Planetary Probe Entry Models for Concurrent and Integrated Interplanetary Mission Design

Purdue Aeronautics & Astronautics

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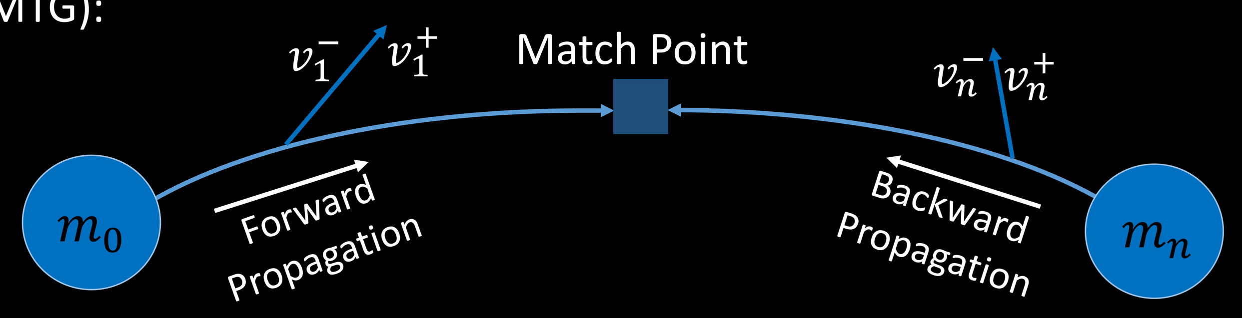
Background

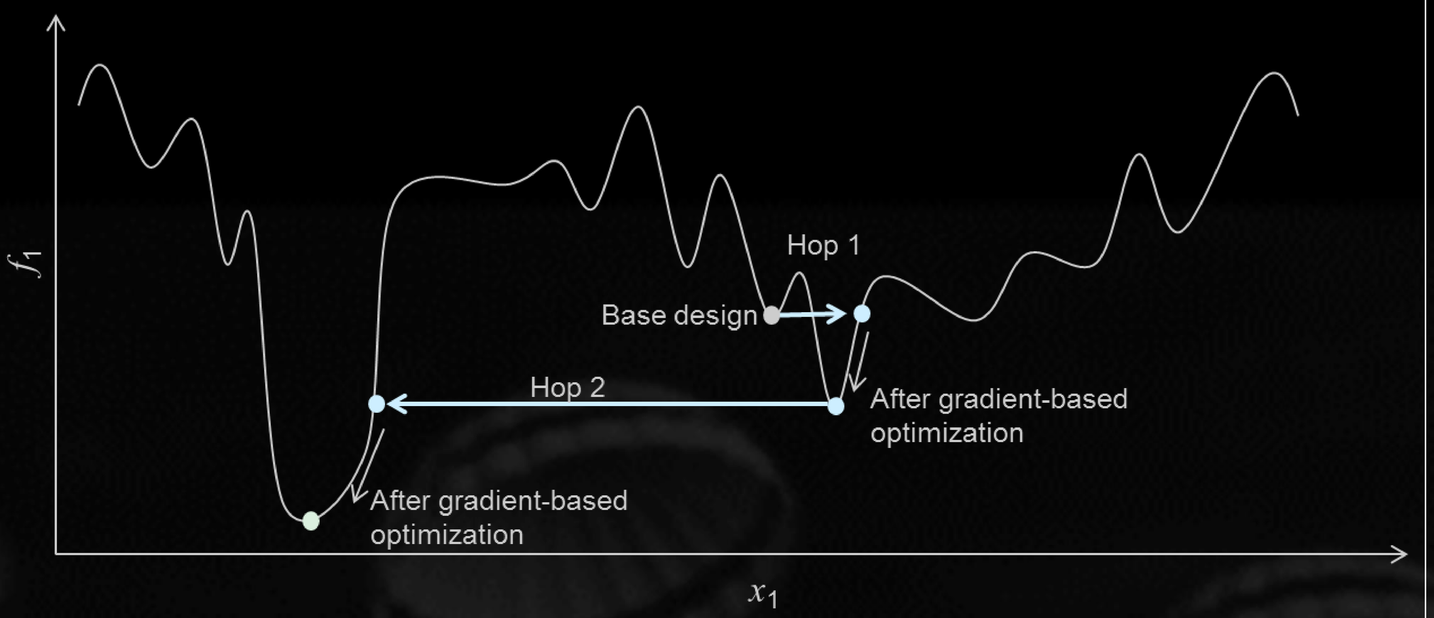
Evolutionary Mission Trajectory Generator (EMTG):

- Rapid interplanetary trajectory design
- Low-thrust and chemical propulsion
- 2-point shooting
- Monotonic basin hopping: searches the design space, no initial guess needed
- Patched conics and integrated solutions



Information Needed for Probe Design:

- Latitude
- Velocity at entry point
- Flight path angle



Entry Probe Design Challenges:

- Nonlinear relationships favor high-fidelity point designs
- What is traditionally beneficial for the interplanetary trajectory can be detrimental to probe design

No low/mid-fidelity rapid-design tool currently exists that simultaneously designs entry probes and interplanetary trajectories.

Thermal Protection System (TPS)

We must account for both convective and radiative heat for the probe. For Earth and Neptune, these are given by[1,2]:

$$\dot{q}_{c,Earth} = c_1 * R_n^{-0.52}\rho^a V^b \qquad \dot{q}_{c,Neptune} = 1.2084e-7 * R_n^{-0.5}\rho^{0.45213}V^{2.6918}$$

$$\dot{q}_{r,Earth} = 3.416e4 * R_n^a \rho^{1.261} f(V) \qquad \dot{q}_{r,Neptune} = 1.279304e-47 * R_n \rho^{0.49814}V^{15.113}$$

We consider 3 different TPS materials: HEEET, PICA, and ACC. Empirical relations are used to determine the TPS thickness at different points along the probe. They follow the general form[3]:

$$t = a(Q/V^2)^b$$

In this expression Q is the integrated heat load along the trajectory and V is the velocity during peak heat rate.

Variable Transformation

We first use a transformation of variables defined by Vinh and Longuski[4,5]:

$$\eta = \frac{\rho S C_D r}{2m\sqrt{\beta r}} \qquad v = V^2/gr \qquad \bar{S} = \sin\gamma_i / \sin\gamma$$

These transformed variables provide the altitude, flight path angle, and velocity along the trajectory for a ballistic entry probe.

Problem Definition

Model Requirements:

1. All equations must be completely analytical
2. All expressions must be continuous functions
3. Every equation must be differentiable

Models Must Provide:

1. Atmospheric trajectory information
2. Probe mass

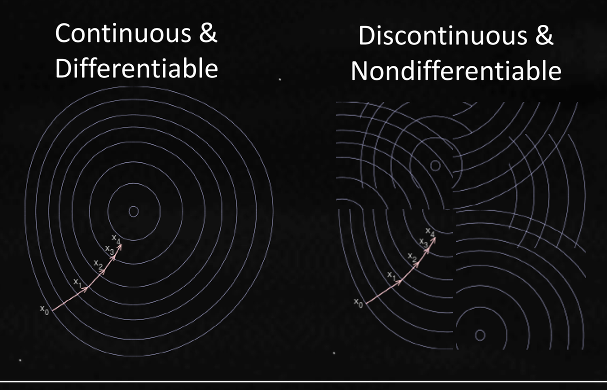
NLP Problem of the form:

Minimize $f(x)$

Subject to: $x_{lb} \le x \le x_{ub}$

$c(x) \le 0$

$Ax \le 0$



Atmospheric Trajectory Model

The transformed variables allow for analytical expressions describing the entry trajectory. These equations were developed by Saikia[6] and provide the foundation for the probe models:

$$v = \bar{v}_i e^{-\eta}[1 + \bar{\epsilon} f_1(\eta) + \bar{\epsilon}^2 f_2(\eta)] \qquad \bar{S} = 1 + \bar{\epsilon} g_1(\eta) + \bar{\epsilon}^2 g_2(\eta)$$

Where

$$f_1 = \bar{v}_i(\eta - \eta_i) - (e^\eta - e^{\eta_i}) - \bar{v}_i(\tan^2\gamma_i + \eta)L + (2\tan^2\gamma_i + \eta)E_0$$

$$f_2 = \bar{v}_i[e^{\eta_i} - \bar{v}_i(3 + 2\tan^2\gamma_i)](\eta - \eta_i) + \frac{1}{2}\bar{v}_i^2(\eta - \eta_i)^2 + \bar{v}_i(3 - \eta)(e^\eta - e^{\eta_i}) - \frac{1}{2}(e^{2\eta} - e^{2\eta_i}) + \bar{v}_i[\bar{v}_i\eta_i(3 + 2\tan^2\gamma_i) - (2 + \tan^2\gamma_i)e^{\eta_i}]L + \bar{v}_i[\bar{v}_i(3 + \tan^2\gamma_i - \eta)(\eta - \eta_i) + (\eta - 2)(e^\eta - e^{\eta_i})]L + \frac{1}{2}\bar{v}_i^2(\tan^4\gamma_i - 3\eta + \eta^2)L^2 + [\bar{v}_i(2 + \tan^2\gamma_i) - 4\bar{v}_i\eta + \bar{v}_i\eta^2 + \bar{v}_i\eta(3 - \eta)L + (2 - \eta)e^\eta]E_0 + \frac{1}{2}\eta(\eta - 3)E_0^2 + 2(\eta - 1)E_0(2\eta) - \bar{v}_i\tan^2\gamma_i(2\tan^2\gamma_i + \eta)F$$

$$g_1 = \bar{v}_i L - E_0$$

$$g_2 = \bar{v}_i(e^\eta - e^{\eta_i}) - \bar{v}_i e^\eta L + \bar{v}_i^2\left(\frac{3}{2} + \tan^2\gamma_i\right)L^2 + [\bar{v}_i - \bar{v}_i\eta_i + e^{\eta_i} + e^\eta - 3\bar{v}_i(1 + \tan^2\gamma_i)L]E_0 + (\frac{3}{2} + 2\tan^2\gamma_i)E_0^2 - 2E_0(2\eta) + \bar{v}_i F tan^2\gamma_i$$

$$\bar{v}_i = v_i e^{\eta_i} \qquad \bar{\epsilon} = \frac{1}{\beta r \bar{v}_i \tan^2\gamma_i} \qquad L = \ln\frac{\eta}{\eta_i} \qquad E_0 = \ln\frac{\eta}{\eta_i} + (\eta - \eta_i) + \sum_{n=1}^{\infty}\frac{\eta^n - \eta_i^n}{n.n!} \qquad F = \frac{1}{2}\left(\ln\frac{\eta}{\eta_i}\right)^2 - \ln\frac{\eta}{\eta_i}\sum_{n=1}^{\infty}\frac{\eta_i^n}{n.n!} + \sum_{n=1}^{\infty}\frac{\eta^n - \eta_i^n}{n^2 n!}$$

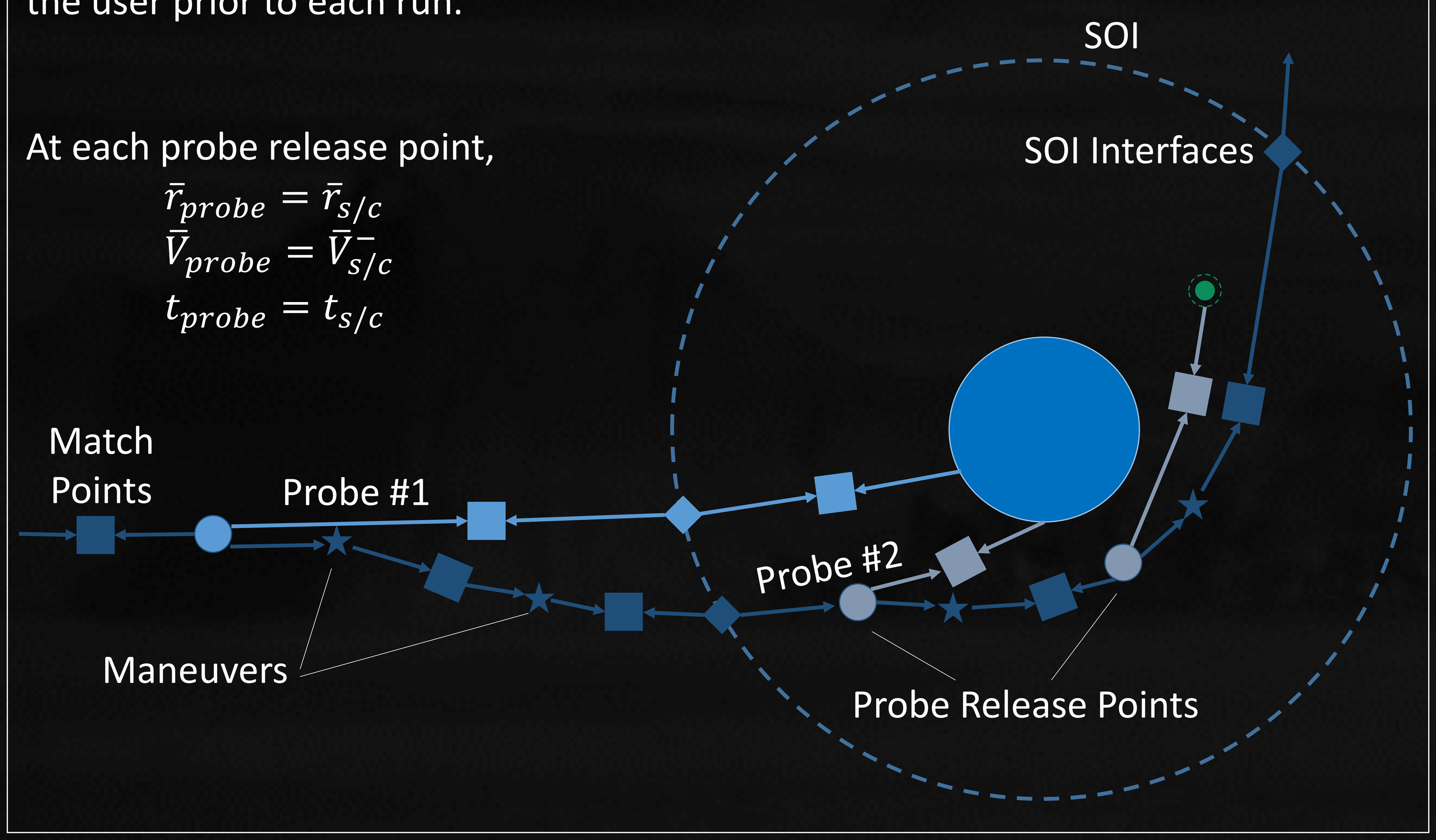
Probe Approach Trajectory Architecture

The Sphere of Influence (SOI) interface allows for the arrival body to be considered in a patched conic model.

The probes must be dropped before or after the SOI. This is a choice made by the user prior to each run.

At each probe release point,

$$\bar{r}_{probe} = \bar{r}_{s/c}$$
$$\bar{V}_{probe} = \bar{V}_{s/c}$$
$$t_{probe} = t_{s/c}$$



Foam Estimation Model

For the purposes of this model, foam is considered to have negligible mass but contributes to the volume of the probe. Assuming a perfectly rigid surface, the stroke and compression strength can be calculated via the following equations provided by Samareh et. al.[3]:

$$Stroke = \frac{v_{terminal}^2}{2\bar{G}g} \qquad FS = \frac{M_{payload} * v_{terminal}^2}{2A_{ref} * Stroke} = \frac{M_{payload}\bar{G}g}{A_{ref}}$$

- $\bar{G}$ is the g-load limit for the science payload and A_{ref} is the reference area of the foam
- Stroke distance is directly proportional to foam thickness and the relationship is determined by the assumed type of acceleration impulse
- Compression strength determines the type of foam required

Probe Geometry and Mass Estimation

There are three mass components in this model which constitute the entirety of the probe's mass:

1. TPS Mass

$$m_{TPS} = A_{cap}t_{cap}\rho_{TPS} + A_{cone}t_{cone}\rho_{TPS}$$

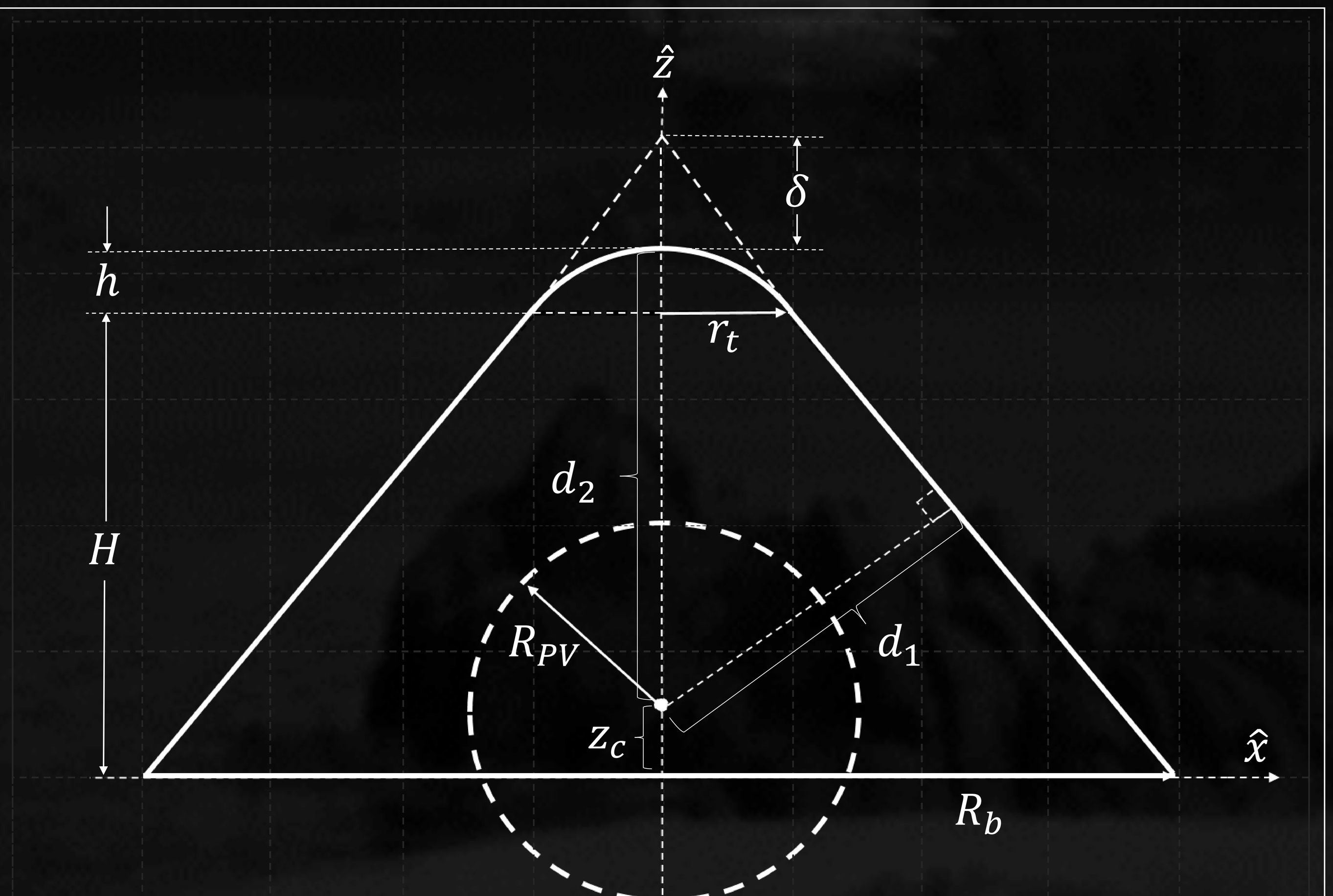
2. Structural Mass (of a spherical pressure vessel)[7]

$$m_{struct} = 4.966\frac{4\pi}{3}R_{PV}^3\frac{\sqrt{p_{env}}}{(\sqrt{E}/\rho)_{material}}$$

3. Payload Mass is an input from the user.

Currently, only a single probe geometry is considered. For this geometry, we assume:

- Axially symmetric sphere-cone geometry
- The pressure vessel is centered along the axis of symmetry
- The cone angle is fixed and tied to the arrival body
- No more than half of the pressure vessel may stick out the back of the probe



Future Work

1. Validation of ballistic entry models and investigate alternative analytical models.
2. Computation of all model partial derivatives for NLP optimization.
3. Incorporate models into EMTG.
4. Develop trajectory models for controlled entry:
   a. Human entry models
   b. Aerocapture trajectories
   c. Aerogravity Assist trajectories

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