

Single Degree-of-Freedom Modeling of SLS Liquid Hydrogen Pre-Valve Flow Guide to Enable Rapid Transient Analysis

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A unique single degree-of-freedom approximation technique has been developed to enable rapid application of a temporally-defined multi-spectral semi-narrow-band loading for generation of realistic stress/cycle values compared to a resonant analysis. The technique uses the harmonic analysis at resonance of a high-fidelity finite element model to produce a transfer function, which is then used to calibrate the response of the SDOF model. A standard numerical ordinary differential equation solver is then used to obtain the temporal response, and its histogram is used in a fatigue/fracture model. This technique is related to other SDOF methods used widely in industry, such as Miles' Equation and the Shock Response Spectra, but it is unique in that it produces a realistic time history of the response. The most obvious error in the process, which is the effect of closely-spaced modes, was also assessed using the parallel application of several SDOF models, and the error is shown to be small. The application of this unique and tractable reduced-order methodology has enabled the SLS program to avoid substantial cost and schedule penalties if a redesign or change of material were required. It has also enabled quick analysis of a number of other structures undergoing the same or similar excitation fields, and quick assessment when the excitation and structural configuration has been altered due to design changes in the system.

Nomenclature

FEM	=	Finite Element Model
FRF	=	Frequency Response Function
m	=	Number of Nodal Diameters in Excitation or Mode Shape
N	=	Engine Rotational Speed
MPS	=	Main Propulsion System
psi	=	pounds per square inch
HOC	=	Higher Order Cavitation
LH2	=	Liquid Hydrogen
SDOF	=	Single Degree of Freedom
SLS	=	Space Launch System
SRS	=	Shock Response Spectra
SSME	=	Space Shuttle Main Engine
TF	=	Transfer Function

I. Introduction

NASA is developing a new launch vehicle, called the Space Launch System (SLS), which is intended on taking humans out of low earth orbit to destinations including the moon, asteroids, and Mars. The propulsion system for the core stage of this vehicle includes four RS-25 Liquid Hydrogen/Oxygen rocket engines. These engines are upgraded versions of the Space Shuttle Main Engines (SSME); the upgrades include higher power levels and affordability enhancements. As with any new vehicle, the Main Propulsion System (MPS), which include the feedlines and ancillary hardware connecting the engines to the fuel and oxidizer tanks, is being redesigned, as the previous MPS for the SSME's was inherently part of the Space Shuttle System, which had a completely different overall configuration.

One of the most notable outstanding problematic issues from the MPS for the SSME that has to be addressed for the SLS is a significant pressure environment propagating upstream from the low pressure fuel pump inducer called Higher Order Cavitation (HOC). HOC is a well-known cavitation-induced oscillatory phenomenon. It occurs at

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particular hydrodynamic conditions, i.e. flow rate, inlet pressure and blade tip speed. Higher order cavitation provides a significant oscillating load on the inducer blades, but also sends spinning pressure waves upstream at frequencies at varying, non-integer multiples of the inducer rotational speed N . During the Space Shuttle program, these upstream-travelling waves were found to be the cause of cracking in a flowliner in the bellows of the feedline. An assessment of all the upstream structures (see figure 1) to this excitation, therefore, was made a requirement of the SLS design.

The first step in this assessment is to characterize the HOC loading itself. This characterization is based both on an extensive measurement and analytical program to gain an understanding of the hydrodynamic field. The results are published for use by the structural design community and include a tabular listing of discrete loadings characterized by a number of parameters. The first of these is the source, which is not only the HOC itself but also other sources found from test, including two times HOC, HOC + 2N, 1N, and 2N. The predominant frequency range of this excitation is then identified, which has approximately a bandwidth equal to 10% of the nominal frequency value, and then a description of the type of spinning wave itself, which is generally a pressure wave with 1 or 2 nodal diameters and 0 or 1 nodal circle, as specified in the equation

$$p(\theta, x, t) = a_{m,n} \cos[m(\theta - 2\pi\pi ft)]e^{-kx} \quad (1)$$

This excitation is actually not a discrete sinusoid, but instead is somewhat narrow-banded, so the magnitude is defined as a psi-rms value. To assess the structural capability for the hardware impacted by this loading, the pressure wave is then applied to the affected structural finite element models as a spinning wave at a single frequency for a number of cases corresponding to different excitations, and responses obtained. As is standard procedure in dynamic analysis, the excitations are applied exactly at structural natural frequencies in their frequency range to obtain the worst case response even if the excitation frequencies are not exactly at those values in the data. The response value is then used in fracture analysis, which initially showed all the hardware in the flowpath to have acceptable stress levels except for the pre-valve flow guide.

An investigation into methods for refining the analysis was therefore initiated. A number of reductions in the loading were identified, but none were large enough to obtain sufficient fracture capability. One other possible conservatism recognized is the characterization of the loading as purely harmonic and the performance of a resonant analysis at that frequency, instead of using the actual semi-narrow-band loading. The most obvious way to address this question would be to apply a transient signal directly onto the FEM. There are a number of problems with this approach, however. First of all, the transient is defined only at four circumferential locations, so an interpolation of these points would be required to adequately define the loading. Secondly, a transient analysis of any large finite

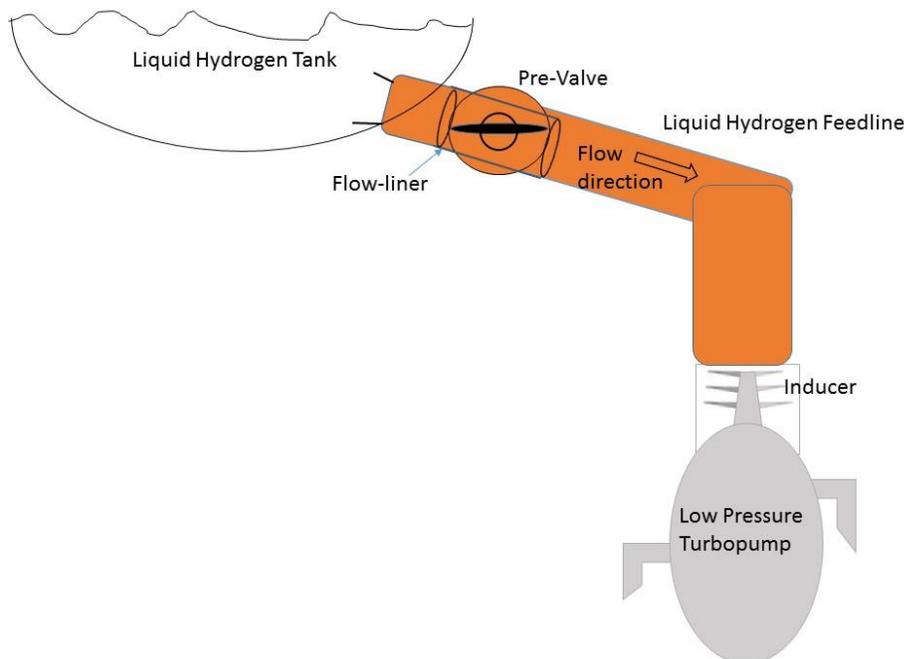


Figure 1. Schematic of LH2 Feedline and Pre-Valve
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element model is extremely computationally intensive and error-prone, and finally, the engineering manpower resources available to devote to a problem of this complexity were limited.

A unique alternative approach described in this paper is therefore developed that can be performed rapidly and still provides a conservative but accurate assessment of the response for the loading cases. This technique approximates the pre-valve structure as a single degree-of-freedom (SDOF) system with a natural frequency at close to the peak frequency in the excitation band. A transient (i.e., specified force time history) analysis using the angular location with the worst case loading can then be directly applied to the SDOF system and a transient response time history quickly calculated using Matlab® numerical techniques. This response history can then be quickly used in a fracture or fatigue life calculation.

II. Literature Survey

SDOF techniques similar to this methodology for rapidly assessing production hardware are frequently used in industry. Mullen, 2016¹, presents a study assessing the accuracy of a software code called SBEDS which is the structural engineering industry standard for analysis of structures undergoing blast loads. He performed an extensive comparison with MDOF methods to assess the error in the process, which is primarily attributed to the SDOF process using the statically deflected shape of the structure as the basis. SBEDS uses a number of factors to account for the spatial and temporal distribution of the loading and boundary conditions for simple beams. The SDOF stiffness parameter K is obtained from the equation for maximum static deflection as a function of the load p , which for a distributed load on a simply supported beam results in $K = 384EI/5L^4$, where E is the Modulus of Elasticity and I is the area moment of inertia. A load factor K_L , which is a factor applied to the total load, and mass factor K_m , which is the factor applied to the total mass, which are integral functions of the modes, loading, and geometry, are also obtained. A test study showing reasonable comparison between test and an SDOF model performed using the same techniques is also presented by Lucia². While valuable, these techniques are used generally when a finite element method is not available, which is not the case here.

A number of methods using equivalent SDOF systems are applied for structures undergoing seismic loads, including the well documented Shock Response Spectra (SRS). The SRS connects the points of maximum acceleration response for SDOF systems with natural frequency at every frequency of a given base acceleration excitation spectra and damping³. It is used frequently in earthquake engineering and in rocket vehicle analysis to give a quick estimate of the response for structures too complex (e.g. electronic boxes) or numerous to perform individual analysis for; for rocket vehicles, it is used to evaluate and later to qualify components mounted on the same panel of a stage which all experience the same excitation. A number of studies have been published to improve the method for specific complicating factors, such as energy loss due to hysteresis⁴. This method generally results in a grossly conservative loading condition, so it isn't appropriate for the pre-valve since weight is a strong consideration.

The most applicable existing method to this situation is probably the "Mile's Equation" methodology used to calculate the response of an SDOF to random excitation of constant amplitude, i.e., white noise⁵. For a force input, the version of the equation is

$$\sigma_{RMS} = \sqrt{\frac{\pi f_n W_o}{4 \zeta k^2}} \quad (2)$$

where f_n is the SDOF natural frequency, W_o is the magnitude of the forcing function power spectral density (PSD), and σ_{rms} is the standard deviation of the response spectra. This technique will account for the difference between a resonant and wide-spectra excitation, but it loses the variation in spectral amplitudes due to the white noise assumption. There are a number of difficulties applying this technique in this situation. First, the PSD is not at all constant, so the choice of W_o would be extremely problematic. Second, the determination of an equivalent stiffness k is also prone to error; the standard technique to calculate k would be to impart a load of one pound onto the location of maximum displacement and simply calculate the displacement, but actual implementation of this is not clear, as a point load in a finite element cylindrical shell model generally causes highly localized displacements and stresses completely unlike the mode shape the SDOF is simulating. In addition, the Miles' method only produces statistics of the response, and a method was sought that would produce a more highly refined time history result since this information is specified in the excitation. Because of these limitations, Mile's Equation methodology is not optimal for this situation.

III. Methodology

As stated previously, the purpose of this methodology is to realize the reduction in overall response due to the fact that the excitation is semi-narrow-band random in nature instead of purely at one frequency, but enable the analysis to be performed rapidly. The outline of the approach, therefore, is to create an equivalent single degree of freedom structure of the FEM, calibrate loading on the SDOF to the FEM response using the harmonic analysis, and then apply the true loading using a Matlab® script to quickly obtain a stress time history. This concept uses the 3-D FEM itself to create the transfer function; this is analogous to the methodology used in the SDOF blast loads methodology, where the transfer function (TF) is obtained from closed form solutions for simple structures except that the TF used here fully captures the maximum dynamic response of a MDOF structure to a harmonic input. This technique is similar to the SRS method, but instead of obtaining the maximum possible response at each individual frequency, the method enables a scaling of the response of the structure from fully resonant response to the much less conservative, but more realistic semi-narrow-band excitation.

The first step in the procedure is to define the forcing function. The harmonic analysis shows the worst case loading on the finite element model (figure 2) to be a 2*HOC, 1 Nodal diameter ($m=1$) wave form of amplitude 2.23 psi-rms at 3684 Hz, resulting in a peak structural resonant response of 12474 psi. The pressure time histories from different engine tests that were used to generate the 2.23 psi-rms forcing function are then identified. These time histories, which are measured at a plane close to the inducer (only 0.4 length/diameter units upstream) for four different circumferential locations for a 60 second period, are filtered with a passband of 3300-3700 Hz since that is the frequency band of the HOC travelling waveform. The pressure power spectral densities (PSD) of a number of these histories is shown in figure 3. Because test 901957 essentially envelopes the other tests, it was chosen for use in the analysis. To reduce excess conservatism, since the 1st nodal diameter waves are the only type that travel all the way upstream to the pre-valve, and there were four measurement locations, Nyquist criteria allow these histories to be spatially decomposed with a two-dimensional (spatial) Fourier Transform into a single complex time history containing the nodal diameter 1 content, plotted in figure 4. This spatially decomposed $m=1$ pressure time history $P_{m=1}(t)$ is complex, so the real and imaginary parts can be resolved to yield the amplitude for any circumferential location.

$$p(t, \theta) = \text{Re}(P_{m=1}(t)) * \cos(\theta) + \text{Im}(P_{m=1}(t)) * \sin(\theta) \quad (3)$$

The next step is to create the SDOF system by assuming unit mass and natural frequency equal to an iterated value to obtain maximum response (further discussion below). The equivalent force F_{eq} necessary to excite the SDOF such that the response equals the rms stress response σ_{FEmax} from the FEM is obtained using the standard equation for peak response at resonance

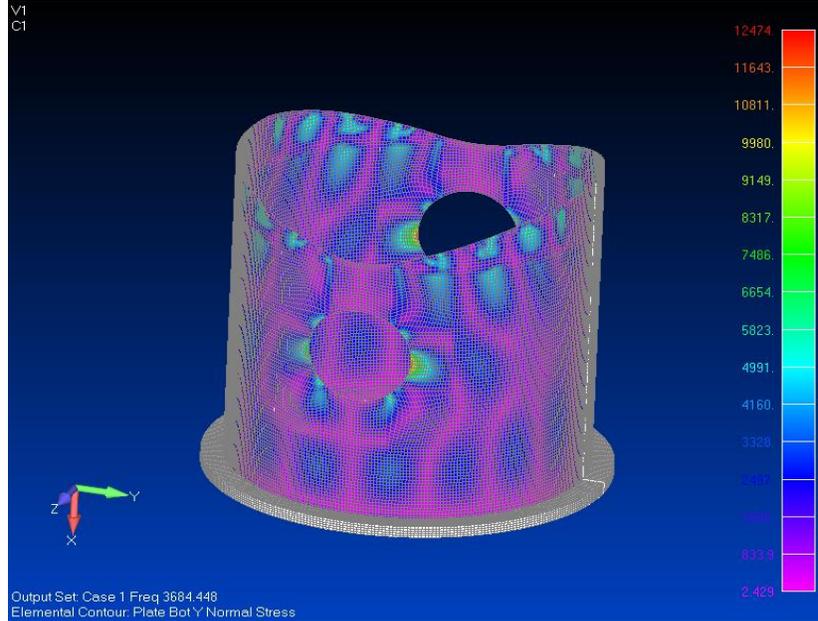


Figure 2. Finite Element Model of Pre-Valve (not to scale)

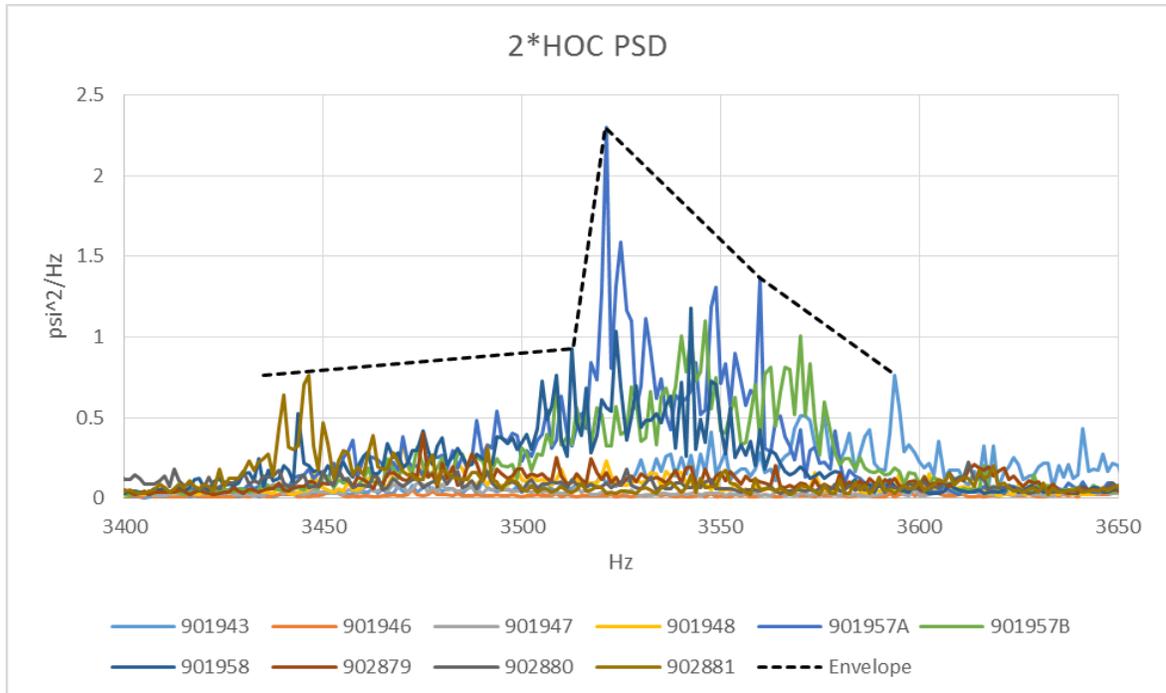


Figure 3. Pressure PSD's of Test Data

$$X_{peak} = \frac{F_{eq}/k}{2\zeta} = \frac{F_{eq}/m\omega^2}{2\zeta} = \frac{F_{eq}}{2\zeta\omega^2} \quad (4)$$

Now, although this response is a displacement, it is assumed that the relationship between the sought-after stress response and displacement is linear, so some constant C times the displacement will yield the stress. So, for $\zeta=.006$, and a natural frequency of 3556 Hz (in this particular example, this is the Fourier peak of a particular time history),

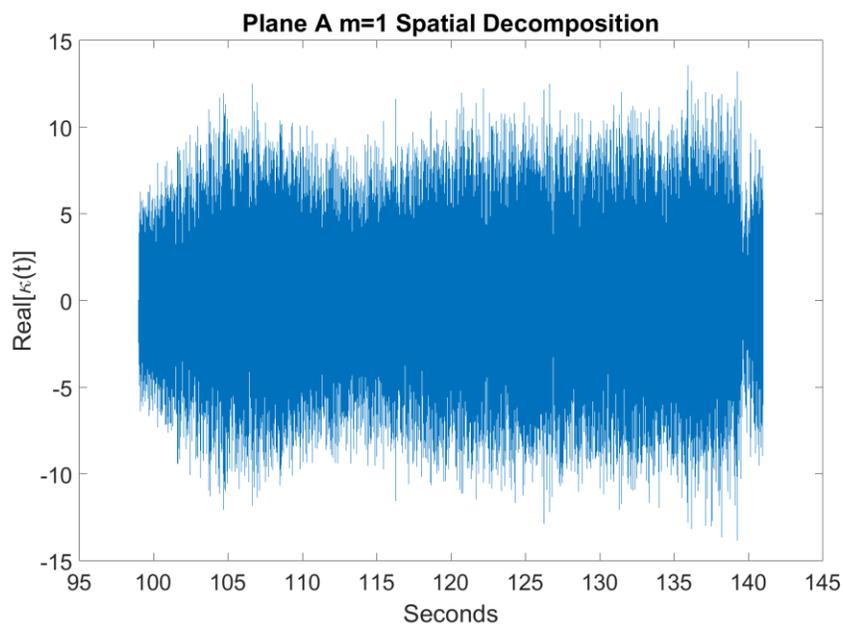


Figure 4. Time History after Spatial Decomposition
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$$F_{eq} = (C)\sigma_{FE_{max}}2\zeta\omega^2 = (C)(12474)(2)(.006)(2*\pi*3556)^2 = 7.4726e10 \quad (5)$$

The constant can be ignored in the rest of the procedure as it will not affect the answer.

At this point, we use the Matlab® script ODE45u to apply the transient excitation to the SDOF system to generate the response. The correct application of the script is verified by generating a harmonic excitation of magnitude F_{eq} , applying it to the SDOF, and obtaining the response, which matches the FEM resonant response of 12474 psi, as intended. Since F_{eq} is equivalent to the harmonic force applied to the FEM, the ratio of F_{eq} to that original harmonic forcing function is then calculated for use as a calibration constant.

$$ratio = \frac{F_{eq}}{Amp_{sin}} = \frac{7.4726e10}{2.23} = 3.3509e10 \quad (6)$$

The finite element model to SDOF simplification requires that the three-dimensional loading be lumped into a single point application, in this case the circumferential location. Since it is unknown a-priori which location will yield the worst stresses, a loop was written to calculate transients for 10 degree increments. To apply this load, these transients are multiplied by the calibration ratio, applied to the SDOF using a Matlab® script, and a stress response generated. In addition, a worst-case response was initially assumed to be when the peak frequency of the transient (as identified by a Fourier transform) is related to the SDOF natural frequency using the precise SDOF relationship

$$\Omega = \omega_n \sqrt{1 - 2\zeta^2} \quad (7)$$

Several iterations showed that the peak was not always at this value though, most likely because of non-stationarity of the data. Another nested loop was therefore written to obtain the response time history for a range of possible natural frequencies.

To determine the worst of all the circumferential and natural frequency cases, the maximum value was obtained from the time history and plotted in Figure 5. The result is a peak response at 180 degrees at a natural frequency of 3543.5 Hz. For illustration, the individual transient excitation, scaling, and stress time history response for that worst case are shown graphically in Figure 6.

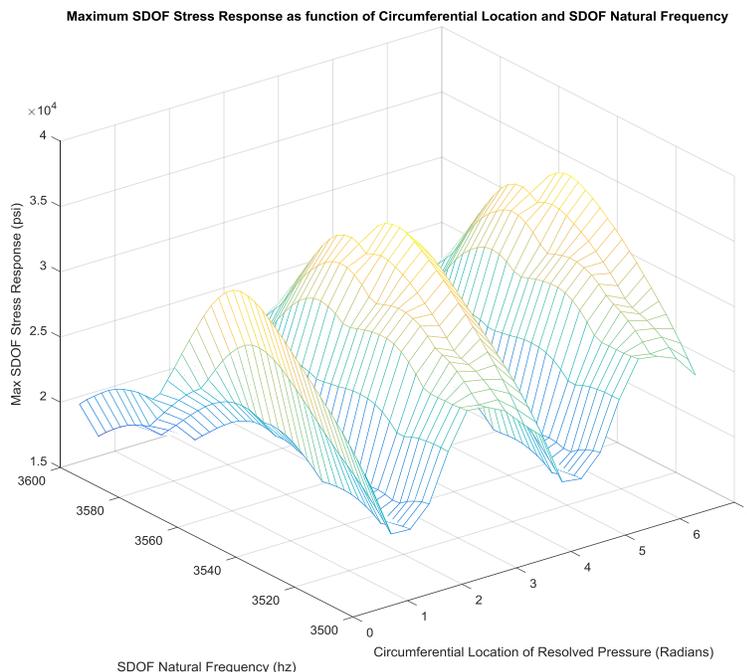


Figure 5. Maximum Transient Stress Response of SDOF as Function of Circumferential Location and SDOF Natural Frequency

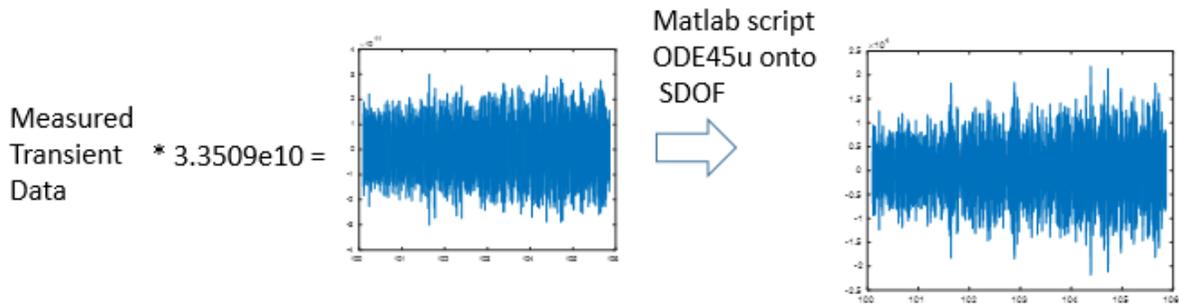


Figure 6. Transient Excitation Multiplied by Scaling Factor, Resulting in Stress Time History

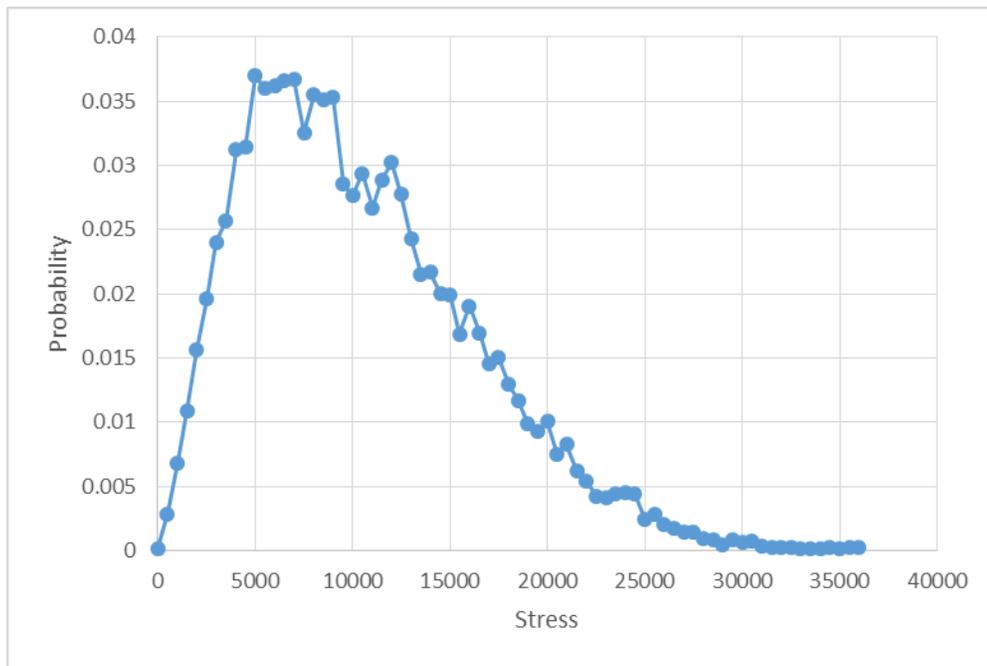


Figure 7. Discrete Probability Density of Peaks of SDOF Response Time History

The benefit of performing this analysis is in the application of the variation of stresses over the life of the component rather than assuming it responds at a single peak stress. This is accomplished by isolating the peaks of every stress cycle and producing a histogram of these values for the above location. This histogram is converted to a discrete Probability Density (figure 7) and used to generate the stress life distribution (i.e., number of cycles for a given value of stress) for the operational life for points in the FEM along possible crack growth paths. In addition, since the HOC is not at full strength during the entire operational life, this stress distribution was scaled further by the strength levels of the HOC which occurred at different durations. With the application of the stress life history from the SDOF analysis and this final convolution with the HOC strength, the final fracture analysis shows the pre-valve has acceptable life and can be fabricated as designed.

IV. Verification

Several different approaches were used to verify the technique. First, a realistic but tractable verification analysis on a full 3-D structure is performed. A simple FEM of a cylinder of approximately the same size at the pre-valve flow guide is created, and Young's Modulus E was adjusted such that the 1ND modal natural frequency equals 3684.45 hz to match the flow guide (see figure 8). As with the flowliner, a frequency response analysis to a spinning wave force excitation is then performed. As expected, defining the axial variation of the spinning wave form is quite difficult, lending credence to one of the motivations for the SDOF analysis. The animation of the

frequency response shows the spinning character of the wave, but the response is a mixture of the 1ND mode and an adjacent 4ND mode; although this interaction is not present for the flow-guide, it should not affect the results.

The second step in the verification is to apply the transient loading on a node-by-node basis and obtain the response. A Matlab® script was used to implement the formula on a node-by-node basis and create the dynamic loading tables for implementation in NASTRAN. This is the step that is approximated by the SDOF analysis, and as expected, generating and running this case is also difficult, even for this simple structure. The loading at one time step is shown in figure 9, and reflects the axially-varying spinning wave character of the forcing function. The Von Mises stress response at a number of time steps is examined to determine several possible peak stress locations, and the time history response at this location is plotted in figure 10. The histogram of this response is shown in figure 11.

Finally, the SDOF technique for the cylinder is performed, and its histogram is shown in figure 12. The comparison with figure 11 indicates that the SDOF technique is conservative, as intended, but not exceedingly so, achieving the goal of the process. This conservatism is due to choosing the worst case natural frequency for the SDOF system, and choosing the worst angular location of the loading.

The largest source of error for the process is obviously the representation of the multi-degree-of-freedom system by a SDOF; the maximum response of an equivalent SDOF system can be shown to be always larger than the

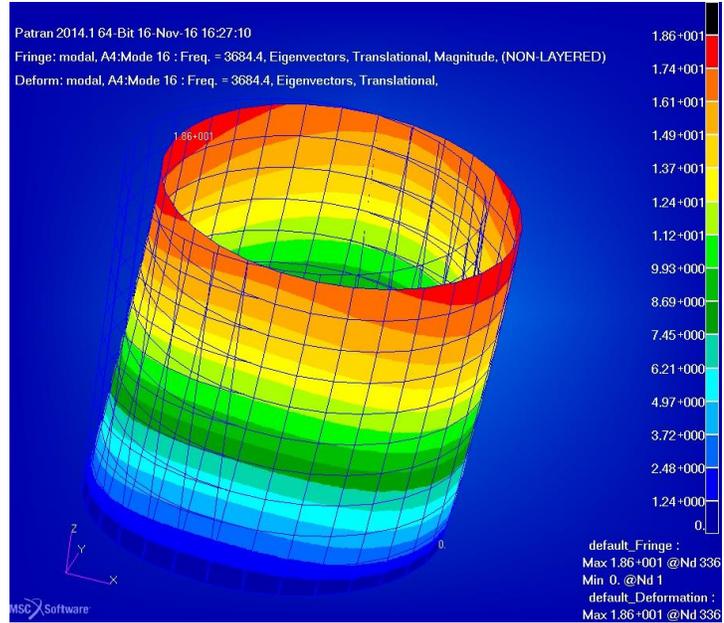


Figure 8. 1ND Mode Shape of Verification Cylinder at 3684Hz

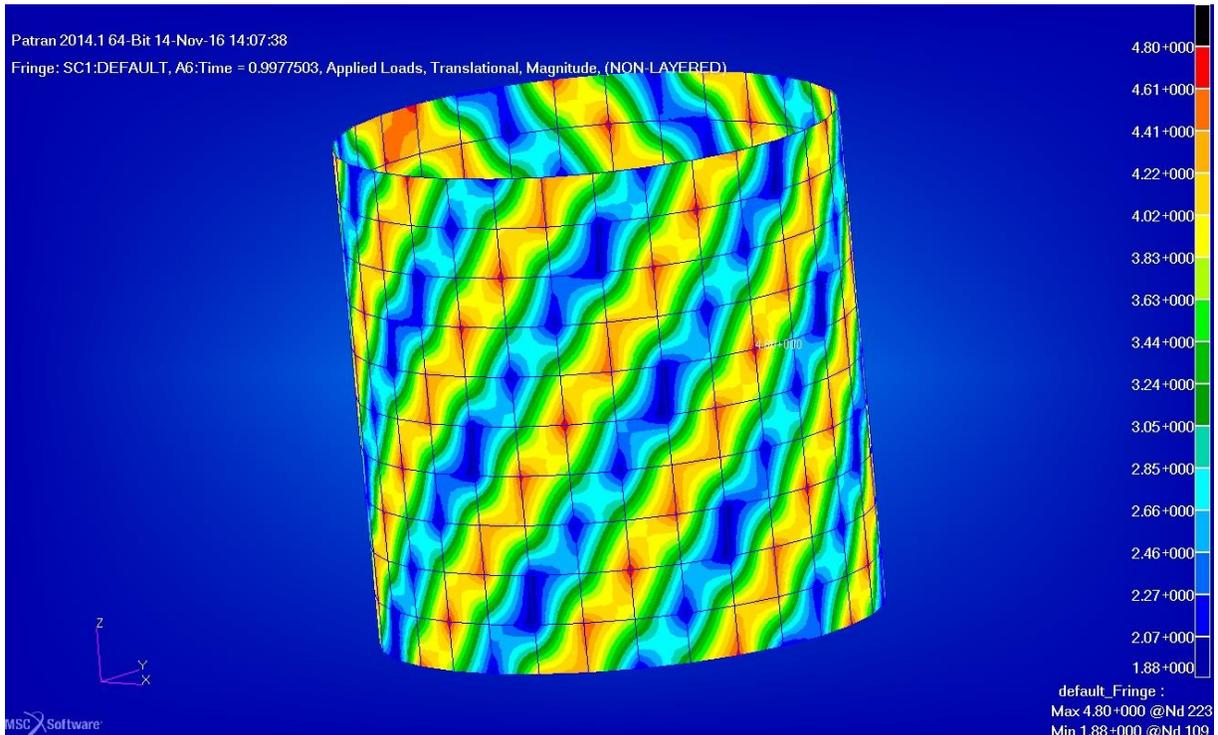


Figure 9. Applied Transient Loading at one time step
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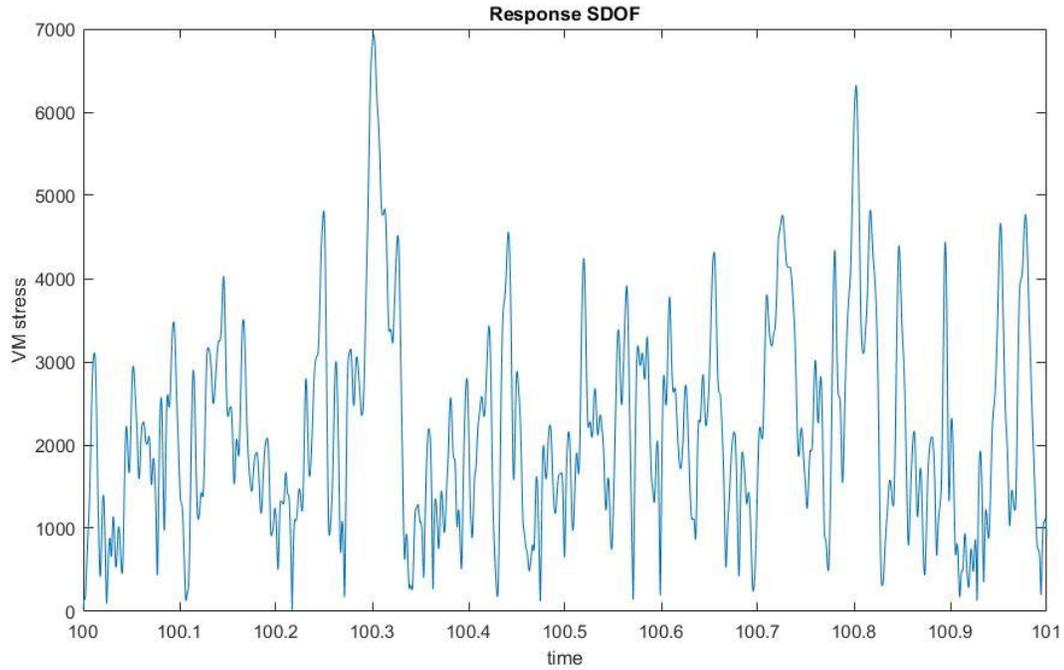


Figure 10. Time History Von Mises Stress for Peak Response Location

maximum response of an MDOF system, but in this case the SDOF maximum response is calibrated to equal the MDOF response. The error in this analysis is due to the response of the non-primary modes to the multi-spectral excitation. This error is a function of the modal density and the similarity of the excitation shape with those non-resonant modes. The relative phase of the modes is also important, as the response of a single location may not be large in an adjacent mode even if it is excited efficiently. A frequency response function (FRF) graph from the FEM for that location will incorporate all of these factors, and can be performed relatively easily.

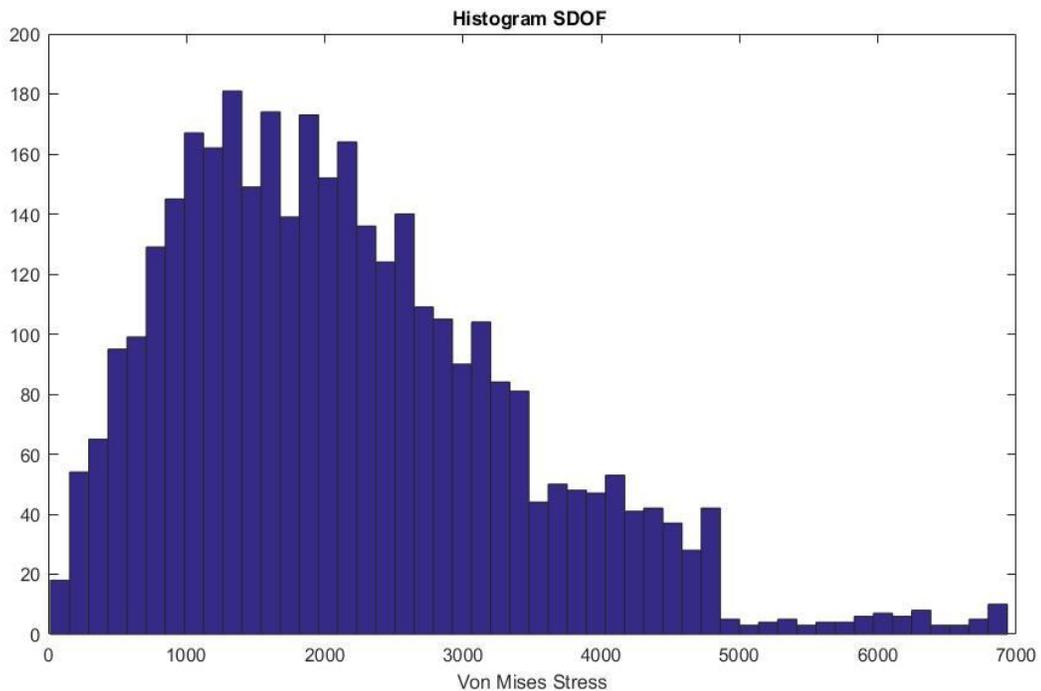


Figure 11. Histogram of Von Mises Stress Response Peak Location for cylinder FEM Transient Analysis
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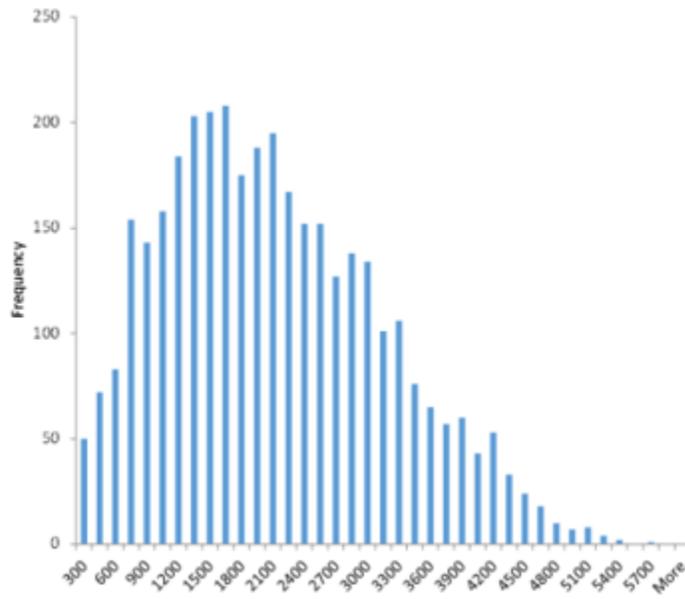


Figure 12. Histogram of VM Stress Response for Cylinder using SDOF technique

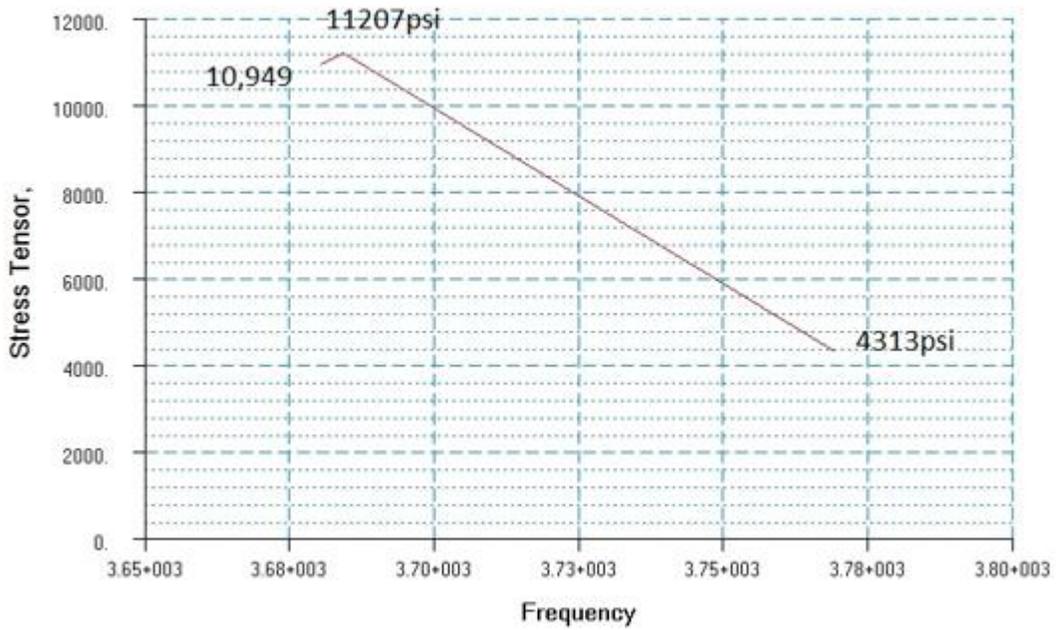


Figure 13. Stress Y Direction Frequency Response

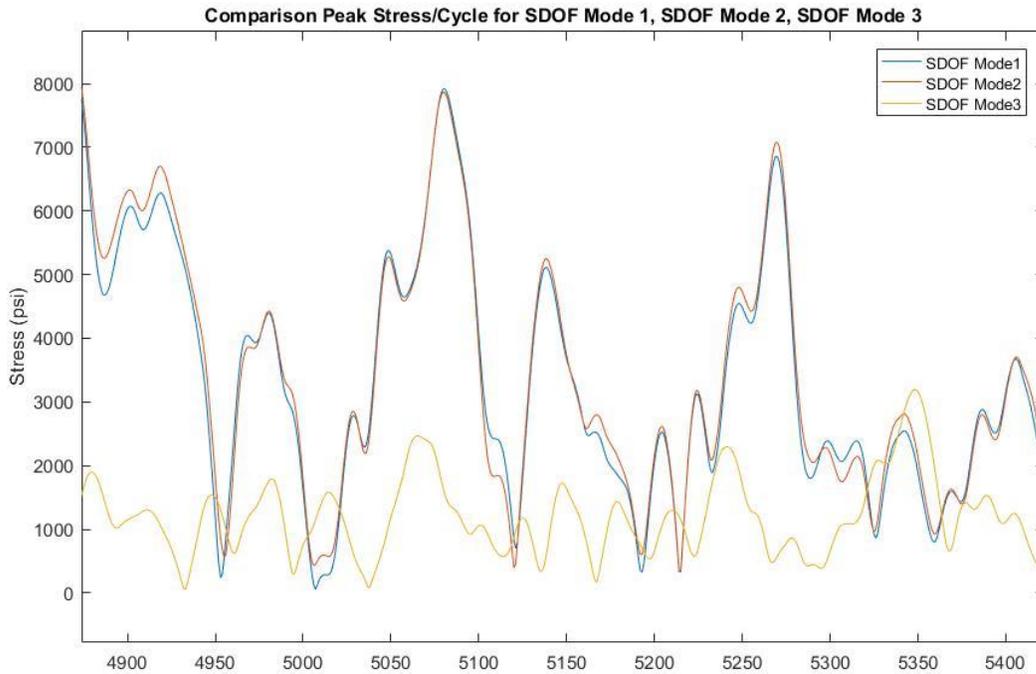


Figure 14. Zoom-in on section of superimposed Peak Stress/Cycle for 3 SDOF models

For the pre-valve, a discrete FRF for the worst case location for the primary response frequency and the two adjacent modes is shown in figure 13 (this analysis uses an updated damping value of $\zeta = .0069$). Unlike the cylinder, which has very well-separated modes, the pre-valve FRF shows a large response of a very similar mode at 3680 hz, only 4 hz from the primary mode at 3684 hz; this situation yields the greatest possible error in the SDOF

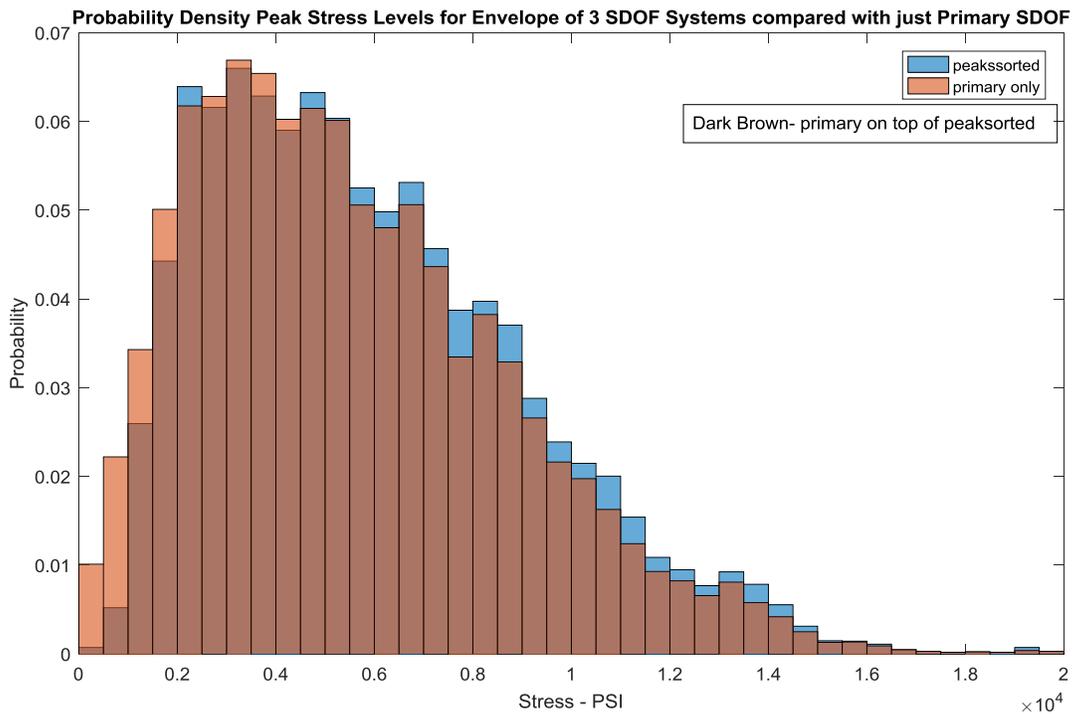


Figure 15. Discrete Probability Density Stress Cycle Peaks for 3-SDOF Envelope compared with Primary SDOF only (envelope of peaks–blue, primary SDOF only- salmon , envelope on top of primary– dark brown)

procedure due to the adjacent mode effect.

To assess the impact of this mode and another mode at a higher frequency, SDOF procedures in addition to the original SDOF using the primary mode are performed using the response values from the FRF; since the primary frequency of 3684 is not actually used in this procedure, the frequency of the secondary calibrated SDOF was set at a value offset by the same amount (-4 hz) from the frequency eventually iterated to, which was 3553 hz, and the next highest mode at 3769 hz (using the delta from the primary to determine the actual value). It is critical to recognize that since each SDOF model is calibrated such that its peak response is equal to the peak response from the FRF of the full model, the maximum response is that SDOF peak, and it includes the effects of the other modes. The total response therefore is no larger than a trace of the envelope of the responses for each SDOF model. To enable the creation of a histogram, though, the envelope is created from a plot of the peak stress for each cycle of the SDOF's (figure 14), which are actually not coincident since the phase relationship with the excitation varies because of the different natural frequencies of those SDOF systems. Comparing the peak stress per cycle in essence assumes the phase is the same, which is conservative. Figure 14 is a zoom of the peak stress/cycle chart, and shows how each of the three SDOF systems can produce the highest response for a particular cycle based on the alignment of that system's natural frequency with the transient spectrum. The histogram of this envelope is then produced (see Figure 15) and overlaid on top of a histogram of the peaks using only the primary SDOF model. As expected, the histogram is slightly skewed to higher values for the total response, showing the larger number of higher stress cycles due to the occasional higher response of the adjacent modes than from the primary mode, but the difference is fairly small. This small error is most likely outweighed by the conservatism previously discussed that are used in the procedure, in addition to the in-phase assumption mentioned just above, validating the application of the single SDOF modeling technique. If additional conservatism is sought, of course, the multiple SDOF method as described above can be easily performed.

V. Conclusion

A single degree-of-freedom approximation technique has been developed to enable rapid application of a temporally-defined multi-spectral semi-narrow-band loading for generation of dynamic stresses. The focus is on quantifying the realistic stress/cycle compared to a resonant analysis to enable acceptable fatigue/fracture life of a component. The technique uses the resonant analysis of a high-fidelity finite element model to produce a transfer function, which is then used to calibrate the response of the SDOF model. The realistic loading can then be applied to the model to obtain a realistic time history of stress, which can then be quantified using a histogram for the calculation of a fatigue spectra along possible fracture paths. The technique is related to the Shock Response Spectra method used for random base excitation approximate analysis, the Miles' Equation technique for random response analysis, and SDOF techniques used for blast analysis of structures, but is quite unique in that the response is calibrated to the modes of a high-fidelity structural model and uses a quick solution of a temporal ordinary differential equation to obtain a complete temporal response history.

Two verification methods are used to validate the process. First, a simpler FEM than the structure in consideration is created and a complete transient, spatially varying loading applied, which would be very difficult to accomplish for the actual structure and is too time-consuming for the assessment of a number of structures in the path of this complicated excitation. The results from this analysis are compared with results from the SDOF technique and show the SDOF results to be conservative yet not excessively so. Second, the effect of adjacent modes found in the frequency response of the component under consideration are assessed. This assessment is performed by creating SDOF models for modes on either side of the primary mode based on the FRF, running the SDOF technique, and enveloping the peak cycle stresses to create a histogram. This comparison does show some underprediction of the single SDOF model, as expected, but the error is small and probably outweighed by several conservatisms used in the SDOF technique.

The application of this unique, tractable reduced-order methodology has enabled the SLS program to avoid substantial cost and schedule penalties if a redesign or change of material were required, as had been considered. It has also enabled quick analysis of a number of other structures undergoing the same or similar excitation fields., and quick assessment when the excitation and structural configuration has been altered due to design changes in the MPS system.

References

- ¹Mullen, Evan M., “Shape Function Limitations within Equivalent Single-Degree-of-Freedom Analysis for Structural Elements Subject to Blasted Loading,” Masters Thesis, Structural Engineering Dept., Lehigh Univ., Bethlehem, PA, 2016
- ²Lucia, F., Papan, D., “Single Degree of Freedom Analysis of Steel Beams under Blast Loading,” *Applied Mechanics and Materials*, Vol. 617, pp. 92-95, 2014, ISSN: 1662-7482.
- ³Craig, R.R, and Kurdila, A. J., *Fundamentals of Structural Dynamics*, 2nd ed., John Wiley & Sons, Hoboken, New Jersey, 2006, pp. 128-136.
- ⁴Rivera, J.L, et. al., “Evaluation of the Response of Post-tensioned Steel Frames with Energy Dissipators Using Equivalent Single-Degree-of-Freedom Systems,” *Advances in Materials Science and Engineering*, Vol. 2014, Article ID 730324, 10 pages, <http://dx.doi.org/10.1155/2014/730324>
- ⁵Wirsching, Paul H., Paez, Thomas L., Ortiz, Keith, *Random Vibrations – Theory and Practice*, John Wiley & Sons, New York, 1995, pp. 204-212.