT$_F$, (3) MinFuel only, (4) MinFuel and MaxT$_F$ and (5) MaxQ and MinFuel. For the cases where more than one algorithm is used, the solution of the first algorithm is served as an initial guess for the second algorithm. Note that MaxT$_F$ requires a good initial guess to start and it cannot be used by itself on a degraded formation state.

4.2. Maneuver Execution and Navigation Errors

Differentiation of the vis-viva equation shows that SMA varies with maneuver error as:

— Eq. 4

where is the semi-major axis, is the spacecraft velocity magnitude, is the gravitational constant of the Earth and is the component of the maneuver error in the orbital velocity direction.

Two maneuver error sources exist for MMS: errors in the maneuver magnitude and errors in its direction. Both of these can cause SMA shifts in differing ways. Magnitude errors along the orbit velocity component and direction errors along the orbit normal component and/or the orbit bi-normal will produce errors in achieved SMA. The importance of these two error sources depends not only upon the error models, their corresponding burns and their location in the orbit.

To quantify the effects of these two mechanisms on SMA, all formation maintenance maneuvers for a large set of Monte Carlo trials of the entire two-year MMS mission were analyzed. For each desired maneuver, a set of 200 random direction and magnitude errors, given by the specified MMS maneuver error model, were generated and applied, and the resulting SMA offsets computed. The results obtained can be summarized as follows: (a) direction errors, for the current MMS maneuver models, are more significant for δa than are magnitude errors; (b) in any given mission phase, δa varies roughly proportionally to formation size; and (c) the results in the two mission phases differ: this appears to be at least partially due to the different maneuver true anomalies used for the two phases as well as the different orbit sizes.

Based on these results, δa values for any given MMS formation size and mission phase were determined. For a 10 km formation, a mean SMA error standard deviation (3σ) value of 280 m was found. This value will be used in the Monte Carlo simulations shown in Figure 3.

4.3 Simulation Validation

A secondary set of Monte Carlo runs was performed using a portion of the ETE simulation to verify and validate the assumptions and simplifications made in the simulations described above. The maneuver execution errors were applied directly to the desired formation and were no longer approximated by the mean SMA errors. Due to computation time considerations, the simulation was restricted to (1) MaxQ only and (2) MaxQ and MaxT_M. This process is captured in the following figure.

Figure 5. Simulation Validation Process.

5. Results

In this section, the results using the baseline setup summarized in Figure 3 are first presented. Lastly, the results from a sensitivity analysis on the collision avoidance limit are discussed.

5.1. Baseline

Table 1 provides a summary of the Monte Carlo statistics for the five initial conditions sampled in this paper for a Phase 1 10 km formation size for all five algorithm combinations. The ‘Monte Carlo’ (MC) statistics correspond to a given initial condition/FDA Monte Carlo simulation (600 trials per simulation for this analysis). When the statistics are computed across all five initial conditions, they are referred to as ‘Ensemble’ statistics (Ens.). The first four rows list the ensemble statistics (mean, standard deviation, minimum and maximum) for the MC mean formation lifetime for all trigger conditions (T_M). All other MC parameters in the table are ensemble mean values.

Table 1. Lifetime Monte Carlo Statistics Summary For a Phase 1, 10-km Formation Size.

MC statistic	Ensemble stat.	MaxQ	MaxQ + maxT_F	MinFuel	MinFuel + MaxT_F	MaxQ + MinFuel
Mean T_M (days)	Ens. Mean	13.7	14.6	13.0	15.1	13.9
	Ens. Std	2.1	2.4	2.3	0.9	2.6
	Ens. Min	11.2	11.6	9.6	13.8	10.6
	Ens. Max	16	16.9	15.4	16.1	16.6
Minimum T_M (days)	Ens. Mean	2.3	3.1	2.7	3.5	2.1
Maximum T_M (days)	Ens. Mean	67.9	74	58.8	68.8	61.1

For the initial conditions considered in this paper, the average lifetime of a Phase 1 10 km formation ranges between 9.6 days and 17 days. The 10 km formation lifetime ranges between 2 days and 69 days depending on the maneuver execution errors. All five algorithm combinations are comparable in terms of their mean formation lifetime performance (within approximately 1 day of each other). The (MinFuel+MaxT_F) formulation seems to consistently perform better with the smallest ensemble standard deviation of its mean formation lifetime. The MaxT_F improves the other FDA solutions by about a day on average. It adds about 10 days on the maximum lifetime and about 1 day on the minimum lifetime. These results were validated in the full maneuver execution error model described in section 4.3; similar statistics were found.

Table 2 lists statistics for only collision-triggered maneuvers. The first row is the ensemble mean of the mean lifetime and the second row is the percentage of cases triggered by a collision versus the total number of trials. Again, MaxT_F improves the lifetime statistics but the number of collision-triggered maneuvers is also increased. This is expected as MaxT_F does not include collision avoidance in its formulation. For small formations such as the 10 km formation the likelihood of violating the minimum range 4 km threshold due to maneuver dispersion is high. The violation tends to occur faster on average than the science metric violation due to the direct relation between range evolution and semi-major axis difference.

Table 2. Lifetime Monte Carlo Statistics Summary for a Phase 1, 10 km Formation Size for Collision Metric-triggered Maneuver Only (Metric T_C).

MC statistic	Ensemble stat.	MaxQ	MaxQ + $MaxT_F$	MinFuel	MinFuel + $MaxT_F$	MaxQ + MinFuel
Mean T_C (days)	Ens. Mean	8.5	10.2	8.9	11.7	8.5
% CA cases	Ens. Mean	40.6	44.3	35.1	43.9	42.1

Table 3 summarizes the ensemble statistics for the total ΔV needed to achieve each FDA formation target state and the run time associated with each FDA solution. The MinFuel+$MaxT_F$ combination has the longest computational time and does not provide any fuel savings advantage over the MaxQ formulation. The MinFuel+$MaxT_F$ total run time is about 20 times longer than the MaxQ formulation, which is a clear drawback as this algorithm would need to be called thousands of times if employed in an ETE trajectory Monte Carlo simulation [6].

Table 3. ΔV and Computational Performance Statistics Summary for a 10-km Formation Size.

MC statistic	Ens. Statistic	MaxQ	MaxQ + $MaxT_F$	MinFuel	MinFuel + $MaxT_F$	MaxQ + MinFuel
ΔV (m/s)	Ens. Mean	1.07	1.28	0.39	1.3	1.07
Run Time (sec)	Ens. Mean	18.7	464.3	51.4	505.7	46.4

5.2 – Collision Avoidance Design Limit Variations

Since the collision-triggered events formed about 50% of the cases for a small formation size such as the Phase 1 10 km formation studied for this paper, the effect of increasing the FDA collision design limit from the 6 km baseline value was subsequently investigated. Collision avoidance design limits of 8 km and 10 km were run for one of the five initial conditions presented in the previous section. Figure 6

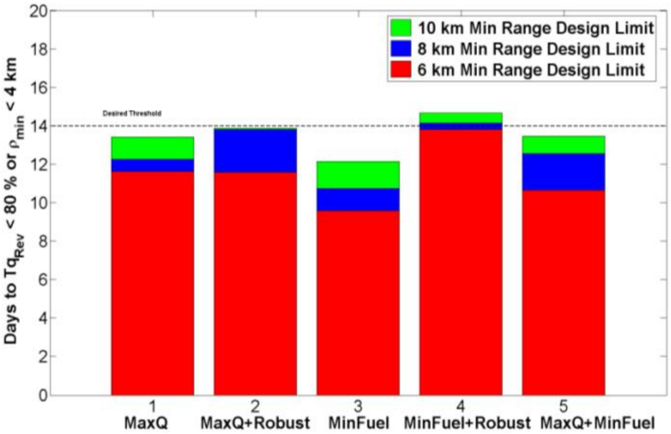

Figure 6. MC Mean Lifetime Statistic (T_M) For Collision Avoidance Limit of 6-km, 8-km and 10-km.

shows the MC Mean formation lifetime T_M for all five algorithm combinations. The red bar corresponds to the lifetime using the baseline collision design limit. The blue bar and green bar corresponds to the increase in mean lifetime for the 8 km and 10 km collision avoidance design limit respectively. The desired operational threshold of 14 days is indicated. As expected, increasing the collision metric design limit provides about 2-days improvement overall in the mean formation lifetime statistic. Consequently, increasing the collision avoidance design limit is highly desirable for smaller size formations where the lifetime statistic is dominated by the collision avoidance maneuvers.

6. Conclusion and Future Work

This paper presented results from a preliminary comparison of the algorithms currently available to determine the MMS desired formation states. Using a Monte Carlo simulation to perturb the desired formation states, each of these algorithms were compared in terms of formation lifetime, total ΔV required to achieve the desired states and overall computation run time. The preliminary study focused on the 10 km formation size for Phase 1.

Overall, the robust optimizer ($MaxT_F$) is shown to improve the mission lifetime statistics of the MaxQ and MinFuel algorithms. However, it increased the run time dramatically. In addition, $MaxT_F$ increased the percentage of cases where collision avoidance maneuvers were needed. Furthermore, in the case of the MinFuel+$MaxT_F$ combination, the $MaxT_F$ algorithm does not preserve the fuel savings achieved by the MinFuel algorithm alone.

Future work will focus on improving the $MaxT_F$ run time to make it a viable option for use in the MMS ETE simulations. In addition, all formation sizes for Phase 1 as well as Phase 2 will be studied to understand which algorithms are better for small formation size versus large formation sizes. Indeed, small formations size are subject to shorter formation lifetimes due to the need for more frequent collision avoidance maneuvers, while larger formations are concerned with larger maintenance maneuvers that require more fuel. Identifying the formation size cross-over point for switching between algorithms (for both Phase 1 and Phase 2 orbits) is important to future formation maintenance studies and, potentially, to flight operations as well. Finally, additional tuning parameters for each individual algorithm will be studied to determine their optimal values for MMS mission studies.

References

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