Computational Fluid Dynamics Analysis of the Stall Characteristics of a Wing Designed Based on Prandtl's Minimum Induced Drag

Seung Y. Yoo

Outline

- Introduction/Background
- Method
- Results
- Conclusion
- Questions

Introduction

- Prandtl's work on minimum induced drag
  - 1929 publication – elliptical spanwise lift distribution, constrain wing span
  - 1933 publication – bell shaped spanwise lift distribution, constrain bending moment
    - 11% less drag, 22% longer span compared to elliptical distribution for wings of identical weight

- Summary of Prandtl's result

  Lift (L): $L = (1 - x^2)^{1.5}$

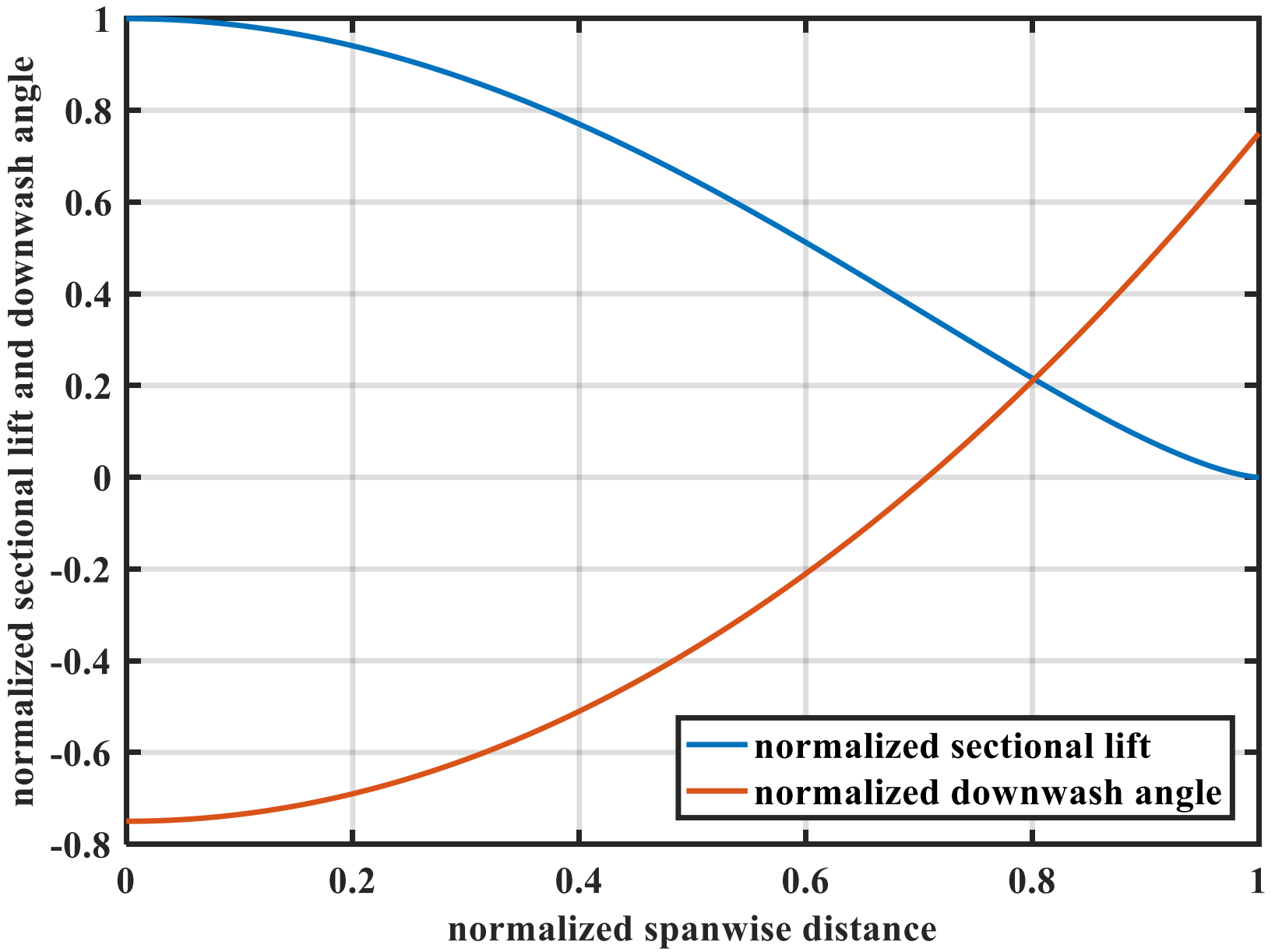
  Downwash angle (DW): $\mathrm{DW} = 1.5 * (x^2 - 0.5)$

  Lift tapers to zero at wing tip: $\lim_{x:0 \to b/2} L(x) = 0, \lim_{x:0 \to b/2} \frac{dL(x)}{dx} = 0$

  Continuous down wash angle at wing tip: $\lim_{x:0 \to b/2} \frac{dDW(x)}{dx} = \lim_{x:\infty \to b/2} \frac{dDW(x)}{dx} = 0$

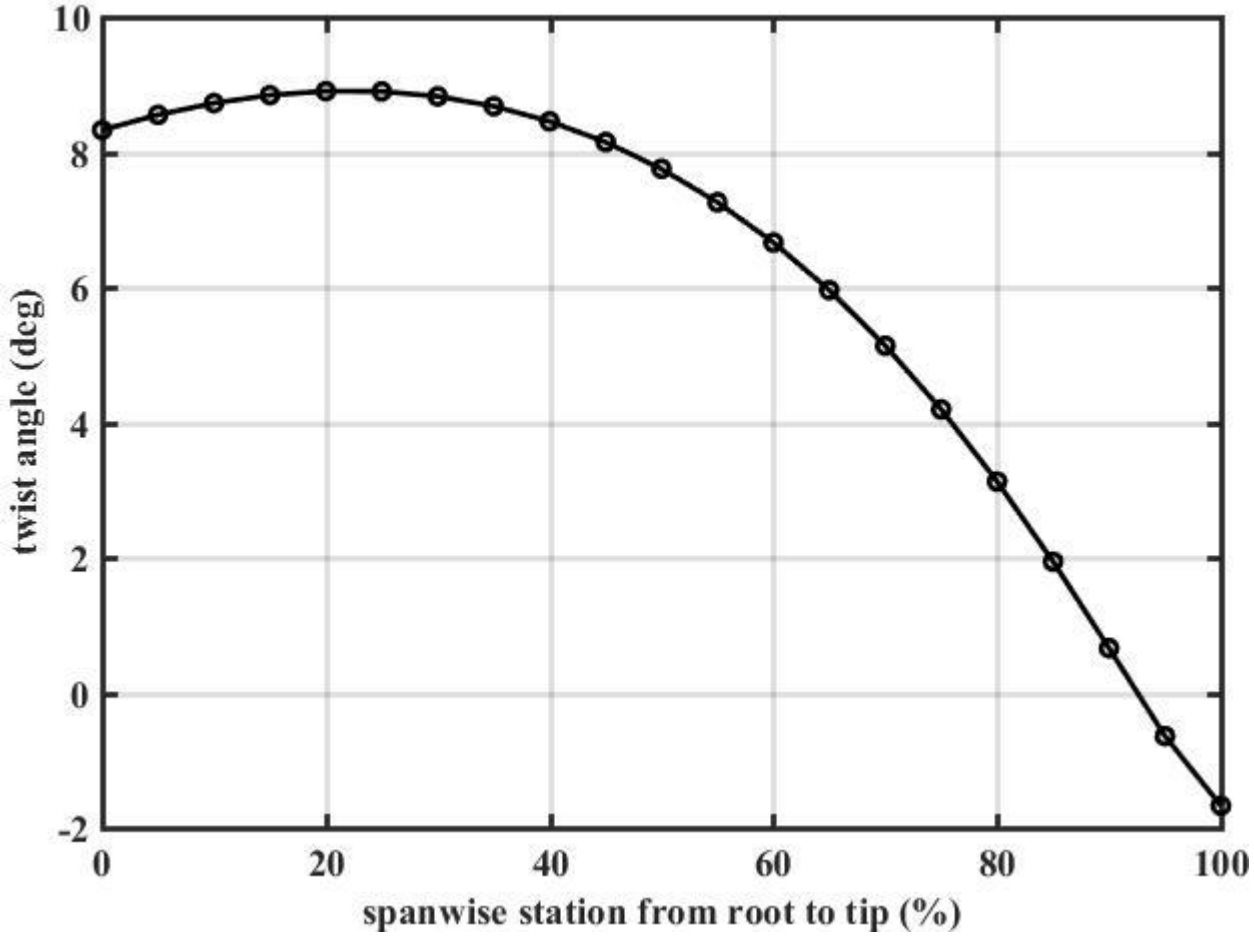
Introduction

- Lift zero at wing tip
- Slope of lift zero at wing tip
- Downwash becomes upwash at 70.7% span
  - Inboard vortex, no wing tip vortex
  - Proverse yaw due to induced thrust at wing tip caused by upwash



Introduction

- Achieve bell shape loading via nonlinear spanwise twist distribution
- Wing tip is at approximately -10° twist relative to root chord



Introduction

- P-3C from the Preliminary Aerodynamic Design To Lower Drag (PRANDTL-D) program
- Span of 24.6 ft
- MAC of 1.969 ft
- Planform area of 40.5 ft^2
- ~30 mph

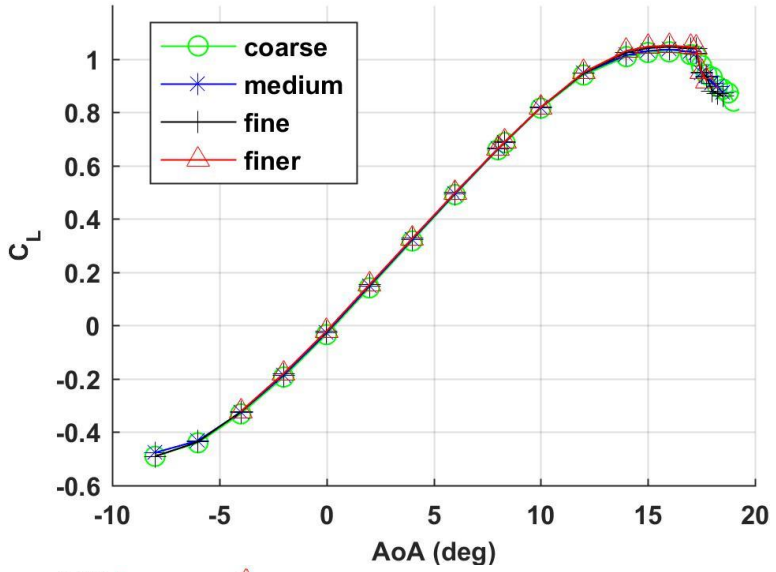
Method

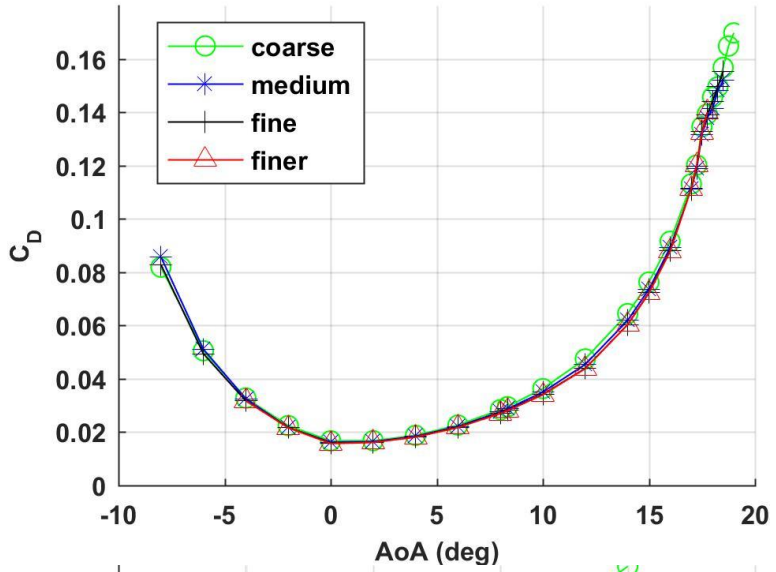
- OVERFLOW version 2.2l
  - 2nd order central differencing scheme
  - Beam-Warming block tridiagonal scheme
  - Low Mach preconditioner
  - Steady state
  - Spalart-Allmaras turbulence model with rotation/curvature correction
- Best practices
  - High lift workshop grid guideline
  - Best practices for overset meshing
- Warm start procedure
  - Sequential restart at stall w/ smaller $\Delta\alpha$, achieve angle of attack resolution of 0.25°

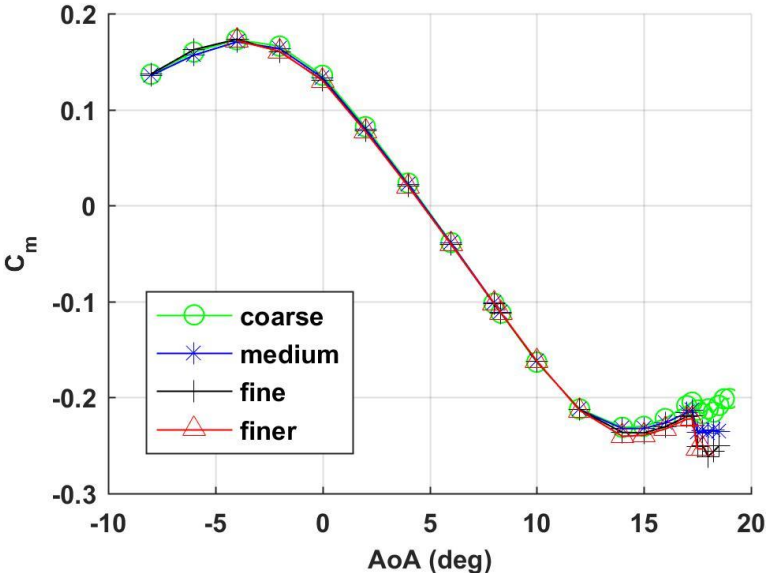
Result – Grid Study

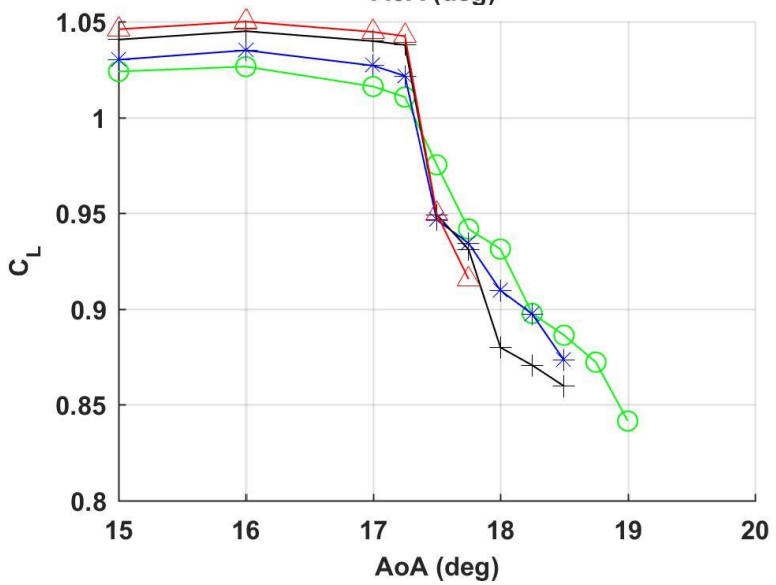
	Parameter	Coarse	Medium	Fine	Finer
Surface	Stretching ratio	1.3	1.2	1.1	1.05
	Maximum spacing, in	20	10	2.5	2.5
	Minimum spacing, in	0.0157	0.00787	0.00197	0.00197
Volume	Stretching ratio	1.3	1.2	1.1	1.05
	Marching distance, in	10.0			
	Initial spacing off of the wall, in	6.50E-04	1.90E-04	6.45E-05	3.23E-05
	Final spacing off in the near field grid, in	1.0	0.5	0.33	0.25
	y+	1.0	0.3	0.1	0.05
	Level-1 spacing, in	0.8	0.4	0.264	0.2
	MINBUF	4	4	6	8
Total number of grid points (millions)		4.48	21.9	79.6	190.3

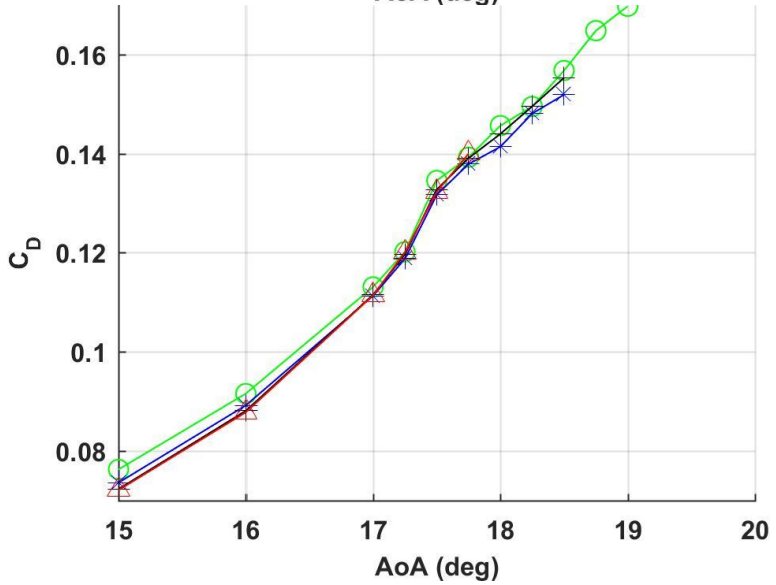
Result – Grid Study

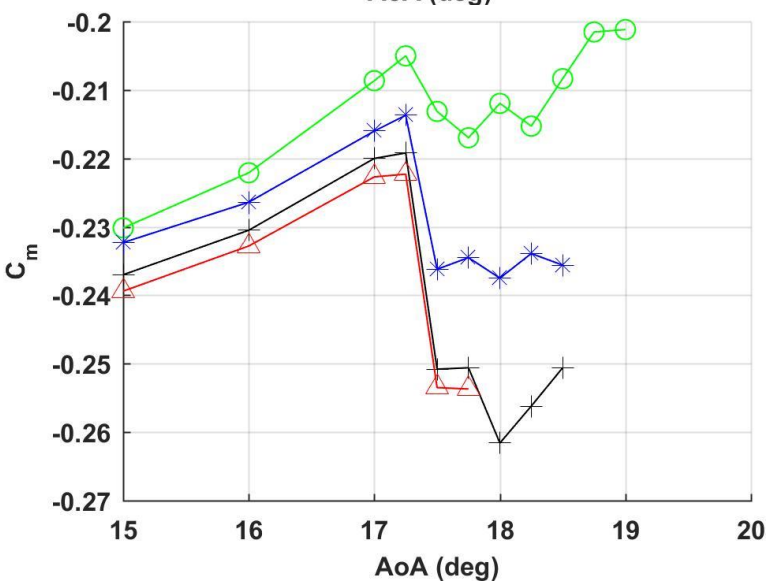




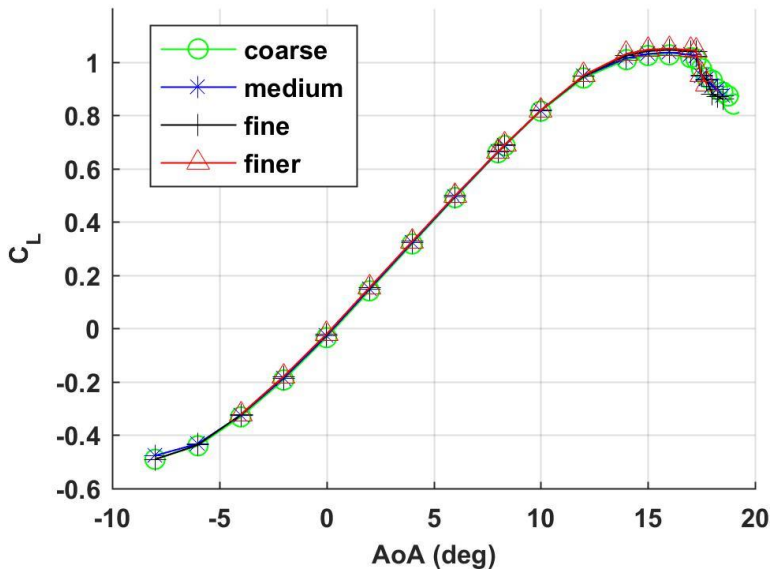


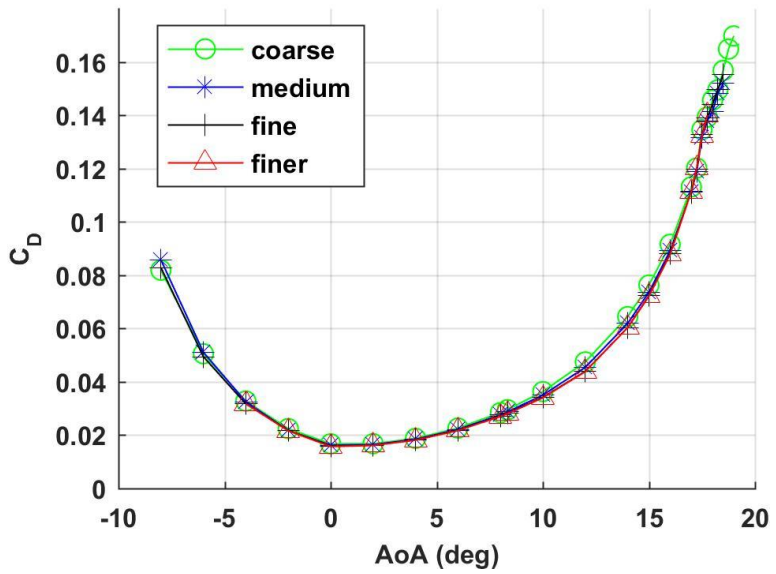


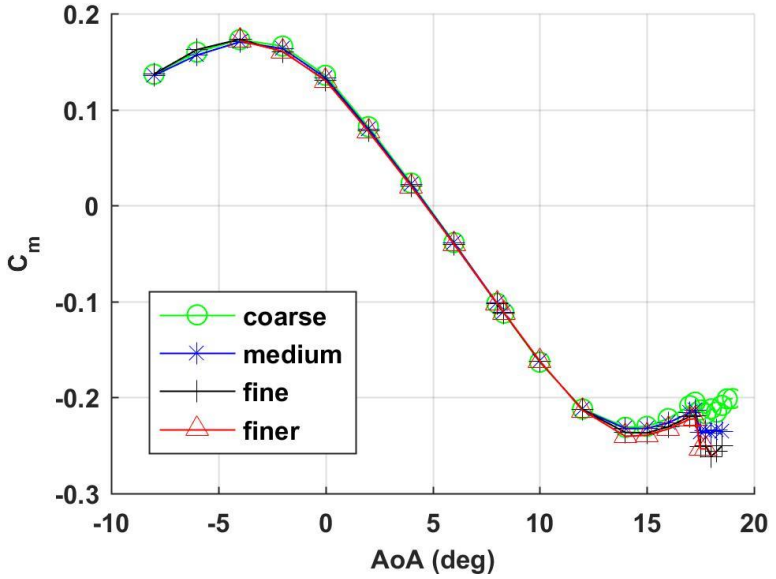




Result – Grid Study

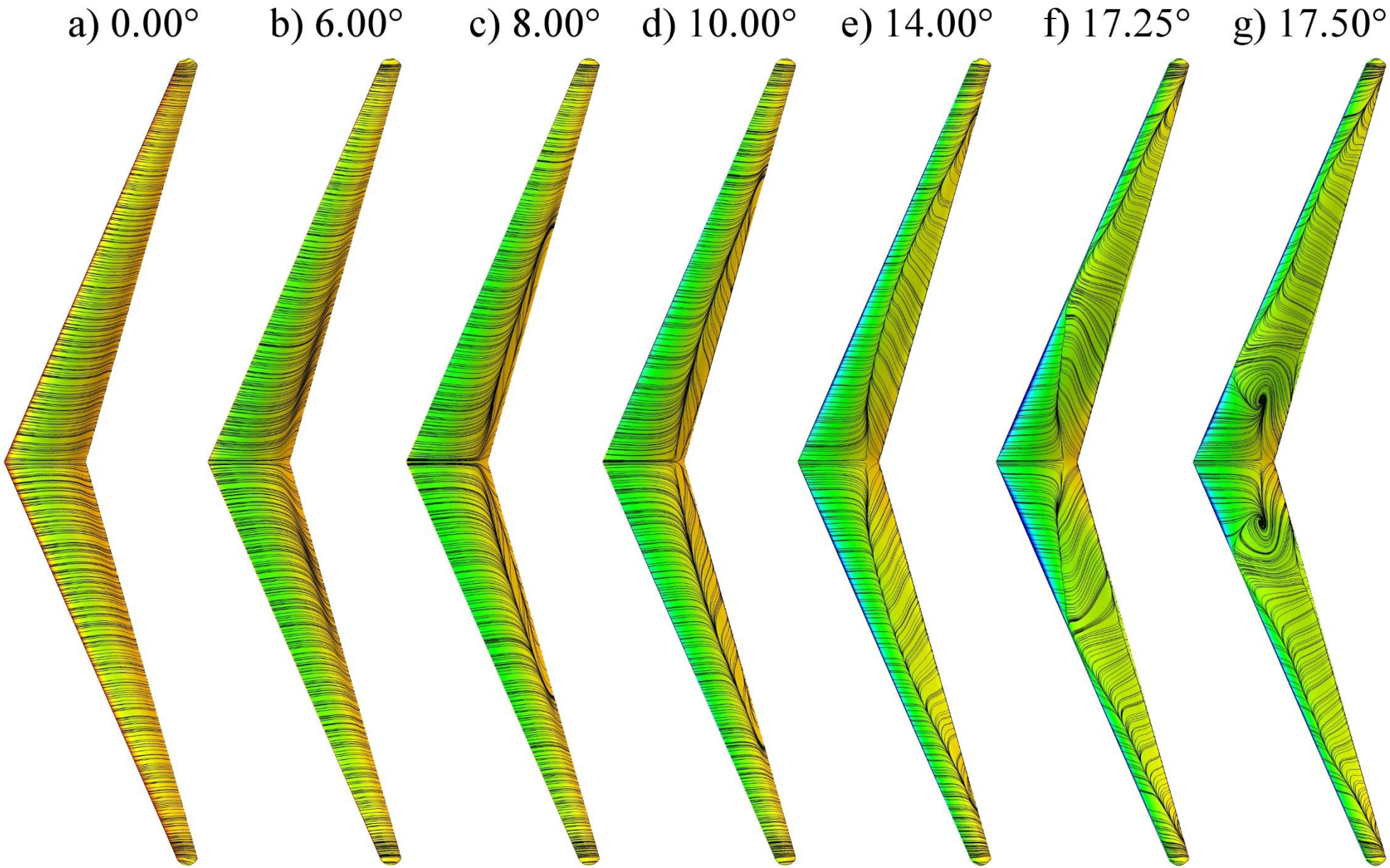






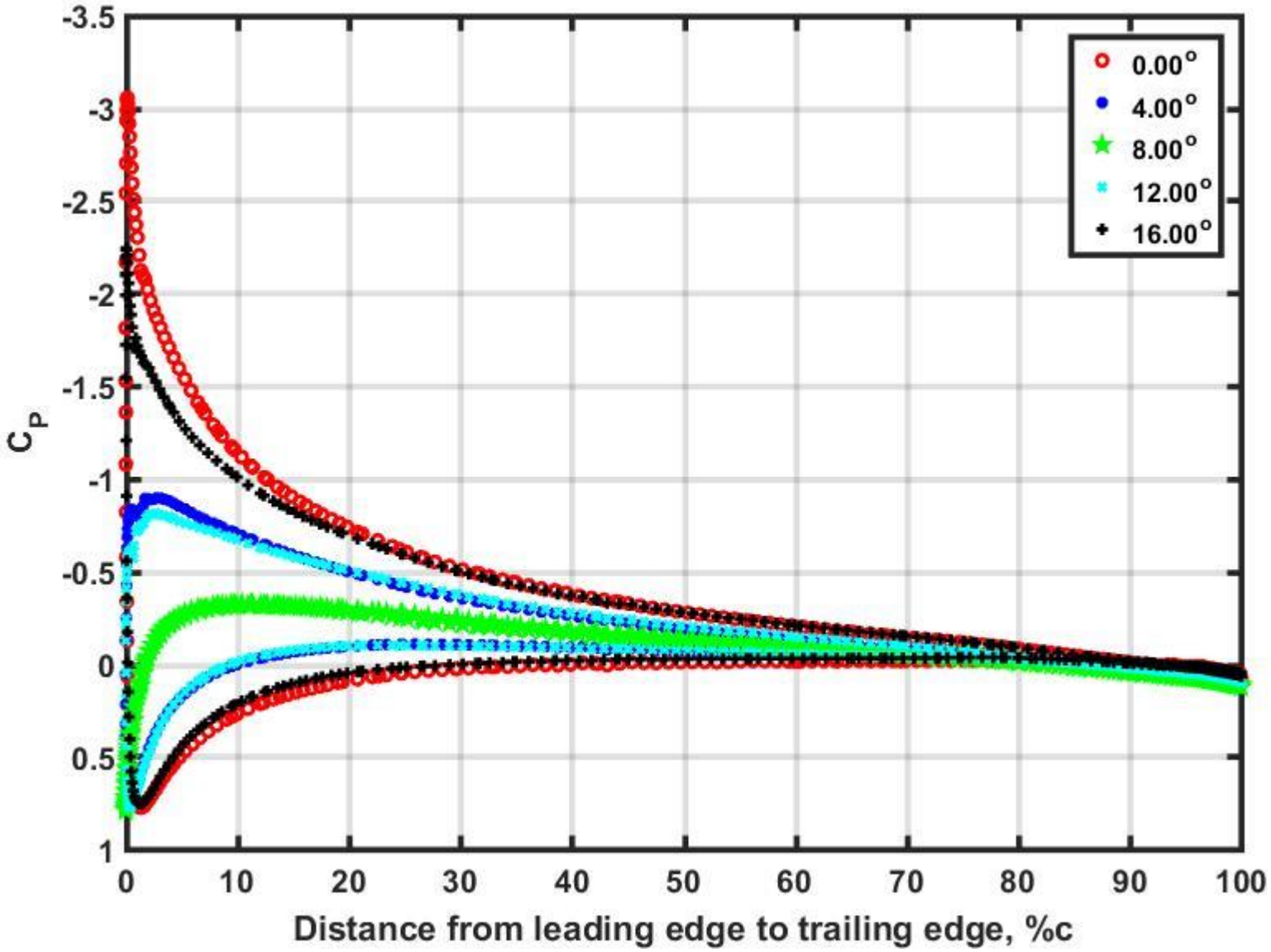
Grid	Stall Angle (deg)	C_{L_stall}	C_{L_stall} Error (%)	C_{L_max}	C_{L_max} Error (%)	C_{D_stall}	C_{D_stall} Error (%)	C_{m_stall}	C_{m_stall} Error (%)
coarse	17.25	1.0106	-3.05	1.0265	2.24	0.12020	0.08	-0.2050	-7.78
medium	17.25	1.0216	-1.99	1.0350	1.43	0.11885	-1.05	-0.2137	-3.87
fine	17.25	1.0378	-0.44	1.0450	0.48	0.11968	-0.36	-0.2192	-1.39
finer	17.25	1.0424	--	1.0500	--	0.12011	--	-0.2223	--

Result – Pressure Contour, Upper Surface



- Flow separations starts at 6.0°
- Most separation at ~40% span
- Flow at tip stays attached for all AoA shown

Result – Surface Pressure at tip



- Tip produces no lift at 8.0° AoA
- Lift varies almost linearly between 0.0° and 16.0°

Conclusion

- Wing designed based on Prandtl's minimum induced drag configuration simulated at high angle of attack
- Adequate grid resolution achieved
- C_L break at 17.25°
- Large flow separation ~40% semi-span
- Flow at wing tip remains attached through the lift break

Acknowledgement

- Albion Bowers
  - NASA-TP-2016-219072 - On Wings of Minimum Induced Drag - Spanload Implications for Aircraft and Birds

QUESTION?