

### A little bit about me...

- I came from a really small fishing village...
- I didn't speak English at all...
- ...but I really loved airplanes...
- ...and I was good at math & science






### My career with NASA

- Intern graduate student (airfoils) 1982
- Hired fulltime as an aerodynamicist 1983
- Aircraft: F-8 Oblique Wing, Deep Stall Schwiezer, hypersonics (M 14.2), F-18 HARV, SR-71A, F-106 Eclipse, X-37, X-38, X-36, X-48, Mars airplanes,...
- Chief of Aerodynamics 2002
- Deputy Director of Research/Engineering 2004
- Washington DC, Special Assistant to the Associate Administrator of Aero 2008
- Project Manager 2009
- Associate Director of Research 2012
- Chief Scientist 2014



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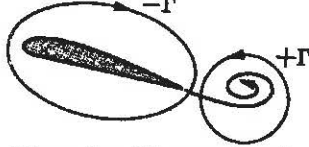
## On the Minimum Induced Drag of Wings

Albion H. Bowers  
NASA Armstrong Chief Scientist


NASA Neil A. Armstrong Flight Research Center

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## Lift









Where does lift come from?  
You should wonder about this...





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## Flying Wings

Flying wings are so elegant and elemental...  
Yet they are difficult to fly and handle poorly. Why?

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## Imagination vs Knowledge

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- Requirements and Assumptions
  
- Concepts and Solutions

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## Questions vs Answers

- "The important and difficult job is never to find the right answers, it is to find the right question." –Peter Drucker, *Concept of the Corporation and Management*
- "...question the unquestionable." –Ratan Tata, CEO Tata Group
- "They get a kick out of screwing with the status quo. They can't bear it. So they spend a tremendous amount of time thinking about how to change the world. As they brain storm, they like to ask: if we did this, what would happen?" –Meg Whitman, cofounder of ebay, PayPal, and Skype

## Personal Air Vehicles



## Flying Wings



Flying wings are so elegant and elemental...  
Yet they are difficult to fly and handle poorly. Why?

## Birds



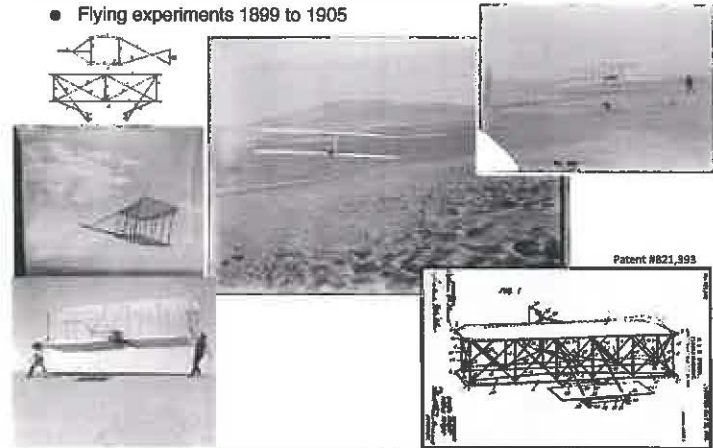
## The Four Ways Birds Differ from Aircraft



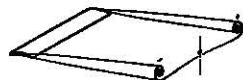
- Birds turn and maneuver without a vertical tail
- Birds have slender tips that carry little load
- Birds gracefully fly formation with overlapped wingtips
- Birds have narrow wingtips without tip stall

## Wilbur & Orville Wright

- Flying experiments 1899 to 1905



## Prandtl Lifting Line Theory



- Prandtl's "vortex ribbons"



- Elliptical spanload for a given span (1920)
- "the downwash produced by the longitudinal vortices must be uniform at all points on the aerofoils in order that there may be a minimum of drag for a given total lift."  $y = c$



Ludwig Prandtl

This is the accepted theory and the standard for the minimum drag of wings. But what is a wing? Is it only aerodynamic? What about the structure?

## Wingtip vortices





**Wingtip vortices**



**Elliptical Spanload**



**Elliptical Spanload**



**Elliptical Spanload and Wingtip Vortices**



## Fundamental Assumptions

For a long time it was difficult to find suitable functions to express the distribution of lift, from which a plausible distribution of  $s$  would be obtained by equation (27). After various attempts it was found that a distribution of lift over the span according to a half ellipse gave the desired solution. According to this, if the origin of coordinates is taken at the center of the wing,

$$\Gamma = \Gamma_0 \sqrt{1 - \left(\frac{y}{b/2}\right)^2}, \text{ hence } \frac{d\Gamma}{dy} = \frac{-\Gamma_0 y}{b \sqrt{\left(\frac{b}{2}\right)^2 - y^2}}$$

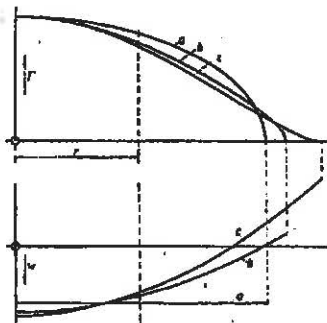
(4) The most important of Betz's theorems, from a practical standpoint, furnishes the complete analogy to Munk's theorem concerning the wing system having the least drag, and, corresponding perfectly to the statements in sections 27 and 28, may be expressed thus: The flow behind a propeller having the least loss in energy is as if the screw surfaces passed over by the propeller blades were solidified into a solid figure and this were displaced backward in the nonviscous fluid with a given axial velocity. The potential difference between the front and rear sides of a screw surface at one and the same point furnishes, then, again the circulation  $\Gamma$  of the corresponding point of the propeller blade.



## Birds



## Minimum Induced Drag & Bending Moment



- Prandtl (1933)  
Constrain minimum induced drag  
Constrain integrated wing bending moment  
22% increase in span with 11% decrease in induced drag

## Fig. 1. Theoretical Minimum Induced Drag

The theoretical minimum induced drag is obtained when the circulation distribution is elliptical. This is shown in the graph above, where the solid line represents the elliptical circulation distribution and the dashed line represents the induced drag distribution. The induced drag is minimized when the circulation is elliptical.

The induced drag coefficient  $C_{Di}$  is given by the equation:

$$C_{Di} = \frac{C_L^2}{\pi e AR}$$

where  $C_L$  is the lift coefficient,  $e$  is the span efficiency factor, and  $AR$  is the aspect ratio. For an elliptical circulation distribution,  $e = 1$ .

The induced drag is also related to the induced velocity  $w$  by the equation:

$$C_{Di} = \frac{2w}{V}$$

where  $V$  is the free stream velocity.

The induced velocity  $w$  is given by the equation:

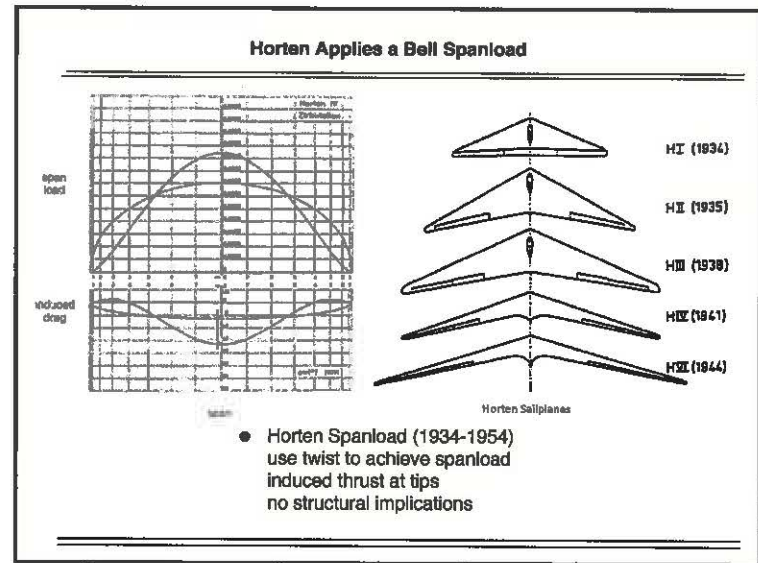
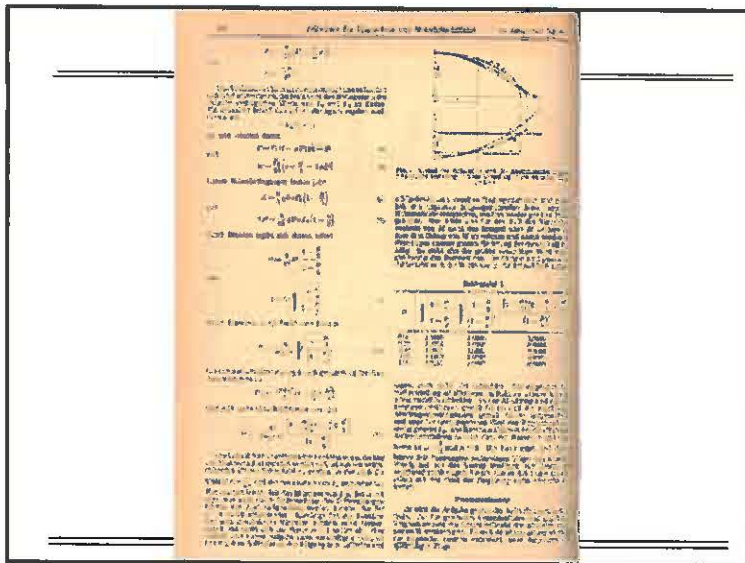
$$w = \frac{C_L}{\pi AR}$$

The induced drag is also related to the induced velocity  $w$  by the equation:

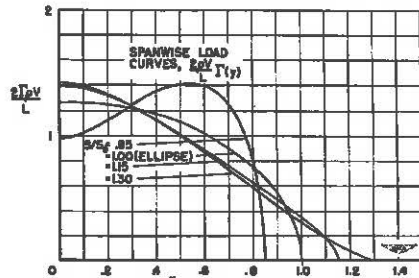
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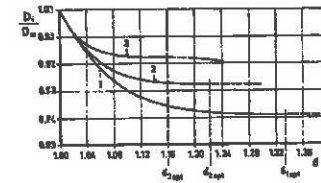


## Jones Spanload



- Minimize induced drag (1950)  
Constrain wing root bending moment  
30% increase in span with 17% decrease in induced drag
- "Hence, for a minimum induced drag with a given total lift and a given bending moment the downwash must show a linear variation along the span."  $y = bx + c$

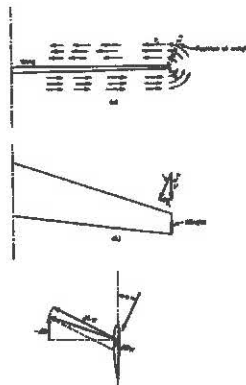
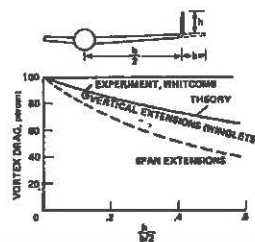
## Klein and Viswanathan



- Minimize induced drag (1975)  
Constrain bending moment  
Constrain shear stress  
16% increase in span with 7% decrease in induced drag
- "Hence the required downwash-distribution is parabolic."  $y = ax^2 + bx + c$

## Winglets

- Richard Whitcomb's Winglets
  - induced thrust on wingtips
  - induced drag decrease is about half of the span "extension"
  - reduced wing root bending stress



## Whitcomb's Winglets



NASA Dryden Flight Research Center Photo Collection  
MD-125A in flight - winglet essay  
NASA Photo SCF9-1324 Date 1978



NASA Dryden Flight Research Center Photo Collection  
MD-125A in flight - winglet attached view  
NASA Photo SCF9-1491 Date 1978



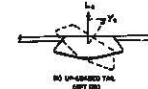
## Bird Flight Model

- Minimum Structure
- Flight Mechanics Implications
- Empirical evidence

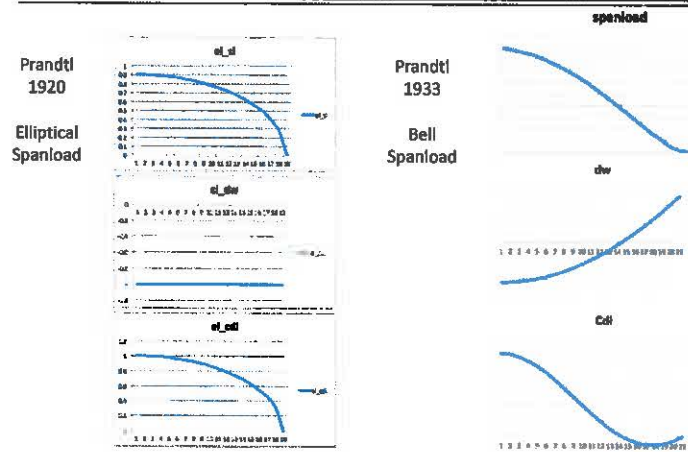
## How do birds turn & maneuver?



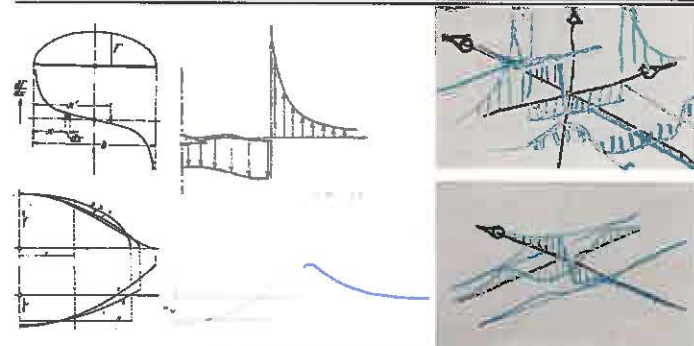
- "First the tail is tilted downward on the side away from the direction of the turn...Perhaps the tail functions as a rudder in starting the turn..." (Koford, 1950)
- Alleviating the load on the tips allows the bird to turn (bank and yaw) correctly (Hoey, 1992)
- "...the tail was loaded upward and the same clockwise tail rotation produced a right force, thus a left turn..." (Hoey, 1992)



## Prandtl(1920) vs Prandtl(1933)



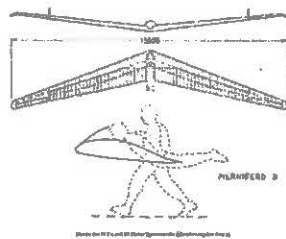
## Spanload, Downwash, Induced Drag



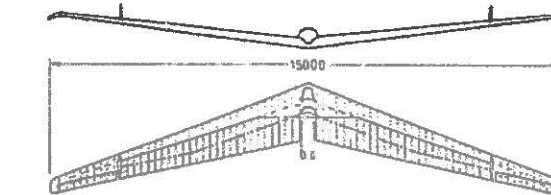
- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

## Horten H Xc Example

- Horten H Xc footlaunched ultralight sailplane 1950
- 24 degree leading edge sweep angle
- Chord:  
root - 63 inches  
tip - 15.75 inches
- Span: 49.2 feet



## Prandtl Wing

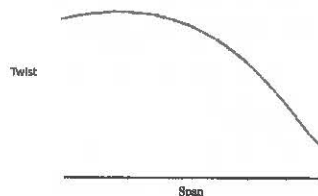


- 24 degree leading edge sweep angle
- Chord:  
root - 15.75 inches  
tip - 3.875 inches
- Span: 147.6 inches

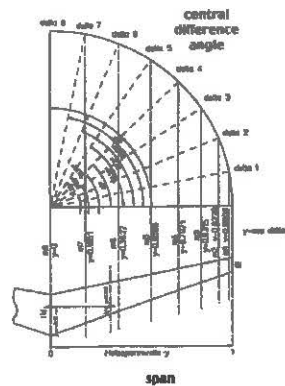
← 3750 →

## Calculation Method

- Taper
- Twist
- Control Surface Deflections
- Central Difference Angle

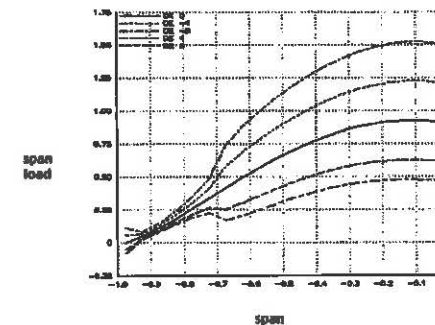


R0 8.9274  
R1 8.5524  
R2 8.7259  
R3 8.8441  
R4 8.9030  
R5 8.8984  
R6 8.8257  
R7 8.6801  
R8 8.4565  
R9 8.1492  
R10 7.7522  
R11 7.2592  
R12 6.6634  
R13 5.9579  
R14 5.1982  
R15 4.1927  
R16 3.1255  
R17 1.9394  
R18 0.6589  
R19 -0.6417  
R20 -1.6726



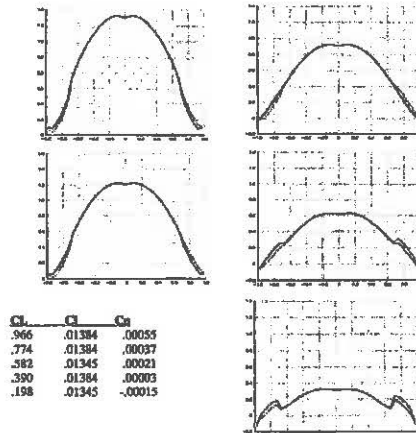
## Symmetrical Spanloads

- Elevon Trim
- CG Location



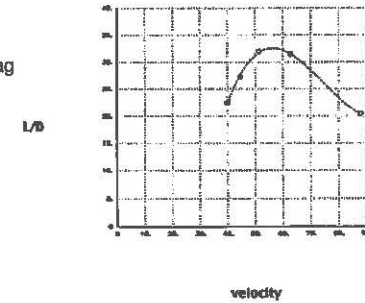
## Asymmetrical Spanloads

- $Cl_{\delta a}$  (roll due to aileron)
- $Cn_{\delta a}$  (yaw due to aileron) induced component profile component change with lift
- $Cn_{\delta a}/Cl_{\delta a}$
- $CL$  (Lift Coefficient)  
Increased lift:  
increased  $C_l$   
increased  $C_n$   
Decreased lift:  
decreased  $C_l$   
decreased  $C_n$



## Performance Comparison

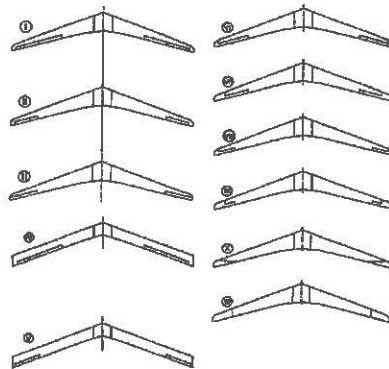
- Max L/D: 31.9
- Min sink: 89.1 fpm
- Does not include pilot drag
- Predicted L/D: 30
- Predicted sink: 90 fpm



## Dr Edward Uden's Results

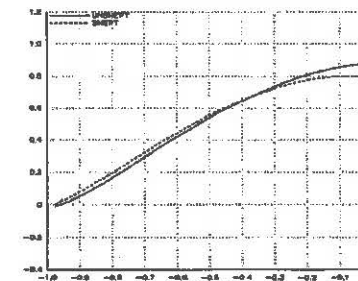
- Spanload and Induced Drag
- Elevon Configurations
- Induced Yawing Moments

Elevon Config	$Cn_{\delta a}$	Spanload
I	-.002070	bell
II	.001556	bell
III	.002788	bell
IV	-.019060	elliptical
V	-.015730	elliptical
VI	.001942	bell
VII	.002823	bell
VIII	.004529	bell
IX	.005408	bell
X	.004132	bell
XI	.005455	bell



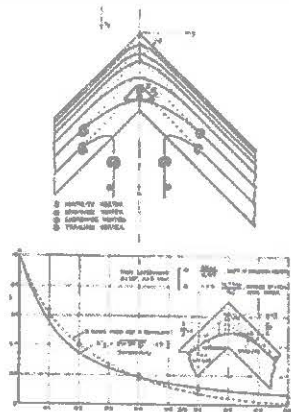
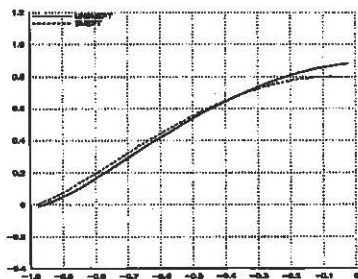
## "Mittleeffekt"

- Artifact of spanload approximations
- Effect on spanloads  
increased load at tips  
decreased load near centerline
- Upwash due to sweep unaccounted for



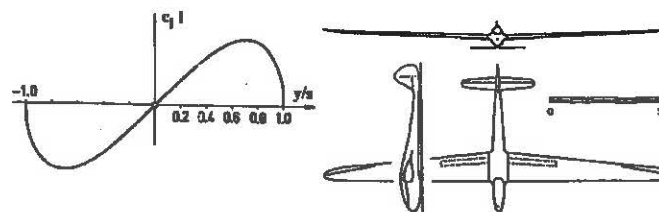
## Kucheman

- Effect on spanloads  
increased load at tips  
decreased load near centerline
- Upwash due to sweep unaccounted for
- Residual (Kucheman)

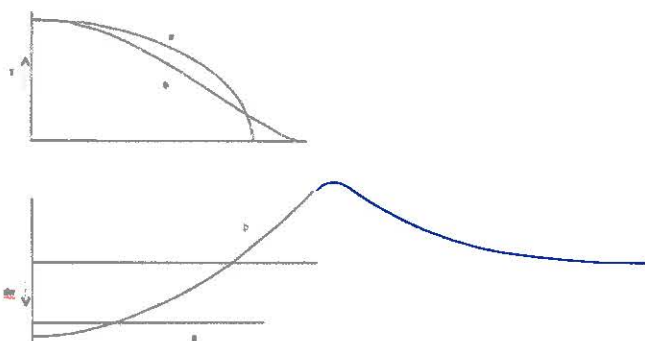


## Elliptical Half-Lemniscate

- Minimum induced drag for given control power (roll)
- Dr Richard Eppler: FS-24 Phoenix



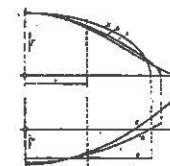
## Prandtl's Bell Spanload



## Prandtl's Spanload

$$sl = (1 - x^2)^{3/2}$$

$$w = 3/2 (x^2 - 1/2)$$



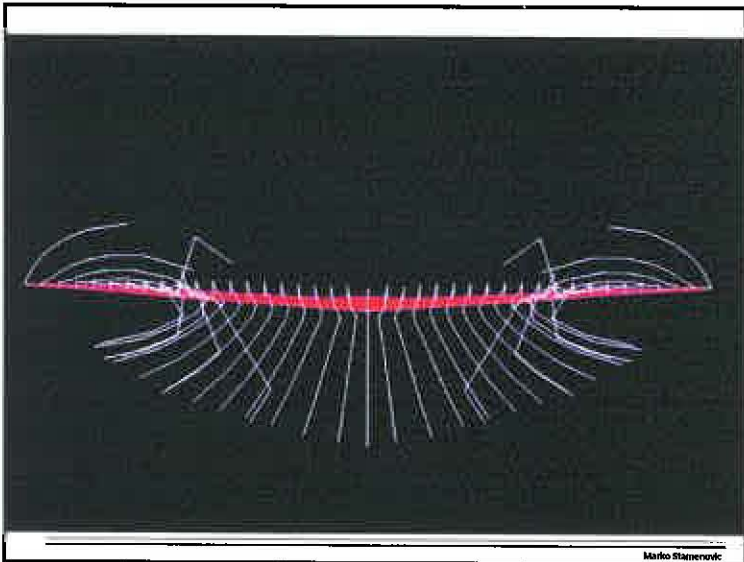
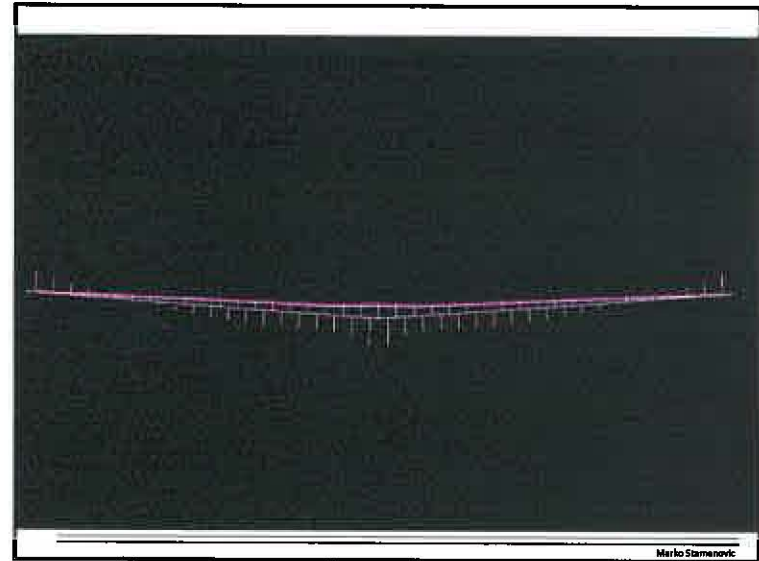
$$\lim_{x: 0 \rightarrow b/2} L(x) = 0 \quad (1)$$

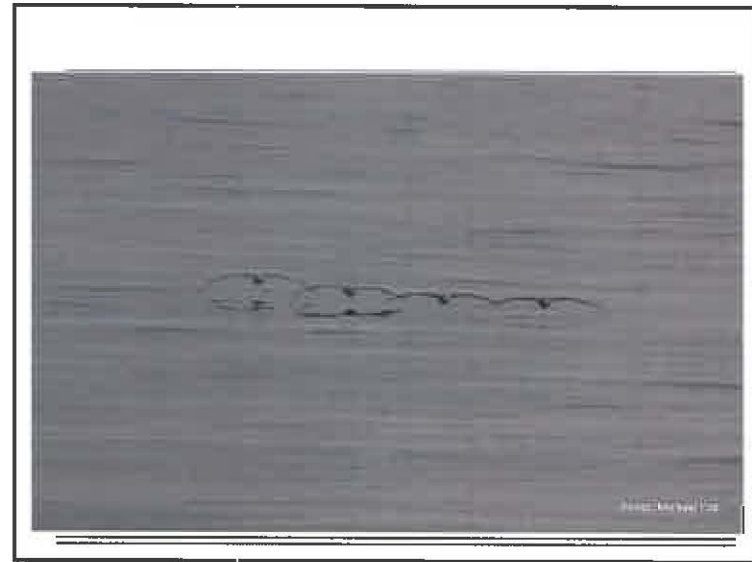
$$\lim_{x: 0 \rightarrow b/2} \frac{dL(x)}{dx} = 0 \quad (2)$$

$$\lim_{x: 0 \rightarrow b/2} \frac{dDW(x)}{dx} = \lim_{x: \infty \rightarrow b/2} \frac{dDW(x)}{dx} \quad (3)$$



**Mike Allen**



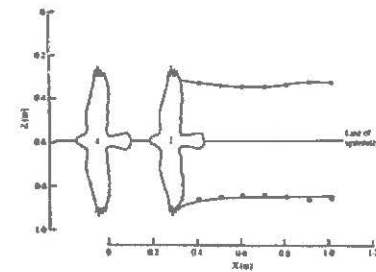


### Spanload

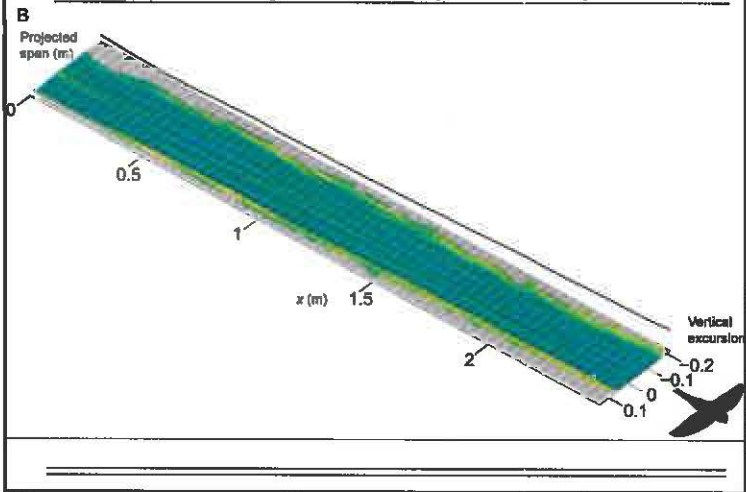


### Spedding's Gliding Falcon

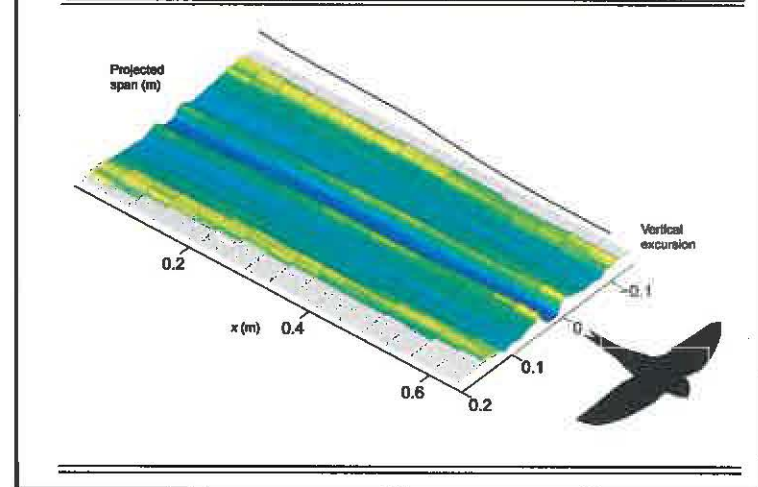
- Spedding photograph's a gliding falcon's wake with He bubbles
- Vortex cores are 0.76 b apart
- Elliptical spanload is assumed, so the vortex cores are assumed to come from the wingtips



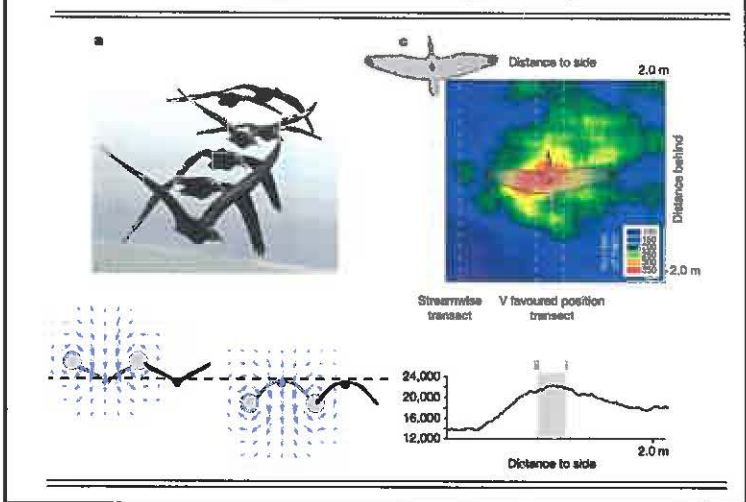
### Henningsson 2014 (PLOS One)



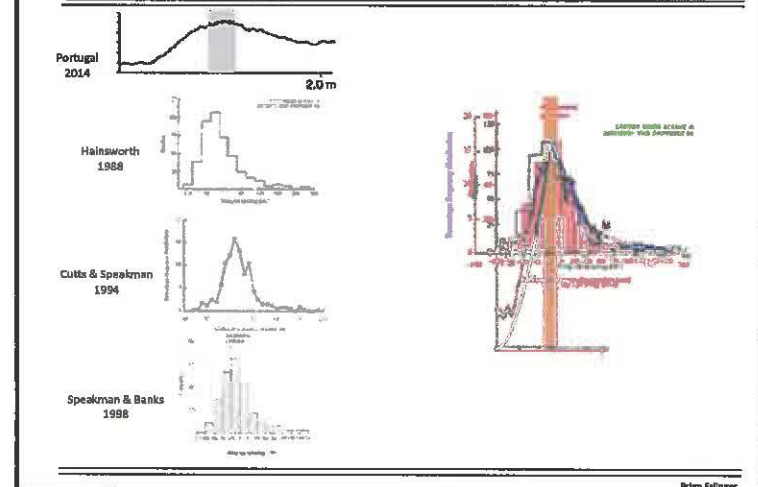
### Henningsson 2014 (PLOS One)

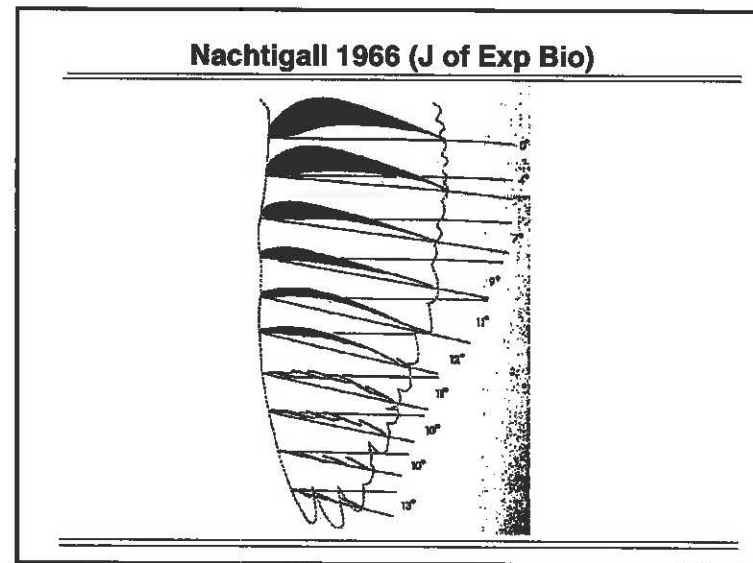
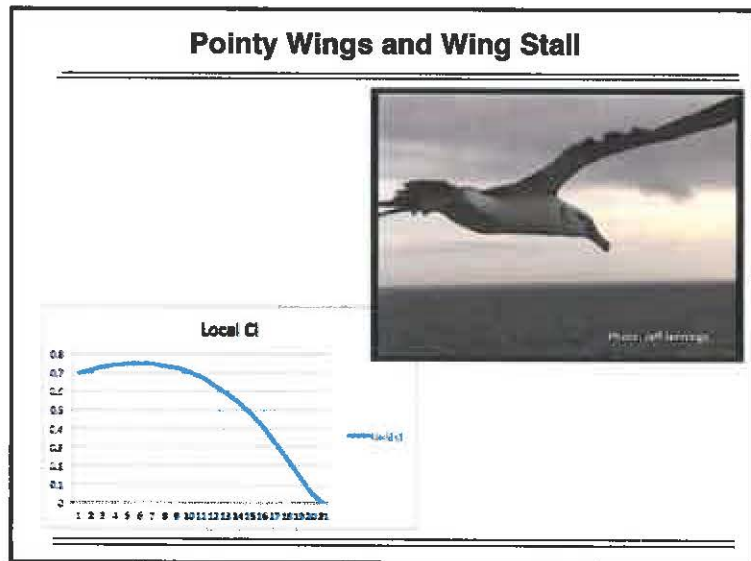
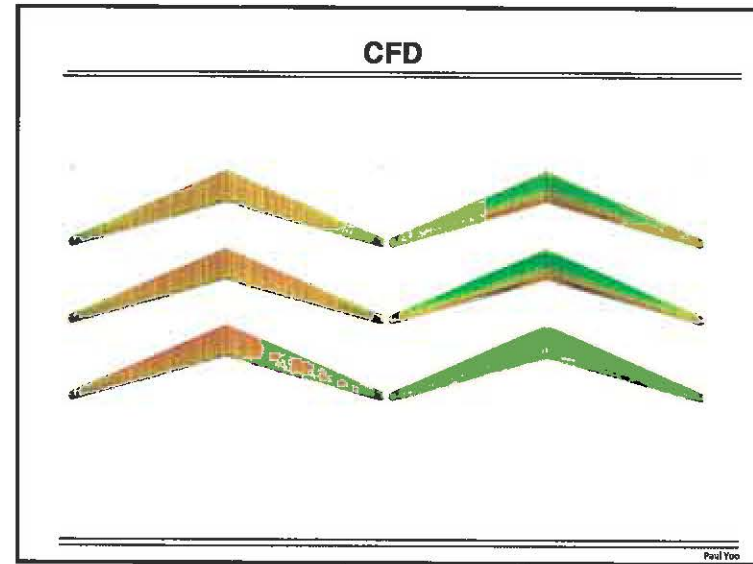
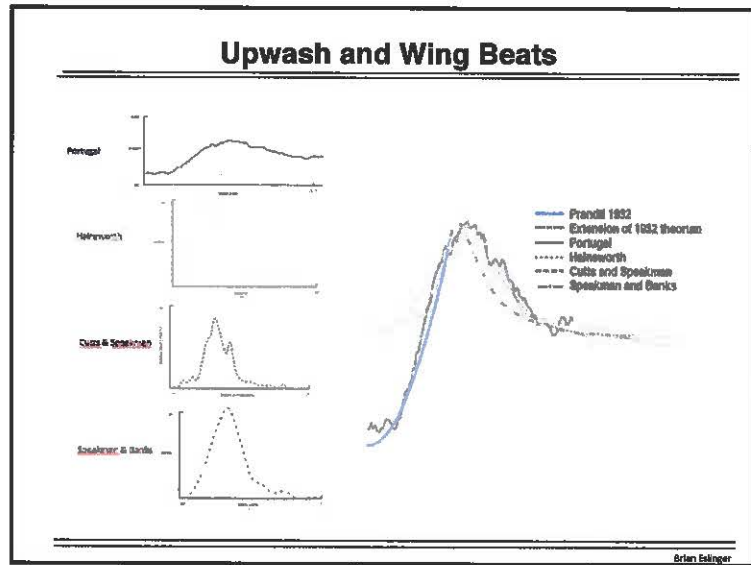


### Portugal, et al 2014 (Nature)



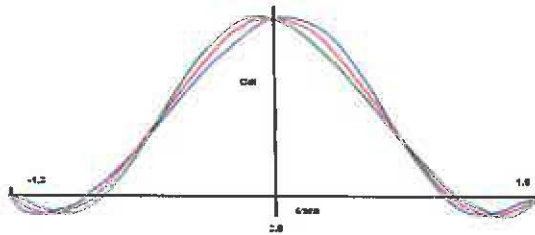
### Upwash and Wing Beats







## Effect of Sideslip



- Wing twist
- Sideslip is imposed
- Distorts the bell spanload and the induced drag/thrust profile



## PRANDTL-D Proverse Yaw?



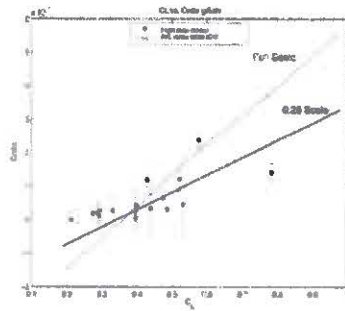
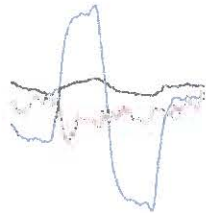
What would Proverse Yaw look like?

## Flight Data

- Measurement of proverse yaw would be the final hurdle to achieve
- Icing on the cake: measure  $C_{nda}$  (yawing moment due to aileron deflection)
- **NOT ONE SECOND OF FLIGHT DATA EXISTS TO PROVE ANY OF THIS IS TRUE**

## Proverse Yaw

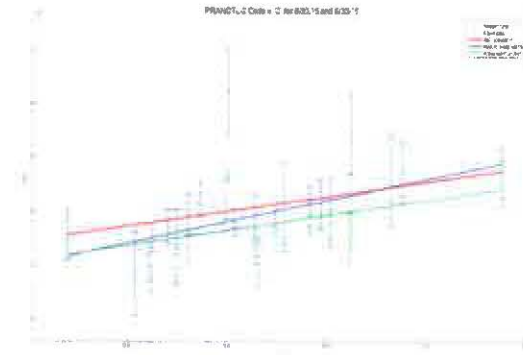
- ...until June 26<sup>th</sup>, 2013
- Roll and Yaw are the same sign
- From Uden:  $C_{nda}$  is +ve
- uncertainty



Inertias; configuration changes, turbulence, and slope of  $C_{nda}$

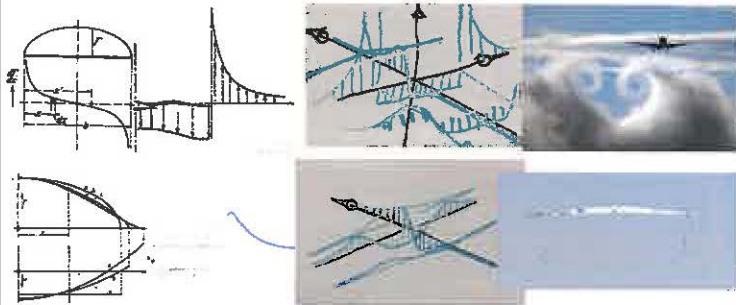
Kristyn Kadala

## Revalidated Proverse Yaw



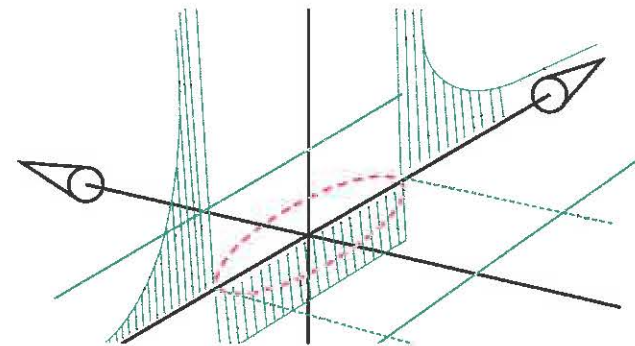
Loren Newton & Meesam Albet

## Spanload, Downwash, Induced Drag

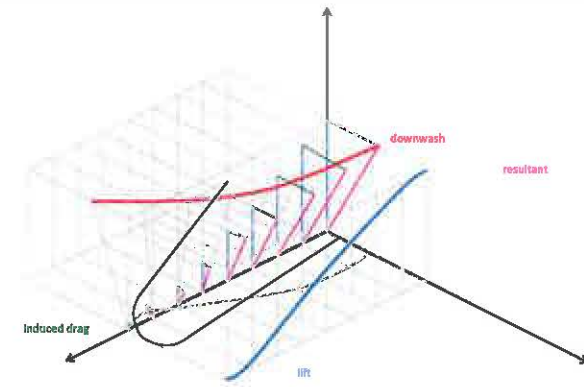
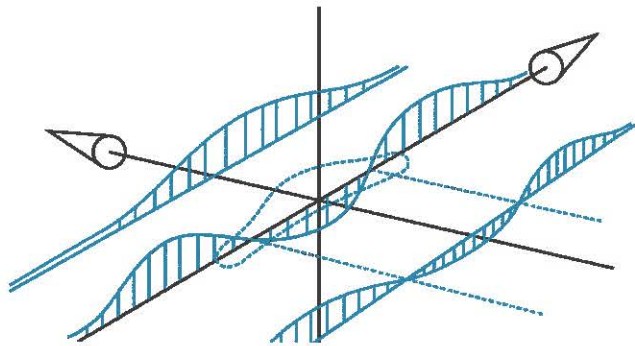


- All wings dictate 3 solutions
- Spanload
- Downwash
- Induced Drag

## Elliptical Spanload Flow

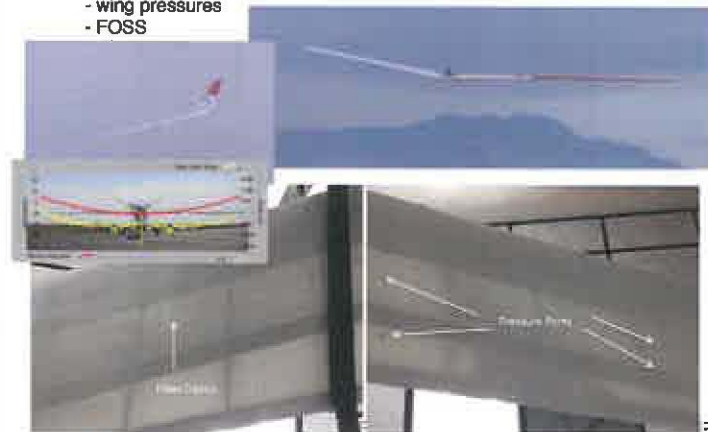


### Bell Spanload Flow



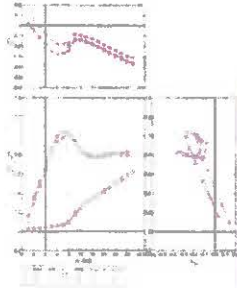
### Prandtl 3

- The aerodynamic testbed
  - wing pressures
  - FOSS

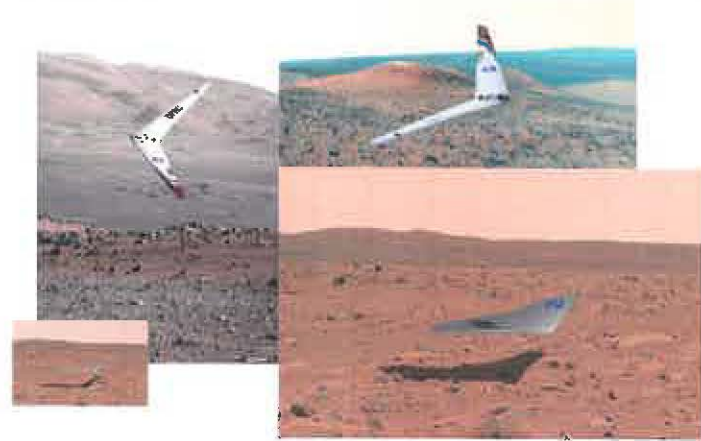


## Wind Tunnel Test

- NASA Langley
- 12-ft low speed tunnel
- 52 runs, 6 component force-moment



## The Future: Prandtl-M on Mars?



## The Future: Prandtl-M on Mars?



Das kleiner  
Prandtl Falke

## The Mars Mission



- Deploy cubesat from large Mars rover
- Use Exobrake parachute for Mars atmosphere entry
- Deploy Prandtl-m (2 lbm, 2 ft span) from cubesat at 15,000 ft agl
- Glide 5 mins, 22 miles, Mach 0.6
- Crash land on Mars
- Transmit images & data back



## The Mars Mission

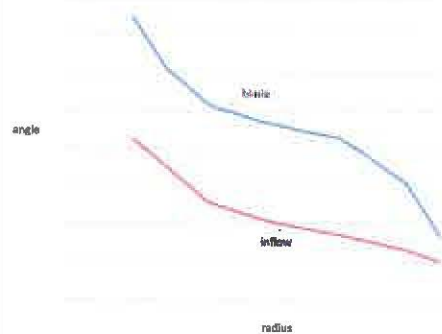


- Deploy cubesat from large Mars rover
- Use Exobrake parachute for Mars atmosphere entry
- Deploy Prandtl-m (2 lbm, 2 ft span) from cubesat at 15,000 ft agl
- Glide 5 mins, 22 miles, Mach 0.6
- Crash land on Mars
- Transmit images & data back



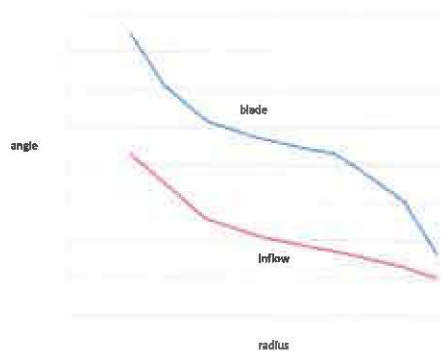
## Prandtl Propulsion

- Propulsion systems currently use "minimum induced loss"
- What if we switched to minimum torque for a given thrust? +15.4%

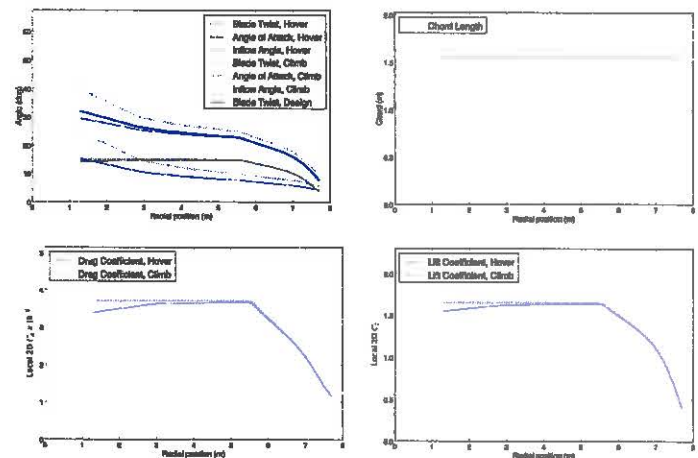


## Prandtl Propulsion

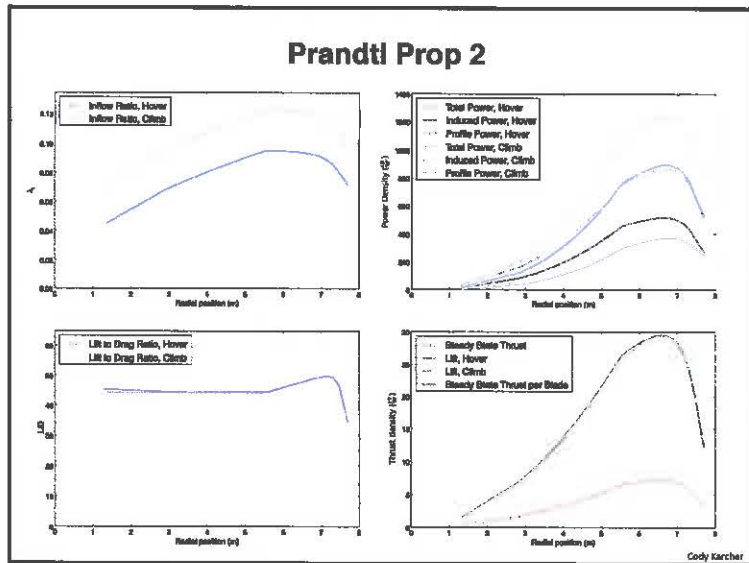
- Propulsion systems currently use "minimum induced loss"
- What if we switched to minimum torque for a given thrust? +15.4%



## Prandtl Prop 1



Coody Karcher



## 2016 Interns & MARTI



## PRANDTL-D

- Videos

- TEDxNASA 2011  
<http://www.youtube.com/watch?v=223OmaQ9uLY>

- NASA Aero Academy 2013  
<http://www.youtube.com/watch?v=Hr0I6wBFGpY>

Red Jensen: pilot,  
engineer



Red Jensen, Justin Hall, & Derek Abramson



## PRANDTL-D Aircraft





## Control of Yaw

- You Have Three Choices:



- 1/ drag a vertical tail around with you all the time to create a yawing moment
- 2/ manipulate drag at the wing tips to control yaw

} Current Design Options

-OR-

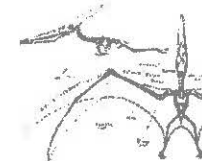
- 3/ manipulate **THRUST** at the wing tips to control yaw



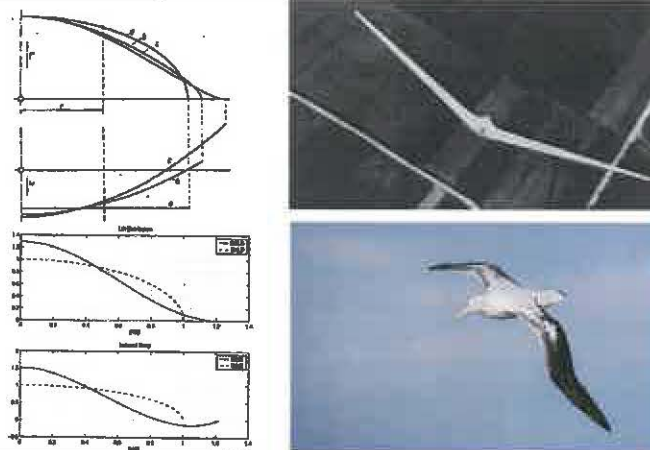
- Biological vs Mechanical Flight

## Biological Flight

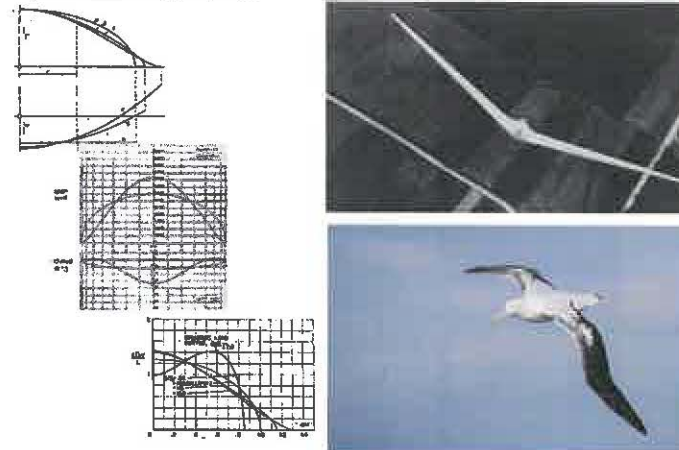
- Mechanical Flight (110 yrs)
- Vertabrate Flight (128 My)



## Prandtl, Horten, Jones, and Birds



## Prandtl, Horten, Jones, and Birds



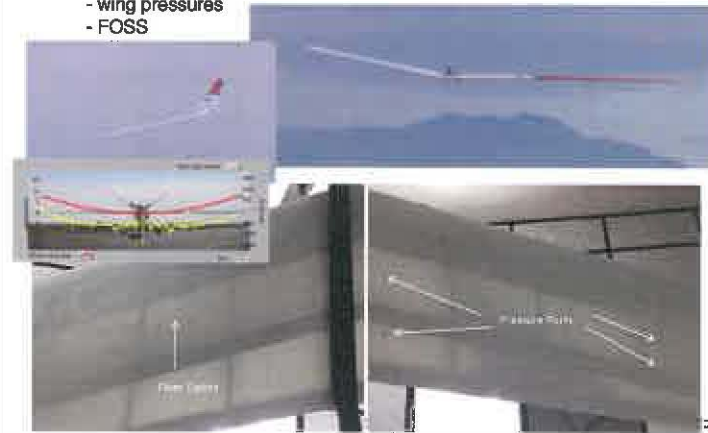


## Efficiency

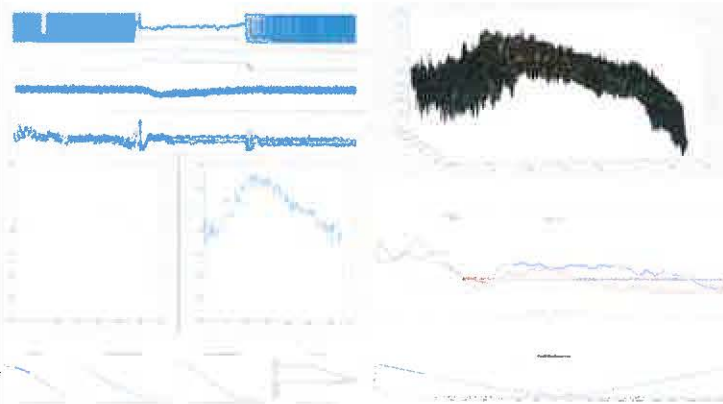
- Efficiency: 12.5% increase in wing efficiency
- 20-30% increase in efficiency by eliminating the tail
- 15.4% increase in propulsive efficiency
- **TOTAL EFFICIENCY INCREASE: 69%**
  
- CY2011: world jet fuel consumption \$134B
- **\$55B in jet fuel saved**
  
- CY2011 World GDP: \$69.7T
- World power production: \$12.0T
- **\$1.85T savings in world power production**

## Prandtl 3 & 3c

- The aerodynamic testbed
  - wing pressures
  - FOSS



## Prandtl 3c cFOSS



Deborah Jackson



Noah Edwards, Abby Weddell, Deborah Jackson & Tari Hawkins

## Concluding Remarks

- Birds as as the first model for flight
- Applied approach gave immediate solutions, departure from bird flight
- Eventual meeting of theory and applications (applied theory)
- Spanload evolution (Prandtl/Horten/Jones/Klein/Viswanathan/Whitcomb/Bowers)
- Solve performance, structure and control with ONE spanload solution!
- 12.5% increase in L/D, ~2% increase in prop efficiency, 20-30% decrease in drag eliminating the tail, ~43-62% reduction in total aero efficiency
- Assumptions and Solutions
- The Wrights disintegrated the flight of birds, and Prandtl/Horten/Jones reintegrated the flight of birds...
- Thanks: Red Jensen, Brian Eslinger, Dr Christian Gelzer, Dr Oscar Murillo, Hayley Foster & Steve Craft, Dr Bob Liebeck, Nalin Ratnayake, Mike Allen, Walter Horten, Georgy Daz-Falvy, Rudi Oplitz, Bruce Carmichael, R.T. Jones, Russ Lee, Bob Hoey, Phil Barnes, Dan & Jan Armstrong, Dr Phil Burgers, Ed Lockhart, Andy Kesckes, Dr Paul MacCready, Reinhold Stadler, Dr Edward Uden, & Dr Karl Nickel

## NASA Aero Academies & Others

- 2014 NASA Aero Academy  
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- 2013 NASA Aero Academy  
- Eric Gutierrez, Louis Edelman, Kristyn Kadala, Nancy Pinon, Cody Karcher, Andy Putch, Hovig Yaralian, Jacob Hall
- 2012 NASA Aero Academy  
- Steffi Valkov, Juliana Plumb (Ulrich), Luis Andrade, Stephanie Reynolds, Joey Wagster, Kimmy Callan, Javier Rocha, Sanel Horozovic, Ronalynn Ramos, Nancy Pinon
- Mike Allen, Alex Stuber, Matt Mohoff, Dave Voracek, Jaiwon Shin, Ross Hathaway, Brian Eslinger, Oscar Murillo, Lesli Monforton, Red Jensen, Aamod Samuel, Brad Neal, Brad Flick, Chris Acuff, Rick Howard (NPS), Marko Stamenovic, Jim Murray, Nalin Ratnayake, Eric Nisbet, Jeromy Robbins, Nelson Brown, Curtis Stump, Andrew Burrell, Anthony MacPherson, Brian Taylor, Chris Miller, Victor Loera, Cameron Law, Koen vander Kerckhove, Bob Hoey, Russ Lee, Reinhold Stadler, Edward Uden, Paul MacCready, Karl Nickel, Walter Horten, Diego Roldan Knollinger, Michael Cox, Jeff Jennings, Phil Barnes

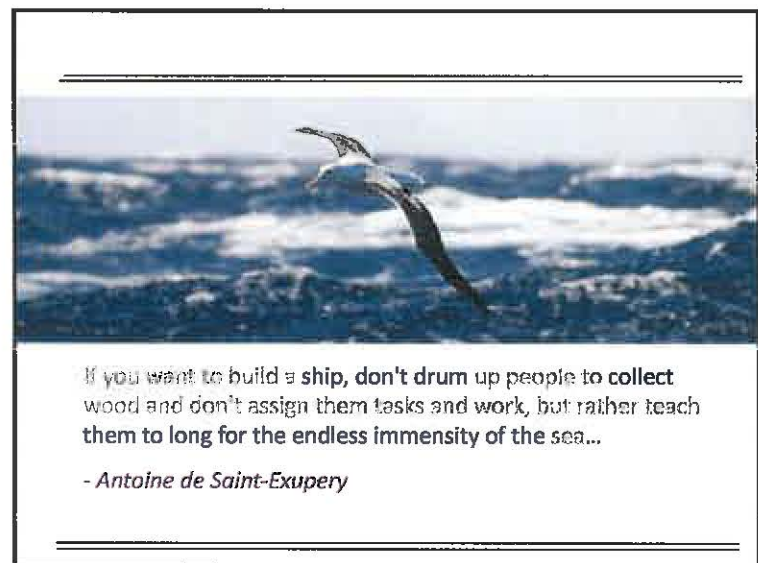
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## Full-Scale Wing


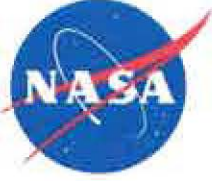


Erich Chase & Steve Dorey






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Questions?



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The End

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Questions?



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