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Shearlet Features for Registration of Remotely Sensed Multitemporal Images

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Problem Description and Outline

- Image registration is a challenging problem in the remote sensing community.
- Specifically, the registration of multimodal and multitemporal images suffers from accuracy and robustness problem.
- In this talk, a novel automatic image registration algorithm for multitemporal images, based on the cutting-edge mathematical construction of *shearlets*, is presented.
- Outline: describe shearlets, summarize our algorithm, and show results on synthetic and real multitemporal data.

Background on Image Registration

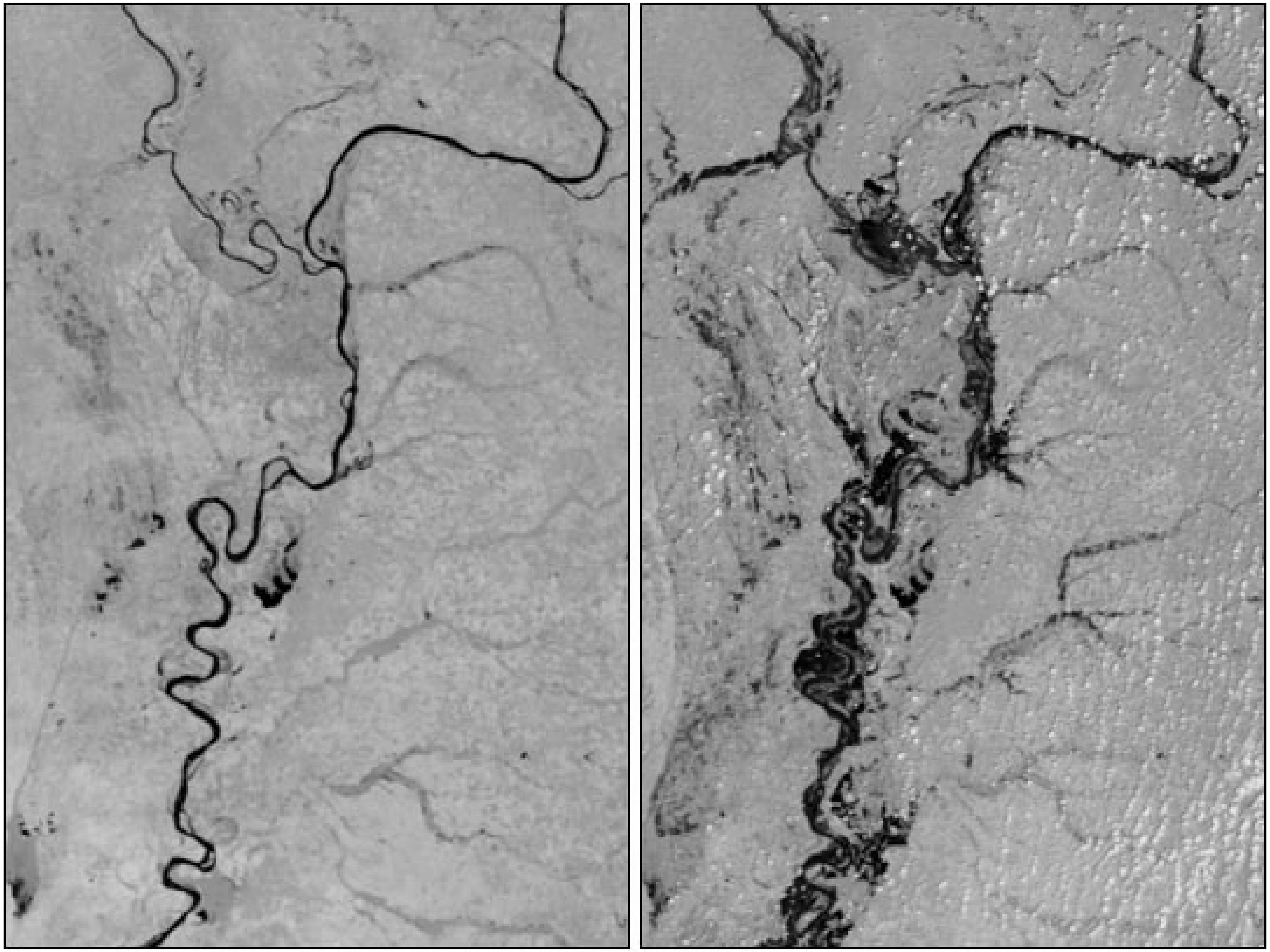
- The process of image registration seeks to align two or more images of approximately the same scene, acquired at different times or with different sensors.

- Image registration may be viewed as the combination of four separate processes:

1. Selecting an appropriate **search space** of admissible transformations.
2. Extracting relevant **features** to be used for matching.
3. Selecting a **similarity metric** in order to decide if a transformed input image closely matches the reference image.
4. Selecting a **search strategy**, which is used to match the images based on maximizing or minimizing the similarity metric.

Multitemporal Images Challenges

Mississippi and Ohio Rivers before & after Flood of Spring 2002 (Terra/MODIS)



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Features for Image Registration: Harmonic Analysis

- The selection of features to use for image registration is a crucial question.

- A huge variety of approaches abound, from selected ground control point algorithms like SIFT and its variants, to transform methods.

- Chief among transform methods are those based on harmonic analysis, in particular *wavelets*, which find global features based on scale.

- That is, wavelet-like algorithms decompose an image into fine and coarse-scale features, which are then used to efficiently register the images.

- Wavelet methods are prominent and have been shown effective in a variety of image registration regimes.

Generalizing Wavelets: Shearlets

- While wavelets have had much success in image registration, they are fundamentally *isotropic*, meaning they have no directional sensitivity.
- This makes capturing edge information with wavelets suboptimal.
- Recently, wavelets have been generalized to be *anisotropic*, meaning directionally sensitive.
- Chief among these generalizations are *shearlets*, which ***refine the wavelet construction by including a directional component.***
- Shearlet mathematical theory is rich, and shearlets are known to *optimally represent* a broad class of image signals, suggesting their use for image registration.

Wavelets and Shearlets - Mathematics

- Wavelets decompose an image with respect to scale and translation.
- For a suitable wavelet ψ, we may decompose a signal f as:

$$f = \Sigma_{m,n} <f,\psi_{m,n}>\psi_{m,n}, \quad \psi_{m,n}(x)=2^{-m/2}\psi(A^m x-n), \text{ where } A=2I.$$

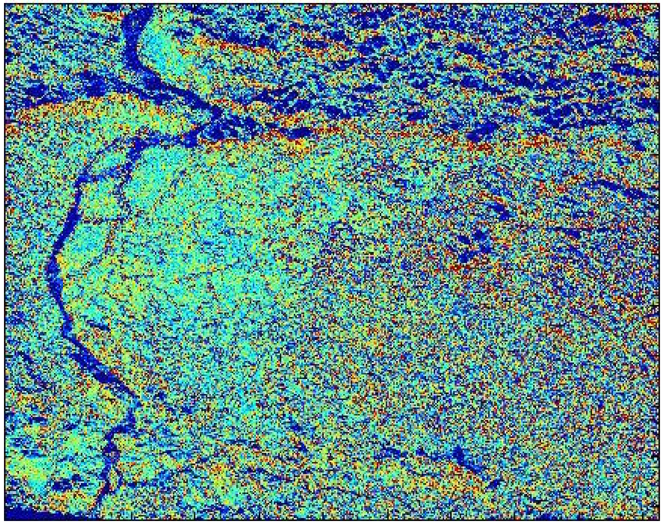
- Shearlets decompose with respect to scale, translation, and direction.

For a suitable shearlet ψ, we may decompose a signal f as:

$f = \Sigma_{m,n,k} <f,\psi_{m,n,k}>\psi_{m,n,k}, \quad \psi_{m,n,k}(x)=2^{-m/2}\psi(S_k A_a x-n)$, where A_a is an anisotropic dilation matrix and S_k is a shearing matrix.

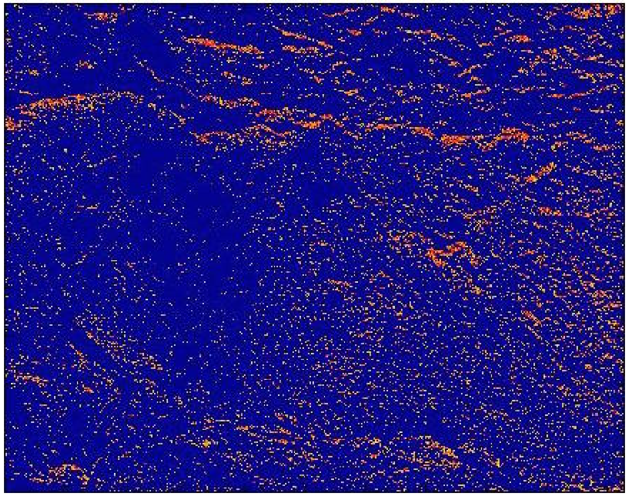
- The shearing matrix S_k focus on a particular direction, making the shearlet decomposition directionally sensitive.

Wavelet Features v. Shearlet Features

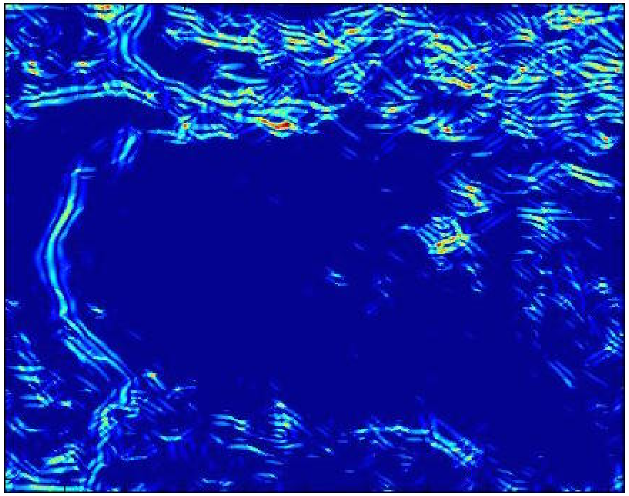


SAR image (1024x1024)

Wavelet Features



Shearlet Features



Registration Algorithm Description

1. Input a reference image, I^r, and an input image I^i. These will be the images to be registered.
2. Input an initial registration guess $(\theta_0, T_{x0}, T_{y0})$. In our experiments, we will vary the initial registration guess relative to the true registration in order to evaluate the robustness of the algorithm.
3. Apply shearlet features algorithm algorithm to I^r and I^i. This produces a set of shearlet features for both, denoted $S_1^r,\ldots,S_n^r$ and $S_1^i,\ldots,S_n^i$, respectively. Here n refers to the level of decomposition chosen.
4. Match S_1^r with S_1^i with a least-squares optimization algorithm and initial guess $(\theta_0, T_{x0}, T_{y0})$ to get a transformation T_1^S. Using T_1^S as an initial guess, match S_2^r with S_2^r to acquire a transformation T_2^S. Iterate this process by matching S_j^r with S_j^i using T_{j-1}^S as an initial guess, for $j=2,\ldots,n$. At the end of this iterative matching, we acquire our final *shearlet-based registration*, call it $T^S=(\theta^S, T_x^S, T_y^S)$.
5. Output T^S.

Experiment Design

- Question: Are the sparse anisotropic features produced by the shearlets algorithm more robust than the wavelet features?

- Experiments: Compare the robustness of shearlet features matching against matching with three types of wavelet features (previously studied):
    - Spline wavelets,
    - Simoncelli band pass features, and
    - Simoncelli low pass features.

- Robustness is tested by running the algorithms with different, worsening initial registration guesses. We perturbed the truth registration parameters by adding artificial translations and rotations.

- A robust algorithm should be able to recover the correct registration transformation, even for a very poor initial guess.

Experiment Evaluation (cont.)

- If (Tx_T, Ty_T,Rot_T) is the "true registration", the initial guess is given in the range:

  $[Tx_T - 50, Tx_T + 50] \times [Ty_T – 50, Ty_T + 50] \times [Rot_T – 50, Rot_T + 50]$

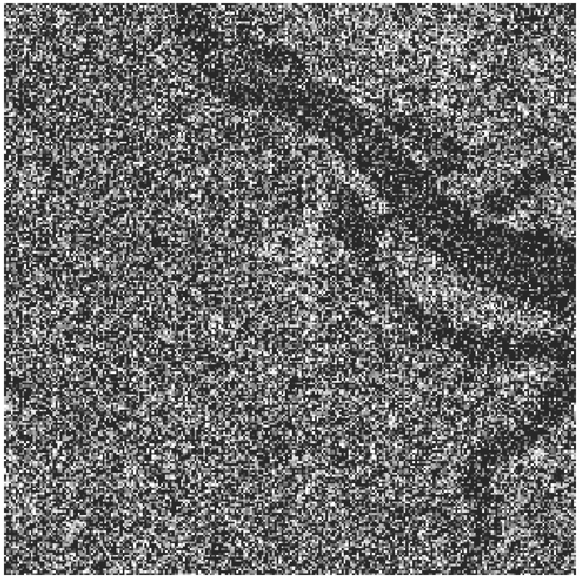
  and

  with a step of .5 pixels and .5 degrees.

- After running the experiment for all the initial guesses in this range, convergence is evaluated:
  - This is based on whether the root mean square error (RMSE) between the computed registration and the correct registration is sufficiently small.
  - *For the purpose of this experiment*, i.e., for a reasonable measure of robustness, we consider the experiment to converge if the RMSE is under a threshold of 5 (Note: these experiments do not intend to measure the accuracy of the algorithms.)

Synthetic Experiments with Noisy Data

Add Gaussian white noise, mean 0, variance .05

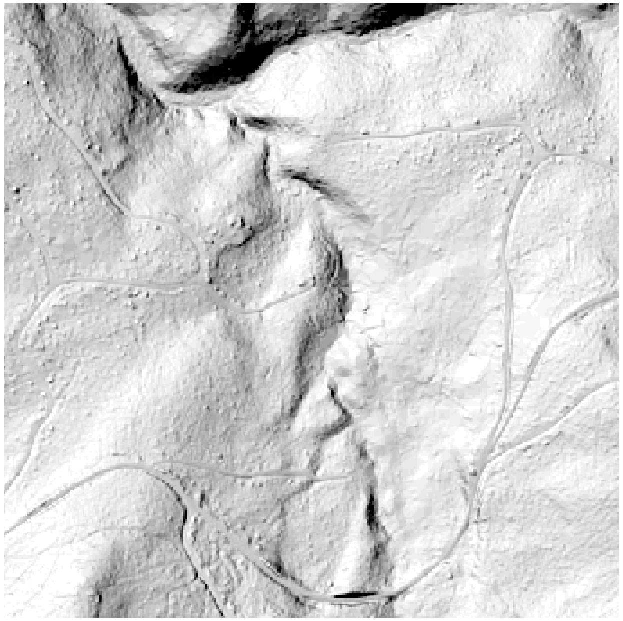


- A noisy version of an ETM+ image of the Washington DC Area, USA, is registered against the original image. The image was captured in 1999. Gaussian white noise with mean 0, variance .05 was added to the original image, to produce the noisy image.
- The “True Registration” is (0,0,0) and to test the robustness of the algorithm, the initial guess is varied from (-50,-50, -50) to (50, 50, 50), stepping by increments of .5 in all coordinates.

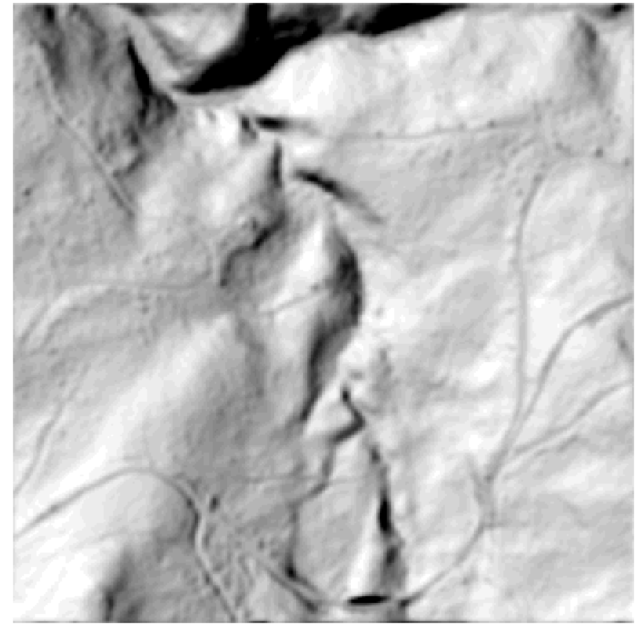
Results for Noisy ETM+ Experiments

Registration Features	*Number Converged Experiments out of 201*	*Percentage Converged Experiments*	*Mean RMSE*
Spline Wavelets	***31***	***15.42%***	***0.0579***
Simoncelli Band Pass	***42***	***20.90%***	***0.0805***
Simoncelli Low Pass	***67***	***33.33%***	***0.0560***
Shearlets	***98***	***48.76%***	***1.8486***

Synthetic Experiments with Radiometrically Warped Data



Apply 512 x 512 PSF, with 0's except to the center 5x5 square of 1's.



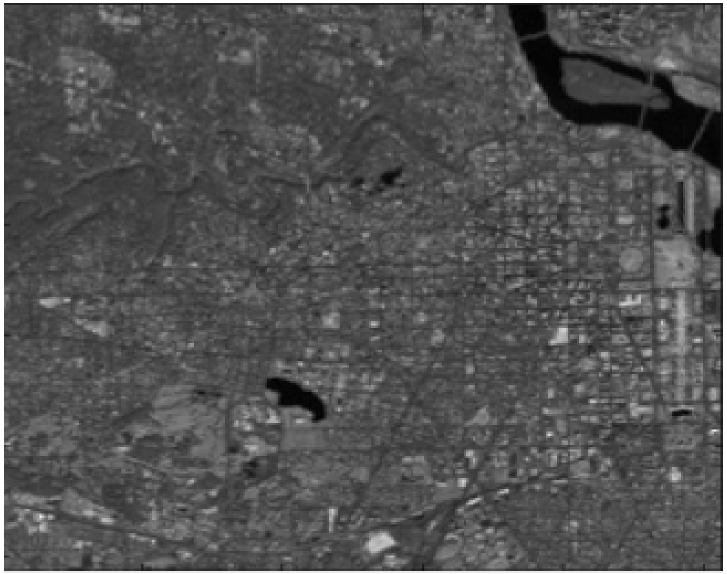
- A radiometrically distorted lidar scene of Mossy Rock, USA, is registered against the original scene. The scene was captured in 2002 using an airborne laser swath mapping conducted by Terrapoint LLC, under contract with the USGS.

- The “True Registration” is (0,0,0) and to test robustness, the initial guess is varied from (-50, -50, -50) to (50, 50, 50), stepping by increments of .5 in each coordinate.

Results for Radiometrically Altered Lidar Experiments

Registration Features	*Number Converged Experiments out of 201*	*Percentage Converged Experiments*	*Mean RMSE*
Spline Wavelets	***74***	***36.82%***	***0.3552***
Simoncelli Band Pass	***42***	***20.90%***	***0.0074***
Simoncelli Low Pass	***72***	***35.82%***	***0.2412***
Shearlets	***108***	***53.73%***	***0.0204***

Multitemporal Experiments



- A Landsat 7 ETM+ (left) and Landsat 5 TM image of the Washington DC area, USA, taken in 1999 and 1996, are registered. Note the substantial differences in the two images.
- The “True Registration” is (103, -8, 0). To test robustness, initial guesses between (0,0,0) and (100,-9,0) are considered.

Results for Multitemporal Images

$(T_{x0}, T_{y0}, \theta_0)$	Simoncelli Band Pass	Spline Waveletts	Simoncelli Low Pass	Shearlet
(0,0,0)	(0.5,3.4, -6.6)	(-1.5, 1.1, -2.4)	(-12.2, 2.2, -14.7)	(-0.1, 0.3, 0.1)
(10, -1, 0)	(10.8, 14.9, -4.5)	(10.2, -0.6, 0.1)	(19.2, 6.8, -10.0)	(62.6, 33.1, 8.54)
(20,-2,0)	(10.8, 14.8, -4.6)	(18.4, -1.8, -1.0)	(41.9, -0.9, -12.3)	(64.8, 30.3, .1)
(30, -3, 0)	(30.1, -3.0, 0)	(29.6, -2.7, -0.2)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(40, -4, 0)	(42.3, -1.8, -13.3)	(39.3, -4.5, -1.3)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(50, -5, 0)	(48.1, 4.9, -3.8)	(39.3, 4.0, -1.3)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(60, -6, 0)	(61.3, -1.2, .6)	(62.9, -1.0, -0.1)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(70, -7, 0)	(60.8, 12.8, .8)	(70.9, -0.2, -1.2)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(80, -9, 0)	(103.5, -8.0, .1)	(103.5, -8.0, 0)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(90, -9, 0)	(103.5, -8.0, .1)	(103.5, -8.0, 0)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)
(100, -9, 0)	(103.5, -8.0, .1)	(103.5, -8.0, 0)	(103.5, -8.0, 0.1)	(103.6, -8.2, .1)

Analysis and Conclusions

- Overall, shearlets improve robustness, but at a cost to registration accuracy.
- Shearlets use edge features well, while wavelets use textural features well.
- Together, they have the potential to perform better than either separately.
- Current work on integrating the robustness of shearlets with the accuracy of wavelets, e.g.:

  1.) Shearlets Registration (on Original or on Compressed Image) => Get Initial Guess

  2.) Wavelets Registration Using Initial Guess from (1) => Get Final Accurate Registration

Current Work with Multimodal Images

- In an upcoming publication, we discuss the value of this hybrid method for a variety of synthetic and multimodal images. In general, this method combines the good robustness from matching with shearlets with the accuracy of wavelet matching.
- In one example of registering large ETM+ Red to ETM+ NIR, we saw an average increase in robustness of 58.29% from using shearlets+wavelets, compared to wavelets-only.
- Other multimodal data types, including lidar-to-optical and MODIS-to-ETM+ shall be investigated as well.

References

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- Ilya Zavorin and Jacqueline Le Moigne, “Use of multiresolution wavelet feature pyramids for auto- matic registration of multisensor imagery,” IEEE Transactions on Image Processing, vol. 14, no. 6, pp. 770–782, 2005.